12. Atomic Nucleus, 3

lecture 36, November 21, 2017

housekeeping

Coming attractions

This week:

chapter 12 homework due Wed 11/29…HW workshop Tue 11/28

no class day after tomorrow

End game:

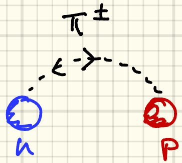
I've made some adjustments to the schedule…stay tuned, now, week by week

exam #3 is Friday, December 1

today

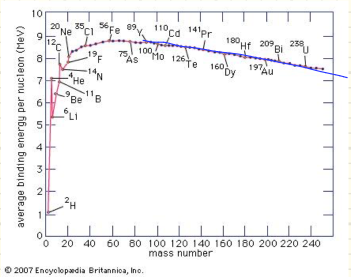
Atomic nucleus - continuing

Where we were:



- ✓ strong force as example of "Exchange Force"
  - → predict pion with $m \sim \underline{\underline{200}}\ MeV/c^2$
  - (found in CRs in 1949 w mass $\underline{\underline{139}}\ MeV/c^2$)

- ✓ nuclear binding



nuclear models

liquid drop & shell model

Radioactivity

What's conserved while stuff is falling apart?

✓ 1. nucleon #, A

✓ 2. electric charge, net

✓ 3. energy

✓ 4. momentum

✓ 5 angular momentum

- 3 kinds of nuclear decay:

		decay product
1.	alpha decay	^4He
2.	beta decay	e^-, e^+, ν's
3.	gamma decay	γ

Generic Language of Decay

$N = \#$ nuclei

Rate at which decay happens:

$$-\frac{\Delta N}{\Delta t} = \text{activity} = \propto N$$

So: $$\frac{dN}{dt} = -\lambda N$$

λ = "decay constant"

or: $$\frac{dN}{N} = -\lambda t$$

$$\ln\left(\frac{N_f}{N_0}\right) = -\lambda t$$

N_0 = # nucleons at $t=0$

N_f = # nucleons after t

$$N_f = N_0\, e^{-\lambda t}$$

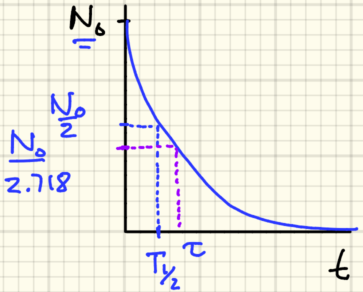
$$N = N_0\, e^{-\lambda t}$$

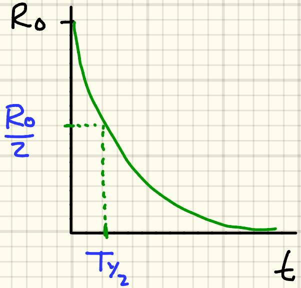
dispense with N_f

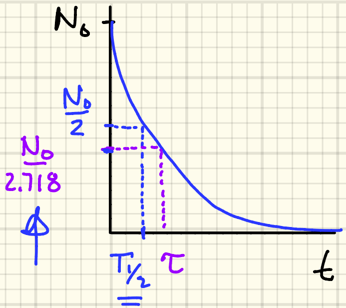
$N = N_0 e^{-\lambda t}$

Decay Rate : $R \equiv \left| \frac{dN}{dt} \right| = N_0 \lambda e^{-\lambda t} = R_0 e^{-\lambda t}$

Two standard representations







"half-life" time for a sample to decay to ½ of its original number, $T_{1/2}$

$$N_{\frac{1}{2}} = \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$e^{\lambda T_{1/2}} = 2$$

$$\lambda T_{1/2} = \ln 2$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

"mean lifetime" aka "lifetime" $\tau = 1/\lambda$

$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-t/\tau}$$

$$N(\tau) = N_0 e^{-1} = \frac{N_0}{2.718} = 0.368 N_0$$

Units

$\rightarrow [T_{1/2}] = s \qquad \rightarrow [\tau] = s \qquad [R] = \text{decays per s} \text{ or } = s^{-1}$

$\downarrow$

$1\ Ci$ = "Curie" = 3.7×10^{10} decays/s

$1\ Bq$ = "Becquerel" = 1 decay/s

mCi and μCi are practical

example: $T_{1/2}$ of $^{226}_{88}Ra$ is 1.6×10^{3} y

what's the activity during this time if $N_0 = 3 \times 10^{16}$ nuclei $\leftarrow$

$$T_{1/2} = (1.6 \times 10^{3}\, y)\left(\frac{\pi \times 10^{7}\, s}{y}\right) = 5 \times 10^{10}\, s$$

$$T_{1/2} = \frac{0.693}{\lambda} \Rightarrow \lambda = 1.4 \times 10^{-11}\ s^{-1}$$

$$R_0 = \lambda N_0 = (\qquad)(\qquad) = 4.2 \times 10^{5} \text{ decays/s} =$$

$$= 11.3\ \mu Ci$$

→ Energies of disintegration

Imagine the following generic decay chain:

$${}^{A}_{Z}X_{N} \rightarrow \text{"}D\text{"} + \text{"}y\text{"} + \text{"}Q\text{"}$$

The "Q" of the decay ⇒ "reaction energy" J

or u's $\longrightarrow Q = \left[M({}^{A}_{Z}X_{N}) - M(D) - M(y) \right] \cdot 931.5$ u

E

u

From before... $Q = -B$

if $B >$ $Q < 0$ ⇒ nucleus is stable ✓

if $B < 0$ $Q > 0$ ⇒ nucleus is unstable and might decay

$\boxed{\alpha \text{ decay}}$

$$ {}^{A}_{Z}X_{N} \longrightarrow {}^{A-4}_{\underline{Z-2}}D_{\underline{N-2}} + {}^{4}_{2}He $$

$\boxed{\beta \text{ decay}}$

$$ {}^{A}_{Z}X_{N} \longrightarrow {}^{A}_{Z+1}D_{N-1} + \beta^{-} $$

$$ {}^{A}_{Z}X_{N} \longrightarrow {}^{A}_{Z-1}D_{N+1} + \beta^{+} $$

can think of as

$$ \underline{n \rightarrow p} + e^{-} $$

$$ p \rightarrow n + e^{+} $$

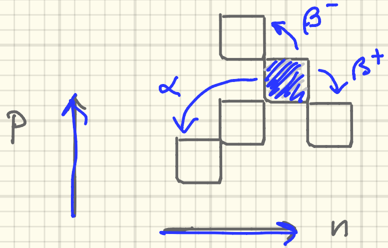
within the nucleus

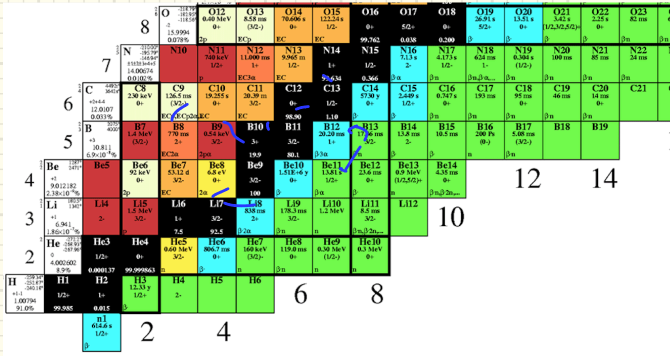
old

$\boxed{\gamma \text{ decay}}$

$$ {}^{A}_{Z}X^{*}_{N} \longrightarrow {}^{A}_{Z}X_{N} + \gamma $$

nuclear excited state...
think shell model





α decay

not all nuclei are α- emitters

Ra Ur are famous

Rn $^{222}_{86}Rn$ in your basement _ _ is also

$\longrightarrow$ $^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}D_{N-2} + {}^{4}_{2}He_{2}$

$\longrightarrow$ $Q = \left[M\left(^{A}_{Z}X_{N}\right) - M\left(^{A-4}_{Z-2}D_{N-2}\right) - M\left(^{4}_{2}He_{2}\right) \right] c^2$

✓ example Radium 226

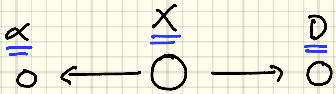
$\longrightarrow M(^{226}Ra) = 226.025406\,u$

$\longrightarrow M(^{222}Rn) = 222.017574\,u$

$\longrightarrow M(^{4}He) = 4.002603\,u$

$Q = (226. \dots - 222 \dots - 4.002603)\,u \;\; 931.5\ MeV/u$

$Q = 4.87\ MeV > 0$ $\Rightarrow$ unstable and decay could happen.



So: $|P_\alpha| = |P_D|$

and $Q = \left[M(X) - M(D) - M(\alpha) \right] c^2 = \underline{K_D} + \underline{K_\alpha}$

Assume non-relativistic

$$Q = \frac{P_D^2}{2M_D} + \frac{P_\alpha^2}{2M_\alpha} = \frac{P_\alpha^2}{2M_\alpha} + \frac{P_\alpha^2}{2M_D}$$

$$= \dots$$

$$Q = K_\alpha \left(1 + \frac{M_\alpha}{M_D} \right)$$

or $K_\alpha = \dfrac{M_D}{M_\alpha + M_D} Q \xrightarrow{\text{here}} \left(\dfrac{222}{226}\right)(4.87) = \underline{\underline{4.8 \text{ MeV}}}$

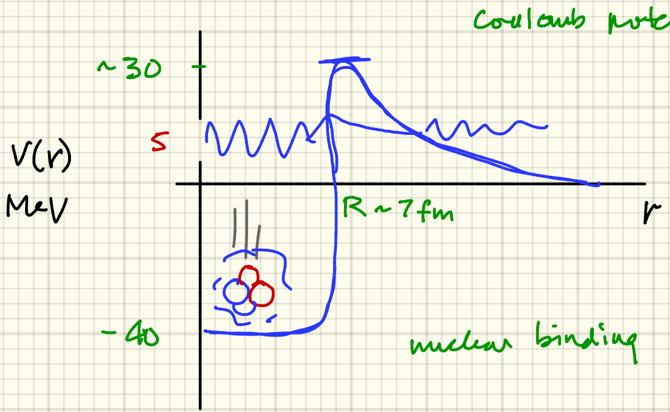
non-rel.

What is α Decay?

a quantum mechanical tunneling phenomenon.

K_α for ^{222}Rn was ~5 MeV

Sometimes  stick and collectively rattle around inside a potential which George Gamow modeled as:



→ $K_\alpha = 5$ MeV ⇒ $v_\alpha \simeq 2\times10^{7}$ m/s

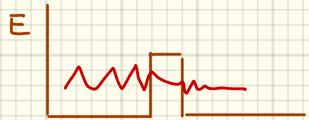
$$\Delta t = \frac{2R}{v_\alpha}$$

$$\Delta t = \sim 7\times10^{-22}\ s$$

$$f = 1.4\times10^{21}\ Hz$$

↳ huge number of "tries" at the potential edge

Remember barrier penetration?



transmission coefficient $T(E) = e^{-(\text{complicated integral of } r)}$

~ not very E-dependent.

For ^{222}Rn with $K_\alpha = 4.7\ \text{MeV}$ $\sim T = 10^{-34}$ ... like a probability...
a likelihood

But the α's are persistant:

$f = 10^{21}$, so 10^{21} tries against a likelihood of 10^{-34}

$$\sim 10^{-34} \times 10^{21}\, s^{-1} \sim 10^{-13}\, s^{-1} \to \lambda$$

half-life: $\lambda = \dfrac{\ln 2}{T_{1/2}} \Rightarrow T_{1/2} = 2\times 10^{12}\ s = 200{,}000\ y$

actual: 160,000 y.

β decay.

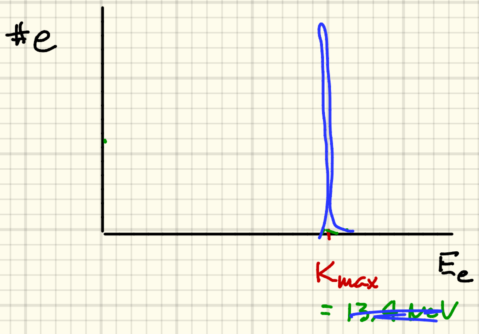
β decay confusing since 19-teens

measurements of $E(\beta)$ well established before Bohr model.

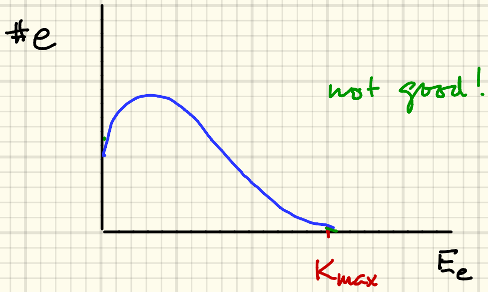
eg $\quad {}^{12}_{5}B \rightarrow {}^{12}_{6}C + e^-$

$\leftarrow \quad C \leftarrow B \rightarrow e \quad \rightarrow$

$Q = 13.4 \text{ MeV}$



Experiment(s):



Pauli's half-hearted idea

ν neutron

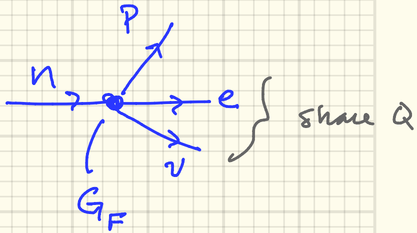
$C \leftarrow B \rightarrow e$

he called it the "neutron" -- nobody paid attention

& when Chadwick found our neutron

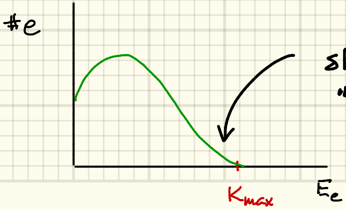
a new name was required.

1934 Enrico Fermi — wrote the complete theory of what he called the "little neutron"... neutrino



insid : ,lo.ns

mass ν? zero... un ... shut. not know. $\sim 10^{-3}$ eV



shape at the "end point" of spectrum sensitive to $m(\nu)$