HOMEWORK # 11

8-32

$\Delta = 0.0168\ \text{nm}$

$\lambda = 422.7\ \text{nm}$

$B = 2\ \text{T}$

$m_\ell = 1$ —— $422.7 + 0.0168\,\text{nm} = 422.7168\,\text{nm}$

0 —— $422.7\ \text{nm}$

-1 ——

From $\Delta E = \mu_0 B \Delta m_\ell$ $\quad \mu_0 = \dfrac{e\hbar}{2m}$ — looking for $2\mu_0$ w/ $\Delta m_\ell = 1$

$$\frac{e\hbar}{m} = 2\frac{\Delta E}{B\,(1)}$$

$$\Delta E = 2\left(\frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}\right) = 2\left(\frac{1240\ \text{eV}\cdot\text{nm}}{422.7\ \text{nm}} - \frac{1240\ \text{eV}\cdot\text{nm}}{422.7168\,\text{nm}}\right)\frac{1}{2\,\text{T}}$$

$$\Delta E = 1.868 \times 10^{-23}\ \text{J/T}$$

8.37 From eq 8.23 the Landé g factor is

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}, \quad B = 1.2\ \text{T};\ m_J = 1$$

for $^2P_{3/2} \longrightarrow$ "2" $= 2S+1 \Rightarrow S = 1/2$; "P" $\Rightarrow L = 1$; "3/2" $\Rightarrow J = 3/2$

$$g = 1 + \frac{3/2(3/2+1) + 1/2(1/2+1) - 1(1+1)}{2\cdot 3/2\,(3/2+1)} = 1.33 = 4/3$$

$$\Delta E = \mu_0 B_{ext}\, g\, m_j = (9.274\times10^{-24}\,\text{J/T})(1.2)\,\tfrac{4}{3}\,(1)$$

$$\Delta E = 6.18\times10^{-24}\ \text{J} = 9.26\times10^{-5}\ \text{eV}$$

C2:

Assign electronic structure for Oxygen

$1s^2 2s^2 2p^4$

$(\uparrow,\downarrow) \vdots (\uparrow,\downarrow)\,;\ (\uparrow,\downarrow)\ (\uparrow\ )\ (\uparrow\ )$

m_ℓ:	-1	0	1
S:	0	$1/2$	$1/2$
L:	0	0	1

max spin $\uparrow\uparrow\uparrow\downarrow$:

$S = 1$

which is symmetric

so space part must be antisymmetric:

$L = 1$

ground state is $\left.\begin{array}{l} L=1 \\ S=1 \\ J=2 \end{array}\right\}\ {}^3P_2$

9-5

$$n = \left\{4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\right\}\int_c^\infty v^2\, e^{-\frac{m}{2\pi kT}}\,dv$$

tiny because of the exponential -- $m=(2)(1)$ for H_2 & remember $kT = \frac{1}{40}\,eV$

$$e^{-\frac{m}{2\pi kT}} \rightarrow -\frac{m}{2\pi kT} = \frac{-(2)(938\times10^{6}\,eV)\cdot 40\,eV^{-1}}{2\pi}$$

$$\cong -(300)\times10^{6}\times40$$

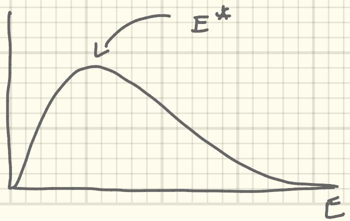
$$\simeq -12\times10^{9}$$ pretty small

9-16 $$f(E) = \left[\frac{2}{\sqrt{\pi}}\,\frac{1}{(kT)^{3/2}}\right]E^{1/2}\,e^{-E/kT}$$

max: $$\frac{df}{dE} = 0 = [\ \ ]\left\{\frac{1}{2}E^{-1/2}e^{-E/kT} - E^{1/2}\frac{1}{kT}e^{-E/kT}\right\}$$

so $$\frac{1}{2}E^{-1/2} = \frac{E^{1/2}}{kT}$$

$$\frac{kT}{2} = E$$



9-21 Fermi energy → $F_{FD} = 0.5$

$$F_{FD} = \frac{1}{B_{FD}\, e^{E_F/kT} + 1} = \frac{1}{2}$$ from book 9.30. At that value $E = E_F$

$$2 = B_{FD}\, e^{E_F/kT} + 1$$

$$1 = B_{FD}\, e^{E_F/kT}$$

$$B_{FD} = e^{-E_F/kT}$$ so we really have

$$F_{FD} = \frac{1}{e^{-E_F/kT}\, e^{E/kT} + 1}$$

$$F_{FD} = \frac{1}{e^{(E-E_F)/kT} + 1}$$

9-25 Silver

density $\rho = 1.05 \times 10^4\ kg/m^3$

$$\frac{N}{V} = \rho\,(kg/m^3)\left[\frac{N_A}{mol}\right]\left[\frac{mol}{\text{molar mass Silver}}\right]$$

molar mass $= 107.87\ g = 0.108\ kg.$

$$\frac{N}{V} = (1.05\times10^4\ kg/m^3)\left(\frac{6.02\times10^{23}}{mol}\right)\left(\frac{mol}{0.108\ kg}\right)$$

$$= 5.9\times10^{28}\ m^{-3}$$

from eq 9.42 $\qquad E_F = \dfrac{h^2}{8m_e}\left(\dfrac{3}{\pi}\dfrac{N}{V}\right)^{2/3}$

$$= \frac{(6.6\times10^{-34}\ J\cdot s)^2}{8(9.1\times10^{-31}\ kg)}\left[\frac{3}{\pi}\cdot 5.9\times10^{28}\ m^{-3}\right]$$

$$E_F = 8.81\times10^{-19}\ J = 5.5\ eV$$

9-26 $\epsilon_F = 5.51$ eV for gold (as derived in lecture)

$$\langle E \rangle = \frac{1}{n} \int E\, n(E)\, dE \quad \text{since} \quad \frac{n(E)\,dE}{n} \text{ is probability}$$

a) book derives $\langle E \rangle = \frac{3}{5} \epsilon_F = 3.31$ eV

b) For that energy, the temperature would correspond to

$$\frac{3}{2} kT = \langle E \rangle$$

$$T = \frac{2}{3} \frac{\langle E \rangle}{k} = \frac{2(3.31\,\text{eV})}{3(8.617 \times 10^{-5}\,\text{eV/K})} = 25{,}600\text{ K}$$

c) Thermal energies (kT - like) are small compared to Fermi energies

9.35 $\quad M = 4.5 \times 10^{30}$ kg.

$r = 10$ km.

From lecture $\quad \epsilon_F = \dfrac{h^2}{2m}\left(\dfrac{3}{8\pi}\right)^{2/3}\left(\dfrac{N}{V}\right)^{2/3}$

neutron, not electron

$$\frac{N}{V} = \frac{4.5\times10^{30}\text{ kg}}{\frac{4}{3}\pi\,(10^4\text{ m})^3}\;\frac{1\text{ neutron}}{1.675\times10^{-27}\text{ kg.}} = 6.41\times10^{44}\text{ m}^{-3}$$

$$\epsilon_F = \frac{(6.62\times10^{-34}\text{ J}\cdot\text{s})^2}{2(1.675\times10^{-27}\text{ kg})}\left[\frac{3}{8\pi}\right]^{2/3}\left(6.41\times10^{44}\text{ m}^{-3}\right)^{2/3}$$

$$1.3\times10^{-40}\cdot 1.8\times10^{29}$$

$$= 2.4\times10^{-11}\text{ J} = 1.5\times10^{8}\text{ eV} = 150\text{ MeV}$$

9.39 T_c comes from eq 9.65

$$T_c = \frac{h^2}{2m_{Ne} k}\left[\frac{N}{V}\frac{1}{2\pi(2.315}\right]^{2/3}$$

$\rho(Ne) = 1200\ kg/m^3$ molar mass $= 20\ g/mol$

$$\frac{N}{V} = \left(1200\ \frac{kg}{m^3}\right)\left(20\times10^{-3}\ \frac{kg}{mol}\right)^{-1}\left(6.02\times10^{23}\ \frac{Ne}{mol}\right)$$

$$= 3.6\times10^{28}\ \frac{Ne}{m^3}$$

$$T_c = \frac{h^2}{2mk}\left[\frac{3.6\times10^{28}}{2\pi(2.315)}\right]^{2/3} = \frac{2.9\times10^{-26}}{m}$$

$$m = 10n + 10p \cong 20p = 20(1.6\times10^{-27}\ kg) = 3.2\times10^{-26}\ kg$$

$$T_c \cong 0.9\ K$$

b) Neon melting point is 24.7 K so it's a solid at temperatures below that ⇒ not a liquid

(He melting point is a fraction of 1 K, so liquid at T_c.)