Physics 558 – Lecture 23

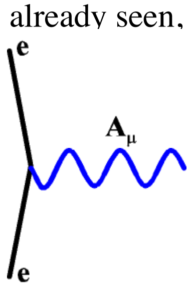
The Standard Model of the Electroweak Interactions Revisited – The Feynman Rules

To effectively summarize what we have learned so far about the electroweak interactions of the leptons let us discuss the Feynman rules for the interaction vertices in this theory that arise from the Lagrangian discussed in Lecture 22. Recall that we can obtain the interaction vertices by considering the terms in the quantity $i\mathcal{L}$ that are cubic, and higher, in the fields.

First consider the vertices describing the interaction of the matter fermions with the gauge bosons. From expanding the Lagrangian for the lepton with the correct definition of the $Z$ and $A$ fields we found in the last lecture that the neutral current sector looks like

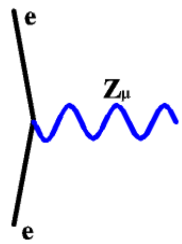
$$\begin{aligned} i\mathcal{L}_{\text{neutral current}} &= ie\,\bar{e}\gamma^{\mu}eA_{\mu} - i\sqrt{\frac{G_F M_Z^2}{2\sqrt{2}}}\,\bar{\nu}\gamma^{\mu}\left(1-\gamma_5\right)\nu Z_{\mu} \\ &- i\sqrt{\frac{G_F M_Z^2}{2\sqrt{2}}}\left[R_e\,\bar{e}\gamma^{\mu}\left(1+\gamma_5\right)e + L_e\,\bar{e}\gamma^{\mu}\left(1-\gamma_5\right)e\right]Z_{\mu}. \end{aligned} \tag{23.1}$$

Thus the electron couples to the photon via the vertex in the figure, which we have already seen, and which has the form



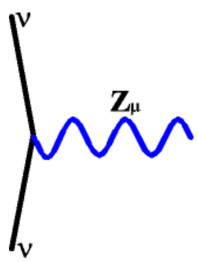

$$-iQ_e\,\bar{e}\gamma^{\mu}e\,A_{\mu} = ie\bar{e}\gamma^{\mu}e\,A_{\mu}. \tag{23.2}$$

We can similarly read off the corresponding coupling to the $Z$ (the neutral weak interaction) and find the following vertex,




$$
\begin{aligned}
&-i\sqrt{\left(\frac{G_F M_Z^2}{2\sqrt{2}}\right)}\,\bar{e}\gamma^\mu\Big[2\sin^2\theta_W\left(1+\gamma_5\right) \\
&\qquad +\left(2\sin^2\theta_W-1\right)\left(1-\gamma_5\right)\Big]e Z_\mu \\
&= -i\frac{g}{4\cos\theta_W}\bar{e}\gamma^\mu\Big[2\sin^2\theta_W\left(1+\gamma_5\right) \\
&\qquad +\left(2\sin^2\theta_W-1\right)\left(1-\gamma_5\right)\Big]e Z_\mu .
\end{aligned}
\qquad (23.3)
$$

The corresponding coupling of the $Z$ to neutrinos is the following,

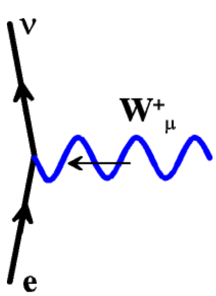



$$
\begin{aligned}
&-i\sqrt{\left(\frac{G_F M_Z^2}{2\sqrt{2}}\right)}\,\bar{\nu}\gamma^\mu\left(1-\gamma_5\right)\nu Z_\mu \\
&= -i\frac{g}{4\cos\theta_W}\bar{\nu}\gamma^\mu\left(1-\gamma_5\right)\nu Z_\mu .
\end{aligned}
\qquad (23.4)
$$

The charged current vertex, which we have discussed already last quarter, arises from the charged current part of the Lagrangian that we introduced in Lecture 22, Eq. (22.33)

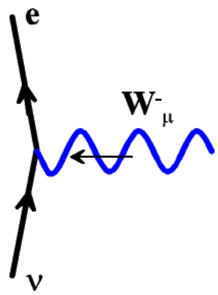
$$
i\mathcal{L}_{\text{charged current}} = -i\frac{g}{2\sqrt{2}}\left[\bar{\nu}\gamma^\mu\left(1-\gamma_5\right)eW_\mu^+ + \bar{e}\gamma^\mu\left(1-\gamma_5\right)\nu W_\mu^-\right]. \qquad (23.5)
$$

This yields the following vertices




$$
\begin{aligned}
&-i\frac{g}{2\sqrt{2}}\bar{\nu}\gamma^\mu\left(1-\gamma_5\right)eW_\mu^+ \\
&= -i\sqrt{\left(\frac{G_F M_W^2}{\sqrt{2}}\right)}\,\bar{\nu}\gamma^\mu\left(1-\gamma_5\right)eW_\mu^+ ,
\end{aligned}
\qquad (23.6)
$$

and the conjugate process




$$-i\frac{g}{2\sqrt{2}}\bar{e}\gamma^{\mu}\left(1-\gamma_5\right)\nu W^-_{\mu}$$
$$= -i\sqrt{\left(\frac{G_F M_W^2}{\sqrt{2}}\right)}\bar{e}\gamma^{\mu}\left(1-\gamma_5\right)\nu W^-_{\mu}. \tag{23.7}$$

In comparing the $Z$ and $W$ vertices, note the missing factor of $1/\sqrt{2}$ and the substitution $M_Z^2 \to M_W^2$ (the factor of cos $\theta_W$).

Next consider the various couplings between the gauge bosons themselves. These arise from the cubic and quartic terms in the original $SU(2)_L$ gauge Lagrangian, $-\frac{1}{4}F^k_{\mu\nu}F^{k,\mu\nu}$, where $F^l_{\mu\nu} = \partial_\mu W^l_\nu - \partial_\nu W^l_\mu - g\varepsilon^{lmn}W^m_\mu W^n_\nu$. Thus the generic cubic term looks like (there is a factor of 4 in the numerator from the 2 in the cross term and the 2 from the two terms in the curl)

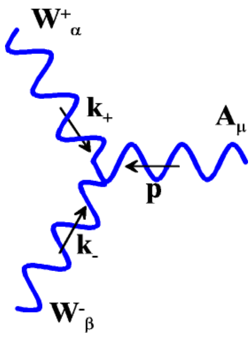
$$i\mathcal{L}_{\text{gauge, cubic}} = -\frac{i}{4}\left(-g\right)\cdot 2\cdot 2\cdot \partial_\mu W^k_\nu g^{\mu\alpha}g^{\nu\beta}\varepsilon^{klm}W^l_\alpha W^m_\beta$$
$$= igg^{\mu\alpha}g^{\nu\beta}\varepsilon^{klm}\partial_\mu W^k_\nu W^l_\alpha W^m_\beta. \tag{23.8}$$

We can define all the momenta as incoming so that $\partial_\mu W^k_\nu \to -iq^k_\mu W^k_\nu$, with $q^k_\mu$ the (incoming) momentum of vector boson with SU(2) index $k$ ($klm$ must all be different due to the antisymmetric structure constant $\varepsilon^{klm}$). There are 3! or 6 terms or coupling structures that we find when expanding out this antisymmetric product. We can write (all momenta defined as incoming)

$$i\mathcal{L}_{\text{gauge, cubic}} = g\Big[g^{\nu\beta}\left(q_1^\alpha - q_3^\alpha\right) + g^{\nu\alpha}\left(q_2^\beta - q_1^\beta\right)$$
$$+ g^{\alpha\beta}\left(q_3^\nu - q_2^\nu\right)\Big]W^1_\nu W^2_\alpha W^3_\beta. \tag{23.9}$$

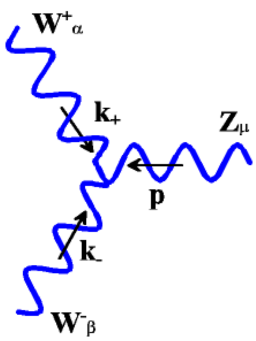
To get to "physically" relevant vertices we must project $W^1$ and $W^2$ onto $W^+$ and $W^-$ and $W^3$ onto $Z$ and $A$. Recall that, since the final photon is a mixture, including a component from the original non-Abelian SU(2) theory, it also participates in the "non-Abelian-like" couplings. The former projection introduces a factor of $i$, while the latter introduces factors of $\cos\theta_W$ and $\sin\theta_W$, respectively.

The photon couples to a W boson pair in the following way. The arrows indicate the assumed flow of momentum for the definition given. We find the following form (the sign is the hard part, depending on definitions of incoming versus outgoing states and many different expressions are provided in the literature),




$$ig\sin\theta_W\left[g^{\alpha\beta}\left(k_+-k_-\right)^\mu+g^{\alpha\mu}\left(p-k_+\right)^\beta +g^{\beta\mu}\left(k_--p\right)^\alpha\right]W_\alpha^+W_\beta^-A_\mu . \tag{23.10}$$

Recall that the coupling for the $A$ vertex can also be written as $g\sin\theta_W=e$ , *i.e.*, the charge of the $W$ is $e$. The corresponding vertex with the photon replaced by a $Z$ differs simply by changing $\sin\theta_W$ to $\cos\theta_W$, (the other component of $W^3$)




$$ig\cos\theta_W\left[g^{\alpha\beta}\left(k_+-k_-\right)^\mu+g^{\alpha\mu}\left(p-k_+\right)^\beta +g^{\beta\mu}\left(k_--p\right)^\alpha\right]W_\alpha^+W_\beta^-Z_\mu . \tag{23.11}$$

Note that the coupling can also be written $g\cos\theta_W=e\cot\theta_W$ .

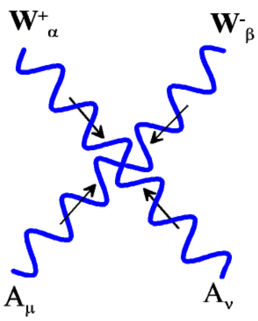
Next we come to the quartic couplings of the gauge bosons. These arise from the square of the purely non-Abelian term in the field strength tensor. The generic form is

$$i\mathcal{L}_{\text{gauge, quartic}}=-i\frac{g^2}{4}\left[\varepsilon^{klm}\varepsilon^{krs}g^{\mu\alpha}g^{\nu\beta}\right]W_\mu^lW_\nu^mW_\alpha^rW_\beta^s . \tag{23.12}$$

With 4 formally identical fields (members of the same multiplet), there are 4! ways to construct this product. Of these, only 6 are actually distinct (canceling the factor of 4 in the denominator) and we can write

$$i\mathcal{L}_{\text{gauge, quartic}} = -ig^2\Big[\varepsilon^{klm}\varepsilon^{krs}\left(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha}\right) + \varepsilon^{klr}\varepsilon^{kms}\left(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\beta}g^{\nu\alpha}\right) + \varepsilon^{kls}\varepsilon^{kmr}\left(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\alpha}g^{\nu\beta}\right)\Big]W_\mu^l W_\nu^m W_\alpha^r W_\beta^s. \tag{23.13}$$

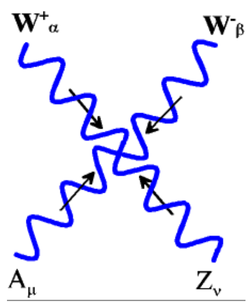
Due to the antisymmetric group factor, at most two of the coupled bosons can be of the same type. Thus there are 2 possibilities, $W^1W^3W^2W^3$ and $W^1W^2W^1W^2$, yielding 4 *physically* distinct cases, $W^+W^-AA$, $W^+W^-AZ$, $W^+W^-ZZ$ and $W^+W^-W^+W^-$. As a result only 4 of the six terms contribute and 2 of these are identical (see below). As we did with the cubic vertex we must project onto the physical states. First consider the $W^+W^-AA$ vertex, where again we define all bosons as incoming




$$-ig^2\sin^2\theta_W\left[2g^{\mu\nu}g^{\alpha\beta} - g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\alpha\nu}\right] \times W_\alpha^+ W_\beta^- A_\mu A_\nu, \tag{23.14}$$

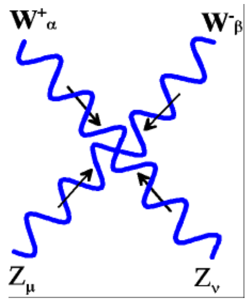
where $g^2\sin^2\theta_W = e^2$.

The $W^+W^-AZ$ vertex looks like ($g^2\sin\theta_W\cos\theta_W = e^2\cot\theta_W$)



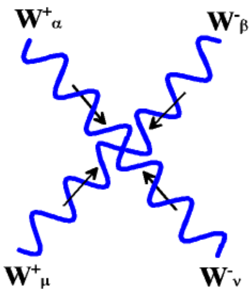

$$-ig^2\sin\theta_W\cos\theta_W\left[2g^{\mu\nu}g^{\alpha\beta} - g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\alpha\nu}\right]W_\alpha^+ W_\beta^- A_\mu Z_\nu. \tag{23.15}$$

For the $W^+W^-ZZ$ and $W^+W^-W^+W^-$ vertices we have




$$-ig^2\cos^2\theta_W\left[2g^{\mu\nu}g^{\alpha\beta}-g^{\mu\alpha}g^{\nu\beta}-g^{\mu\beta}g^{\alpha\nu}\right]W_\alpha^+W_\beta^-Z_\mu Z_\nu\,, \tag{23.16}$$

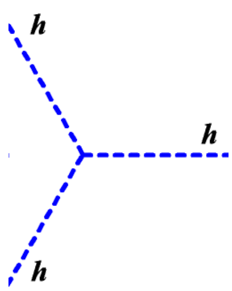
where $g^2\cos^2\theta_W = e^2\cot^2\theta_W$ , and




$$-ig^2\left[2g^{\mu\nu}g^{\alpha\beta}-g^{\mu\alpha}g^{\nu\beta}-g^{\mu\beta}g^{\alpha\nu}\right]W_\alpha^+W_\beta^-W_\mu^+W_\nu^-\,, \tag{23.17}$$

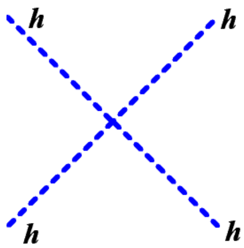
where $g^2 = e^2/\sin^2\theta_W$ .

Finally we want to consider the couplings of the scalar Higgs boson. Due to the purely Higgs part of the Lagrangian, the Higgs boson has both a cubic and a quartic coupling from the expansion of the term $-\lambda\left[\left(\mathrm{v}+h\right)/\sqrt{2}\right]^4$. In this case the n! factor, due to all the ways the n identical bosons can be identified with the external particles, is explicit. Thus the cubic term has a factor of 4 in the denominator from the $\sqrt{2}$ normalization factor, a factor 4 in the numerator from the expansion of the quartic polynomial and the 3! factor to yield,



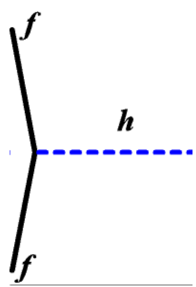

$$-6i\lambda\mathrm{v}hhh = -3im_h^2\sqrt{\sqrt{2}G_F}\,hhh. \tag{23.18}$$

The corresponding quartic terms has no extra factor of 4 from the expansion but the symmetry factor is now 4! to yield,




$$-6i\lambda hhhh = -3im_h^2\sqrt{2}G_F hhhh. \tag{23.19}$$

As we noted in the last lecture, since the coupling of fermions to the scalar vacuum expectation value via the Yukawa term provides the mass of the fermion, the coupling of a fermion to the physical Higgs particle is proportional to the fermion's mass. The vertex has the general form

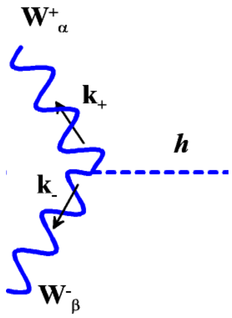



$$-i\frac{m_f}{\mathrm{v}}\bar{f}f = -im_f\sqrt{\sqrt{2}G_F}\,\bar{f}f. \tag{23.20}$$

The coupling of the original scalar doublet to the gauge bosons means that the physical Higgs boson also should have such couplings. We can read these terms off by returning to the terms in the Higgs Lagrangian of Lecture 22 that gave masses to the vector bosons and replacing v by v + $h$,

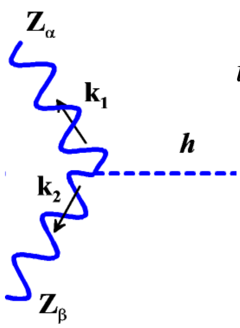
$$\begin{aligned} i\mathcal{L}_{\text{scalar,vector bosons}} &= i\frac{\mathrm{v}^2}{8}\left[g^2\left|W_\nu^+\right|^2 + g^2\left|W_\nu^-\right|^2 + \left(g'^2+g^2\right)\left(Z_\nu\right)^2\right] \\ &\to i\frac{\left(\mathrm{v}+2\mathrm{v}h+h^2\right)}{8}\left[g^2\left|W_\nu^+\right|^2 + g^2\left|W_\nu^-\right|^2 + \left(g'^2+g^2\right)\left(Z_\nu\right)^2\right]. \end{aligned} \tag{23.21}$$

Thus we have couplings of a single Higgs boson to pairs of both the $W$'s and the $Z$. Noting that the two $W$ terms both contribute to the same vertex and the $Z$ vertex has a 2! identical particle factor, we find




$$i\frac{g^2 v}{2} g^{\alpha\beta} W_\alpha^{+\dagger} W_\beta^{-\dagger} h = igM_W g^{\alpha\beta} W_\alpha^{+\dagger} W_\beta^{-\dagger} h$$
$$= 2iM_W^2 \sqrt{\sqrt{2}G_F}\, g^{\alpha\beta} W_\alpha^{+\dagger} W_\beta^{-\dagger} h \qquad (23.22)$$

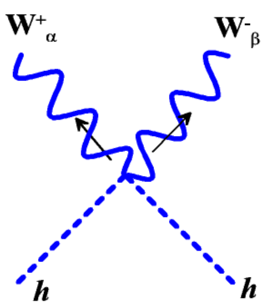
and




$$i\frac{g^2 + g'^2}{2} v g^{\alpha\beta} Z_\alpha^\dagger Z_\beta^\dagger h = ig\frac{M_Z}{\cos\theta_W} g^{\alpha\beta} Z_\alpha^\dagger Z_\beta^\dagger h$$
$$= 2iM_Z^2 \sqrt{\sqrt{2}G_F}\, g^{\alpha\beta} Z_\alpha^\dagger Z_\beta^\dagger h. \qquad (23.23)$$

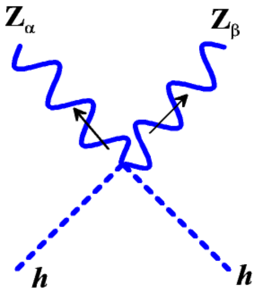
These vertices are written for an incoming Higgs and outgoing vector bosons but they are essentially the same if the vector bosons are incoming (only the $\dagger$ goes away).

Finally the $h^2$ term yields quartic couplings between Higgs pairs and vector bosons pairs. These have the form (with the extra 2! for the identical Higgs bosons)




$$i\frac{g^2}{2} g^{\alpha\beta} hh W_\alpha^{+\dagger} W_\beta^{-\dagger}$$
$$= 2iM_W^2 \sqrt{2}G_F\, g^{\alpha\beta} hh W_\alpha^{+\dagger} W_\beta^{-\dagger} \qquad (23.24)$$

and




$$i\frac{g^2+g'^2}{2}g^{\alpha\beta}hhZ_\alpha^\dagger Z_\beta^\dagger = i\frac{g^2}{2\cos^2\theta_W}g^{\alpha\beta}hhZ_\alpha^\dagger Z_\beta^\dagger$$
$$= 2iM_Z^2\sqrt{2}G_F g^{\alpha\beta}hhZ_\alpha^\dagger Z_\beta^\dagger . \qquad (23.25)$$

In the next lecture we will return to the question of calculating with these Feynman rules.

For completeness let us restate here also the forms of the relevant propagators:

$$\text{fermion} \sim \frac{i}{\not{k} - m} = i\frac{\not{k}+m}{k^2-m^2},$$
$$\text{massless vector} \sim -i\frac{g^{\mu\nu}}{k^2}, \qquad (23.26)$$
$$\text{massive vector} \sim -i\frac{\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{M^2}\right)}{k^2-M^2}.$$