1. (total for the problem: 10 pts) The element scandium has $Z = 21$.

a. (3 pts) I want you to construct the ground state electronic configuration (you know, the $1s^2 2s^2 \ldots$ designation.). The Noble gas that's just before Sc in the Periodic Table is Ar ($1s^2 2s^2 2p^6 3s^2 3p^6$). This is one of those elements for which a high n, S state has lower energy than a lower n, but higher L state. So, the outer shells for Sc, above those of Ar, are $4s^2 3x^n$ What are x and n?

$Z(Ar) = 2 + 2 + 6 + 2 + 6 = 18$

$Z(Sc) = 21 \Rightarrow$ 3 electrons beyond the full Ar shell.

So $3x^n 4s^2 = 3d^1 4s^2$

b. (2 pts) Those values of x and n define the overall angular momentum and spin quantum numbers for Sc (that is, the inner shells contribute 0 to the overall L and S.) Using your values of x and n from a., what are L and S for the Sc ground state?

$l = d \Rightarrow l = 2$

$s = 1/2$

1., cont.

c. (3 pts) There is a spin-orbit interaction. Using your values of x and n from a., what are the possible values of J for Sc?

$J = L \oplus S \Rightarrow$

$L+S,\ L+S-1 \ldots\ L-S$

$2+1/2,\ 2+1/2-1,$

$J = 5/2,\ 2,\ 3/2$

d. (2 pts) Using your values of x and n from a., which is the lowest energy J state for Sc?

$J = 3/2$

2. (10 pts) A gas of atomic hydrogen is at room temperature ($T = 293\ K$). Remember that the ground state energy is $E_G = -13.6$ eV.

a. (2 pts) In the ground state, $n = 1$, the degeneracy factor is $g(E_G) = 2$. Why is that?

$\pm \frac{1}{2}$ spin.

a. (3 pts) In the first excited state state, $n = 2$, the degeneracy factor is $g(E_1) = 8$. Why is that? Your explanation might be in terms of n, ℓ, and/or s.

$2s^2 + 2p^6 = 8$ electronic states

a. (5 pts) What is the relative population of the first excited state for which $E_1 = -3.4$ eV? (Hint: Boltzmann statistics apply.) The answer is a number.

$$\frac{P(E_1)}{P(E_G)} = \frac{g(E_1)\, e^{-E_1/kT}}{g(E_G)\, e^{-E_G/kT}}$$

$$= \frac{g(E_1)}{g(E_G)} e^{-(E_1 - E_G)/kT}$$

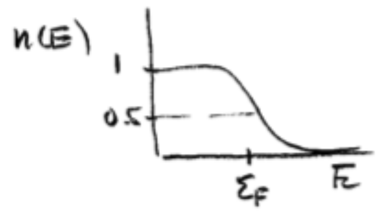
$$= \frac{8}{2} e^{-(-3.4 + 13.6)/kT} = 4 e^{-10.2/kT}$$

$$kT = (8.617 \times 10^{-5}\ \text{eV/K})(293) = 0.0252\ \text{eV}$$

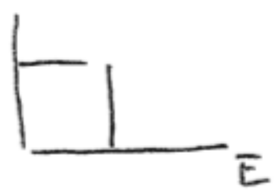
$$= 0.6 \times 10^{-176} \ !$$

3. (total for the problem 10 pts) Consider a system of fermions.

a. (5 pts) Draw the Fermi-Dirac distribution function as a function of energy for a temperature not too far above T=0. Label the vertical and horizontal axes and point out the location of the Fermi Energy, E_F.



b. (3 pts) Draw the FD distribution as a function of energy for a temperature, T=0. Label the vertical and horizontal axes and point out the location of the Fermi Energy, E_F.



c. (2 pts) For this last situation, what is the occupation level of states with energies $E > E_F$?

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4. (total for problem: 10 pts) Plutonium $^{236}_{94}Pu$ is unstable and decays into $^{232}_{92}U$. $M\left(^{236}_{94}Pu\right) = 236.046071\ u$ and $M\left(^{232}_{92}U\right) = 232.037168\ u$

a. (3 pts) Is this an alpha decay? beta decay? or gamma decay? and why? Justify your choice by explicitly counting the numbers of protons and neutrons in each element of the initial and final states of the decay and by conserving electric charge.

$$^{236}_{94}Pu \rightarrow {}^{232}_{92}U + {}^{4}_{2}He \quad \rightarrow \alpha.$$

b. (3 pts) Using the choice you made in a., show that this isotope of plutonium is indeed unstable by calculating its Q value in MeV.

$$Q = (236.046071\,u - 232.037168\,u - 4.00151\,u)(931.5\,\tfrac{MeV}{u})$$

$$Q = 6.89\ MeV > 0 \quad \checkmark$$

c. (4 pts) Calculate the binding energy per nucleon in MeV for the plutonium isotope.

$$B = (Z m_p + N m_n - M_{Pu})c^2$$

$$= \left((94)(1.007277\,u) + (142)(1.008665\,u) - 236.046071\,u\right)c^2$$

$$= 1.868\,u$$

$$= 1740.4\ MeV.$$

$$B/A = 7.4\ MeV.$$

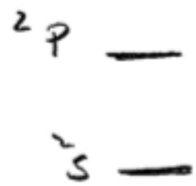
You may choose to do one or both of the next pair of problems for extra credit. In order to qualify, you must have made an attempt on all of the previous problems.

5. (total for problem: 10 pts) Let's walk through the level splitting for Sodium, starting with the simplest situation of no interaction between the electron spins and the orbital angular momentum.

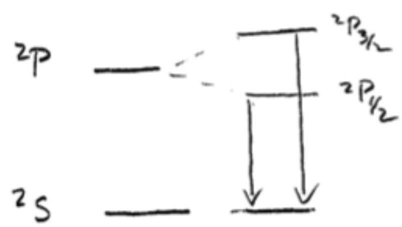
a. (2 pts) Na has $Z = 11$. First, what is the electronic configuration (the $1s^2 2s^2 \dots$ designation)?

$1s^2 2s^2 2p^6 3s^1$

b. (2 pts) Draw the level diagram for the ^2P and ^2S levels (no external magnetic field, no spin-orbit coupling).



c. (2 pts) Now, redraw the diagram from b. and add the spin-orbit splitting for $B_{\text{external}} = 0$. Label the levels properly as ^nX_j Draw the allowed transitions.



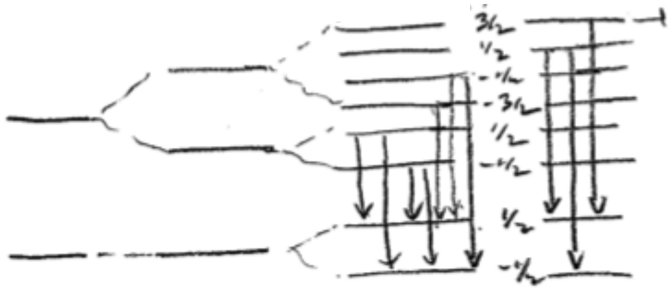
2 => doublet => s = 1/2 => j = 1+1/2, 1-1/2

2P <

9

5., cont.

d. (2 pts) Now, redraw the diagram from c. and add the level splittings for the inclusion of an external magnetic field, $B_{\text{external}} \neq 0$.



splits according to m_j

$^2P_{3/2}$, $m_j = 3/2, 1/2, -1/2, -3/2$

$^2P_{1/2}$ $m_j = 1/2, -1/2$

$^2S_{1/2}$ $1/2, -1/2$

$\Delta m_j = \pm 1, 0$ $\Delta \ell = 1$

e. (2 pts) How many allowed transitions are there for the situation in d.?

10

6. (total for problem: 10 pts) The half-life of ^{22}Na is 2.6 years.

a. (3 pts) What is the decay constant in years?

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{2.6\,y} = 0.27\,y^{-1}$$

b. (7 pts) How much time in years will be required for a 5 mg sample of ^{22}Na to be reduced to 1 mg?

$$N = N_0 e^{-\lambda t}$$

$$N/N_0 = e^{-\lambda t}$$

$$\ln(N/N_0) = -\lambda t$$

$$t = -\frac{\ln(N/N_0)}{\lambda} = +\frac{\ln(N_0/N)}{\lambda}$$

$$t = \frac{\ln 5/1}{0.27\,y^{-1}} = 6.04\,y$$