# 12. Atomic Nucleus, 2

lecture 35, November 20, 2017

# housekeeping

## Coming attractions

### Next week:

*lecture today and tomorrow*

*chapter 12 homework due Wed 11/29…HW workshop Tue 11/28*

*no class day after tomorrow*

### End game:

*I've made some adjustments to the schedule…stay tuned, now, week by week*

*exam #3 is Friday, December 1*

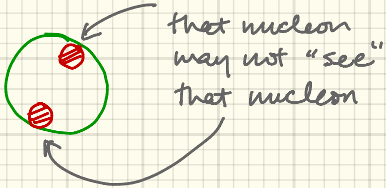
*I've not given any quizzes…have you noticed? I'll add that percentage to the homework fraction*

# today

Atomic nucleus - continuing

Notice: $R \propto A^{1/3}$ , $\rho$ independent of $A$.

$\Rightarrow$ as $A$ goes $\uparrow$ , $\rho$ doesn't $\Rightarrow$ nuclear force short ranged.




Forces of attraction $\left.\begin{array}{l} n\text{-}n \\ n\text{-}p \\ p\text{-}p \end{array}\right\}$ all about the same & very compact.

$\Rightarrow$ hard to remove N

## Spin

Nuclei have spin, "I" bosons or fermions

$\downarrow$ ${}^4He$ $\downarrow$ ${}^3He$

As usual:

$$|\vec{I}| = \hbar\sqrt{I(I+1)} \Rightarrow \text{magnetic moments also.}$$

New unit: "nuclear magneton"

$$\mu_N \equiv \frac{e\hbar}{2m_p} = 5.05 \times 10^{-27}\ J/T \qquad \ll \mu_B$$

$$\mu_p = 2.7928\ \mu_N$$

$$\mu_n = -1.9135\ \mu_N\ ! \qquad \Rightarrow \text{structure}$$

Apply a field...

get level splitting & $\mu$ precession.

for $I = 1/2$ nucleus in magnetic field $B$


Boltzmann: $$\frac{N_{up}}{N_{dn}} = e^{-(E_{up} - E_{dn})/kT}$$

and this population difference can be exploited:

"NMR" ... now called "MRI"

Normal

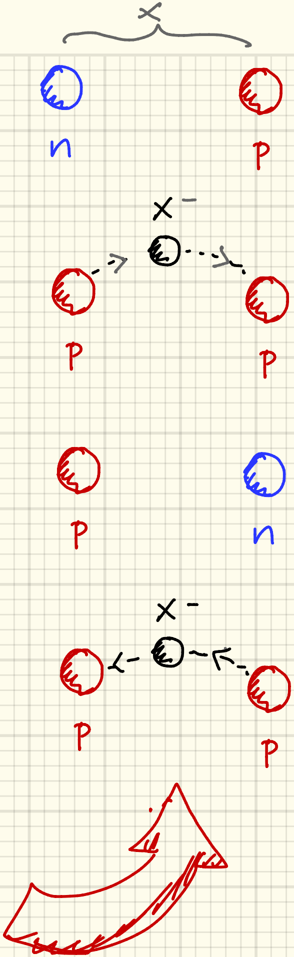
Acute tear

# Nuclear Forces

strong $\quad p \rightarrow \leftarrow p$

!

what does this?




quantum x spontaneously handed off...

How?

Uncertainty.

$$\Delta E \, \Delta t \sim \hbar$$

too short to be observed.

$\Rightarrow$ energy violation

"Exchange Force"

original idea of Heisenberg

$$\Delta E = m_x c^2$$

$$\Delta E \, \Delta t = \hbar$$

$$\Delta t = \frac{\hbar}{m_x c^2} \Rightarrow \text{shortest time}, \quad \text{fastest speed} = c$$

$$x = c\,\Delta t$$

$$x = c \, \frac{\hbar}{m_x c^2}$$

$$m_x c^2 = \frac{\hbar c}{x} \qquad x \sim 1 \text{ fm}.$$

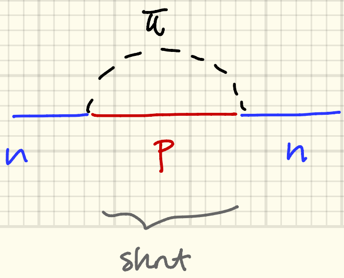
$$m_x c^2 = \frac{\hbar c}{10^{-15}\,\text{m}} = \frac{197.3 \text{ eV-nm}}{10^{-6}\,\text{nm}}$$

$$\simeq 200 \text{ MeV}$$

Predicted in 1935 by Hideki Yukawa

He called it Y, I called it X,

now called pion, $\pi$



Remember "binding energy" for atoms?

$$m_e c^2 + m_p c^2 = m_H c^2 + 13.6\,\text{eV} \quad ?$$

$$m_e c^2 + m_p c^2 - m_H c^2 = B$$

to liberate e & p, must supply B

Ditto for nuclei, but more.

Simplest compound nucleus, $^2D$

$$m_n = 1.008665\,u$$

$M(^1H) = 1.007825\,u$ hydrogen p

$M(^2H) = 2.014102\,u$ deuterium pn

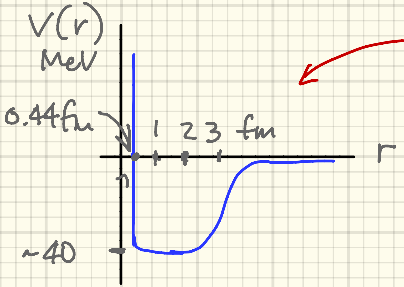
$$m_n + M(^1H) - M(^2H) = 0.002388\,u = B/c^2$$

$$= (0.002388)\left(\frac{931.5\,\text{MeV}/c^2}{u}\right)$$

$$B = 2.224\,\text{MeV}$$

very loosely bound deuterium

# How to make a Deuteron




p n

get 'em close &
they stitch
&
hand-off pions

But $^2H$ is wimpy

# HOW STRONG CAN YOU GO?

$$B = m_n c^2 + m_p c^2 - m_D c^2$$

( aside... $m_{atom} c^2 = m_{nucleus} c^2 + Z m_e c^2 +$ electromagnetic binding

$\downarrow$ $\sim 10^9 - 10^{11}$ eV  $\downarrow$ $10^6 - 10^8$ eV  $\downarrow$ $10 - 10^5$ eV

nuclear mass of hydrogen: $m_H - m_e$

nuclear mass of deuterium: $m_D - m_e$ )

$$B = m_n c^2 + [m(^1H) \underline{-m_e}]c^2 - [m(^2H) \underline{-m_e}]c^2$$

cancel

$$B = [m_n + m(^1H) - m(^2H)]c^2$$

cancelation of e's always

So for ${}^A_Z X_N$: $B = [N m_n + Z m({}^1_1H_0) - m({}^A_Z X_N)]c^2$

These binding energies per nucleon vary.

$^9_4Be_5$ $\quad m(^9Be) = 9.0121\,u$ -- from $\quad B = [Nm_n + Zm(^1_1H_0) - m(^A_ZX_N)]c^2$

$$B = [5(1.008665u) + (4)(1.007276u) - 9.0121u]c^2 \; 931.5 \frac{MeV}{c^2 \cdot u}$$

$$B = (0.060329)(931.5\,MeV) = 56.196\,MeV$$

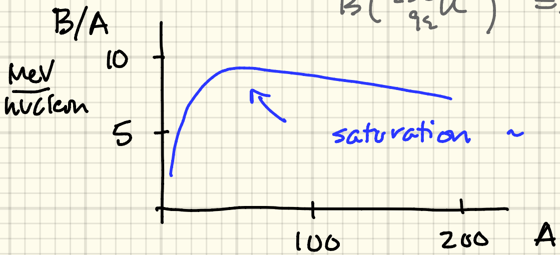
( $^2H$ was ~ 2 MeV... so 1 MeV/nucleon...

Be is ≈ 6 MeV/nucleon, 6.24 MeV/A ) ↑ as A ↑

Likewise $\quad B(^{56}_{26}Fe) \Rightarrow B/A = 8.791$ MeV/nucleon

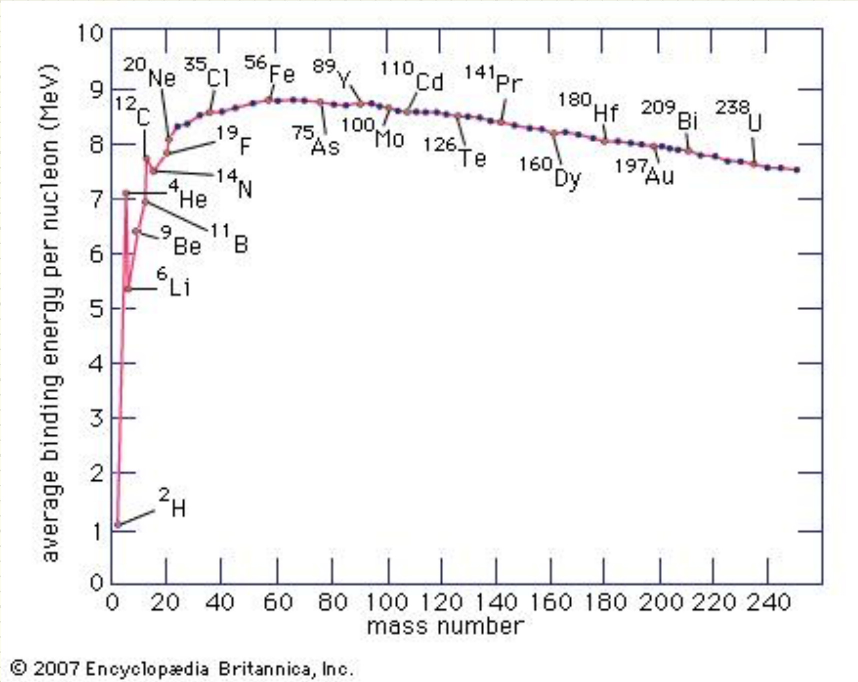
$B(^{238}_{92}U) \Rightarrow B/A = 7.571$ MeV/nucleon

↓ as A ↑




saturation ~ 8 MeV/nucleon

Iron – most tightly bound




Models need to explain this

# Nuclear Models

very much a phenomenological exercise

Historically ... roughly two broad classes:

Liquid Drop Model (Bohr 1936)

Independent Particle Model aka Shell Model (Wigner, Jensen, Meyer ~ 1948)

→ amusingly Aage Bohr received the Nobel Prize for working out some reconciliation between the models ~ 1952

then ... his Ph.D. 1954

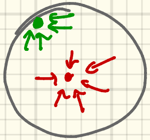
Liquid Drop Model — 3 pieces of evidence suggestive

1. Volume effect

since $\bar{B}/A \sim$ constant $\quad B \sim A \propto V$

2. Surface effect

nuclei near surface will reduce overall binding



3. Coulomb repulsion

total Coulomb energy $\longrightarrow$ work required to assemble Z protons from $\infty$ to the volume

$$\propto \frac{Z(Z-1)}{A^{1/3}}$$

→ "semi-empirical binding formula"

$$B\left({}^{A}_{Z}X_N\right) = a_V A - a_A A^{2/3} - \frac{3}{5}\frac{Z(Z-1)}{4\pi\epsilon_0 r}e^2 - a_s\frac{(N-Z)^2}{A} + \delta$$

~14 MeV "volume"

~13 MeV "surface"

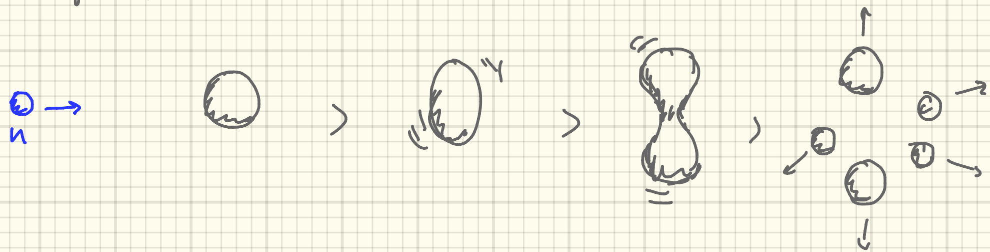
~19 MeV "symmetry"

+Δ even-even
0 even-odd
-Δ odd-odd
$\Delta = 33\text{ MeV } A^{-3/4}$
"pairing"

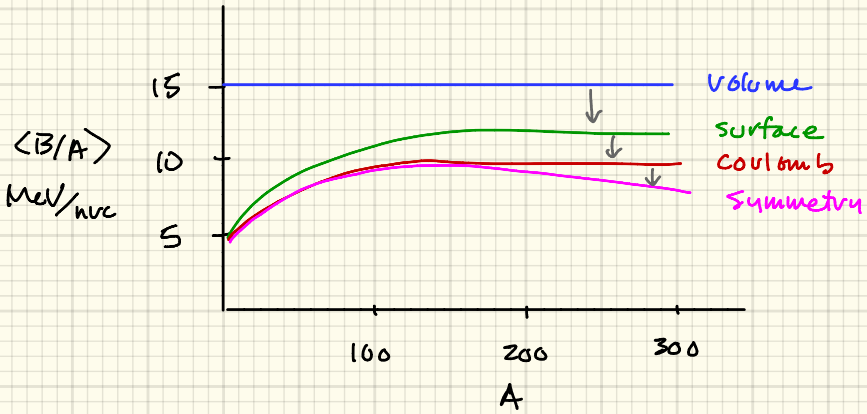
⇒ from fits to data

~ $A \gtrsim 15$

sort of helps to understand fission



Sorta goes like this



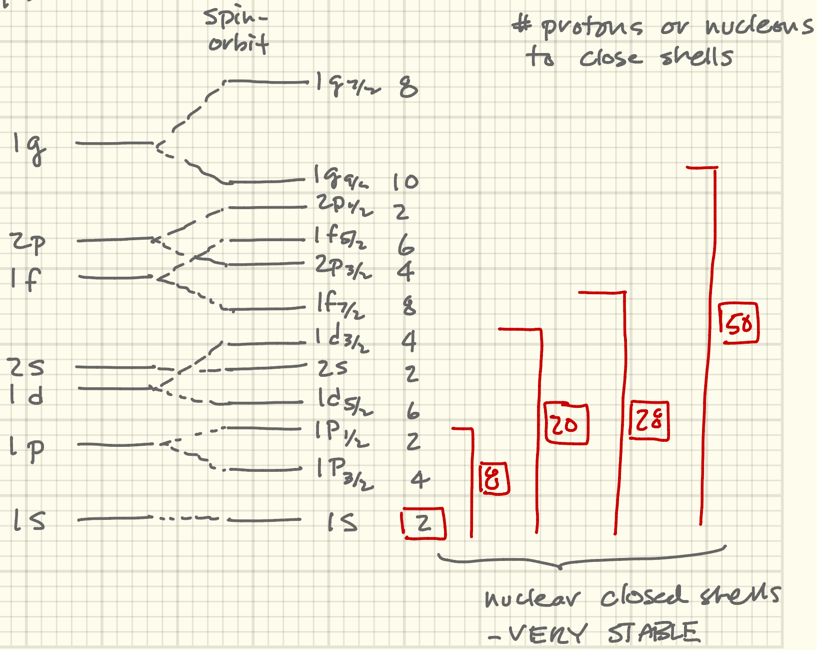
# Shell Model

nucleons in a well-defined quantum-mechanical quantized! orbit

– each nucleon in an average potential created by others.

... like a Fermi gas

A square-well



nber of identical nucleons in a state $j$
a total spin $j$ and a magnetic moment
single particle in that state.
nucleus the "pairing energy" of the
ame orbit is greater for orbits with

ption leads to the prediction that the
ppears less often as the spin of odd
nergy order of Table II predicts. For
$_{1/2}$ level has slightly lower energy than
iring energy of $h_{11/2}$ exceeds that of $s_{1/2}$
s difference, the spin 11/2 would not
ei, but 1/2 would be observed instead.
heoretical justification for assumptions
his will be discussed in the next paper.
as the consequence that all even-even
zero. The main testing ground for the
consists then in the spins and mag-
the nuclei of odd $A$. According to the
vill adopt for these nuclei the extreme
re, ascribing both spin and magnetic
st odd proton or neutron.

C MOMENTS OF ODD A NUCLEI

3 were exactly correct, the magnetic
dd nuclei could be computed by the
m the known gyromagnetic ratios of
ron. The two possible cases, $l=j-\frac{1}{2}$
given $j$ value lead to two computed
f magnetic moment $\mu$ against $j$ for
neutron number and two (different)
ith odd proton number. These theo-
e referred to as "Schmidt lines."[5] The
es lie in between the Schmidt lines,
ide with them. For each $j$ value the
ts seem to fall into two groups, one
to the line corresponding to $l=j+\frac{1}{2}$,
d from near the line corresponding to
halfway. It turns out that the assign-
de attributes to the first group an odd
$l=j+\frac{1}{2}$, to the second one $l=j-\frac{1}{2}$. In
on $l$-values as derived from magnetic

TABLE II. Order of energy levels obtained from those of a square well potential by spin-orbit coupling.

| Osc. no. | Square well | Spin term | No. of states | Shells | Total no. |
|---|---|---|---|---|---|
| 0 | 1s | $1s_{1/2}$ | 2 | 2 | 2 |
| 1 | 1p | $1p_{3/2}$ | 4 | | |
| | | $1p_{1/2}$ | 2 | 6 | 8 |
| 2 | 1d | $1d_{5/2}$ | 6 | | |
| | | $1d_{3/2}$ | 4 | 12 | |
| | 2s | $2s_{1/2}$ | 2 | | 20 |
| 3 | 1f | $1f_{7/2}$ | 8 | 8 | 28 |
| | | $1f_{5/2}$ | 6 | | |
| | 2p | $2p_{3/2}$ | 4 | 22 | |
| | | $2p_{1/2}$ | 2 | | |
| | | $1g_{9/2}$ | 10 | | 50 |
| 4 | 1g | $1g_{7/2}$ | 8 | | |
| | 2d | $2d_{5/2}$ | 6 | | |
| | | $2d_{3/2}$ | 4 | 32 | |
| | 3s | $3s_{1/2}$ | 2 | | |
| | | $1h_{11/2}$ | 12 | | 82 |
| 5 | 1h | $1h_{9/2}$ | 10 | | |
| | 2f | $2f_{7/2}$ | 8 | | |
| | | $2f_{5/2}$ | 6 | | |
| | 3p | $3p_{3/2}$ | 4 | 44 | |
| | | $3p_{1/2}$ | 2 | | 126 |
| | | $1i_{13/2}$ | 14 | | |
| 6 | 1i, 2g, 3d, 4s | $1i_{11/2}$ | | | |

Maria Goeppert Mayer
1906 - 1972

# Shell Model & Stability

it makes some sense

| Nucleus Z - N | # stable nuclei | # very long-lived nuclei |
| --- | --- | --- |
| even-even | 155 | 11 |
| even-odd | 53 | 3 |
| odd-even | 50 | 3 |
| odd-odd | 4 | 5 |

Want to be really stable? Be a nucleus with an even # p & even # n

why? Pauli exclusion.