6. Quantum Mechanics 2, 1

lecture 21, October 16, 2017

housekeeping

exam 2: Friday, October 27

Tomorrow

- The department has an "Investiture"…I have to be an adult and give a speech down the hall
- I'll figure out some way for the "HW Workshop" to happen.

today

finish Uncertainty Principle

Schroedinger Quantum Mechanics

wave functions

Heisenberg Uncertainty Principle

$$\Delta p \Delta x \geq \frac{\hbar}{2} \quad \& \quad \Delta E \Delta t \geq \frac{\hbar}{2}$$

- properties of all waves
- de Broglie makes it seem strange

A few years ago...

pulled over for doing 105 mph*

State police radar: ~20 GHz , $\lambda \sim 14$ cm

Could he resolve my speed? Really?

$$\Delta x \Delta p \geq \frac{\hbar}{2} \qquad \Delta p = m \Delta v$$

$m \hat{=} 1500$ kg

$$\Delta p = \frac{\hbar}{2 \Delta x}$$

$$\Delta v m = \frac{\hbar}{2 m \Delta x} =$$

$$\Delta v = \frac{\hbar}{2 (\Delta \lambda)(m)} = \frac{1.05 \times 10^{-34}\ \text{J} \cdot \text{s}}{2 (0.14 \text{m})(1500 \text{ kg})}$$

$$\Delta v \hat{=} 3.5 \times 10^{-37}\ \text{m/s}$$

* it was another black BMW... not me!

Electron in 1^{st} Bohr orbit...

remember, I calulated $v \simeq 2 \times 10^{6}$ m/s

calculate ΔE

$$\Delta x \Delta p \geq \frac{\hbar}{2} \qquad \Delta p \sim mv$$

$$\Delta x = \frac{\hbar}{2\Delta p} = \frac{\hbar}{2mv} = \frac{1.05 \times 10^{-34}\,\mathrm{J \cdot s}}{(2)(9 \times 10^{-31}\,\mathrm{kg})(2 \times 10^{6}\,\mathrm{m/s})}$$

$$\Delta x = 3 \times 10^{-11}\,\mathrm{m} \doteq 0.3\,\text{Å} \sim a_0$$

HOW DO WE THINK ABOUT THIS?

Sometimes Uncertainty is a part of the job

Lifetimes... see below

Sometimes Uncertainty is an epistemological.. challenge

If it is impossible to precisely measure a position of an object

can we claim that an absolute PLACE is an attribute that objects possess?

Determinism: classically.. give me $\vec{x}$ and $\vec{p}$ & I can predict precisely the future

now – you can't give me $\vec{x}$ or $\vec{p}$

Vacuum: cannot be state of zero energy... nothing can have $E=0$

LIFETIMES

in decays - atomic, nuclear, elementary particles.

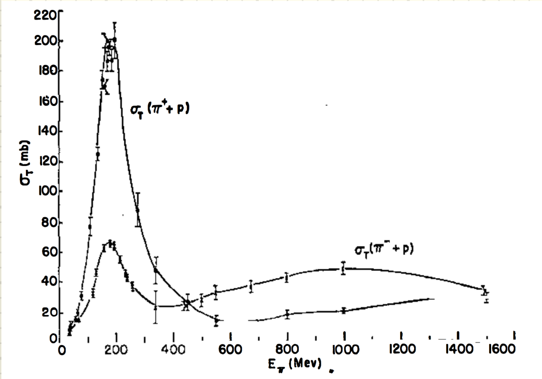
$\Delta E \Delta t \geq \frac{\hbar}{2}$ call Δt the lifetime of an unstable particle, τ

$\tau \geq \frac{\hbar}{2} \frac{1}{\Delta E}$ a short lifetime $\Rightarrow$ large uncertainty in energy (energy level or invariant mass)

Fermi's Discovery of the Δ^{++} resonance

$\pi^+ p \to \underbrace{\pi^+ p}$

increase K_π and measure cross section



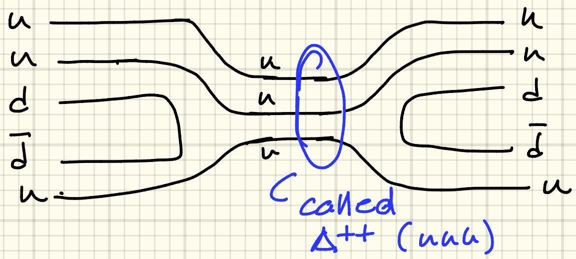
$\uparrow K \simeq 195$ MeV

interpreted as

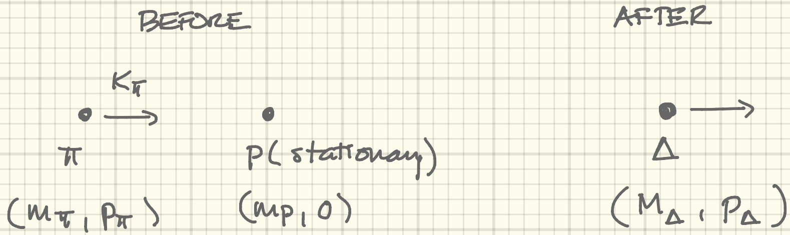
$\pi^+ p \to S^* \to \pi^+ p$

an excited-bound-state that decays into $\pi^+ p$

The modern quark view: $\pi^+(u\bar{d})$ $p(uud)$



From energy and momentum conservation, the situation:



$$M_\Delta^2 c^4 = m_\pi^2 c^4 + m_p^2 c^4 + 2(K_\pi + m_\pi c^2)(m_p c^2)$$

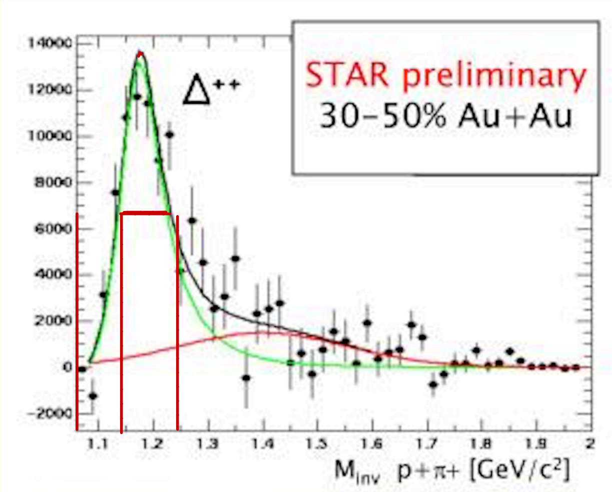
show this

C1

8 problem-like points

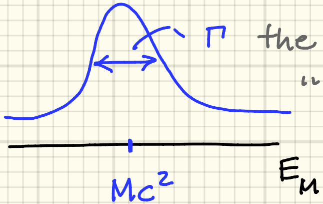
$$= (0.139)^2 + (0.938)^2 + 2(0.195 + 0.139)(0.938)$$

$$\simeq 1235 \text{ MeV}$$



$\Gamma \sim 100\,\text{MeV}$

Arbitrary data from Westfall's experiment at RHIC



the natural width
... energy uncertainty

Mass of Δ^{++}: $M_\Delta \pm \Delta E$

$M_\Delta \pm \frac{\Gamma}{2}$

Measure Γ, calculate τ

$\Delta E \Delta t \geq \frac{\hbar}{2}$ call $\Delta\tau$

$\tau \geq \frac{\hbar}{2}\frac{1}{\Delta E}$ a short

before

$$\tau_\Delta = \frac{\hbar}{2}\frac{1}{\Delta E} = \frac{\hbar}{2}\frac{1}{\Gamma/2} = \frac{\hbar}{\Gamma}$$

$$\tau_\Delta = \frac{6.6\times10^{-16}\,\text{eV}\cdot\text{s}}{1\times10^{8}\,\text{eV}} \simeq 6\times10^{-24}\,\text{s}$$

Delta baryons

Particle name	Symbol	Quark content	Rest mass (MeV/c²)	I_3	J^P	$Q(e)$	S	C	B'	T	Mean lifetime (s)	Commonly decays to
Delta[1]	$\Delta^{++}(1232)$	uuu	1,232 ± 2	$+3/2$	$3/2^+$	+2	0	0	0	0	$(5.63 \pm 0.14)\times10^{-24}$[a]	$p^+ + \pi^+$
Delta[1]	$\Delta^{+}(1232)$	uud	1,232 ± 2	$+1/2$	$3/2^+$	+1	0	0	0	0	$(5.63 \pm 0.14)\times10^{-24}$[a]	$\pi^+ + n^0$, or $\pi^0 + p^+$
Delta[1]	$\Delta^{0}(1232)$	udd	1,232 ± 2	$-1/2$	$3/2^+$	0	0	0	0	0	$(5.63 \pm 0.14)\times10^{-24}$[a]	$\pi^0 + n^0$, or $\pi^- + p^+$
Delta[1]	$\Delta^{-}(1232)$	ddd	1,232 ± 2	$-3/2$	$3/2^+$	−1	0	0	0	0	$(5.63 \pm 0.14)\times10^{-24}$[a]	$\pi^- + n^0$

[a] ^ PDG reports the resonance width (Γ). Here the conversion $\tau = \frac{\hbar}{\Gamma}$ is given instead.

Conjugate variables

Have you noticed a pattern?

$x \longleftrightarrow p_x$

$y \longleftrightarrow p_y$

$z \longleftrightarrow p_z$

$t \longleftrightarrow E$

Relativity: interval ... $s^2 = (ct)^2 - x^2 - y^2 - z^2$ ← always invariant

energy – $(m_0c^2)^2 = E^2 - p_x^2c^2 - p_y^2c^2 - p_z^2c^2$ ← always invariant

now Uncertainty: $\Delta x \Delta p \geq \frac{\hbar}{2}$

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Quantum Mechanics born of some anxiety

the lack of radiation of Bohr's accelerating electrons was still a problem: Bohr knew it and figured there would be a more complete answer.

what in the world is an electron in deBroglie's scheme?

There was much that was ad hoc and not believable

both in Bohr's approach and deBroglie's

however, the experimental situation made it clear that the broad suppositions of both had to be a part of the truth.

Quantum Mechanics, proper was the child of 3+1 people:

Werner Heisenberg - 1925; invention #1

Erwin Schrödinger - 1926; invention #2

Paul Dirac - 1925; showed #1 and #2 are equivalent

Max Born - 1926; gave the modern interpretation

the breakthrough

from an unlikely source

either Erwin Schrödinger

or Erwin Schrodinger

or Erwin Schroedinger

Erwin Schroedinger 1887-1961

where do you look for your keys in the dark?

Schroedinger was an expert

in the mathematics of waves

EM waves, material waves, fluids, elastic media, sound...

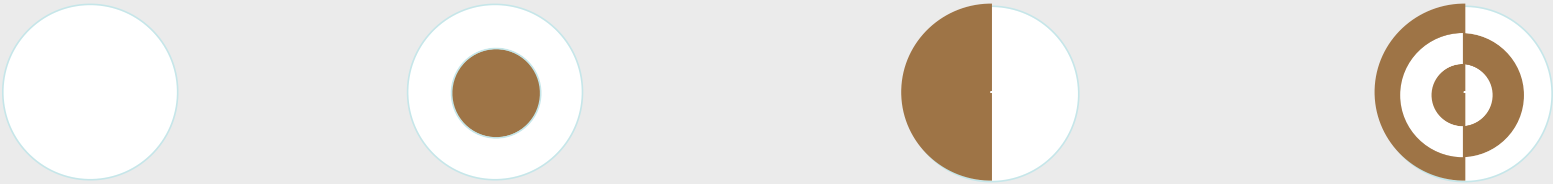
integers again

$$u(r,t) = \sum_{m=0}^{m=\infty}\sum_{k=0}^{k=\infty}[(A_{mk}\cos\omega_{mk}t + B_{mk}\sin\omega_{mk}t)\cos\theta + (A_{mk}\cos\omega_{mk}t + B_{mk}\sin\omega_{mk}t)\sin\theta]J_m(\frac{\omega_{mk}r}{v})$$

Solutions for the vibrations of a drumhead, or a violin string, or that vibrating hoop...

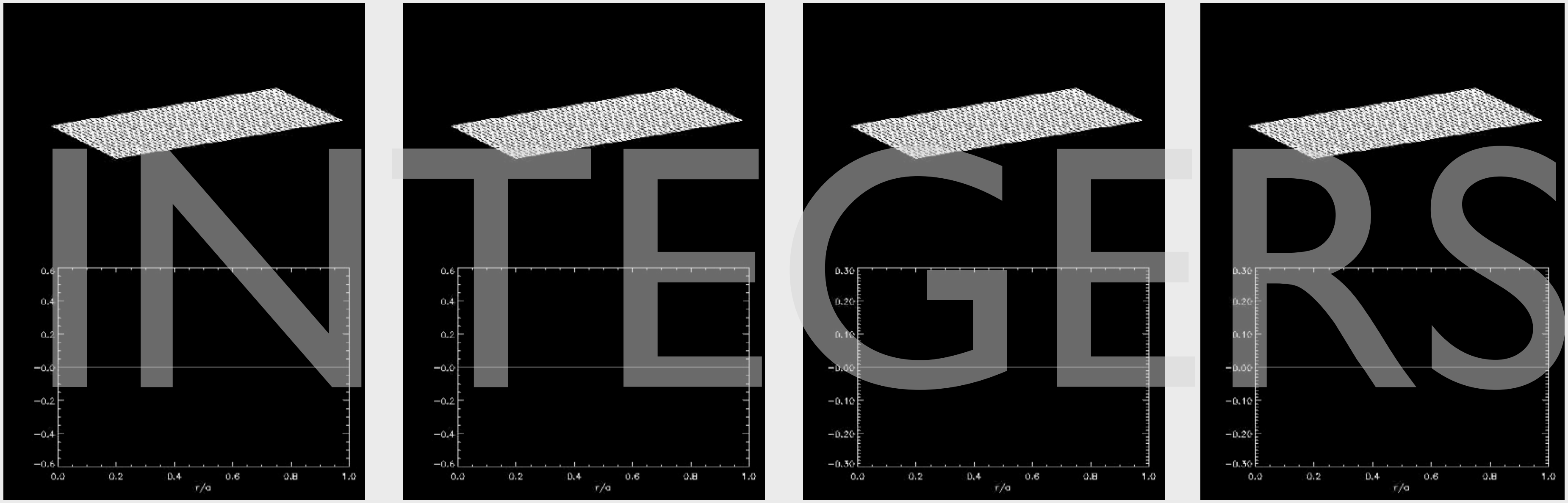
Forget the details...just notice the mixing of lots of waves...the m's and k's? **Integers**.

Here are some of these infinite modes of vibration as described by some of the functions (white and brown are moving in opposite directions (the drum is clamped down at the edges)



these are both m=0 modes

these are both m=1 modes



I found these nice movies at: http://photon.phys.clemson.edu/brad/courses.dir/movies.dir/phys841-01.dir/movies.html terrific

what's waving???

Schroedinger "solved" a drum-head-like equation for the hydrogen atom

Discrete, vibrational modes...of a something.

However, he was in for a surprise -

Brave guy: worked in the alps over Christmas 1925 with his girlfriend while his wife stayed in Zurich.

The surprise, is that the mathematics required that the **state** of such a system had to be

imaginary!!

Solutions: the Bohr atom bang-on.
but with a twist.

the ‘‘quantum field’’

“psi”…also called the “wavefunction”

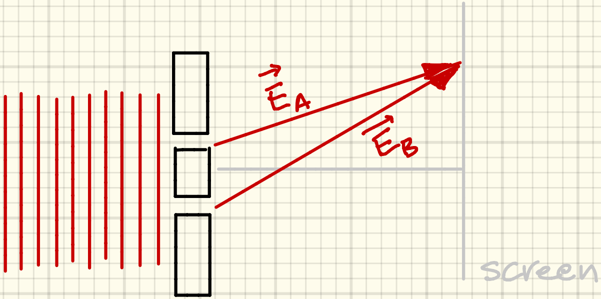
the “state” of something.

The “Schroedinger Equation”

predicts its behavior in space and time

$$\psi(x,t)$$

REPLAY: OPTICS



Brightness ⇒ intensity

$$I = c\epsilon_0 E_T^2$$

$$E_T = E_A + E_B$$

$$E_T^2 = E_A^2 + E_B^2 + 2E_A E_B$$

constructive & destructive interference

$$I = c\epsilon_0 E^2 = Nhf$$

get a classical analogy

$$N(\gamma) \propto E^2$$

Electrons are waves

They diffract: and you saw it!

What's "waving" for electrons?

THE WAVEFUNCTION $\psi(x,t)$ Brainchild of Erwin Schrödinger...

In quantum mechanics we don't use sines or cosines, we use both for genality:

$$Ae^{ix} = A\cos x + A i \sin x \quad \text{Euler's Relation}$$

A general wavefunction:

$$\psi(x,t) = A e^{2\pi i\left(\frac{x}{\lambda} - ft\right)}$$

Remember: $E = hf$ & $2\pi f = \omega$

$E = \frac{2\pi}{\tau} hf = \hbar\omega$; $2\pi f = \frac{E}{\hbar}$

and: $p = \frac{h}{\lambda}$, $\frac{p}{h} = \frac{1}{\lambda}$

$\frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{p}{\hbar}$

so

$$\psi(x,t) = A e^{i(p_x x - Et)/\hbar}$$

Motion of our electron .. in wavepackets...

has a particle $\leftrightarrow$ wave ... tentative relationship

So think "particle" for a minute

Energy:

$$\underbrace{\frac{p^2}{2m} + U = E}$$

non-relativistic

(there's a story there)

whatever Schroedinger was to do... this should be.. somewhere!

Some "arbitrary" differentiation ...

$$\frac{\partial \psi}{\partial x} = \frac{i}{\hbar} p_x \psi \qquad \psi(x,t) = A e^{i(p_x x - Et)/\hbar}$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{1}{\hbar^2} p_x^2 \psi$$

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar} E \psi$$

from the right

Take the energy equation and multiply by $\leftarrow \psi(x,t)$

$$\frac{p^2}{2m} \psi(x,t) + U \psi(x,t) = E \psi(x,t)$$

and look above. $p_x^2 \psi = -\hbar^2 \frac{\partial^2 \psi}{\partial x^2}$ & $E\psi = -\frac{\hbar}{i} \frac{\partial \psi}{\partial t}$

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x,t) + U(x) \psi(x,t) = -\frac{\hbar}{i} \frac{\partial}{\partial t} \psi(x,t)$$

THE TIME-DEPENDENT SCHROEDINGE EQUATION

A RECIPE: how to go from classical physics to quantum physics

DEFINE THE MOMENTUM OPERATOR $\hat{p}_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$

DEFINE THE ENERGY OPERATOR $\hat{E} \equiv -\frac{\hbar}{i}\frac{\partial}{\partial t}$

TAKE ANY CLASSICAL EQUATION AND MAKE SUBSTITUTIONS:

$$\left.\begin{aligned} p_{x,y,z} &\longrightarrow \hat{p}_{x,y,z} \\ E &\longrightarrow \hat{E} \end{aligned}\right\} \text{ and "operate on" } \psi$$

CALLED "FIRST QUANTIZATION"   this "to quantize"