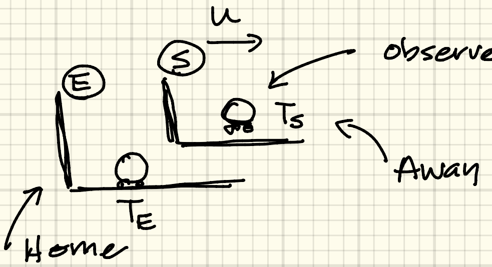
PHY 215 HOMEWORK #1

CHAPTER 2

23.



$$T_A = \gamma T_E$$

$$\gamma = T_E / T_A = \frac{T_E}{\frac{3}{5} T_A} = \frac{5}{3}$$

$$\frac{1}{\sqrt{1 - u^2/c^2}} = \frac{5}{3}$$

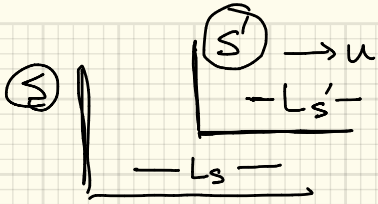
$$\frac{3}{5} = \sqrt{1 - u^2/c^2}$$

$$9/25 = 1 - \beta^2 \Rightarrow \beta^2 = 1 - 9/25$$

$$\beta = \sqrt{\frac{25-9}{25}} = \sqrt{\frac{16}{25}}$$

$$\beta = 4/5$$

24.



L_s' observed to be 20 m

$L_s = 40$ m.

$$L_s' = \frac{L_s}{\gamma} \Rightarrow \gamma = \frac{L_s}{L_s'}$$

$$\gamma = \frac{40}{20} = 2$$

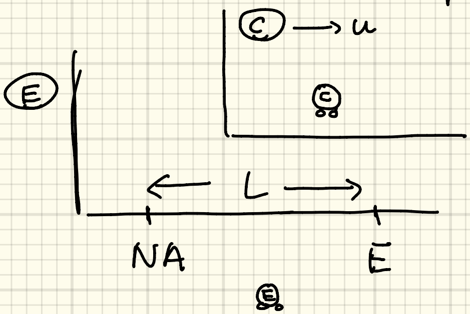
$$\frac{1}{\sqrt{1-\beta^2}} = 2$$

$$\sqrt{1-\beta^2} = \frac{1}{2} \Rightarrow \frac{1}{4} = 1-\beta^2$$

$$\beta^2 = 1 - 1/4 = 3/4$$

$$\beta = \sqrt{3}/2$$

25.



$u = 375$ m/s

$$t_E = \frac{L}{u} = \frac{8000 \times 10^3 \text{ m}}{375 \text{ m/s}}$$

$t_E = 21.3 \times 10^3$ s elapsed

Atomic clock on Concord will appear to run slower

$T_C = \gamma T_E$ — need γ

Whats β?

$$\beta = u/c = \frac{375}{3 \times 10^8} = 1.25 \times 10^{-6} \rightarrow \text{too tiny for calculator!}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \cong 1 + \frac{1}{2}\beta^2 \quad \text{so}$$

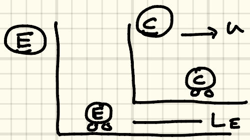
$$\Delta T = T_C - T_E = \gamma T_E - T_E$$

$$= T_E(\gamma - 1) = T_E\left(1 + \tfrac{1}{2}\beta^2 - 1\right)$$

$$\Delta T = T_E\left(\tfrac{1}{2}\beta^2\right) \cong 2 \times 10^{-8} \sim 20 \text{ ns}$$

28. $c = 100\ m/s$ $\qquad$ $u = 125\ \frac{km}{h} = 125\left(\frac{1000\,m}{1\,km}\right)\left(\frac{1\,h}{3600\,s}\right)$

a)



$u = 34.7\ m/s$

$\beta = \frac{34.7}{100} = 0.35$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.35)^2}}$

$\gamma = 1.07$

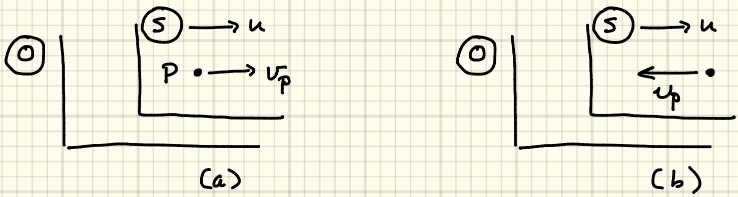
$\Delta T = T_C - T_E = \gamma T_E - T_E = T_E(\gamma - 1) = T_E(1.07 - 1) = 0.07\,T_E$

~ 7% slower

b) L_E is 1m in E and shorter as observed by C

$L_C = \frac{L_E}{1.07} \simeq 93\ cm$

31.



$u = 0.84c$ $\qquad$ $|v_p| = 0.62c$ in S

$\beta = 0.84$

a) $v_o = \frac{v_p + u}{1 + \frac{v_p}{c}\frac{u}{c}} = \frac{0.62c + 0.84c}{1 + 0.62 \cdot 0.84} = c\left(\frac{1.46}{1.52}\right)$

$v_o = 0.96c$

b) $v_o = \frac{-v_p + u}{1 - \frac{v_p}{c}\frac{u}{c}} = c\left(\frac{0.22}{0.48}\right) = 0.46c$

51. $f = \sqrt{\frac{1-\beta}{1+\beta}}\, f_0$ $\beta = 0.95$

$f_0 = 1400$ kHz

$$f = \sqrt{\frac{1-0.95}{1+0.95}}\,(1400) = 224 \text{ kHz}$$

54. $\lambda = \lambda_0 \sqrt{\frac{1-\beta}{1+\beta}}$ moving away so λ will appear to increase

$\downarrow$

$$\frac{1-\beta}{1+\beta} = \frac{\lambda^2}{\lambda_0^2} = \left(\frac{700\text{nm}}{589\text{nm}}\right)^2 \Rightarrow \beta = -0.17$$

$\uparrow$ astronauts looking back at earth

$$t \simeq \frac{|v|}{a} = \frac{(0.17)(3\times10^8)}{29.4}$$

$$= 1.7\times10^6 \text{ s}$$

(note. Edition #3 of TR uses a different acceleration so $t = 2\times10^6$ s)

66. need speed of proton, given KE:

$$E_T = m_o \gamma c^2$$

$$K = E - m_o c^2 = m_o \gamma c^2 - m_o c^2 = m_o c^2 (\gamma - 1)$$

$$\gamma = \frac{K}{m_o c^2} + 1$$

Cockcroft Walton $K = 750\text{ keV}$. $m_o c^2$ of proton $= 938\text{ MeV} = 938 \times 10^3\text{ keV}$

$$\gamma = \frac{750\text{ keV}}{938 \times 10^3\text{ keV}} + 1 = 1.0008$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.04$$

you might have appreciated

http://hyperphysics.phy-astr.gsu.edu/hbase/Relativ/relmom.html

Here they all are:

stage	K (GeV)	γ	β
CW	0.750	1.0008	0.04
Linac	0.4	1.43	0.71
Booster	8	9.53	0.994
MR	150	160.9	0.99998
Tevatron	1000	1067	0.9999996

D1. From G.T. we found $F = F'$ (2nd law)

and $v = v' + u$

to find that $E = E' + vB'$ ①

$B = B'$

spoiled the Lorentz Force invariance wrt G.T.

From $\vec{\nabla} \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$

we picked out a particular direction:

$$\frac{\partial B}{\partial x} = -\frac{1}{c^2} \frac{\partial E}{\partial t} \qquad (A)$$

(the - sign comes from the curl n the direction I chose.)

We need to transform derivatives, so we'll need:

$$\frac{\partial B}{\partial x} = \frac{\partial B}{\partial x'} \frac{\partial x'}{\partial x} + \frac{\partial B}{\partial t'} \frac{\partial t'}{\partial x}$$

$$\frac{\partial E}{\partial t} = \frac{\partial E}{\partial t'} \frac{\partial t'}{\partial A} + \frac{\partial E}{\partial x'} \frac{\partial x'}{\partial t} \qquad ②$$

From the G.T.

$$x' = x - ut$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

$$\frac{\partial x'}{\partial x} = 1 \qquad \frac{\partial x'}{\partial t} = -u \qquad \frac{\partial t'}{\partial t} = 1 \qquad \frac{\partial t'}{\partial x} = 0 \qquad (3)$$

From (2) & (3)

$$\frac{\partial B}{\partial x} = \frac{\partial B}{\partial x'}$$

$$\frac{\partial E}{\partial t} = \frac{\partial E}{\partial t'} - u\frac{\partial E}{\partial x'}$$

And, we need to transform the fields using (1)

$$\frac{\partial B}{\partial x} = \frac{\partial B'}{\partial x'} \qquad (4)$$

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial x'}(E' + uB') - u\frac{\partial}{\partial x'}(E' + uB')$$

$$= \frac{\partial E'}{\partial x'} + u\frac{\partial B'}{\partial x'} - u\frac{\partial E'}{\partial x'} - u^2\frac{\partial B'}{\partial x'} \qquad (5)$$

and from (A), (4), and (5):

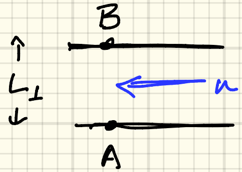
$$\frac{\partial B'}{\partial x'} = -\frac{1}{c^2}\frac{\partial E'}{\partial x'} - \frac{1}{c^2}\left(u\frac{\partial B'}{\partial x'} - u\frac{\partial E'}{\partial x'} - u^2\frac{\partial B'}{\partial x'}\right)$$

$$\frac{\partial B'}{\partial x'} = -\frac{1}{c^2}\frac{\partial E'}{\partial x'} + \frac{1}{c^2}\left(u\frac{\partial E'}{\partial x'} - u\frac{\partial B'}{\partial x'} + u^2\frac{\partial B'}{\partial x'}\right)$$

which is not form-invariant.

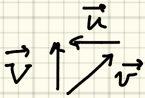
So, Maxwell's Equations are not invariant with respect to a Galilean Transformation

D2



$$T_\perp(RT) = T_\perp(AB) + T_\perp(BA)$$

AB :



$$v^2 = V^2 + u^2$$

$$V = \sqrt{v^2 - u^2} \quad \Rightarrow \quad T_\perp(AB) = \frac{L_\perp}{V} = \frac{L_\perp}{\sqrt{v^2 - u^2}}$$

BA: same

$$T_\perp(ABA) = \frac{2L_\perp}{\sqrt{v^2 - u^2}} = \frac{2L_\perp}{\sqrt{v^2(1 - u^2/v^2)}}$$

$$T_\perp(ABA) = \frac{2L_\perp}{v} \frac{1}{\sqrt{1 - u^2/v^2}}$$