



## ISP220, fall 2021: In-Class Project #12; 15 pts

### Quarks, Spacetime, and the Big Bang

Thursday, October 14, 2021

Name: KEY Student # \_\_\_\_\_

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#### 1 Speeds, big and small, 5 points

Something you've seen, but it's worth passing it through your fingers to get it into your head. Trust me, I'm still a doctor.

Answer (1 points): Take the model for speeds as compared between a Home and Away frame. What happens to  $v_H$  if the speed of the Away frame,  $u$ , is very much less than the speed of light? Show in the box what you get.  $u \ll c$

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

$$u \ll c \Rightarrow \frac{u}{c^2} v_A \sim 0$$

$$v_H \rightarrow v_A + u \quad \checkmark$$

Answer (1 points): Take the model for speeds as compared between a Home and Away frame. What happens to  $v_H$  if the speed of the Away frame,  $u$ , is at the speed of light? Show in the box what you get.

$$\begin{aligned}
 v_H &= \frac{v_A + u}{1 + \frac{u}{c^2} v_A} \Rightarrow \frac{v_A + c}{1 + \frac{c}{c^2} v_A} \\
 &= \frac{v_A + c}{1 + \frac{v_A}{c}} = \frac{\cancel{v_A + c}}{c + v_A} \cdot c \\
 &= c
 \end{aligned}$$

Answer (2 points): Take the model for speeds as compared between a Home and Away frame. What happens to  $v_H$  if the speed of an object moving with velocity  $v_A$  within the Away frame, is itself moving at the speed of light? Show in the box what you get. Does it depend on the speed of the frame in which that object moving at  $c$  is traveling?

$$\begin{aligned}
 v_H &= \frac{v_A + u}{1 + \frac{u}{c^2} v_A} \Rightarrow \frac{c + u}{1 + \frac{u}{c^2} c} \\
 &= \frac{c + u}{1 + \frac{u}{c}} = \frac{\cancel{c + u}}{c + u} \cdot c \\
 &= c
 \end{aligned}$$

Answer (1 point): Take the model for speeds as compared between a Home and Away frame. What happens to  $v_H$  if the speed of an object moving with velocity  $v_A$  within the Away frame, is itself moving at the speed of light? Show in the box what you get. Does it depend on the speed of the frame in which that object moving at  $c$  is traveling? Circle your answer:

yes

no

## 2 Speed comparisons, 5 points

The interactive plot in Lesson 17 just before Section 3.4.1 shows the comparison of the Home frame observation of a speed of an object in the Away frame, each as a fraction of the speed of light, so  $v_H$  versus  $v_A$  for different values of the Away frame's velocity as a fraction of the speed of light...its  $\beta$ . Plot that same quantity versus  $\beta$  of the frame as a function of  $v_A/c = 0.8$  and  $v_A/c = 0.2$

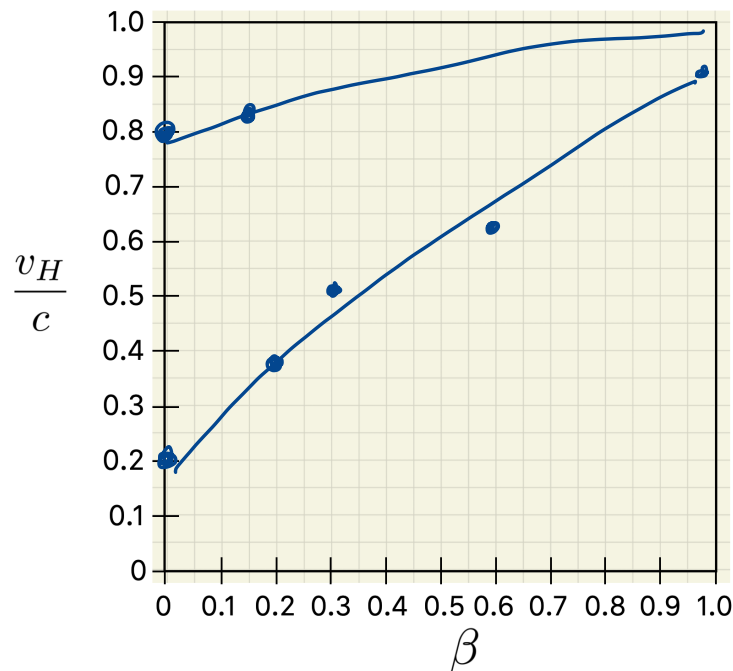


Figure 1: The plane as viewed from the airport.

### 3 Muon Decay, 5 points

Remember our muon decay from the last project?

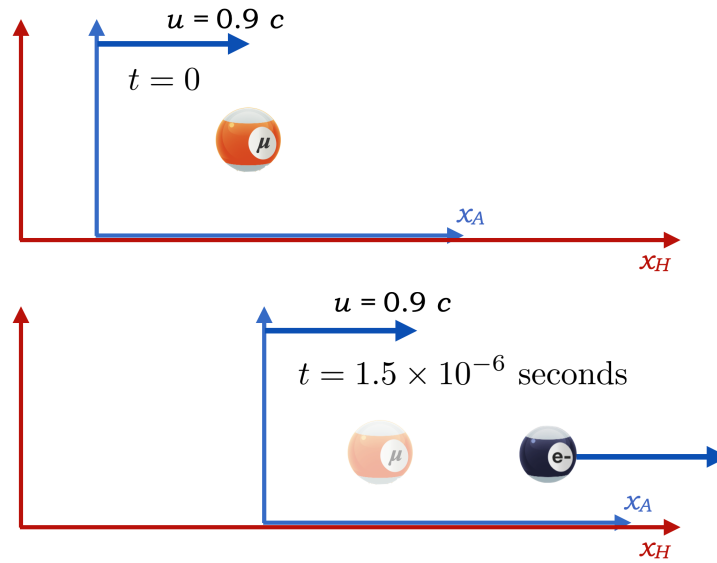


Figure 2: Muon decaying into an electron

Answer (5 points): If the electron in that decay is moving at  $0.6c$  in the muon's frame of reference, what is the electron's speed in the laboratory's frame of reference? Show your work and feel free to use the tools in Lesson 17!

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

$$= \frac{0.6c + 0.9c}{1 + \frac{(0.9c)}{c} 0.6c} = \frac{1.5c}{1 + 0.54} = \frac{1.5c}{1.54}$$

$$v_H = 0.97c$$

$$u = 0.9c$$

$$v_A = 0.6c$$