4. Structure of the Atom, 3

lecture 18, October 10, 2017

housekeeping

I got nothin'

exam 2: Friday, October 27

This week:

- lecture MTW…we'll see about Friday
- HW4 due Friday

Honors option

- Go to: https://qstbb.pa.msu.edu/storage/PHY215/honors/
- read the MinervaInstructions1_2017_215 document

continuing

The “Bohr Atom”

Rutherford not disposed kindly

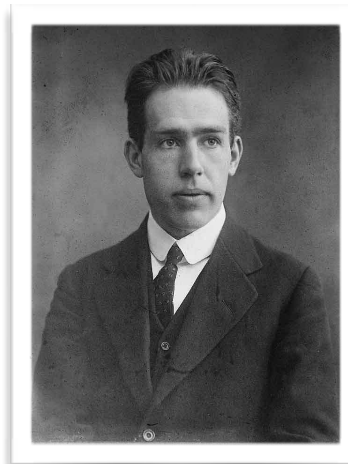
towards theoretical physicists

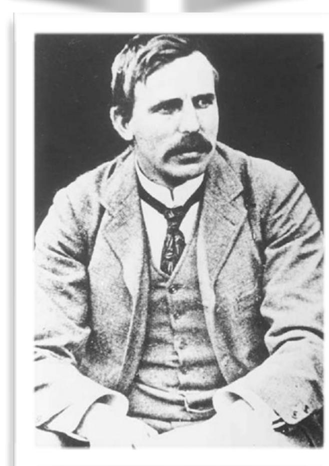
but he saw something in young Bohr

and in 1912 hired him to Manchester

away from a grumpy JJ Thompson

"He's different! He's a football player!"





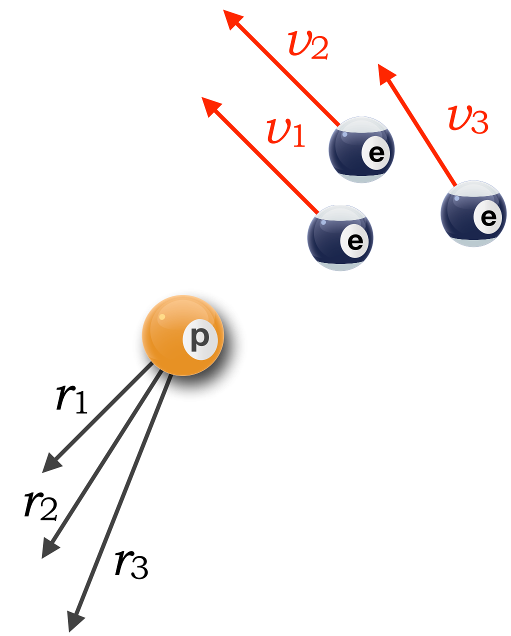
In 1913 Bohr simply asserted

That at atomic distances...

- there are electron orbits that simply don’t radiate - “stationary states”
- fixed “quantized” orbital radii and orbital velocities

The Bohr Model

The Hydrogen Atom (the proton not yet discovered!)



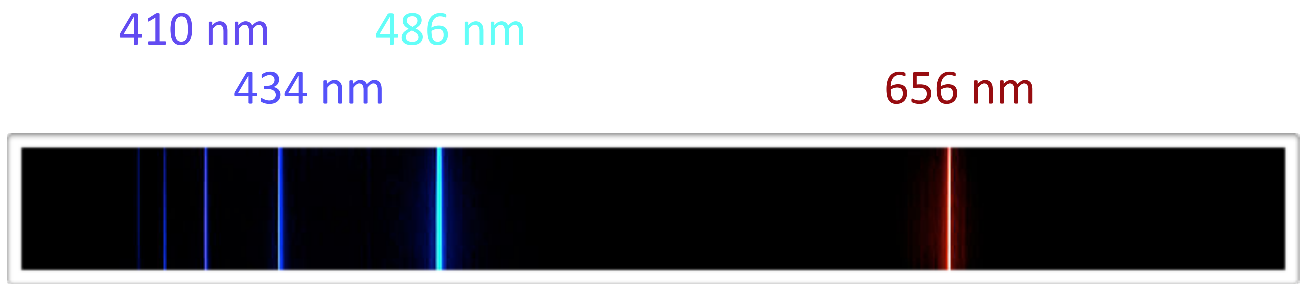
for any atom with one electron on the outside shell

With each radius and velocity...comes a distinct energy.

$$E_n = -\underbrace{\frac{1}{2}\frac{4\pi^2 k^2 e^4}{h^2}}_{\text{just numbers...}}\frac{1}{n^2} = -C\left(\frac{1}{n^2}\right)$$

$$E_n = -(13.6)\frac{1}{n^2}\ \mathrm{eV}$$

Hydrogen spectrum



light emitted by Hydrogen was at particular wavelengths...

in 1885 Johann Balmer played and found a pattern:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2} - \frac{1}{n^2} \right) \quad \text{n = 3, 4, 5...}$$

1.09737 x 10^7 m^{-1}

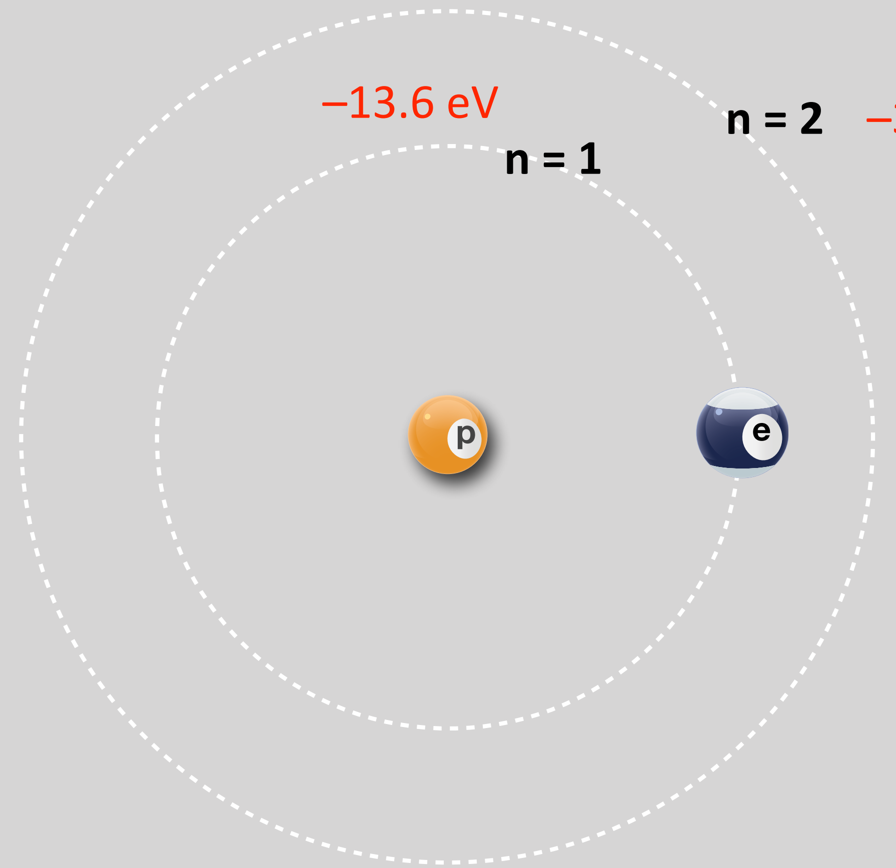
already known

but apparently not by Bohr!

When Bohr learned of the old Balmer idea

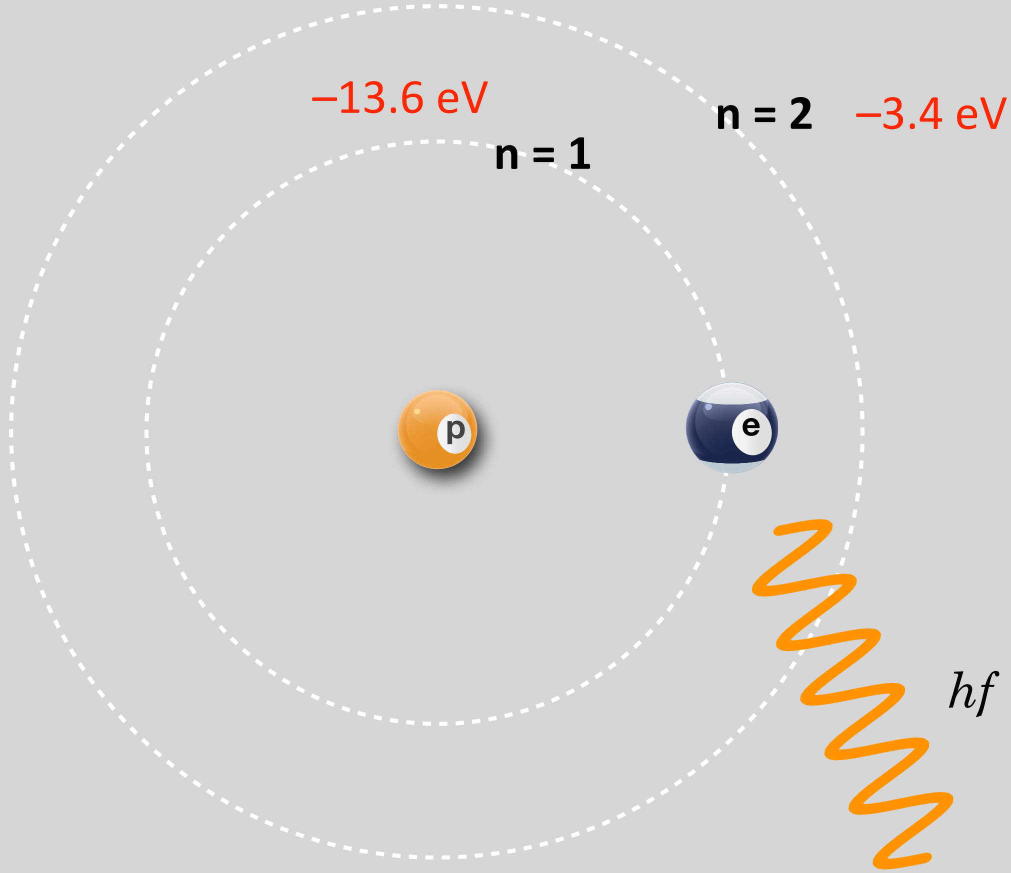
aha! moment

- energy differences could matter



the magic:

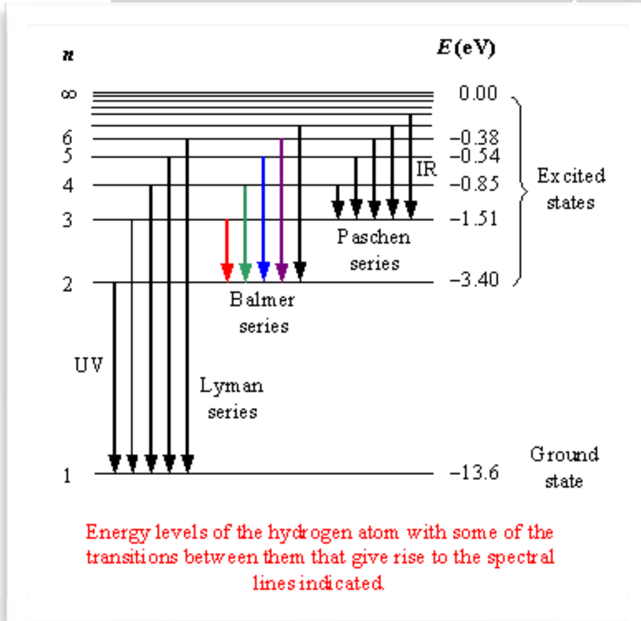
the idea of an atomic transition

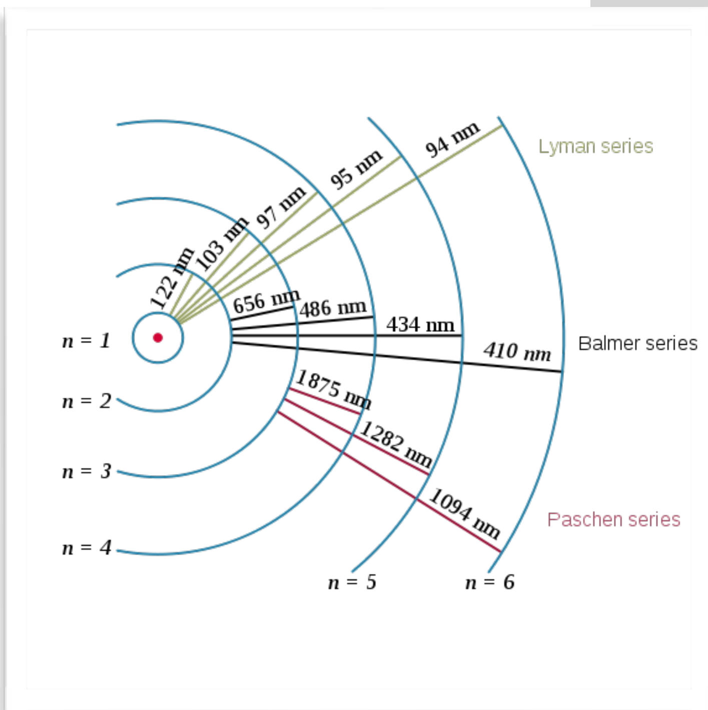


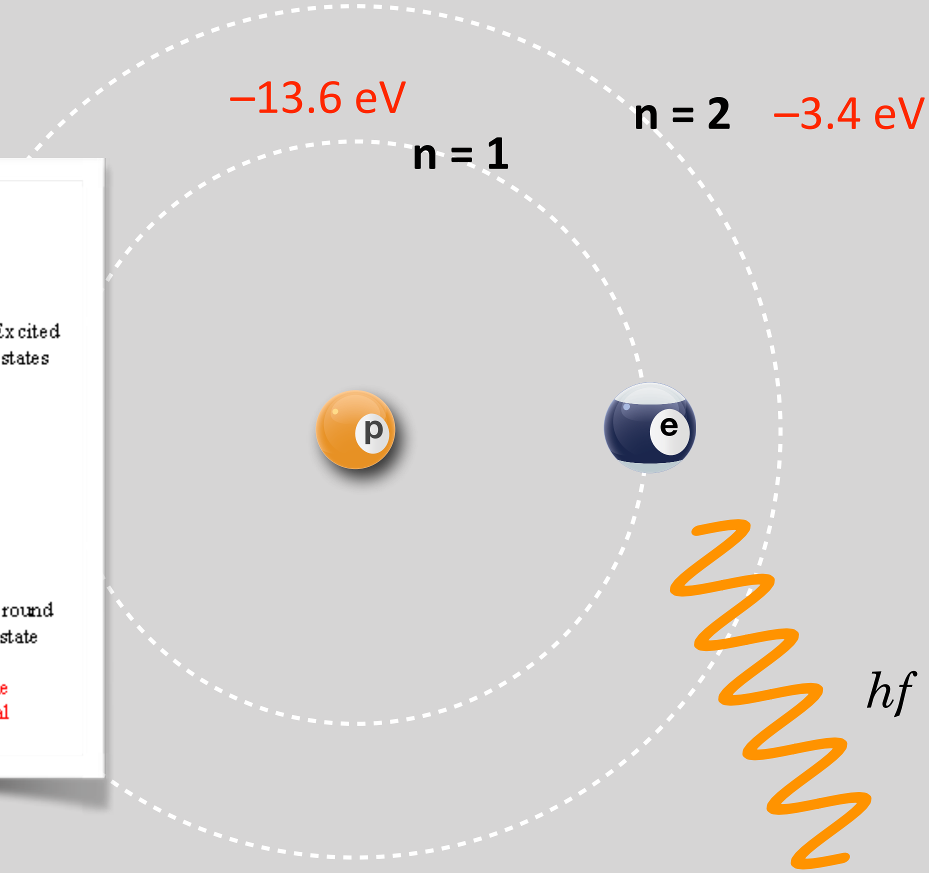
The idea: transition of electrons results in the released energy of a photon...of a particular energy

imagine his surprise

1913: his way.







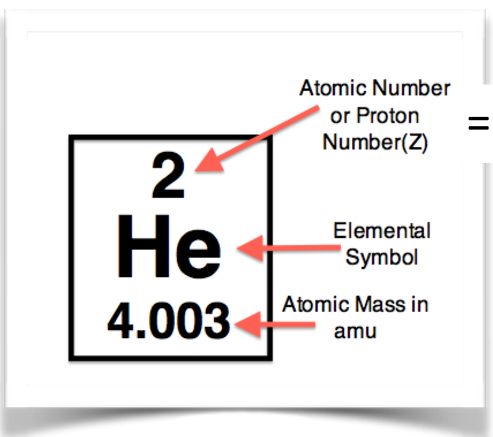
$$E_2 - E_1 = (13.6\ \text{eV})\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = hf$$

$$E_2 - E_1 = 10.1\ \text{eV} \longrightarrow \lambda = 122\ \text{nm}$$

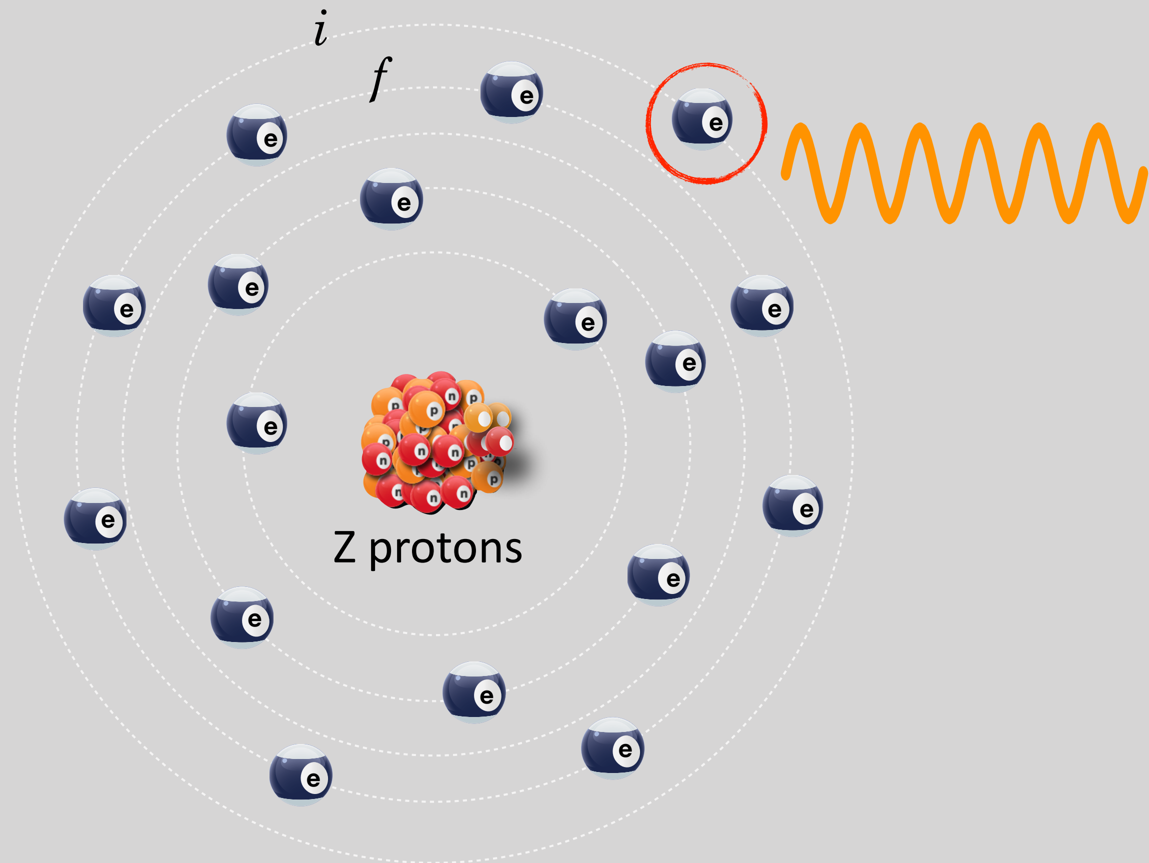
hydrogen, fine

how about more complex elements?

Higher atomic number, Z?



= # of electrons also!



lots of electrons, but as long as there's one lone one..the Bohr Formula still works.

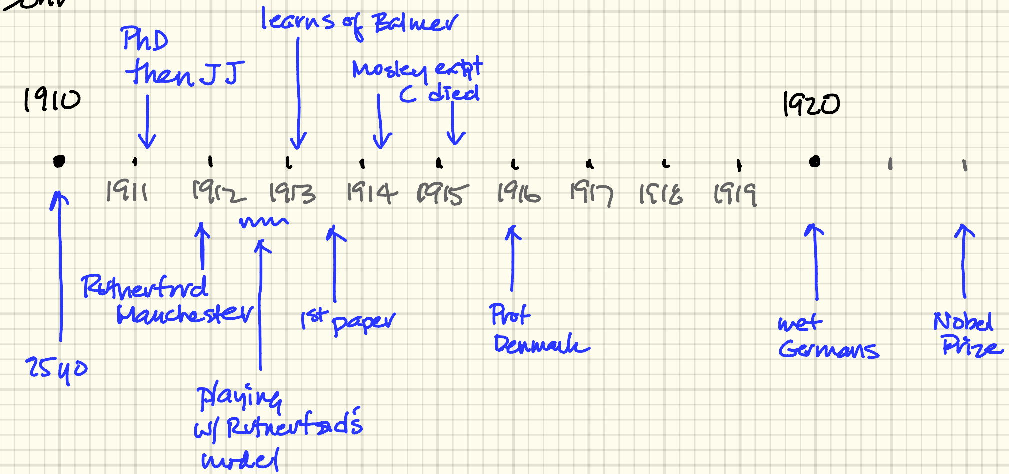
$$E_f - E_i = -\frac{1}{2}\frac{4\pi^2 k^2 Z^2 e^4}{h^2}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = -hf$$

Go looking for new elements....

yup, 1922

actually with Einstein’s delayed prize

Niels Bohr



PhD
then JJ
learns of Balmer
Mosley expt
C died
1910
1920
1911
1912
1913
1914
1915
1916
1917
1918
1919
25 yo
Rutherford
Manchester
playing
w/ Rutherford's
model
1st paper
Prof
Denmark
met
Germans
Nobel
Prize

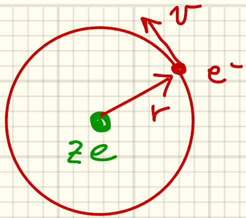
Bohr Model

- electron in atom moves in circular orbits – classical dynamics
- only specific orbits allowed
- electrons do not radiate away energy.
- energy radiated only when changing orbit from high to low

He quantized angular momentum.

$$L = n\hbar \qquad \hbar = \frac{h}{2\pi} \qquad n = 1, 2, 3 \ldots$$

$$hf = E_i - E_f \qquad E_i \text{ energy of } i\text{th orbit}$$



Assume $M_N \gg m_e$

$$\sum F_e = m_e a_e$$

$$\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2} = \frac{mv^2}{r}$$

from $L = n\hbar = mvr$

$$v = \frac{n\hbar}{mr}$$

$$\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r_n^2} = \frac{m}{r_n}\left(\frac{n\hbar}{mr}\right)^2$$

$$r_n = 4\pi\epsilon_0 \frac{n^2\hbar^2}{mZe^2}$$

n: "quantum number"

$$v_n = \frac{n\hbar}{mr_n} = \frac{n\hbar}{m}\frac{mZe^2}{n^2\hbar^2 4\pi\epsilon_0} = \frac{Ze^2}{4\pi\epsilon_0 n\hbar}$$

Energy

$$U = -\int_r^{\infty} \frac{Ze^2}{4\pi\epsilon_0 r^2}\, dr = -\frac{Ze^2}{4\pi\epsilon_0 r}$$

$$K = \frac{1}{2}mv^2 = \frac{Ze^2}{4\pi\epsilon_0 2r}$$

$$E = K + U = -\frac{Ze^2}{4\pi\epsilon_0 2r} = -K \qquad r_n = 4\pi\epsilon_0 \frac{n^2\hbar^2}{mZe^2}$$

use r_n:

$$E_n = -\frac{mZ^2e^4}{(4\pi\epsilon_0)^2 2\hbar^2}\,\frac{1}{n^2}$$

For hydrogen and $n=1$:

$$E_1 = -\frac{(9\times10^9)^2(9.11\times10^{-31})(1.6\times10^{-19})^4}{(2)(1.05\times10^{-34})^2}$$

$$E_1 = -2.17\times10^{-18}\ \text{J} = -13.6\ \text{eV}$$

Radiation ?

$$f = \frac{E_i - E_f}{h}$$

$$f = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{m Z^2 e^4}{4\pi\hbar^3} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$f\lambda = c$$

$$f = \frac{c}{\lambda}$$

$$\frac{c}{\lambda} = \left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{m Z^2 e^4}{4\pi\hbar^3} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\frac{1}{\lambda} = \underbrace{\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{m Z^2 e^4}{4\pi\hbar^3 c}}_{R_\infty} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

R_∞ ... calculated R_H

Rydberg Formula:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n^2} - \frac{1}{n^2}\right)$$

Rydberg Constant for hydrogen $1.0967 \times 10^7\ m^{-1}$

radius:

$$r_n = \frac{n^2}{Z}\left(\frac{\hbar}{mc\alpha}\right)$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}$$

"fine structure constant"

$\simeq \frac{1}{137} \simeq 0.007$

measure of strength of E&M interaction

r_n for hydrogen, $Z=1$

$$r_n = n^2\left(\frac{\hbar}{mc\alpha}\right)$$

$$\underbrace{\left(\frac{\hbar}{mc\alpha}\right)}_{\equiv\, a_0}$$

$a_0 \equiv$ Bohr radius $= \dfrac{1.05\times10^{-34}}{(9.1\times10^{-31})(3\times10^{8})(1/137)}$

$a_0 = 0.53\times10^{-10}\,\text{m}$

so smallest diameter of H is $2a_0 \hat{=} 10^{-10}\,\text{m} \simeq 1\,\text{Å}$

known...

In general:

$$a_n = \frac{n^2}{Z}\,a_0$$

Energies again..

$$E_n = -\frac{e^2 Z^2}{8\pi\epsilon_0 a_0 n^2} = -\frac{\hbar^2 Z^2}{2m a_0^2 n^2}$$

$$E_n = -\frac{1}{2} m \left(\frac{Z\alpha c}{n}\right)^2$$

or

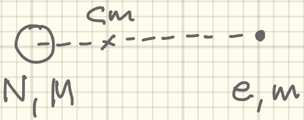
$$E_n = -13.6 \frac{Z^2}{n^2} \text{ eV}$$

Seemed to not work for ionized He.

$$\frac{R_\infty(He)}{R_\infty(H)} = \frac{\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{m Z_{He}^2 e^4}{4\pi\hbar^3 c}}{\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{m Z_H^2 e^4}{4\pi\hbar^3 c}} = \frac{Z_{He}^2}{Z_H^2} = 4$$

expt: 4.0016 ?

Bohr repled:



deal with reduced mass, μ

$$\frac{1}{\mu} = \frac{1}{m} + \frac{1}{M}$$

$$\mu = \frac{mM}{M+m} = \frac{m}{1+m/M}$$

$$\mu \sim m\left(1 - \frac{m}{M}\right)$$

So

$$\frac{R_\infty(\mathrm{He})}{R_\infty(\mathrm{H})} = \frac{\left(1 - \frac{m}{4M_p}\right)}{\left(1 - \frac{m}{M_p}\right)} \frac{Z_{He}^2}{Z_H^2} = 4.00162$$

$\uparrow$
modern -- Bohr: 4.00163

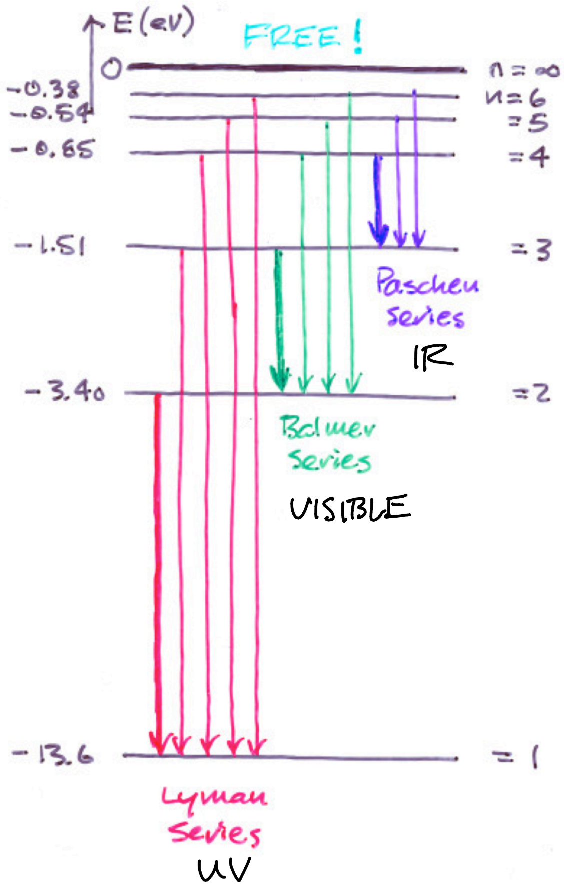
$\Rightarrow$ Instantly! Bohr is a major figure in physics

BOHR'S MODEL:

predicted all possible H spectra... plus He^+

After 1913.. this was all the rage.

how? why?



One particular transition

434nm 486nm 656nm

410nm

Balmer Series

3→2 in H what is λ?

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$= R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\frac{1}{\lambda} = \frac{5}{36} R \Rightarrow \lambda = \frac{36}{5} R^{-1} = \frac{36}{5}\left(1.097 \times 10^{7}\ \mathrm{m}^{-1}\right)^{-1}$$

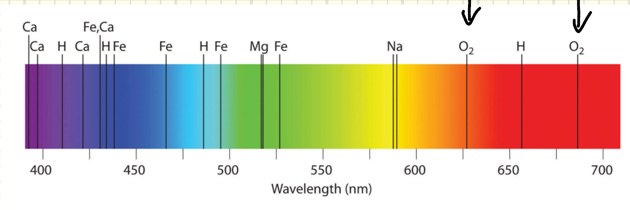
$\lambda = 656$ nm red Balmer α

Its energy?

$$E_\gamma = hf = \frac{hc}{\lambda}$$

$$= \frac{1239.8\ \mathrm{eV\text{-}nm}}{656\ \mathrm{nm}} = 1.89\ \mathrm{eV}$$

earth's atm.



Sun's Absorption spectrum

H.G.J. Moseley

Lecturer at Manchester University

expert in x-ray emission & absorption

Moseley & Bohr talked a lot...

Periodic Table:

Thought to be organized according to atomic mass, A.

So potassium A = 39.10 and argon A = 39.95 should be ordered:

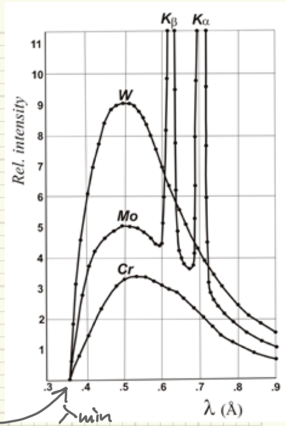
K then Ar

Heavy Atoms — high $Z \Rightarrow$ tightly bound

characteristic x-rays:
(1909)

Remember:

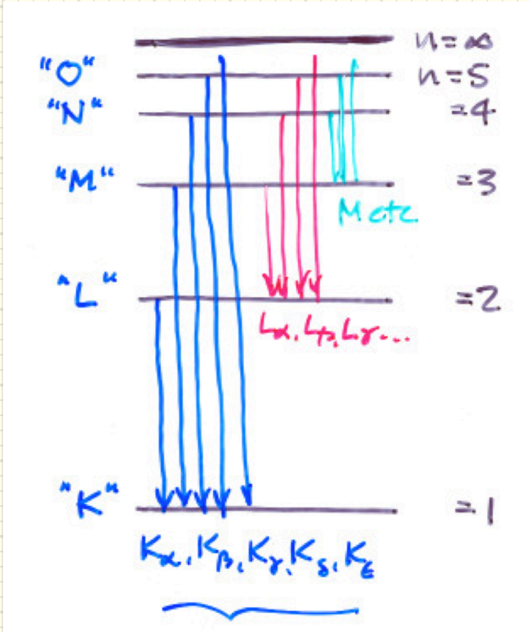
for electron to give up all KE to E_γ



λ_{min}

0.35×10^{-10} m

$\sim 3 \times 10^{-2}$ nm



now we know

Reihen	Gruppo I. — R^2O	Gruppo II. — RO	Gruppo III. — R^2O^3	Gruppo IV. RH^4 RO^2	Gruppo V. RH^3 R^2O^5	Gruppo VI. RH^2 RO^3	Gruppo VII. RH R^2O^7	Gruppo VIII. — RO^4
1	H=1							
2	Li=7	Be=9,4	B=11	C=12	N=14	O=16	F=19	
3	Na=23	Mg=24	Al=27,3	Si=28	P=31	S=32	Cl=35,5	
4	K=39	Ca=40	—=44	Ti=48	V=51	Cr=52	Mn=55	Fe=56, Co=59, Ni=59, Cu=63.
5	(Cu=63)	Zn=65	—=68	—=72	As=75	Se=78	Br=80	
6	Rb=85	Sr=87	?Yt=88	Zr=90	Nb=94	Mo=96	—=100	Ru=104, Rh=104, Pd=106, Ag=108
7	(Ag=108)	Cd=112	In=113	Sn=118	Sb=122	Te=125	J=127	
8	Cs=133	Ba=137	?Di=138	?Ce=140	—	—	—	— — — —
9	(—)	—	—	—	—	—	—	
10	—	—	?Er=178	?La=180	Ta=182	W=184	—	Os=195, Ir=197, Pt=198, Au=199.
11	(Au=199)	Hg=200	Tl=204	Pb=207	Bi=208	—	—	
12	—	—	—	Th=231	—	U=240	—	— — — —

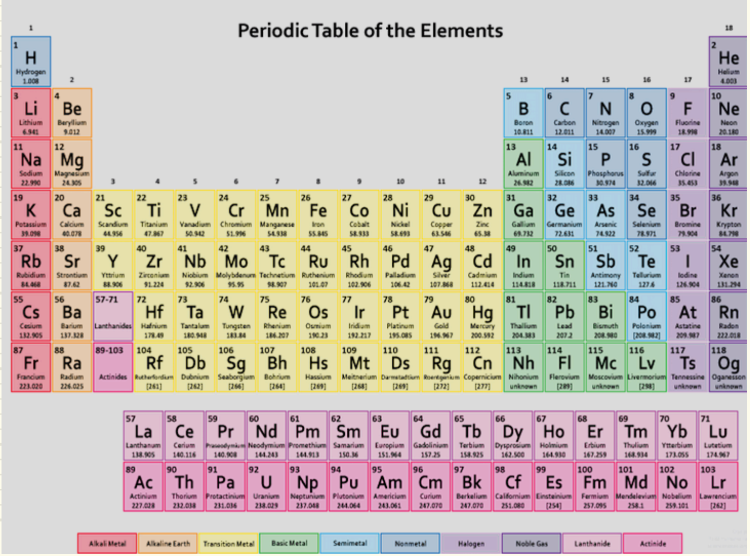
& when Ar included, wrong relative to K

How did we get from here

organized by A

to here

organized by Z

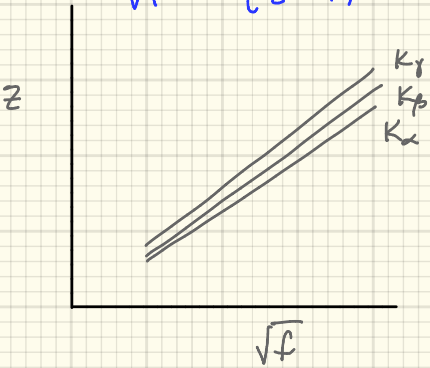


Moseley

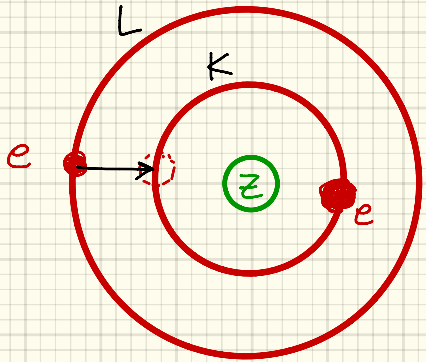
Moseley was without exception or exaggeration the most brilliant man – and the hardest worker I have ever met. There were of course no regular meals, and work often went on for most of the night. Indeed one of Moseley's expertises was the knowledge of where one could get a meal in Manchester at 3 o'clock in the Morning.
Charles Galton Darwin in a 1916 obituary of Harry.

Moseley found empirically

$\sqrt{f} \propto (Z-1)^2$?



linear



Bohr: $f = \text{stuff} \times Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$

clearly informed by Bohr.

$(Z-1)^2$??

Argued that when, say, K electron ejected, L electron "sees" an "effective charge":

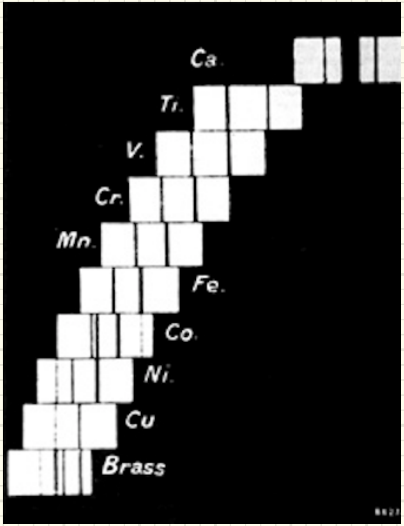
$(Z-1)$

Moseley's Law

predicted 4 elements

reordered a handful

Z rather than A



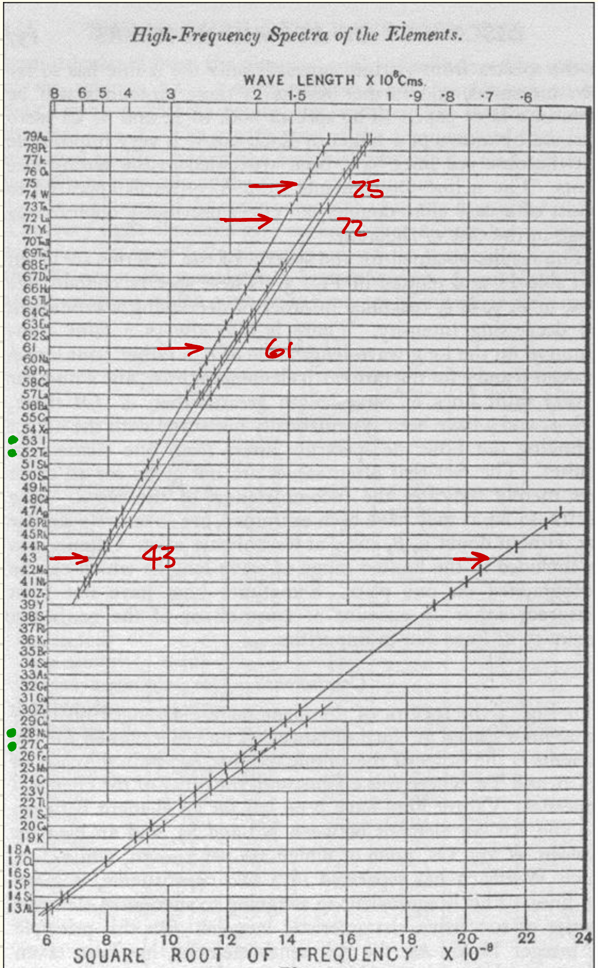


Fig. 3.

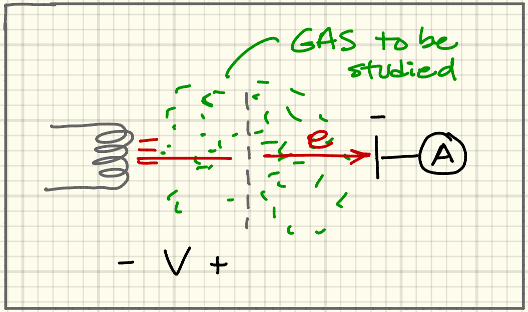
MADNESS OF WWI

14 July 1915
We moved yesterday to a place where the road is worse than the flies. Sand in boots clothes mouth eyes hair. Sand in the food and the water and the air. All my kit is in danger of being buried. The men are not bearing the heat very well.
Letter from Harry to his mother sent from Gallipoli

	Dead	Wounded	Missing or POW	Total
Ottoman Empire	**56,643**	**97,007**	**11,178**	**164,828**
United Kingdom	34,072	78,520	7,654	120,246
France	9,798	17,371	–	27,169
Australia	8,709	19,441	–	28,150
New Zealand	2,721	4,752	–	7,473
British India	1,358	3,421	–	4,779
Newfoundland	49	93	–	142
Total Allies	**56,707**	**123,598**	**7,654**	**187,959**

FRANK HERTZ EXPERIMENT 1914

(James) (Gustav)



Electron states are quantized.

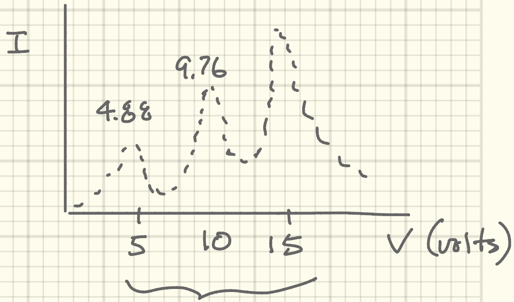
Hg

V ↑ then I ↑

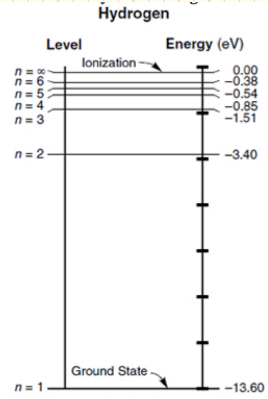
until V = 4.88V

current drops...

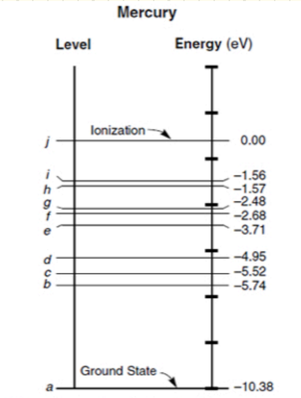
repeatedly...



multiples of 4.88V



Energy Levels for the Hydrogen Atom



A Few Energy Levels for the Mercury Atom

BOHR CORRESPONDENCE PRINCIPLE

"Old quantum theory" — entirely Bohr — a hodge-podge of weird ideas and claims... that worked. Unsatisfying

began a troubling relationship with quantum mechanics in 1913...

1st attempt:

In the limiting case of large quantum numbers, frequencies and intensities of radiation:

quantum results $\rightarrow$ classical results

$$F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r}$$

$$\frac{1}{2}mv^2 = \frac{1}{2\cdot 4\pi\epsilon_0}\frac{e^2}{r}$$

Classically:

frequency of electron in an orbit:

$$f_c = \frac{v}{2\pi r} \Rightarrow \qquad K = \frac{1}{2}mv^2 = \frac{1}{8\pi\epsilon_0}\frac{e^2}{r}$$

$$v = \sqrt{\frac{2e^2}{8\pi\epsilon_0 m r}}$$

$$f_c = e\sqrt{\frac{1}{4\pi\epsilon_0 m r}}\ \frac{1}{2\pi r}$$

$$= e\sqrt{\frac{1}{4\pi\epsilon_0 m r}\cdot\frac{1}{4\pi^2 r^2}}$$

$$f_c = \frac{e}{\sqrt{16\pi^3\epsilon_0 m r^3}}$$

the allowed orbits: $r_n = \dfrac{4\pi\epsilon_0\hbar^2}{me^2}n^2$

$$f_c = \frac{e}{\sqrt{16\pi^3\epsilon_0 \cancel{m}\cdot \dfrac{48\,\epsilon_0^3\hbar^6 n^6\pi^3}{m^3e^6}}} = \frac{e^4 m}{32\,\epsilon_0^2\pi^3\hbar^3 n^3}$$

frequency of classical electron at this orbit, r_n

$$\boxed{f_c = \frac{e^4 m}{32\,\epsilon_0^2\pi^3\hbar^3 n^3}}$$

quantum frequency for large n transitioning to n-1

$$f_Q = \frac{e^4}{4\pi(4\pi\epsilon_0)^2}\,\frac{m_e}{\hbar^3}\left[\frac{1}{(n-1)^2}-\frac{1}{n^2}\right]$$

$$f_Q = \frac{e^4}{4\pi(4\pi\epsilon_0)}\,\frac{m_e}{\hbar^3}\,\frac{2n-1}{n^2(n-1)^2}$$

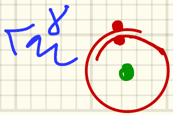
n is very large -- $n-1 \sim n$ & $2n-1 \sim 2n$

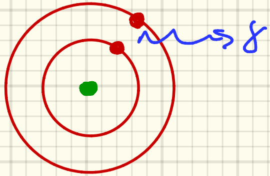
$$f_Q \to \frac{e^4}{4\pi(4\pi\epsilon_0)}\,\frac{m_e}{\hbar^3}\,\frac{2n}{n^4} = \frac{me^4}{32\pi^3\epsilon_0^2\hbar^3}\,\frac{1}{n^3}$$

this guy jumps

$$\boxed{f_c = \frac{e^4 m_e}{32\,\epsilon_0^2\pi^3\hbar^3 n^3}}$$

← this guy radiates because it spirals





Became a guide later...