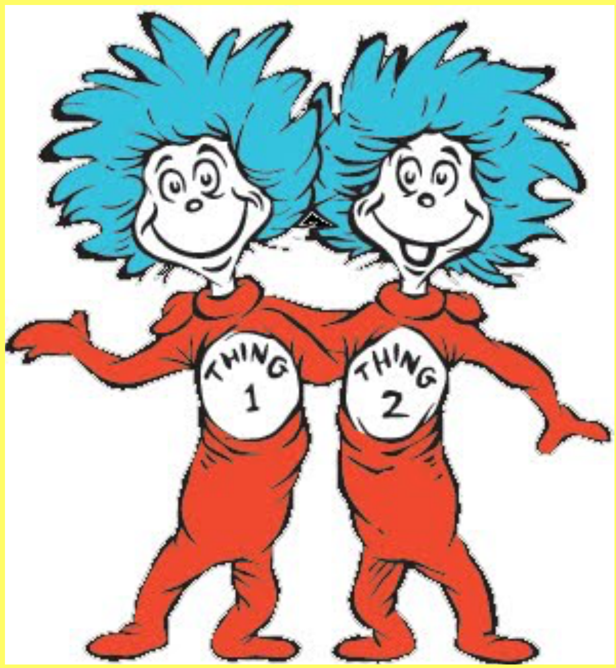

THING
1
THING
2

THING
Elmer

12. Atomic Nucleus, 4

13. Nuclear Interactions, 1

lecture 37, November 27, 2017

housekeeping

Coming attractions

This week:

chapter 12 homework due Wed 11/29

HW workshop tomorrow 11/28

End game:

I’ve made some adjustments to the schedule…stay tuned, now, week by week

exam #3 is Friday, December 1: chapters 7,8,9,12,13

today

Atomic nucleus - finishing

Nuclear Interactions - beginning

β decay.

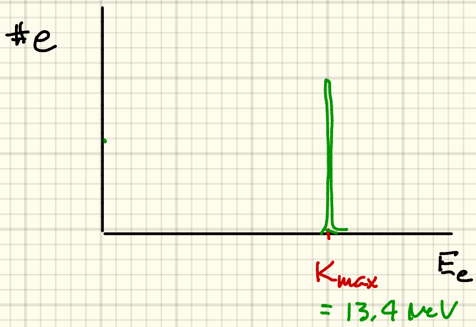
β decay confusing since 19-teens

measurements of $E(\beta)$ well established before Bohr model.

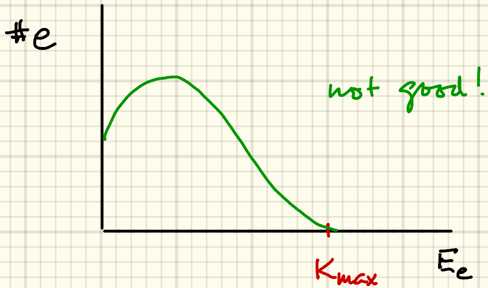
eg $$\underbrace{{}^{12}_{5}B \to {}^{12}_{6}C + {}^{0}_{-1}e}_{C \longleftarrow B \longrightarrow e}$$

$$Q = \left[M({}^{12}_{5}B) - M({}^{12}_{6}C)\right]c^2$$

$$Q = 13.37 \text{ MeV}$$



Experiment(s):



Pauli's half-hearted idea

$C \leftarrow B \rightarrow e$ ↗ ν

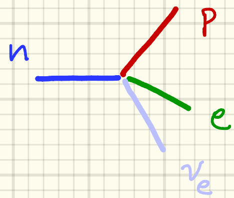
something else... invisible

he called it the "neutron" -- nobody paid attention

& when Chadwick found our neutron

a new name was required.

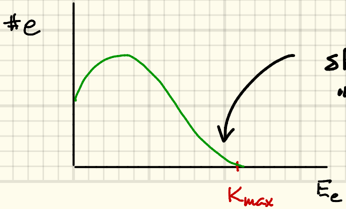
1934 Enrico Fermi – wrote the complete theory of what he called the "little neutron"... neutrino



share Q

inside the nucleus

mass ν? zero... we thought. not now. $\sim 10^{-3}$ eV



shape at the "end point" of spectrum sensitive to $m(\nu)$

Inside nucleus... other β decay happen.

$$p^+ + (\text{energy from nucleus}) \longrightarrow n + e^+ + \nu_e$$

positron

$${}^{14}_{6}C \longrightarrow {}^{14}_{7}N + \beta^- + \nu$$ beta decay. → important for carbon-dating

$${}^{13}_{7}N \longrightarrow {}^{12}_{6}C + \beta^+ + \nu$$ positron decay

$$n \rightarrow p + \beta^- + \nu$$ free neutron decay.

$${}^{A}_{Z}X_N + {}^{0}_{-1}e \longrightarrow {}^{A}_{Z-1}X + \nu$$ electron capture

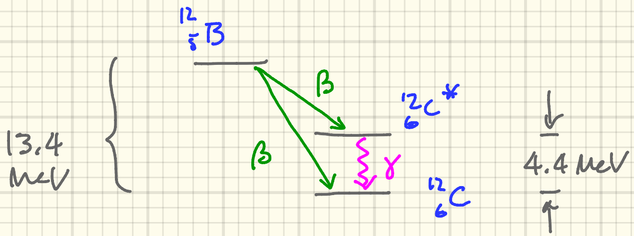
Gamma Decay.

often: $X \to D^* + \alpha$ — left in an excited state

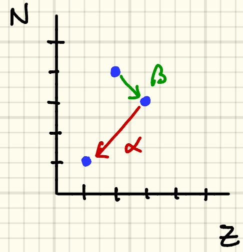
$\qquad\qquad\hookrightarrow D + \gamma$

and, collisions can excite a nucleus

Example... just like atomic spectra.



cascades:



and so-on

Radioactivity.

- natural $\longrightarrow$ found in nature... stars
- artificial $\longrightarrow$ lab-created, reactors, nuclear explosions

change A : α decay $A \to A-4$

change Z : α decay $Z \to Z-2$

β decay $Z \to Z+1$

3 naturally occuring radioactive "chains"

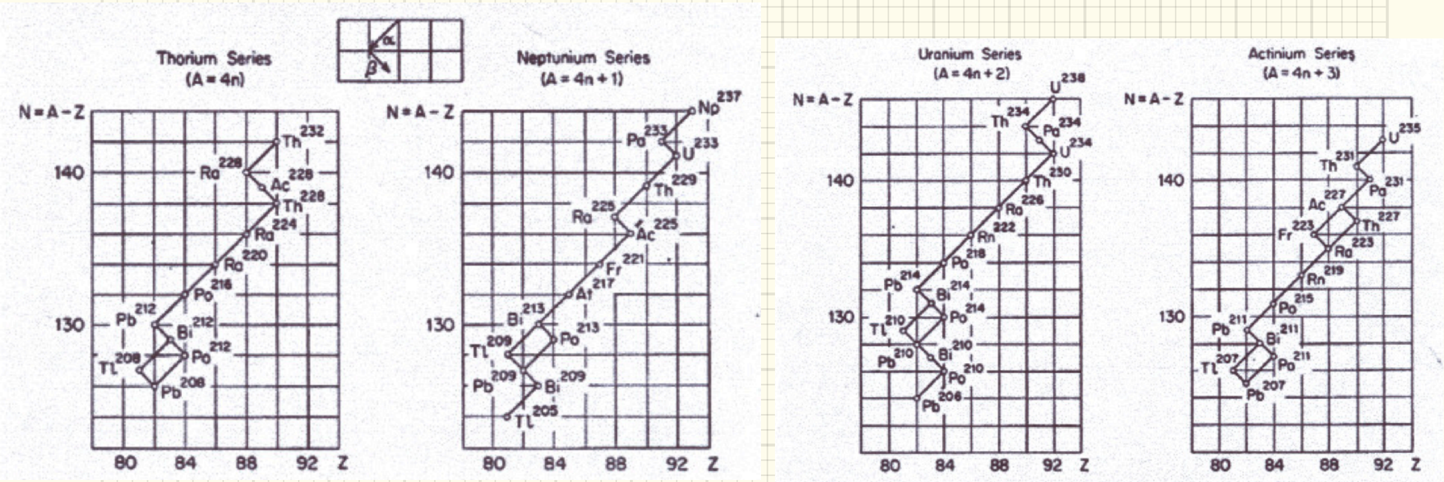
$^{238}_{92}U$ $T_{1/2} = 4.47 \times 10^{9}\,y \longrightarrow {}^{206}_{82}Pb$ "uranium chain"

$^{235}_{92}U$ $T_{1/2} = 7.04 \times 10^{8}\,y \longrightarrow {}^{207}_{82}Pb$ "actinium chain"

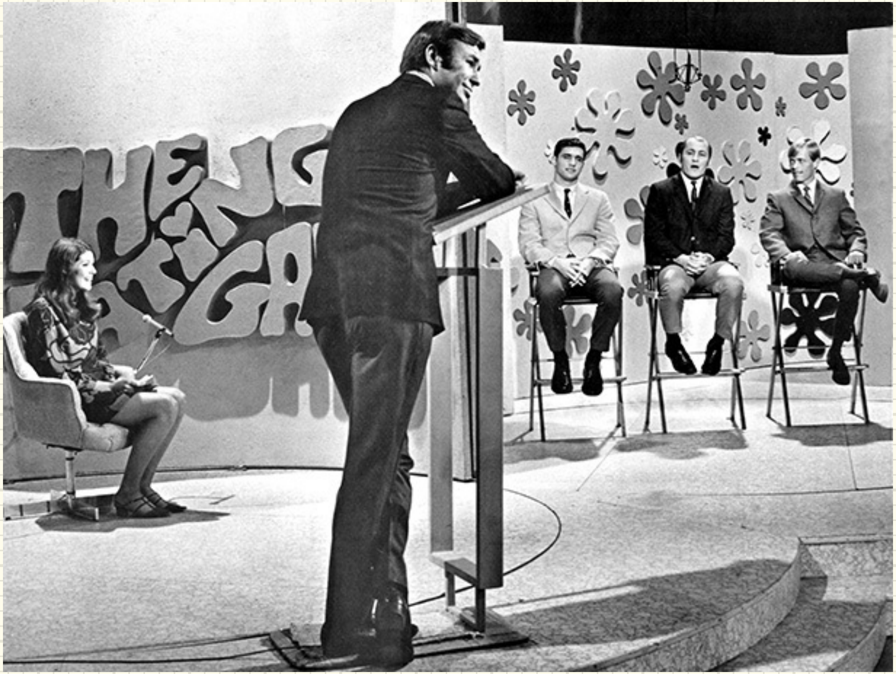
$^{232}_{90}Th$ $T_{1/2} = 1.41 \times 10^{10}\,y \longrightarrow {}^{208}_{82}Pb$ "thorium chain"

one artificial chain – "trans-uranic" Np

$^{239}_{93}Np$ $T_{1/2} = 2.14 \times 10^{6}\,y \longrightarrow {}^{209}_{83}Bi$ "neptunium chain"



not found in nature – artificially produced.



Radioactive Dating

^{14}C ... in a nutshell

$\downarrow$

$^{14}_{6}C \rightarrow {}^{14}_{7}N + \beta + \nu$

$T_{1/2}(^{14}C) = 5730\,y$

measure Activity.

cosmic rays

n

$P \leftarrow$ ^{14}N

^{14}C

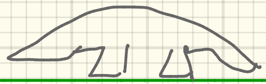
constant rate

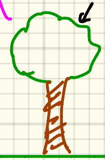
O_2

$^{14}CO_2$ (1 in 10^{12})

$^{12}CO_2$ (99%)

$\frac{^{14}C}{^{12}C} = 1.3 \times 10^{-12}$ in atmosphere





die - stops making new C

Example Caputure a sample of CO_2 from atmosphere into

$V = 200\ cm^3$

$P = 2.0 \times 10^4\ Pa \sim 10^{-5}\ atm$

$T = 295K$

assume ^{14}C counting efficiency is perfect → after 1 week, what is count?

$$n = \frac{PV}{RT} = \#\text{ moles} = \frac{(2\times10^{4} N/m^2)(2\times10^{-4} m^3)}{(8.314\ J/_{mol\,K})(295K)} = 1.63\times10^{-3}\ mol$$

$$N = N_A n = 9.82\times10^{20}\text{ molecules } CO_2 = \#\text{ atoms of } {}_6C$$

$$\#\,{}^{14}C = (1.3\times10^{-12})(9.82\times10^{20}) = 1.28\times10^{9}\text{ atoms } {}^{14}_{6}C$$

$$R_0 = N_0\lambda = (1.28\times10^9)\left(\frac{\ln 2}{T_{1/2}}\right) = (1.28\times10^9)\frac{(0.693)}{(5730y)}\frac{(1y)}{([illegible]\times10^7 s)}$$

$R_0 = 4.91\times10^{-3}$ decay/s

$R = 2969$ decays / wk.

Example put piece of old wood into same device...

after 1 min, 1420 counts are recorded. How old is the wood?

$$R = R_0 e^{-\lambda t}$$

$R_0 = 2969\ s^{-1}$ now

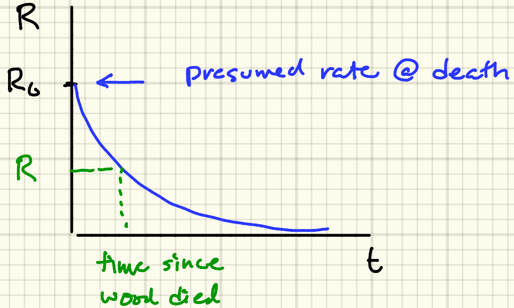
$R = 1420\ s^{-1}$

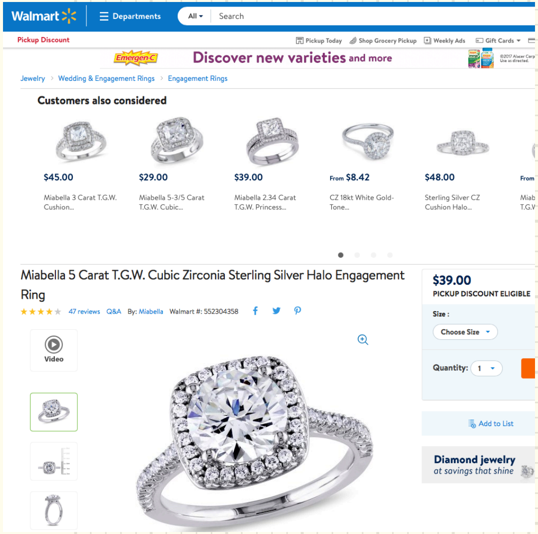
$$t = \frac{1}{\lambda} \ln\left(\frac{2969}{1420}\right)$$

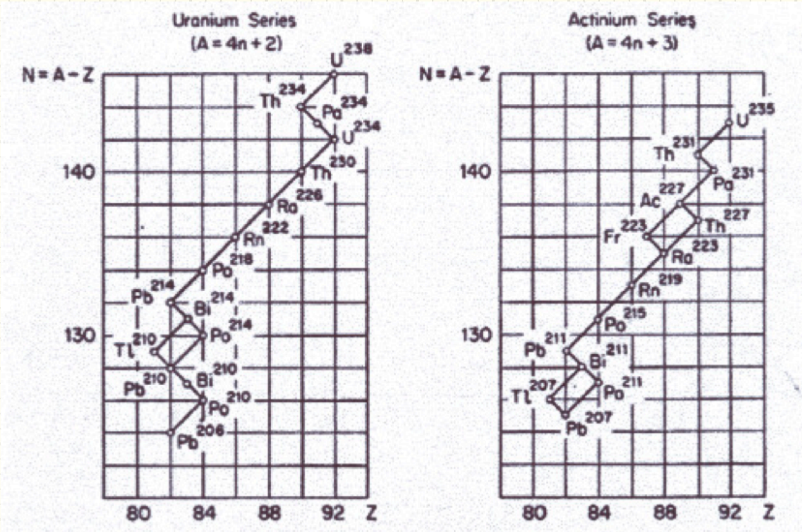
$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

$$t = \frac{5730}{0.693} \ln\left(\frac{2969}{1420}\right)$$

$$t = 6099\ y$$

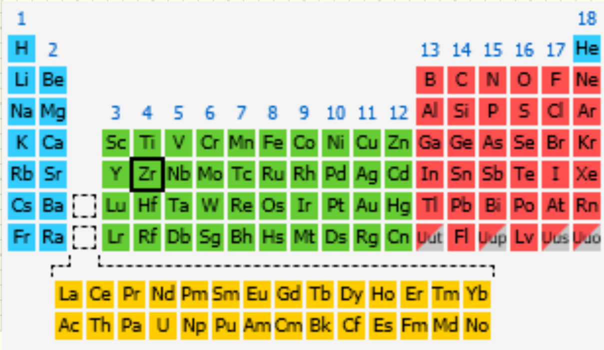






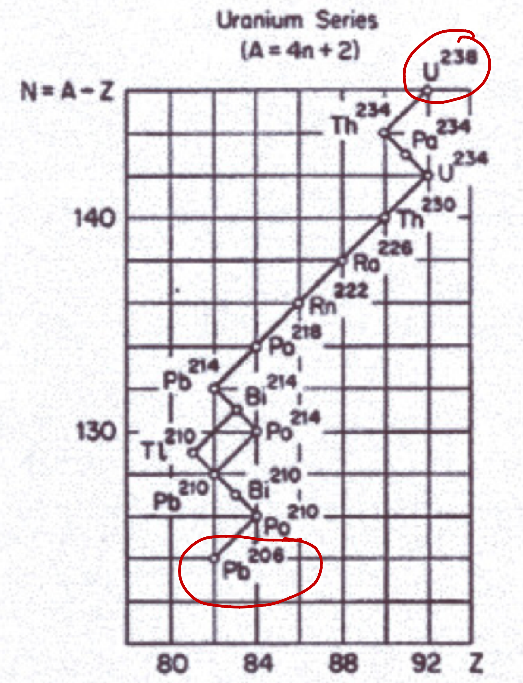
Zircon

$ZrSiO_4$



Zr: $[Kr]\,4d^2 5s^2$ U: $[Rn]\,5f^3 6d^1 7s^2$

Uranium - Lead dating



Assume all ^{206}Pb in a sample comes from ^{238}U decay...*

$\#\ ^{238}U$ originally $-\ \#\ ^{238}U$ today $= \#\ ^{206}Pb$

$N_0 = \#\ ^{238}U$ originally.

$N_0 e^{-\lambda t} = \#\ ^{238}U$ today

$$R = \frac{\#\ ^{238}U}{\#\ ^{206}Pb} = \frac{N_0 e^{-\lambda t}}{N_0 - N_0 e^{-\lambda t}}$$

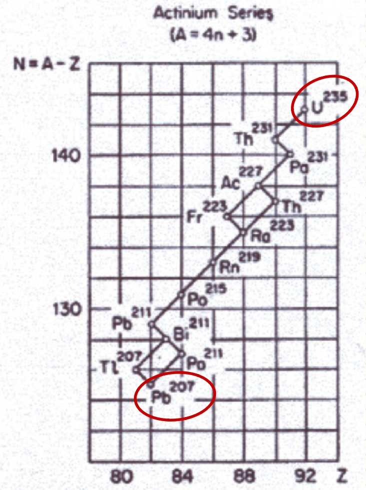
solve for t

$$t = \frac{1}{\lambda} \ln\left(\frac{1}{R} + 1\right) \qquad \lambda = \frac{T_{1/2}}{0.693}$$

$$t = \frac{T_{1/2}}{0.693}\left(\frac{1}{R} + 1\right) \qquad T_{1/2}(^{238}U) = 4.5 \times 10^9\ y$$

* probably... there are other techniques

Pb dating, another way ^{204}Pb : stable   nothing decays to it
it doesn't decay.   abundance constant



But: $^{238}U \quad T_{1/2} = 4.47 \times 10^9 y \longrightarrow {}^{206}Pb$

$^{235}U \quad T_{1/2} = 0.7 \times 10^9 y \longrightarrow {}^{207}Pb$

Earth is about $4.5 \times 10^9 y \longrightarrow$

$6 \times T_{1/2}({}^{235}U) \Rightarrow$ most is gone

^{238}U is still decaying from original abundance

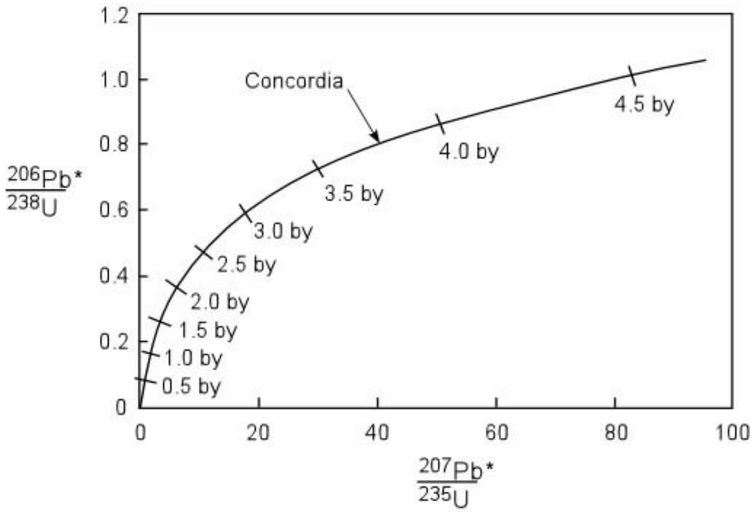
$\frac{^{207}Pb}{^{204}Pb}$ = constant & $\frac{^{206}Pb}{^{204}Pb}$ still increasing

compared with

abundance

dates lead ore
meteorites ~ 4.5 By
oldest rocks ~ 4 By

Still another way



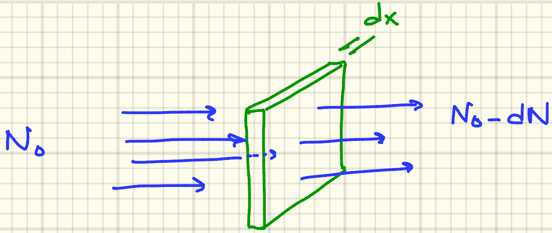
NUCLEAR REACTIONS

scattering particles from nuclei

- sometimes altering the trajectory
- sometimes completely changing the particles

Rutherford ... again ... proton discovery

$$\alpha + {}^{14}_{7}N \longrightarrow p + {}^{17}_{8}O \qquad \longrightarrow \text{aka " } {}^{14}N(\alpha, p)^{17}O \text{ "}$$



total # target nuclei

nAx

$\frac{\#}{\text{volume}}\,[m^{-3}]$ having cross section σ

total "area" presented to the beam

$\sigma n A x$

$$\frac{\text{collisions}}{\text{beam particles}} = \frac{\text{Rate of collisions } [s^{-1}]}{\text{Rate of beam } [s^{-1}]} = \frac{R}{R_0} = \frac{\sigma n A x}{A} = \sigma n x$$

$$-\frac{dN}{N} = \frac{n A \sigma dx}{A} = n\sigma dx$$

$$-\int_{N_0}^{N} \frac{dN}{N} = -n\sigma \int_0^x dx \implies \ln N/N_0 = -n\sigma x$$

$$N = N_0 e^{-n\sigma x}$$

remember $[\sigma]$ = barn

Reaction kinematics ... low energy.

$$a + B \to c + D$$

B at rest $\Rightarrow$ "lab frame"

$$E_0 = E_f$$

$$m_a c^2 + K_a + m_B c^2 + \underbrace{K_B}_{=0} = m_c c^2 + K_c + m_D c^2 + K_D$$

rearrange mass energies = kinetic energies.

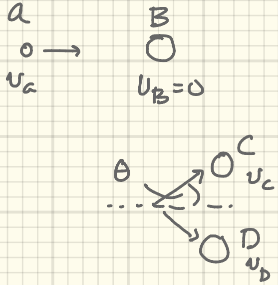
$$m_a c^2 + m_B c^2 - (m_c c^2 + m_D c^2) = K_c + K_D - K_a \equiv \text{"}Q\text{"}$$

$Q > 0 \Rightarrow$ final $K >$ initial K
(exothermic) $\Rightarrow$ mass deficit

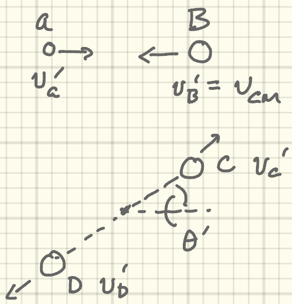
$Q < 0 \Rightarrow$ K converted into mass energy
(endothermic)

"threshold" required – K_a center of mass frame – cm – analysis

LAB



CM

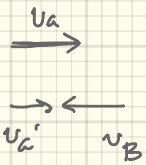


transformation:

$$\vec{P}_a' = -\vec{P}_B'$$

$$m_a |v_a'| = m_B |v_B'| \qquad v_B = v_{cm}$$

$$\downarrow$$



$$\left\{ \begin{array}{l} v_a' = v_a - v_B \\ v_a' = v_a - v_{cm} \end{array} \right. \longrightarrow$$

$$m_a |v_a'| = m_B |v_B'|$$

$$v_a' = v_a - v_{cm} \longrightarrow v_B' = \frac{m_a}{m_B} v_a' = \frac{m_a}{m_B}(v_a - v_{cm}) = v_{cm}.$$

$$v_{cm}\left(1 + \frac{m_a}{m_B}\right) = \frac{m_a}{m_B} v_a = v_{cm}\left(\frac{m_B + m_a}{m_B}\right)$$

$$\Rightarrow v_{cm} = \left(\frac{m_B}{m_B + m_a}\right)\frac{m_a}{m_B} v_a$$

$$\underline{v_{cm} = \frac{m_a}{m_a + m_B} v_a}$$

In cm frame: $E_o' = E_f'$

$$m_a c^2 + \tfrac{1}{2} m_a v_a'^2 + m_B c^2 + \tfrac{1}{2} m_B v_{cm}^2 = m_c c^2 + \tfrac{1}{2} m_c v_c'^2 + m_D c^2 + \tfrac{1}{2} m_D v_D'^2$$

$$\underline{\tfrac{1}{2} m_a (v_a - v_{cm})^2} + m_a c^2 + m_B c^2 + \underline{\underline{\tfrac{1}{2} m_B v_{cm}^2}} = m_c c^2 + \tfrac{1}{2} m_c v_c'^2 + m_D c^2 + \tfrac{1}{2} m_D v_D'^2$$

"Threshold" $\equiv$ C & D are produced with no kinetic energies $v_c' = v_D' = 0$

$$\underline{\tfrac{1}{2} m_a \left(v_a - \frac{m_a}{m_a + m_B} v_a\right)^2} + \underline{\underline{\tfrac{1}{2} m_B \left(\frac{m_a}{m_a + m_B} v_a\right)^2}} = m_c c^2 + m_D c^2 - m_a c^2 - m_B c^2 = -Q$$

$$\tfrac{1}{2} m_a\left(v_a - \frac{m_a}{m_a+m_B} v_a\right)^2 + \tfrac{1}{2} m_B\left(\frac{m_a}{m_a+m_B} v_a\right)^2 = m_C c^2 + m_D c^2 - m_a c^2 - m_B c^2 = -Q$$

$$\tfrac{1}{2} m_a\left(v_a - \frac{m_a v_a}{m_a+m_B}\right)^2 + \frac{1}{2} m_B \frac{m_a^2 v_a^2}{(m_a+m_B)^2} = -Q$$

$$\tfrac{1}{2} m_a\left[v_a^2 + \frac{m_a^2 v_a^2}{\Delta^2} - 2 v_a^2 \frac{m_a}{\Delta}\right] + \frac{1}{2} \cdots$$

$$\tfrac{1}{2} v_a^2\left[m_a + \frac{m_a^3}{\Delta^2} - \frac{2 m_a^2}{\Delta} + \frac{m_B m_a^2}{\Delta^2}\right]$$

$$\tfrac{1}{2} \frac{v_a^2}{\Delta^2}\left[m_a \Delta^2 + m_a^3 - 2 m_a^2 \Delta + m_B m_a^2\right]$$

$$\frac{1}{2} \frac{v_a^2 m_a}{\Delta^2}\left[(m_a+m_B)^2 + m_a^2 - 2 m_a(m_a+m_B) + m_B m_a\right]$$

$$\frac{1}{2} \frac{v_a^2 m_a}{\Delta^2}\left[\cancel{m_a^2} + m_B^2 + \cancel{2 m_a m_B} + \cancel{m_a^2} - \cancel{2 m_a^2} - \cancel{2 m_a} m_B + m_B m_a\right]$$

$$\frac{1}{2} \frac{v_a^2}{\Delta} m_a m_B\left[m_B + m_a\right] = \frac{1}{2} v_a^2 \frac{m_a m_B}{m_a+m_B} = -Q$$

$$\Rightarrow \quad -Q\left(\frac{m_a+m_B}{m_B}\right) = \frac{1}{2} m_a v_a^2 = K_a^{\text{threshold}}$$

example: Q for ${}^{16}_{8}O(\gamma, p){}^{15}_{7}N$?

a, b, c, D

$$Q = [m_a + m_B - (m_c + m_D)]c^2$$

$$= [15.994915u - 0u - (15.000108u + 1.007825u)]c^2 \quad 931.5 \frac{MeV}{c^2 u}$$

$$Q = -12.13 \text{ MeV}$$

Threshold γ kinetic energy ?

$$K_\gamma^{th} = -Q\left(\frac{m_a + m_B}{m_B}\right)$$

$$K_\gamma^{th} = (12.13)\left(\frac{15.994915 + 0}{15.99415}\right)u = 12.13 \text{ MeV.}$$

a B D c

$${}^{16}_{8}O\,(p,d)\,{}^{15}_{8}O \quad - \quad Q\,?$$

$$Q = \left[m_a + m_B - (m_c + m_D) \right] c^2$$

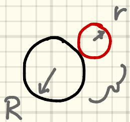
$$= \left[15.994915u + 1.007825u - 15.003070u - 2.014102u \right] c^2 \; \frac{931.5\,\text{MeV}}{c^2 u}$$

$$Q = -13.44 \text{ MeV}$$

$$K_p^{th} \cong -Q \left(\frac{16u + 1u}{16u} \right) = 13.44 \text{ MeV} \left(\frac{17}{16} \right) = 14.28 \text{ MeV}.$$

There's actually more: proton must overcome electrostatic repulsion

estimate...



$$\delta = R + r \qquad R = r_0 A^{1/3}$$

$$\delta \simeq r_0 (A^{1/3} + 1) \qquad r_0 = 1.4 \text{ fm}.$$

$$E_c = \frac{1}{4\pi\varepsilon_0} \frac{Ze \cdot e}{\delta} = \frac{1.44 \text{ MeV} \cdot \text{fm}}{1.4 \text{ fm}} \; \frac{Z}{A^{1/3}+1}$$

${}^{16}_{8}O(p,d){}^{15}_{8}O$: $\quad E_c = 2.34$ MeV so K_p^{th} ? $> E_c$ ✓ will happen!

Or $\;{}^{209}_{83}\mathrm{Bi}\,(p,d)\,{}^{208}_{83}\mathrm{Bi}$

$$\left.\begin{aligned} Q &= -5.23\ \mathrm{MeV} \\ K_p^{th} &= 5.26\ \mathrm{MeV}. \\ \text{but}\quad E_c &= 12.33\ \mathrm{MeV} \end{aligned}\right\}\ ?$$