# 1. Special Relativity, 6

lecture 7, September 13, 2017

# housekeeping

remember to check the course page:

chipbrock.org

and sign up for the feedburner reminders

Homework due Friday, remember?

I'll adjust thermo homework if necessary

for my verbosity

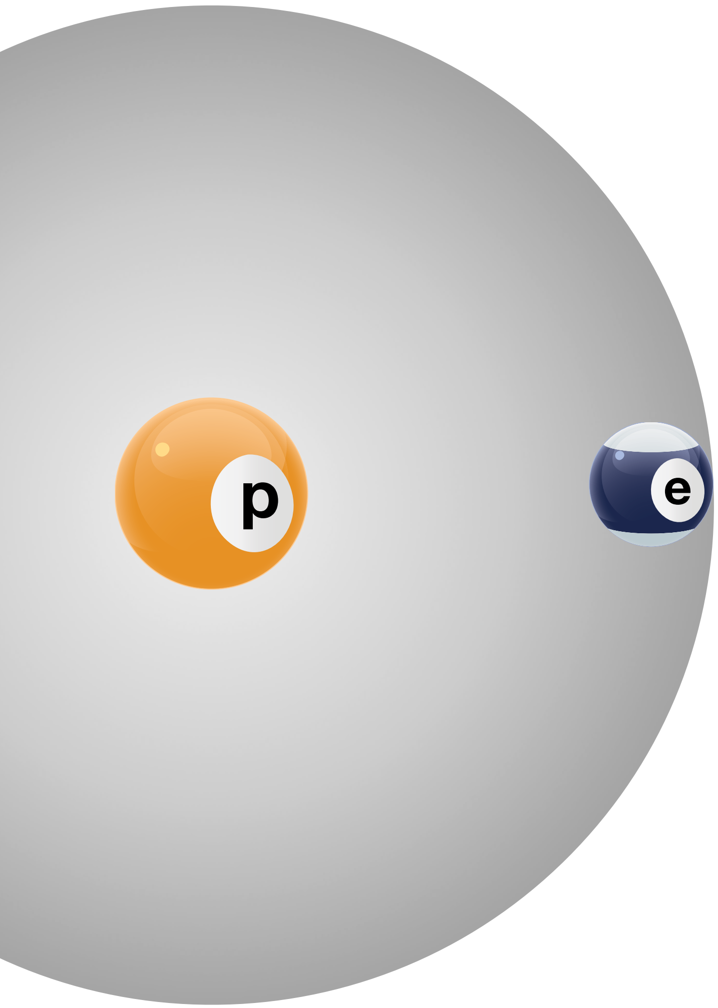
H

# suppose we have a bound system

What holds the electron to the proton?

Hydrogen Atom

Last week:
the electrostatic force, or the Electric field, right?



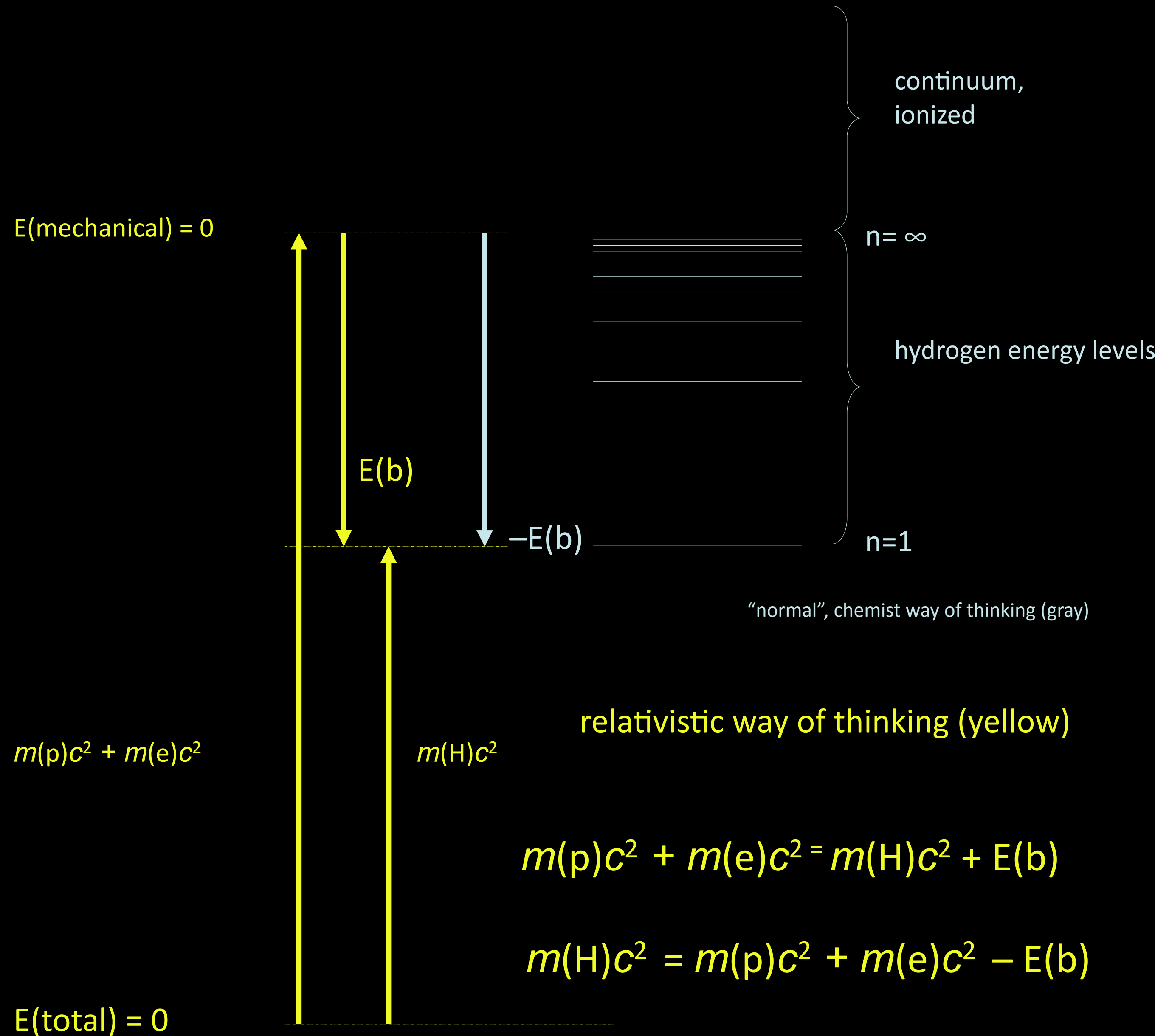

## Remember from Chemistry:

What's it take to ionize* Hydrogen?

You must supply 13.6 eV

*make the electron free of the proton's influence

# energy diagram for H




The mass of a hydrogen atom is LESS than the sum of $m_p + m_e$
No negative binding energy...just a "mass deficit" in the attraction of the P and e.
The energy is in the field.

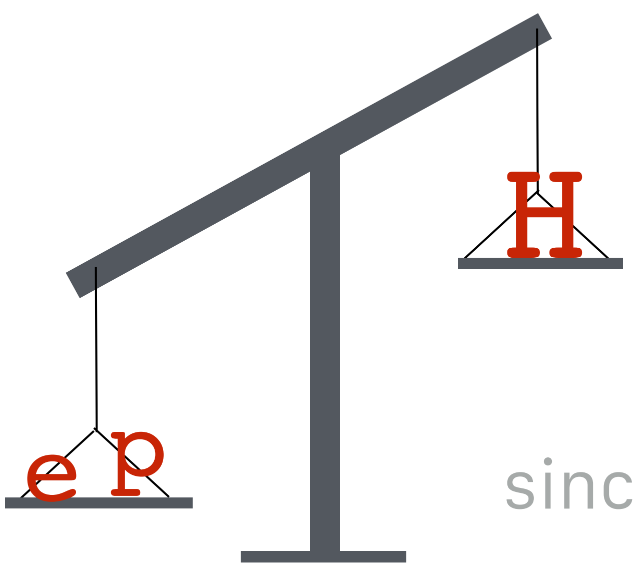
# a hydrogen atom, take 2

weighs less than the components of a hydrogen atom

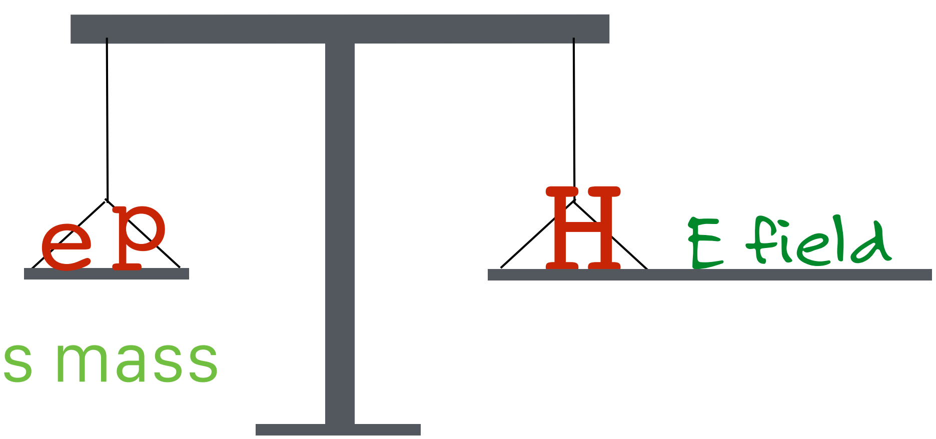
so it can't fall apart into its components

where is that "missing mass"?

in the energy of the Electric Field,

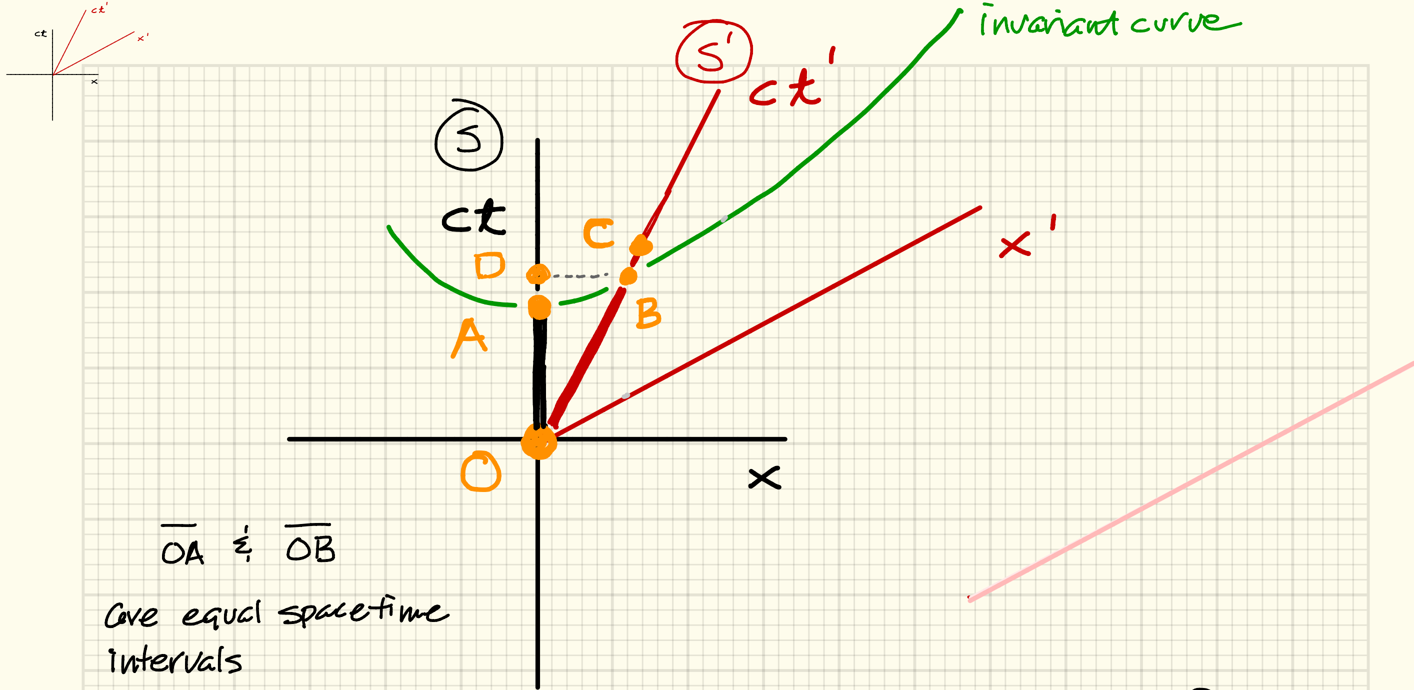



since Energy is mass



# the ‘‘mass deficit’’ in nuclei

is observable and works for good and for ill.




$\overline{OA}$ & $\overline{OB}$

have equal spacetime intervals

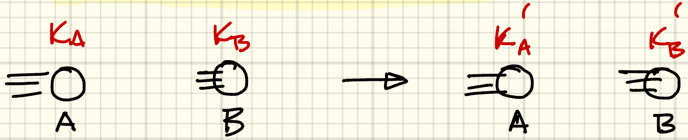
$\overline{AC}$ are simultaneous in S'

but not in S

$\overline{DB}$ are simultaneous in S but

not in S'

# Energy Conservation in Relativity

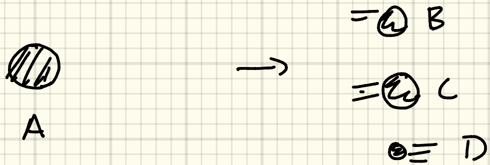



ENERGY BEFORE = ENERGY AFTER

$$K_A + m_A c^2 + K_B + m_B c^2 = K_A' + m_A c^2 + K_B' + m_B c^2$$

cancel, so a hidden relativity process

"Decay"




how about energy now?

how much?

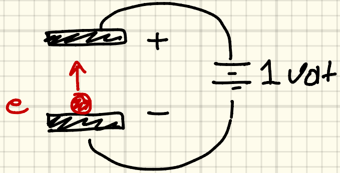
$$m_A c^2 = K_B + m_B c^2 + K_C + m_C c^2 + K_D + m_D c^2$$

$$(m_A c^2 - m_B c^2 - m_C c^2 - m_D c^2) = K_B + K_C + K_D$$

mass-energy difference = energy of motion

Units are no fun $\quad m_p \simeq 10^{-27}$ kg. $\Rightarrow$ mistakes waiting to happen

"Electron Volts"




$q_e = -1.6\times10^{-19}$ C

accelerated over 1 Volt $\rightarrow$

$$E = K = qV$$

from work done by $\vec{E}$ : $\quad E = (1.6\times10^{-19}\,C)(1\ J/C) = 1.6\times10^{-19}\,J$

$$\equiv 1\ eV$$

Furthermore, masses: $\quad$ " $eV/c^2$ "

rest energies $\quad$ " eV "

$m_e = 9.1\times10^{-31}$ kg

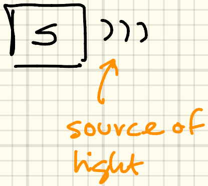
$$E_0 = m_e c^2 = (9.1\times10^{-31})(3\times10^{8})^2\left(\frac{1\,eV}{1.6\times10^{-19}\,J}\right)$$

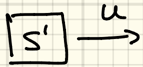
$$E_0(e) = 511{,}900\ eV$$

$$= 0.511\times10^{6}\ eV = 0.511\ MeV$$

so $\quad m_0(e) = \dfrac{E_0}{c^2} = m_0(e) = 0.511\ \dfrac{MeV}{c^2}$ $\quad$ "MeV over $c^2$"

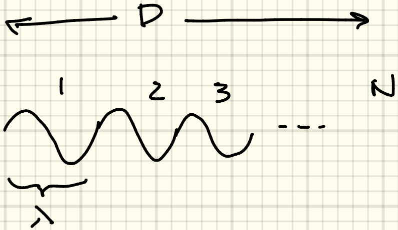
# A Little Doppler




from S

$D$ = total distance = $N\lambda$




emits $N$ waves @ frequency $f$

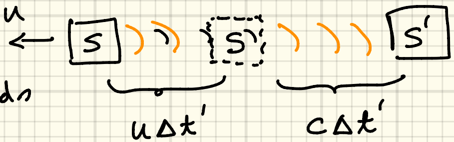
$T = \dfrac{c}{\lambda}$

total time

$\Delta t_0 = NT = \dfrac{N}{f}$

from $S'$:

for $\Delta t'$ seconds




$$\lambda' = \frac{D'}{N} = \frac{u\Delta t' + c\Delta t'}{N}$$

← same in S & S'

$$\lambda' = \frac{u\Delta t' + c\Delta t'}{f\Delta t_0} \Rightarrow \frac{c}{f'} = \frac{\Delta t'(u+c)}{f\Delta t_0}$$

in $S'$: $f\lambda = c \Rightarrow f' = \dfrac{c}{\lambda'}$

$$f' = c\,\frac{f\,\Delta t_0}{\Delta t'(u+c)} = f\,\frac{\Delta t_0}{\Delta t'}\,\frac{1}{1+u/c}$$

$$f' = c\,\frac{f\,\Delta t_0}{\Delta t'(u+c)} = f\,\frac{\Delta t_0}{\Delta t'}\,\frac{1}{1+u/c}$$

$$\Uparrow$$

$$\frac{1}{\gamma} = \sqrt{1-u^2/c^2}$$

$$f' = f\,\frac{\sqrt{1-u^2/c^2}}{1+u/c}$$

aside: $(1-\beta)(1+\beta) = 1+\beta-\beta-\beta^2$

$$= 1-\beta^2$$

so $\dfrac{\sqrt{(1-\beta)(1+\beta)}}{1+\beta} = \dfrac{\sqrt{1-\beta^2}}{1+\beta}$

$$f' = f\sqrt{\frac{1-\beta}{1+\beta}}$$

Relativistic Doppler Shift

speed of sound in "medium"

Different from "regular" Doppler Shift: $f' = f\left(\dfrac{\text{"}c\text{"} \pm v_s}{\text{"}c\text{"} \pm v_s}\right)$

observed

actual

$$f' = f\sqrt{\frac{1-\beta}{1+\beta}} \Rightarrow f' > f \quad \text{or} \quad \underbrace{\lambda' < \lambda}_{\text{a "red shift"}}$$

example:

Galaxy moving away from earth such that $\lambda(H) = 434\,nm$ appears to be at $\lambda(H) = 600\,nm$ → what's $u$?

$$f = \frac{c}{\lambda} \quad \& \quad f' = \frac{c}{\lambda'}$$

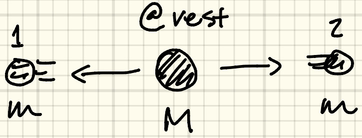
$$\frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{1-\beta}{1+\beta}} = \frac{c}{600} = \frac{c}{434}\sqrt{\frac{1-\beta}{1+\beta}}$$

⋮

$$\beta = 0.31$$

# Binding Energy

→ "kinds of energy"?  2 kinds: energy of mass
energy of motion.




firecracker? sure where chemical energy might be counted

$$\vec{p}_1 = -\vec{p}_2$$

$$Mc^2 = E_1 + E_2 = 2E$$ where $E = m\gamma c^2 = K + m_o c^2$

or $K = m_o c^2(\gamma - 1)$

$$Mc^2 = 2K + 2m_o c^2$$

$$2K = Mc^2 - 2m_o c^2$$

$$2K = c^2 \underbrace{(M - 2m)}_{\text{old idea of conservation of mass? nope}}$$

J

If a system is to stay together.... say 2 components $M$ & $m$

$$E(\text{system}) < \underbrace{M_0c^2 + m_0c^2}$$

Previously... like in Chemistry? $\rightarrow$ $E(\text{system}) = M_0c^2 + m_0c^2 - B$

binding energy.
$\Rightarrow$ need to add to separate them.

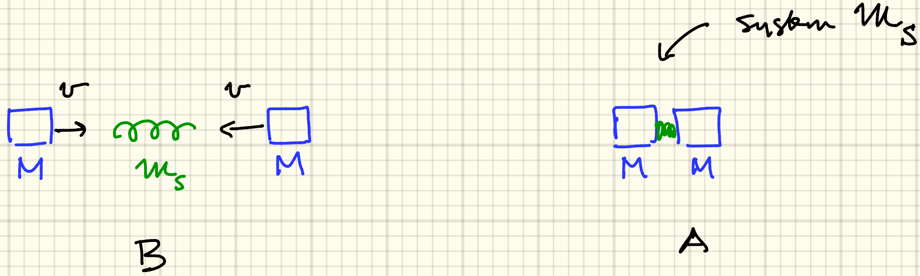
In Relativity... $E_S(\text{system}) = M(\text{system})\,c^2$

$$M_S(\text{system}) = M_0 + m_0 - B/c^2 < M_0 + m_0$$

A hydrogen atom: $M(H) < M_0(\text{proton}) + m_0(\text{electron})$

so it stays together... "mass deficit"
wrt "binding energy"

one step... over the edge.




Classically: $2\left(\frac{1}{2}Mv^2\right) = P(\text{spring})$

Relativistically: $2M_0c^2 + 2M_0c^2(\gamma-1) + m_sc^2 = 2M_0c^2 + P + m_sc^2$

$$2M_0c^2(\gamma-1) = P = E = \delta m c^2$$

$$\cancel{2M_0c^2} + \underbrace{2M_0c^2(\gamma-1)}_{K} + m_sc^2 = \cancel{2M_0c^2} + \delta m c^2 + m_sc^2$$

$$K + m_sc^2 = \delta m c^2 + m_s c^2 = (\delta m + m_s)c^2$$

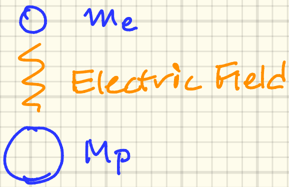
$$\left(\frac{K}{c^2} + m_s\right)c^2 = (\delta m + m_s)c^2$$

spring gets heavier.. no "P"

E's 4th paper: "Does the Inertia of a Body Depend on Its Energy Content"

yup.

Hydrogen:




$$M(H) = M(p) + m(e) - \frac{E(\text{field})}{c^2}$$

weighs less than a proton + an electron.

mass in field

$\delta M_E$

Raise a mass away from earth? $\longrightarrow$ It gets heavier... no "P"

Cook an egg in the morning? $\longrightarrow$ it gets heavier

Heat a gas? $\longrightarrow$ It gets heavier

Charged Pion Decay: $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}$

pion $\longrightarrow$ muon + neutrino

Facts $m_{\pi} = 139.57 \ MeV/c^2$

$m_{\mu} = 105.45 \ MeV/c^2$

$\tau_{\mu} = 2.2 \times 10^{-6} \ s$

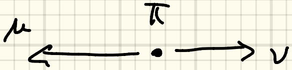
$m_{\nu} \simeq 0$

$\vec{p}_{\pi} = 0 \Rightarrow$ at rest

a) What is momentum of $\mu$?

b) on average how far does $\mu$ travel before it decays?

a)




conserve momentum:

$$\vec{P}_\pi = \vec{P}_\mu + \vec{P}_\nu$$

$$= 0$$

$$\vec{P}_\mu = -\vec{P}_\nu$$

conserve energy:

$$E_\pi = E_\mu + E_\nu$$

$$m_\pi c^2 + \underset{0}{\underset{\uparrow}{K_\pi}} = \underbrace{m_\mu c^2 + K_\mu}_{\sqrt{E_\mu^2}} + \underset{0}{\underset{\uparrow}{m_\nu c^2}} + K_\nu$$

$$E^2 = p^2c^2 + m^2c^4 \;!$$

$$E_\nu^2 = p_\nu^2 c^2 \qquad E_\mu^2 = p_\mu^2 c^2 + m_\mu^2 c^4$$

but $|p_\mu| = |p_\nu| \equiv p$

$$E_\nu = pc \qquad E_\mu = \sqrt{p^2c^2 + m_\mu^2 c^4}$$

$$E_\pi = E_\mu + E_\nu$$

$$m_\pi c^2 + \underset{\uparrow \atop 0}{K_\pi} = \underbrace{m_\mu c^2 + K_\mu}_{\sqrt{E_\mu^2}} + m_\nu c^2 + \underset{\uparrow \atop 0}{K_\nu}$$

$$\underline{m_\pi c^2} = \sqrt{p^2c^2 + \underline{m_\mu^2 c^4}} + pc \longrightarrow \text{solve for } pc$$

Solution

$$A = \sqrt{x^2 + B^2} + x$$

$$\sqrt{x^2+B^2} = A - x$$

$$\cancel{x^2} + B^2 = (A-x)^2 = A^2 - 2Ax + \cancel{x^2}$$

$$B^2 = A^2 - 2Ax$$

$$2Ax = A^2 - B^2$$

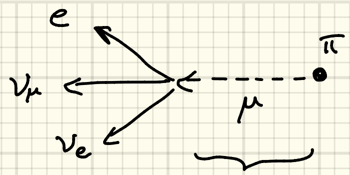
$$x = \frac{A^2 - B^2}{2A} = \frac{(m_\pi c^2)^2 - (m_\mu c^2)^2}{2(m_\pi c^2)} = \frac{(139.6)^2 - (105)^2}{2(139.6)}$$

$$x = 30.3$$

$$pc = 30.3 \text{ MeV}$$

$$p = 30.3 \text{ MeV}/c \quad ✓$$

b)




d on average

"classically" $d = v\tau$ $\longrightarrow$ let it be as fast as conceivable $= \underline{\underline{c}}$

$$d = c\tau = (3\times10^{8}\,m/s)(2.2\times10^{-6}\,s)$$

$$d = 660\ m$$

## AN ASIDE

we are bombarded by $\mu$'s which have lived through the entire atmosphere $\sim$ 50,000 m @ $\sim 1\mu/cm^2/min$

muon decay

called the "lifetime"

$$N(t) = N(0)\, e^{-t/\tau}$$

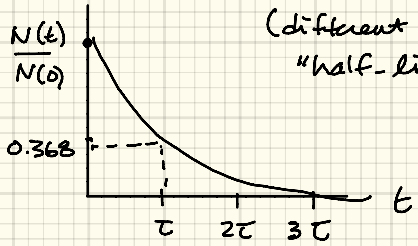
#$\mu$ after $t$ seconds

#$\mu$ @ $t=0$

time for a decay to reduce a sample size by factor

$$\frac{1}{e} = \frac{1}{2.72} = 0.368$$

(different slightly from "half-life" --- stay tuned)




In $\mu$ rest frame —

a likelihood of decay

$\tau = 2.2\times 10^{-6}\,s = 2.2\,\mu s$

K

On earth: we see the $\mu$'s "clock" dilated.

$$\tau = \gamma \tau_\mu$$

us $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $\mu$

$\gamma$ is about $\underline{7}$

So for us... it travels - on average -

$$d = \gamma \underbrace{u\tau} = 7\,(650\,m) = 4600\,m$$

us

$\mu$... 650 m

$\mu$ see's earth's atmosphere rushing toward it &
length - contracted.

$$d_\mu = \frac{1}{\gamma} d_e$$

muon $\quad\quad$ us

DONE WITH ASIDE $\rightarrow$ our original $\pi$ decay:

Back to b)

$$d = \gamma u \tau \qquad p = \gamma m_\mu u$$

$$d = \frac{p}{m_\mu u} u \tau \frac{c^2}{c^2}$$

$$d = \frac{(pc)(c\tau)}{m_\mu c^2} = \frac{(30.\text{ MeV})(3\times10^8)(2.2\times10^{-6})}{(105.45\text{ MeV})}$$

$$d \simeq 185\text{ m}$$

Handy rules of thumb:

$$E = \gamma m c^2 \qquad E^2 = \frac{1}{(1-\beta^2)} m^2 c^4$$

$$\gamma = \frac{E}{mc^2} \qquad \Rightarrow \beta^2 = \frac{p^2 c^2}{E^2}$$

$$\beta = \frac{pc}{E}$$