Homework

HOM-WORK # 8 CHAPTER 6

6-5 $\psi(r) = A r e^{-r/\alpha}$

$$\int_0^\infty \psi^*(r)\psi(r)\,dr = 1 = A^2\int_0^\infty r^2 e^{-2r/\alpha}\,dr$$

$$= A^2\Big[$$

$$\int x^2 e^{-x/a}\,dx = 2a^3$$

$$a = \frac{\alpha}{2}$$

$$\vdots$$

$$\int \quad dx = \frac{2\alpha^3}{8} \qquad 1 = A^2\frac{\alpha^3}{4} \Rightarrow A^2 = \frac{4}{\alpha^3}$$

$$A = \frac{2}{\alpha^{3/2}}$$

6-11 $\quad A\sin x \quad 0 < x < \pi$

$\quad 0 \quad$ elsewhere

$$\int_0^{\pi} \psi^* \psi \, dx = 1 = A^2 \int_0^{\pi} \sin^2 x \, dx$$

$$1 = A^2 \frac{\pi}{2} \quad \Rightarrow \quad A^2 = \frac{2}{\pi}$$

$$A = \sqrt{2/\pi}$$

a) prob $[0, \pi/4]$

$$\frac{2}{\pi} \int_0^{\pi/4} \sin^2 x \, dx = \frac{2}{\pi} \left[\frac{x}{2} - \frac{1}{4} \sin 2x \right]_0^{\pi/4}$$

$$= \frac{2}{\pi} \left[\frac{\pi}{8} - \frac{1}{4} \sin \pi/2 + \frac{1}{4} \sin 0 \right]$$

$$= \frac{2}{\pi} \left[\frac{\pi}{8} - \frac{1}{4} \right] = \frac{1}{4} - \frac{1}{2\pi} = 0.09$$

b) $$\frac{2}{\pi}\int_0^{\pi/2} \sin^2 x\,dx = \frac{2}{\pi}\left[\frac{x}{2} - \frac{1}{4}\sin 2x\right]_0^{\pi/2}$$

$$= \frac{2}{\pi}\left[\frac{\pi}{4} - \frac{1}{4}\sin\pi + \frac{1}{4}\sin 0\right]$$

$$= \frac{1}{2}$$

6-17 $$\psi_1 = \sqrt{2/L}\,\sin\frac{\pi x}{L}$$

$$P_1 = \int_0^{L/3}\left(\frac{2}{L}\right)\sin^2\frac{\pi x}{L}\,dx = \frac{2}{L}\left[\frac{x}{2} - \frac{L}{4\pi}\sin\left(\frac{2\pi x}{L}\right)\right]_0^{L/3}$$

$$P_1 = 2\left[\frac{1}{6} - \frac{\sqrt{3}}{8\pi}\right] \simeq 0.2$$

$$P_2 = \int_{L/3}^{2L/3}\left(\frac{2}{L}\right)\sin^2 x\,dx = 2\left[\frac{1}{6} + \frac{\sqrt{3}}{4\pi}\right] = 0.61$$

$$P_3 = 2\left[\frac{1}{6} - \frac{\sqrt{3}}{8\pi}\right] = 0.2$$

$$P_1 + P_2 + P_3 \simeq 1$$

6-21

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad n=4 \qquad L = 0.7\,nm$$

$$\pi^2\hbar^2 = \pi^2\frac{h^2}{4\pi^2} = \frac{h^2}{4}$$

$$E_1 = \frac{h^2}{8mL^2} = \frac{(hc)^2}{8mc^2L^2} = \frac{(1240\ eV\cdot nm)^2}{8(511\times10^3 eV)(0.7\,nm)^2}$$

$$E_1 = 0.77\ eV$$

others: $E_n = n^2E_1$

$$E_4 - E_3 = (16-9)(0.77) = 5.37\ eV$$

$$E_4 - E_2 = 9.2\ eV$$

$$E_4 - E_1 = 11.5\ eV$$

$$E_3 - E_1 = 6.14\ eV$$

$$E_2 - E_1 = 2.3\ eV$$

$$E_3 - E_2 = 3.84\ eV$$

6-35

$$E_n = (n+\tfrac{1}{2})\hbar\omega = (n+\tfrac{1}{2})hf \qquad m = 2.32\times10^{-26}\,\text{kg}.$$

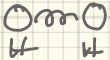
$$E_n = (4.1\times10^{-15}\,\text{eV-s})(1\times10^{13}\,\text{s}^{-1})(n+\tfrac{1}{2})$$

$$E_n = (4.14\times10^{-2}\,\text{eV})(n+\tfrac{1}{2})$$

$$k = \omega^2 m = 4\pi^2 f^2 m = 4\pi^2(1\times10^{-13}\,\text{s}^{-1})^2(2.32\times10^{-26}\,\text{kg})$$

$$k = 91.6\ \text{N/m}$$

6-39



$$k = 1.1\times10^{3}\ \text{N/m}$$

$$E_n = (n+\tfrac{1}{2})\hbar\omega \qquad \text{from 3rd}$$

$$\omega = \sqrt{k/m} = \sqrt{\frac{k(m_1+m_2)}{m_1 m_2}} = \sqrt{\frac{2k}{m}} \quad \text{since} \quad (m_1 = m_2)$$

reduced mass

$E_3 - E_2 \qquad \Delta E_{32} = (3+\frac{1}{2})\hbar\omega - (2+\frac{1}{2})\hbar\omega = \hbar\omega$

$E_3 - E_1 \qquad \Delta E_{31} = (3-1)\hbar\omega = 2\hbar\omega$

$E_2 - E_1 \qquad \Delta E_{21} = (2-1)\hbar\omega = \hbar\omega$

$E_3 - E_0 \qquad \Delta E_{30} = (3-0)\hbar\omega = 3\hbar\omega$

$E_2 - E_0 \qquad \Delta E_{20} = (2-0)\hbar\omega = 2\hbar\omega$

$E_1 - E_0 \qquad \Delta E_{10} = (1-0)\hbar\omega = \hbar\omega$

$\Delta E_{32} = \Delta E_{10} = \Delta E_{21} \qquad \hbar\omega = \hbar\sqrt{\frac{2k}{m}} = (6.5\times10^{-16}\ \mathrm{eV\cdot s})\sqrt{\frac{2(1.1\times10^{3}\ \mathrm{N/m})}{1.67\times10^{-27}\ \mathrm{kg}}}$

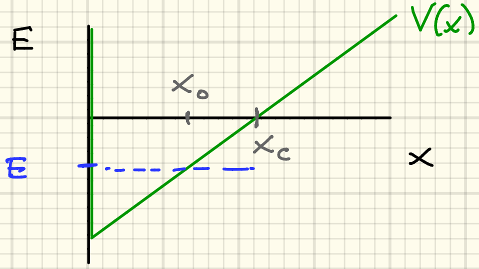
$\hbar\omega = 0.76\ \mathrm{eV}$

$\lambda = \frac{hc}{E} = \frac{1240\ \mathrm{eV\cdot nm}}{0.76\ \mathrm{eV}} = 1640\ \mathrm{nm}$

$\Delta E_{31} = \Delta E_{20} \qquad 2\hbar\omega = 1.51\ \mathrm{eV} \qquad \lambda = 820\ \mathrm{nm}$

$\Delta E_3 - E_0 \qquad 3\hbar\omega = 2.26\ \mathrm{eV} \qquad \lambda = 549\ \mathrm{nm}$

6-43



For $x < x_0$, ψ is oscillatory.

but $E - V_0$ gets smaller as x increases, so

$$k = \sqrt{\frac{(E-V)2m}{\hbar}}$$ changes

so that λ gets longer.

For $x > x_0$, ψ is penetrating the barrier

So it should look something like