1. Special Relativity, 6.5
2. Thermodynamics, 1

lecture 8, September 15, 2017

housekeeping

- remember to check the course page:
    - chipbrock.org
    - and sign up for the feedburner reminders
- Remember the textbook for thermodynamics is Bauer and Westfall
    - you have it, or you can buy just chapters 17-21…see syllabus
- After today I’ll adjust thermo homework
    - look tomorrow for a subset of the homework
    - chapter 17, surely. the question is how much of chapter 18

is Relativity

the case?

so, how was this all received?

According to Einstein's sister,

> …he anticipated a large reaction with much criticism
>
> What he got at first was silence.

oh, a nice note from Max Planck asking for some clarification

then a seminar by Planck in Berlin which touched on Relativity…

- only then… a little professional attention, to "Prof. Einstein, University of Bern"

The first paper published on Relativity by not-Einstein:

> also by Planck, who derived the relativistic momentum relation, $p = \gamma m v$

"Raum und Zeit"

“M. H.! [ladies and gentleman!] The views of space and time, which I would like develop, have sprung from the experimental-physical soil. Therein lies their strength. They tend to be radical. Henceforth space by itself and time by itself, fade away completely into shadow, and only a kind of union of the two will preserve independent permanency.”

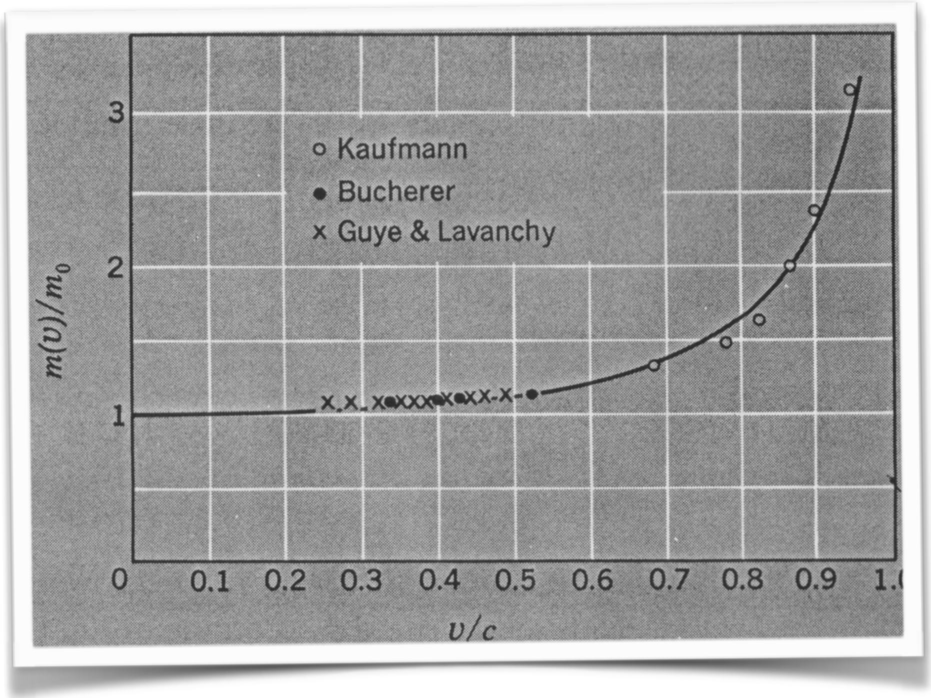
> Hermann Minkowski, September 21, 1908, in the 80th annual general meeting of the German Society of Scientists and Physicians (Gesellschaft Deutscher Naturforscher und Ärzte) at Cologne

What about experiment?

the first experimental confirmation

New experiments were done,

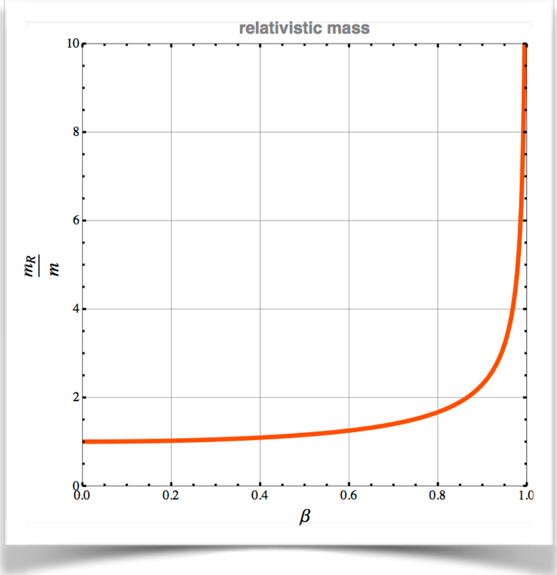
and by 1910, the results were:



These results are from 1910 for three experiments, and the curve is the special relativity prediction

From this point on relativity has become a part of everyday scientific and engineering life

Kaufmann lost again…Max Planck corrected his analysis

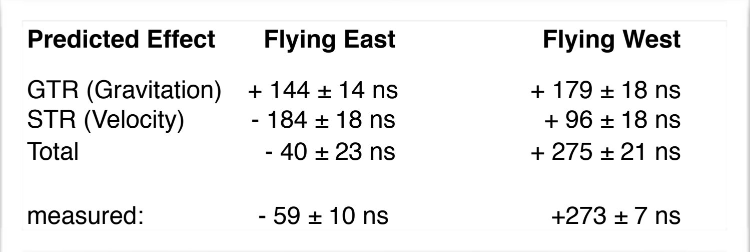


This has been measured many times:

an atomic clock was carefully carried around the world in 1972 and carefully calibrated and compared with ground-based clocks

J. Hafele and R. Keating

There are a number of corrections: accelerations, decelerations, the rotation of the orbit, the fact that the earth is not inertial - but relativity was absolutely correct



Predicted Effect	Flying East	Flying West
GTR (Gravitation)	+ 144 ± 14 ns	+ 179 ± 18 ns
STR (Velocity)	- 184 ± 18 ns	+ 96 ± 18 ns
Total	- 40 ± 23 ns	+ 275 ± 21 ns
measured:	- 59 ± 10 ns	+273 ± 7 ns

redone twice more in airplanes and rockets/satellites

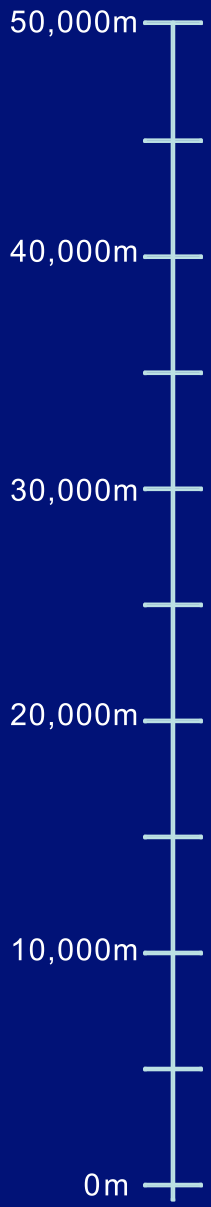
‘‘muons’’: μ

are unstable particles which are easily made in an accelerator lab and shown to have a half life of 1.56 μs…

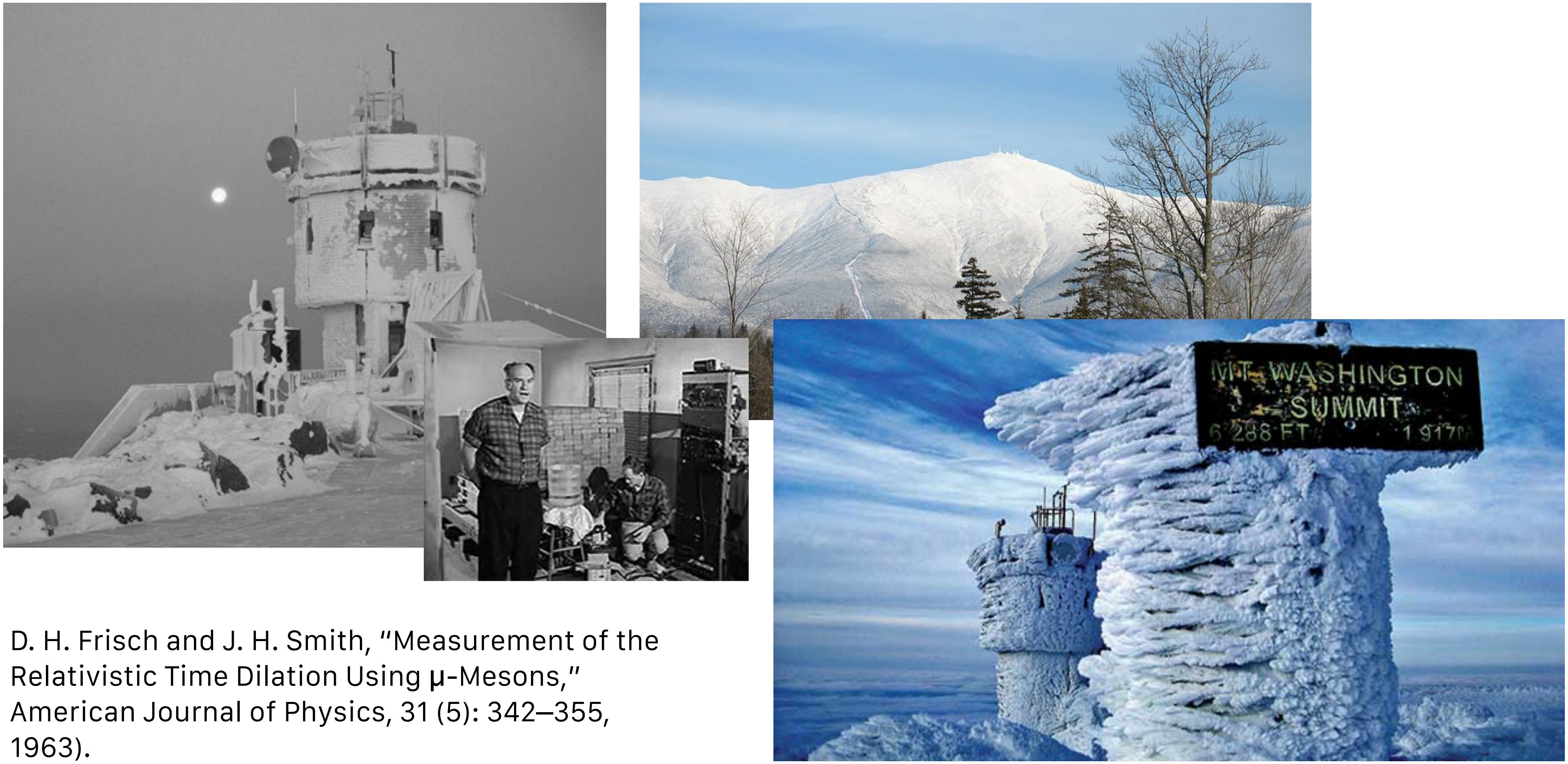
1.56×10^{-6} seconds

stand-up cosmic

~20 particles/cm/s

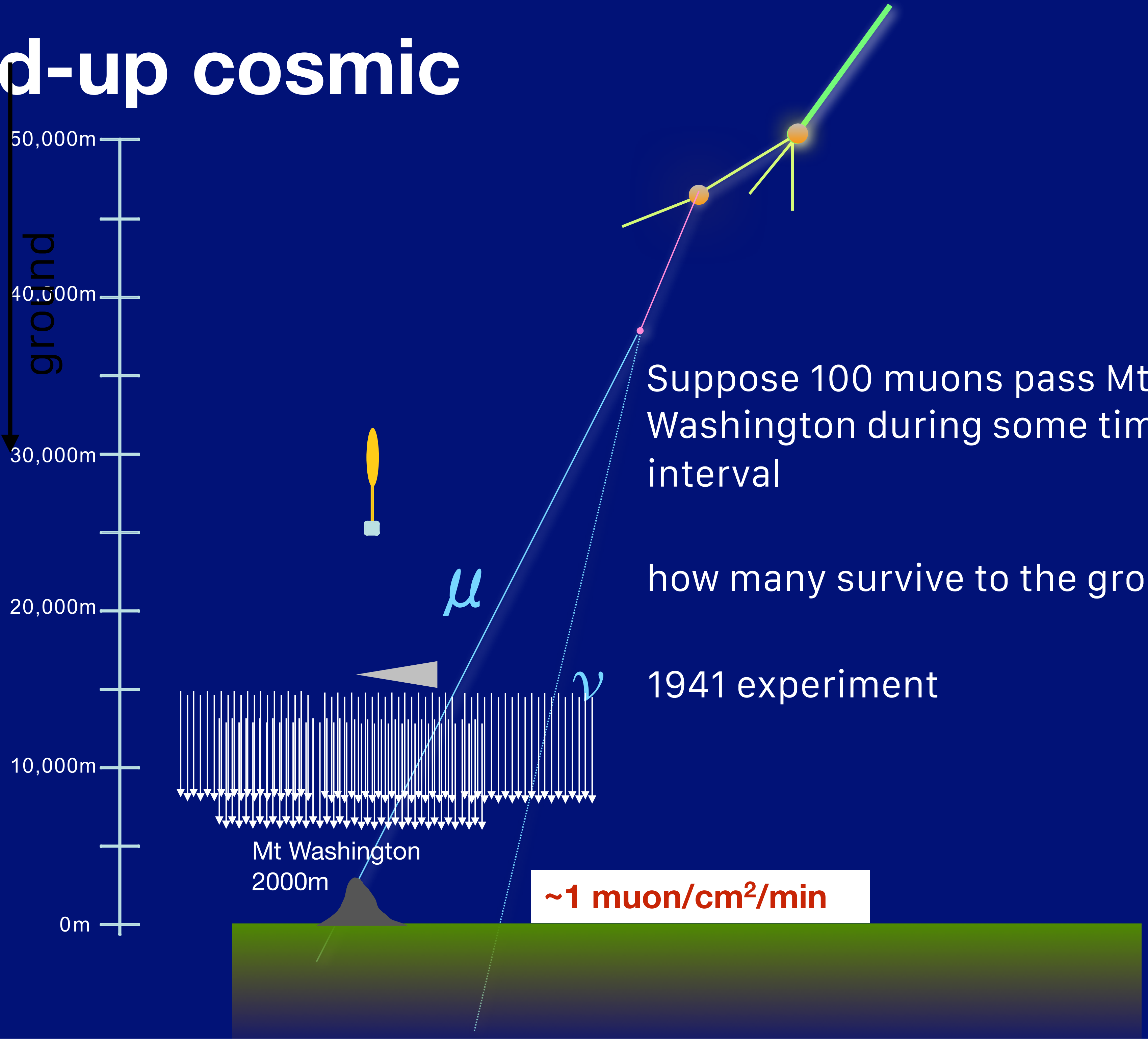


Mount Washington Observatory
New Hampshire



D. H. Frisch and J. H. Smith, “Measurement of the Relativistic Time Dilation Using μ-Mesons,” American Journal of Physics, 31 (5): 342–355, 1963).

stand-up cosmic



Suppose 100 muons pass Mt Washington during some time interval

how many survive to the ground?

1941 experiment

~1 muon/cm^2/min

home and away

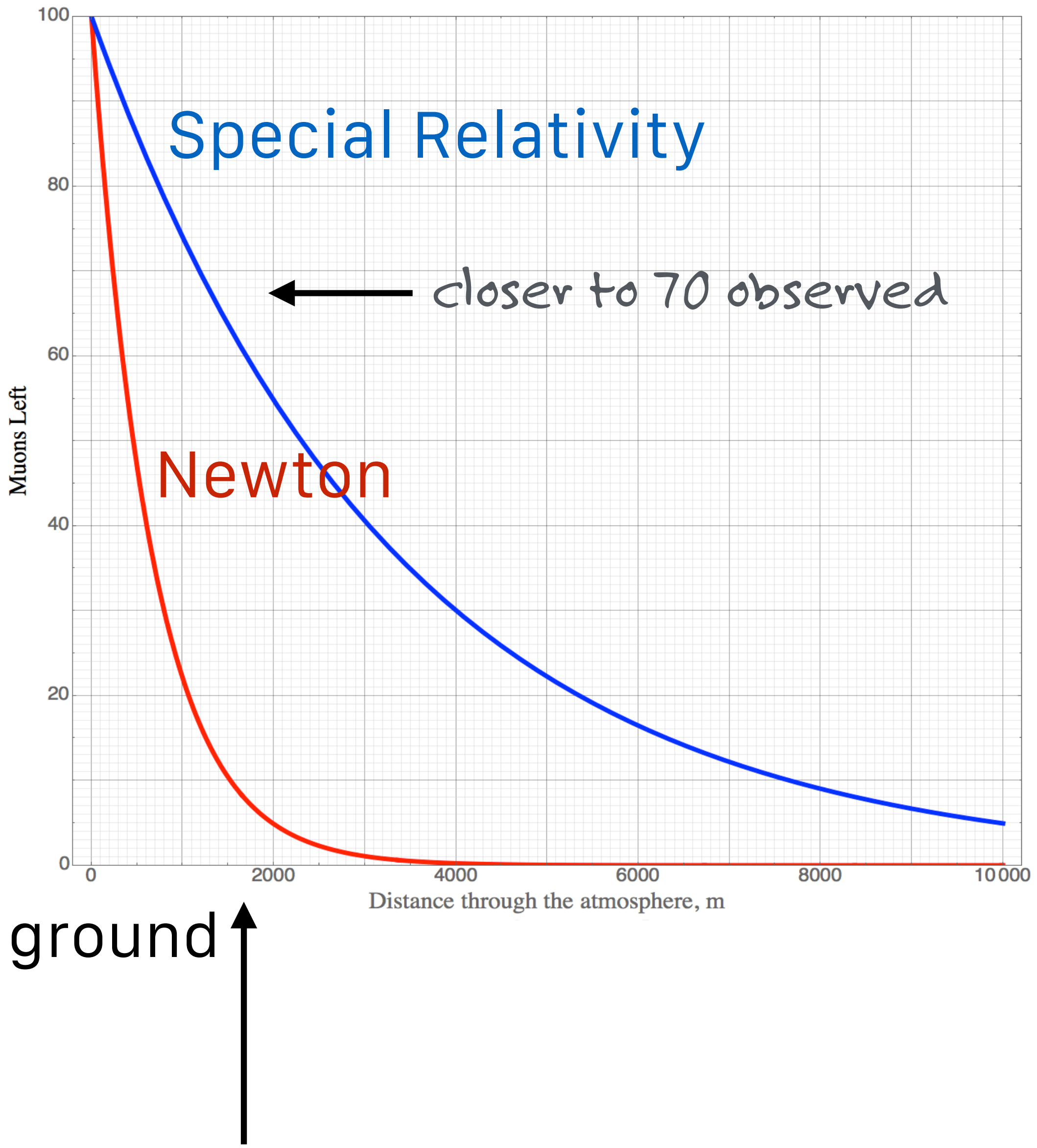
in the muon’s rest frame

its “clock” is 1.6 microseconds of life

in the mountain’s rest frame

for the muon moving with $\beta = 0.99$

its clock slows to be γ times that, or 7 x 1.6 microseconds



closer to 70 observed

ground

how can it decay and not decay?

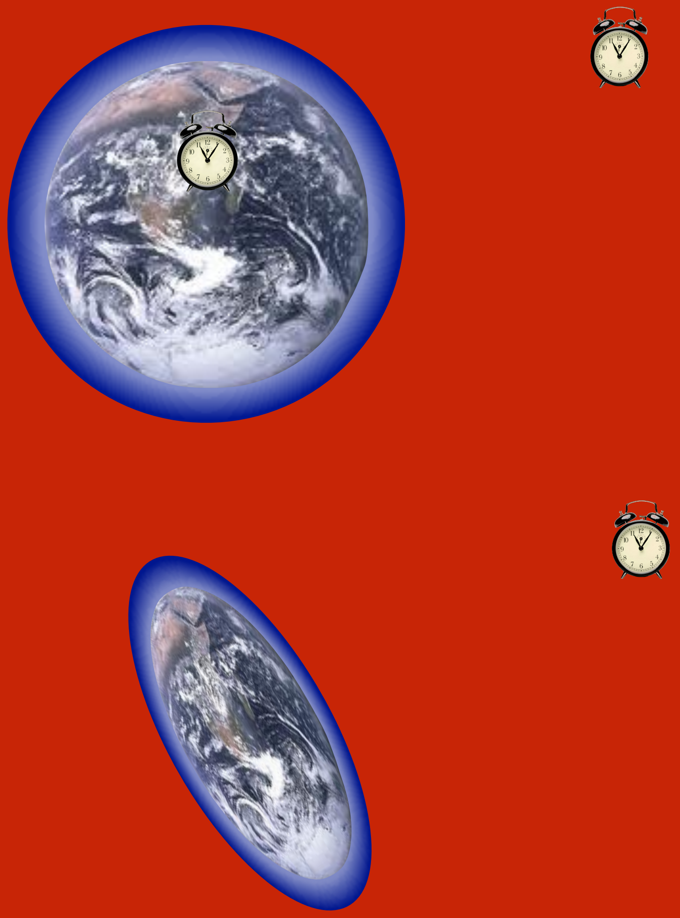
reciprocity

while it decays in 1.5μs in its rest frame...

it sees the atmosphere coming toward it at nearly c

which, to the muon, is Length Contracted

shorter by the same factor that the lifetimes differed



Charged Pion Decay: $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}$

pion ⟶ muon + neutrino

Facts $m_{\pi} = 139.57 \text{ MeV}/c^2$

$m_{\mu} = 105.45 \text{ MeV}/c^2$

$\tau_{\mu} = 2.2 \times 10^{-6} \text{ s}$

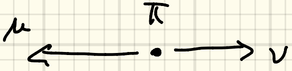
$m_{\nu} \simeq 0$

$\vec{p}_{\pi} = 0 \Rightarrow$ at rest

a) What is momentum of μ?

b) on average how far does μ travel before it decays?

a)



conserve momentum:

$$\vec{p}_\pi = \vec{p}_\mu + \vec{p}_\nu$$

$$= 0$$

$$\vec{p}_\mu = -\vec{p}_\nu$$

conserve energy:

$$E_\pi = E_\mu + E_\nu$$

$$m_\pi c^2 + \underset{\substack{\uparrow \\ 0}}{K_\pi} = \underbrace{m_\mu c^2 + K_\mu}_{\sqrt{E_\mu^2}} + \underset{\substack{\uparrow \\ 0}}{m_\nu c^2} + K_\nu$$

$$E^2 = p^2c^2 + m^2c^4 \;!$$

$$E_\nu^2 = p_\nu^2 c^2 \qquad E_\mu^2 = p_\mu^2 c^2 + m_\mu^2 c^4$$

but $|p_\mu| = |p_\nu| \equiv p$

$$E_\nu = pc \qquad E_\mu = \sqrt{p^2c^2 + m_\mu^2 c^4}$$

$$E_\pi = E_\mu + E_\nu$$

$$m_\pi c^2 + \underset{\overset{\uparrow}{0}}{K_\pi} = \underbrace{m_\mu c^2 + K_\mu}_{\sqrt{E_\mu^2}} + \underset{\overset{\uparrow}{0}}{m_\nu} c^2 + K_\nu$$

$$\underline{m_\pi c^2} = \sqrt{p^2 c^2 + \underline{m_\mu^2} c^4} + pc \longrightarrow \text{solve for } pc$$

Solution

$$A = \sqrt{x^2 + B^2} + x$$

$$\sqrt{x^2 + B^2} = A - x$$

$$\cancel{x^2} + B^2 = (A - x)^2 = A^2 - 2Ax + \cancel{x^2}$$

$$B^2 = A^2 - 2Ax$$

$$2Ax = A^2 - B^2$$

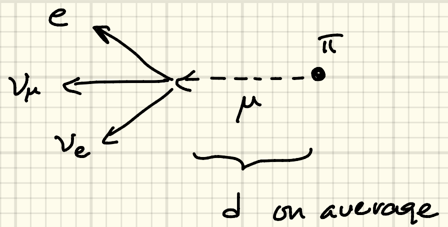
$$x = \frac{A^2 - B^2}{2A} = \frac{(m_\pi c^2)^2 - (m_\mu c^2)^2}{2(m_\pi c^2)} = \frac{(139.6)^2 - (105)^2}{2(139.6)}$$

$$x = 30.3$$

$$pc = 30.3 \text{ MeV}$$

$$p = 30.3 \text{ MeV}/c$$ ✓

b)



"classically" $d = v\tau$ $\longrightarrow$ let it be as fast as conceivable $= \underline{\underline{c}}$

$$d = c\tau = (3\times10^8\,m/s)(2.2\times10^{-6}\,s)$$

$$d = 660\ m$$

AN ASIDE

we are bombarded by μ's which have lived through the entire atmosphere ~ 50,000 m @ ~ $1\mu/cm^2/min$

muon decay

called the "lifetime"

$$N(t) = N(0)\, e^{-t/\tau}$$

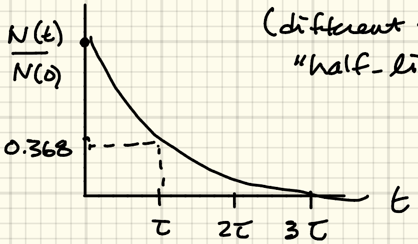
$\uparrow$ #μ after t seconds

$\uparrow$ #μ @ $t=0$

time for a decay to reduce a sample size by factor

$$\frac{1}{e} = \frac{1}{2.72} = 0.368$$

(different slightly from "half-life" --- stay tuned)



In μ rest frame —

a likelihood of decay $\nearrow$ $\tau = 2.2 \times 10^{-6}\,\text{s} = 2.2\,\mu\text{s}$

K

On earth: we see the μ's "clock" dilated.

μ

$$\tau = \gamma \tau_\mu$$

us

γ is about $\underline{7}$

So for us... it travels — on average —

$$d = \gamma \underbrace{u\tau} = 7\,(650\,\text{m}) = 4600\,\text{m}$$

us

μ... 650 m

μ sees earth's atmosphere rushing toward it & length-contracted.

$$d_\mu = \frac{1}{\gamma}\, d_e$$

muon

us

DONE WITH ASIDE → our original π decay:

Back to b)

$$d = \gamma u \tau \qquad p = \gamma m_\mu u$$

$$d = \frac{p}{m_\mu u} u \tau \, \frac{c^2}{c^2}$$

$$d = \frac{(\underline{pc})(c\tau)}{m_\mu c^2} = \frac{(30.\ \mathrm{MeV})(3\times10^{8})(2.2\times10^{-6})}{(105.45\ \mathrm{MeV})}$$

$$d \simeq 185\ \mathrm{m}$$

Handy rules of thumb:

$$E = \gamma m c^2 \qquad E^2 = \frac{1}{(1-\beta^2)} m^2 c^4$$

$$\underline{\underline{\gamma = \frac{E}{mc^2}}} \qquad \Rightarrow \beta^2 = \frac{p^2c^2}{E^2}$$

$$\underline{\underline{\beta = \frac{pc}{E}}}$$

THERMODYNAMICS

n.b. Chapters 17-20 in Bauer & Westfall → see syllabus!

17: Temperature
measuring temperature
properties of materials

18: Heat & 1st Law of Thermodynamics
heat & work
specific heats, latent heat, phase transitions
energy transfer

19: Ideal Gases
empirical relations
Ideal Gas Law
Equipartition
Kinetic Theory

20: Second Law of Thermodynamics
Reversability – Carnot cycle
Entropy

Thermo
→ phenomenological theory... empirical
→ atomic, kinetic

SOLIDS	LIQUIDS	GASES
mass, volume, density, shape:	mass, volume, density:	mass:
constant	constant	constant

→ most properties* of materials <u>are</u> temperature-dependent

exceptions: mass, charge, lifetime

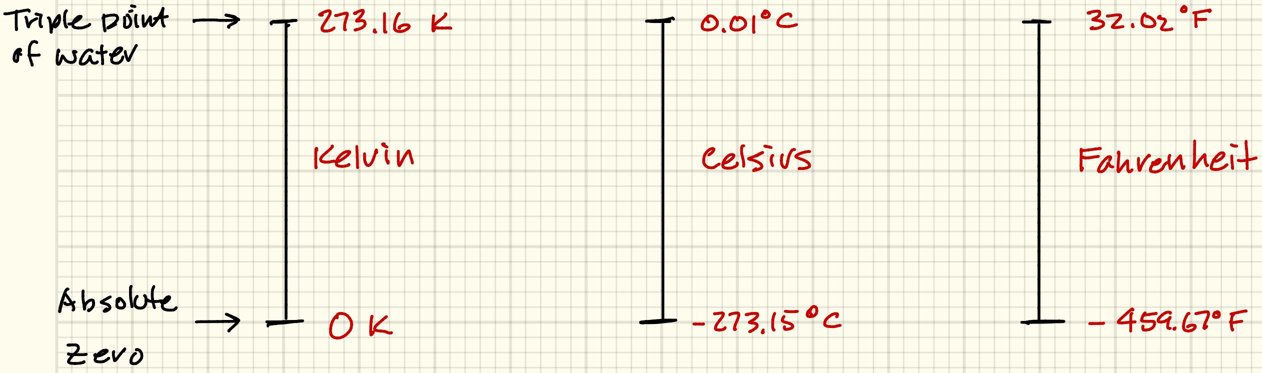
* length, volume, density, resistivity, magnetization, index of refraction

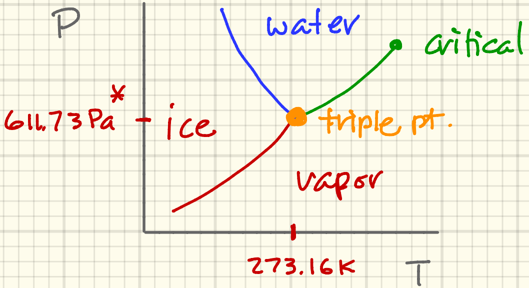
"Zeroth Law of Thermodynamics"

Temperature exists & it's transitive: If $T_A = T_B$ & $T_B = T_C \Rightarrow T_A = T_C$

Heat is a form of energy and can be transferred from one object to another if they are a) in "contact" and b) are @ different T's.

Temperature Scales





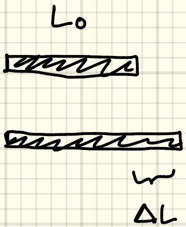
$* \ 1\,\text{atm} = 1.01 \times 10^5\ \text{Pa}$

$= 760\ \text{Torr}$

$= 14.7\ \text{lb/in}^2$

$1\ \text{Pa} = \frac{1\text{N}}{\text{m}^2}$

Linear Thermal Expansion



T

$T + \Delta T$:

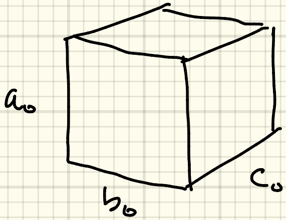
$$\left.\begin{array}{l} \Delta L \propto \Delta T \\ \Delta L \propto L_0 \end{array}\right\} \quad \begin{array}{l} \Delta L = \alpha L_0 \Delta T \\ L = L_0 (1 + \alpha \Delta T) \end{array}$$

coefficient of linear expansion

$$[\alpha] = \frac{1}{[T]}$$

Copper : $1.62 \times 10^{-5} \,/\, C^\circ$

Volume Thermal Expansion



$$V_0 = a_0 b_0 c_0$$

each length expands / contracts

$$\left.\begin{aligned} a &= a_0(1+\alpha\Delta T) \\ b &= b_0(1+\alpha\Delta T) \\ c &= c_0(1+\alpha\Delta T) \end{aligned}\right\} \quad V = abc = a_0 b_0 c_0 (1+\alpha\Delta T)^3$$

$$V = V_0\left(1 + 3\alpha\Delta T + \text{two small terms}\right)$$

α^2 & α^3

$$V \simeq V_0(1 + \underbrace{3\alpha}_{\equiv \beta}\Delta T)$$

coefficient of volume expansion

$$\Delta V = \beta V_0 \Delta T$$

$$[\beta] = \frac{1}{[T]}$$

examples

Petroleum $9.6\times10^{-4}\,/C^\circ$

Water $2.0\times10^{-4}\,/C^\circ$

Heat & Mechanical Work

long L O N G history.

First model:

PHLOGISTON released.

(George Stahl 1600-1734)

Antoine Laurant Lavoisier (1743-1794)

combustion = oxygen + substance

actually measured air, reaction products carefully & first.

"Conservation of Mass"

"The state needs no scientists." ... bad end.

Antoine Lavoisier and Marie-Anne Paulze Lavoisier

How much heat to raise a substance temperature?

Joseph Black (1728-1799) defined "specific heat"

Quantity of Heat required to raise temperature of 1g by 1 C°.

$$[c] = \frac{[Q]}{[M][T]} \longrightarrow \frac{\text{calorie}^{*}}{g\ C^{\circ}}$$

$$Q = c m \Delta T$$

ΔQ	1g substance	ΔT	
1 cal	water	1°C	← high
1 cal	mercury	30°C	
1 cal	iron	9°C	

* lower-case "c" ... calories for physicists and engineers

Upper Case "C" .. Calories for food   C = 1000 c = 1 kcal

Subtle point: heat capacity: just proportionality

$$Q = C\,\Delta T$$

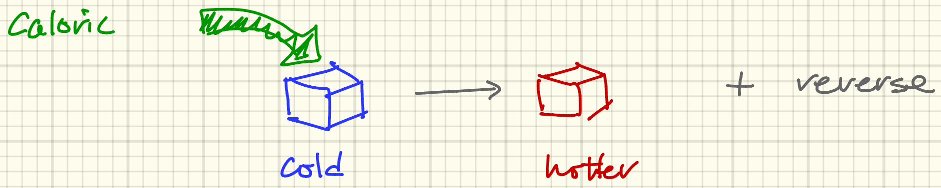
⇒ depends on mass

specific heat better: $[c] = \frac{1\,\text{cal}}{\text{g}\,^\circ\text{C}} = \frac{1\,\text{cal}}{\text{g}\,\text{K}} = \frac{1\,\text{Btu}}{\text{lb}\,^\circ\text{F}} = 4190\,\frac{\text{J}}{\text{kg}\,\text{K}}$

$$c = \frac{C}{m}$$

Why?

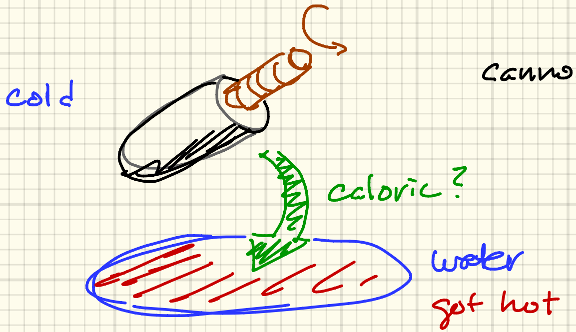
Second Model: "Caloric" a fluid that some substances have an affinity for.



So heat is a material substance.

TRAITOROUS THEORY: Benjamin Thompson (1753 - 1814)

↳ Count Rumford



cannon boring

Submerged whole thing in water... continued boring...

water would not stop boiling... "inexhaustible..."

caloric never seemed to get used up

⟶ related to the effort of horses which powered drill?

Antoine Lavoisier and Marie-Anne Paulze Lavoisier

Benjamin Thompson

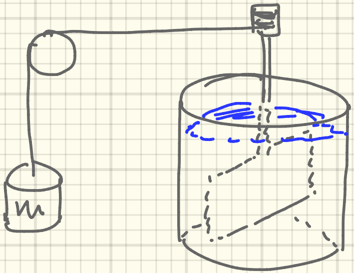
Heat *not* a substance?

50 years--- James Prescott Joule (1818 - 1889)

Manchester brewer

Became convinced:
- electrical source of heat (Joule heating)
- mechanical source of heat

mechanical work ⟷ heat



very precise measurements

4.184 Joules = 1 Calorie

Somehow work "in" ... heat out?

Absorption of heat ... an empirical history.

Sometimes: absorbed according to c and $T\uparrow$

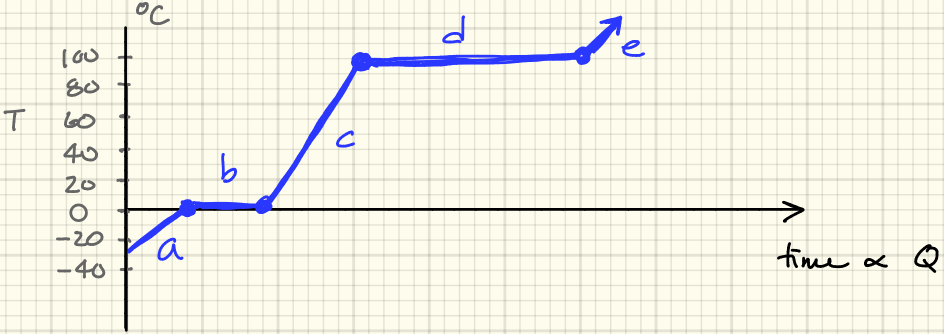
Sometimes: a phase transition happens & $\Delta T = 0$

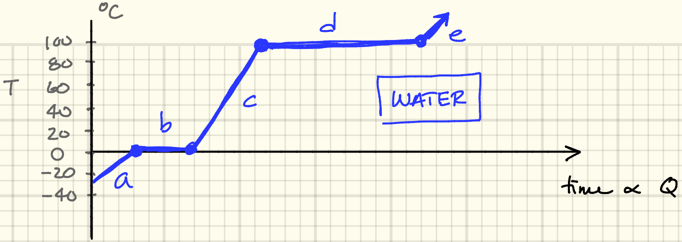
↳ described by substances' "latent heat"

$$Q = mL$$

L_V : latent heat of vaporization → boiling & condensing

L_F : latent heat of fusion → melting & freezing





(1 cal = 4.186 J)

a. temperature of ice increases: $\Delta T = \frac{Q}{c_I m}$ $c_I = 2.06\ kJ/kg\,K$

$Q_a = c_I m \Delta T$

b. temperature doesn't change: $\Delta T = 0$

$Q_b = m L_F$ $L_F = 333\ kJ/kg$

c. temperature of water: $\Delta T = \frac{Q}{c_w m}$ $c_w = 4.19\ kJ/kg\,K$

$Q_c = c_w m \Delta T$

d. temperature of steam doesn't change: $\Delta T = 0$

$Q_d = m L_v$ $L_v = 2256\ kJ/kg$

e. temperature of steam increases: $\Delta T = \frac{Q}{c_s m}$ $c_s = 2.01\ kJ/kg\,K$

$Q_e = c_s m \Delta T$

Cultural Consideration...

not always: grams, kilograms ...

sometimes: moles

1 mole = 6.02×10^{23} "units" = N_A

1 mole Aluminum = 6.02×10^{23} atoms

hydrogen = 6.02×10^{23} molecules.

$$m(\text{substance}) = \overset{\#\text{ moles}}{\overset{\downarrow}{n}}\, \underset{\underset{\text{atomic or molecular weight}}{\uparrow}}{A}$$

1 mole ^{12}C = 12 gm.

If masses are ⟶ moles

↳ molar specific heats

molar heats of vaporization and fusion

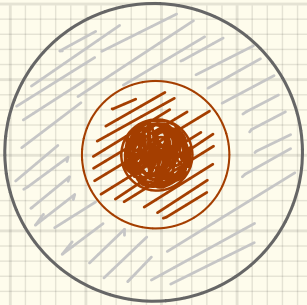
18: Heat & 1st Law of Thermodynamics

heat & work

specific heats, latent heat, phase transitions

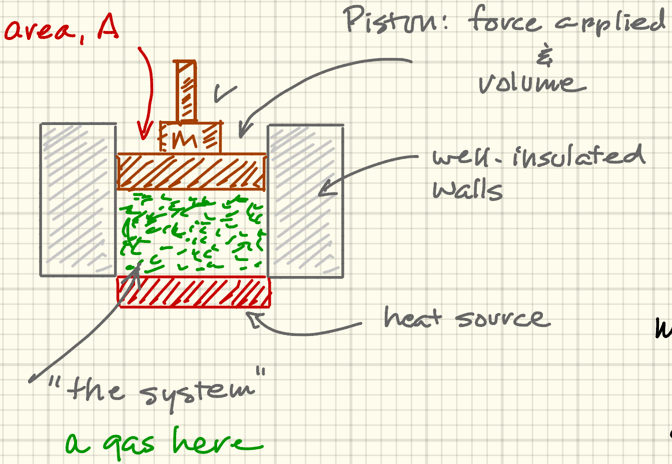
energy transfer

Element	Specific Heat J/kg K	A (g) average	Mol Specific Heat J/mole K
Lead	128	207	26.5
Tungsten	134	184	24.8
Silver	236	108	25.5
Copper	386	63.5	24.5
Aluminum	900	27	24.4
Water	4,190	18	75.2



HEAT & WORK

Always interested in
interactions among: temperature
work
heat
pressure
volume



$$P_G = \frac{F_G}{A}$$

move the piston a bit: $d\vec{s}$ by the gas

$$dW(\text{gas}) = \vec{F}_G \cdot d\vec{s} = P_G \vec{A} \cdot d\vec{s}$$

$$= P\,dV$$