9. Quantum Statistics, 3.5

12. Atomic Nucleus, 1

lecture 34, November 17, 2017

housekeeping

Coming attractions

Next week:

lecture M and T

chapter 12 homework due 11/22…HW workshop 11/21

Wed before Thanksgiving is a lost cause, so the schedule always indicated no class

End game:

watch the schedule link as I'll do some adjustment

exam #3 is Friday, December 1

I've not given any quizzes…have you noticed? I'll add that percentage to the homework fraction

today

statistical physics - Bose Condensation

Atomic nucleus - introduction

STILL MORE BOSONS   Helium

Consider now a gas of bosons → not spin 1, so 1 d.o.f.

$$g(E)\,dE = \frac{8\sqrt{2}\,\pi m^{3/2}}{h^3} E^{1/2}\,dE$$

... but take back that factor of 2

$\downarrow$

$$g_B(E)\,dE = \frac{4\sqrt{2}}{h^3}\pi\, m_B^{3/2}\, E^{1/2}\,dE$$

$$n_B(E)\,dE = g_B(E)\,F_{BE}(E)\,dE \qquad F_{BE}(E) = \frac{1}{B e^{E/kT} - 1}$$

$$B \leftarrow e^{-\mu/kT}$$

$$N = \int_0^\infty n(E)\,dE = \frac{V \cdot 4\sqrt{2}\,\pi\, m_B^{3/2}}{h^3} \int_0^\infty \frac{E^{1/2}\,dE}{e^{(E-\mu)/kT} - 1}$$

$$N = \int_0^\infty n(E)\,dE = V \cdot \frac{4\sqrt{2}\,\pi\, m_B^{3/2}}{h^3} \int_0^\infty \frac{E^{1/2}\,dE}{e^{(E-\mu)/kT} - 1}$$

N can't be negative!

$e^{(E-\mu)/kT}$ must$_{\mu}$ be > 0 -- E is > 0 ... so $(T) < 0$ $\exists\, T$

change variables $x = E/kT$ & let $T \downarrow$

$$N = V \cdot \frac{4\sqrt{2}}{h^3}\,\pi\, m^{3/2}\, k^{3/2}\, T^{3/2} \int_0^\infty \frac{x^{1/2}}{e^{x-\mu/kT} - 1}\,dx$$

constant as $T \downarrow$

goes $\downarrow$ as $T \downarrow$

must $\uparrow$ as $T \downarrow$ $\Rightarrow$ $|\mu(T)| \downarrow$

$\Rightarrow$ $\mu(T) \longrightarrow 0$ as $T \downarrow$

But: problem. At some point

$\mu(T) = 0 \longrightarrow$ at $T_c \longleftarrow$ "critical"

$$N = V \cdot \frac{4\sqrt{2}\,\pi m^{3/2} k^{3/2} T^{3/2}}{h^3} \int_0^\infty \frac{x^{1/2}}{e^{x-\mu/kT} - 1}\,dx$$

but: $\displaystyle\int_0^\infty \frac{x^{1/2}\,dx}{e^x - 1} = 2.315$ ... ie, when $\mu(T_c) = 0$

$$N = V \cdot \frac{4\sqrt{2}\,\pi m^{3/2} k^{3/2} T^{3/2}}{h^3} (2.315)$$

can't compensate to keep N constant

Problem:

can't continue $T \downarrow$

Below T_c? Trouble. Einstein to the rescue in 1924

Einstein's way out: separate out the ground state, $E = 0$

$$N = \int_0^\infty n(E)\,dE = \frac{V \cdot 4\sqrt{2}\,\pi\, m_B^{3/2}}{h^3} \int_0^\infty \frac{E^{1/2}\,dE}{e^{(E-\mu)/kT} - 1}$$

only applies for particles not in g.s.: $E > 0$, not $E = 0$

Below T_c ...

$$N = N_0 + N_n$$

normal LHe: He_I

ground state of He_{II}

as $T \downarrow$ ... N_n is depleted and N_0 increases.

$$T < T_c < T < T_v = 4.2\,K$$

He_{II} — He_I a liquid satisfying $N(T)$

something different

phase transition

$$N = V \cdot \frac{4\sqrt{2}\,\pi m^{3/2}}{h^3} k^{3/2} T_c^{3/2} (2.315)$$

$\leftarrow$ can solve for T_c

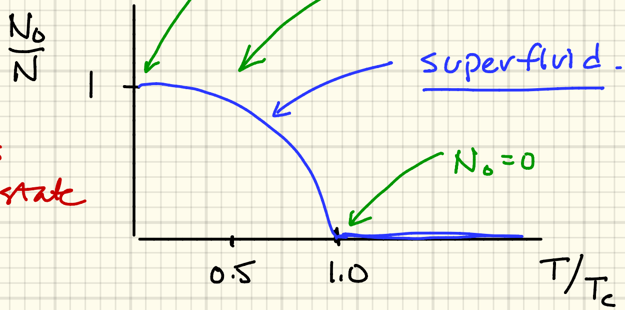
$$m = m_{He} = 2m_p + 2m_n$$

$$T_c = \frac{h^2}{2m_{He}k}\left[\frac{N}{V}\frac{1}{2\pi(2.315)}\right]^{2/3} \sim 3\,K \quad (\text{actually } 2.17\,K)$$

$N_n = 0$

Two kinds of liquid He

$N = N_0 + N_n$



superfluid ... ground state fills with everything.

every atom is in the ground state

$N_0 = 0$

"Bose Einstein Condensation"

ground state ... all atoms, single ψ

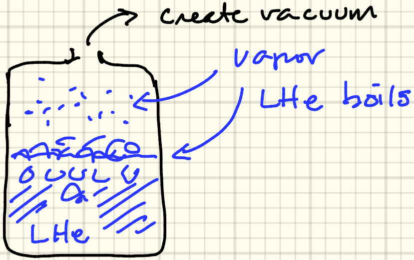
MACROSCOPIC QUANTUM SYSTEM ... zero viscosity.

Liquid Helium condensate is... unusual.

He atoms interact with one another -- closely spaced

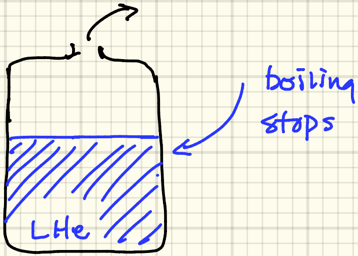
$T > T_c$

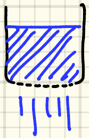
$T < 4.2\,K$



porous -- viscosity of LHe prevents leakage

$T \leq T_c$

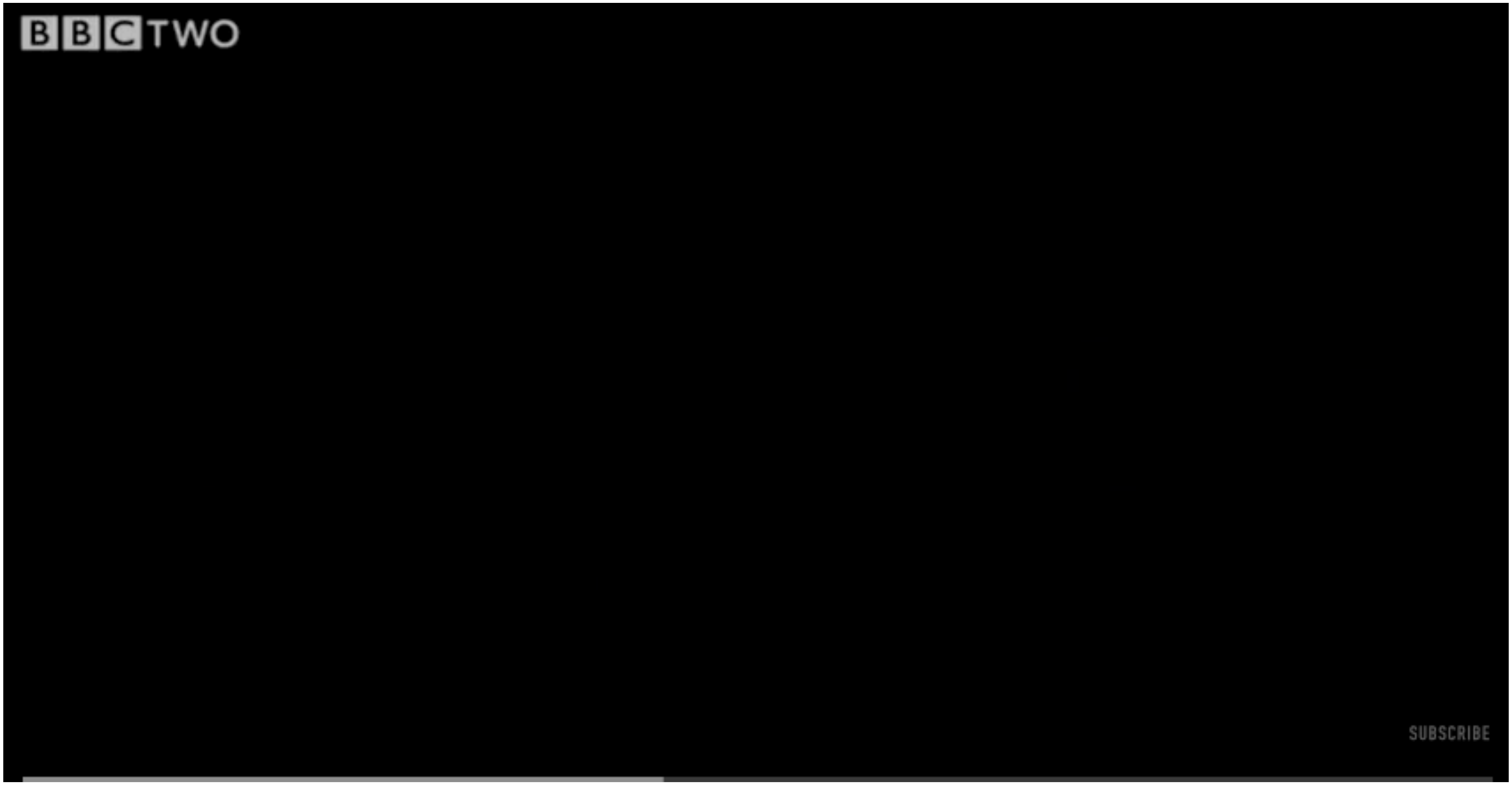




viscosity = 0 -- leaks

MOVIE $\longrightarrow$

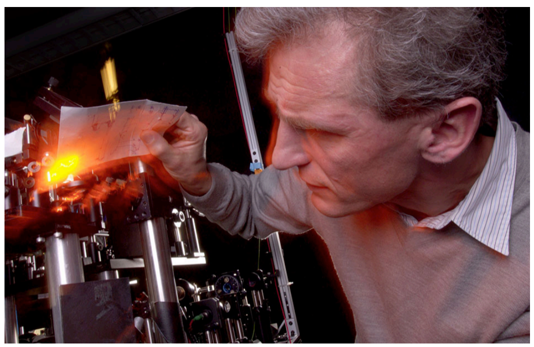
liquid Helium

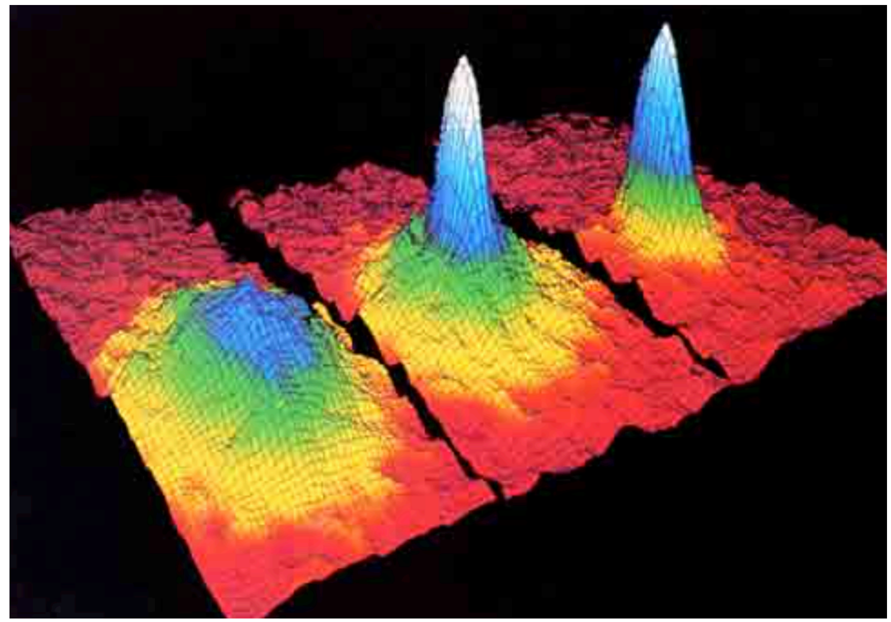


https://www.youtube.com/watch?v=9FudzqfpLLs

BEC - 1995

Rb atoms…cooled to 50nK





nuclear physics hits:

1896	radioactivity discovered, Becquerel
1898	Radium/Polonium, Curies
1899	alpha/beta rays, Rutherford
1902	natural "transmutation," Rutherford and Soddy
1909	alpha rays = helium nuclei, Rutherford and Royds
1911	nucleus of + charge, Rutherford, Marsden, Geiger
1920	proton discovered, Rutherford
1921	neutron predicted, Rutherford
1921	laboratory transmutation, Rutherford and Chadwick
1929	proton accelerator, (Rutherford) Cockcroft and Walton
1930	neutrino predicted, Pauli
1931	Deuterium discovered, Urey
1932	neutron discovered, (Rutherford) Chadwick
1933	proton-neutron model of nucleus, Heisenberg
1933	artificial radioactivity, I. Curie and Joliot
1934	pion predicted, Yukawa
1935	neutrino model, Fermi
1938	nuclear fission, Hahn and Strassmann
1942	controlled nuclear fusion, Fermi
1947	pion and muon discovered, Occhialini and Powell

NUCLEAR PHYSICS

Just the facts, ma'am:

Nuclei → neutrons & protons (to first approximation)

"protons" discovered and named by -- Rutherford, of course

"neutrons" predicted by and discovered in the lab of ... Rutherford ... of course

Notation

Z "Atomic number" = # protons in nucleus

N "neutron number" = ... # of neutrons

A "mass number" = $Z+N$

Symbol $^{A}_{Z}X_N$ ... often $^{A}X_N$

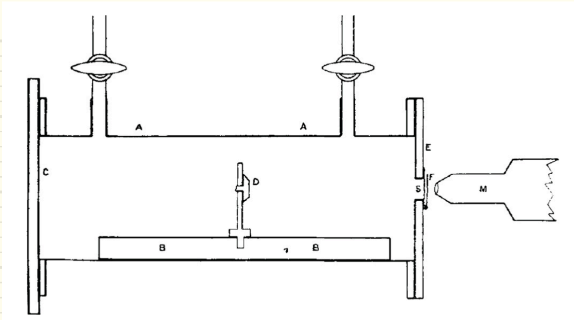
eg: Nitrogen: $^{14}N_7$, Oxygen $^{17}O_8$, α $^{4}\alpha_2$ or $^{4}He_2$

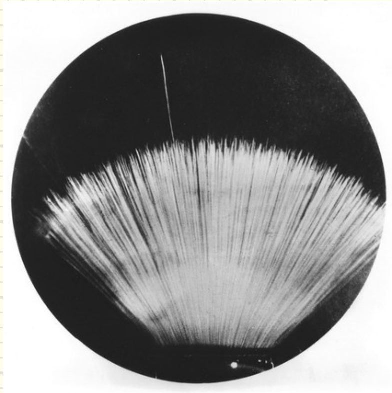
Rutherford's Discovery measurement

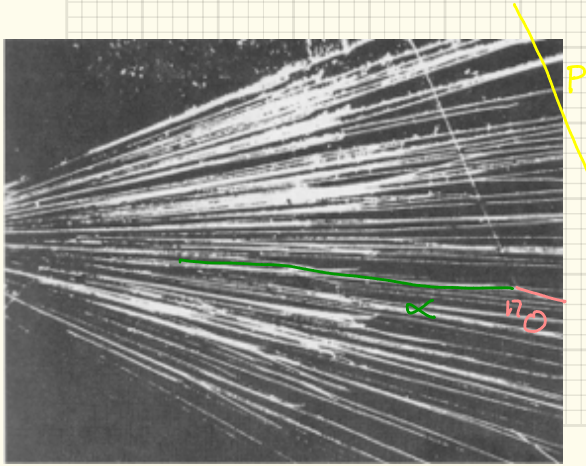
$$^{14}N_7 + {}^{4}\alpha_2 \longrightarrow {}^{1}H_1 + {}^{17}O_8$$

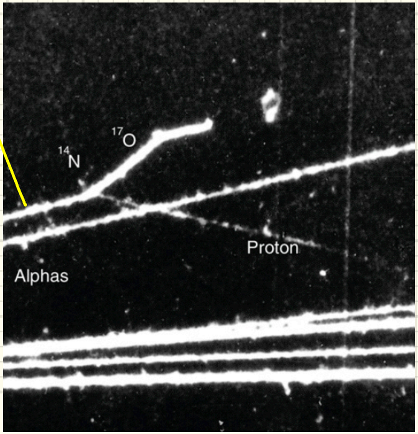
or: $\quad {}^{14}N_7\,(\alpha, p)\,{}^{17}O_8$

spoke of "H atoms" later named
"proton" => "first one"
speculated that a neutral partner existed









Neutron – 1932

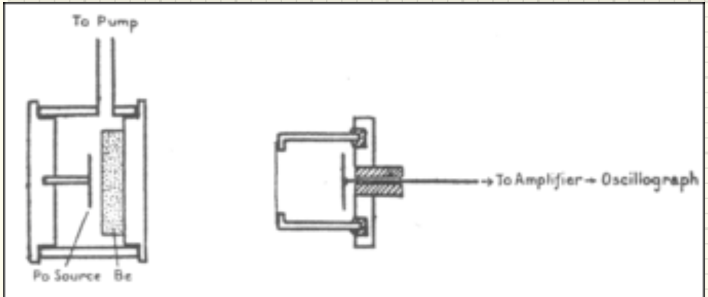
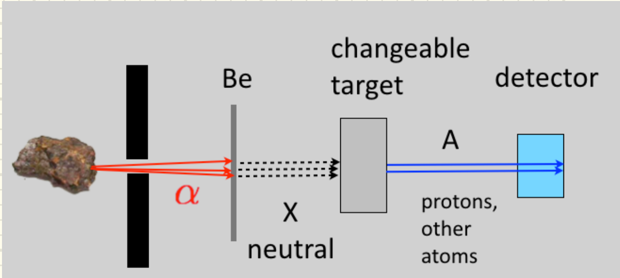


Fig. 1.



Chadwick compared

$X + T \rightarrow X + T$

$X + N \rightarrow X + N$

$X + p \rightarrow X + p$

$\left.\right\}$ $m_T = 14$ and 1

max recoil velocity

$$v_T = \frac{2m_x}{m_x + m_T}$$

v_{x_0}? so ratios:

$$\frac{v_H}{v_N} = \frac{m_x + 14}{m_x + 1} = \frac{33 \times 10^6 \text{ m/s}}{4.7 \times 10^6 \text{ m/s}} \Rightarrow m_x \simeq 1.1 \; (\times m_p)$$

actually found:

$m_n = 938 \pm 1.8$ MeV/c^2

then

$m_n = 939.57$ MeV/c^2

Now:

used:

$m_p = 938.27\ MeV/c^2$ $\quad {}^1_1p_1 \quad = 1.007276\,u$

$m_n = 939.57\ MeV/c^2$ * $\quad {}^1_0n_1 \quad = 1.008665\,u$

$m_e = 0.511\ MeV/c^2$ $\quad {}^0_{-1}e$

but almost never.

Generically: "nucleon" = proton or neutron

"A" is the # of nucleons in a nucleus

Mass convention:

unified mass unit, u or "Atomic Mass Unit"

defined for $^{12}C = 12u$ exactly. $\Rightarrow 1u = 1.660559 \times 10^{-27}\ kg$

$= 931.5\ kg.$

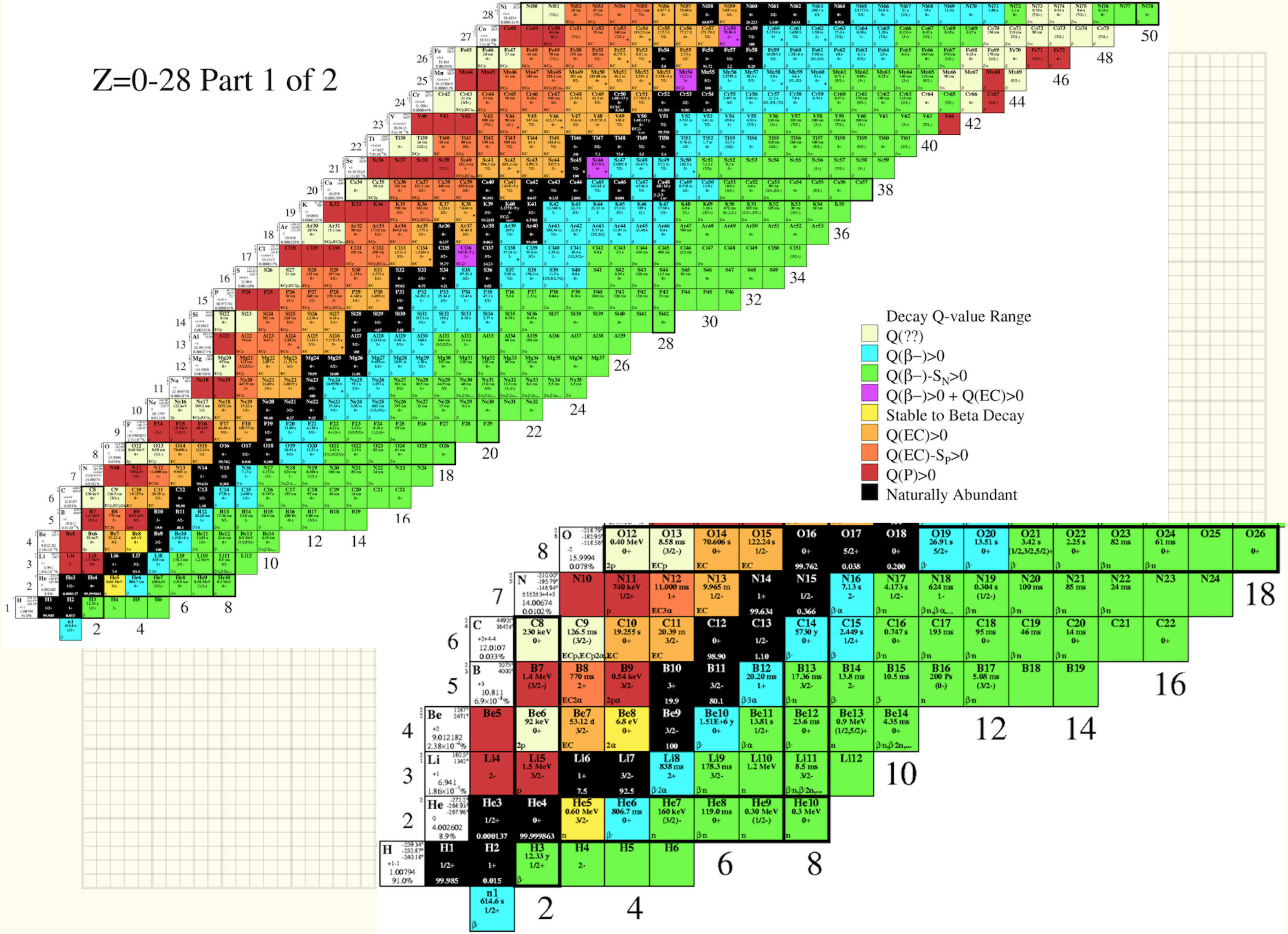
* took a while

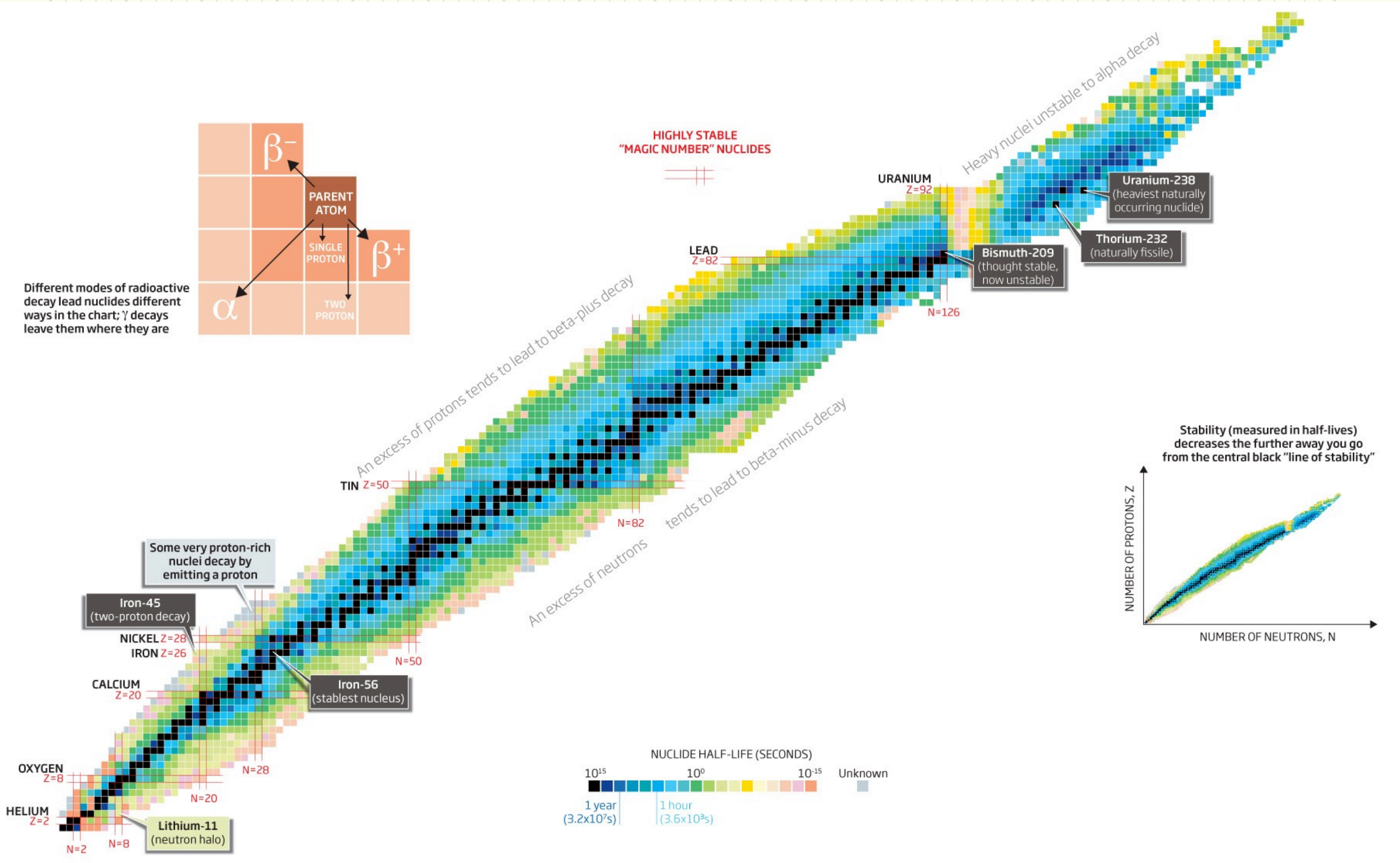
Isotope : elements with different # neutrons & same # protons
(um... same element, huh)

Natural abundances vary

Carbon	${}^{11}_{6}C_5$	($= {}^{11}C = {}^{11}C_6$)	trace %
	${}^{12}_{6}C_6$	98.6%	
	${}^{13}C_7$	1.1 %	
	${}^{14}C_8$	trace %	

Z=0-28 Part 1 of 2

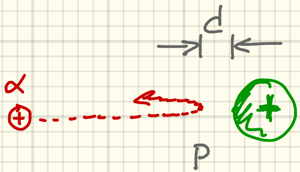




Different modes of radioactive decay lead nuclides different ways in the chart; γ decays leave them where they are
β−
PARENT ATOM
SINGLE PROTON
β+
α
TWO PROTON
HIGHLY STABLE "MAGIC NUMBER" NUCLIDES
Heavy nuclei unstable to alpha decay
URANIUM Z=92
Uranium-238 (heaviest naturally occurring nuclide)
Thorium-232 (naturally fissile)
LEAD Z=82
Bismuth-209 (thought stable, now unstable)
N=126
An excess of protons tends to lead to beta-plus decay
An excess of neutrons tends to lead to beta-minus decay
TIN Z=50
N=82
Some very proton-rich nuclei decay by emitting a proton
Iron-45 (two-proton decay)
NICKEL Z=28
IRON Z=26
N=50
CALCIUM Z=20
Iron-56 (stablest nucleus)
N=28
OXYGEN Z=8
N=20
HELIUM Z=2
N=8
N=2
Lithium-11 (neutron halo)
Stability (measured in half-lives) decreases the further away you go from the central black "line of stability"
NUMBER OF PROTONS, Z
NUMBER OF NEUTRONS, N
NUCLIDE HALF-LIFE (SECONDS)
10^{15}
10^{0}
10^{-15}
Unknown
1 year (3.2×10^7s)
1 hour (3.6×10^3s)

SIZES AND SHAPES OF NUCLEI

Started with Rutherford & his model



@ turning point, P , $KE(\alpha) \longrightarrow PE(\vec{E})$

$$\frac{1}{2} m v^2 = \frac{1}{4\pi\epsilon_0} \frac{Q_\alpha Q_{Au}}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2 Z_\alpha Z_{Au}}{d}$$

$$d = \frac{4}{4\pi\epsilon_0} \frac{Z_{Au} e^2}{m v^2} \sim 3.2 \times 10^{-14} \text{ m}$$

a maximum size for Au nucleus

For Ag ... he found $d \sim 2 \times 10^{-14}$ m

New unit: femtometer (fm)

$$1 \text{ fm} = 10^{-15} \text{ m}$$

$R = r_0 A^{1/3}$ — Emperical

also called "Fermi".. fm!

$r_0 = 1.2 \times 10^{-15} \text{ m} = 1.2 \text{ fm}$

NUCLEAR DENSITIES

Assume spherical .. nuclear matter densities:

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_0^3 A \qquad \text{so} \quad V \propto A$$

$$m_N = m_p \simeq m_n$$

$$\rho = \frac{M}{V} = \frac{A m_N}{\frac{4}{3}\pi r_0^3 A} = \frac{3 m_N}{4\pi r_0^3} \sim 2.3 \times 10^{17}\ \mathrm{kg/m^3}$$

$$2 \times 10^{14} \times \rho(\text{water})$$

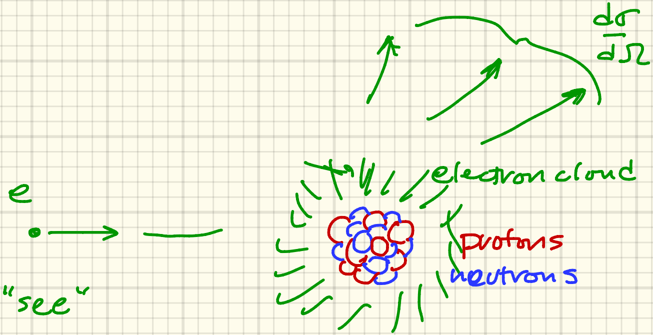
not bad! density of neutron star $\sim 10^{8}\ \mathrm{kg/m^3}$

Sophistication of electron beam production

→ Long program at Stanford "size" is not a clean and distinct concept.

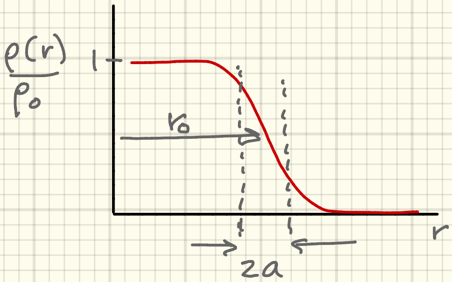
would "map" the charge distribution of nuclei.

$\frac{d\sigma}{d\Omega}$ can be converted into proton charge distribution



doesn't "see" atomic electrons

$$\rho(r) = \frac{\rho_0}{\left(1 + e^{r - r_0/a}\right)}$$



$r_0 \simeq 1.07\, A^{1/3}$ fm

$2a \simeq 1$ fm

pretty good for $Z > 20$

Looks like F_{FD} doesn't it

Electron scattering at Stanford 1954 - 57

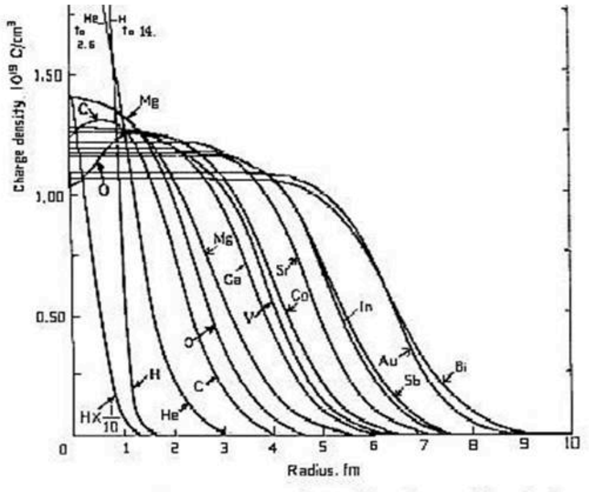
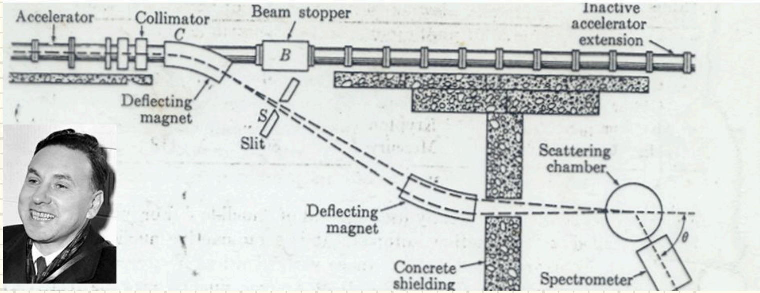
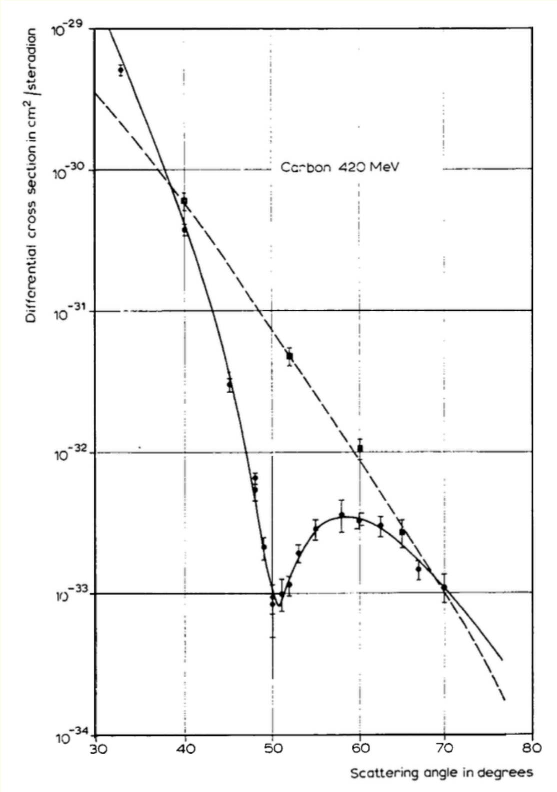


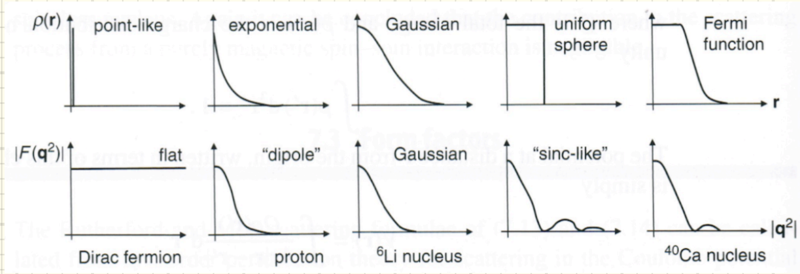
Figure 3. Charge distributions for various nuclei as determined from electron-scattering experiments. [From R. Hofstadter, Ann. Rev. Nucl. Sci. 7, 231 (1957).]





↑ electron diffraction

θ_1



$\sin\theta_1 \simeq \dfrac{1.22\,\lambda}{D}$ ← here, λ_e

← diameter

$D = \dfrac{1.22\,\lambda_e}{\sin\theta_1}$ for 450 MeV e's

$\lambda_e \simeq 3\times10^{-15}$ m

$D = \dfrac{(1.22)(3\times10^{-15})}{\sin 50^\circ} = 4.8\times10^{-15}$ m

so $r_c = 2.4\times10^{-15}$ m

$= 2.4$ fm