ISP220, fall 2021: In-Class Project #6; 15 pts

Quarks, Spacetime, and the Big Bang

Thursday, September 23, 2021

Name: KEY Student # ______________

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1 International Space Station, 8 pts

Figure 1 on the next page shows cartoons of distances measured away from the earth using the diameter or radius of the earth as a measure. The radius of the earth is 6380 km and for these pictures I've pretended to use $R_E \sim 6,000$ km. The bottom figure is just a blowup of the top one.

The International Space Station (ISS) is at an altitude above sea level of 400 km.

Mark the ISS orbit on both pictures with a big "X." (4 points, 2 each)

Are you surprised at how low it is? This is for two reasons, at least:

- It already takes hours for a Russian Soyuz spacecraft to get all the way to the ISS. So it would take much more energy and require more fuel.
- To go farther out would begin to get too close to the Van Allen Radiation belts, which are the shaded orange donut around the earth in the pictures.

The Van Allen belts (there are two) are regions created by the earth's magnetic field (thanks, Mr Gilbert...you'll see next week) which actually trap charged particles, which in the near belt—the one pictured as orange—are mostly protons. They're produced by cosmic rays and particles that come from the sun. They would damage electronics and be a hazard for humans...so best to stay under the belt, or above it.

Let's look at some practical "orbital mechanics" of the 21st century. We've done this in

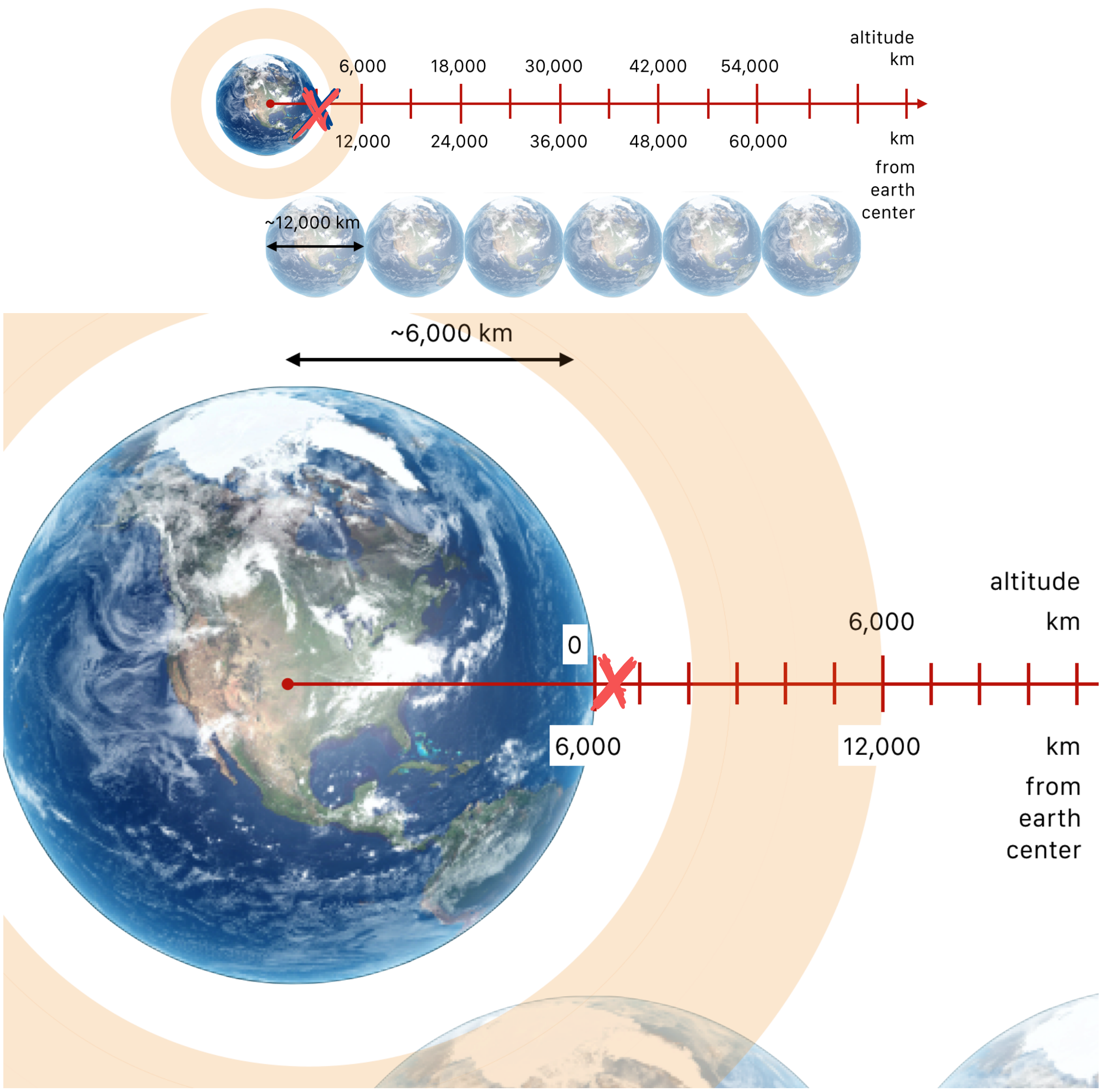

Figure 1: The top figure shows altitude and distances from the center of the earth. Bottom figure shows a blowup of the region near the earth.

class:

$$v^2 = \frac{GM_E}{R} \text{ so:}$$

$$v = \sqrt{\frac{GM_E}{R}} \tag{1}$$

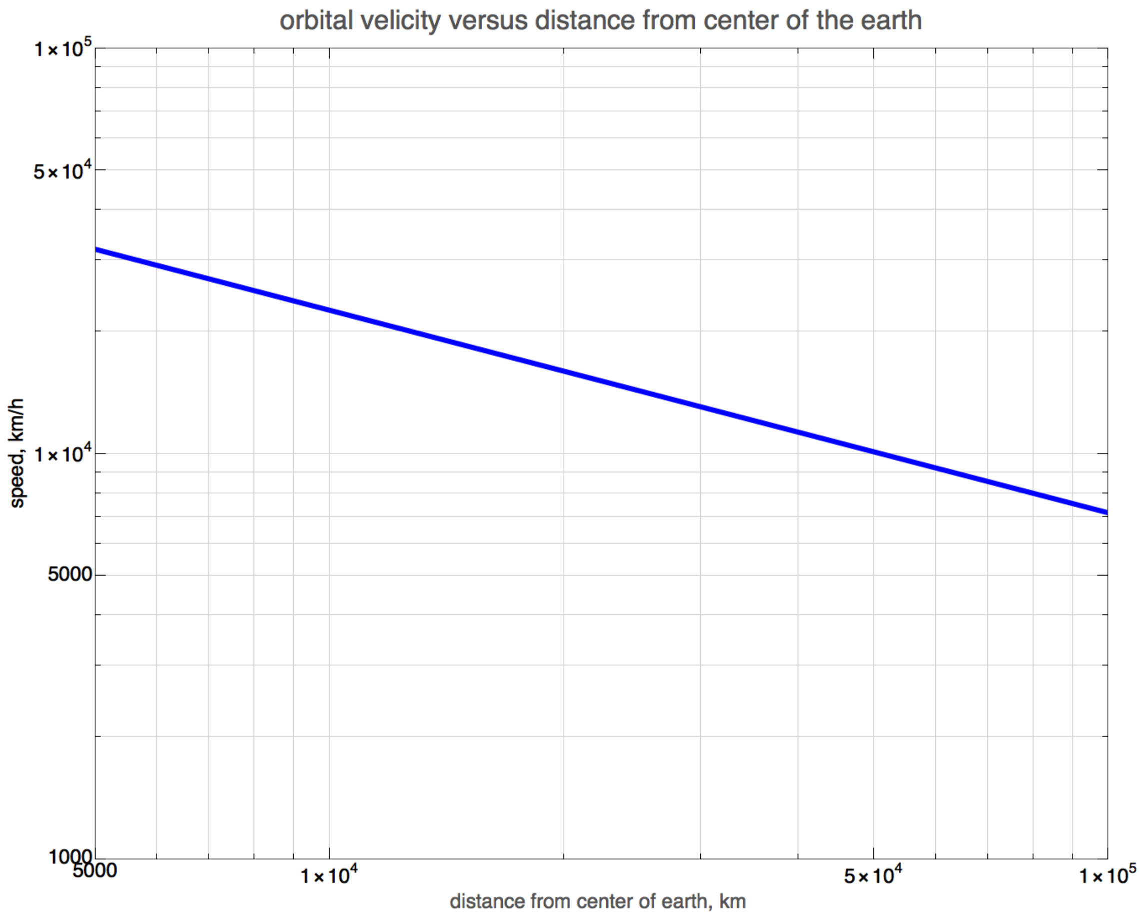

Figure 2: The orbital speed of a satellite of earth in a circular orbit versus the distance from the center of the earth.

I've plotted Eq. 1 in Figure 2 where I've manipulated the units in order to get the velocity in km/h and the distance in km. Figure 3 shows two circular orbits and various distance relationships of use to us. Let's figure out how fast the ISS is actually moving. First, remember that the ISS altitude is $A = 400$ km. If the average radius of the earth is 6380 km, how far is the ISS from the center of the earth, R_L (for **R**adius of **L**ow earth orbit)?

(1 point) 6780 km

With that information, Eq. 1 tells us what the speed of the ISS is in general, and Figure 2 shows us that function evaluated in km/h.

$$v = \sqrt{\frac{GM_E}{R_L}} = \sqrt{\frac{(6.7\times10^{-11})(6\times10^{24})}{6.8\times10^{3}\times10^{3}}} = \sqrt{5.9\times10^{-11+24-6}} = \sqrt{5.9\times10^{7}} = 7700 \text{ m/s}$$

Mark a big X on the Figure 2 for the ISS parameters. (2 points)

Fast, right? How fast in 'merican units...The conversion from km/h to mph is about 5/8ths.[1] Or, you can ask Mr Google. So what is the approximate speed in mph?

(1 point) v(ISS) = $\boxed{17,200}$ mph

$M_E = 6\times10^{24}$ kg

$G = 6.7\times10^{-11} \frac{\text{N m}^2}{\text{kg}^2}$

Equation 1 points at a really funny thing about Newtonian orbital mechanics that really caught the space program off-guard. If the only force that acts on a spacecraft (or an actual moon, or anything!) is that force of gravity, there are counter-intuitive maneuvers necessary in order to dock two spacecraft. What's landing on the moon, or on mars? It's "docking" a spacecraft with those objects—called a space rendezvous. So we had to figure out how to do that before going to the moon and much of the Gemini program was spent trying to get that right. There were failures!

What happens is that the Soyuz takes off from Kazakhstan riding on top of the most reliable launch vehicle in history, the Soyuz Launcher. The spacecraft is boosted into a low, "parking" orbit and then begins the docking process. Look at Eqn. 1 and Figure 3. If the lower orbit, R_L is the Soyuz spacecraft's parking orbit and R_H is the ISS's orbit, **is the Soyuz going faster or slower than the ISS? (circle)**

faster   slower

(Hint...it's faster.) (We will have talked about this before the project.) Somehow these two objects feeling only Newton's Universal Gravitation have to come together and that's remarkably difficult. Suppose you're in a lower orbit than the space station...wouldn't you just point your spacecraft at it and briefly fire your booster rockets like shooting a bow and arrow? That impulse of force would instantly put you into an *elliptical* orbit and cause you to gain so much altitude that you'd move *away* from the ISS...and then because your R is larger, you'd be going *slower* than ISS at that higher altitude! *That's right: aiming at the target moves you further away from the target.* That, and a lack of depth perception caused no end of trouble until NASA engineers (yup, including the ladies from the great

[1] Look at your speedometer and you'll see that when you're driving 80 km/h in Canada...it's about the same speed as 50 mph in the US. That's how I do the conversion...I run out to my car and look at my speedometer. Now you can too.

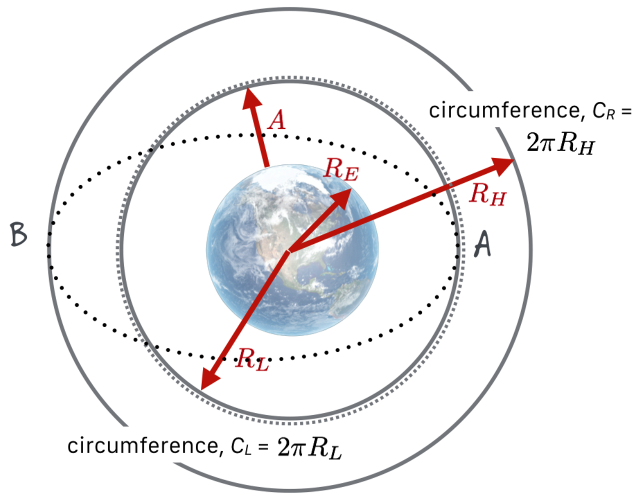

Figure 3: The orbital speed of a satellite of earth in a circular orbit versus the distance from the center of the earth.

movie, *Hidden Figures*) figured it out.

(We just watched a movie about this.) They dock using the "Hohmann transfer" ...that's what the dotted line in Figure 3 represents. In order to dock at point B, the boosters need to be fired exactly on the other side of the earth at point A in order to precisely put the spacecraft into a particular elliptical orbit to match up with the ISS at about that point...but just ahead of it. Then there's a careful series of burns to let the ISS come up from behind and dock to the waiting Soyuz...all of which are traveling at more than 15,000 mph.

Back to work.

2 Kepler's 3rd Law & Newton's Gravitation, 7 pts

Just before this we looked Kepler's Third Law and found:

$$T^2 = \frac{4\pi^2}{GM_E}R^3$$

$$T = \sqrt{\frac{4\pi^2}{GM_E}R^3} \tag{2}$$

I've plotted this function in Figure 4 for earth satellites. Here's a famiiar question: **how long does it take for the ISS to circle the earth?** You'll need R_L from before and you can then mark a big X on Figure 4 and transfer your answer here:

Roughly, it takes

(1 point) 90 minutes for ISS to go around the earth once.

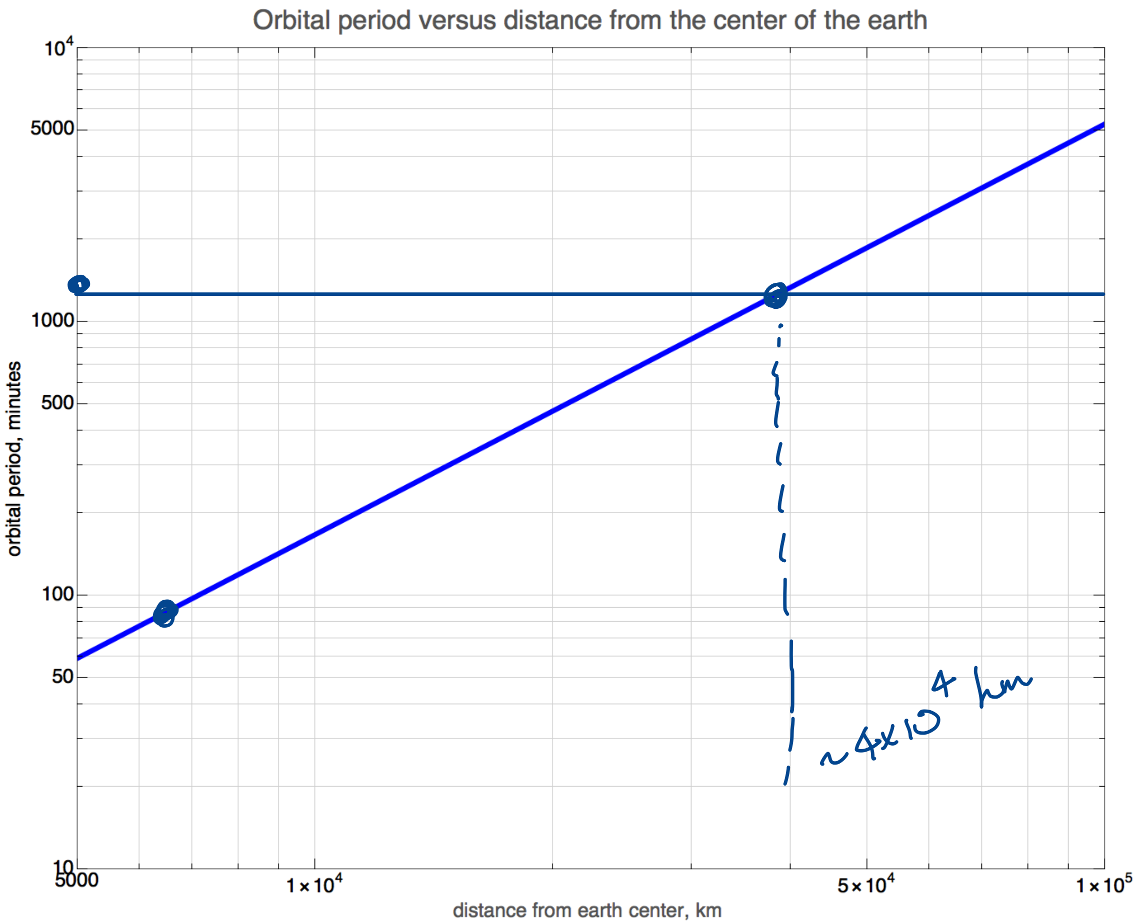

Figure 4: Kepler's 3rd Law represented by the Period (T) in hours plotted against the distance a satellite is from the center of the earth in km.

There is a particularly famous kind of orbit, called "geosynchronous" in which a satellite has a period that is exactly 1 earth day, 24 hours. These satellites can be viewed from earth without the ground equipment having to track the its movement: it's always at in the same place in the sky. If you have DirecTV or Dish Network, then that's what your dish is looking at. The satellite has a big "footprint" in its down-broadcast so many customers view the same satellite. DirecTV has 13 satellites (made by Hughes Aircraft, Boeing, and Space Systems/Loral) and Dish Network has 16 (mostly owned by EchoStar).

Our question is: how far up are these satellites? We'll pick on a particular kind of geosynchronous orbit called a geostationary orbit, which is one that's directly over the earth's equator. I'll tell you that there are 1440 minutes in a day, so you now know the period of a geocentric satellite.

Put a big dot on Figure 4 at the geostationary point (1 point) and put an arrow on Figure 5 cartoon (repeated from above) pointing at its location. (1 point)

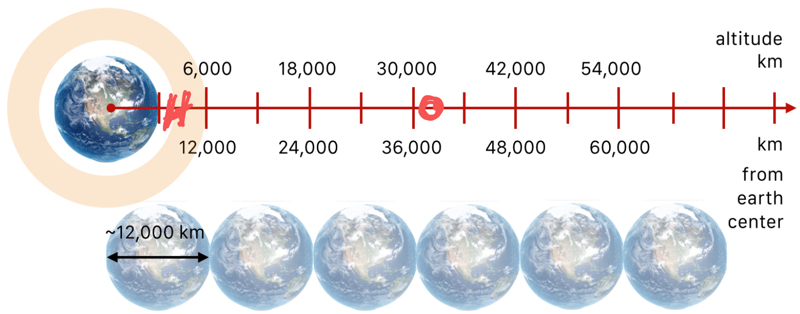

Figure 5: The top figure shows altitude and distances from the center of the earth.

Tell me how many kilometers from the center of the earth are those satellites.

(2 points) $R_L =$ $\boxed{4 \times 10^4 \text{ km}}$ km

Just to complete the story of satellites....there are a half million pieces of stuff orbiting the earth, from actual satellites to man-made debris. Goddard Space Flight Center in Maryland catalogs 2,271 actual functioning satellites now. Of course the satellites that we all deal with throughout our waking days are the Global Positioning Satellites (also called Navstar GPS), the US-funded GPS system. There are 32 GPS satellites in orbit in a medium earth orbit of around 20,000 km.

Put an arrow on Figure 5 cartoon pointing at that GPS altitude and label it "GPS." (1 point)

Ah...you already did that in the quiz, didn't you. Do it again.

The Hubble Space Telescope's distance from the center of the earth is about 6,919 km.

Put an arrow on Figure 5 cartoon pointing at that Hubble altitude and label it "H." (1 point)

The James Webb Space Telescope is the next generation, and it will be spectacular and will launch in March of 2021. It's orbit will be...sit down...1.5 million km. Can't plot that on the cartoons...rather you'd need 22 more pages side by side in order to get to that altitude!

Among many cool websites for satellite inventory is `http://stuffin.space`. You'll be surprised at the amount of "stuff" in especially low earth orbit.