1. Special Relativity, 4

lecture 5, September 8, 2017

housekeeping

remember to check the course page:

chipbrock.org

and sign up for the feedburner reminders

testing, testing:

someone please go to the syllabus and create a fake pdf according to the instructions and then try to upload it to the dropbox in the D2L site. Let me know friday. Okay?

someone did and it worked for us

Homework workshop is Tuesday

your homework “attempts” due midnight Monday night into D2L

Postulates of Special Relativity

1. All laws of physics – mechanical and electromagnetic – are identical in co-moving inertial frames.

    taking Galileo seriously, and then adding Maxwell

    called "The Principle of Relativity"

2. The speed of light is the same for all inertial observers.

    taking Maxwell seriously…that "c" in M.E. is a constant.

“The introduction of a “luminiferous ether” will prove to be superfluous inasmuch as the view here to be developed will not require an “absolutely stationary space” provided with special properties, nor assign a velocity-vector to a point of the empty space in which electromagnetic processes take place.”

TIME DILATION

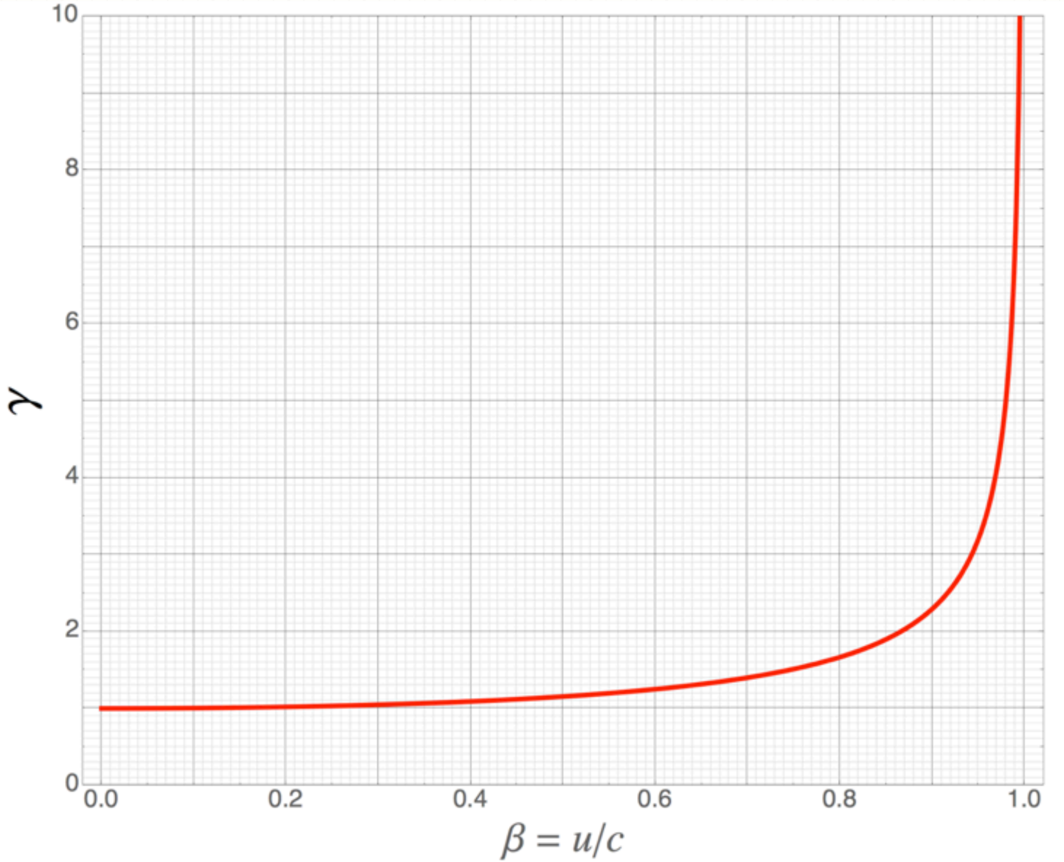
$$t_H = \gamma t_A$$

Moving clocks appear to run slower as seen by a relatively stationary observer

$$L_H = \frac{L_A}{\gamma}$$

LENGTH CONTRACTION

Moving lengths appear shorter to a relatively stationary observer



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γ
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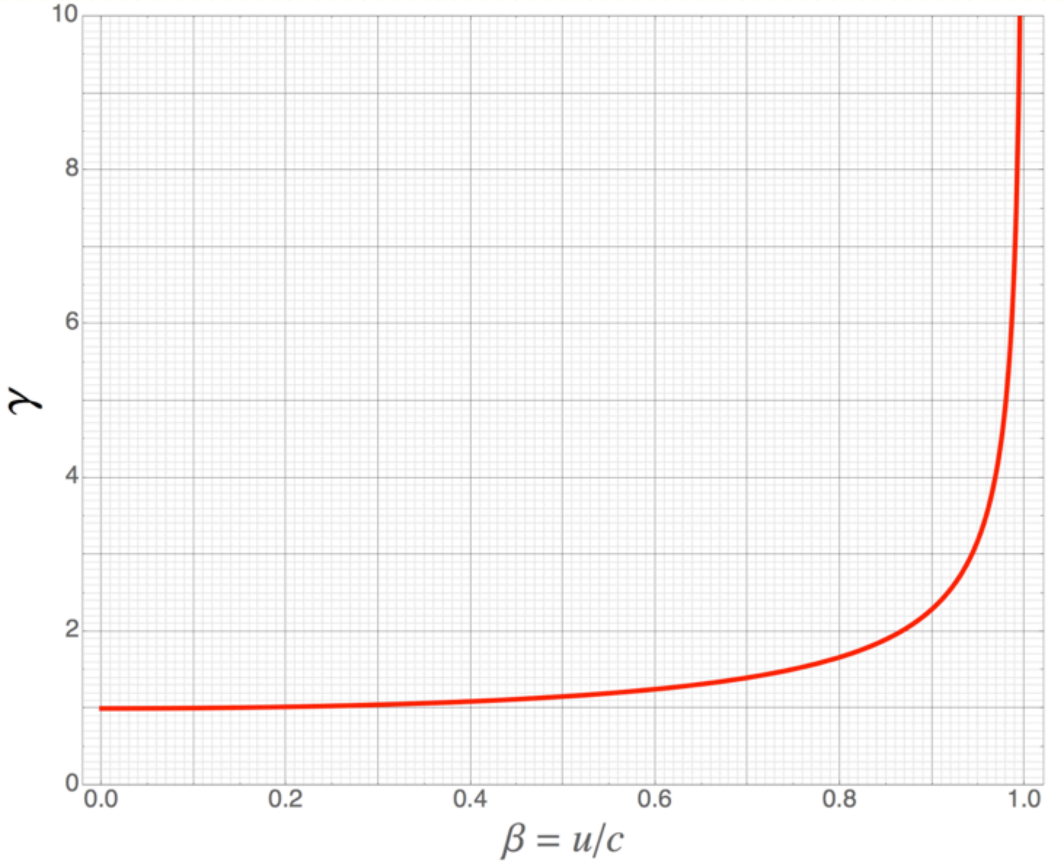
β = u/c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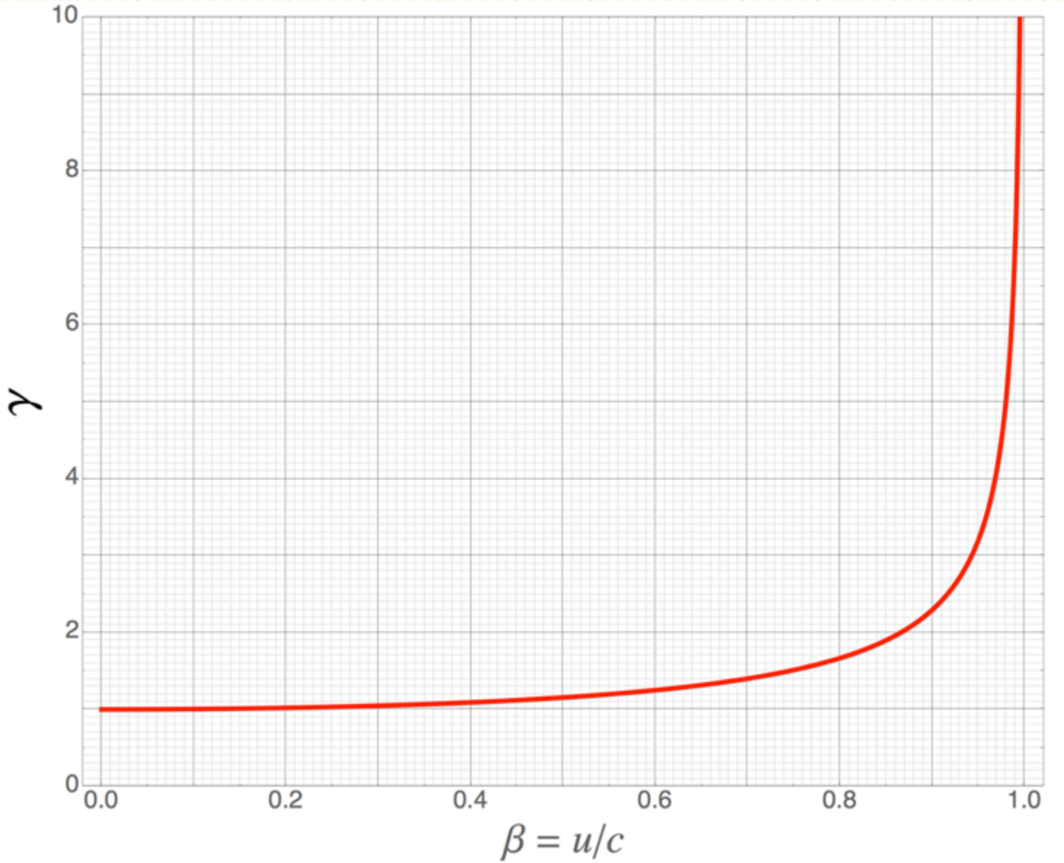
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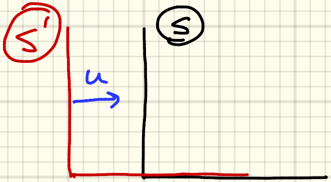
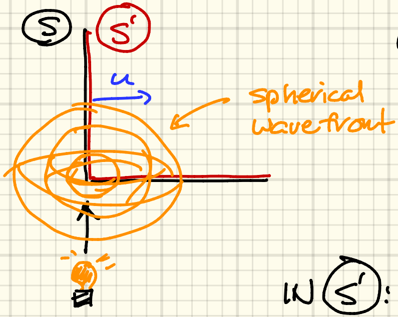
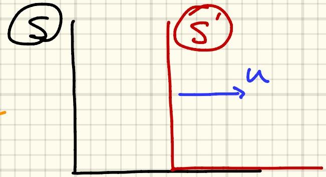
$\beta = u/c$

Galilean Transformations → "Lorentz Transformations"

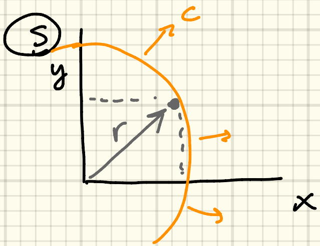
E's derivation







IN S:



after time t

$$r = ct$$

$$ct = \sqrt{x^2 + y^2}$$

$$c^2t^2 = x^2 + y^2$$

$$x^2 + y^2 - ct^2 = 0$$

IN S':

$$x'^2 + y'^2 - (ct')^2 = 0$$

also

How to connect them?

Try a G.T.: $x' = x - ut$

$t' = t$

$$x'^2 + y'^2 - (ct')^2 = 0$$

$$(x - ut)^2 + y^2 - (ct)^2 = 0$$

$$x^2 + \underbrace{(ut)^2 - 2xut}_{\text{spoils invariance}} + y^2 - (ct)^2 = 0$$

E: search for a linear transformation that preserves Postulate #2

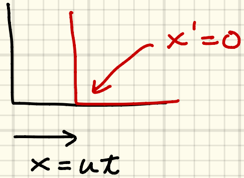
$$x' = \alpha x + \eta t$$

$$t' = \epsilon x + \delta t$$

determine $\alpha, \eta, \epsilon, \delta$

$x' = \alpha x + \eta t$ When $x' = 0$

$t' = \epsilon x + \gamma t$ $x = ut$



So: $x' = 0 = \alpha u \cancel{t} + \eta \cancel{t}$

$$\eta = -\alpha u$$

$$x' = \alpha x - \alpha u t = \alpha(x - ut)$$

from $x'^2 + y'^2 - (ct')^2 = 0$

$$t' = \epsilon x + \gamma t$$

$$\alpha^2 (x - ut)^2 - c^2 (\epsilon x + \gamma t)^2 = 0$$

$$\alpha^2 x^2 + \alpha^2 u^2 t^2 - 2\alpha^2 x u t - c^2 \epsilon^2 x^2 - c^2 \gamma^2 t^2 - 2c^2 \epsilon x \gamma t = 0$$

$$x^2(\alpha^2 - c^2\epsilon^2) - t^2(-\alpha^2 u^2 + c^2\gamma^2) - 2\alpha^2 x u t - 2c^2 \epsilon x \gamma t = 0$$

$$x^2(\alpha^2 - c^2\epsilon^2) - t^2(-\alpha^2 u^2 + c^2\gamma^2) - \underbrace{2\alpha^2 xut - 2c^2\epsilon x\gamma t}_{-2tx(\alpha^2 u + c^2\epsilon\gamma)} = 0$$

compare with $\quad x^2 + y^2 - (ct)^2 = 0$

$$\left.\begin{aligned} \alpha^2 - c^2\epsilon^2 &= 1 \\ c^2\gamma^2 - \alpha^2 u^2 &= c^2 \\ \alpha^2 u + c^2\epsilon\gamma &= 0 \end{aligned}\right\}$$

solve:

$$\alpha = \gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

$$\epsilon = -\frac{\gamma u}{c^2} = \frac{-u/c^2}{\sqrt{1 - u^2/c^2}}$$

so we end up with.

$$\boxed{\begin{aligned} x' &= \gamma(x - ut) \\ t' &= \gamma\left(t - \frac{ux}{c^2}\right) \\ \gamma &= \frac{1}{\sqrt{1-\beta^2}} \end{aligned}}$$

Lorentz Transformations

Why? Because Lorentz worked backwards from Maxwell's Equations:

Q. What transformations on x & t world leave Maxwell's Equations invariant?

A. $x' = \gamma(x - ut)$

$t' = \gamma\left(t - \frac{ux}{c^2}\right)$

1895 named by Poincaré 1904

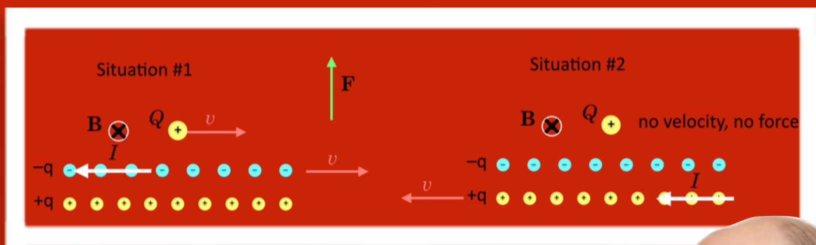
→ how about the other way? "inverse transform"?

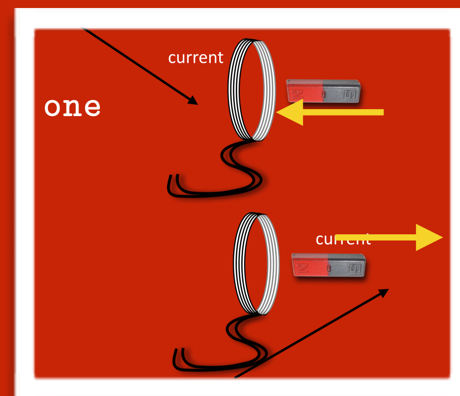
$u \to u'$

$' \to$ u-t $'$

G1

remember?

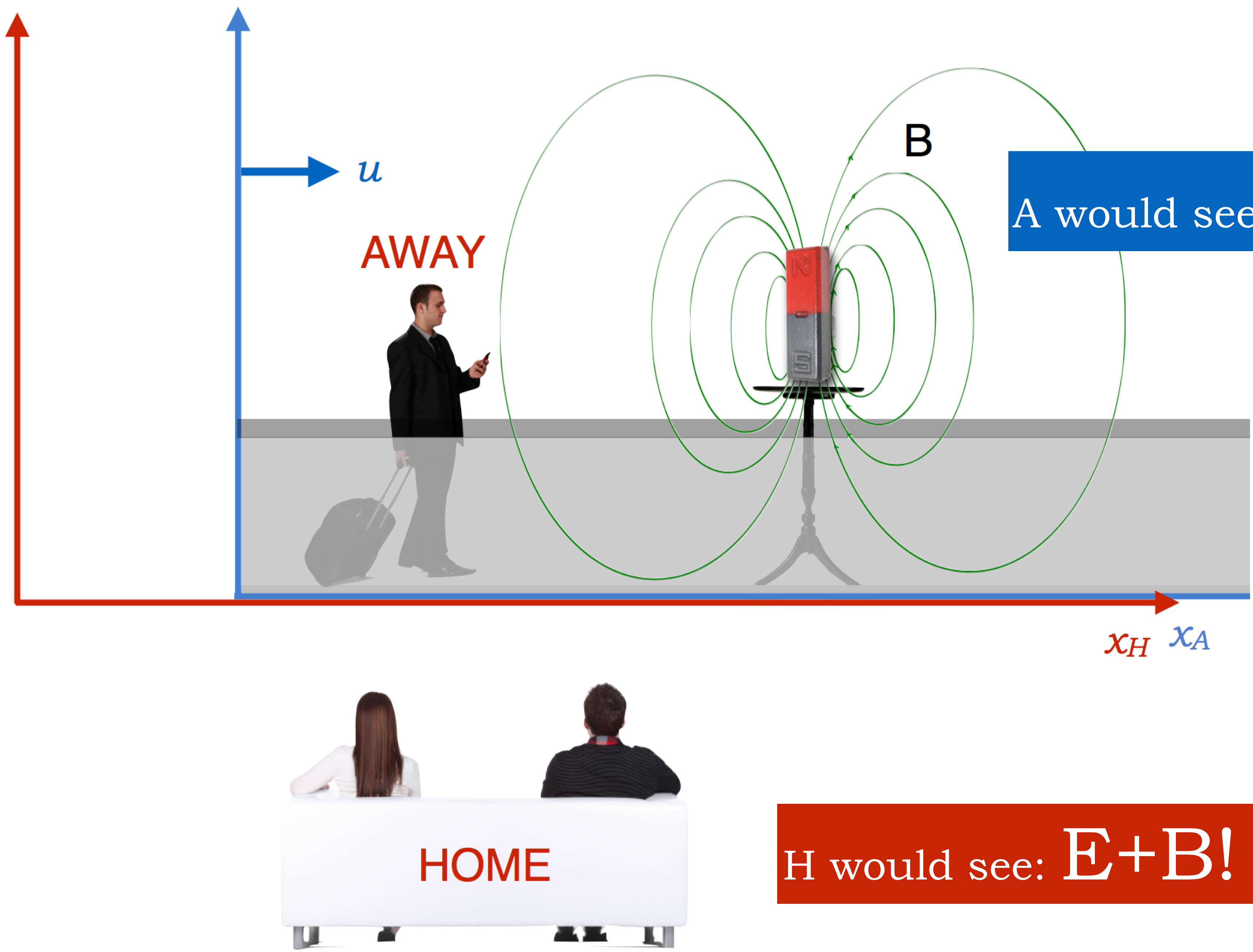




Weird alert #1:
Two different physical outcomes...
for situations which differ only by the frame of reference

Weird alert #2:
Two identical physical outcomes...
from entirely different physical causes for situations which

back to the airport



A would see: B

H would see: E+B!

so the original problems are solved by:

the Lorentz transformations in *x* and *t*

actually **mix** electric and magnetic fields

so

A **magnetic field** in one frame

is a **mixture of magnetic and electric fields** in another frame

An **electric field** in one frame

is a **mixture of electric and magnetic fields** in another frame so the original problems are solved by:

E and B are two manifestations of one thing: the Electromagnetic Field

the Lorentz transformations in x and t

is a mixture of magnetic and electric fields in another frame

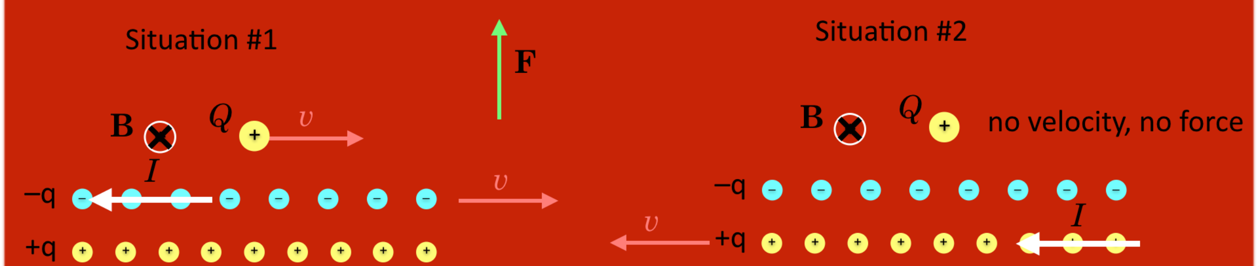
An electric field in one frame

is a mixture of electric and magnetic fields in another frame

remember:

more simple questions

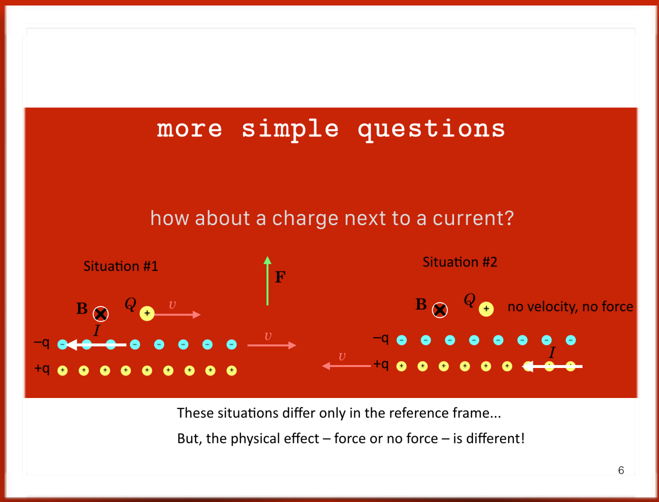
how about a charge next to a current?



These situations differ only in the reference frame...

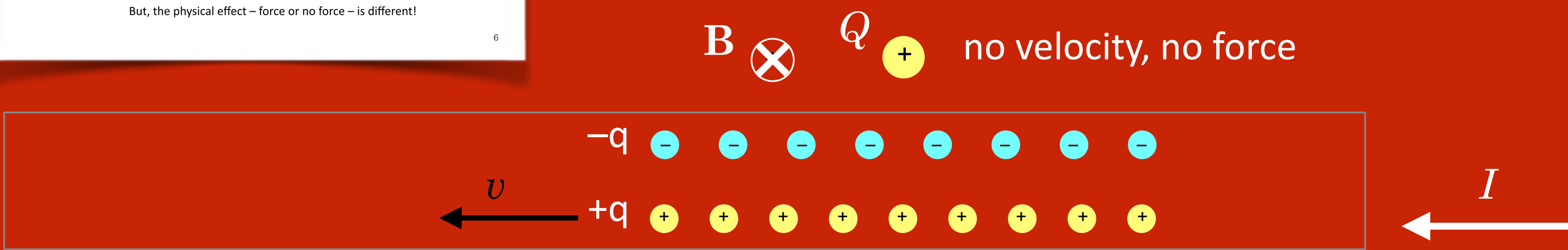
But, the physical effect – force or no force – is different!

remember?



but there should have been a force!

Situation #2

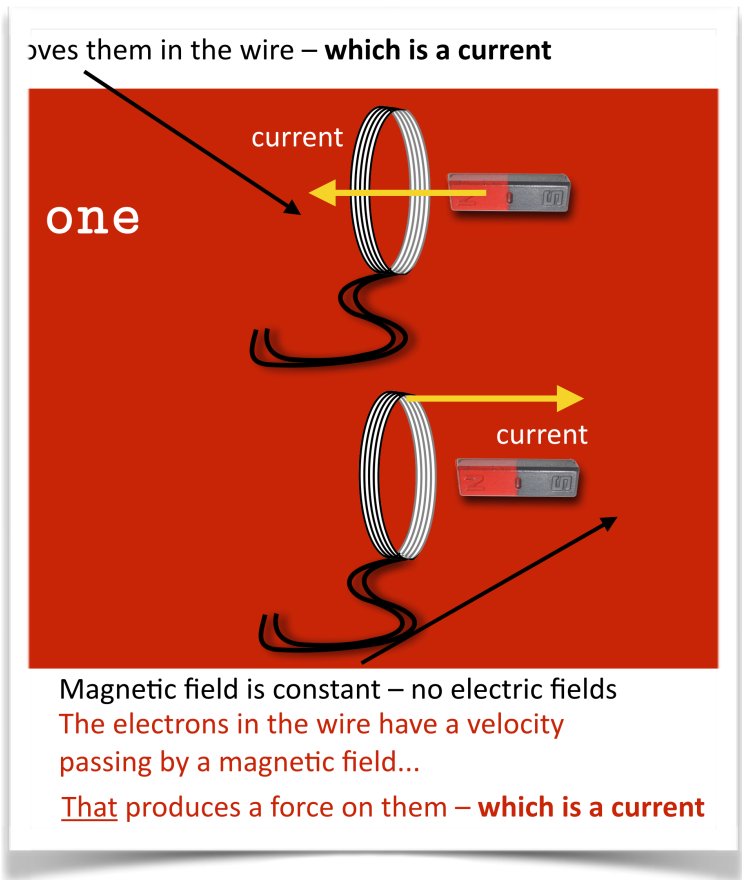


ow

and the coil?

yup. right observation all along.

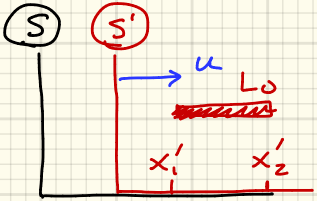
Electric and magnetic fields, depending on the relative frames



the punch line.

Maxwell’s Equations were good all along.

Length Contraction redux

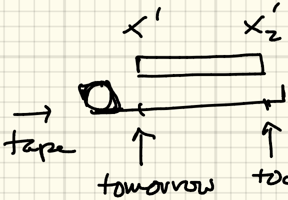


In S' $L_0 = x_2' - x_1'$ fixed

what about S ?

tricky.. think about times for length measurement

1) S' stick sitting still =>

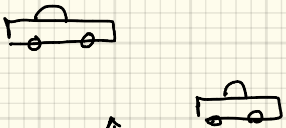


=> same length.

$x_2' - x_1' = L_0$

↑ proper

2) S watching S' go by...



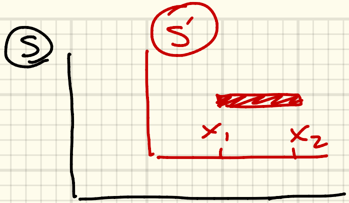
measure

measure

} different times? wrong length.

S must make L @ $t_2 = t_1 = 0$

simple



$$L = x_2 - x_1 \qquad @ \quad t_1 = t_2 = 0$$

$$L = \gamma(x_2' - ut_2') - \gamma(x_1' - ut_1')$$

$$L = \gamma\left[x_2' - x_1' - ut_2' + ut_1'\right]$$

but also: $t_2 = \gamma\left(t_2' - \frac{u}{c^2}x_2'\right) = 0 \Rightarrow t_2' = \frac{u}{c^2}x_2'$

$$t_1 = \gamma\left(t_1' - \frac{u}{c^2}x_1'\right) = 0 \qquad t_1' = \frac{u}{c^2}x_1'$$

$$L = \gamma\left[x_2' - x_1' - \frac{u^2}{c^2}x_2' + \frac{u^2}{c^2}x_1'\right]$$

$$L = \gamma\left[(x_2' - x_1')\left(1 - \frac{u^2}{c^2}\right)\right] = \frac{1}{\sqrt{1-u^2/c^2}}\left[1 - \frac{u^2}{c^2}\right](x_2' - x_1')$$

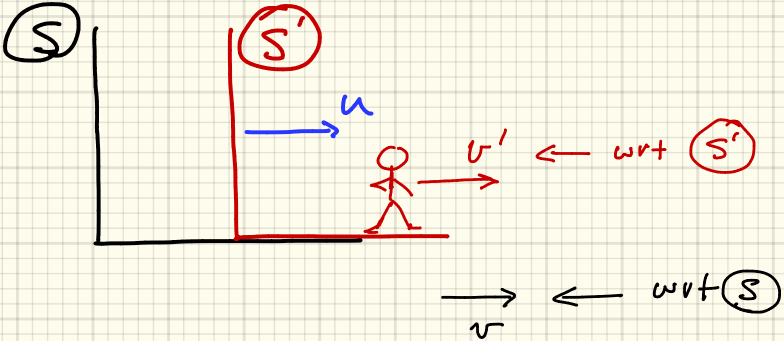
$$L = \sqrt{1 - u^2/c^2}\; L_0$$

$L = \dfrac{L_0}{\gamma} \Rightarrow$ L measures stick to be less than L_0

what does S' see if stick in S?

TRANSFORMATION OF VELOCITIES

the sidewalk evolves



3 velocities going on here

In G.T.: what you expect $v' = v - u$

speed of something in S'

speed of S' relative to S

speed of something in S

But now we do L.T.

$$x' = \gamma(x - ut)$$

$$\frac{dx'}{dt} = \gamma\left(\frac{dx}{dt} - u\frac{dt}{dt} - \frac{du}{dt}t\right) = \gamma\Big(\underbrace{\frac{dx}{dt}}_{v} - u\Big)$$

$\beta = \frac{u}{c}$

but $t' = \gamma\left(t - \frac{\beta}{c}x\right)$

$$\frac{dt'}{dt} = \gamma\left(1 - \frac{\beta}{c}v\right) \quad \leftarrow !$$

$$\frac{dx'}{dt'} = v' = \frac{dx'}{dt}\frac{dt}{dt'} = \frac{\gamma(v-u)}{\gamma\left(1 - \frac{\beta}{c}v\right)}$$

$$v' = \frac{v-u}{1 - \frac{\beta}{c}v}$$

Relativistic Velocity Transformation

But wait... there's more

v's transverse to u?

they transform also $\rightarrow$ t & t'

$$v_x = \frac{v_x' + u}{1 + \frac{\beta}{c} v_x'} \quad \longleftarrow \text{ along } \vec{u}$$

$$v_y = \frac{v_y'}{\gamma\left[1 + \frac{\beta}{c} v_x'\right]}$$

$$v_z = \frac{v_z'}{\gamma\left[1 + \frac{\beta}{c} v_x'\right]}$$

$$v' = \frac{v - u}{1 - \frac{\beta}{c} v} \quad \& \quad v = \frac{v' + u}{1 + \frac{\beta}{c} v'}$$

extremes of: $\quad v = \dfrac{v' + u}{1 + \frac{\beta}{c} v'}$

1) u very small $\quad \beta = u/c$ so $\beta \to 0 \Rightarrow v = v' + u \quad$ G.T.

2) v very small $\quad \frac{\beta}{c} v' = \frac{u}{c^2} v'$ so $\frac{v'}{c} \to 0 \Rightarrow v = v' + u \quad$ G.T.

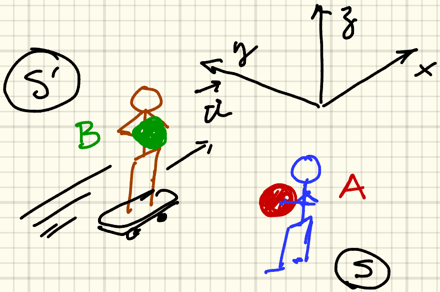
3) $v' = c$ a light beam in S'

$$v = \frac{c + u}{1 + \frac{\beta}{c} c} = \frac{c+u}{c+u} \cdot c = c \quad \checkmark \quad 2^{\text{nd}}$$

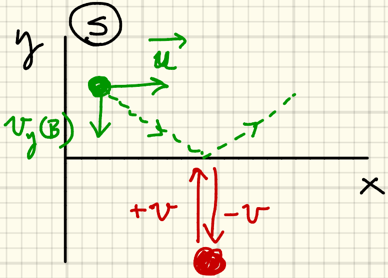
$$1 + \beta = 1 + \frac{u}{c}$$

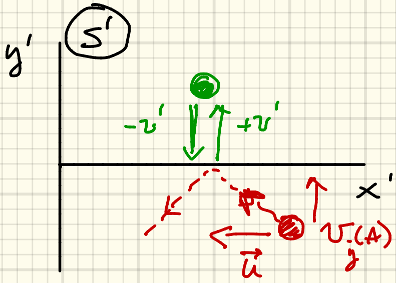
Mass is complicated.

→ not willing to part with momentum conservation



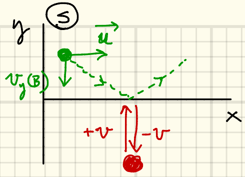
- each throws their identical A, B ball at one another
- they collide elastically & recoil

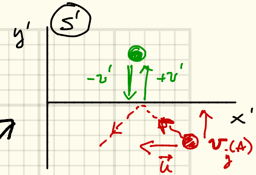




after lots of practice: $|v| = |v'|$ frame $\pm\beta = \frac{\pm u}{c}$

in S calculate S' quantities





$$v_y(B) = \frac{v_y'(B)}{\gamma\left(1 + \frac{u v_x'(B)}{c^2}\right)}$$

$$v_y = \frac{v_y'}{\gamma\left[1 + \frac{\beta}{c} v_x'\right]}$$

$$v_x'(B) = 0$$

$$v_y'(B) = v'$$

$$v_y(B) = \frac{v'}{\gamma}$$

MOMENTUM CONSERVATION! $\Rightarrow$ along y axis

$$\underbrace{\sum P_y(\text{before})}_{\text{in } S} = \underbrace{\sum P_y(\text{after})}_{\text{in } S}$$

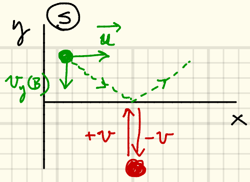
$$m_A v - m_B v_y(B) = -m_A v + m_B v_y(B)$$

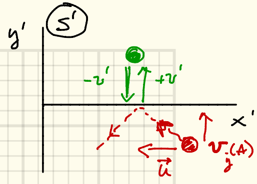
$$m_A v - m_B \frac{v'}{\gamma} = -m_A v + m_B \frac{v'}{\gamma}$$

$|v| = |v'| \equiv v$

$$2 m_A v = 2 m_B \frac{v}{\gamma}$$

$$m_B = \gamma m_A$$





MOMENTUM CONSERVATION! ⇒ along y

$$\underbrace{\Sigma P_y \text{ (before)}}_{\text{in } S} = \Sigma P_y \text{ (after)} \quad \text{in } S$$

$$m_A v - m_B v_y(B) = -m_A v + m_B v_y(B)$$

$$m_A v - m_B \frac{v'}{\gamma} = -m_A v + m_B \frac{v'}{\gamma}$$

$|v| = |v'| \equiv v$

$$2 m_A v = 2 m_B \frac{v}{\gamma}$$

$$m_B = \gamma m_A$$

if u is vanishingly small...

$m_B = m_A \rightarrow = $ "m_0" "rest mass"

but as u increases $m_B > m_A$, $m_B > m_0$

$$m(u) = \frac{m_0}{\sqrt{1 - u^2/c^2}}$$

called "relativistic mass"

not popular.. but consistent.

* ahem *

continuing $\qquad \vec{p} = m\vec{u}$

$$\vec{p} = m_0 \gamma \vec{u} \qquad \text{relativistic momentum}$$

Transform-a Force

$$\vec{F} = \frac{d}{dt}\vec{p} \qquad \text{still}$$

$$= m\frac{d\vec{u}}{dt} + \vec{u}\frac{dm}{dt}$$

$$= m\vec{a} + \vec{u}\frac{dm}{dt}$$

$$m(u) = \frac{m_0}{\sqrt{1-u^2/c^2}}$$

differentiate $\frac{d}{dt}$

$$\vec{F} = m_0\gamma\left[\frac{d\vec{u}}{dt} + \frac{u\frac{du}{dt}}{c^2-u^2}\vec{u}\right]$$

ugh. worse.. if $\vec{u}$ is not along $\vec{F}$? really ugly.