8. Atomic Physics, 3

9. Quantum Statistics, 1

lecture 30, November 8, 2017

housekeeping

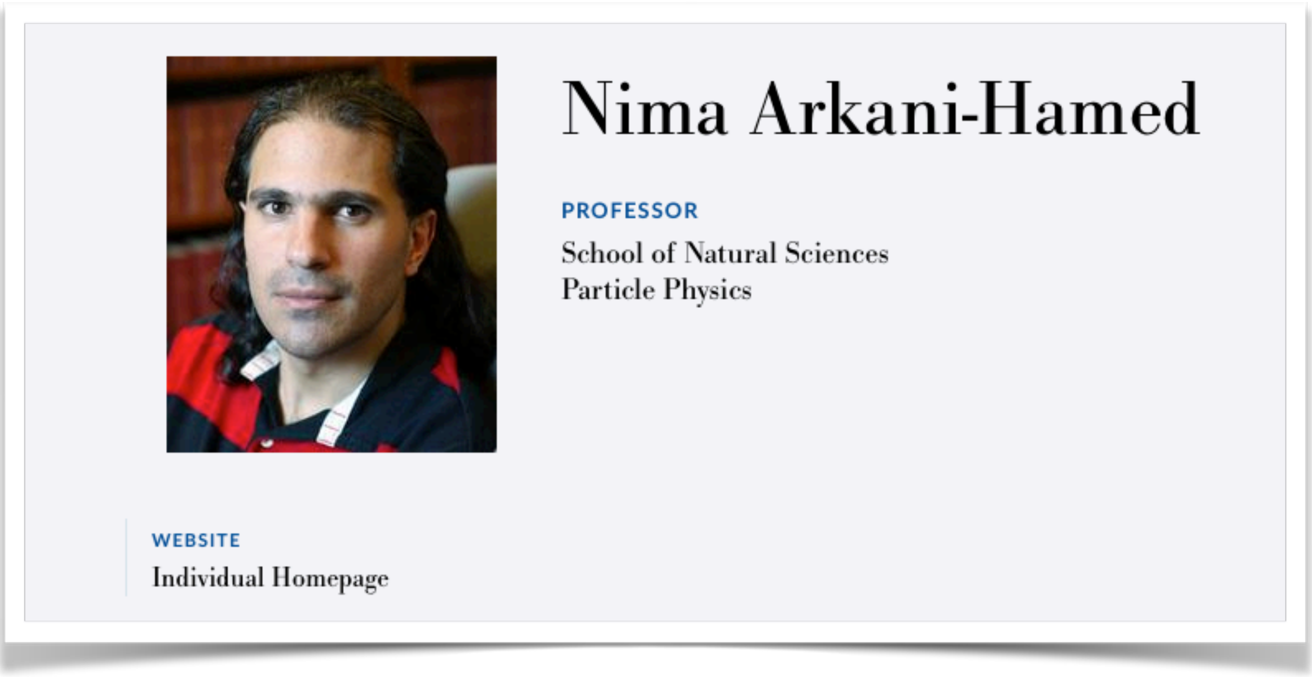
Honors project

two nights ago

By ~~tonight~~ I need to know via email:

if you’re doing the honors project

The Tom Story



Nima Arkani-Hamed

PROFESSOR

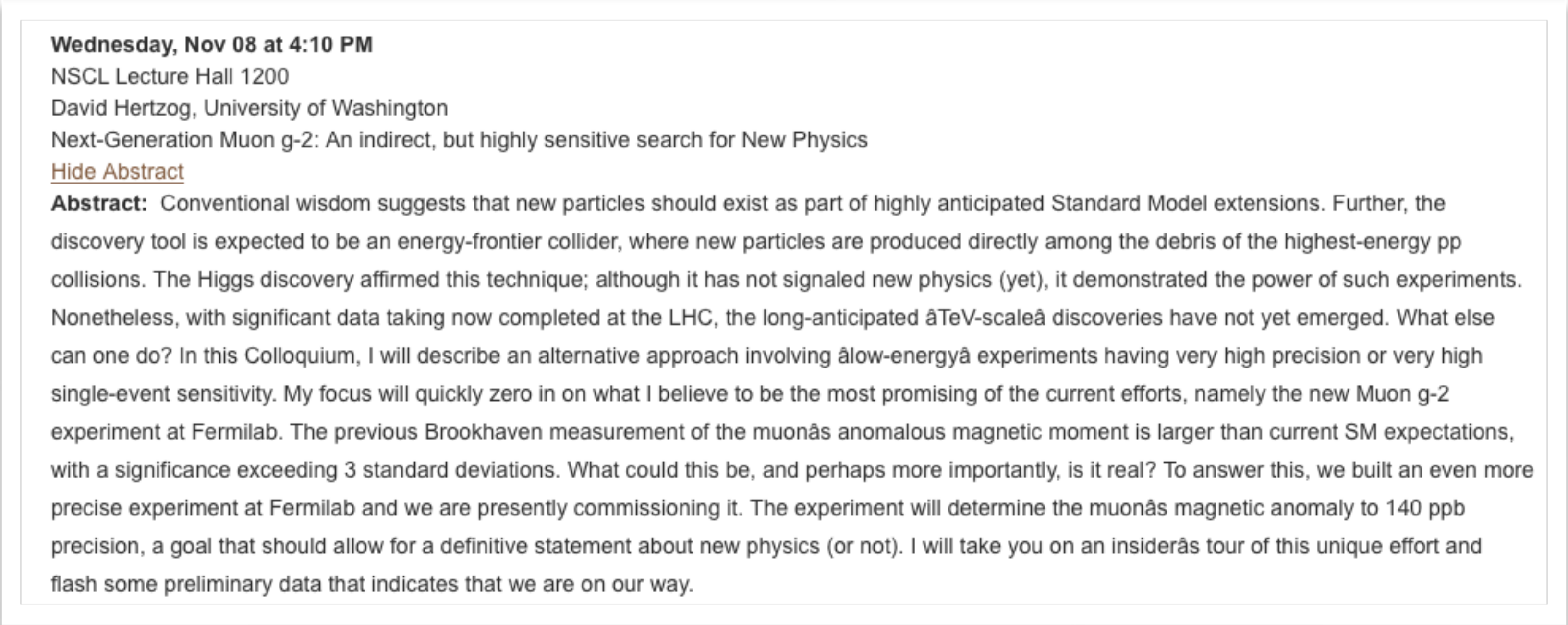
School of Natural Sciences
Particle Physics

WEBSITE

Individual Homepage

Colloquium Thursday, 4:10pm
Public Lecture Thursday, 8:00pm
Both in BPS1415

https://www.quantamagazine.org/nima-arkani-hamed-and-the-future-of-physics-20150922/



Wednesday, Nov 08 at 4:10 PM
NSCL Lecture Hall 1200
David Hertzog, University of Washington
Next-Generation Muon g-2: An indirect, but highly sensitive search for New Physics
Hide Abstract

Abstract: Conventional wisdom suggests that new particles should exist as part of highly anticipated Standard Model extensions. Further, the discovery tool is expected to be an energy-frontier collider, where new particles are produced directly among the debris of the highest-energy pp collisions. The Higgs discovery affirmed this technique; although it has not signaled new physics (yet), it demonstrated the power of such experiments. Nonetheless, with significant data taking now completed at the LHC, the long-anticipated âTeV-scaleâ discoveries have not yet emerged. What else can one do? In this Colloquium, I will describe an alternative approach involving âlow-energyâ experiments having very high precision or very high single-event sensitivity. My focus will quickly zero in on what I believe to be the most promising of the current efforts, namely the new Muon g-2 experiment at Fermilab. The previous Brookhaven measurement of the muonâs anomalous magnetic moment is larger than current SM expectations, with a significance exceeding 3 standard deviations. What could this be, and perhaps more importantly, is it real? To answer this, we built an even more precise experiment at Fermilab and we are presently commissioning it. The experiment will determine the muonâs magnetic anomaly to 140 ppb precision, a goal that should allow for a definitive statement about new physics (or not). I will take you on an insiderâs tour of this unique effort and flash some preliminary data that indicates that we are on our way.

today

Hydrogen, Sodium, Helium, oh my

the beginnings of quantum statistics

Last Time

Spin-orbit coupling

"internal" magnetic field seen by orbiting electron

- $$\Delta E = -\vec{\mu}_e \cdot \vec{B}_{int}$$

spin magnetic moment of electron

splits the ℓ states into two & couples $\vec{L}$ & $\vec{S}$:

$$\vec{J} = \vec{L} + \vec{S}$$

and new quantum numbers (n, ℓ, j, m_j)

$$|J| = \hbar\sqrt{j(j+1)}$$

$$J_3 = m_j \hbar$$

- Rules for combining in tables
- Selection rules:
  - $\Delta n =$ anything
  - $\Delta j = 0, \pm 1$ (no $0 \rightarrow 0$)
  - $\Delta m_j = 0, \pm 1$
  - $\Delta \ell = \pm 1$

Hydrogen

remember $S \cdot L$ coupling only $\geq p$ states

Table

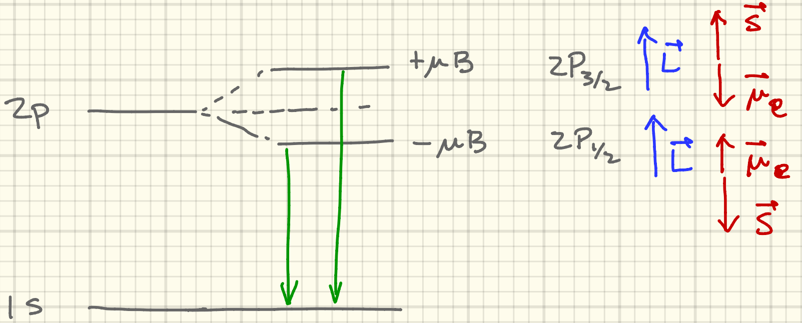
$S = A = 1/2$ spin

$L = B = 1$ P state L

J	m_j
$1/2$	$-1/2, 1/2$
$3/2$	$-3/2, -1/2, 1/2, 3/2$

$J = 1/2$ and $3/2$

→ each P state splits into doublet

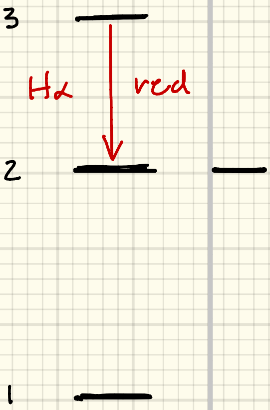


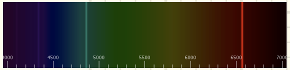
$\Delta E = 5 \times 10^{-5}$ eV

EVOLUTION OF HYDROGEN SPECTRA

... you're welcome

Bohr | including L & B_{ext} | including S-O, but $B_{ext} = 0$

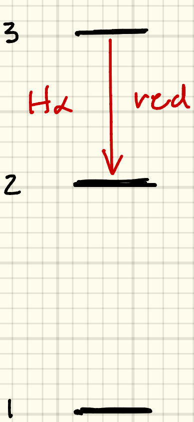


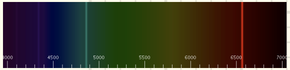


EVOLUTION OF HYDROGEN SPECTRA

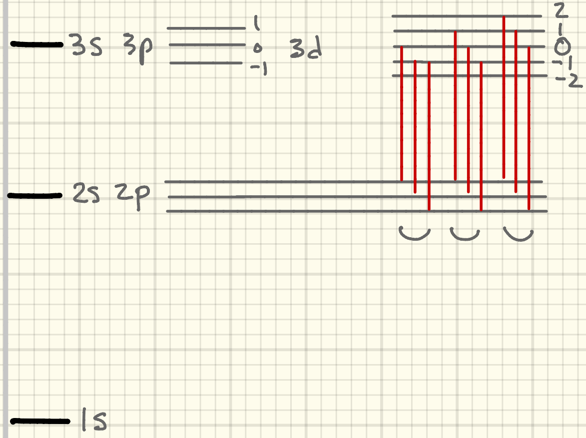
... you're welcome

Bohr





including L & B_{ext}

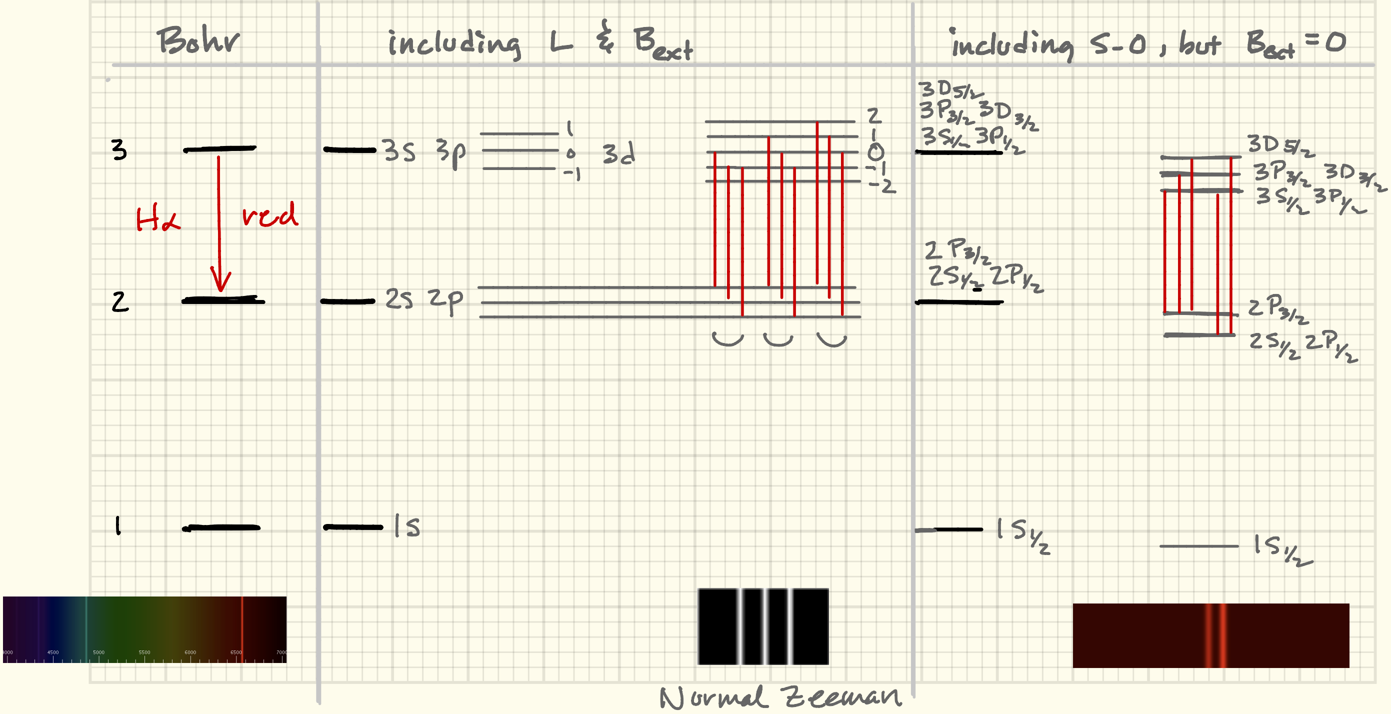


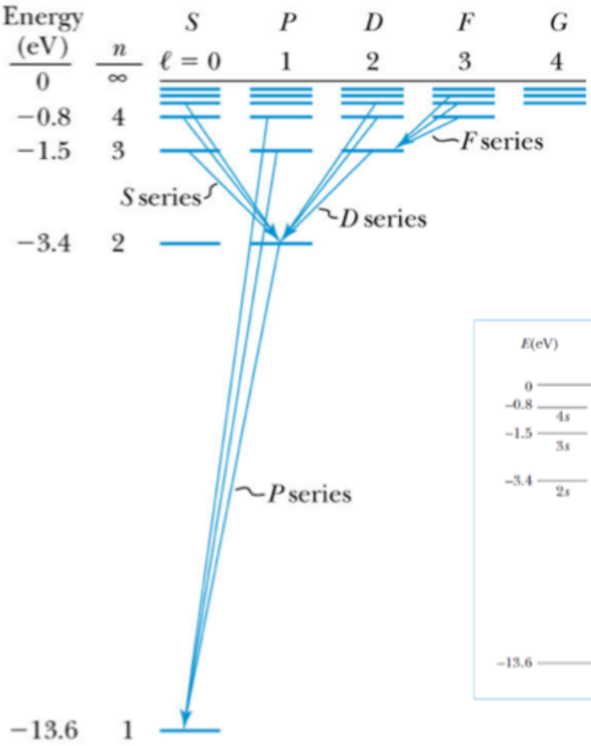
Normal Zeeman

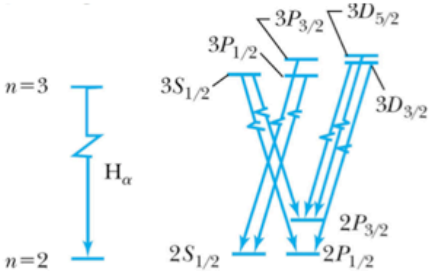
including S-O, but $B_{ext} = 0$

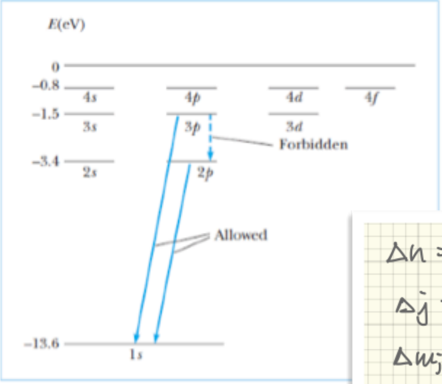
EVOLUTION OF HYDROGEN SPECTRA

... you're welcome









Δn = anything

Δj = 0, or ±1 (no 0 → 0)

Δm_j = 0 or ±1

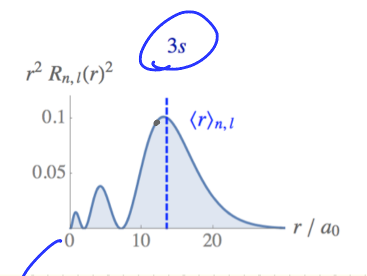
$\Delta \ell$ = ±1

Everyone's favorite: sodium

Na $Z = 11$ $1s^2 2s^2 2p^6 \underline{\underline{3s^1}}$

1 e outside of a closed shell

hydrogen-like single electron... excitations above this → 3p



Bohr	Spin Orbit $B_{ext} = 0$	Spin orbit + B_{ext}
3s ——		
3s ——		

that little bit

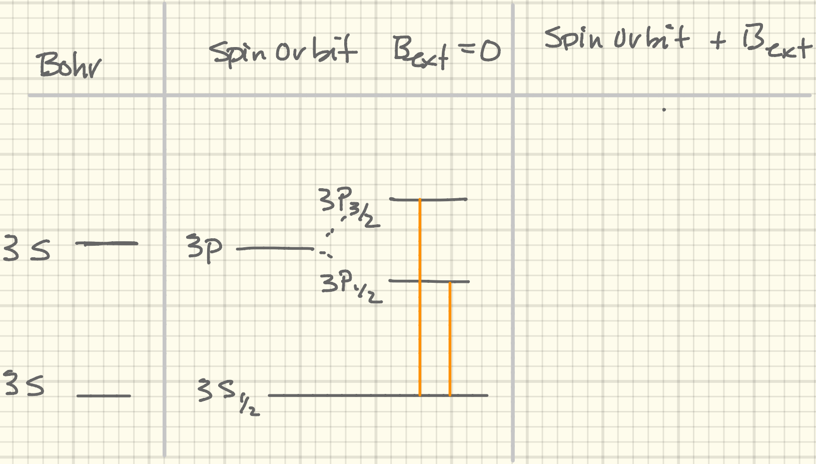
— overlap of

3s - 1s — increases 3s binding relative to 3p

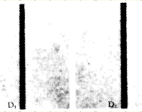
Everyone's favorite: sodium

Na $Z = 11$ $1s^2 2s^2 2p^6 \underline{\underline{3s^1}}$ ← 1 e outside of a closed shell

hydrogen-like single electron... excitations above this → 3p



Zeeman's images



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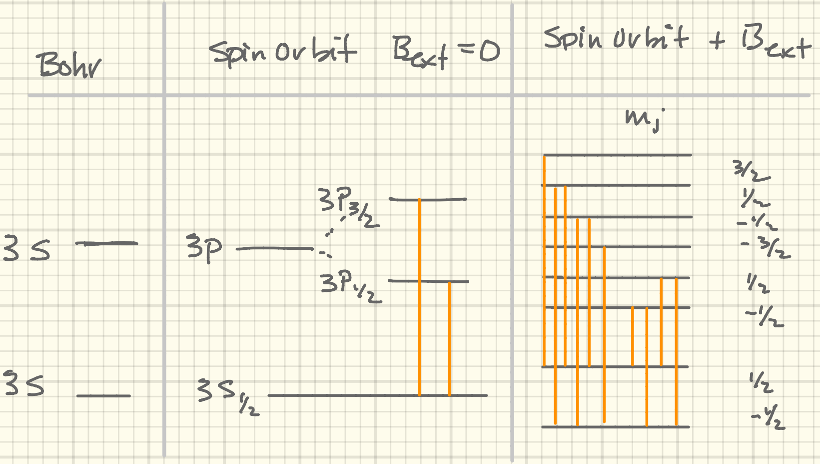

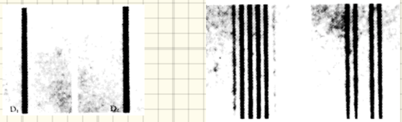

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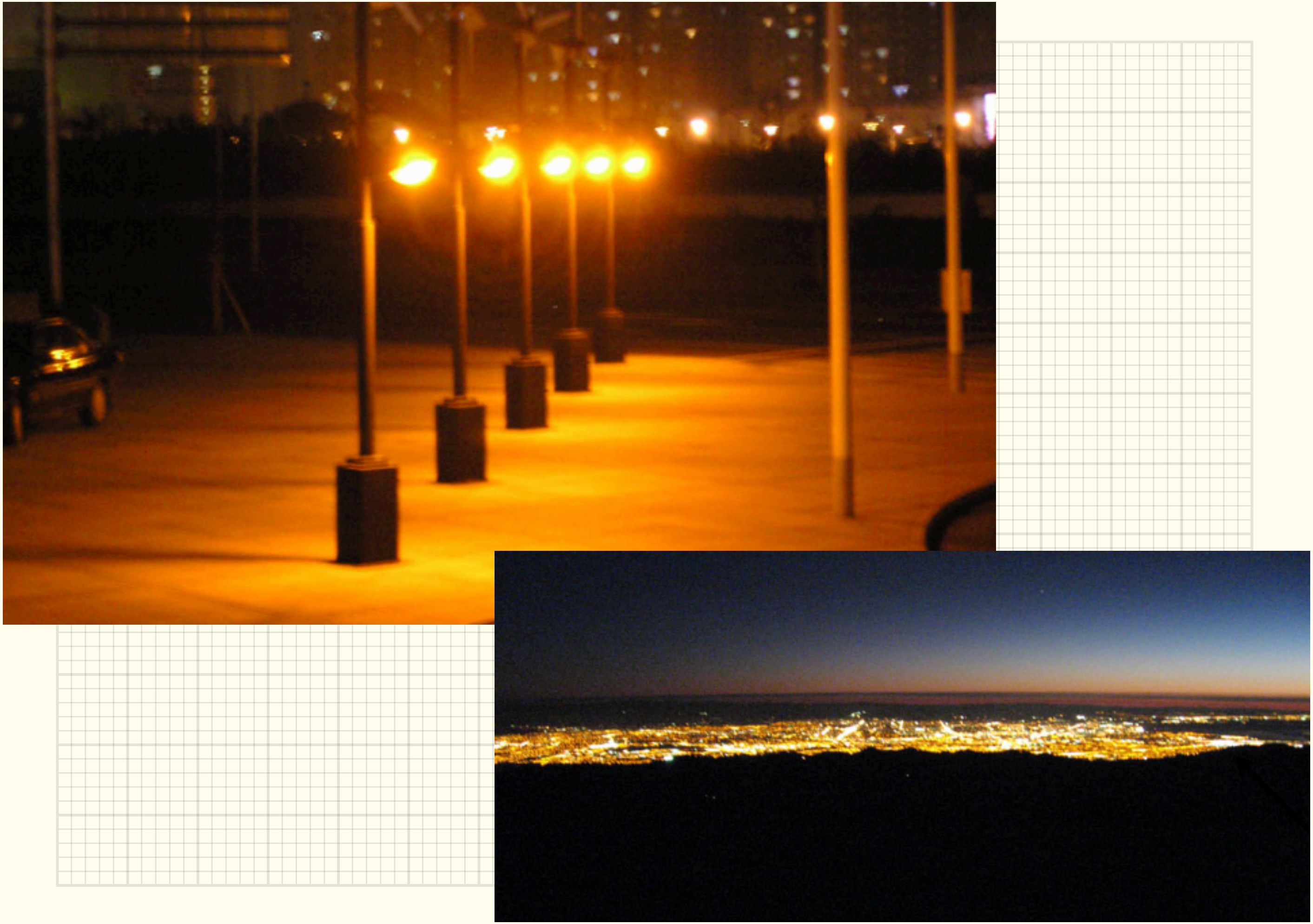
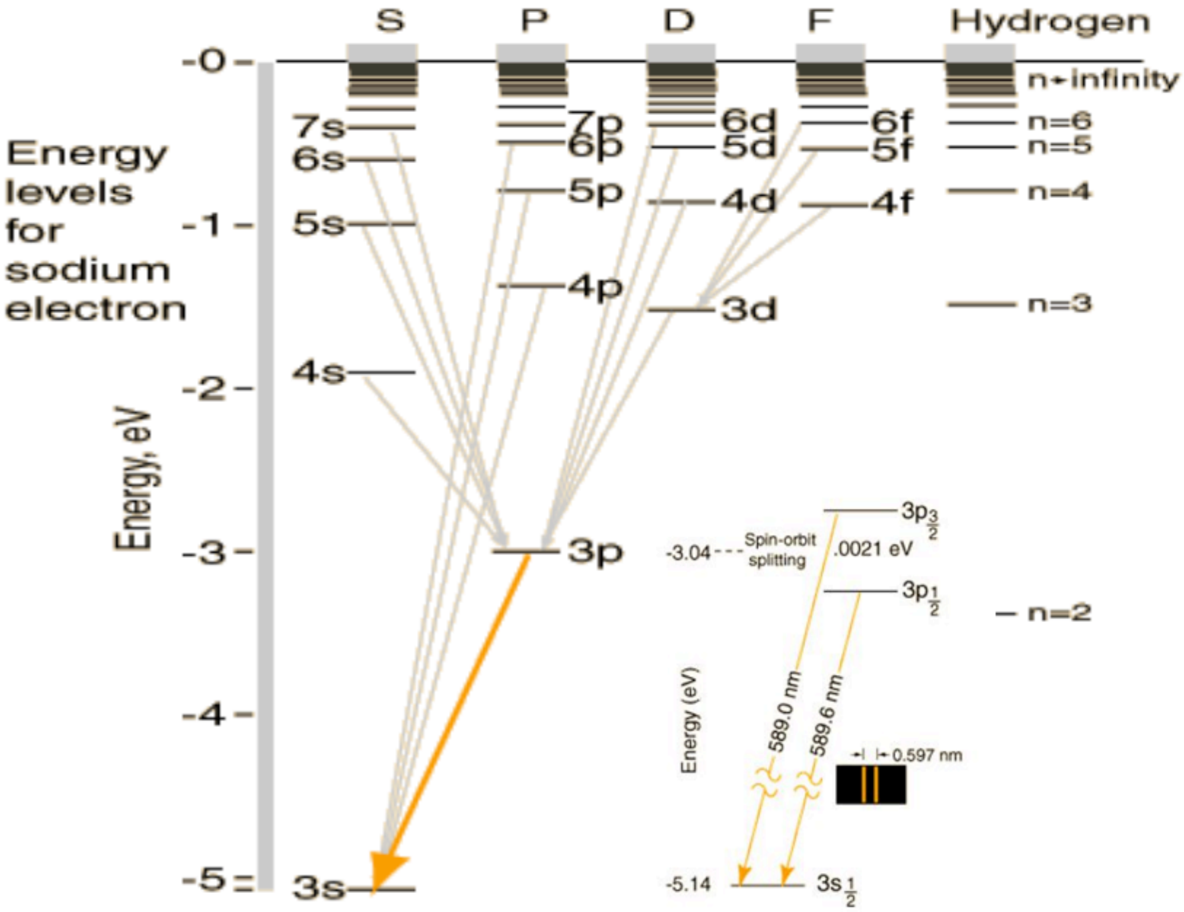
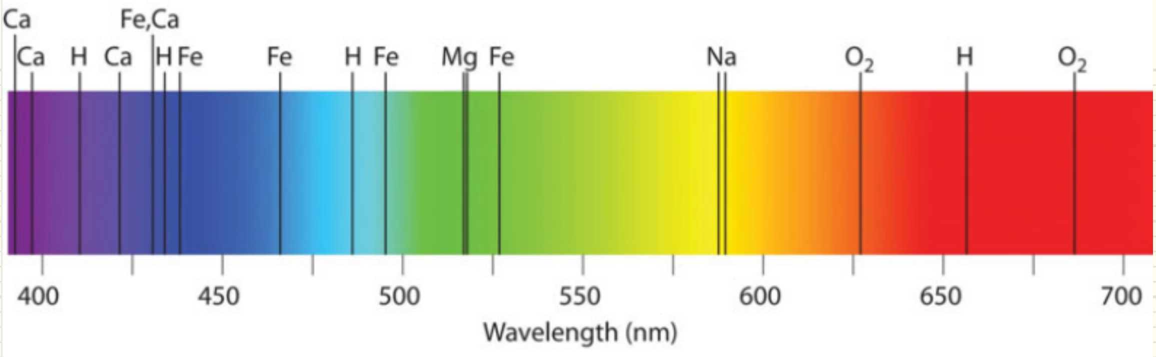
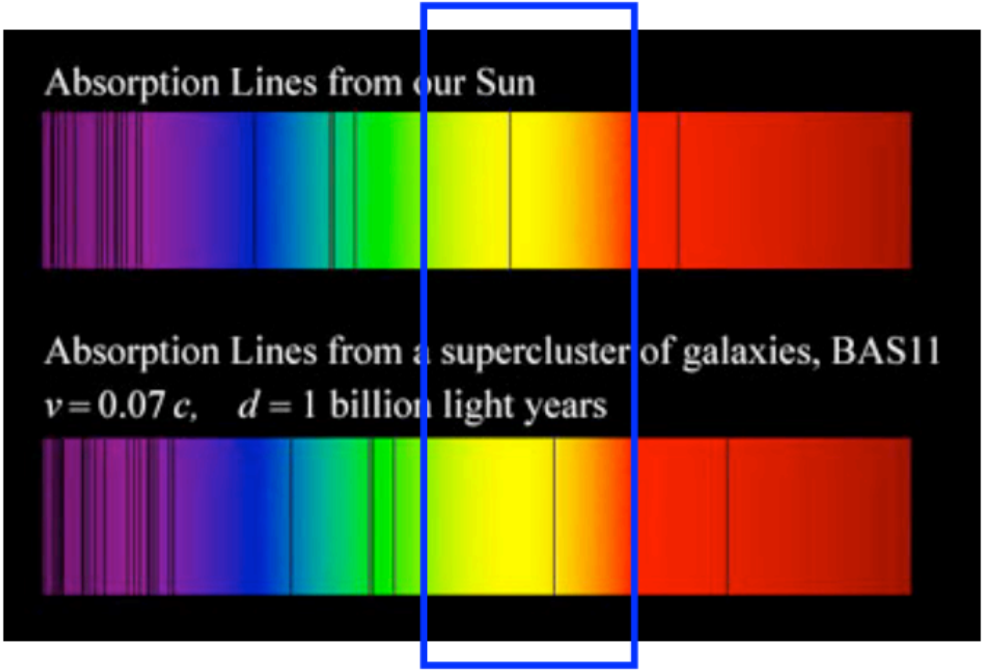



S
P
D
F
Hydrogen
Energy levels for sodium electron
Energy, eV
-0
-1
-2
-3
-4
-5
n→infinity
7s
6s
5s
4s
3s
7p
6p
5p
4p
3p
6d
5d
4d
3d
6f
5f
4f
n=6
n=5
n=4
n=3
n=2
-3.04
Spin-orbit splitting
.0021 eV
$3p_{\frac{3}{2}}$
$3p_{\frac{1}{2}}$
Energy (eV)
589.0 nm
589.6 nm
0.597 nm
-5.14
$3s_{\frac{1}{2}}$





Absorption Lines from our Sun

Absorption Lines from a supercluster of galaxies, BAS11

$v = 0.07\,c$, $d = 1$ billion light years

Multielectron Atoms

even more complicated

spin-orbit couplings

spin-spin couplings

orbit-orbit couplings

Hund's Rule (of thumb)

Likelihood for quantum numbers to be ordered within a subshell

1. Total spin should be maximized
2. after rule 1, total angular momentum should be maximized

Plausibility argument:

max spin: ↑↑ (for 2) BUT can't happen in same shell → Pauli

so each in different ℓ

Suppose we have 2.

Total orbital momentum $\vec{L} = \vec{L}_1 + \vec{L}_2$

$$|L| = |L_2 - L_1| \; --- \; L_2 + L_1$$

$$L_{max} = L_1 + L_2$$

$$L_{min} = |L_1 - L_2|$$

$$L_z = L_{1z} + L_{2z}$$

$$m_L = m_{L_1} \oplus m_{L_2} = |m_{L_1} - m_{L_2}| \; --- \; m_{L_1} + m_{L_2}$$

ditto for spin $\vec{S} = \vec{S}_1 + \vec{S}_2$ $\qquad s_1 = s_2 = 1/2$

$$S_{max} = 1$$

$$S_{min} = 0$$

$$M_S = 0, 1$$

Hund's Rule language:

1. arrange for $S = M_{S,MAX}$ 2. then arrange for $L = M_{L,MAX}$

Carbon: $1s^2 2s^2 2p^2$

$$(\uparrow,\downarrow) \vdots (\uparrow,\downarrow);\quad (\ ,\ )\quad (\ ,\ )\quad (\ ,\ )$$

$$M_L = -1 \qquad 0 \qquad 1$$

Rule 1.:

$S = S_{MAX} \Rightarrow S = 1 \Rightarrow \uparrow\uparrow$ --- not same, but where?

Rule 2.:

$$(\uparrow,\downarrow) \vdots (\uparrow,\downarrow);\quad (\ ,\ )\quad (\uparrow,\ )\quad (\uparrow,\ )$$

$$M_L = -1 \qquad 0 \qquad 1$$

$\Rightarrow M_L = 1 + 0 = 1 \Rightarrow L = 1$

so ground state is $S = 1$, $L = 1$

Nitrogen $1s^2 2s^2 2p^3$

Rule 1: max spin $\Rightarrow$ $\uparrow \; \uparrow \; \uparrow$ $M_S = 3/2$

Rule 2: max L $\Rightarrow$ $M_L = -1, 0, 1$

$$(\uparrow, \downarrow) \vdots (\uparrow, \downarrow) ; \quad (\uparrow, \;\;) \quad (\uparrow, \;\;) \quad (\uparrow, \;\;)$$

$M_L = -1$ $\qquad$ 0 $\qquad$ 1

$$M_L = -1 + 0 + 1 = 0 \quad \Rightarrow \quad L = 0$$

ground state is $L = 0$, $S = 3/2$ state

C2 Do this for oxygen
next homework set

2 standard ways to combine S and L an atom with n electrons

"L-S coupling" ("Russell-Saunders Coupling")

when spin-orbit is negligible ... relatively light elements

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \ldots \vec{L}_n$$

$$\vec{S} = \vec{S}_1 + \vec{S}_2 \ldots \vec{S}_n$$

Quantum numbers are then

$$\vec{J} = \vec{L} + \vec{S}$$

"J-J coupling"

relatively heavy elements where spin-orbit can be significant

$$\vec{J}_1 = \vec{L}_1 + \vec{S}_2$$

$$\vec{J}_2 = \vec{L}_2 + \vec{S}_2$$

$$\vdots$$

$$\vec{J}_n = \vec{L}_n + \vec{S}_n$$

Quantum numbers come from

$$\vec{J} = \vec{J}_1 + \vec{J}_2 \cdots \vec{J}_n$$

L-S coupling -- some bits. _

each electron has an L_i and S_i

whole state has combined L and S

$$\vec{L} = \vec{L_1} + \vec{L_2} + \dots \vec{L_n}$$

$L =$ set of $|L_n - L_{n-1}| \dots L_{n-1} + L_n$

$S =$ set of $|S_n - S_{n-1}| \dots S_{n-1} + S_n$

when $L \geq S$, the number of J states is $2S+1$

$\longrightarrow$ notation $n\,{}^{2S+1}L_J$

Back to He

ground state $1s^2$

$L_1 = 0 \quad L_2 = 0 \Rightarrow L = 0$

$S_1 = 1/2 \quad S_2 = 1/2 \Rightarrow S = 0, 1$

But $S = 1 \Rightarrow \uparrow\uparrow$ .. so forbidden by Pauli

So $S = 0$ for He : $1\,{}^1S_0$

Excited States of Helium

$\boxed{1s^1 2s^1}$

$(\uparrow,\ ) \,\vdots\, (\uparrow,\ )$

$$\underbrace{\begin{matrix} S = 1 \text{ triplet} \\ L = 0 \end{matrix}}$$

$J = |L-S|, \cdots L+S$

$J = 1$

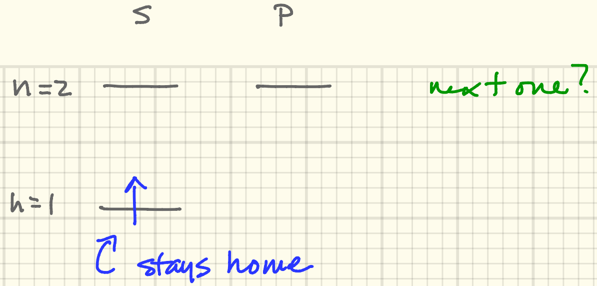
OR

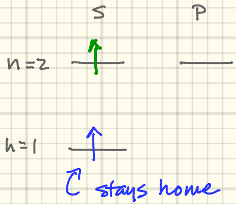
$(\uparrow,\ ) \,\vdots\, (\downarrow,\ )$

$$\underbrace{\begin{matrix} S = 0 \text{ singlet} \\ L = 0 \end{matrix}}$$

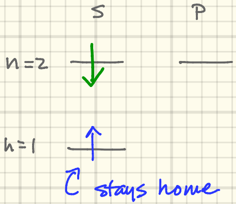
$J = |L-S| \cdots L+S$

$J = 0$





$^3S_1$



$^1S_0$

$\boxed{1s^1 2p^1}$

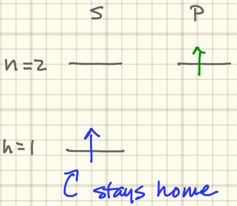
$(\uparrow, \ ) \vdots (\ , \ ) ; (\uparrow, \ )(\ , \ )(\ , \ )$

S = 1 triplet

$\underbrace{L = 1}$

$J = |L-S|, \ldots L+S$

$J = 0, 1, 2$



$^3P_{0,1,2}$

OR

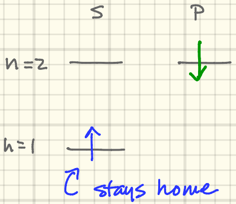
$(\uparrow, \ ) \vdots (\ , \ ) ; (\downarrow, \ )(\ , \ )(\ , \ )$

S=0 singlet

$\underbrace{L = 1}$

$J = |L-S| \ldots L+S$

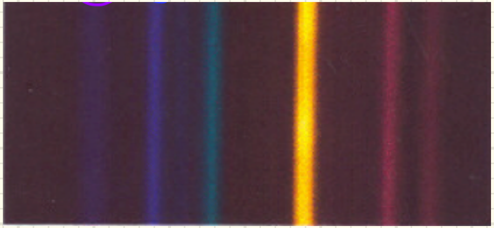
$J = 1$

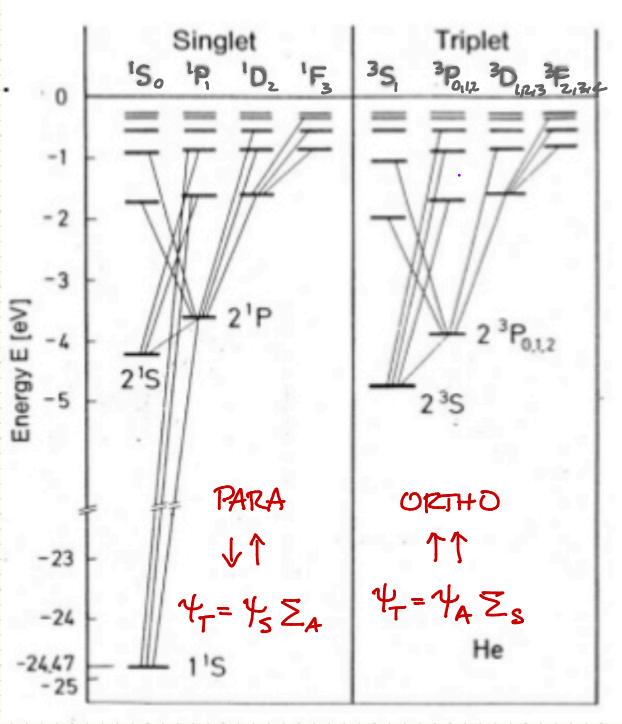


$^1P_1$

Helium discovered 1868 in Sun

during an eclipse ⟶ yellow was not sodium!



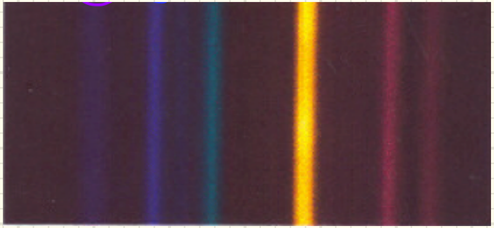


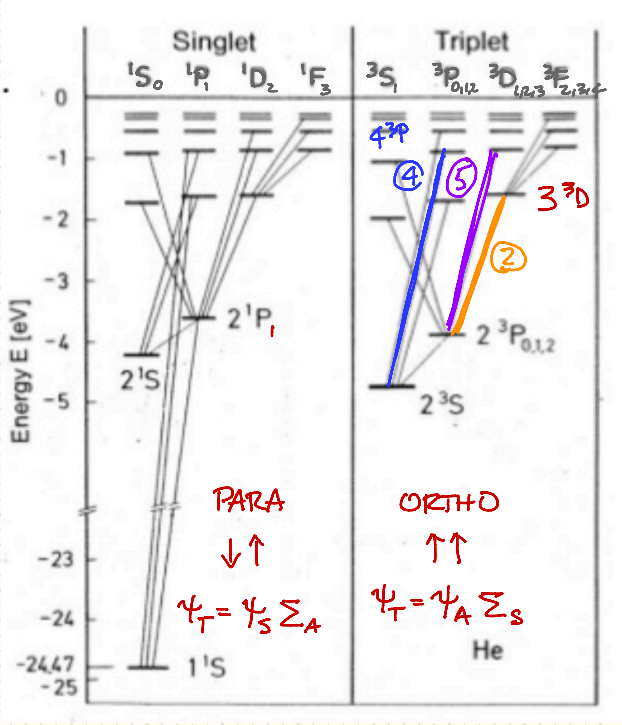
Helium discovered 1868 in Sun

during an eclipse → yellow was not sodium!

$4^3D \to 2^3P$ $4^3P \to 2^3S$ $3^3D \to 2^3P$

⑤ ④ ②





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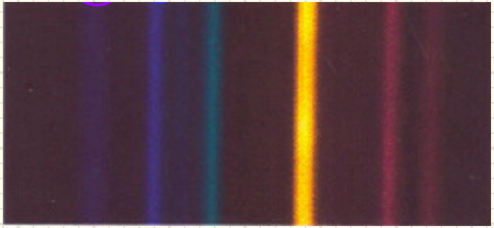
$4^3D \to 2^3P$ $4^3P \to 2^3S$ $3^3D \to 2^3P$

⑤ ④ ②



③ ①

$3^1P \to 2^1S$ $3D^1 \to 2^1P$