Name: KEY

Student #

PHY215, fall 2017

Modern Physics and Thermodynamics

Exam #2, Code-Name: Wavy Warrior
Friday, October 27, 2017: 7 questions, 50 points

You must show all of your work.

Constants

1 calorie = 4.186 J
1 atmosphere = 1.01×10^5 Pa
Gas Constant: $R = 8.3145$ J/mol·K
Bolzmann's Constant: $k = 1.38 \times 10^{-23}$ J/L
Sefan-Boltzmann's constant: $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4
Avagadro's Number: $N_A = 6.023 \times 10^{23}$ mol^{-1}
Speed of Light: $c = 3 \times 10^8$ m/s
Charge of the electron: $-e = -1.6 \times 10^{-19}$ C
Mass of the electron: $m_e = 9.1094 \times 10^{-31}$kg = 511 keV/c^2
Mass of the proton: $m_p = 1.6726 \times 10^{-27}$kg = 938.3 MeV/c^2
Mass of the neutron: $m_n = 1.6749 \times 10^{-27}$kg = 939.6 keV/c^2
Mass of the alpha particle: $m_\alpha = 3727.4$ MeV/c^2
Planck's Constant: $h = 6.63 \times 10^{-34}$J-s = 4.14×10^{-15} eV-s
...times c: $hc = 1.9864 \times 10^{-25}$J-m = 1239.8 eV-nm
Reduced h: $h/2\pi = \hbar = 1.0546 \times 10^{-34}$J-s = 6.5821×10^{-16} ev-s
...times c: $\hbar c = 3.162 \times 10^{-28}$J-m = 197.33 eV-nm
Electrostatic constant: $\dfrac{1}{4\pi\epsilon_0} = 8.9876 \times 10^9$ N-m^2-C^{-2}
...times e^2: $\dfrac{e^2}{4\pi\epsilon_0} = 2.3071 \times 10^{-28}$ J-m = 1.4400×10^{-9}eV-m
Bohr radius: $a_0 = \dfrac{\hbar}{m_e c\alpha} = 0.5292 \times 10^{-10}$ m
Fine structure constant: $\alpha = \dfrac{e^2}{4\pi\epsilon_0 \hbar c} = 1/137.036$
Hydrogen Rydberg constant: $R_H = 1.097 \times 10^7$ m^{-1}

Formulae

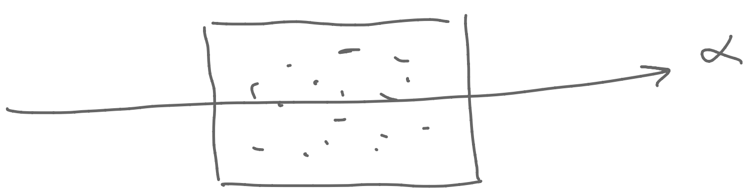
$$\begin{aligned} \text{reduced mass: } \mu &= \frac{mM}{m+M} \\ \text{mean velocity for an ideal gas: } < v > &= \frac{4}{\sqrt{2\pi}}\sqrt{\frac{kT}{m}} \end{aligned}$$

Integrals

$$\begin{aligned} \int \sin x dx &= -\cos x \\ \int \cos x dx &= \sin x \\ \int \sin^2 x dx &= \frac{1}{2}x - \frac{1}{2}\sin 2x \\ \int x \sin^2 x dx &= \frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8} \\ \int x^2 \sin^2 x dx &= \frac{x^3}{6} - \left(\frac{x^2}{4} - \frac{1}{8}\right)\sin 2x - \frac{x \cos 2x}{4} \\ \int e^{-ax} dx &= -\frac{1}{a}e^{-ax} \end{aligned}$$

1. (5 pts) Thomson's "Plum Pudding" model of the atom imagined a "pudding" of positive charge interspersed with "plums" of particulate electrons. It could not explain what experiment done by his old student, Rutherford, and why? A sketch might help.

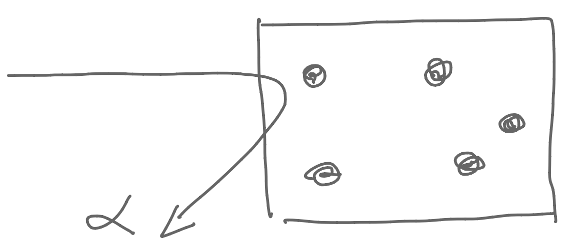
Model presumed that positive charge was distributed in atom. Large angle scattering of α's could not happen from anything but a hard, positive core.



Plum Pudding

dots: electrons

background: uniform + charge



Rutherford

• + nuclei

2. (total for problem: 10 pts) The cosmic ray particle, the "muon" (μ) has a rest energy of 106 MeV. It can be produced in the lab and actually be captured by a proton to form a "muonic atom" in which the μ takes the place of an electron in an otherwise hydrogen-looking atom. So, the bound system is one of proton-muon.

a. (2 pts) Show that the reduced mass of the $\mu - p$ system is 95.2 MeV/c^2.

$$\mu = \frac{mM}{m+M} = \frac{(106)(938)}{106+938}$$

$$\mu = 95.2 \text{ MeV}/c^2$$

b. (5 pt) What is the smallest radius for the "orbiting" muon according to the Bohr model?

$$a_0^\mu = \frac{4\pi\epsilon_0}{e^2}\frac{\hbar^2}{\mu} \times \frac{c^2}{c^2} = \left(\frac{4\pi\epsilon_0}{e^2}\right)\frac{(\hbar c)^2}{\mu c^2}$$

$$a_0^\mu = \left(\frac{1}{1.44\times10^{-9}\text{ eV-m}}\right)\frac{(197\times10^{-9}\text{ eV-m})^2}{(95.2\times10^6\text{ eV})}$$

$$= 2.8\times10^{-13}\text{ m}$$

200x smaller than hydrogen

c. (3 pts) What is the binding energy of the muon-proton system in the lowest Bohr orbit compared to that of "normal" hydrogen atom?

$$E = \frac{e^2}{8\pi\epsilon_0 a_0} = \frac{(1.44\times10^{-9}\text{ eV-m})}{2(2.8\times10^{-13}\text{ m})} = 2540\text{ eV}$$

3. (total for problem: 5 pts) Air is mostly Nitrogen. On a warm summer day, we'll assume that $T = 37°$C. Treating the molecule as a part of an ideal gas leads to a mean molecular speed of $< v >= 484.2$ m/s. Remember that the Nitrogen molecule is diatomic. Assume that the mass of its neutrons is the same as the mass of its protons and there are 7 neutrons and 7 protons in each Nitrogen atom.

a. (3 pts) What is the DeBroglie wavelength of N_2?

$$m(N_2) = 28 \cdot m_p$$

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ \mathrm{J \cdot s}}{(28)(1.67 \times 10^{-27}\ \mathrm{kg})(484.2\ \mathrm{m/s})}$$

$$\lambda \cong 3 \times 10^{-11}\ \mathrm{m}$$

$$= 0.03\ \mathrm{nm}$$

b. (2 pts) Is it's DeBroglie wavelength bigger than or smaller than its own diameter, which is about 1nm?

smaller

4. (total for problem: 10 pts) A wavefunction has the value $\psi = A \sin x$ between 0 and 2π and zero elsewhere.

a. (5 pts) What is the normalization constant, A?

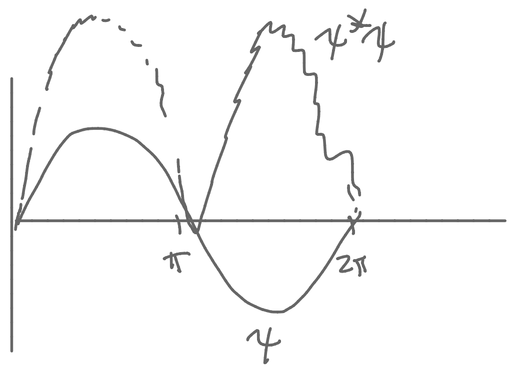
$$1 = \int_0^{2\pi} A^2 \sin^2 x \, dx$$

$$1 = A^2 \left(\tfrac{1}{2}x - \tfrac{1}{2} \sin 2x \right) \Big|_0^{2\pi}$$

$$1 = A^2 \pi$$

$$A = \sqrt{1/\pi}$$

b. (5 pts) Sketch the wavefunction and the probability density on the same graph. don't worry about an absolute vertical scale, but show the relative sizes of the two curves in your sketch.



5. (5 pts) A electron moves with a speed of $v = 10^{-4}c$ inside a one-dimensional, infinite potential well of length 48.5 nm. The potential is zero elsewhere and *the electron may not escape* the box. Treating the electron as nonrelativistic, its kinetic energy is $E = 0.002555$ eV.

What is the approximate quantum number of the electron?

$$E = \frac{1}{2} m v^2 = \frac{1}{2} m \beta^2 c^2 = \frac{1}{2}(0.511 \times 10^6)(10^{-4})^2$$

$$E = 0.0025 \text{ eV}$$

6. (5 pts) A gamma ray of 700 keV energy Compton-scatters from an electron. Find the energy of the scattered photon at 110° and the energy of the scattered electron.

$$\lambda' = \lambda + \frac{h}{m_e c}(1-\cos\theta)$$

$$= \frac{hc}{E} + \frac{h}{m_e c}(1-\cos\theta) \times \frac{c}{c}$$

$$= \frac{1240\ \text{eV.nm}}{700 \times 10^3\ \text{eV}} + \frac{1240\ \text{eV.nm}}{0.511 \times 10^6\ \text{eV}}(1-\cos 110°)$$

$$\lambda' = 0.00177 + 0.0033 = 5 \times 10^{-3}\ \text{nm}$$

$$\lambda' = 5\ \text{pm}$$

The scattered photon energy

$$E' = \frac{hc}{\lambda'} = \frac{1240\ \text{eV.nm}}{5 \times 10^{-3}\ \text{nm}}$$

$$E' = 246\ \text{keV}$$

Scattered electron energy from energy conservation

$$E_\gamma + m_e c^2 = E_\gamma' + E_e'$$

$$E_e' = E_\gamma - E_\gamma' + m_e c^2$$

$$= 700\ \text{keV} - 246\ \text{keV} + 511\ \text{keV}$$

$$E_e' = 965\ \text{keV}$$

If KE

$$E_e' = K_e + m_e c^2$$

$$K_e = E_e' - m_e c^2 = 965 - 511 = 454\ \text{keV}$$

7. (total for problem: 10 pts) The electron in the second excited state ($n = 3$) of a Bohr atom of hydrogen jumps to the ground state.

a. (7 pts) What is the wavelength of the emitted photon in nm if the electron goes directly from $n = 3 \to n = 1$?

$n=3 \to n=1$:

$$\lambda(3\to1) = \left[R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)\right]^{-1}$$

$$= \left[(1.097\times10^{7}\,m^{-1})\left(\frac{1}{1^2} - \frac{1}{3^2}\right)\right]^{-1}$$

$$\lambda(3\to1) = 102.6\,nm$$

a. (3 pts) What other transitions to the ground state could the electron take besides $n = 3 \to n = 1$ and how many photons would be emitted in total for those other transitions?

$n=3 \to n=2$ $\quad 1\gamma$

$n=2 \to n=1$ $\quad 1\gamma$

$\overline{\quad 2\gamma \quad}$