5. Quantum Mechanics 1, 1

lecture 19, October 11, 2017

housekeeping

exam 2: Friday, October 27

This week:

- lecture MTW...**we will meet on Friday**
- HW4 due Friday

Honors option

- Go to: https://qstbb.pa.msu.edu/storage/PHY215/honors/
- read the MinervaInstructions1_2017_215 document

today

tiny bit more of Bohr

“Correspondence Principle”

Quantum Mechanics 1

BOHR CORRESPONDENCE PRINCIPLE

"Old quantum theory" — entirely Bohr — a hodge-podge of weird ideas and claims... that worked. Unsatisfying

began a troubling relationship with quantum mechanics in 1913...

1st attempt:

In the limiting case of large quantum numbers, frequencies and intensities of radiation:

quantum results → classical results

$$F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r}$$

$$\frac{1}{2}mv^2 = \frac{1}{2\cdot 4\pi\epsilon_0}\frac{e^2}{r}$$

Classically:

frequency of electron in an orbit:

$$f_c = \frac{v}{2\pi r} \Rightarrow \qquad K = \frac{1}{2}mv^2 = \frac{1}{8\pi\epsilon_0}\frac{e^2}{r}$$

$$v = \sqrt{\frac{2e^2}{8\pi\epsilon_0 mr}}$$

$$f_c = e\sqrt{\frac{1}{4\pi\epsilon_0 mr}}\ \frac{1}{2\pi r}$$

$$= e\sqrt{\frac{1}{4\pi\epsilon_0 mr}\cdot\frac{1}{4\pi^2 r^2}}$$

$$f_c = \frac{e}{\sqrt{16\pi^3\epsilon_0 m r^3}}$$

the allowed orbits: $r_n = \frac{4\pi\epsilon_0\hbar^2}{me^2}n^2$

$$f_c = \frac{e}{\sqrt{16\pi^3\epsilon_0 m\cdot \frac{48\,\epsilon_0^3\hbar^6 n^6\pi^3}{m^3e^6}}} = \frac{e^4m}{32\,\epsilon_0^2\pi^3\hbar^3n^3}$$

frequency of classical electron at this orbit, r_n

$$f_c = \frac{e^4m}{32\,\epsilon_0^2\pi^3\hbar^3n^3}$$

quantum frequency fn large n transitioning to n-1

$$f_Q = \frac{e^4}{4\pi(4\pi\epsilon_0)^2} \frac{m_e}{\hbar^3} \left[\frac{1}{(n-1)^2} - \frac{1}{n^2} \right]$$

$$f_Q = \frac{e^4}{4\pi(4\pi\epsilon_0)} \frac{m_e}{\hbar^3} \frac{2n-1}{n^2(n-1)^2}$$

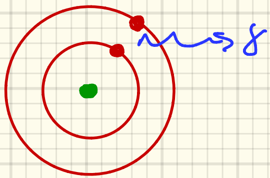
n is very large -- $n-1 \sim n$ & $2n-1 \sim 2n$

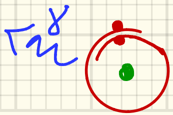
$$f_Q \rightarrow \frac{e^4}{4\pi(4\pi\epsilon_0)} \frac{m_e}{\hbar^3} \frac{2n}{n^4} = \frac{me^4}{32\pi^3\epsilon_0^2\hbar^3} \frac{1}{n^3}$$

↖ this guy jumps

$$f_c = \frac{e^4 m_e}{32\epsilon_0^2\pi^3\hbar^3 n^3}$$

← this guy radiates because it spirals





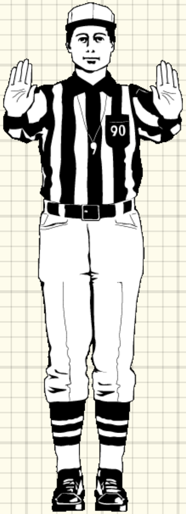
Became a guide later...

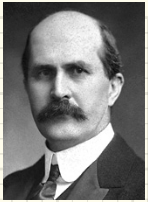
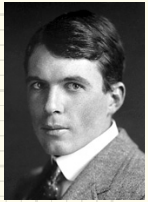
X-ray Diffraction

The definitive characteristic that makes a wave... a wave:

interference

obstruction size ~ wavelength



x-rays: are they waves? 1912 – sure

but how to show it?

known that $\lambda \sim 10^{-11}$ m or so   Mo $K_\alpha \rightarrow \lambda = 0.63\,\text{Å} = 6.3 \times 10^{-11}$ m

How to demonstrate diffraction?

1912   Max von Laue (theory) → Walter Friedrich, Paul Knipping (experiment)   demonstration

Nobel 1914

1912 – 1914   W.L Bragg (son: theory)
W.H Bragg (father: experiment)

Nobels 1915

simplified analysis & a true spectrograph instrument

- study materials
- measure x-ray characteristics

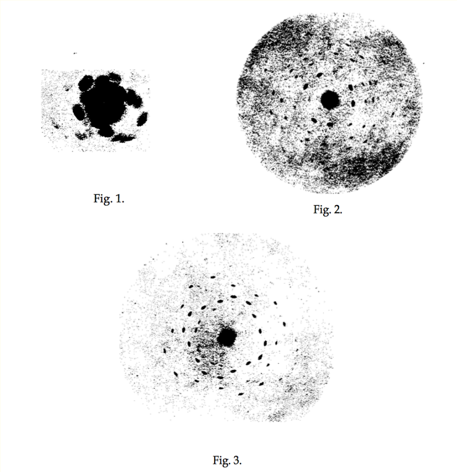


Fig. 1.

Fig. 2.

Fig. 3.

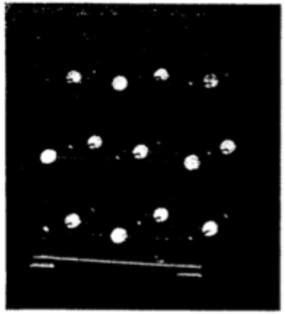


Fig. 1.

model of NaCl from Bragg's Nobel lecture

Bragg Spectrometer

Demonstrated both wave-like nature of x-rays but also confirmation of crystaline nature of solids

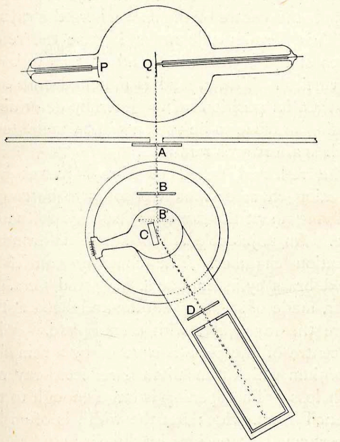
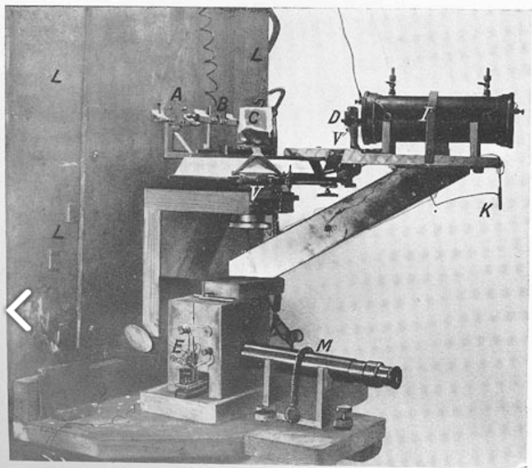
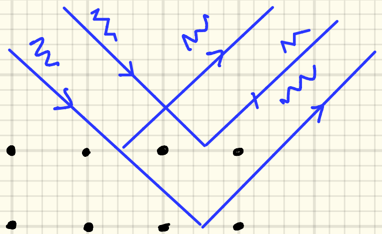


FIG. 4.

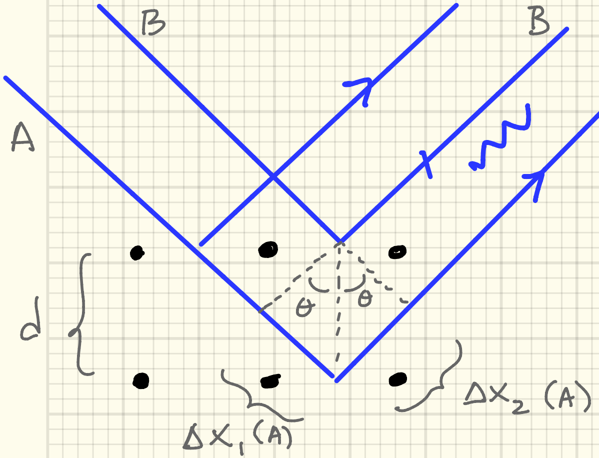


X-RAY SPECTROMETER.

LLL,	Lead box.	*V,*	Vernier of crystal table.
A, B, D,	Slits.	*V',*	Vernier of ionisation chamber.
C,	Crystal.	*K,*	Earthing key.
I,	Ionisation chamber.	*E,*	Electroscope.
		M,	Microscope.



when these emerge in phase, bright spots



$x(A) = x(B) + \Delta x_1 + \Delta x_2$

$$\frac{\Delta x_1}{d} = \sin\theta = \frac{\Delta x_2}{d}$$

$\Delta x = 2d\sin\theta = n\lambda$ for constructive interference

$n\lambda = 2d\sin\theta$

"Bragg Condition" "Bragg Law"

- pick angle → determine λ
- measure ∡'s of maxima → determine crystal geometry

Ni

$d = 2.15$ Å

Mo K_α $\lambda = 0.63$ Å

} what angles?

$n\lambda = 2d\sin\theta$

$$\sin\theta_n = n\frac{\lambda}{2d} = n\frac{0.63}{4.3} = \frac{n}{6.7}$$

first angle, $n=1$ $\sin\theta_1 = 0.15 \Rightarrow \theta_1 = 8.6°$

second $n=2$ $\theta_2 = 17°$

note when $n \geq 7$? no diffraction: $\sin\theta_7 > 1$

$\Rightarrow$ 6 angles available for interference

1923... what's known?

h explains: Blackbody radiation, photoelectricity

h appears to explain: Hydrogen, Bohr atom

h strongly suggests: light is quantized in spite of being a wave

either: particle and wave?

or: particle or wave?

Into this: a 16 page Ph.D. thesis

from Prince Louis de Broglie

1924

"Becaus photons have wave properties and particle properties, perhaps all forms of matter have wave as well as particle characteristics."

remember: $E = pc = h\nu$

$$p = \frac{h\nu}{c} = \frac{h}{\lambda} \Rightarrow \lambda = \frac{h}{p}$$

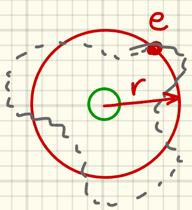
for light.

de Broglie suggested that this is generally true.

non-relativistic: $\lambda = \frac{h}{mv}$

relativistic: $\lambda = \frac{h}{mv\gamma}$

Not completely arbitrary



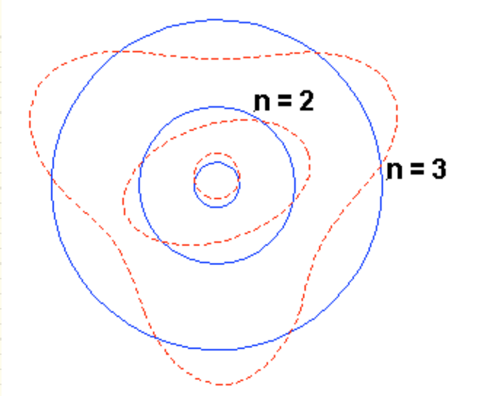
think of e as a trapped wave
^
standing

circumference $= n\lambda$

$$2\pi r = n\lambda = n\frac{h}{p}$$

$$L = rp = \frac{nh}{2\pi} = n\hbar$$ → Bohr's condition!

deBroglie picture



the nodes move as the electron waves at each Bohr orbit

3rd: what's λ_e?

$2\pi r = n\lambda$ $\quad r_n = n^2 a_o$

$\lambda = \frac{2\pi r_n}{n}$

$\lambda = \frac{2\pi n^2 a_o}{n} = 2\pi n a_o = (2\pi)(3)(0.53\times10^{-10} m)$

$\lambda = 10\times10^{-10} m = 1\ nm$

v_e?

$\lambda = \frac{h}{p}$ $\quad$ NR: $p = mv$

$v = \frac{h}{m\lambda} = \frac{h}{m 2\pi r}\cdot n = \frac{n\hbar}{m r_n}$

$$v = \frac{h}{m\lambda} = \frac{h}{m 2\pi r} \cdot n = \frac{n\hbar}{m r_n} \qquad r_n = n^2 a_0$$

$$v = \frac{n\hbar}{m n^2 a_0} \left(\frac{c^2}{c^2} \right) = \frac{1}{n} \frac{\hbar c^2}{a_0 m c^2}$$

$$\frac{v}{c} = \beta = \frac{1}{n} \frac{\hbar c}{a_0 m c^2} \qquad mc^2 = 0.511 \text{ MeV}$$

$$\hbar c = 197.3 \text{ eV} \cdot \text{nm}$$

$$= \frac{1}{n} \frac{197.3 \text{ eV} \cdot \text{nm}}{(0.053 \text{ nm})(0.511 \times 10^6 \text{ eV})}$$

$$\beta = \frac{1}{n} 0.00729 \Rightarrow v = \frac{1}{n} 2.18 \times 10^6 \text{ m/s}$$

$$v_3 = 7.3 \times 10^5 \text{ m/s}$$

How come you're not waving?

Aroldis Chapman:

fastest pitch recorded in MLB
105 mph
~ 47 m/s

9/24/2010

what's λ?

$m = 140\,g$

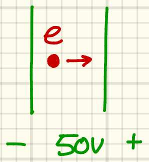
$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\,J\cdot s}{(0.140\,kg)(47\,m/s)} \sim 1.01\times10^{-34}\,m$$

$10^{-30}\times$ proton radius

no chance for macroscopic objects to diffract with anything

What about tiny matter objects?

How about an electron through modest potential difference?



$$\tfrac{1}{2}mv^2 = eV$$

$$\frac{p^2}{2m} = eV$$

$$p = \sqrt{2meV}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} = \frac{6.63\times10^{-34}\ \mathrm{J\cdot s}}{\left[(2)(9.11\times10^{-31}\,\mathrm{kg})(1.6\times10^{-19}\,\mathrm{C})(50\,\mathrm{V})\right]^{1/2}}$$

or

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{hc}{\sqrt{2mc^2 eV}} = \frac{1240\ \mathrm{eV\cdot nm}}{\left[(2)(0.511\times10^{6}\,\mathrm{eV})(50\,\mathrm{eV})\right]^{1/2}}$$

$$\lambda = 0.17\ \mathrm{nm} = 1.7\times10^{-10}\ \mathrm{m}$$

↳ crystal-sized

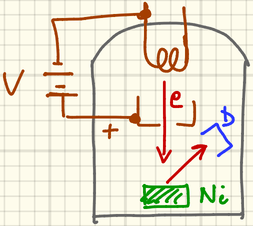
THE PHONE COMPANY

1927 Bell Labs

Clinton Davisson &

Lester Germer

goofed



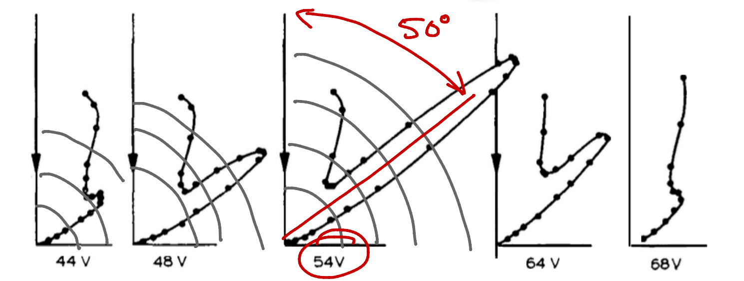
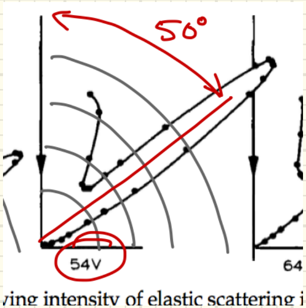


Fig. 2. Polar diagram showing intensity of elastic scattering in A-azimuth (Fig. 1) as function of latitude angle, for series of primary-beam voltages.



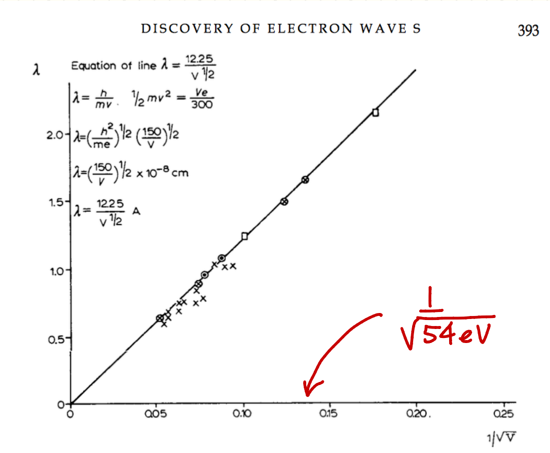
Studied surface with x-rays:

$$\lambda(Ni)_{xrays} = 1.65\ \mathring{A}$$

Considered de Broglie λ:

$$\lambda_e = \frac{h}{\sqrt{2Vem}} = \frac{1240\ eV.m}{\sqrt{(2)(54)(0.511\times10^6)}}$$

$$= 1.67\ \mathring{A}$$



Remember J.J.?

His son G.P. ...

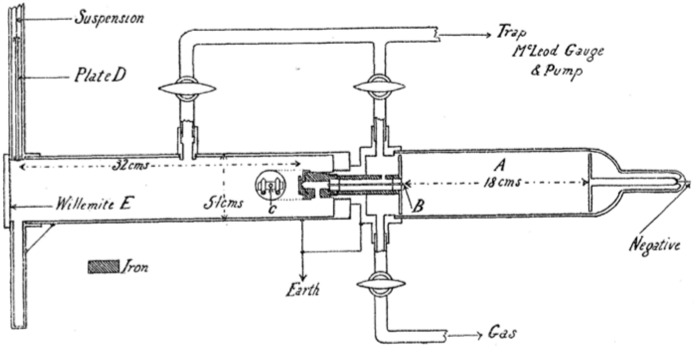
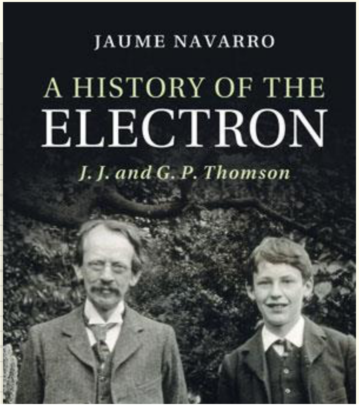


Fig. 1.



xperiments on the Diffraction of Cathode Rays.

omson, M.A., Fellow of Corpus Christi College and Professor of Natural Philosophy in the University of Aberdeen.

(Communicated by Sir Joseph Thomson, F.R.S.—Received November 4, 1927.)

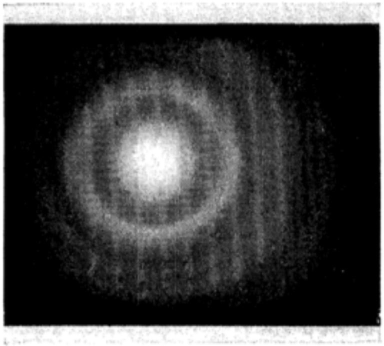
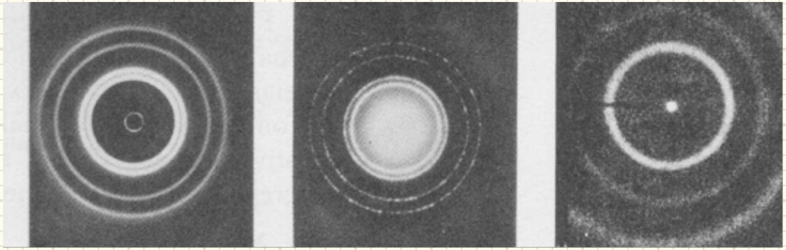


Fig. 4.—Gold.



0.071nm X-ray diffraction on a polycrystal	600 Ev electron diffraction on a polycrystal	0.057 ev neutron diffraction on a polycrystal

JJ received the Nobel Prize for discoverying that the electron is a particle

GP received the Nobel Prize for discovering that the electron is a wave.

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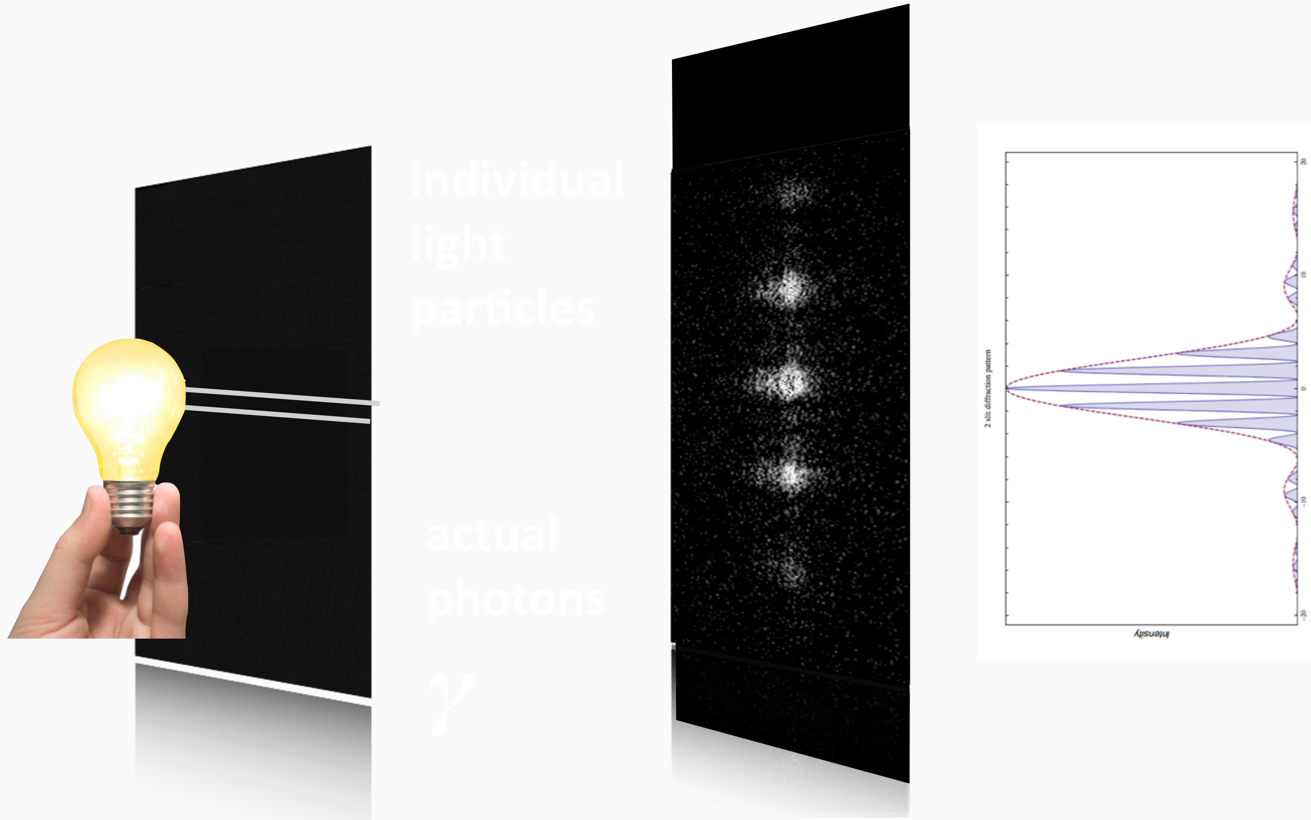
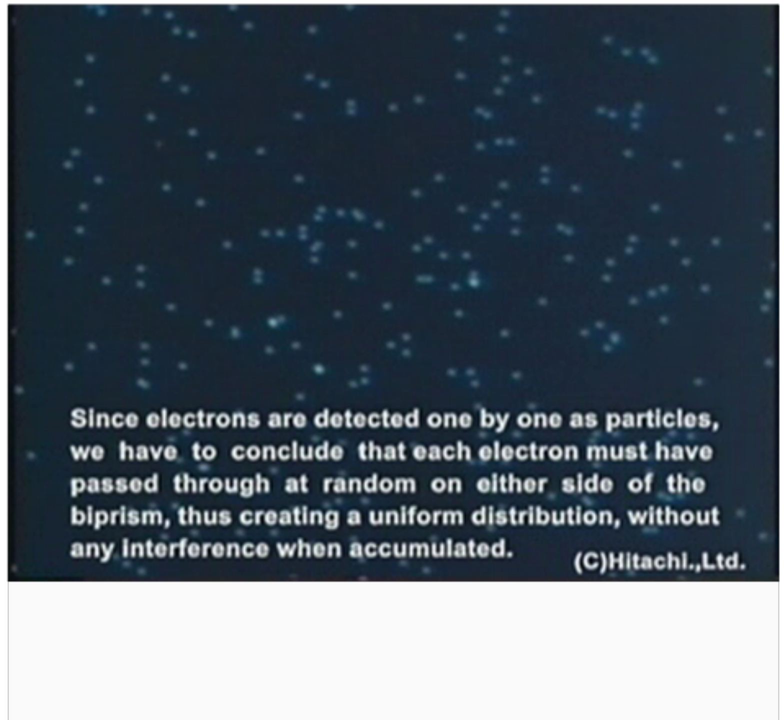
GP received the Nobel Prize for discovering that the electron is a wave.

How does this work?

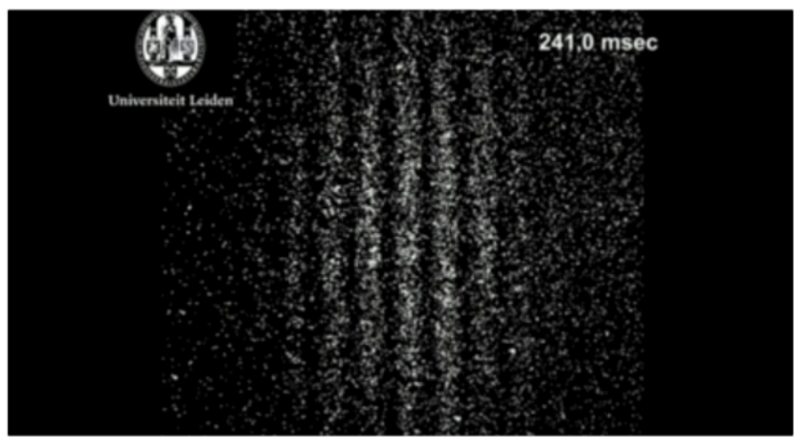


David Dykstra, Steven Busch, Wouter Peeters, Martin vanExter, Leiden University, 2008

http://www.youtube.com/watch?v=MbLzh1Y9POQ lets count electrons



electrons!



photons!