

Homework 3

Chapter 19, 20

B&W

29. Gauge pressure excludes the atmospheric pressure — "the gauge".

$$p_i(g) = 300 \text{ kPa} \quad T_i = 15^\circ\text{C} \quad V_i = V_f = V$$

$$p_f(g) = ? \quad T_f = 45^\circ\text{C} \quad n_i = n_f = n$$

$$P_i = p_i(g) + p(\text{atm})$$

$$P_f = p_f(g) + p(\text{atm})$$

Consider air to be ideal, then $P_i V = n R T_i$

$$\frac{P_i}{T_i} = \frac{nR}{V} = \frac{P_f}{T_f}$$

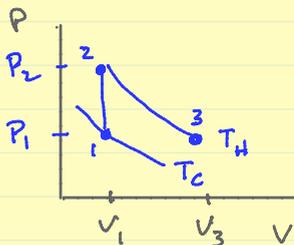
(also called the Gay-Lussac Law of Pressure and Temperature.)

$$P_f = P_i \left(\frac{T_f}{T_i} \right)$$

$$p_f(g) = [p_i(g) + p(\text{atm})] \left(\frac{T_f}{T_i} \right) - p(\text{atm})$$
$$= (300 \text{ kPa} + 101.3 \text{ kPa}) \left(\frac{273 + 45}{273 + 15} \right) - 101.3 \text{ kPa}$$

$$p_f(g) = 342 \text{ kPa}$$

19.32



$$V_1 = 1L = V_2 \quad V_3 = ?$$

$$P_2 = 2P_1 \quad T_2 = T_3 = T \quad \text{isothermal}$$

$$P_3 = P_1$$

assume ideal or just use
Boyle's Law

first leg: 1-2

$$P_1 V_1 = nRT_1 \quad V_1 = V_2 = V$$

$$P_2 V_2 = nRT_2$$

$$\frac{P_1}{T_1} = \frac{nR}{V} = \frac{P_2}{T_2}$$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right) = T_1 \left(\frac{2P_1}{P_1} \right) = 2T_1$$

second leg: 2-3

$$P_2 V_2 = nRT_2 \quad T_2 = T_3$$

$$P_3 V_3 = nRT_3$$

$$P_2 V_2 = P_3 V_3 \quad P_2 V_2$$

$$V_3 = V_2 \left(\frac{P_2}{P_3} \right) = V_2 \left(\frac{2P_1}{P_1} \right) = 2V_2 = 2V_1 = 2L$$

19.37

$$V_1 = V_2 = 2L$$

$$n = 1 \text{ mol}$$

$$P_2 - P_1 = \Delta P = ?$$

$$T_2 - T_1 = 100^\circ\text{C}$$

 $P_2 - P_1$

$$\Delta PV = nR\Delta T$$

$$\Delta P = \frac{nR\Delta T}{V} = \frac{(1 \text{ mol})(8.314 \text{ J/mol K})(100 \text{ K})}{(2 \times 10^{-3}) \text{ m}^3}$$

$$\Delta P = 416 \text{ kPa}$$

19.45

$$\eta = \text{number density} = 1 \text{ atom/cm}^3 \left(\frac{10^2 \text{ cm}}{\text{m}} \right)^3 = \frac{10^6 \text{ atoms}}{\text{m}^3}$$

$$T = 2.73 \text{ K}$$

a) P?

$$PV = NkT$$

$$P = \frac{NkT}{V} = \left(\frac{N}{V} \right) kT$$

$$P = \left(\frac{10^6}{\text{m}^3} \right) (1.38 \times 10^{-23} \text{ J/K})(2.73 \text{ K})$$

$$P = 3.77 \times 10^{-17} \text{ N/m}^2$$

$$P = 3.77 \times 10^{-17} \text{ Pa}$$

b)

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m_H}}$$

$$m_H = 1.67 \times 10^{-27} \text{ kg}$$

$$v_{\text{rms}} = \sqrt{\frac{(3)(1.38 \times 10^{-23} \text{ J/K})(2.73 \text{ K})}{1.67 \times 10^{-27} \text{ kg}}}$$

$$v_{\text{rms}} = 260 \text{ m/s}$$

$$c. \quad E_T = \# \text{ atoms} \times \langle KE/\text{atom} \rangle = 1 \text{ J}$$

$$E_T = N \left(\frac{3}{2} \right) kT \quad \text{from equipartition}$$

$$\frac{N}{V} = 10^6 \text{ m}^{-3}, \text{ remember?}$$

$$E_T = (10^6)(V) \left(\frac{3}{2} \right) kT$$

$$V = \frac{E_T}{(10^6) \left(\frac{3}{2} \right) (k)(T)}$$

$$V = \frac{1 \text{ J}}{(10^6 \text{ m}^{-3}) \left(\frac{3}{2} \right) (1.38 \times 10^{-23} \text{ J/K}) (2.73) \text{ K}}$$

$$V = 1.77 \times 10^{16} \text{ m}^3$$

$$L = (1.77 \times 10^{16} \text{ m}^3)^{0.33}$$

$$L = 230 \text{ km}$$

19.54 $V = 8 \times 10 \times 3 \text{ m}^3$ ' \rightarrow want heat.

$$T_1 = 20 \quad T_2 = 22^\circ \text{C} \quad \Delta T = 2 \text{ K}$$

$$P = 101 \text{ kPa}$$

$$C_V = \frac{5}{2} R \quad \text{for diatomic molecule}$$

$$Q = n C_V \Delta T = n \frac{5}{2} R \Delta T$$

need n , moles — which will stay constant
so calculate at initial temperature

$$PV = nRT$$

$$n = \frac{PV}{RT_1}$$

$$Q = \frac{PV}{RT_1} \left(\frac{5}{2}\right) R \Delta T$$

$$Q = \frac{(1.01 \times 10^5 \text{ Pa})(8)(10)(3) \text{ m}^3 \left(\frac{5}{2}\right)(2 \text{ K})}{293 \text{ K}}$$

$$Q = 414 \text{ kJ}$$

19.57

$$V_2 = 2V_1 \rightarrow \text{adiabatically } V_1 = 15 \text{ L}$$

$$P_1 = 1.5 \times 10^5 \text{ hPa}, \text{ monatomic}$$

a) P_2 ? adiabatic, no $PV^\gamma = \text{constant}$

$$P_1 V_1^\gamma = P_2 V_2^\gamma \quad \gamma = 5/3 \text{ for monatomic.}$$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^\gamma = (1.5 \times 10^5 \text{ hPa}) \left(\frac{15}{30}\right)^{5/3}$$

$$P_2 = 4.7 \times 10^4 \text{ hPa}$$

b) $T_1 = 300 \text{ K}$ $T_2 = ?$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (300) \left(\frac{1}{2}\right)^{2/3} = 190 \text{ K}$$

19.59

Diesel

$$T_1 = 29^\circ\text{C} \quad T_2 = ?$$

$$P_1 = 1 \text{ atm} \quad P_2 = ?$$

$$V_1 = 600 \text{ cm}^3 \quad V_2 = 45 \text{ cm}^3$$

$$\text{air - diatomic} \rightarrow \gamma = 7/5$$

→ adiabatic by design.

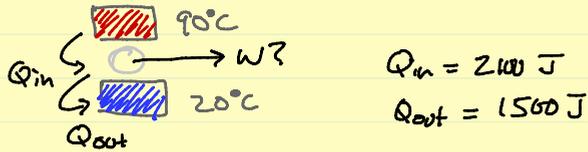
$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = (1 \text{ atm}) \left(\frac{600}{45} \right)^{7/5} = 37.6 \text{ atm.}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = 293 \text{ K} \left(\frac{600}{45} \right)^{2/5} = 826 \text{ K}$$

20.26



$$W = Q_H - Q_C = 2100 - 1500 = 600 \text{ J}$$

$$\epsilon = \frac{W}{Q_H} = \frac{600}{2100} = 0.3$$

$$P = 2500 \text{ W} = 2500 \text{ J/s}$$

$$t = \frac{W}{P} = \frac{600}{2500} \text{ s} = 0.24 \text{ s}$$

20.33

$$T_H = 1000 \text{ K}$$

$$T_C = 300 \text{ K}$$

$$\langle P \rangle = 1 \text{ W/cycle}$$

$$a) \quad \epsilon = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{1000} = 0.7$$

$$b) \quad \epsilon \text{ also} = \frac{W}{Q_H} \Rightarrow Q_H = \frac{W}{\epsilon} = \frac{1000 \text{ J}}{0.7}$$

$$Q_H = 1430 \text{ J}$$

$$c) \quad W = Q_H - Q_C$$

$$Q_C = Q_H - W = 1430 - 1000 = 430 \text{ J}$$

20.45



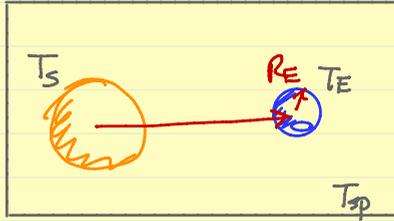
$$a) \quad \Delta S_H = \frac{Q}{T_H} = -\frac{8500}{700} = -12.1 \text{ J/K}$$

$$\Delta S_C = \frac{Q}{T_C} = \frac{8500}{160} = 85 \text{ J/K}$$

$$b) \quad \Delta S_P = 0 \quad \dots \text{ no net heat in or out.}$$

$$c) \quad \begin{aligned} \Delta S_S &= \Delta S_H + \Delta S_C + \Delta S_P \\ &= -12.1 + 85 + 0 \\ \Delta S_S &= 72.9 \text{ J/K} \end{aligned}$$

20.53



$$T_s = 5779 \text{ K}$$

$$T_E = 278.9 \text{ K}$$

$$T_{sp} = 50 \text{ K}$$

$$K = 1370 \text{ W/m}^2$$

$$R_E = 6371 \times 10^3 \text{ m}$$

$$\Delta S = \frac{\Delta Q}{T}$$

$$\frac{\Delta S}{t} = \frac{1}{T} \left(\frac{\Delta Q}{t} \right) \text{ generally.}$$

earth absorbs, sun: $\frac{\Delta Q}{t} = \pi R_E^2 K$

earth absorbs, space: S-B... $\frac{\Delta Q}{t} = -4\pi R_E^2 \sigma T_{sp}^4$

earth radiates: S-B... $\frac{\Delta Q}{t} = -4\pi R_E^2 \sigma T_E^4$

$$\left(\frac{\Delta S}{t} \right)_{\text{total}} = \frac{\pi R_E^2 K}{T_s} - 4\pi R_E^2 \sigma T_{sp}^3 - 4\pi R_E^2 \sigma T_E^3$$

$$= \pi R_E^2 \left[\frac{K}{T_s} - 4\sigma (T_{sp}^3 - T_E^3) \right]$$

$$= \pi (6.371 \times 10^6) \left[\frac{1370}{5779} - 4(5.67 \times 10^{-8}) (50^3 - 278.9^3) \right]$$

$$\frac{\Delta S}{t} \sim -5.9 \times 10^{14} \text{ J/sK} = -5.9 \times 10^{14} \text{ W/K}$$

D3 Adiabatic, ideal gas...

$$P_i V_i^\gamma = P_f V_f^\gamma$$

$$P_f = P_i \left(\frac{V_i}{V_f} \right)^\gamma$$

$$P_i V_i = nRT_i$$

$$P_i = \frac{nRT_i}{V_i} \quad \text{and for } P_f$$

$$P_f = \frac{nRT_f}{V_f} = \frac{nRT_i}{V_i} \left(\frac{V_i}{V_f} \right)^\gamma$$

$$T_f = T_i \left(\frac{V_i}{V_f} \right)^\gamma \frac{V_f}{V_i}$$

$$T_f = T_i \left(\frac{V_i}{V_f} \right)^{\gamma-1}$$

D4

net work:

$$nRT_2 \ln(V_B/V_A) - nRT_1 \ln(V_D/V_C)$$

$PV = nRT \Rightarrow$ since T is constant, then

$$\left. \begin{aligned} P_A V_A &= P_B V_B \\ P_C V_C &= P_D V_D \end{aligned} \right\}$$

but also

$$\begin{aligned} P_B V_B^\gamma &= P_C V_C^\gamma \\ P_D V_D^\gamma &= P_A V_A^\gamma \end{aligned}$$

↓

$$\frac{P_A V_A}{V_B} = P_B$$

$$\frac{P_D V_D}{V_C} = P_C$$

$$\frac{P_D V_D^\gamma}{V_A^\gamma} = P_A$$

$$\frac{P_A V_A V_B^\gamma}{V_B} = \frac{P_D V_D V_C^\gamma}{V_C}$$

$$P_A V_A V_C V_B^\gamma = P_D V_B V_D V_C^\gamma$$

$$P_D V_D^\gamma V_A V_C V_B^\gamma = P_D V_A^\gamma V_B V_D V_C^\gamma$$

$$V_A V_C (V_B V_D)^\gamma = V_B V_D (V_A V_C)^\gamma$$

$$(V_B V_D)^{\gamma-1} = (V_A V_C)^{\gamma-1}$$

$$V_A V_C = V_B V_D$$

$$\frac{V_C}{V_D} = \frac{V_B}{V_A}$$

from previous page

net work:

$$nRT_2 \ln\left(\frac{V_B}{V_A}\right) - nRT_1 \ln\left(\frac{V_D}{V_C}\right) \leftarrow \frac{V_C}{V_D} = \frac{V_B}{V_A}$$

$$nRT_2 \ln\left(\frac{V_B}{V_A}\right) - nRT_1 \ln\left(\frac{V_B}{V_A}\right)$$

$$nR \ln\left(\frac{V_B}{V_A}\right) [T_2 - T_1] \quad \checkmark$$

same for net Q added.