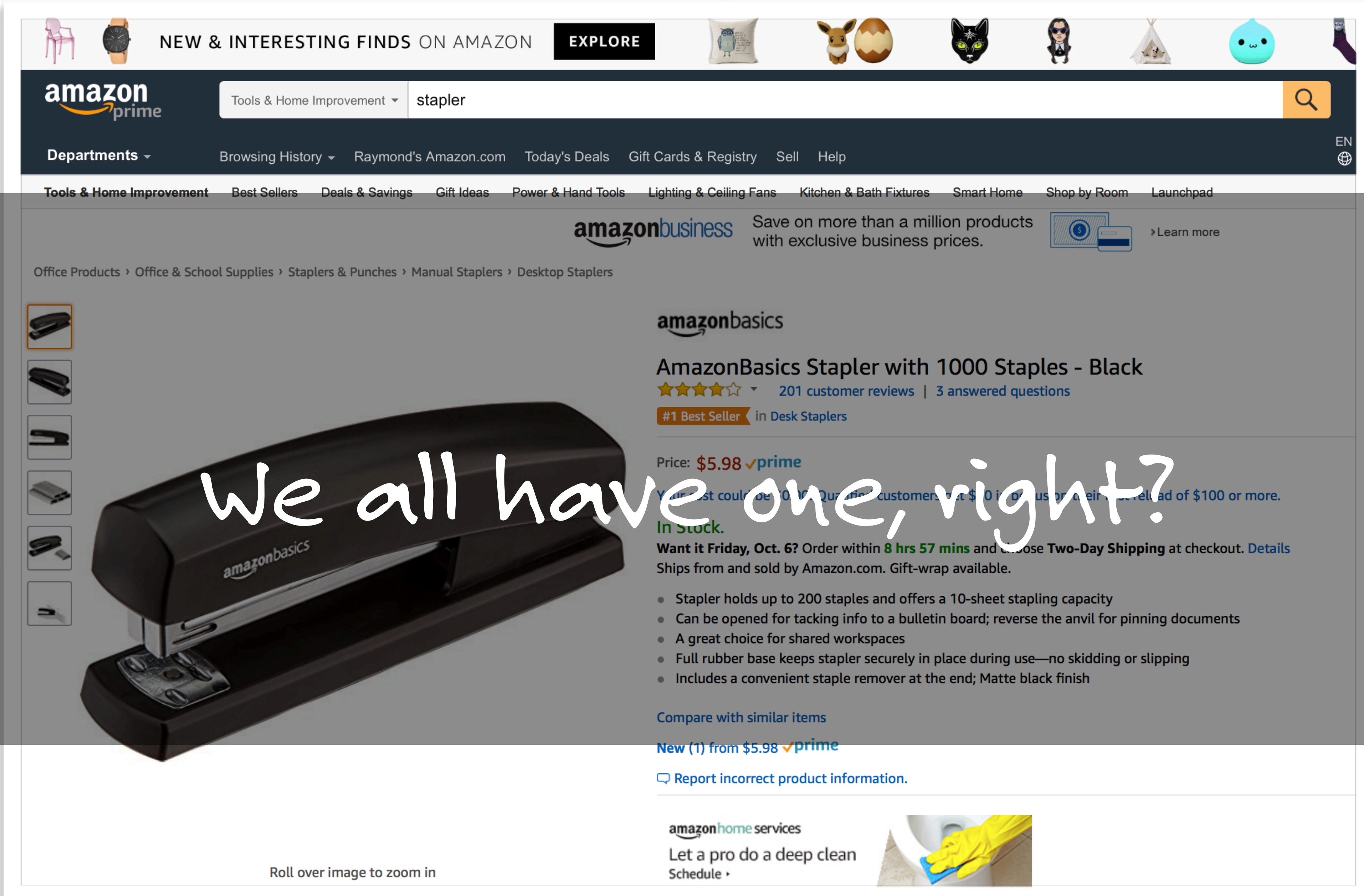
4. Structure of the Atom, 2

lecture 17, October 9, 2017



We all have one, right?

housekeeping

exam 2: Friday, October 27

This week:

- lecture MTW...we'll see about Friday
- HW4 due Friday

Honors option

- Go to: https://qstbb.pa.msu.edu/storage/PHY215/honors/
- read the MinervaInstructions1_2017_215 document

finish

Rutherford Scattering

soliloquy on “cross sections”

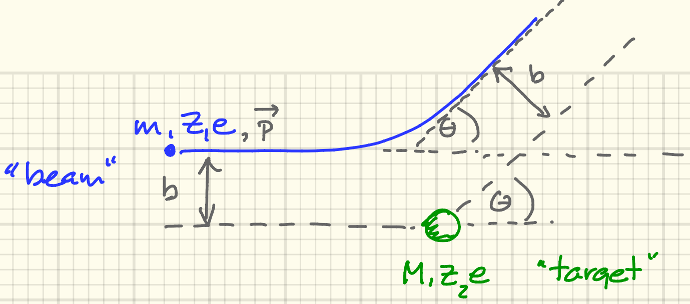
The “Bohr Atom”

finish

Rutherford Scattering

soliloquy on “cross sections”

The “Bohr Atom”



$\theta = 0 \quad \cot\theta/2 = \infty \quad \Rightarrow b = \infty$ forward

$\theta = \pi/2 \quad \cot\theta/2 = 1 \quad b = \dfrac{z_1 z_2 e^2}{4\pi\epsilon_0 m v_0^2}$

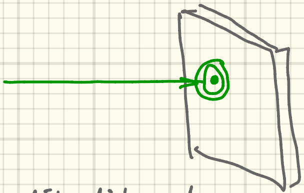
$$b = \frac{z_1 z_2 e^2}{4\pi\epsilon_0 m v_0^2} \cot\theta/2$$

↑ for a single nucleus...

$\theta = \pi \quad \cot\theta/2 = 0 \quad \Rightarrow b = 0$ backward.

smaller b ⇒ larger θ

if b is between b and $b + db$, θ will be between θ and $\theta + d\theta$



What's the likelihood of scattering into a particular direction?

glad you asked.

"cross section"

What impact parameter will cause scattering for α of 7.7 MeV on gold

1° ?

90° ?

$$b = \frac{z_1 z_2 e^2}{4\pi\varepsilon_0 m v_0^2} \cot \theta/_2$$

$K = 7.7$ MeV ... assume classical

$K = \frac{1}{2} m v^2$

$mv^2 = 2K$

$$b = \frac{z_1 z_2 e^2}{4\pi\varepsilon_0 2K} \cot \theta/_2$$

$z_2 = 79$ $\quad z_1 = 2$ $\quad 7.7\text{ MeV} = 1.2 \times 10^{-12}\text{ J}$

$$b = \frac{(2)(79)(1.6\times10^{-19})}{(4\pi)(8.85\times10^{-12})(1.2\times10^{-12}\text{ J})(2)} (\cot \theta/_2) = 1.48\times10^{-14} \cot \theta/_2$$

for 1° $\quad b = 1.7\times10^{-12}$ m

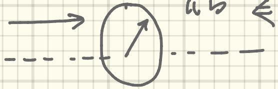
for $\pi/_2$ $\quad b = 1.4\times10^{-14}$ m

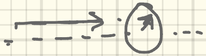
What is the probability for deflection $> 1°$ compared to $> 90°$?

for $1°$ $b = 1.7 \times 10^{-12}\ m$ so $> 1°$, $b < 1.7 \times 10^{-12}\ m$ or

for $\pi/2$ $b = 1.4 \times 10^{-14}\ m$ so $> 90°$, $b < 1.4 \times 10^{-14}\ m$.

or <u>areas</u>

$> 1°$  $\pi b^2 \leq \pi (1.7 \times 10^{-12}\ m)^2$

$> 90°$  $\pi b^2 < \pi (1.4 \times 10^{-14}\ m)^2$

the area corresponding to a given deflection $\equiv$ cross section , σ

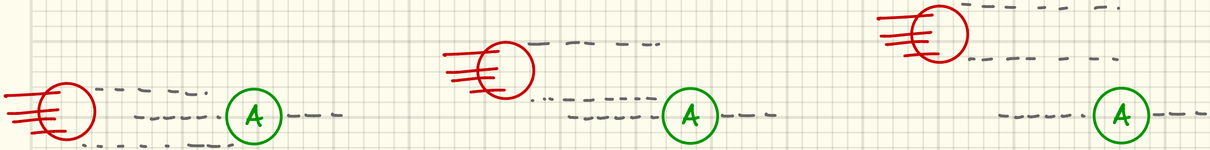
beams are randomly aimed at atom, so

$$\frac{P(>1°)}{P(>90°)} = \frac{\pi (1.7 \times 10^{-12}\ m)^2}{\pi (1.4 \times 10^{-14}\ m)^2} = \frac{14{,}740}{1}$$

so: if 14,740 α's are scattered $> 1°$ ... <u>one</u> will scatter $> 90°$

Cross sections, σ

Imagine two classical billiard balls:

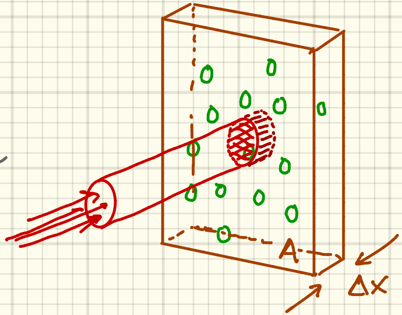


The Area of A and the impact parameter of  determine whether scattering will happen. $\sigma \propto A$

This "idea" $\longrightarrow$ σ in QM scattering

CROSS SECTIONS

slab of material of thickness Δx and area A_t



impinging on it are beam particles confined to an area area A_b

inside are some # of scattering centers with projected areas, $\sigma\,(cm^2)$

the density of scattering centers is $n_t\ (\#tgt/cm^3)$

beam

$$j_b \equiv \text{flux density of beam over } A_b \ \left(\#beam/s\cdot cm^2\right) = n_b\left(\frac{\#beam}{cm^3}\right) v_b\left(\frac{cm}{s}\right)$$

$$J_b \equiv \text{total flux} = j_b A_b = n_b v_b A_b \ \left(\#beam/s\right)$$ .. an intensity.

target

within the beam area A_b, the number of scatterers is

$$N_t = n_t A_b \Delta x$$

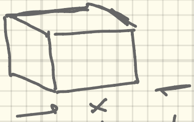
beam particles interact when they overlap with target scatterers

a very classical picture...
"overlap" => some force at work
to cause interaction

probability of scattering

$$dp = \frac{(\text{area of each scatterer})(\text{number of scatters that overlap beam})}{\text{area of the beam}}$$

$$dp = \frac{\sigma N_t}{A_b} = \frac{n_t A_b \, dx \, \sigma}{A_b} = \sigma n_t \, dx$$



change in
number of interacting beam particles $\quad N dp$

$$dN = -N dp = -N \sigma n_t dx$$

$$N(x) = N_0 e^{-\sigma n_t x} = N_0 e^{-x/\lambda} \qquad \lambda \equiv \frac{1}{\sigma n_t}$$

"interaction length"

number left from
original N_0

The rate of interaction $\dot{N}_e = \frac{dN_e}{dt}$ for a flux J_b is

$$\dot{N}_e \left(\frac{\text{events}}{s}\right) = J_b \left(\frac{\#b}{s}\right) \sigma\,(cm^2)\, n_t \left(\frac{\#\text{tgt}}{cm^3}\right) \Delta x\,(cm)$$

$\dot{N}_e = \mathcal{L}\,\sigma$ where $\mathcal{L}$ = "instantaneous luminosity"

$$\mathcal{L} \equiv J_b\left(\frac{\#b}{s}\right) n_t \left(\frac{\#\text{tgts}}{cm^3}\right) \Delta x\,(cm)$$

L = total or "integrated luminosity"

$$L \equiv \int \mathcal{L}\,dt \rightarrow \mathcal{L}\,\Delta t$$

Δt ↳ some time interval

$$N_e = \dot{N}_e \Delta t = \mathcal{L}\,\sigma\,\Delta t$$

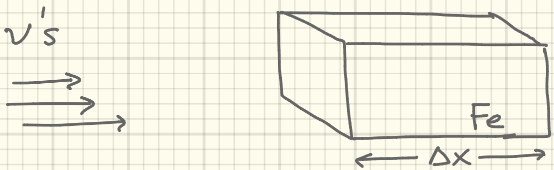
total events for $ba \rightarrow c$ depends on

1) Nature $\sigma(ba \rightarrow c)$

2) Experimental arrangement J_b, n_t, Δt_e, ...

$$N_e = \dot{N}_e \Delta t = L\,\sigma(ba \rightarrow c) = \mathcal{L}\,\sigma(ba \rightarrow c)\,\Delta t$$

$$N_e = J_b(b)\, n_t(a)\, \sigma(ba \rightarrow c)\, \Delta x\, \Delta t_e$$



- How thick must an iron detector be to get 1 neutrino interaction per minute?
  - What's the instantaneous luminosity?

$\sigma(\nu N) = 10^{-40}\ \text{cm}^2$ $\qquad \rho$ iron $7.87\ \text{g/cm}^3$

$J = 10^{10}\ \frac{\text{neutrinos}}{\text{s}}$ $\qquad \Delta t = 60\ \text{s}$

$$N_e = \dot{N}_e \Delta t = J_b\, n_t\, \Delta x\, \Delta t\, \sigma(\nu N)$$

$$\Delta x = \frac{N_e}{J_b\, n_t\, \Delta t\, \sigma(\nu N)}$$

$$\Delta x = \frac{1}{(10^{10})(8.5\times10^{22})(60)(10^{-40})}$$

$$= \frac{1}{5.1\times10^{-6}}$$

$$\Delta x = 196{,}078\ \text{cm} \simeq 2000\ \text{m} = 2\ \text{km}$$

need n_t for iron.

$$n_t = \frac{(\text{density})}{(\text{mass/scatterer})} \longrightarrow \left(\frac{\text{mass}}{\text{mole}}\right)\left(\frac{\text{mole}}{\#\text{tgt}}\right)$$

$$= \frac{\rho}{(M_a/N_A)}$$

$$= \frac{(7.87\ \text{g/cm}^3)}{(56\ \text{g/mole})\left(\frac{1}{6.02\times10^{23}\ \text{tgt/mole}}\right)}$$

$$n_t = 8.5\times10^{22}\ \#\text{tgt}/\text{cm}^3$$

The luminosity $\mathcal{L} = J n_t \Delta x$

$$= (10^{10})(8.5\times10^{22})(196,078)$$

$$\mathcal{L} = 1.67\times10^{38}\ cm^{-2}s^{-1}$$

integrated $L = \mathcal{L}(60) = 9.9\times10^{39}\ cm^{-2}$

Standard units are "barns" for cross sections

$$1\ b = 10^{-28}\ m^2 \left(\frac{100\ cm}{m}\right)^2 = 10^{-24}\ cm^2$$

so our $\sigma(\nu N) = 10^{-40}\ cm^2 \left(\frac{1b}{10^{-24}\ cm^2}\right) = 10^{-16}\ b$

10^{-3}	mb
10^{-6}	μb
10^{-9}	nb
10^{-12}	pb
10^{-15}	fb
10^{-18}	ab

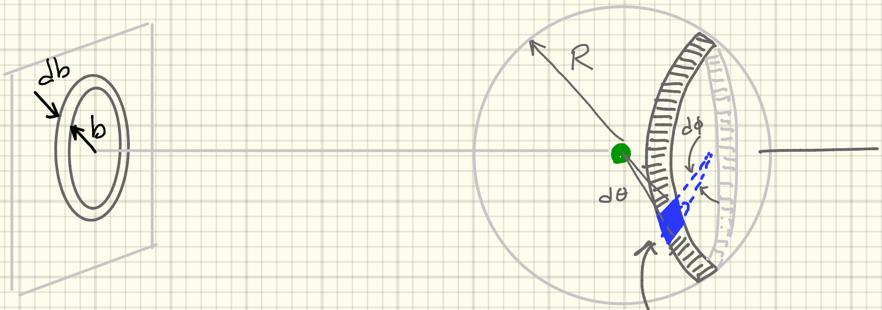
$\sigma(\nu N) = 0.1\ fb = 100\ ab$ tiny.

Standard units for L... inverse-barn-units

$$L\left(\frac{1}{cm^2}\right)\left(\frac{10^{-24}\ cm^2}{b}\right) = L\times10^{-24}\ b^{-1}$$

so: $9.9\times10^{39}\frac{1}{cm^2}\left(\frac{10^{-24}\ cm^2}{b}\right)\left(\frac{10^{-15}\ b}{fb}\right) = 9.9\ fb^{-1}$

What about a real detector?



subtends a solid angle patch

area of patch: $dA = (R\sin\theta\, d\varphi)(R\, d\theta)$

differential solid angle: $d\Omega = \dfrac{dA}{R^2} = \sin\theta\, d\theta\, d\varphi$

Scattering rate into $\Delta\Omega$

$$\dot{N}_e = J_b\, n_t\, \Delta x\, \frac{d\sigma}{d\Omega}\, \Delta\Omega$$

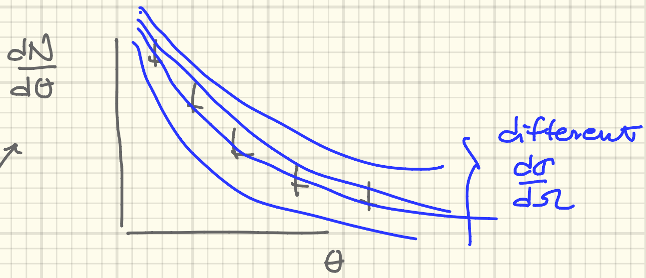
calculated or measured.

$$\dot{N}_e = J_b \, n_t \, \Delta x \, \frac{d\sigma}{d\Omega} \, \Delta\Omega$$

calculated or measured.

$$\Delta\Omega = \frac{\text{area of detector face}}{R^2}$$

$$\frac{dN_e}{d\Omega} = J_b \, n_t \, \Delta x \, \frac{d\sigma}{d\Omega}$$



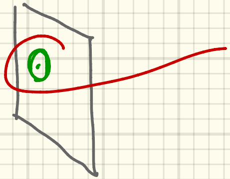
and so on...

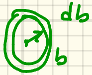
Let's finish our calculation and aim for $\frac{d\sigma}{d\Omega}$ for Rutherford Scattering.

We had: $\dot{N}_e \left(\frac{\text{events}}{s}\right) = J_b \left(\frac{\# b}{s}\right) \sigma n_t \Delta x$ & $b = \frac{c}{2} \cot \theta/2$

$$c = \frac{Z_1 Z_2 e^2}{4\pi \epsilon_0 K}$$

$$p = \frac{\dot{N}_e}{J_b} = \sigma n_t \Delta x$$ $\longrightarrow$ now thin target...





Lots of these little areas around scattering center.

$$\sigma = \pi b^2$$

$$d\sigma = 2\pi b \, db$$

$$db = \frac{c}{4}\left(-\csc^2 \theta/2\right) d\theta$$

$$dp = n_t \Delta x \, d\sigma$$

$$dp = n_t \Delta x \, 2\pi b \, db$$

$$= -n_t \Delta x \, 2\pi \, \frac{c}{2} \cot \theta/2 \, \frac{c}{4} \csc^2 \theta/2 \, d\theta.$$

$$dp = -\frac{n_t \Delta x \, \pi}{4} \frac{\cos \theta/2}{\sin^3 \theta/2} c^2 \, d\theta$$

$$\dot{N}(\theta)_e \, d\theta = -J_b \, dp$$

a decrease in b $\rightarrow$ increase in $d\theta$

1 $$\dot{N}(\theta)_e \, d\theta = J_b n_t \Delta x \frac{\pi}{4} \frac{\cos \theta/2}{\sin^3 \theta/2} c^2$$

$$= J_b n_t \Delta x \frac{\pi}{8} \frac{\sin\theta}{\sin^4 \theta/2} c^2 d\theta$$

remember: 3

Scattering rate into $\Delta\Omega$

$$\dot{N}_e = J_b n_t \Delta x \frac{d\sigma}{d\Omega} \Delta\Omega$$

calculated or measured

plus: $d\Omega = 2\pi \sin\theta \, d\theta = \sin\theta \, d\theta \, d\phi$ 4

we can

2 identify $$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \left(\frac{z_1 z_2 e^2}{4K}\right) \frac{1}{\sin^4 \theta/2}$$

go from 1 to 2 using 3 and 4

D5

Rutherford Cross Section

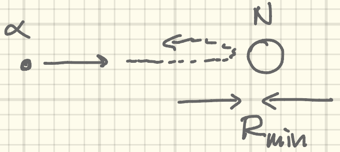
A touchstone for scattering

a taste...

→ assumes the "atoms" are point-like

- classical → can be extended to QM... same form
- non-relativistic → can be made relativistic

How close could Rutherford et al. go?



$E_B = E_A$

just stops

$KE \rightarrow PE$

$$K_\alpha = \frac{1}{4\pi\epsilon_0}\frac{z_1 z_2 e^2}{R_{min}}$$

$$R_{min} = \frac{z_1 z_2 e^2}{4\pi\epsilon_0 K_\alpha}$$

α's : $K = 7.7\ \text{MeV}$

target: gold

$$R_{min} = \frac{(8.99\times10^{9})(2)(79)(1.6\times10^{-19})^2}{(7.7\times10^{6})(1.6\times10^{-19})}$$

$$\simeq 3\times10^{-14}\ \text{m}$$

$$R(\text{gold}) \sim 0.7\times10^{-14}\ \text{m}$$

<u>not</u> penetrating gold nucleus

Relativistic Rutherford: Mott Scattering

still point-like

1960's... deviates from Mott

starting to "see" finite-sized proton

$$\frac{d\sigma}{d\Omega} \propto \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left(\text{form factors}\right)$$

"size" of proton

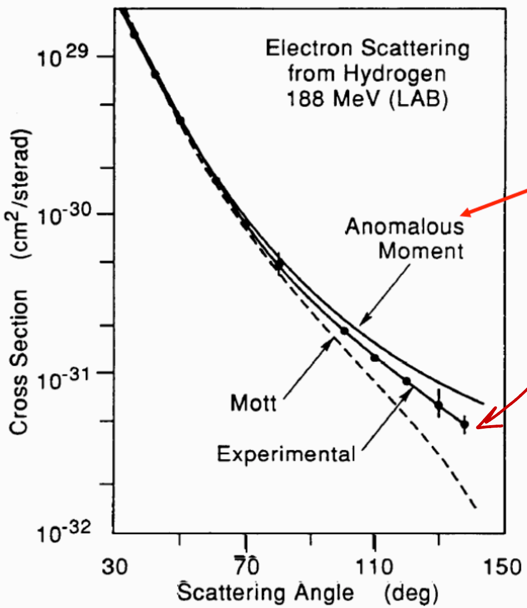


Fig. 5. **Elastic electron scattering cross sections from hydrogen compared with the Mott** scattering formula (electrons scattered from a particle with unit charge and no magnetic moment) and with the Rosenbluth cross section for a point proton with an anomalous magnetic moment. The data falls between the curves, showing that magnetic scattering is occurring but also indicating that the scattering is less than would be expected from a point proton.

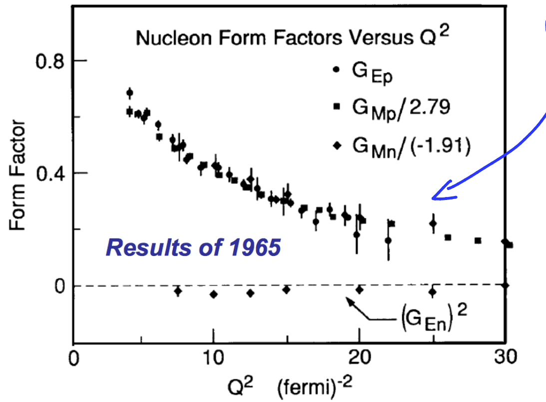


Fig. 23. Summary of results on nuclear form factors presented by the Stanford group at the 1965 "International Symposium on Electron and Photon Interactions at High Energies". (A momentum transfer of 1 GeV^2 is equivalent to 26 Fermis^{-2}.)

$\frac{\sigma}{\sigma_{Mott}}$

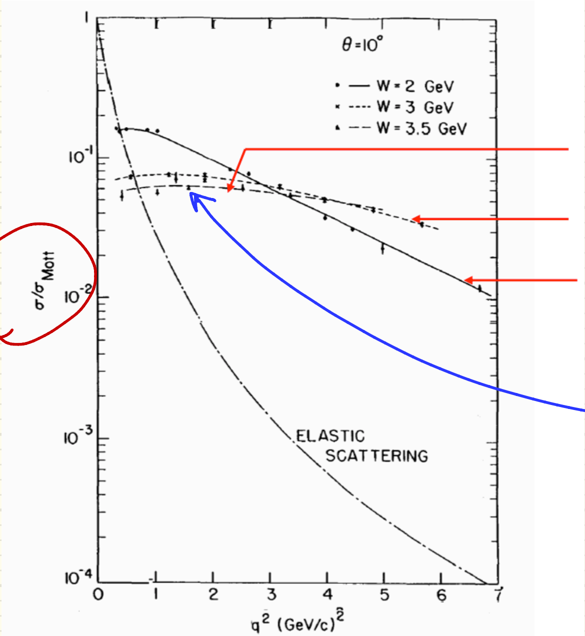


Fig. 1: $(d^2\sigma/d\Omega dE')/\sigma_{Mott}$, in GeV^{-1}, vs. q^2 for W = 2, 3 and 3.5 GeV. The lines drawn through the data are meant to guide the eye. Also shown is the cross section for elastic e-p scattering divided by σ_{Mott}, $(d\sigma/d\Omega)/\sigma_{Mott}$, calculated for $\theta = 10°$, using the dipole form factor. The relatively slow variation with q^2 of the inelastic cross section compared with the elastic cross section is clearly shown.

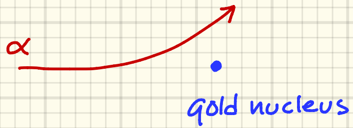
high energy.

medium energy

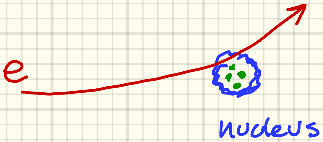
low energy

Mott-like coming back!

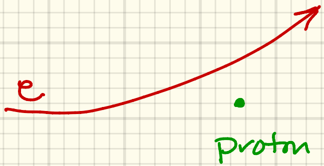
point-like scattering centers inside of proton



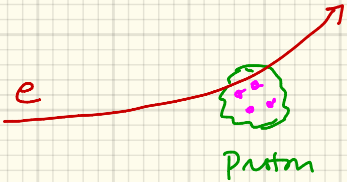
Rutherford — Mott



nucleus ... big — "see" nucleons ... protons & neutrons
↑ !



proton tiny



proton .. big — "see" quarks

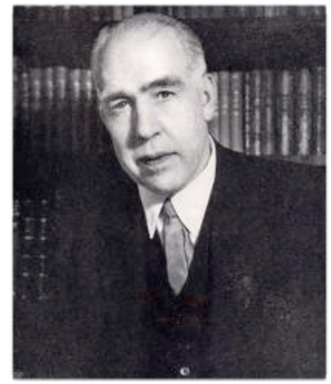
1913

Niels Bohr

1885 – 1962

a talker.

> "The opposite of a profound truth may very well be another profound truth.



Rutherford not disposed kindly

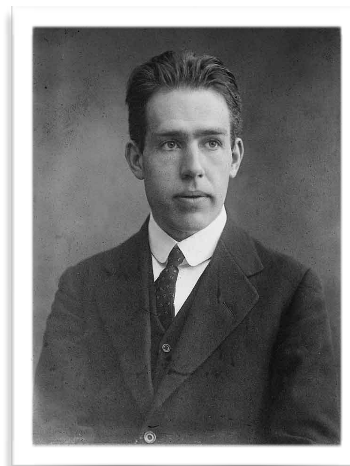
towards theoretical physicists

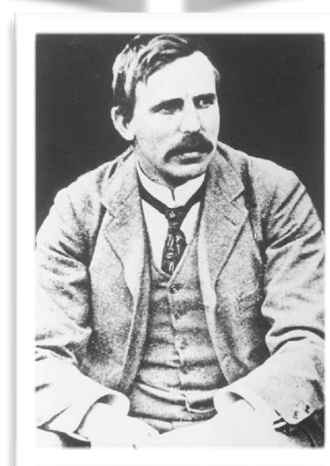
but he saw something in young Bohr

and in 1912 hired him to Manchester

away from a grumpy JJ Thompson

"He's different! He's a football player!"





In 1913 Bohr simply asserted

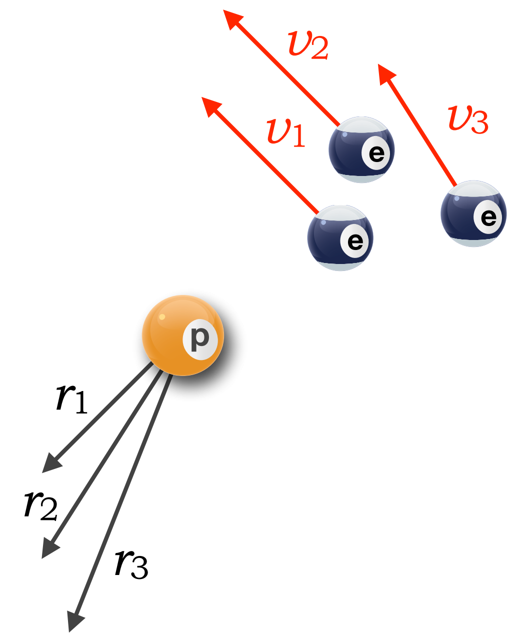
That at atomic distances...

- there are electron orbits that simply don’t radiate - “stationary states”
- fixed “quantized” orbital radii and orbital velocities

The Bohr Model

for any atom with one electron on the outside shell

The Hydrogen Atom (the proton not yet discovered!)



With each radius and velocity...comes a distinct energy.

$$E_n = -\underbrace{\frac{1}{2}\frac{4\pi^2 k^2 e^4}{h^2}}_{\text{just numbers...}}\frac{1}{n^2} = -C\left(\frac{1}{n^2}\right)$$

$$E_n = -(13.6)\frac{1}{n^2}\ \text{eV}$$

Hydrogen spectrum

light emitted by Hydrogen was at particular wavelengths...

in 1885 Johann Balmer played and found a pattern:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2} - \frac{1}{n^2} \right) \quad \text{n = 3, 4, 5...}$$

$1.09737 \times 10^{7}\ \mathrm{m}^{-1}$

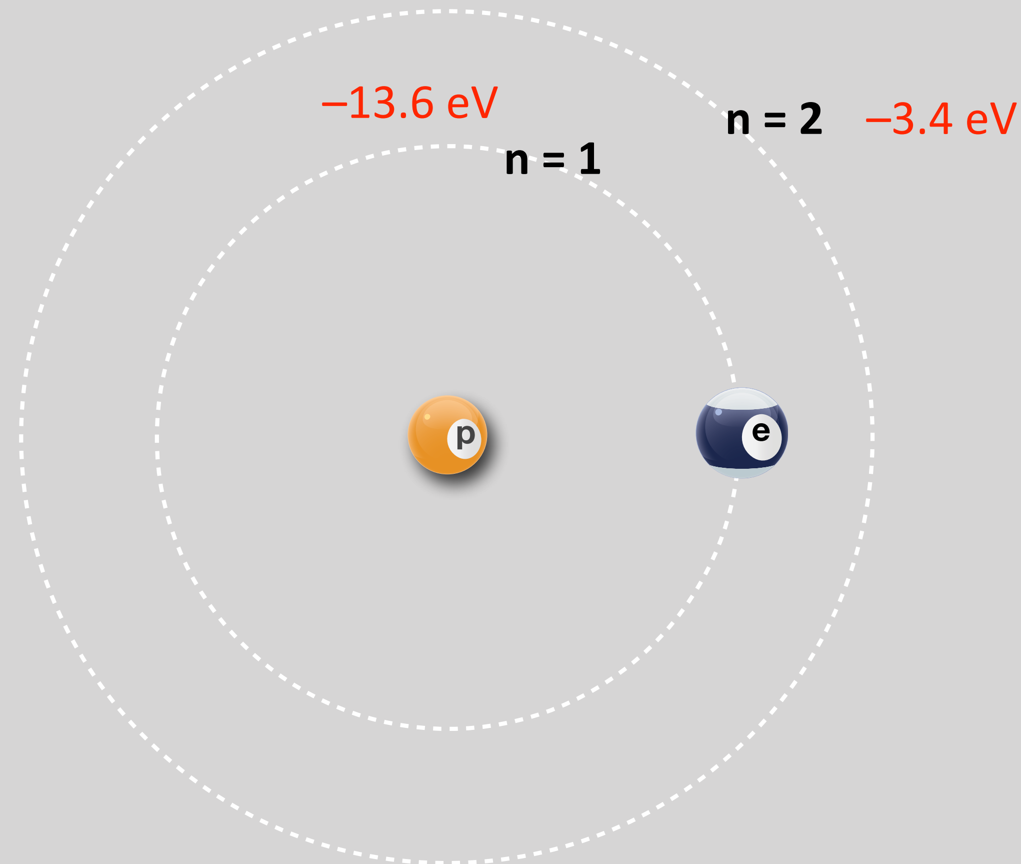
already known

but apparently not by Bohr!

When Bohr learned of the old Balmer idea

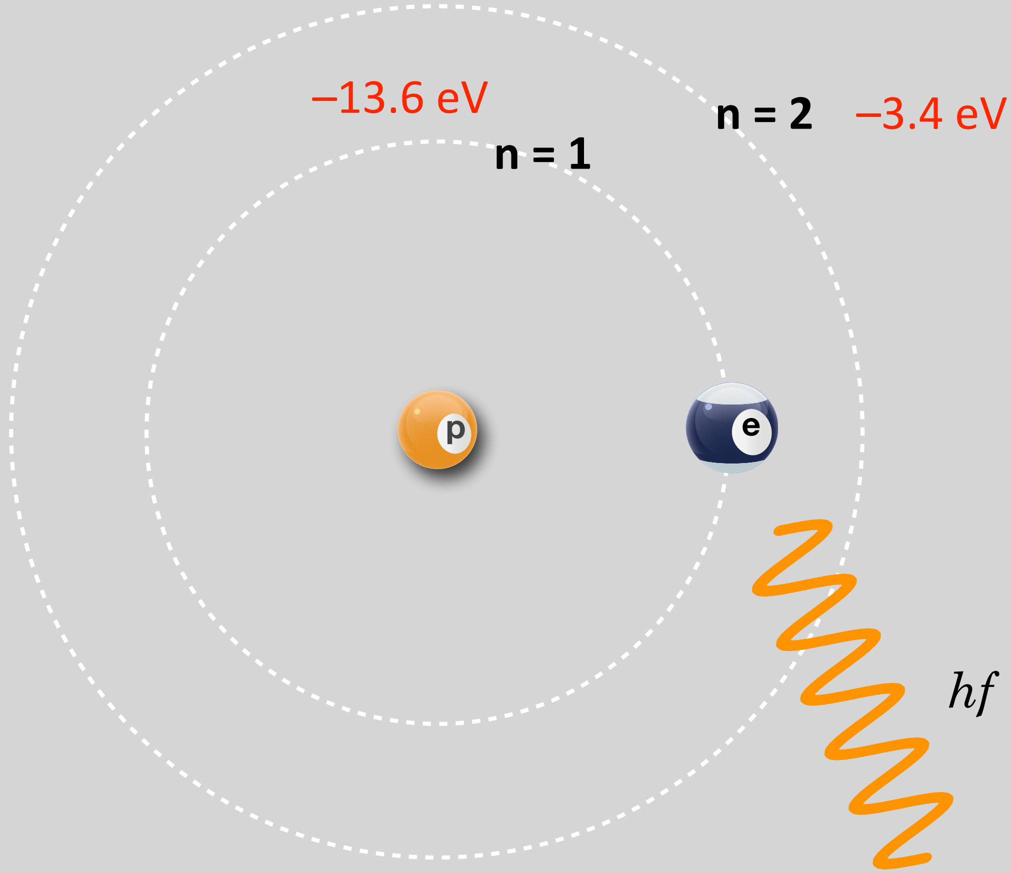
aha! moment

- energy differences could matter



the magic:

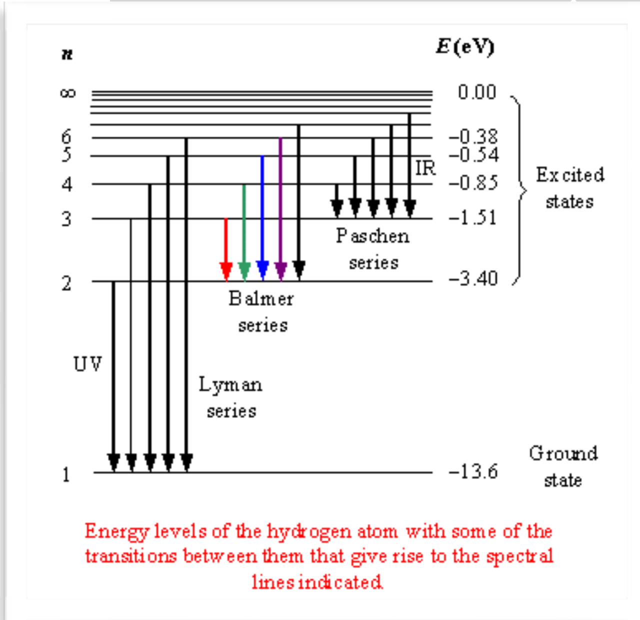
the idea of an atomic transition



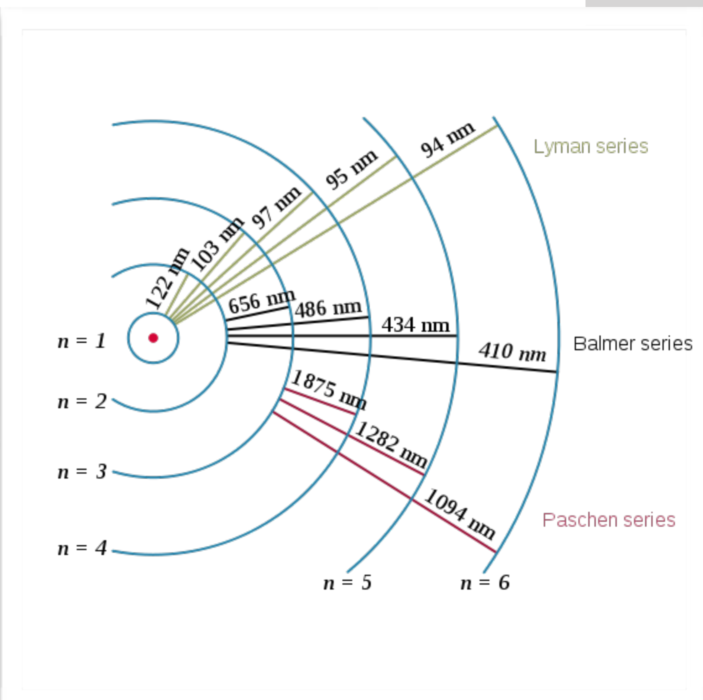
The idea: transition of electrons results in the released energy of a photon...of a particular energy

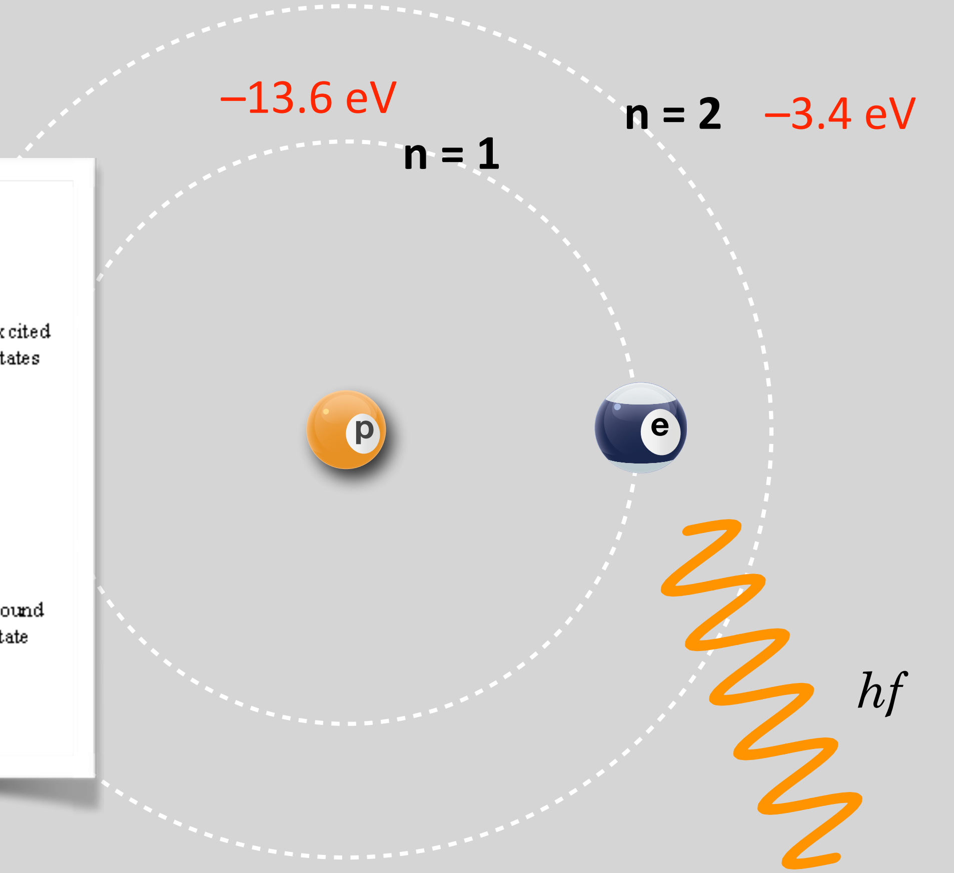
imagine his surprise

1913: his way.



Energy levels of the hydrogen atom with some of the transitions between them that give rise to the spectral lines indicated





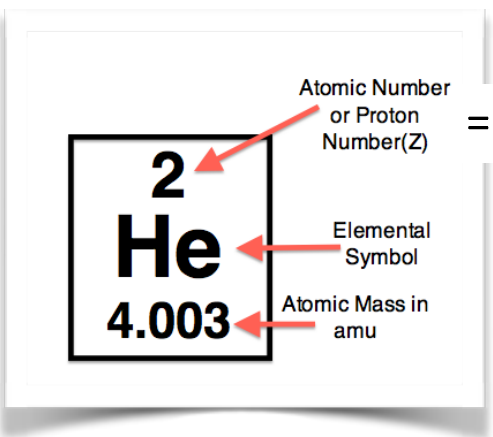
$$E_2 - E_1 = (13.6\ \text{eV})\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = hf$$

$$E_2 - E_1 = 10.1\ \text{eV} \longrightarrow \lambda = 122\ \text{nm}$$

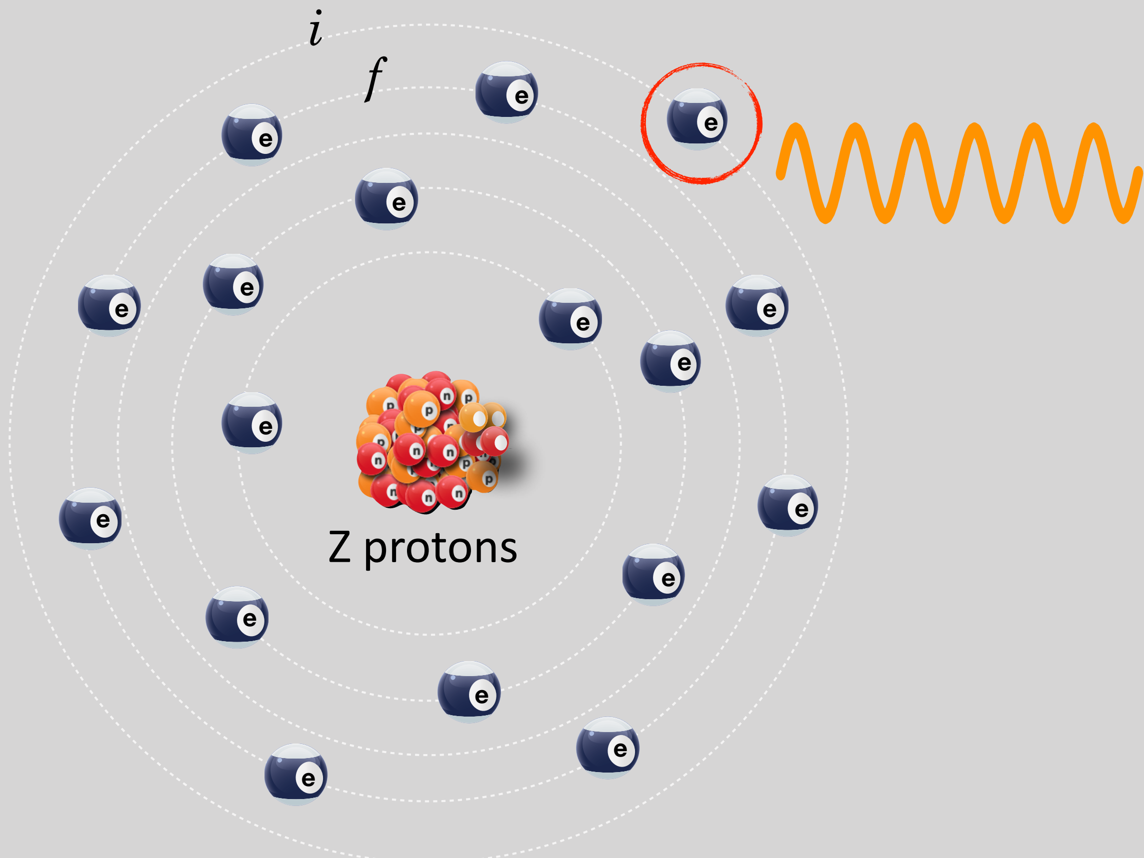
hydrogen, fine

how about more complex elements?

Higher atomic number, Z?



= # of electrons also!



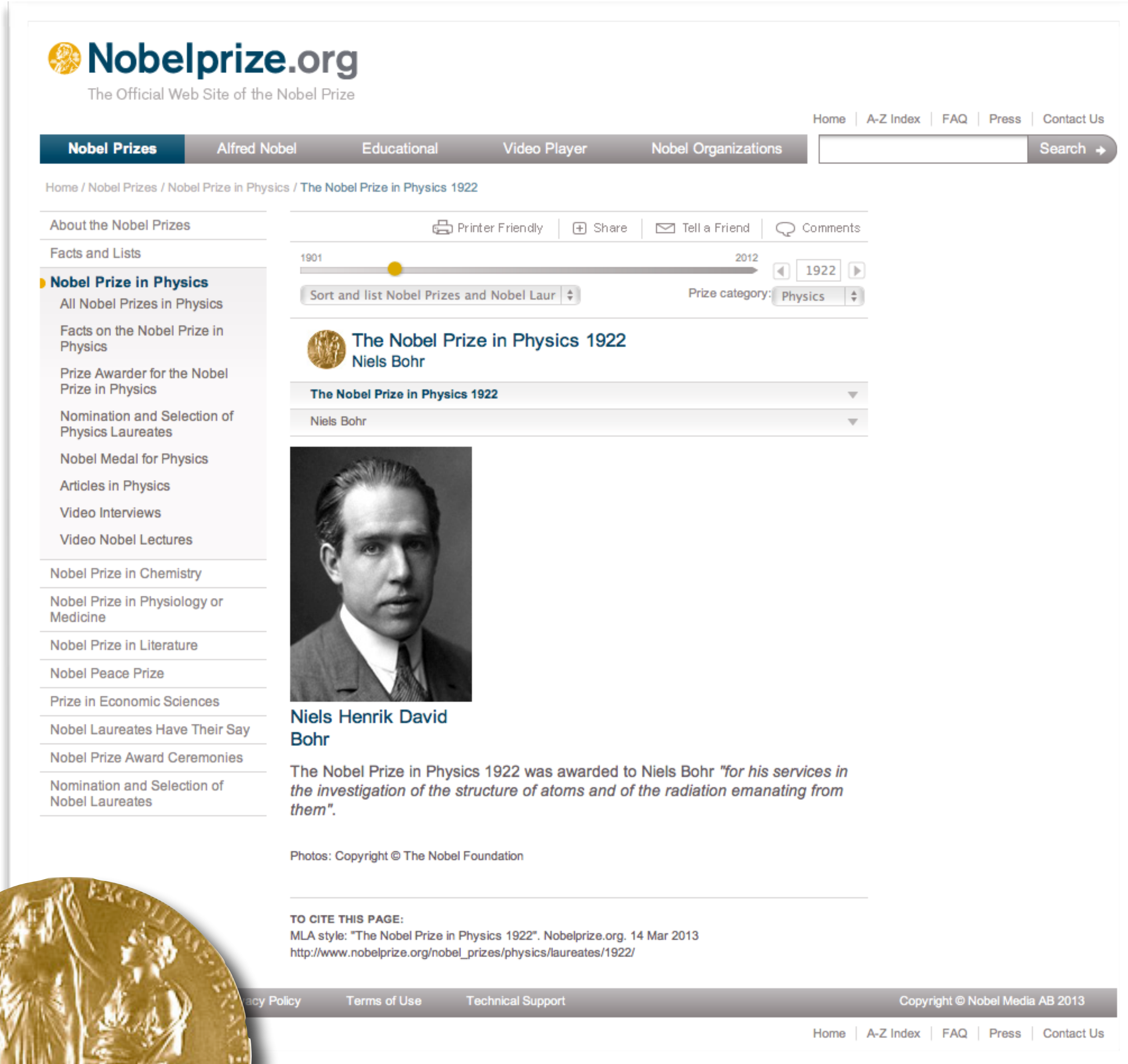
lots of electrons, but as long as there's one lone one..the Bohr Formula still works.

$$E_f - E_i = -\frac{1}{2}\frac{4\pi^2 k^2 Z^2 e^4}{h^2}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = -hf$$

Go looking for new elements....

yup, 1922

actually with Einstein's delayed prize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Niels Henrik David Bohr

The Nobel Prize in Physics 1922 was awarded to Niels Bohr *"for his services in the investigation of the structure of atoms and of the radiation emanating from them".*

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