7. Hydrogen Atoms,4

lecture 27, October 32, 2017

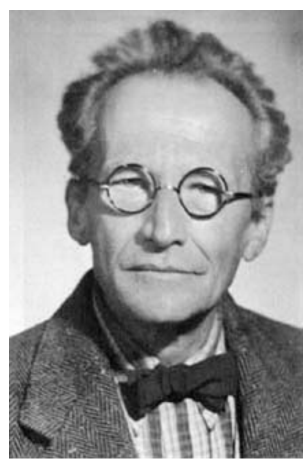
I crack myself up sometimes

housekeeping

I got nothin’

today

Hydrogen atom, more



WHERE WE WERE!

Schrodinger equation for 3 dimensional configuration

$$-\frac{\hbar^2}{2\mu}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]\psi(r,\theta,\phi)$$

reduced mass

$$-\frac{e^2}{4\pi\epsilon_0 r}\psi(r,\theta,\phi)=E\psi(r,\theta,\phi)$$

$$R_{n\ell}(r)=-\left[\left(\frac{2}{na_0}\right)\frac{(n-\ell+1)!}{2n[(n+\ell)!]^3}\right]^{\frac{1}{2}}\left(\frac{2r}{na_0}\right)^{\ell}e^{-r/na_0}L_{n+\ell}^{2\ell+1}\left(\frac{2r}{na_0}\right)$$

$$\psi_{n\ell m_\ell}(r,\theta,\phi)=R_{n\ell}(r)Y_\ell^{m_\ell}(\theta,\phi)$$

"eigenfunctions"

$$E_n=\frac{e^4}{(4\pi\epsilon_0)^2}\frac{\mu}{\hbar^2 2n^2}$$

"eigenvalues"

QUANTUM NUMBERS of the QUANTUM CENTRAL FORCE SOLUTION

$\boxed{n:}$ Principle Quantum Number

$n = 1, 2, 3 \ldots \infty$

$\boxed{\ell:}$ Orbital Angular Momentum Quantum Number

$\ell = 0, 1, 2 \ldots (n-1)$ $\qquad 0 \leq \ell \leq n$

connected to R

$\boxed{m_\ell:}$ Maqnetic Quantum Number

$m_\ell = -\ell, -\ell+1, -\ell+2, \ldots, -1, 0, 1, \ldots \ell-1, \ell$

$-\ell \leq m_\ell < \ell$

a few: $Y_{\ell m}$

l = 0[1] [edit]

$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}}$$

l = 1[1] [edit]

$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{-i\varphi}\cdot\sin\theta = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot\frac{(x-iy)}{r}$$

$$Y_1^{0}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cdot\cos\theta = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cdot\frac{z}{r}$$

$$Y_1^{1}(\theta,\varphi) = -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot e^{i\varphi}\cdot\sin\theta = -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\cdot\frac{(x+iy)}{r}$$

l = 2[1] [edit]

$$Y_2^{-2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot e^{-2i\varphi}\cdot\sin^2\theta = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x-iy)^2}{r^2}$$

$$Y_2^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot e^{-i\varphi}\cdot\sin\theta\cdot\cos\theta = \frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x-iy)z}{r^2}$$

$$Y_2^{0}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}\cdot(3\cos^2\theta-1) = \frac{1}{4}\sqrt{\frac{5}{\pi}}\cdot\frac{(2z^2-x^2-y^2)}{r^2}$$

$$Y_2^{1}(\theta,\varphi) = -\frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot e^{i\varphi}\cdot\sin\theta\cdot\cos\theta = -\frac{1}{2}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x+iy)z}{r^2}$$

$$Y_2^{2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot e^{2i\varphi}\cdot\sin^2\theta = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\cdot\frac{(x+iy)^2}{r^2}$$

l = 3[1] [edit]

$$Y_3^{-3}(\theta,\varphi) = \frac{1}{8}\sqrt{\frac{35}{\pi}}\cdot e^{-3i\varphi}\cdot\sin^3\theta = \frac{1}{8}\sqrt{\frac{35}{\pi}}\cdot\frac{(x-iy)^3}{r^3}$$

$$Y_3^{-2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{105}{2\pi}}\cdot e^{-2i\varphi}\cdot\sin^2\theta\cdot\cos\theta = \frac{1}{4}\sqrt{\frac{105}{2\pi}}\cdot\frac{(x-iy)^2z}{r^3}$$

$$Y_3^{-1}(\theta,\varphi) = \frac{1}{8}\sqrt{\frac{21}{\pi}}\cdot e^{-i\varphi}\cdot\sin\theta\cdot(5\cos^2\theta-1) = \frac{1}{8}\sqrt{\frac{21}{\pi}}\cdot\frac{(x-iy)(4z^2-x^2-y^2)}{r^3}$$

$$Y_3^{0}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{7}{\pi}}\cdot(5\cos^3\theta-3\cos\theta) = \frac{1}{4}\sqrt{\frac{7}{\pi}}\cdot\frac{z(2z^2-3x^2-3y^2)}{r^3}$$

$$Y_3^{1}(\theta,\varphi) = -\frac{1}{8}\sqrt{\frac{21}{\pi}}\cdot e^{i\varphi}\cdot\sin\theta\cdot(5\cos^2\theta-1) = \frac{-1}{8}\sqrt{\frac{21}{\pi}}\cdot\frac{(x+iy)(4z^2-x^2-y^2)}{r^3}$$

$$Y_3^{2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{105}{2\pi}}\cdot e^{2i\varphi}\cdot\sin^2\theta\cdot\cos\theta = \frac{1}{4}\sqrt{\frac{105}{2\pi}}\cdot\frac{(x+iy)^2z}{r^3}$$

$$Y_3^{3}(\theta,\varphi) = -\frac{1}{8}\sqrt{\frac{35}{\pi}}\cdot e^{3i\varphi}\cdot\sin^3\theta = \frac{-1}{8}\sqrt{\frac{35}{\pi}}\cdot\frac{(x+iy)^3}{r^3}$$

l = 4[1] [edit]

$$Y_4^{-4}(\theta,\varphi) = \frac{3}{16}\sqrt{\frac{35}{2\pi}}\cdot e^{-4i\varphi}\cdot\sin^4\theta = \frac{3}{16}\sqrt{\frac{35}{2\pi}}\cdot\frac{(x-iy)^4}{r^4}$$

$$Y_4^{-3}(\theta,\varphi) = \frac{3}{8}\sqrt{\frac{35}{\pi}}\cdot e^{-3i\varphi}\cdot\sin^3\theta\cdot\cos\theta = \frac{3}{8}\sqrt{\frac{35}{\pi}}\cdot\frac{(x-iy)^3z}{r^4}$$

$$Y_4^{-2}(\theta,\varphi) = \frac{3}{8}\sqrt{\frac{5}{2\pi}}\cdot e^{-2i\varphi}\cdot\sin^2\theta\cdot(7\cos^2\theta-1) = \frac{3}{8}\sqrt{\frac{5}{2\pi}}\cdot\frac{(x-iy)^2\cdot(7z^2-r^2)}{r^4}$$

$$Y_4^{-1}(\theta,\varphi) = \frac{3}{8}\sqrt{\frac{5}{\pi}}\cdot e^{-i\varphi}\cdot\sin\theta\cdot(7\cos^3\theta-3\cos\theta) = \frac{3}{8}\sqrt{\frac{5}{\pi}}\cdot\frac{(x-iy)\cdot z\cdot(7z^2-3r^2)}{r^4}$$

$$Y_4^{0}(\theta,\varphi) = \frac{3}{16}\sqrt{\frac{1}{\pi}}\cdot(35\cos^4\theta-30\cos^2\theta+3) = \frac{3}{16}\sqrt{\frac{1}{\pi}}\cdot\frac{(35z^4-30z^2r^2+3r^4)}{r^4}$$

$$Y_4^{1}(\theta,\varphi) = \frac{-3}{8}\sqrt{\frac{5}{\pi}}\cdot e^{i\varphi}\cdot\sin\theta\cdot(7\cos^3\theta-3\cos\theta) = \frac{-3}{8}\sqrt{\frac{5}{\pi}}\cdot\frac{(x+iy)\cdot z\cdot(7z^2-3r^2)}{r^4}$$

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$$Y_4^{4}(\theta,\varphi) = \frac{3}{16}\sqrt{\frac{35}{2\pi}}\cdot e^{4i\varphi}\cdot\sin^4\theta = \frac{3}{16}\sqrt{\frac{35}{2\pi}}\cdot\frac{(x+iy)^4}{r^4}$$

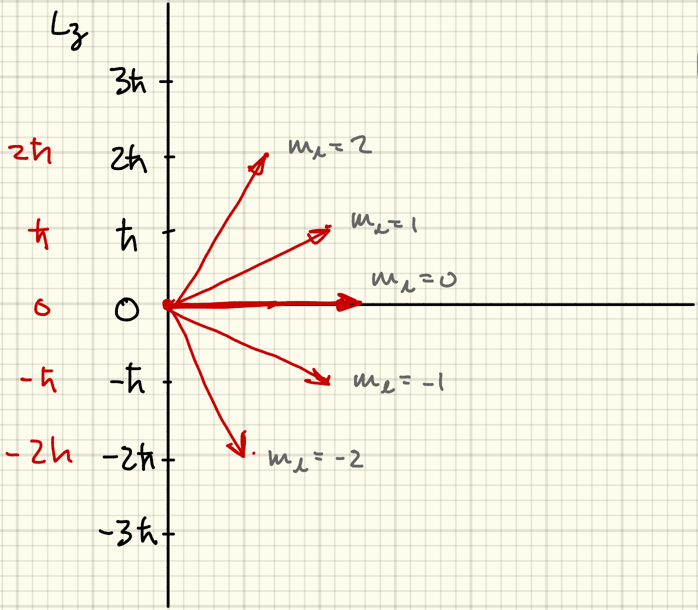
MATHEMATICA

$$l = 2 \qquad L = \hbar\sqrt{l(l+1)}$$

$$L = \hbar\sqrt{6}$$

$$L_z = m_l \hbar$$

$$= 0,\ 1,\ -1,\ 2,\ -2$$



$$m_l = -l,\ -l+1,\ -l+2, \ldots, -1,\ 0,\ 1, \ldots\ l-1,\ l$$

Normalization:

$$1 = \iiint_{\text{all space}} |R(r)|^2 |Y(\theta,\phi)|^2 r^2 dr \sin\theta \, d\theta \, d\phi$$

$$1 = \int_0^\infty |R(r)|^2 dr \int_0^\pi \int_0^{2\pi} |Y(\theta,\varphi)|^2 \sin\theta \, d\theta \, d\varphi$$

can sort of visualize 2 different probability distributions:

$$r^2 |R(r)|^2 dr$$

$$|Y_\ell^m(\theta,\varphi)|^2 \sin\theta \, d\theta \, d\varphi$$

Most probable? in hydrogen, $Z=1$

$$P_{10} = \frac{4}{a_0^3} r^2 e^{-2r/a_0}$$

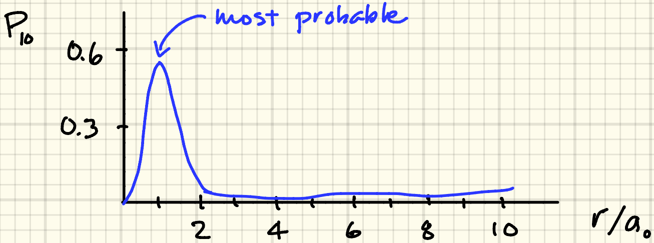
$$\frac{dP_{10}(r)}{dr} = \frac{4}{a_0^3}\left[2r\, e^{-2r/a_0} - r^2\left(\frac{2r}{a_0}\right) e^{-2r/a_0}\right]$$

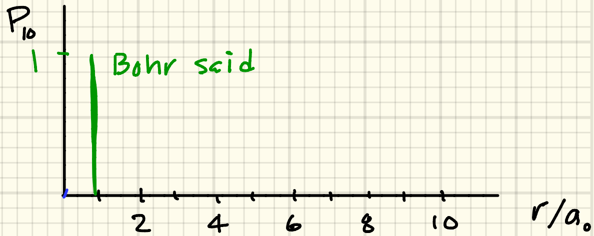
$$= 0 \text{ for extremum}$$

$$= \frac{4}{a_0^3} e^{-2r/a_0}\left[2r - \frac{2r^2}{a_0}\right]$$

$$\overset{\|}{0} \Rightarrow r = a_0$$

HOW COOL IS THAT?





Average "place" for electron?

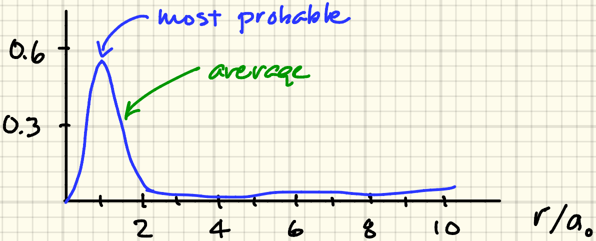
$$\langle r \rangle = \int_0^\infty r\, P_{10}(r)\, dr = \frac{4}{a_0^3} \int_0^\infty r^3 e^{-2r/a_0}\, dr$$

like $\int z^n e^{-z} dz = n!$

change variables: $z = 2r/a_0$

$$\langle r \rangle = \frac{a_0}{4}(3!)\ P_{10}$$

$$\langle r \rangle = \frac{3}{2} a_0$$



WHERE IS THE ELECTRON?

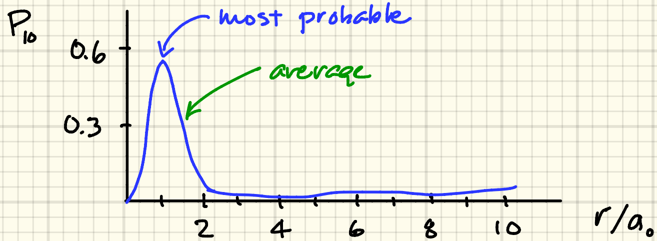
Average "place" fn electron?

$$\langle r \rangle = \int_0^\infty r\, P_{10}(r)\, dr = \frac{4}{a_0^3} \int_0^\infty r^3 e^{-2r/a_0}\, dr$$

like $\int z^n e^{-z}\, dz = n!$

change variables: $z = 2r/a_0$

$$\langle r \rangle = \frac{a_0}{4}(3!)$$



$$\langle r \rangle = \frac{3}{2} a_0$$

WHERE IS THE ELECTRON?

Probability of electron outside the 1st Bohr orbit?

assume: 1s

$$P(r>a_0) = \int_{a_0}^{\infty} P_{10}(r)\,dr$$

$$= \frac{4}{a_0^3} \int_{a_0}^{\infty} r^2 e^{-2r/a_0}\,dr$$

$$= \frac{1}{2} \int_2^{\infty} x^2 e^{-x}\,dx$$

another one of those integrals.

$$\int x^2 e^{-x}\,dx = -\frac{1}{2}(x^2+2x+2)e^{-x}$$

$x = 2r/a_0$

$$P(r>a_0) = 5e^{-2} \sim 0.68$$

$\Rightarrow$ 2/3 of the time!

How about clearly outside of its atom?

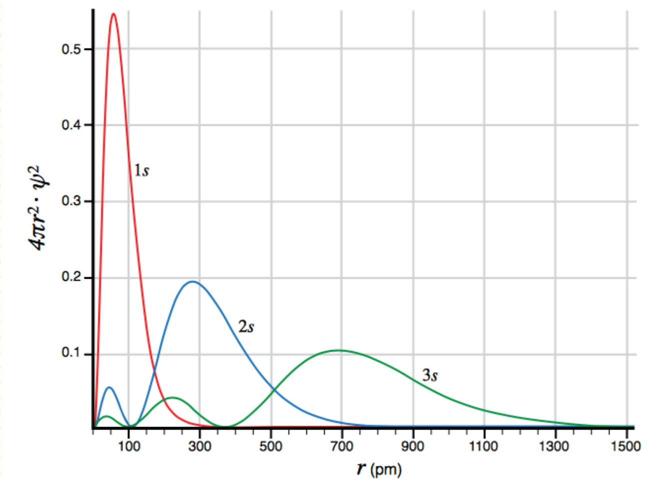
$> r = 100\,a_0$

$$P(r>100\,a_0) \simeq 10^{-83} \neq 0$$

How about 2s state...

$Y_2^0 \rightarrow$ symmetric.

$$R_{20} = \frac{1}{\sqrt{(2a_0)^3}} \underbrace{\left(2 - \frac{r}{a_0}\right)}_{\text{density peaks}} e^{-r/2a_0}$$

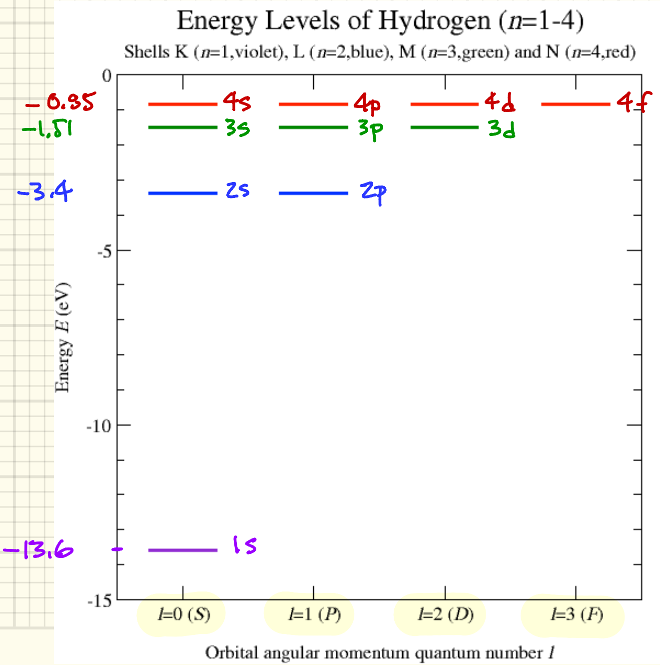


2s electrons spend a significant time in 1s land

Energy depends on n alone: $E_n = \frac{e^4}{(4\pi\epsilon_0)^2}\frac{\mu}{\hbar^2 2n^2}$ THE BOHR ENERGY

But each n can have many ℓ's & m_ℓ's

substantial degeneracy.



states

$1 + 3 + 5 + 7 = 16$

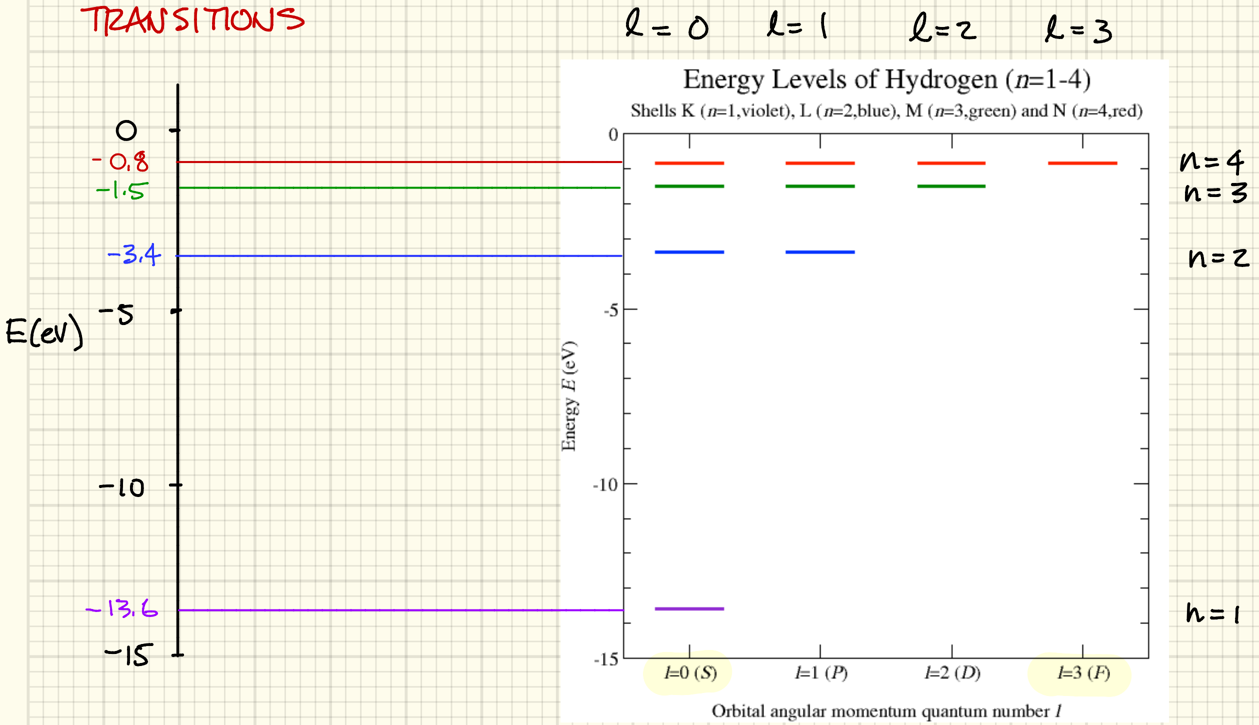
$1 + 3 + 5 = 9$

$1 + 3 = 4$

$\underbrace{1}$

degeneracies $= n^2$ for H

TRANSITIONS

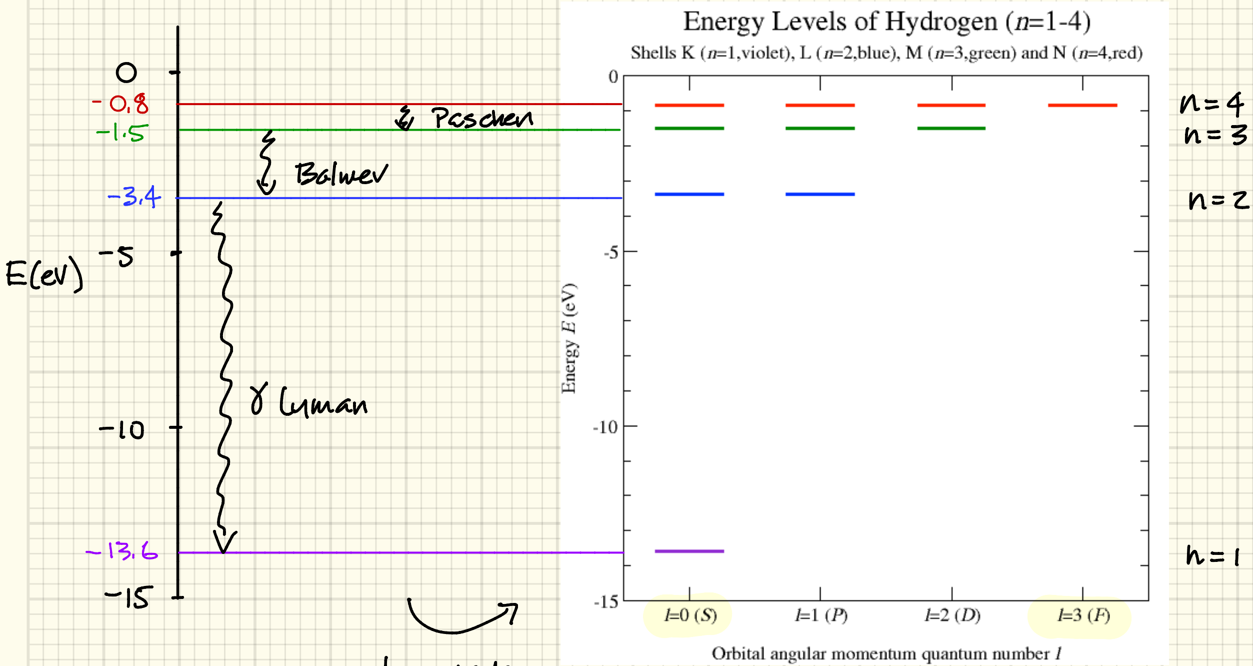


BOHR

SCHROEDINGER

TRANSITIONS

$\ell = 0$ $\ell = 1$ $\ell = 2$ $\ell = 3$



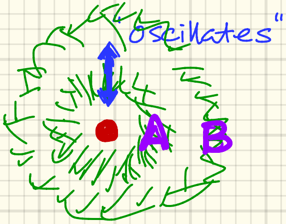
translate to $\ell = 0$ in S.E.

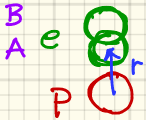
BOHR

SCHROEDINGER

TRANSITIONS … the almost real way

external radiation or collisions $\Longrightarrow$





like an oscillating electric dipole
$\vec{p} = e\vec{r}$

$\psi_A(r,\theta,\varphi)$ & $\psi_B(r,\theta,\varphi)$ each wavefunctions for "levels" A & B

$\longrightarrow$ for a transition, they must "overlap" in the "presence" of an electric dipole operator.

$$\langle p \rangle = \iiint \psi_A^*(r,\theta,\varphi)\, q\hat{r}\, \psi_B(r,\theta,\phi)\, r^2 dr \sin\theta\, d\theta\, d\phi$$

If integrals are zero $\Rightarrow$ no transition from B $\rightarrow$ A

non-zero $\Rightarrow$ transitions from B $\rightarrow$ A can happen & calculation yields prob & rate (intensities)

$$\langle p \rangle = \underbrace{\int r^2 R(r)_A^* \, R(r)_B \, dr}_{\text{complicated.. but always non-zero}} \; \underbrace{\iint Y_{\ell_A}^{m_{\ell A}}(\theta,\phi)^* \, Y_{\ell_B}^{m_{\ell B}}(\theta,\varphi) \sin\theta \, d\theta \, d\varphi}_{\text{control the "allowed" and "forbidden" transitions}}$$

$\downarrow$ which become

particular $\Delta \ell$'s

always† $\Delta \ell = \pm 1$

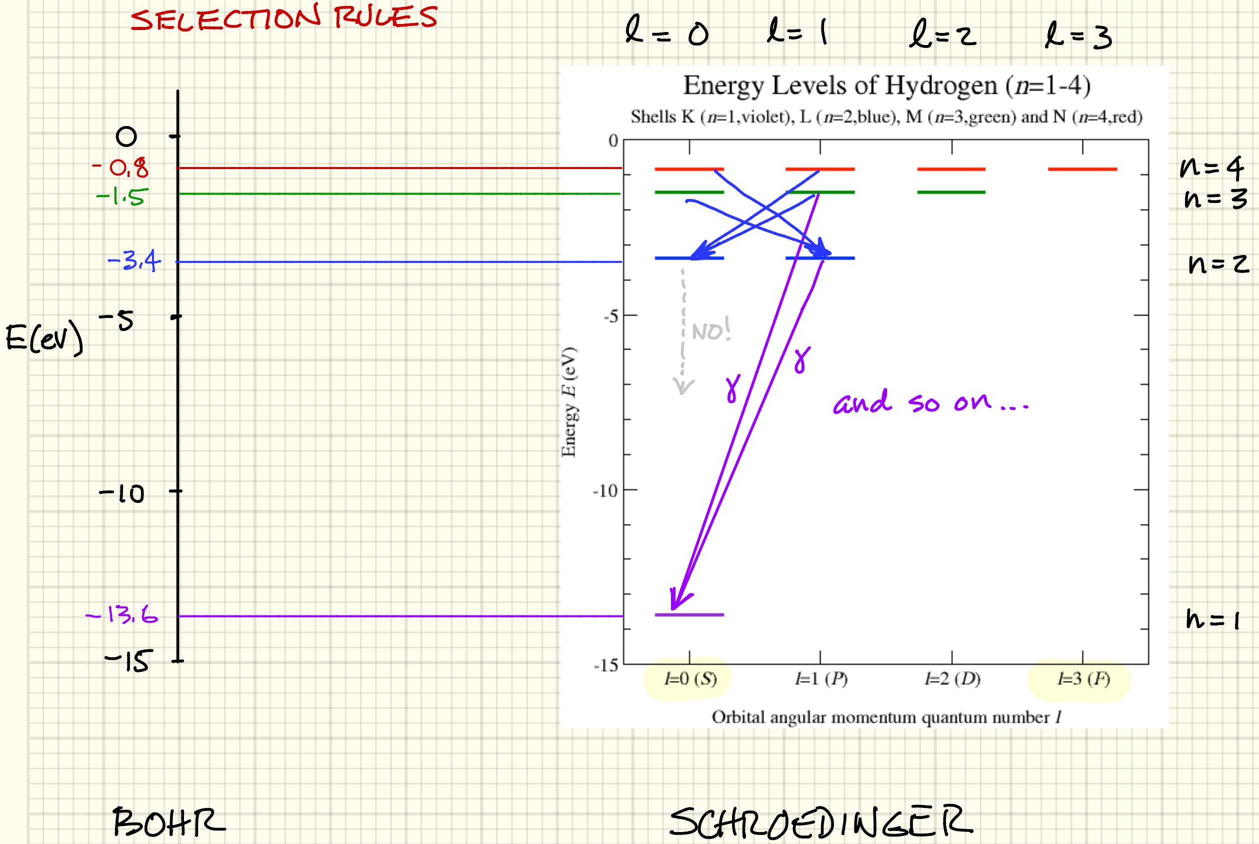
so: $p \rightarrow s$? sure

$s \rightarrow s$, $p \rightarrow p$ ... ? nope

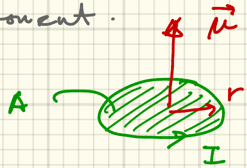
"Selection Rules"

† not quite

SELECTION RULES



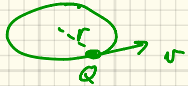
Orbiting electric charges $\longrightarrow$ a magnetic moment.



Classical E & M: current loop

magnetic moment $|\mu| = IA$ (R.H. rule)

If an electric charge going in a circle $I = \frac{dQ}{dt} = \frac{Q}{T}$

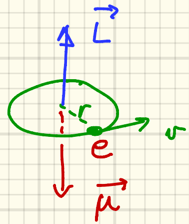


So $L = mvr$ + $v = \frac{2\pi r}{T}$ + $A = \pi r^2$

$$L = m\left(\frac{2\pi r}{T}\right)r = 2m\frac{\pi r^2}{T} = 2m\frac{A}{T}$$

$$\mu = IA = \frac{Q}{T}A = Q\frac{L}{2m}$$ or $$\vec{\mu} = \frac{Q}{2m}\vec{L}$$

$$\vec{\mu}_e = -\frac{e}{2m}\vec{L}$$



Strange superposition of classical & quantum mechanical notions:

We found:

$$L = \hbar\sqrt{\ell(\ell+1)}$$

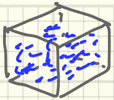
a vector w/ quantized projection onto L_z axis → $L_z = m_\ell \hbar$

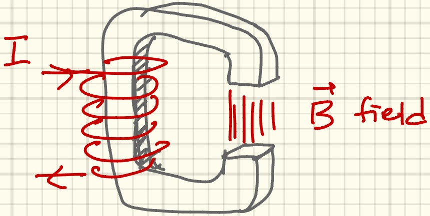
We have no preferred direction... $\vec{L}$ could be anywhere

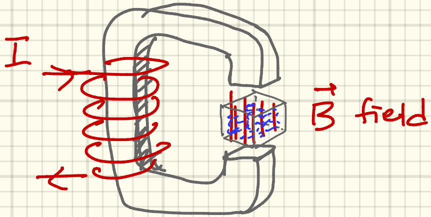
something must pick a particular direction and break that symmetry

& define the "axis of quantization"

Magnetic fields do that.

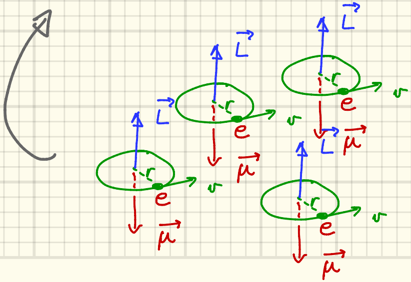






$\rightarrow$ defines the z axis
so, an axis of quantization

all of these
moments align
according to $\vec{L}$



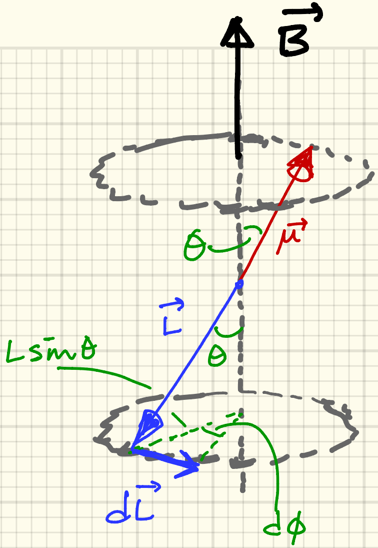
$$\mu_z = \frac{-e}{2m} L_z = \frac{-e\hbar}{2m} m_\ell$$

$\mu_z = \frac{-e}{2m} \hbar \, m_\ell$ defines **Bohr Magneton** $\mu_B \equiv \frac{e\hbar}{2m} = 9.3 \times 10^{-24} \, J/T$

$\mu_z = -\mu_B \, m_\ell \quad \Rightarrow$ atomic magnetic moments can be thought of as multiples of μ_B.

Remember: $\vec{\tau} = \vec{\mu} \times \vec{B} \quad \rightarrow$ seeks to align $\vec{\mu}$ and $\vec{B}$

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{Q}{2m} \vec{L} \times \vec{B}$$



dL perpendicular to plane of $\vec{B}$ & $\vec{L}$

$$|dL| = L\sin\theta\, d\phi$$

$$d\phi = \frac{|dL|}{L\sin\theta}$$

$$\frac{|dL|}{dt} = \tau$$

$$d\phi = \frac{|\tau|\, dt}{L\sin\theta}$$

$$d\phi = \frac{Q/2m\,(LB\sin\theta)\, dt}{L\sin\theta}$$

$$d\phi = \frac{BQ}{2m}\, dt$$

$\Rightarrow$ precession @ angular frequency:

$$\omega_L = \frac{d\phi}{dt} = \frac{QB}{2m}$$

Larmor Frequency

Got a torque? through an angle?

$\longrightarrow$ you got work done

$$dW = \tau \, d\theta$$

$$dW = -\mu B \sin\theta \, d\theta \qquad \text{(opposes } B\text{)}$$

$$dW = d(\vec{\mu} \cdot \vec{B})$$

work stored as potential energy: $dW = -dV$

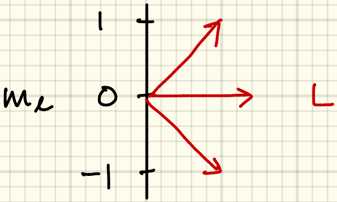
an induced magnetic potential energy $V = -\vec{\mu} \cdot \vec{B}$

depends on:

- $\vec{B}$
- the more aligned, the less is V

BUT... μ is quantized according to L_z $\qquad \mu_z = -\mu_B m_\ell$

$\ell = 1$, for example:



from $V = -\vec{\mu} \cdot \vec{B}$

V is indeed quantized

The magnetic field defines a particular direction in space

$\longrightarrow$ the quantization axis, along $\vec{B}$ ... $\hat{z}$

$$V = \frac{e}{2m} \vec{L} \cdot \vec{B} = \frac{e\,|B|\,L_z}{2m}$$

$$V = \mu_B m_\ell B$$

m_ℓ ranges over $-\ell \rightarrow +\ell$

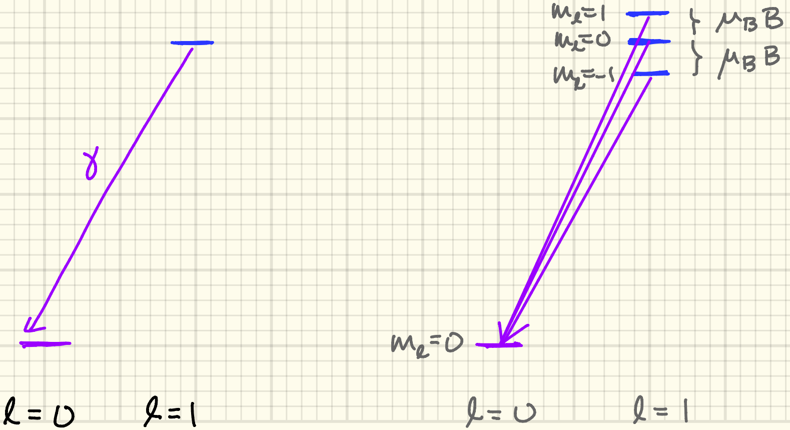
Bottom line:

apply a magnetic field → energy levels change
degeneracy lifted

Lorentz predicted, Zeeman discovered → before Q.M.

Spectra:

$B = 0$ $\qquad$ $B \neq 0$



SELECTION RULE

$\Delta m_\ell = 0, \pm 1$

"NORMAL" ZEEMAN EFFECT

$1 \to 3$ splitting .. "normal" because classical picture predicts rough behavior

"ANOMALOUS" ZEEMAN EFFECT

$\left.\begin{array}{l} 1 \to 4 \\ \text{or} \\ 1 \to 6 \end{array}\right\}$ experimentally confusing, classically, disturbing

"Stern-Gerlach Experiment"

$\longrightarrow$ slides!

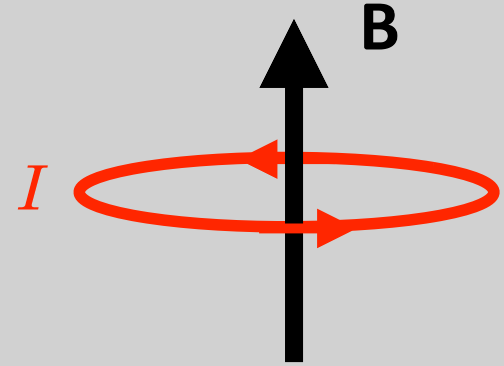
“Spin”

sometimes we use “normal language”

to refer to a physical property without a common-sense analog

and sometimes, just to be silly

Remember a current loop: where’s the B field?

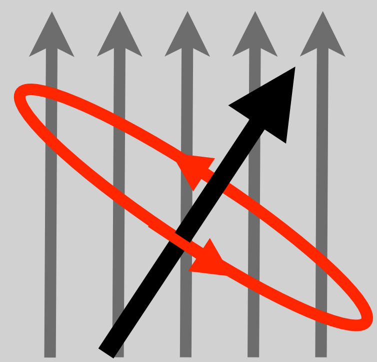


called a “magnetic dipole”

Like a bar magnet, with the north pole up...

Remember that Oersted found that a compass needle followed the B field, so this setup would also

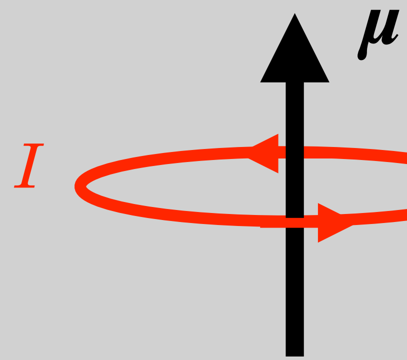
external B field



dipole moment

characterizes the magnetism of a current loop

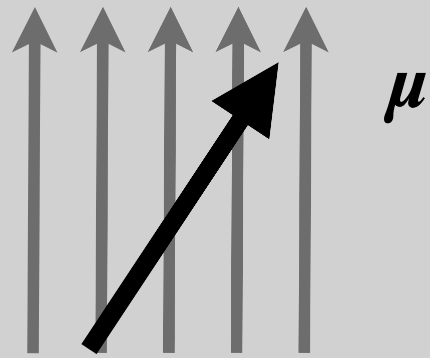
A current loop's magnetism can be characterized



μ is called the "magnetic dipole moment"

more current, higher the moment – the μ the more torque

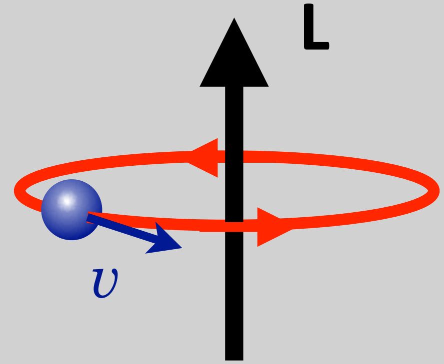
external B field



Bohr was not the whole story

but his quantization condition was mostly correct

Orbital angular momentum - the s, p, d, f, g, h, etc originates with Schroedinger's evolution of Bohr's original idea:



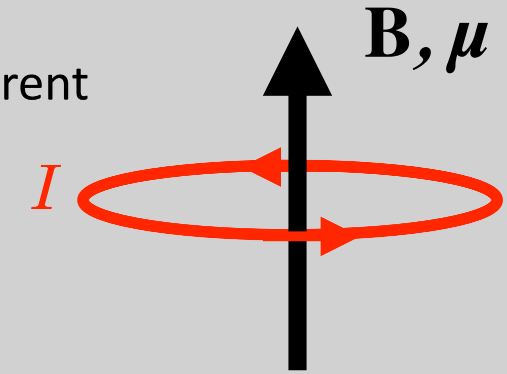
$$L = mvr = \frac{nh}{2\pi}$$

In Schroedinger's model

$$L_z = m_\ell \frac{h}{2\pi}$$

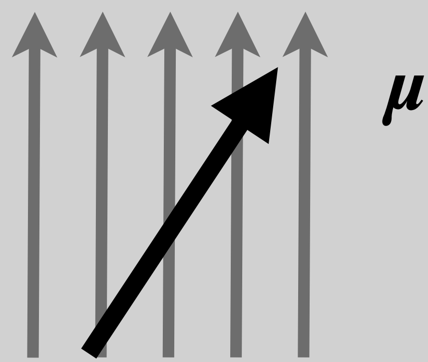
$$\begin{cases} = 0:\ \mathrm{s} \\ = 1:\ \mathrm{p} \\ = 2:\ \mathrm{d} \\ \text{etc} \end{cases}$$

But, the electron's orbit is a little current



So, atomic orbits should twist

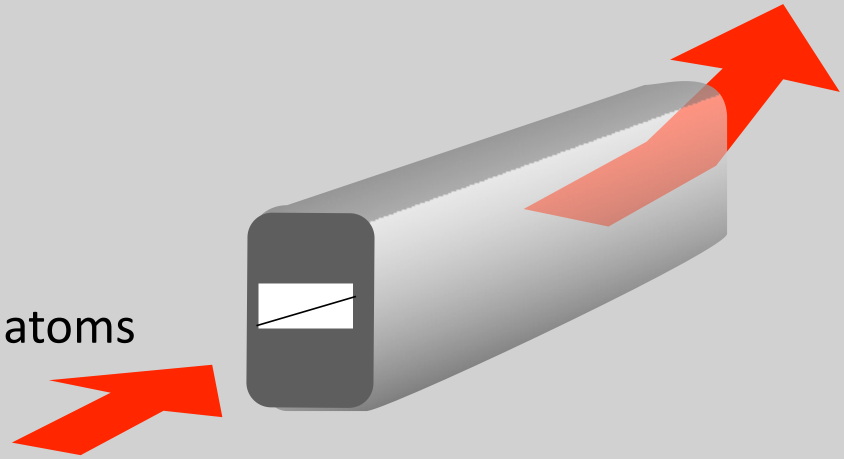
external
B field



remember
spectroscopic notation
from Chemistry?

That’s what Otto Stern and Walther Gerlach set about to measure in 1922...

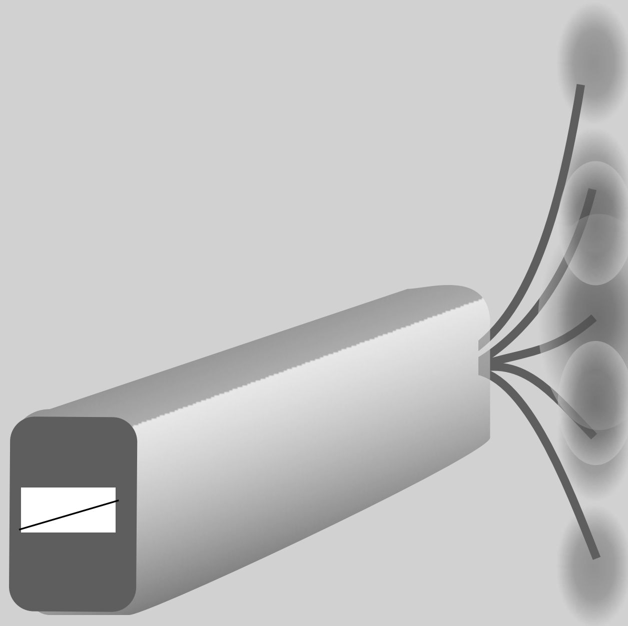
silver atoms



the Bohr Model predicted a bunch of spots

since Silver is a complicated atom:

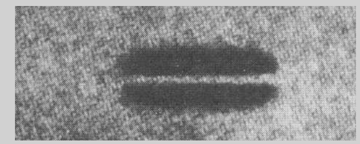
$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6\ 4d^{10}\ 5s^1$



If Bohr was right

What they saw

If Bohr was wrong



Separation into 2 spots - evidence of “quantization” of some sort…why 2?

Separation into 2 locations, which made no sense.

Something is “quantized”

you can’t always do what you’re supposed to do

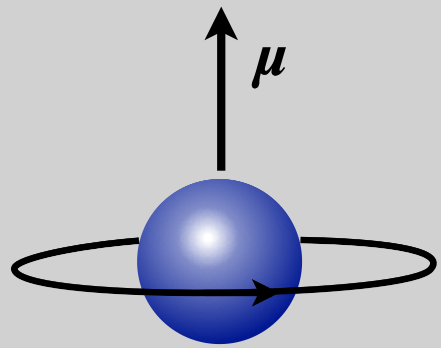
young Dutch experimenters George Uhlenbeck* and Sam Goudsmit had an idea

told by their advisor “don’t publish”

they did.

* eventually professor at UofM

The electron is *like* a spinning charge...



Like for orbital angular momentum,

$$L_z = m_\ell \frac{h}{2\pi}$$

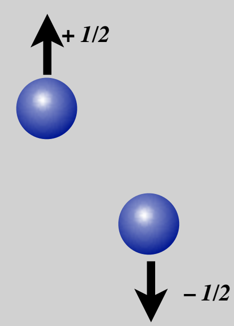
Electrons have an **intrinsic** angular momentum

$$S_z = m_s \frac{h}{2\pi}$$

But, the “spin” can only take on two values:

$$m_s = +\frac{1}{2} \quad \text{or} \quad m_s = -\frac{1}{2}$$

We say
“spin, plus 1/2” or “spin up”
and
“spin, minus 1/2” or “spin down”



Stern & Gerlach got a hodge-podge of results

Oxygen 5 spots $2s^2 2p^4$

Silver 2 spots ... $5s$

Interpreted as an angular-momentum-like selection rule w/ "ℓ" $= 1/2$

You're not much if you're not Dutch...

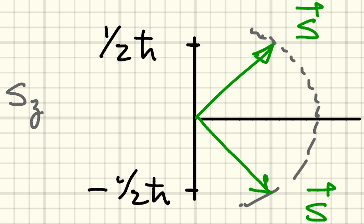
Goudsmit & Uhlenbeck proposed

electron has an inherent property: "spin"

$S = 1/2$ with projections like L: $|S| = \hbar\sqrt{s(s+1)}$

$$= \hbar\sqrt{\tfrac{1}{2}\cdot\tfrac{3}{2}}$$

$$|S| = \frac{\sqrt{3}}{2}\hbar$$



$S_z = \pm \frac{1}{2}\hbar = m_s \hbar$

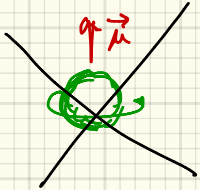
Spin

has features like angular momentum -- magnetic-like

NO CIRCULAR CURRENTS!

NO CHARGES GOING AROUND ANYTHING!

NOT A BALL OF CHARGE ROTATING!



SPIN IS INHERENT TO ELECTRONS

↑

lousy name!
no-thing is "spinning"

DEFINE AN ELECTRON!

- An excitation of the electron field
- having mass = 0.511 MeV/c^2
- having electric charge = -1.6×10^{-19} C
- having intrinsic spin = ½

Magnetic Moment

$$\vec{\mu}_s = -g_e \frac{e}{2m_e} \vec{S}$$

a fudge factor.. measurable: gyromagnetic ratio (stay tuned)

No reason or "implementation" of spin in Quantum Mechanics

Until 1928: Paul Dirac → first relativistic quantum mechanics

The Dirac Equation designed to deal with negative E

- 4 wave functions resulted

spin falls out from mathematics
2 with + energies → electron
2 with − energies → anti-electron

Spin is an inherently Relativistic property of electrons...

... and neutrons, protons → all "fermions"

Also a quantum mechanics of integer-spin excitations → all "bosons"

$S = 0$ pions, kaons, Higgs boson ...

$S = 1$ photon, rho meson, W, Z ...

that's why $\Delta l = \pm 1$

Angular momentum is conserved ... γ radiates away and takes angular momentum of $\hbar$ from system so, Δl is reduced.

ABOUT g.

We have 2 moments now:

Orbital angular momentum: $\vec{\mu}_\ell = -\frac{\mu_B}{\hbar}\vec{L} \quad \left(= -\frac{g_\ell \mu_B \vec{L}}{\hbar} \right)$

Spin "intrinsic" angular momentum: $\vec{\mu}_s = -\frac{2\mu_B}{\hbar}\vec{S} \left(= -\frac{g_s \mu_B}{\hbar}\vec{S} \right)$

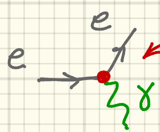
g_s is strictly a relativistic quantity.

$$g_s = 2$$

g_ℓ is sometimes included.

$$g_\ell = 1$$

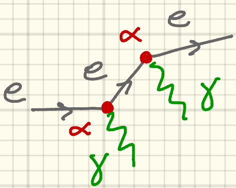
MORE g.



$$\alpha = \frac{e^2}{\hbar c\, 4\pi\epsilon_0} \sim \frac{1}{137}$$

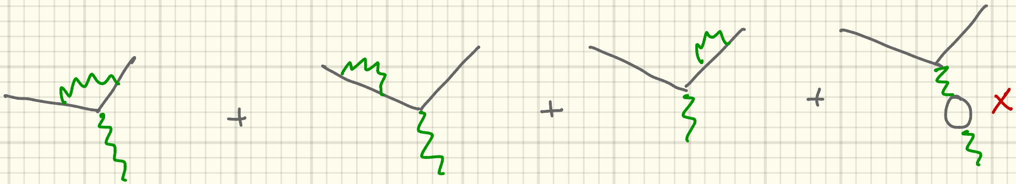
"strength of EM interaction"

Relativistic Quantum Field Theory

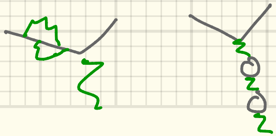


since α small, each additional "vertex" is smaller than predecessor.

Full treatment of electrons requires



+ combinations & additional



etc.

Contributions add up and modify

static properties of the electron

$$\vec{\mu}_s(e) = g_e \frac{e}{2m_e c} \vec{S}$$

net effect is to modify $g_e = 2$

"g-2" ... "gee minus two" ... a race for precision

between theory ($\sim 10^3$ individual Feynman diagrams)

and experiment (heroic Penning Trap experiments)

Conventionally, what's reported is

$$a_e \equiv \frac{g-2}{2}$$

experiment: $a_e^{EXP} = 115\,965\,218\,07.3\,(2.8) \times 10^{-13}$

$= 0.00115\,965\,218\,073$

$\pm\, 0.00000000000028$

theory $a_e^{SM} = 0.00115965218178$

$\pm\, 0.000000000000006$ (4 loop uncertainties)

$\pm\, 0.000000000000004$ (5 loop uncertainties)

$\pm\, 0.000000000000002$ δa_e^{HAD}

$\pm\, 0.000000000000076$ $\delta a_e^{Rb}(\alpha)$

$a_e^{EXP} - a_e^{SM} = -10.5\,(8.1) \times 10^{-13}$ 1.1σ difference

The most precise enunciation about nature in the history of...everything