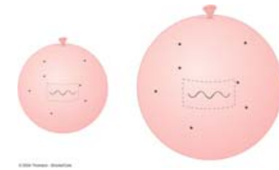
Lecture 2: Selected Powerpoint Slides

The Cosmic Scale Factor (*R*)

The redshift of distant galaxies is produced by the *expansion of space*, NOT the motion of galaxies *through space*.



Scale factor (*R*): A measure of the size of the universe as a function of time. It can be thought of as the ratio of the average separation of, e.g., galaxies, compared with the present separation. (Note: $R_0 \equiv 1.0$)

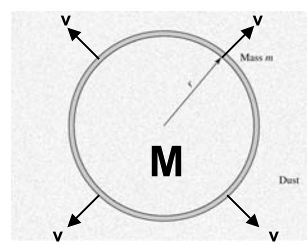
$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}} = \frac{R_{\text{obs}} - R_{\text{emitted}}}{R_{\text{emitted}}}$$

$$\boxed{\frac{R_{\text{obs}}}{R_{\text{emitted}}} = 1 + z} \qquad \text{If } R_{\text{obs}} \equiv 1, \text{ then } R(z) = \frac{1}{1+z}$$

Cosmology in 5 Easy Pieces

- **Cosmology with Newton**
- **The Cosmic Microwave Background and the Early Universe.**
- **Cosmology with Einstein: General Relativity and the Cosmological Constant**
- **Observational Cosmology: The Supernova Result**
- **The (Very) Early Universe**

Introducing the Dust-Filled Universe



r(t) = radius of sphere; r_0 = radius now
v(t) = velocity of material on surface of sphere
R(t) = scale factor of Universe; R_0 = scale factor today
M = mass interior to shell
m = mass of shell

Note: $\dot{R} \equiv \frac{dR}{dt}$

Coordinate distance (r): $r(t) = r_0 \frac{R(t)}{R_0}$; $\frac{r_0}{R_0} = \frac{r(t)}{R(t)} = \text{const.} \; (\equiv \varpi)$

Comoving coordinate (ϖ): $\varpi = \frac{r(t)}{R(t)}$

$$v(t) \equiv \frac{dr(t)}{dt} = \frac{r_0}{R_0}\dot{R}(t) = \frac{r(t)}{R(t)}\dot{R}(t) = \frac{\dot{R}(t)}{R(t)} r(t)$$

Compare: $v(t) = H(t)d$

Thus: $\boxed{H(t) = \frac{\dot{R}(t)}{R(t)}}$

$M = \frac{4}{3}\pi r^3(t)\rho_m(t)$; Note: $\rho r^3 =$ constant

Since $\rho r^3 = \text{const.}$, $r \propto R$, and $R(t_0) \equiv 1$:

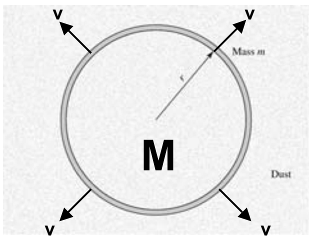
$$R^3(t)\rho(t) = R^3(t_0)\rho(t_0) = \rho_0$$

Thus: $\rho = \frac{\rho_0}{R^3}$

Since $R = \frac{1}{1+z}$:

$$\rho(z) = \rho_0 (1+z)^3$$

Dynamics of the Expanding, Dust-Filled Universe



Conservation of energy for shell:

$$K + U = E_{tot}\ ;\ E_{tot} = \text{const.}$$

If:
- $E_{tot} > 0$ -- shell expands forever.
- $E_{tot} = 0$ -- expansion rate approaches 0 as t approaches infinity.
- $E_{tot} < 0$ -- expansion will someday halt, and reverse itself.

$$K + U = E_{\text{tot}}$$

$$\frac{1}{2}mv^2 + \frac{-GMm}{r} = E_{\text{tot}};\ (E_{\text{tot}} = \text{const.},\ v = \frac{r\dot{R}}{R},\ M = \frac{4}{3}\pi r^3\rho)$$

$$\frac{1}{2}r^2\left(\frac{\dot{R}}{R}\right)^2 + \frac{-G4\pi r^2\rho}{3} = \text{const.; (multiply by 2, } r = \frac{R}{R_0}r_0)$$

$$\left(\frac{r_0}{R_0}\right)^2 R^2\left(\frac{\dot{R}}{R}\right)^2 + \frac{-8\pi G\rho}{3}\left(\frac{r_0}{R_0}\right)^2 R^2 = \text{const.; (}\left(\frac{r_0}{R_0}\right)^2 = \text{const.)}$$

$$\left[\left(\frac{\dot{R}}{R}\right)^2 - \frac{8\pi G\rho}{3}\right]R^2 = \text{const.; Let const.} \equiv -kc^2:$$

The **Friedmann Equation**:

$$\boxed{\begin{array}{l}\left[\left(\frac{\dot{R}}{R}\right)^2 - \frac{8\pi G\rho}{3}\right]R^2 = -kc^2 \\ \left[H^2 - \frac{8\pi G\rho}{3}\right]R^2 = -kc^2\end{array}}\ ;\ \text{Since}\left(\frac{\dot{R}}{R}\right) = H:$$

The Friedmann Equation

(1) $$\left[\left(\frac{\dot{R}}{R}\right)^2 - \frac{8\pi G\rho}{3}\right]R^2 = -kc^2$$

(2) $$\left[H^2 - \frac{8\pi G\rho}{3}\right]R^2 = -kc^2$$

k determines the fate of the Universe:

- **$k > 0$: Total energy negative, will recollapse. Universe is "bounded and closed".**
- **$k < 0$: Total energy positive, will expand forever. Universe is "unbounded and open".**
- **$k = 0$: Total energy zero. Expansion slows to 0 as t approaches infinity. Universe is "flat".**

Since $R^3\rho = \rho_0$ **, eq. (1) can be rewritten:**

(3) $$\dot{R}^2 - \frac{8\pi G\rho_0}{3R} = -kc^2$$

Setting $k = 0$ in the Friedmann equation (2) yields:

$$\boxed{\rho_c(t) = \frac{3H^2(t)}{8\pi G}}$$

Critical Density (ρ_c)

The density of matter that would allow the universe to expand forever, but at a rate that would decrease to zero at infinite time.

Setting $k = 0$ in the Friedmann equation yields:

$$\boxed{\rho_c(t) = \frac{3H^2(t)}{8\pi G}}$$

Density Parameter

The ratio of a measured density to the critical density.

$$\boxed{\Omega(t) \equiv \frac{\rho(t)}{\rho_c(t)} = \frac{8\pi G\rho(t)}{3H^2(t)}}$$

The value today:

$$\boxed{\Omega_0 \equiv \frac{\rho_0}{\rho_{c,0}} = \frac{8\pi G\rho_0}{3H_0^2}}$$

Ω_0
- **>1 closed, recollapses**
- **=1 flat, critical**
- **<1 open, expands forever**

Two Ways to Determine the Fate of the Universe

1) Count up the mass, to determine the average density.

→ **By 1997, "darn close" to critical universe (i.e., within factor of 4).**

Relations for a Flat Universe (Matter Only)

Scale factor:

$$R = \left(\frac{3}{2}\right)^{2/3}\left(\frac{t}{t_H}\right)^{2/3}$$

Age:

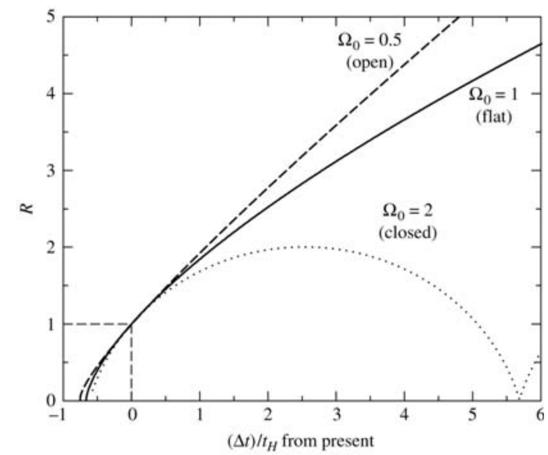
$$\frac{t(z)}{t_H} = \frac{2}{3}\cdot\frac{1}{(1+z)^{3/2}}$$

$$t_0 = \frac{2}{3}t_H \qquad \begin{array}{l} \text{open}: \ \frac{2}{3}t_H < t_0 < t_H \\ \text{closed}: \ 0 < t_0 < \frac{2}{3}t_H \end{array}$$

Lookback time:

$$t_L = \frac{2}{3}t_H\left(1 - \frac{1}{(1+z)^{3/2}}\right)$$

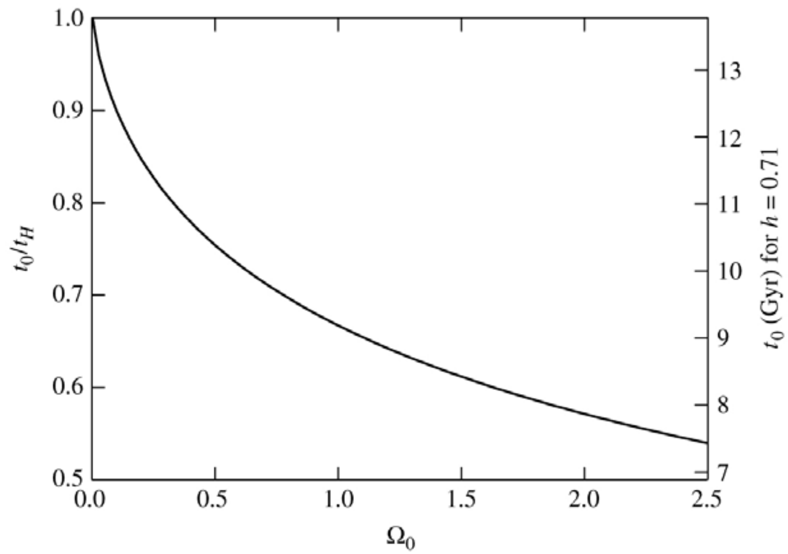
Evolution of the Scale Factor (matter only)



(Fig. 15.5)

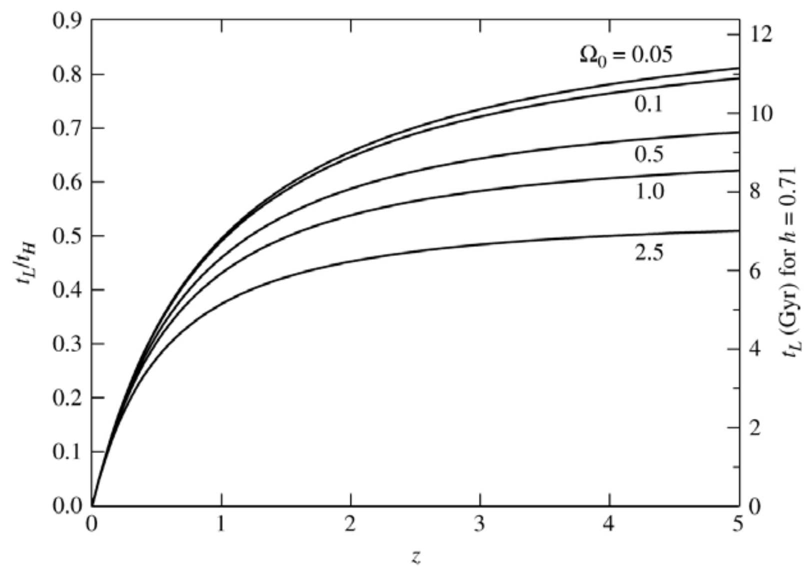
$$R_{\text{flat}} = \left(\frac{3}{2}\right)^{2/3}\left(\frac{t}{t_H}\right)^{2/3}$$

Present Age of Universe as a Function of Density Parameter (matter only)



(Fig. 15.6)

Lookback Time as a Function of Redshift



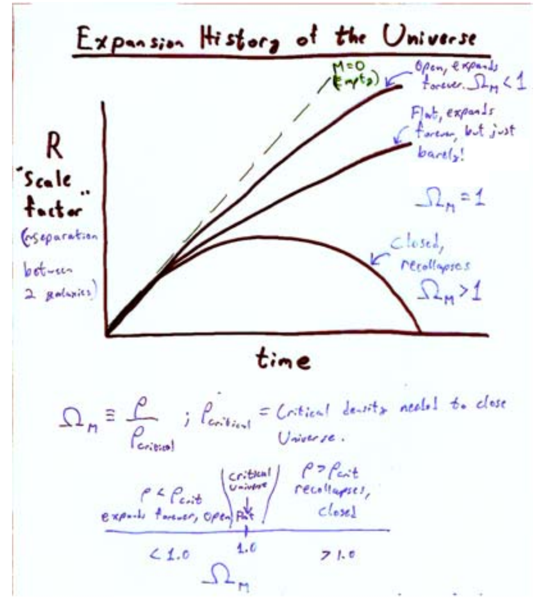
(Fig. 15.7)

How H and Ω Vary with Redshift

$$H = H_0(1+z)(1+\Omega_0 z)^{1/2}$$

$$\Omega = 1 + \frac{\Omega_0 - 1}{1 + \Omega_0 z}$$

$$\Omega - 1 = \frac{\Omega_0 - 1}{1 + \Omega_0 z}$$



When only the gravity of pressureless matter acts, the *geometry* and *fate* of the Universe are directly linked.

Astronomy 660 Toolkit

Relativistic Energy

- **Total relativistic energy of a massive particle:**

$$\boxed{E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}}$$

- **General equation for total relativistic energy of *any* particle (massive or massless):**

$$\boxed{E^2 = p^2c^2 + m^2c^4}$$

➢ **For photons:**

$$\boxed{E = h\upsilon = \frac{hc}{\lambda} = pc}$$

→ **Both the energy of photons and the kinetic energy of massive particles contribute to the gravity of the universe (along with the mass of massive particles).**

Note: "Equivalent mass density", ρ, for any form of energy with energy density u, is just $\frac{u}{c^2}$.

First Law of Thermodynamics

A statement of the *conservation of energy*: The increase in the internal energy of a thermodynamic system is equal to the amount of heat energy added to the system minus the work done by the system on the surroundings.

$$dU = dQ - dW$$

***dU*: change in internal energy**
***dQ*: heat flow into (positive) or out of (negative) the system**
***dW*: work done by (positive) or on (negative) the system**

$$dU = dQ - PdV$$

For expanding universe, *dQ* = 0 (adiabatic), so:

$$dU = -PdV$$