2. Thermodynamics, 2

lecture 9, September 18, 2017

housekeeping

exam 1 is in about 2 weeks

- Friday, 29 September
- Relativity and Thermodynamics

Remember the textbook for thermodynamics is Bauer and Westfall

- you have it, or you can buy just chapters 17-21…see syllabus

Some changes:

- you saw that I deleted a problem that I didn’t think was useful in chapter 18
- we will skip sections 19.6 and 19.7 as they will be dealt with in a more grown-up way in Thornton and Rex later

Shameless plug:

- ISP220

THERMODYNAMICS

n.b. Chapters 17-20 in Bauer & Westfall → see syllabus!

17: Temperature
- measuring temperature
- properties of materials

18: Heat & 1st Law of Thermodynamics
- heat & work
- specific heats, latent heat, phase transitions
- energy transfer

19: Ideal Gases
- empirical relations
- Ideal Gas Law
- Equipartition
- Kinetic Theory

20: Second Law of Thermodynamics
- Reversability – Carnot cycle
- Entropy

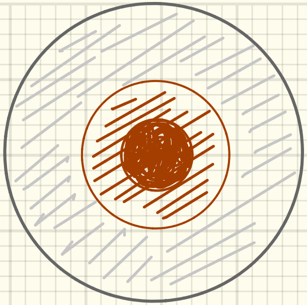
18: Heat & 1st Law of Thermodynamics

heat & work

specific heats, latent heat, phase transitions

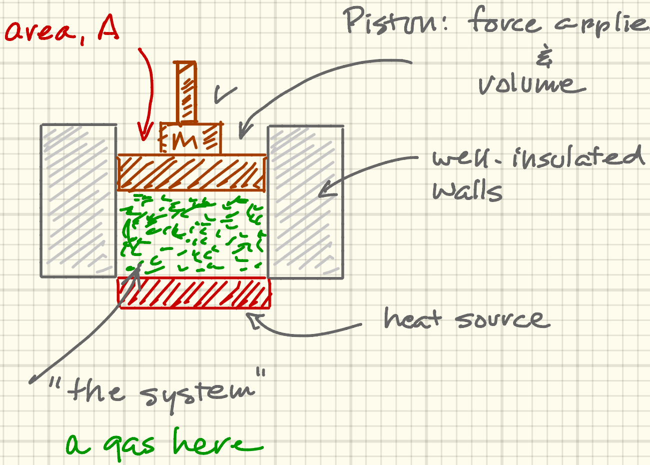
energy transfer

Element	Specific Heat J/kg K	A (g) average	Mol Specific Heat J/mole K
Lead	128	207	26.5
Tungsten	134	184	24.8
Silver	236	108	25.5
Copper	386	63.5	24.5
Aluminum	900	27	24.4
Water	4,190	18	75.2



HEAT & WORK

Always interested in interactions among:
- temperature
- work
- heat
- pressure
- volume



$$P_G = \frac{F_G}{A}$$

move the piston a bit: $d\vec{s}$ by the gas

$$dW(\text{gas}) = \vec{F}_G \cdot d\vec{s} = P_G \vec{A} \cdot d\vec{s}$$

$$= P\,dV$$

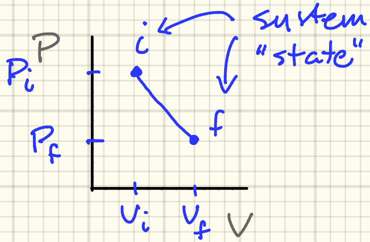
total work done by the gas:

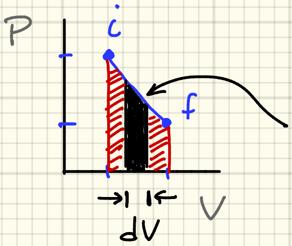
$$W = \int_{V_i}^{V_f} P\,dV$$

raise piston: $+W_G$

lower piston: $-W_G$

P-V diagrams



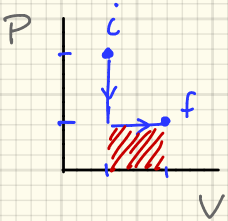


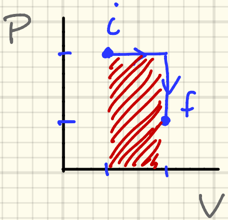
$dW = P\,dV$

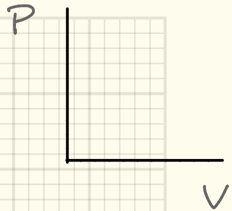
&

$W = \int P\,dV$ area under P-V curve

Lots of ways to go from i to f





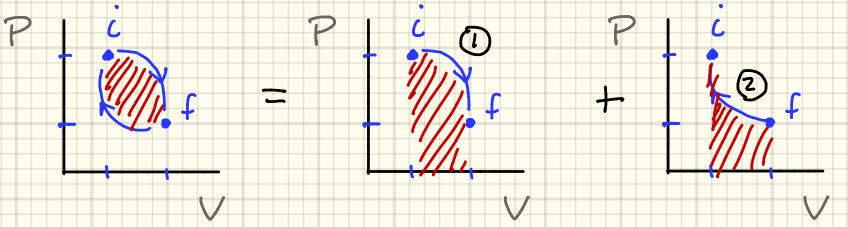


Less More Work

Increase V $\Rightarrow$ gas moves piston up $\Rightarrow$ $+W_G$

Managing thermodynamic cycles

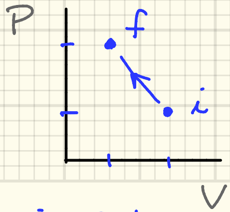
a "cycle":



$$W(\text{net}) = \int_{V_i}^{V_f} P_{①}\, dV + \int_{V_f}^{V_i} P_{②}\, dV$$

$$W(\text{net}) = W^{①}_{if} + W^{②}_{fi}$$

$$> 0$$



$i \to f$ here

$V \downarrow$

piston $\downarrow$

$-W_G$ by gas

($+W_G$ by piston)

Adding heat to a substance:

- raise temperature (increase "internal energy")
- do work

But something stays constant: ← empirical.

$\Delta Q - \Delta W$

$\equiv \Delta U$ internal energy

$$\Delta U = \Delta Q - \Delta W$$

← FIRST LAW OF THERMODYNAMICS

a statement of energy conservation

↑ change in internal energy (raises temperature)

↑ change in heat input

↑ work done by system

State Functions

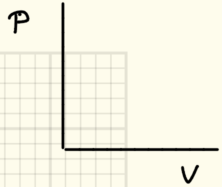
a property of the state of a system: State Function

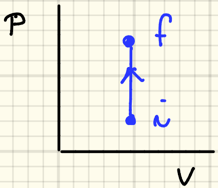
P, V, T, U are state functions ← path independent → a point on P-V

Q, W are not ← path-dependent

Different Paths

$\Delta Q = \Delta U + \Delta W$





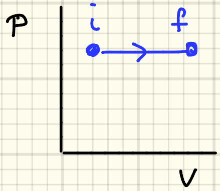
isochoric

$\Delta W = \int P\,dV = 0$

$\Rightarrow \quad \Delta Q = \Delta U + \Delta W$

$\Delta Q = \Delta U$

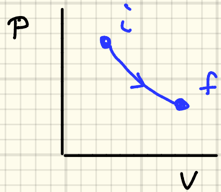
heat $\rightarrow$ T ↑



isobaric

$\Delta W = \int P\,dV > 0$

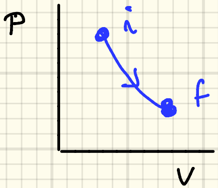
$\Delta Q = \Delta U + \Delta W$



constant temperature:

isothermal

$\Delta U = 0 \Rightarrow \Delta Q = \Delta W$

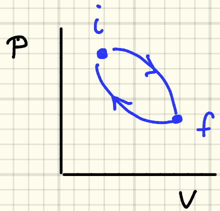


constant heat:

adiabatic

$\Delta Q = 0 \Rightarrow \Delta U = -\Delta W$

work done by reducing internal energy



cycle

$\Delta U = 0 \Rightarrow \Delta Q = \Delta W$

heat goes into work

↳ a useful engine!

All are "quasi-static"

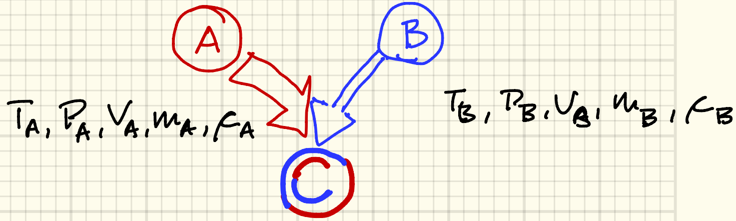
$\Rightarrow$ slow enough to have thermal equilibrium at each point... all times

Also: $\Delta Q = 0$, $\Delta W = 0$ $\Rightarrow \Delta U = 0$

"adiabatic, free expansion"

Heat & Calorimetry

Often:



coffee
$m_{co} = 0.2\,kg$
$T_{co} = 70°C$

Copper cup
$m_{cu} = 0.1\,kg$
$T_{cu} = 20°C$

↳ what's the final temperature, T of the two when thermal equilibrium is reached?

$Q = cm\Delta T$ lost, or gained

Q_{cu} (gained) $= Q_{co}$ (lost)

$$c_{cu} m_{cu} (T_{cu} - T) = c_{co} m_{co} (T_{co} - T)$$

algebraic eq. fn T

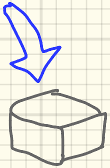
so here with

$c_{cu} = 390\,J/kg°C$
$c_{co} = 4186\,J/kg°C$

} $T = 67.8\,°C$

another

water $m_w = 300g$, $T_w = 30°C$



ice

$m_i = 50g$, $T_i = -5°C$

→ final temperature, T of water + water (ice)

↑ assuming it all melts

$$\Delta Q_w = \Delta Q_i$$

$$m_w c_w \Delta T_w = m_i c_i \Delta T_{i2w} + m_i L_F + m_i c_w \Delta T_{wi}$$

$\Delta T_w = (30 - T)$

↑ heating ice to melting — ↑ melting ice — ↑ heat resulting water from ice

$\uparrow T_{i2w} = (0 - -5)$

$\uparrow T_{wi} = (T - 0)$

$$\int_{T_i}^{T_i + \Delta T} c\, dT$$

$T_i + \Delta T - T_i$

$T_F - T_i$

solve for T: $T = 13.9°C$

suppose

T-30 0 - -5 T-0

Energy Transfer

SYSTEMS ARE USUALLY NOT ISOLATED

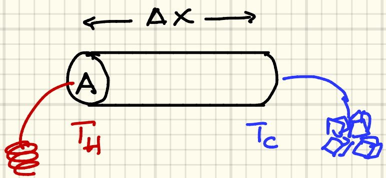
heat migrates

conduction
convection
radiation

} transfer mechanisms

all around you

Conduction



- atoms of adjacent – "touching" – media move, rotate, and/or vibrate
- collide & transfer these motions (energy moves... not atoms)

$$H = \frac{\Delta Q}{\Delta t} = k A \frac{\Delta T}{\Delta x}$$

H: heat conduction rate, J/s, cal/s ↑ Watts

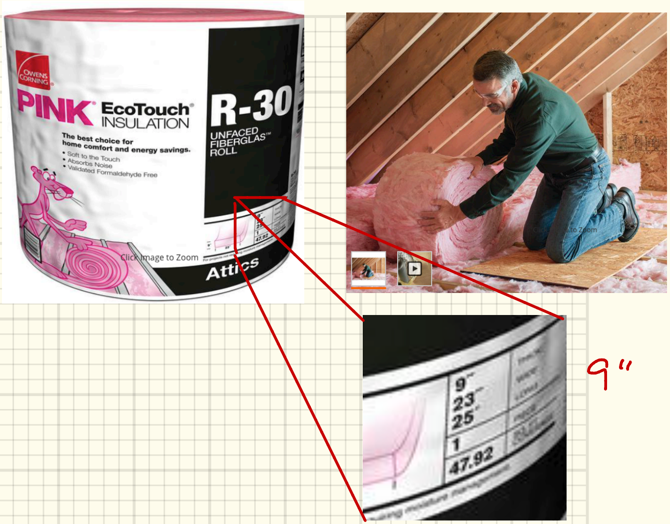
k: "Thermal Conductivity" W/mK

$$H = \frac{kA(T_H - T_c)}{\Delta x}$$

$$H = \frac{A \Delta T}{\Delta x / k}$$

$\Delta x / k$ — R, thermal resistance

material	$k\ (W/mK)$
stainless steel	14
lead	35
copper	401
fiberglass	0.048
window glass	1.0
dry air	0.026
He	0.15
H_2	0.18



9"

$$R\left(ft^2\,{}^\circ F\,h/BTU\right) = 5.678\,R\left(\frac{m^2K}{W}\right)$$

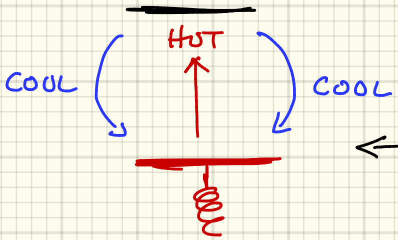
$$R = \frac{\Delta x}{k} = \frac{(9'')(2.54\,cm/in)\left(\frac{1\,m}{100\,cm}\right)}{0.048\,W/mK}$$

$$R = 4.8 \xrightarrow{US} \sim 27$$

series of conductors: $R_{eq} = R_1 + R_2 + \cdots$

Convection

material moves ⇒ fluid, gas or liquid



← more dense ↓

← less dense ↑

Fireplaces

Why Britian is not frozen

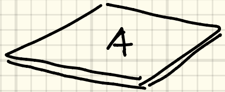
atmospheric Highs and Lows

Solar convection

Radiation

electromagnetic radiation ⟶ everything radiates

(thermal radiation)



Rate $\propto T^4$

P_R power radiated -- energy / time ... Watts

$P_R \propto$ (details of surface) $A T^4$

$P_R = \sigma \varepsilon A T^4$

"emissivity" $(0 < \varepsilon < 1)$

Stefan - Boltzmann constant

$\sigma = 5.6703 \times 10^{-8}\ W/m^2K^4$

radiation:

$P_R = \sigma \varepsilon A T^4$ Stefan-Boltzmann Law

1879 experiment → Stefan

1884 theory → Boltzmann

absorption:

$P_{abs} = \sigma \varepsilon A T_{env}^4$

env ← environment

$0 < \varepsilon < 1$

light objects, ε small

dark objects, ε large

$\varepsilon = 1$, total absorption Blackbody Radiator

19: Ideal Gases

empirical relations

Ideal Gas Law

Equipartition

Kinetic Theory

Ideal Gases

Astonishingly: simple, versitile, enlightening

Few assumptions:
- large N
- point-like objects
- identical objects
- no forces among objects

} most gases approach this for small P, ρ

Historical roots:

Boyle's Law $PV = \text{constant}$ (@ fixed T & N)

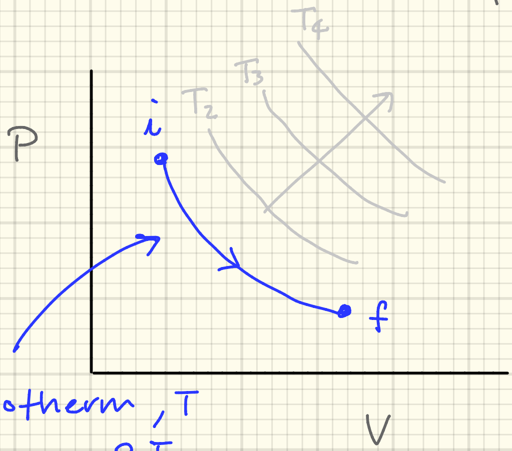
Charles' Law $V/T = \text{constant}$ (@ fixed N & P)

Ideal Gas Law (s)

$PV = nRT$ or $PV = NkT$

n = # moles, R = gas constant, $R = 8.31\ J/mol\cdot K$

N = # molecules, k = Boltzmann's Constant, $k = 1.38 \times 10^{-23}\ J/K$



isotherm, T

$$P = \frac{nRT}{V}$$

Work done by gas:

$$\Delta W = \int_i^f P\,dV$$

$$\Delta W = \int_i^f \left(\frac{nRT}{V}\right) dV = nRT \ln\left(V_f / V_i\right)$$

$V_f > V_i$ expansion $\Delta W > 0$

$V_i > V_f$ compression $\Delta W < 0$

example 2.1 Calculate work done by external agent on 1 mole of O_2 from $V = 22.4\,L$ @ $0\,°C$ and $P = 1\,atm$ to $V = 16.8\,L$ → isothermal compression

$$W = nRT \ln\left(V_f / V_i\right) \quad \leftarrow \text{by gas.}$$

$$= (1\,mol)(8.31\,J/mol\cdot K)(273\,K) \ln\left(16.8/22.4\right)$$

$$W = -653\,J$$

$$W(\text{by agent}) = +653\,J$$

example 2.2

Ideal gas @ $T = 10°C$ & $P = 100\,kPa$ has $V = 2.5\,m^3$.

a) How many moles of the gas are there?

b) If $P \to 300\,kPa$ and $T \to 30°C$ what is new V?

Units!

atmospheric pressure

$1\,atm = 1.013 \times 10^5\,N/m^2$ (= $14.7\,lb/in^2$)

$= 1.013 \times 10^5\,Pa$ "Pascals" SI

$= 1013$ millibars

$= 76.0\,cm\,Hg$.

So, $100\,kPa = 10^5\,Pa$

about 1 atm.

a) moles:

$$PV = nRT$$

$$n = \frac{PV}{RT}$$

$R = 8.31\ J/mol \cdot K$

$$= \frac{(10^5\ N/m^2)(2.5\ m^3)}{(8.31\ J/mol \cdot K)(10 + 273)K}$$

$N \cdot m \rightarrow J$

$$n = 106\ mol$$

b)

P: 100 kPa → 300 kPa

T: 10°C → 30°C

V: 2.5 m³ → ?

what stays the same? n

$$P_1 V_1 = n_1 R T_1 \quad \& \quad P_2 V_2 = n_2 R T_2$$

$$n_1 = n_2 = n$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$V_2 = \left(\frac{T_2}{T_1}\right)\left(\frac{P_1}{P_2}\right) V_1$$

$$V_2 = 0.892\ m^3$$

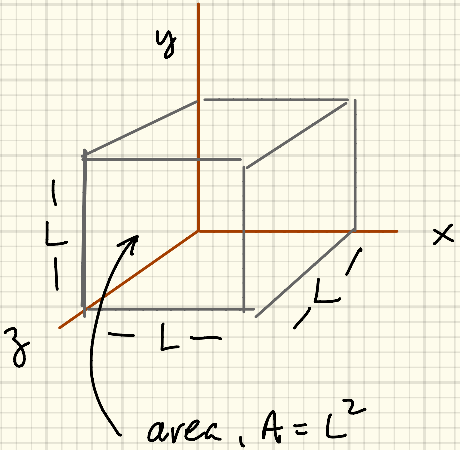
"Kinetic Theory"

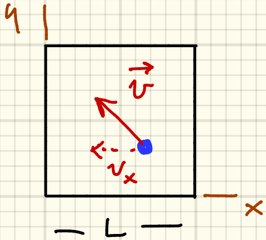
very old & very cool

Imagine a volume filled with molecules

- Identical, m
- no size
- no interactions
- numerous $\Rightarrow$ ignore fluctuations $\sim \frac{1}{\sqrt{N}}$

Sounds like?





collides elastically from wall

$\leftarrow v_x$

$\rightarrow v_x'$

Δt = average time between collisions at $x = 0$ & $x = L$, round trip

$$\Delta t = \frac{2L}{|v_x|}$$

momentum transfered to wall

$$\Delta p_x = 2m|v_x|$$

rate at which momentum is transfered

$$\frac{\Delta p_x}{\Delta t} = \frac{2m|v_x|}{2L/|v_x|} = \frac{mv_x^2}{L} = \langle F_x \rangle$$ by one molecule ← "average"

total force applied to $x=0$ or $x=L$ walls:

$$\langle F \rangle = \frac{Nmv_x^2}{L}$$

pressure:

$$P = \frac{\langle F \rangle}{A} = \frac{Nmv_x^2}{L^2 L} = \frac{Nmv_x^2}{V}$$

use averages: $v_x^2 \to \langle v_x^2 \rangle$

$$P = \frac{Nm\langle v_x^2 \rangle}{V}$$

$$P = \frac{Nm\langle v_x^2\rangle}{V}$$

nothing special about x .. on average

$$\langle v_x^2\rangle + \langle v_y^2\rangle + \langle v_z^2\rangle \equiv \langle v^2\rangle$$

so $\langle v_x^2\rangle = \frac{1}{3}\langle v^2\rangle$

$$P = \frac{Nm\langle v^2\rangle}{3V}$$

$$PV = \frac{1}{3} Nm \langle v^2\rangle \quad = \text{Constant ... Boyle's Law}$$

Now remember ideal gas law: $PV = NkT$

THE MAGIC!

$$PV = \frac{1}{3} Nm\langle v^2\rangle = NkT \quad \text{for an ideal gas}$$

$$kT = \frac{1}{3} m\langle v^2\rangle = \frac{2}{3} \cdot \frac{1}{2} m\langle v^2\rangle = \frac{2}{3} K$$

WHOA.