1. Special Relativity, 2

lecture 2, September 1, 2017

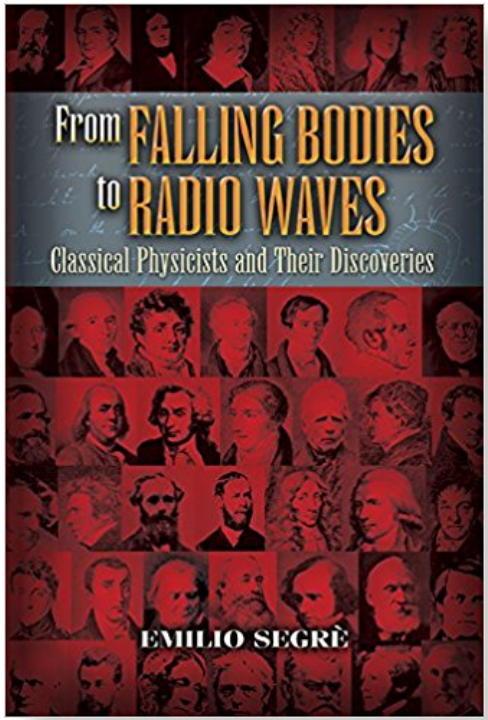
housekeeping

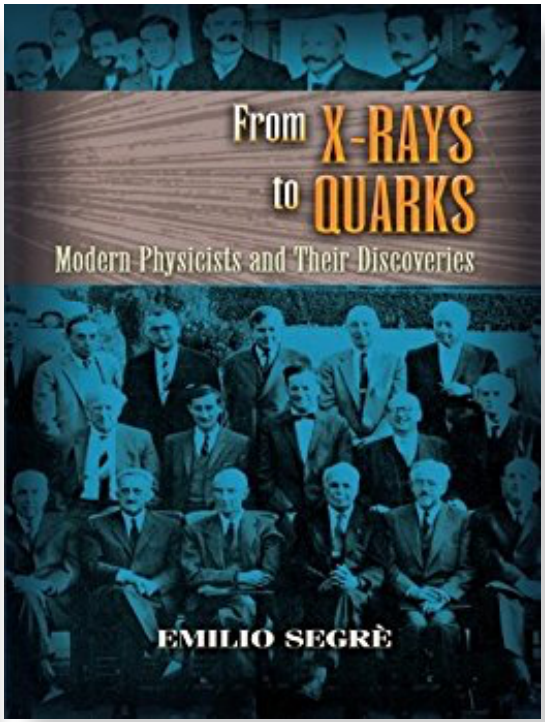
remember to check the course page:

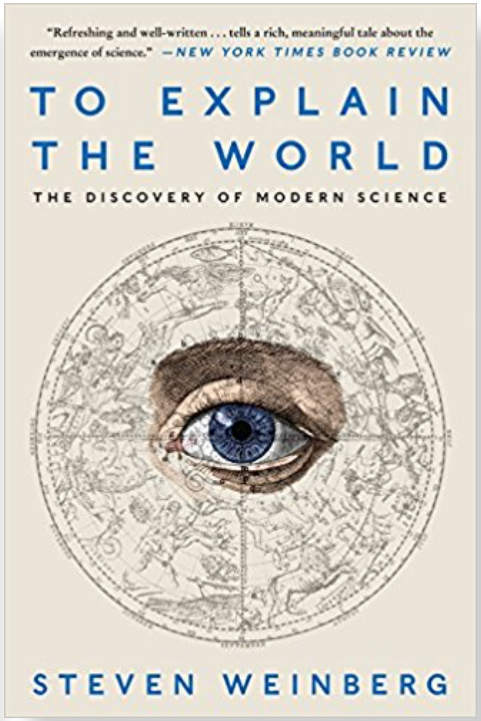
chipbrock.org

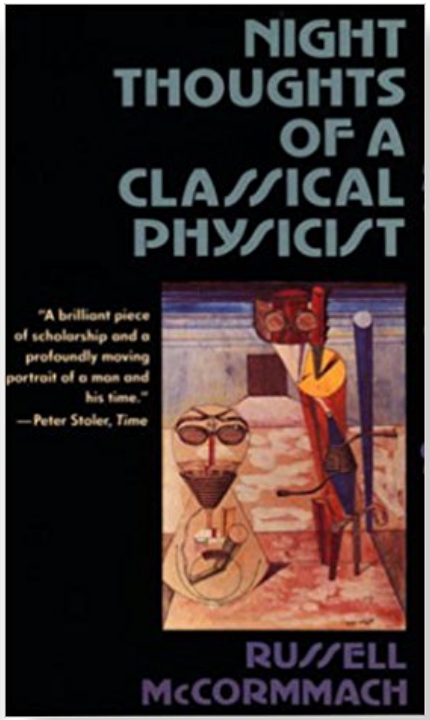
and sign up for the feedburner reminders

I'll post something tonight...tell me if you don't get a message on Tuesday


From FALLING BODIES to RADIO WAVES
Classical Physicists and Their Discoveries
EMILIO SEGRÈ


From X-RAYS to QUARKS
Modern Physicists and Their Discoveries
EMILIO SEGRÈ


"Refreshing and well-written . . . tells a rich, meaningful tale about the emergence of science." —NEW YORK TIMES BOOK REVIEW
TO EXPLAIN THE WORLD
THE DISCOVERY OF MODERN SCIENCE
STEVEN WEINBERG


NIGHT THOUGHTS OF A CLASSICAL PHYSICIST
"A brilliant piece of scholarship and a profoundly moving portrait of a man and his time."
—Peter Stoler, Time
RUSSELL McCORMMACH

Maxwell's Equations + 1

1. $$\oint_S \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon_0}$$ Gauss' Law for electrostatics

2. $$\oint_S \vec{B} \cdot d\vec{S} = 0$$ Gauss' Law for magnetostatics

3. $$\oint_\ell \vec{E} \cdot d\vec{\ell} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$ Ampere's Law

4. $$\oint_\ell \vec{B} \cdot d\vec{\ell} = -\frac{d\Phi_E}{dt}$$ Faraday's Law

5. $$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$ Lorentz Force

$\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2N^{-1}m^{-2}}$ vacuum permittivity $\frac{1}{4\pi\epsilon_0} = k_e = 8.9 \times 10^9\ \mathrm{Nm^2C^{-2}}$

$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{JA^{-2}m^{-1}}$ vacuum permeability

tracking

theory and experiment
circa 1900

differential form of Maxwell's Equations

In general: $\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$

and so

$$\vec{E} = -\left(\frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right)$$

Standard calculus operator: <u>gradient</u>

$$\vec{\nabla} \equiv \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$$

so $\vec{E} = \vec{\nabla} V$

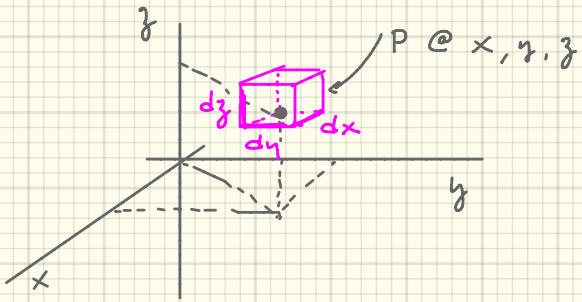
Also... <u>divergence</u>

$$\vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

and... <u>curl</u>

$$\vec{\nabla} \times \vec{A} = \hat{i} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{j} \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{k} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

Take a tiny volume



Imagine an $\vec{E}(x, y, z)$

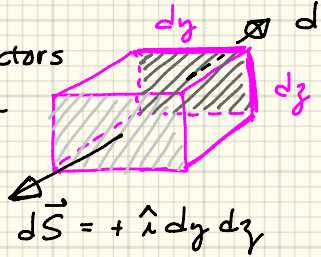
@ <u>back</u> <u>face</u>

which changes at

the <u>front</u> <u>face</u>

from $x \to x + dx$

Surface vectors point <u>out</u>

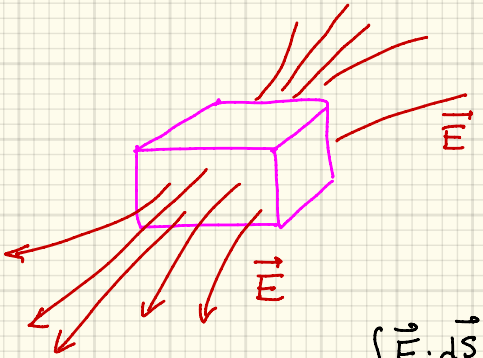


$d\vec{S} = -\hat{i}\, dy\, dz$

$d\vec{S} = +\hat{i}\, dy\, dz$

$$\vec{E}(x, y, z) + \left(\frac{\partial \vec{E}}{\partial x}\right) dx$$

back + change



Look at the flux through the volume $\oint \vec{E}\cdot d\vec{S}$ due to these two faces.

$$\underset{\text{back, front}}{\int \vec{E}\cdot d\vec{S}} = \underbrace{\vec{E}\cdot(-\hat{i}\,dy\,dz) + \left(\vec{E} + \frac{\partial \vec{E}}{\partial x}dx\right)\cdot(\hat{i}\,dy\,dz)}_{\text{cancel}}$$

$$= dx\,dy\,dz\,\frac{\partial}{\partial x}(\vec{E}\cdot\hat{i})$$

$$= dx\,dy\,dz\,\frac{\partial E_x}{\partial x}$$

other faces? similar.

$$\oint \vec{E}\cdot d\vec{S} = dx\,dy\,dz\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right)$$

$$= dx\,dy\,dz\,(\vec{\nabla}\cdot\vec{E})$$

$$\oint \vec{E}\cdot d\vec{s} = dx\,dy\,dz\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right)$$
$$= dx\,dy\,dz\,(\vec{\nabla}\cdot\vec{E})$$

Suppose there had been a charge inside?

$q = \rho\, dx\,dy\,dz$

↑ charge density

OK:

$$\oint \vec{E}\cdot d\vec{s} = \frac{q}{\epsilon_0}$$

$$\Downarrow$$

$$dx\,dy\,dz\,(\vec{\nabla}\cdot\vec{E}) = \frac{dx\,dy\,dz\,\rho}{\epsilon_0}$$

$$\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}}$$

Maxwell's Equation for Gauss' Law

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

"current density"

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

FREE SPACE $\longrightarrow$ no sources.

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ $\longrightarrow$ time changing B? get $\perp \vec{E}$

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$ $\longrightarrow$ time changing E? get $\perp \vec{B}$

$$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

pick out a particular component : x

$$\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = \mu_0\epsilon_0\frac{\partial E_x}{\partial t}$$

remember this

now:

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \longrightarrow \underbrace{\vec{\nabla}\times\vec{\nabla}\times\vec{E}}_{\text{an identity}} = \vec{\nabla}\times\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$= \vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{E}}_{=0\ \text{Gauss' Law.}}) - \nabla^2\vec{E}$$

$$\nabla^2\vec{E} = \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\vec{\nabla}^2\vec{E} = \mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2} \longrightarrow \nabla^2\vec{E} - \mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

like $\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2}$

another for $\vec{B}$ also

$$\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

another for $\vec{B}$ also

look at $$\frac{\partial^2 \phi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 \phi}{\partial t^2} = 0$$

$\rightarrow$ a wave equation for ϕ with velocity v.

$\Downarrow$

a wave equation for $\vec{E}$

with velocity $v = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$

ONE OF THOSE WOW MOMENTS IN THEORETICAL PHYSICS

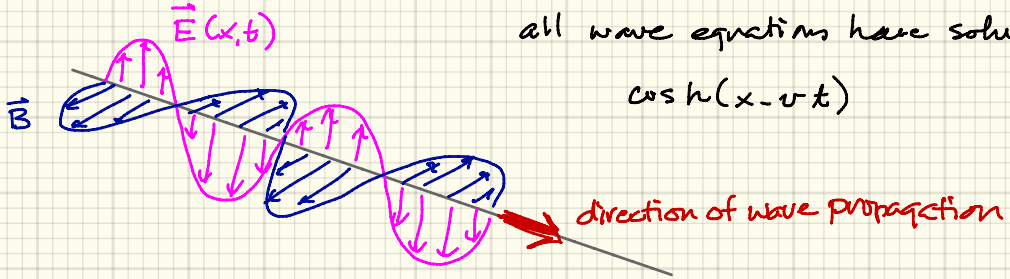
$\mu_0 = 4\pi \times 10^{-7}\ N \cdot C^{-2} s^2$

$\epsilon_0 = 8.8542 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

so $$\sqrt{\frac{1}{\mu_0 \epsilon_0}} = \sqrt{\frac{1}{\left(4\pi \times 10^{-7}\ \frac{N s^2}{C^2}\right)\left(8.8542 \times 10^{-12}\ \frac{C^2}{N\ m^2}\right)}}$$

$$v = 2.998 \times 10^8\ m/s = c$$

and $|B| = |E|/c$



all wave equations have solutions like

$$\cos h(x - vt)$$

the $\frac{\partial E}{\partial t}$ causes the B

$\updownarrow$

the $\frac{\partial B}{\partial t}$ causes the E

As you know $\quad \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ $\quad$ Poynting Vector

$$[S] = \frac{\text{energy}}{(\text{area})(\text{time})}$$

$$\text{Intensity} = |\langle S \rangle|$$

$$= \frac{1}{\mu_0} B_{rms} E_{rms} = \frac{E_{rms}^2}{\mu_0 c} = \epsilon_0 c E_{rms}^2$$

Summary

1. $\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$

2. $\oint \vec{B} \cdot d\vec{S} = 0$

3. $\oint \vec{E} \cdot d\vec{l} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

4. $\oint \vec{B} \cdot d\vec{l} = -\frac{d\Phi_E}{dt}$

These are for "bulk" regions of space and extended sources... "Integral form"

Useful are versions which are infinitesimal $\rightarrow$ "Differential form"

Useful: <u>potential</u>

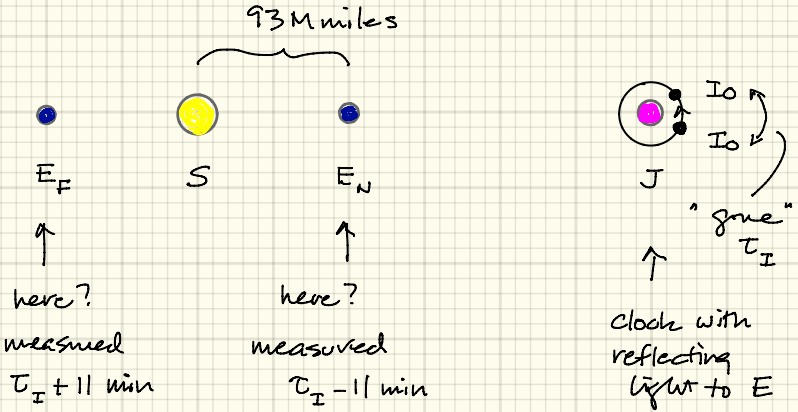
$$E_l = -\frac{dV}{dl} \qquad l = x, y, \text{ or } z$$

$$V = V(x, y, z)$$

Now: experiments on light.

1. Speed of light

1676 Ole Römer



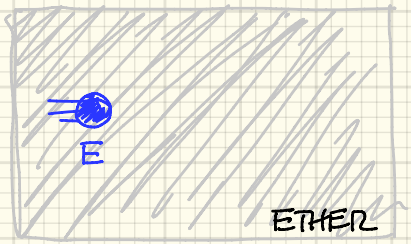
knew how long for Io to eclipse

Newton or Huygens → 16 min total

$$c = \frac{93\times10^6}{8\text{ min}}\left(\frac{1\text{ min}}{60\text{ s}}\right) = 200{,}000\text{ mi/s} \sim 186{,}000\text{ mi/s}$$

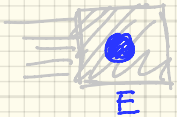
2. Motion relative to the ether: Stellar Aberation

Ether:



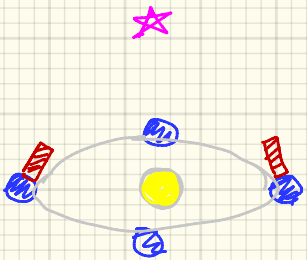
earth moves through Ether

or:



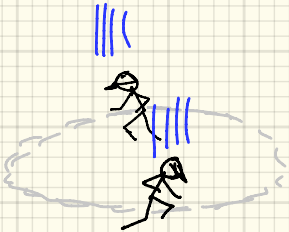
earth drags Ether

Abberation of starlight:



Suppose:

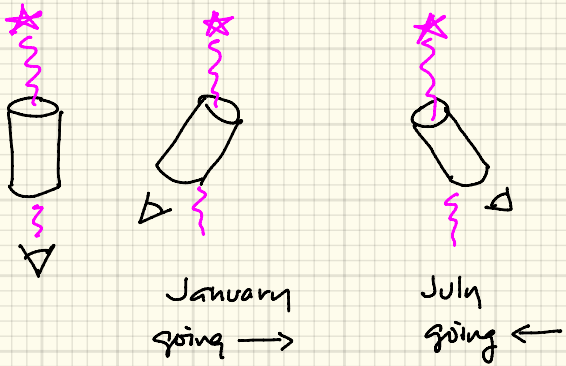
face dry



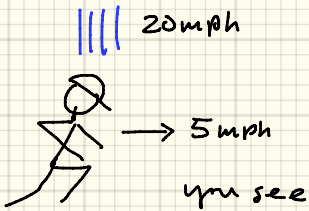
you would tilt your head to keep rain off your face

Analogy:

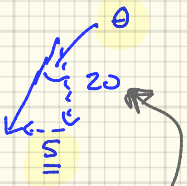
light : rain
earth orbit : track
telescope : hat



you say the star is the direction of telescope



you see

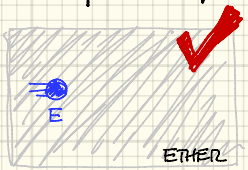


you could determine

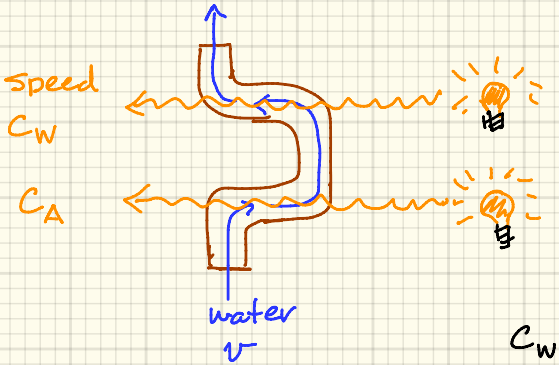
James Bradley 1728

↳ observed angle

⇒ earth moving through ether



2. Motion relative to the ether: Fizeau's Ether-Drag Experiment



with current

against current

$\frac{c}{n}$ speed of light in stationary water

$$c_W = \frac{c}{n} + f v$$

$$c_A = \frac{c}{n} - f v$$

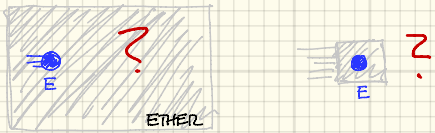
if ether dragged by water...

$f \neq 1$

H. Fizeau 1851: $f \neq 0$

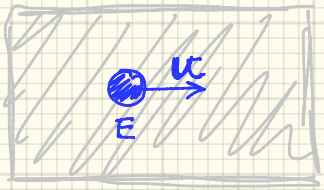
$f \neq 1$!

$\Rightarrow$ some drag of Ether by water ?



2. Motion relative to the ether: Michelson-Morley Experiment

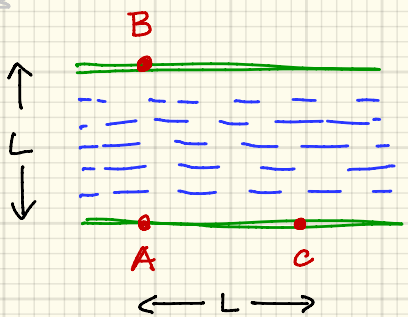
Measure the speed of earth relative to ether



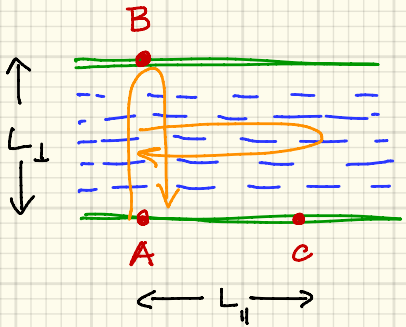
on earth:

Ether "wind"

clever idea:



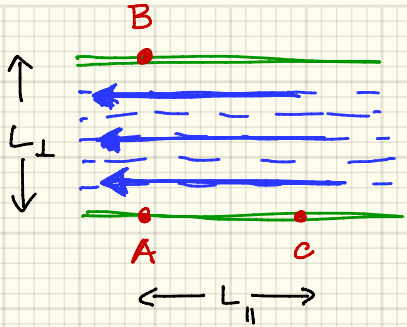
swimmers capable of speed v wrt still water



ABA : "$\perp$"

ACA : "$\parallel$"

still water: $t_\perp(\text{still}) = \frac{2L_\perp}{v}$ $\quad t_\parallel(\text{still}) = \frac{2L_\parallel}{v}$



river flows u

$t_\parallel$: AC ... $\xleftarrow{u}\ \xrightarrow{v}$ $\quad V_{AC} = v - u$

CA ... $\xleftarrow{u}\ \xleftarrow{v}$ $\quad V_{CA} = v + u$

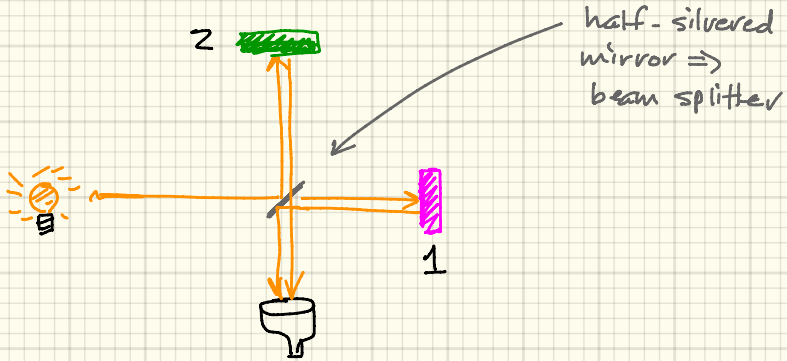
round trip: $t_\parallel = \dfrac{L_\parallel}{V_{AC}} + \dfrac{L_\parallel}{V_{CA}} = \dfrac{L_\parallel}{v-u} + \dfrac{L_\parallel}{v+u}$

$$t_\parallel = \frac{2L_\parallel}{v} \frac{1}{1 - u^2/v^2}$$

$$t_\perp = \frac{2L_\perp}{v} \frac{1}{\sqrt{1 - \frac{u^2}{v^2}}}$$

show this D2

Michelsons Idea



B

B

1887

Albert Michelson (1852-1931)

and

Edward Morley

The Nobel Prize in Physics 1907
Albert A. Michelson

Share this:

The Nobel Prize in Physics 1907

Albert Abraham Michelson
Prize share: 1/1

The Nobel Prize in Physics 1907 was awarded to Albert A. Michelson *"for his optical precision instruments and the spectroscopic and metrological investigations carried out with their aid"*.

Photos: Copyright © The Nobel Foundation

Share this:

To cite this page
style: "The Nobel Prize in Physics 1907". *Nobelprize.org.* Nobel Media AB 2014. Web. 28
<http://www.nobelprize.org/nobel_prizes/physics/laureates/1907/>

The prize went to Michelson for the instrument: the Michelson interferometer

the same idea
as the gravitational wave instrument

We remember him for the most important null measurement in 200 years

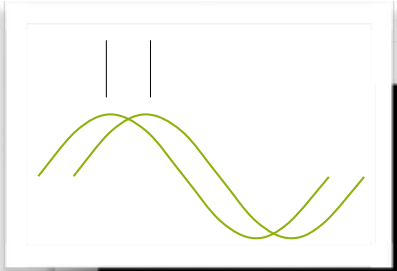
‘‘Michelson Morley Experiments’’

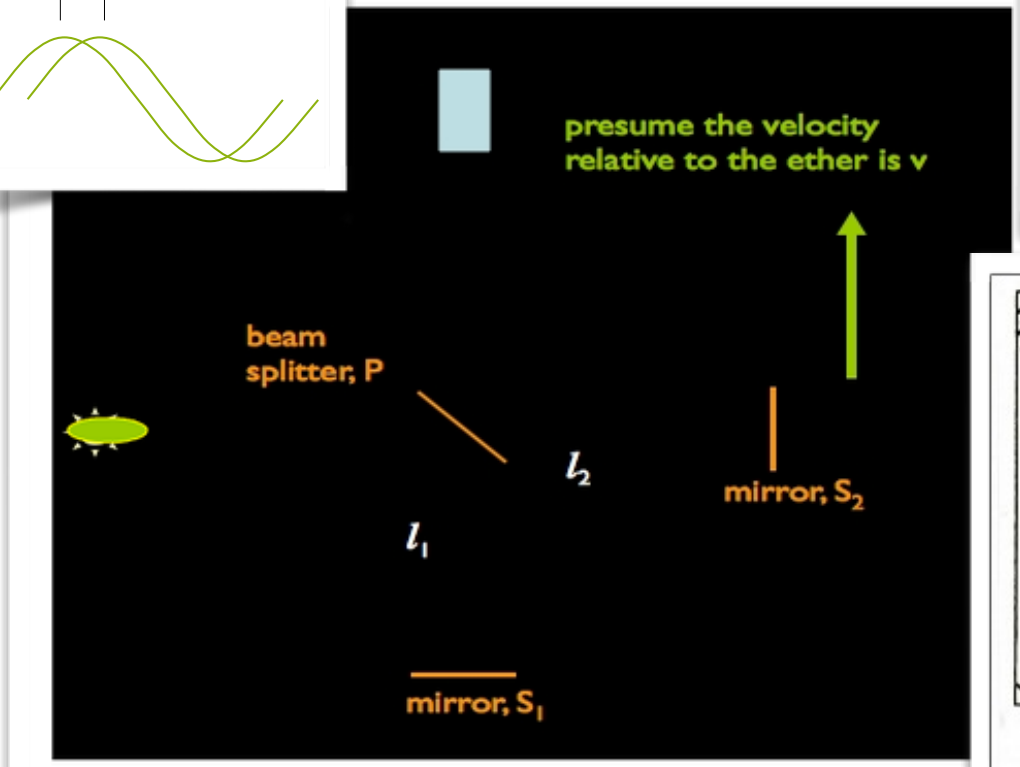
trying to measure the speed of Earth relative to Ether

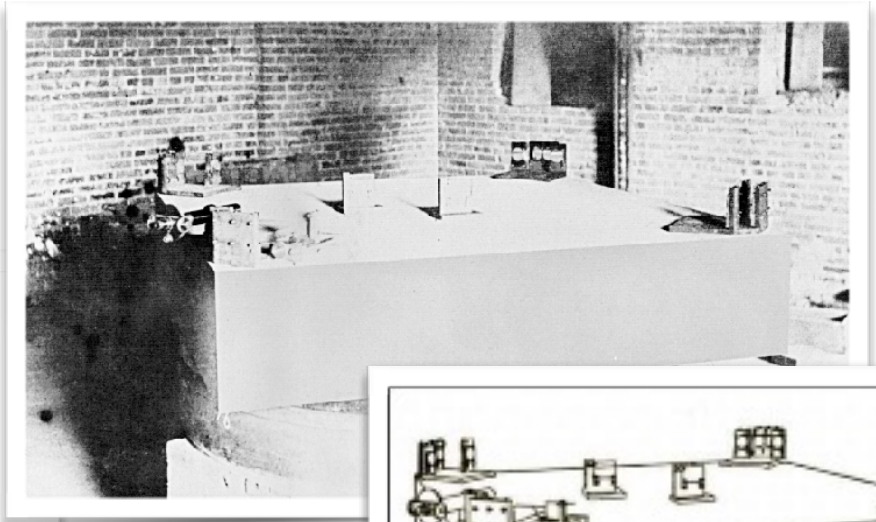
This technique was perfected by cowboy, Albert Michelson and eventually his sidekick, Edward Morley at Case Western Reserve in Cleveland between 1880 and 1888

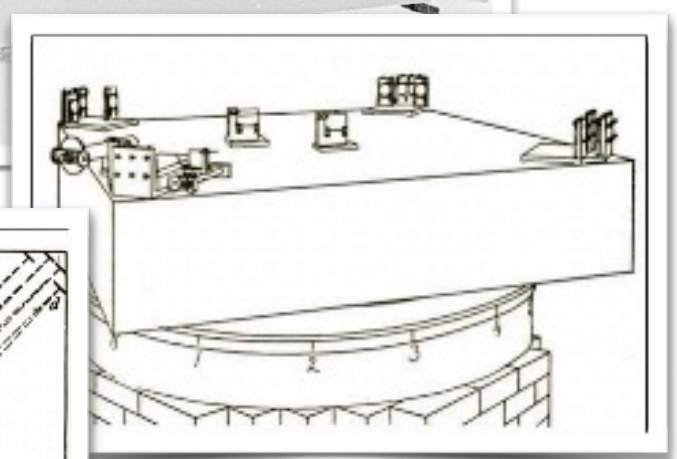
measure the fringes in light interfering from the two paths...then rotate the instrument 90 degrees - and do it again.

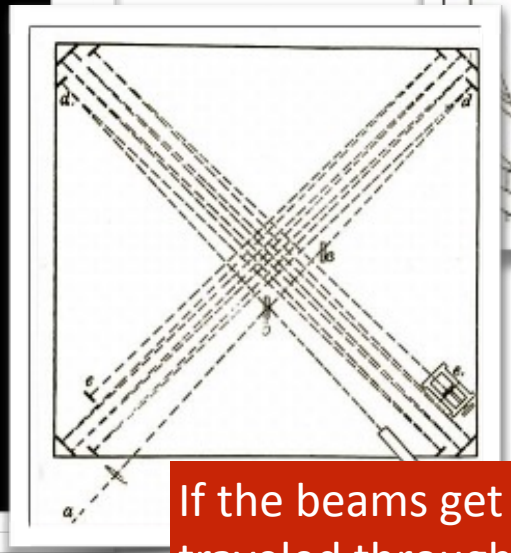
The differences between the two configurations is related to the time difference







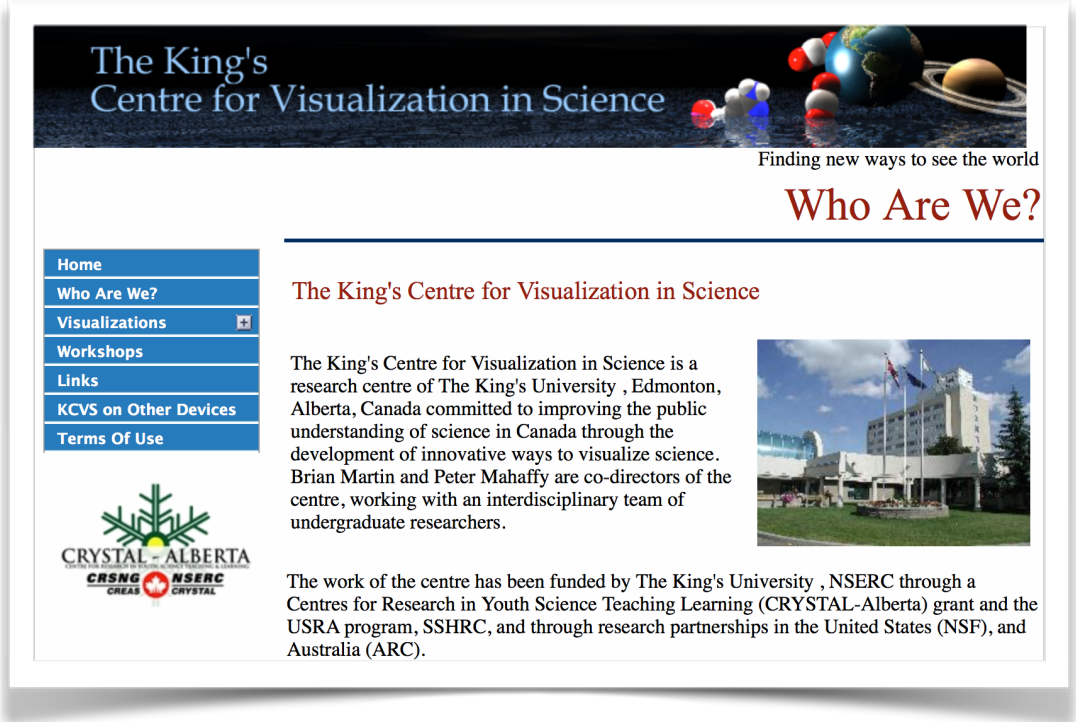




If the beams get back out of phase...one traveled through the ether differently from the other.

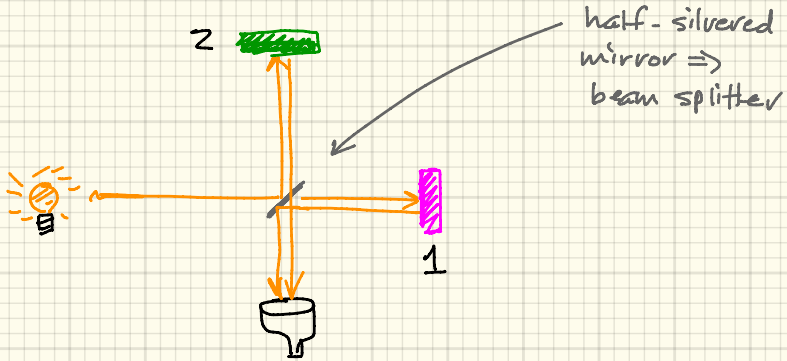
neat simulation

http://www.kcvs.ca/site/projects/physics_files/specialRelativity/michelsonMorley/mmExperiment.swf



C

Michelsons Idea

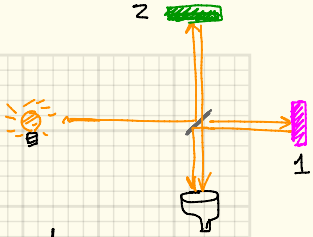


B

C

u, ether : water, u

c, light : swimmer, v



$v = c$, u = speed of ether wind

$$t_1 = \frac{2L_1}{c}\frac{1}{1-u^2/c^2} \qquad t_2 = \frac{2L_2}{c}\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$t_1 - t_2 = \frac{2L_1}{c}\frac{1}{1-u^2/c^2} - \frac{2L_2}{c}\frac{1}{\sqrt{1-u^2/c^2}}$$

$$\left.\begin{aligned} v_e &= 3\times10^4\ \mathrm{m/s} \\ c &= 3\times10^8\ \mathrm{m/s} \end{aligned}\right\} \quad v_e/c \sim 10^{-4}$$

Taylor Expand...

$$\frac{1}{1-u^2/c^2} \simeq 1 + \frac{u^2}{c^2} + \dots \qquad \frac{1}{\sqrt{}} \simeq 1 + \frac{1}{2}\frac{u^2}{c^2} + \dots$$

$$\underbrace{t_1 - t_2}_{\text{tiny}} \simeq \frac{2L_1}{c}\left(1+\frac{u^2}{c^2}\right) - \frac{2L_2}{c}\left(1+\frac{1}{2}\frac{u^2}{c^2}\right) \equiv \Delta t(0)$$

$$\Delta t(0) = \frac{2L_1}{c}\left(1+\frac{u^2}{c^2}\right) - \frac{2L_2}{c}\left(1+\frac{1}{2}\frac{u^2}{c^2}\right)$$

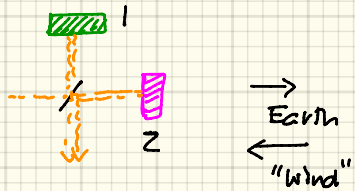
$$= \frac{2L_1}{c} - \frac{2L_2}{c} + \frac{2L_1 u^2}{c^3} - \frac{L_2 u^2}{c^3}$$

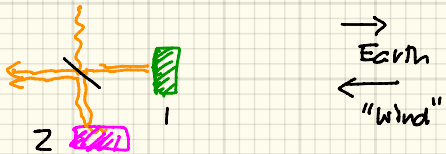
$$\Delta t(0) = \frac{2}{c}(L_1 - L_2) + \frac{u^2}{c^3}(2L_1 - L_2) = t_1(0) - t_2(0)$$

how be clever

hard to measure precisely.

ROTATE EVERYTHING!





Redo the calculation

$$\Delta t(\pi/2) = t_1(\pi/2) - t_2(\pi/2)$$

$$\Delta t(0) = \frac{2}{c}(L_1 - L_2) + \frac{u^2}{c^3}(2L_1 - L_2) = t_1(0) - t_2(0)$$

$$\Delta t(\pi/2) = t_1(\pi/2) - t_2(\pi/2)$$

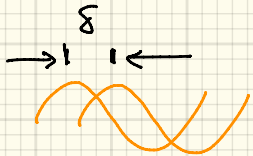
$$= \frac{2}{c}(L_1 - L_2) + \frac{u^2}{c^3}(L_1 - 2L_2)$$

$$\Delta T = \Delta t(0) - \Delta t(\pi/2)$$

$$= \cancel{\frac{2}{c}(L_1 - L_2)} + \frac{u^2}{c^3}(2L_1 - L_2) - \cancel{\frac{2}{c}(L_1 - L_2)} - \frac{u^2}{c^3}(L_1 - 2L_2) \quad !$$

$$= \frac{u^2}{c^3}(2L_1 - L_1 - L_2 + 2L_2)$$

$$\Delta T = \frac{u^2}{c^3}(L_1 + L_2)$$



$$c = \frac{\delta}{\Delta T}$$

$$\delta = c\Delta T$$

$$\delta = \frac{u^2}{c^2}(L_1 + L_2) \quad \checkmark$$