HOMEWORK 10

chapters 8 and 7

8-21 $\lambda_1 = 766.41\,nm$ $\lambda_2 = 769.9\,nm$

$$E_i = \frac{hc}{\lambda_i} \Rightarrow \Delta E = E_1 - E_2 = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$$

$$= 1240\ eV\cdot nm \left(\frac{1}{766.41} - \frac{1}{769.9}\right)$$

$$\Delta E = 7.3\times10^{-3}\ eV$$

the field is from

$$V = -\vec{\mu}_e \cdot B = -g_e \frac{e\hbar}{2m} B \qquad g_e = 2$$

$$V = \frac{e\hbar}{m} B$$

$$B = \frac{m}{e\hbar}\Delta E = \frac{(9.1\times10^{-31}\,kg)}{(1.6\times10^{-19}\,C)(6.582\times10^{-16}\,eV\cdot s)}$$

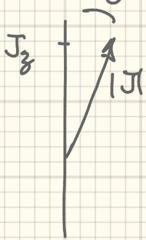
$$B = 63.4\ T \quad !!!$$

8-24 Same idea.

$$\Delta E = \frac{e\hbar}{m} B = \frac{(1.6\times10^{-19}C)(6.582\times10^{-16}\,eV\cdot s)(2.55T)}{9.1\times10^{-31}\,kg}$$

$$\Delta E = 2.95\times10^{-4}\,eV$$

8-26 $\theta \rightarrow$ minimum, so J_z is maximum



$\theta = 32.3°$

so $\cos\theta = \frac{J_z}{|J|} = \frac{m_j \hbar}{\hbar\sqrt{j(j+1)}}$

$$\cos\theta = \frac{j}{\sqrt{j(j+1)}}$$

solve for j: $\cos^2\theta = \frac{j^2}{j(j+1)} \equiv x$

$$j^2 = j^2 x + j x$$

$$j = jx + x \qquad j = \frac{x}{x-1} = \frac{\cos\theta}{\cos\theta - 1} = 2.5 = 5/2$$

7.25 5f in H $\quad$ f: $m_\ell = -3, -2, -1, 0, 1, 2, 3$

$B = 3T$ $\quad$ 7 states in all

w/o magnetic field, energy from Bohr formula:

$$E = -\frac{13.6\,eV}{n^2} = -\frac{13.6\,eV}{25} = -0.544\,eV$$

w/ magnetic field:

$$\Delta E = \mu_B B \,\Delta m_\ell$$

$m_\ell = 0$, $\Delta E = (5.788\times10^{-5}\,eV/T)(3\,T)\,m_\ell = (1.74\times10^{-4}\,eV)\,m_\ell$

$m_\ell = \pm1$, $\Delta E = (1.74\times10^{-4}\,eV)(\pm1) = \pm\,1.74\times10^{-4}\,eV$

$m_\ell = \pm2$, $\Delta E = (1.74\times10^{-4}\,eV)(\pm2) = \pm\,3.5\times10^{-4}\,eV$

$m_\ell = \pm3$, $\Delta E = (1.74\times10^{-4}\,eV)(\pm3) = \pm5.21\times10^{-4}\,eV$

7.29 4f in H

$n=4 \quad l=3$ so $2l+1 = 6+1=7$ possible m_l states:

$m_l = -3, -2, -1, 0, 1, 2, 3$

But there are 2 possible spin states for each so,

total degeneracy is $2 \times 7 = 14$

7.31 $\Delta E = 5.9 \times 10^{-6}$ eV ... this is a hyperfine transition

$E = \frac{3}{2} kT$

3 d.o.f.

$T = \frac{2}{3} E/k = \left(\frac{2}{3}\right)\left(5.9\times10^{-6}\,\text{eV}\right)\left(\frac{1}{8.62\times10^{-5}\,\text{eV/K}}\right)$

$T = 0.046$ K

7.34 (n, ℓ, m_ℓ, m_s):

a) $(5,2,1,\tfrac{1}{2}) \rightarrow (5,2,1,-\tfrac{1}{2})$ $\Delta\ell = 0$ forbidden.

b) $(4,3,0,\tfrac{1}{2}) \rightarrow (4,2,1,-\tfrac{1}{2})$ Δn is not forbidden... but there is no energy difference unless there is a B field

c) $(5,2,-2,-\tfrac{1}{2}) \rightarrow (1,0,0,-\tfrac{1}{2})$ $\Delta\ell = 2$ is forbidden.

d) $(2,1,1,\tfrac{1}{2}) \rightarrow (4,2,1,\tfrac{1}{2})$ this satisfies the selection rules and involves the absorption of a photon of energy

$$\Delta E = E_0\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 2.55\,eV$$

2-38 Ground state $P(r) = r^2 \frac{4}{a_0^3} e^{-2r/a_0}$

could integrate as in class or approximate since

proton radius is ~ 1.2×10^{-15} m $<< a_0 \Rightarrow e^{-2r/a_0} \simeq 1$

$$\frac{4}{a_0^3} \int_0^{1.2\times10^{-15}\,m} r^2 dr = \frac{4r^3}{a_0^3}\Big|_0^{1.2\times10^{-15}\,m} = 1.55 \times 10^{-14}$$