“Superfluous”: The Stories of Einstein’s Special Relativity

Draft

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Chapter 1

It's All Greek To Me : The Greeks

"There is a land called Crete, in the midst of the wine-dark sea, a fair, rich land begirt with water; and therein are many men past counting, and ninety cities." Homer, *The Odyssey*

"The safest general characterization of the European philosophical tradition is that it consists of a series of footnotes to Plato." A.N Whitehead (1861-1947), mathematician and philosopher

"We are all Greeks. Our laws, our literature, our religion, our arts have their root in Greece." Percy Bysshe Shelley (1792-1822), poet

"The more I read the Greeks, the more I realize that nothing like them has ever appeared in the world since." Albert Einstein (1879-1955), theoretical physicist, Nobel Laureate

1.1 A Little Bit of The Greeks

It may have once been the case that all roads lead to Rome, but for most of western philosophy, physical science, and mathematics, all roads lead *from* Greece. This chapter is the first stop in our path towards Einstein's Special Relativity: our MOTION themes start with the Greeks, eventually centered on Plato and Aristotle and friends. Aristotle in particular informed—and infected—science for 2000 years. His models of motion on and of the Earth became Medieval and Renaissance Church dogma, but are wrong in almost every respect. By pushing back scientists learned what was better and why.

Since this is a book on physics, and since you can only invent something once, I want to tell you how physics started. This first of two chapters on the Greeks will be different from the ones that follow as I'll talk about lots of people, rather than one. Here we'll learn about the first steps to new habits of mind that evolved two centuries before Plato and drive us still. This is the beginning of asking two of our three questions: how (or whether) the Earth itself moves and how (or whether—looking at you, Zeno) things on the Earth move. When you see the tags MOTION BY THE EARTH and MOTION ON THE EARTH it signifies an important step on our path to developing our MOTION theme.

The quotes above are a small sampling of how we modern scientists look back at the Greeks. In many ways my field of particle physics is relentlessly Platonic (but don't tell anyone that I said that!). Plato (and to a lesser extent, Aristotle) continues to challenge us: *What* can we know? And, *how* do we know we know that something is true? And, of course, *how* do things move?

The next chapter will deal with them—but Plato was reacting to the thinkers who came before him, traditionally called "Presocratics," obviously meant to cover those who came before Socrates. Now, "Presocratics" is an all-purpose label that apples to people before Socrates, but also those who were contemporary to Socrates, and even some who where younger than Socrates. For all practical purposes, it essentially means: pre-plato. This chapter is about the Presocratics.

Here I will lay out four Greek Research Programs which still motivate us today, but which were really first identified by the Presocratics. Each was seeded before Plato and Aristotle and then they watered and then harvested those themes. I'll highlight them as we move along. They are:

1. **Is the universe constructed of fundamental building blocks?** And, might those fundamental entities behave together according to rules? This is the nature of physics today: my field of particle physics is dedicated to finding and characterizing the fundamental entities that make up everything else. Quarks and Leptons are those entities. But just stockpiling particles is merely stamp-collecting. They have to interact with one another and so the rules are deeply important. We call them the four fundamental forces today.

2. **Is the universe inherently mathematical?** It's long been appreciated that the universe seems to operate according to rules that are mathematical. Discoveries in physics and mathematics have each influenced the other. Why that relationship exists isn't understood and is so persuasive to some theoretical physicists, that they postulate—still— that the universe is not just mathematical, but *is* mathematics. I'll have a lot to say about this as it underpins not only MOTION and LIGHT but all of modern science.

3. **How can we reconcile permanence with change?** This is a tricky issue and one that bedeviled not only the Greeks, but much of philosophy to the present day. Unraveling this tension is intimately connected to theories of knowledge: what can we know and what can we trust? The permanent part of physics today refers to the various "conservation laws"...the Conservation of Energy, for example. But our elementary particles move around, they mix together, they annihilate and are born out of the vacuum. All the time. Change and permanence, agonized over by the Presocratics, are firmly a part of our modern story.

4. **How is the Universe structured and what are the rules that govern its beginning and current state?** "Cosmology" is the Greek word for this study that mashes together their word *cosmos* for "the world" or "universe," and *logos*, the word or "study of." It's now a modern term and Cosmology is an entire discipline in physics and astronomy. It started with the Greeks and their ideas became, just like motion, mangled by Aristotle's authority. It took 2000 years to get it right.

Around 2800 years ago a proto-science began by asking modern-sounding questions. We'll concern ourselves with our scientific parents: the Milesians (in Ionia, on the modern day west coast of Turkey) who invented the idea of substructure and natural rules, the Pythagoreans (in Italy) who emphasized the fundamental nature of mathematics, the Eleatics (in Italy) who fleshed out the tension between change and permanence, and the Pluralists (in Italy and Ionia), who found a rational alternative to the most persuasive and extreme of the Eleatics.

Greeks reveled in drama and it's within the turmoil and bloodshed between the Persian Wars and Alexander the Great that western philosophy and nascent science had its beginnings. So we'll picture this as a play in eight acts. The curtain rises...on a catastrophe.

1.1.1 ACT I: Light Followed By Dark and the Heroic Greeks

I imagine that it started out like any bright day on the northern coast of Crete. A lazy afternoon in this peaceful paradise.

Then, total darkness.

Without warning, the **loudest sound** ever experienced by humans was followed on the northern horizon by a hint of fire and smoke erupting tens of miles into the previously clear sky. Slowly the sun dimmed, and then the sky became black as six inches of ash fell all over the island like a dirty rain. In fact, debris fell as far as the whole of modern Turkey, northern Egypt, and the middle east. Following that sooty deluge, tidal waves fifty feet high engulfed the sea-side areas of Crete and destroyed everything for kilometers inland.

That terrifying −1650 day[1] in the capital city of Knossos was the consequence of a massive volcanic eruption on the island of Santorini, about 100 miles to the north. Look at your map application and navigate to 36°23′41.46" N 25°23′57.55" E. There you'll see a little Packman-like, backwards "C" feature in the Aegean Sea. That's the scar—the caldera from the "Minoan Eruption"—left behind by the opening act in what might have been the story of us in the West.

Our tragic Minoa—modern-day Crete—was a refined culture of master architects, mariners, and traders, an apparently relaxed and leisure-loving people. Their cities didn't seem to need much fortification—they seem to be secure among themselves and were rulers of the sea. They were literate and created the first *symbolic*, written language—two of them, actually. Their ancestors were pre-Bronze-Age migrants from the north, seasoned with Egyptian influence from about –3000.

Over the next thousand years, Minoans and Phoenicians became Mediterranean, international sea-going powerhouses trading across its entire breadth. Think about that: 1000 years of prosperity! Trading partners inclusive of hundreds of different cultures. After the volcano, they rebuilt but were never the same and were likely absorbed by a rougher crowd from the Greek mainland (which is called the "Peloponnese"). I like to think of the Minoans as our literate ancient scientific ancestors, influencing the Greek culture even though they ceased to exist.

That "rougher crowd" were the Mycenaeans who evolved into the heroic Greeks of Homer's Iliad, made perhaps slightly more civilized by their Minoan acquisition. The centuries following were eventful and then blank: Iron-weapon-wielding northerners created chaos with the Mycenaeans and eventually initiated a multi-century dark age. What emerged around −800 included the still-standing Athens, Sparta and Corinth; the singing and eventual writing of the Homeric sagas; and an explosive emigrant population prominently on the Aegean islands, western Ionian

[1]Wait...Negative years? I'm sorry, but the timeline of history is linear with an origin (albeit, without a zero) on which one side is BC or BCE and the other, AD, or CE. Too clumsy. I prefer almost-straight-up arithmetic for years since it makes comparing one year with another a breeze.

shores, and the southern boot of Italy. Established by −650, these colonies were active traders, especially in Melitus in Ionia. Figure 1.1 shows the Greek colonial expanse and details of the immediate Aegean and Italian city-states.

Brief relative peace in the Ionian colonies might have led to a secure period, eventually suitable for growth of a new culture. And this was it: The beginning of western philosophy. The time of the "Presocratics," literally those early philosophers who came before (or overlapped with) Socrates. These folks and their "Postsocratics (?)" asked modern-sounding questions of their surroundings.[2]

I like to think of those long-gone cultured Minoans as the polite part of our western scientific ancestors—the smart side of the family. But the famously disagreeable, Homeric Greeks came from that side of the family that you'd like to hide from your friends.

The timeline in Figure 1.2 shows roughly three distinct periods with names you might recognize. There are the Presocratics (from about –600 to about –430), the classic philosophers (from about –430 to about –250), and then the Hellenistic philosophers and scientists (from about -250 to +165). Notice that each of these periods overlap with war: Greeks fighting Persians, Greeks fighting Greeks (after the Persian wars, an over-confident Athens precipitated a dozen conflicts with Corinth and Sparta until the major Peloponnesian war), Macedonians fighting Greeks, and Greeks fighting the rest of the Mediterranean and Middle East. Notice that the whole of western history since the Magna Carta in 1215 would fit within a tick mark and a half in that top timeline.

1.1.2 ACT II: Is Nature Made From Stuff Governed By Rules?

Over my career I've published hundreds of scientific articles. Every publication has a common element: a bibliography with references to dozens or even more than a hundred other scientific works. Science doesn't happen in isolation as we're constantly building on, disputing, or confirming work of other scientists.

Take out a piece of blank paper. In many ways what your looking at is the bibliography of the first western philosopher, or even proto-scientist: **Thales of Miletus** (ca –624 to –547). Plato and Aristotle (and neo-Platonic philosophers who came centuries later) tell stories of him which form a lot of what we know. The fellow who invented history, Herodotus, also is a source.[3] Thales left no first-hand writings but stories about him abound.

Here's one: my favorite *New Yorker* cartoon is a Robert Weber's 1981 image of professorial-looking, tweedy fellow with pipe on a NYC street corner asking a cop, "Excuse me, Officer. I'm an academic. Where am I?" That image of us academics didn't originate in a fancy magazine. Plato told the story that Thales was walking

[2]But the next century would see Ionia ruled by Persian-installed kings and tyrants.

[3]Herodotus was the first to tell about the past by trying to justify his assertions and find reasons for events. He's best known for his detailed history of the Greco-Persian Wars.

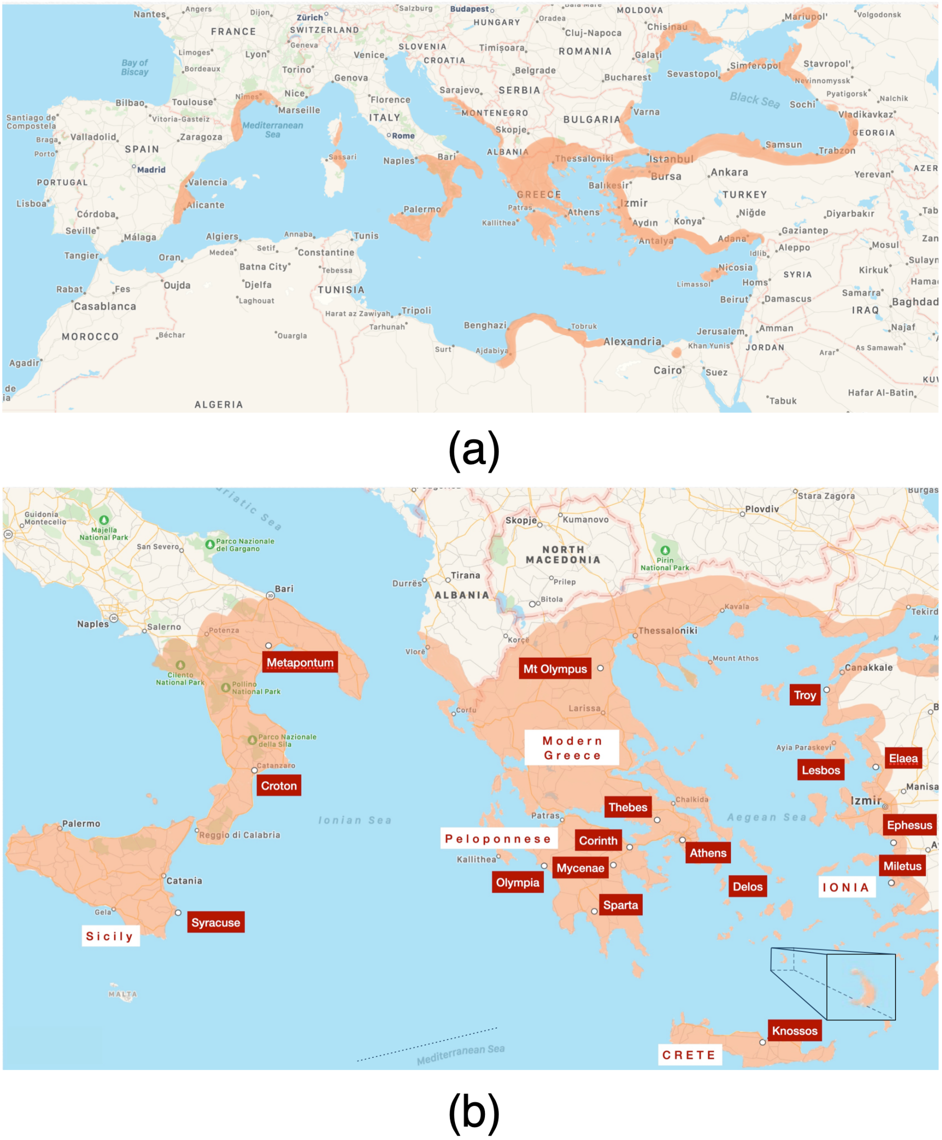

Figure 1.1: (a) The Greek colonial empire was vast, although I hesitate to use the word "empire" which implies cohesiveness since Greek colonies were only loosely connected to the mother ship. The Egyptian port (to be called), Alexandria became the final storehouse of Greek learning, outside of Baghdad. All of this came at a price. Greeks were almost constantly at war. (b) The regions around Home Base show the eastern Ionian and western Italian Greek cities where the Presocratics lived. The inset highlights the island of Santorini, the caldera left from the massive "Minoan Eruption" of approximately –1600.

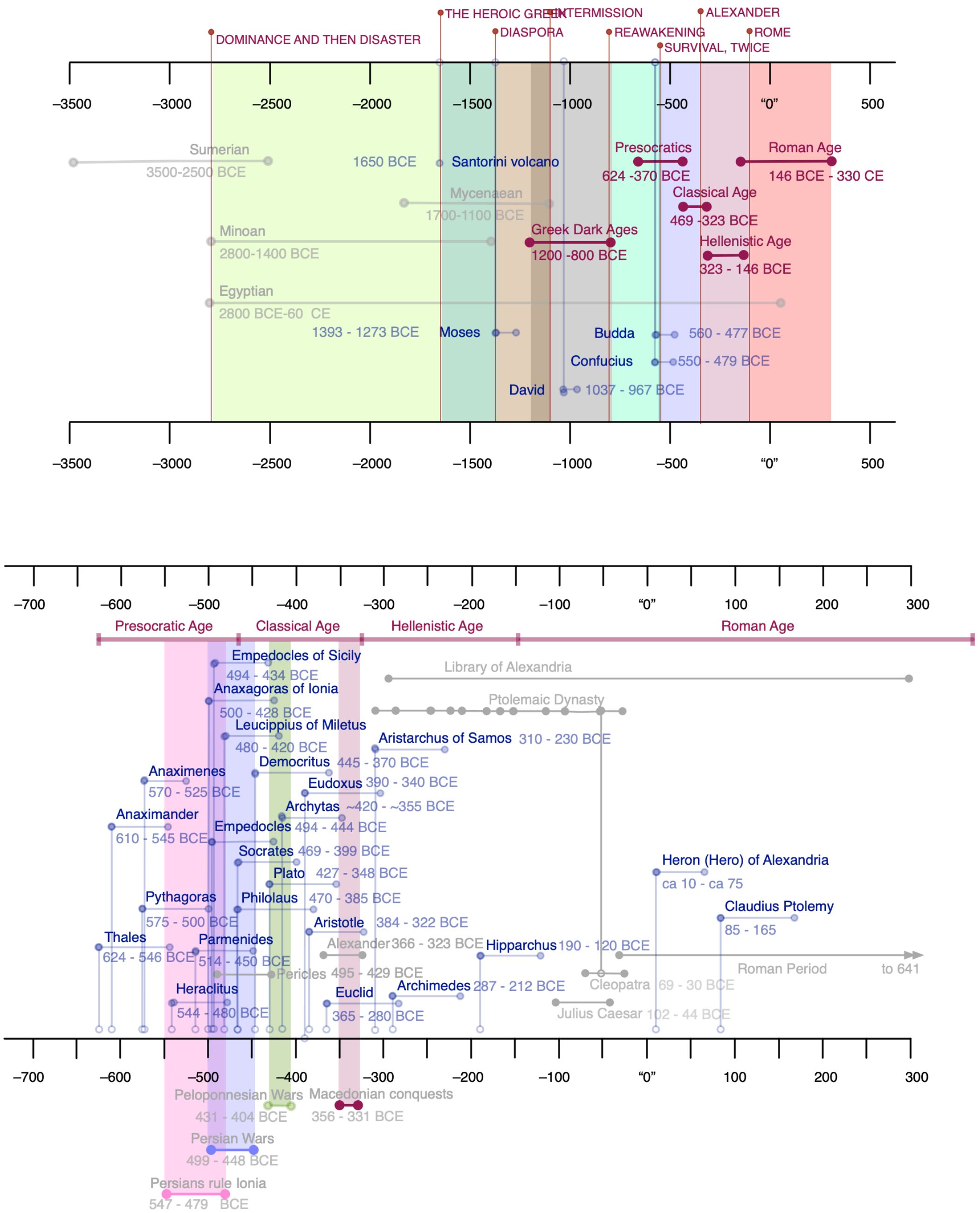

Figure 1.2: On the top, a Mediterranean timeline stretches from pre-biblical times to the end of the Roman empire. The bottom lays out the life spans of all of the Greeks you probably ever heard of...and the overlapping disasters that surrounded their lives.

along looking at the stars and deep in thought and dropped straight into a well that he didn't see in his path. That embarrassment wasn't enough, as Plato also notes that a passing servant girl was on-hand to make fun of him in his reduced state.[4] But we also know that he was savvy enough to predict some weather changes and a possible bumper olive crop so he bought up all of the olive-presses in Miletus, and made a fortune selling them back.[5]

Maybe that happened. Here's another. It was suggested by Herodotus that Thales studied in Egypt, learned geometry and astronomy sufficiently to be able to predict an eclipse of the Sun on (our dating) May 28, -585 that pretty much stunned everyone, including causing a battle to pause. How did he do that?

Well, he couldn't have. That didn't happen. Available data would not have allowed anyone to make such a prediction. It's trivial *now* to point back to the line of totality (the swath on Earth that would be dark) which would maybe have indeed been over the historical battle site, at that time. But a prediction? No.

Determining the veracity of stories like these is an example of a detective-story-approach to unraveling Thales and the other Presocratics: The eclipse fable suggests that Thales might have been an adult in -585 and thought by Herodotus to be a well-enough respected personage that his "predictions" might have mattered. So this story, while fiction, did contribute to the picture of the man called Thales and his reputation and his timeline. Thales was a real person.

What's not in dispute is that he initiated, or was a part of, a new way of asking questions *and* a new standard of what constitutes acceptable answers. Nobody thought like him and his immediate successors, and now we all do.

1.1.2.1 The World Before Thales & Co.

Why does it rain? Why are there earthquakes? Why are some people honest and others not? Why did my crop succeed and yours fail? Why is the Earth suspended under the sky? If you're Greek before about -500, there's a god for that. Why are there clouds? Yup, a god for that too. Why does the Sun shine? Another god. I tried to count all of the Greek gods, titans, minor deities, spirits, sea gods, agricultural gods, "rustic" gods, plus health and sleep gods. Oh, plus almost 30 mortals who earned promotion to god-like eternal life. It's hundreds. There is the varsity team—the 12 gods of Olympus and the 12 Titans. But the god-team bench is really deep.

Take earthquakes. Currently, Greece ranks fifth or sixth in propensity for seismic activity. So Greeks were used to their ground moving. What everyone knew was that earthquakes happen because Poseidon (the god of the sea) is irritated. Without

[4] Plato's references to the Presocratics are often to make fun of them.

[5] He was also an astronomer of note and a mathematician with theorems to his credit. An all-around academic.

enough attention, he bangs his trident on the ground from Olympus and they get an earthquake. Or rain. If water falls from the sky it's also the case that another petulant god is unhappy, this time Zeus (the god of a lot, including the weather) using his lightening bolt symbol to make trouble.

There's a madness to this, but also some sort of understandable urge to assign every human experience to an outside influence. While Homer's tales include the gods as major actors, it was Homer's contemporary, Hesiod who thought that the story of the gods needed a rational and believable origins story. His *Theogony* is basically the story of the origins of the world including the genealogy of the gods. There's also a cosmology in these myths: the gods are themselves born. . . they have not always been around. And they have lives—outrageous ones.

That's interesting. They could have just "been there" outside of time, like in other religions, but Greek myth seemed to require a logical structure: Earth (Gaia) and Sky (Ouranos) were the first and their union is followed by scenes from *Animal House*. . . no, much worse. Infanticide, incest, fratricide, cannibalism, mutilation, and betrayal follow among the gods and the Titans, and between them and regular humans. Murders are the most light-hearted events in Hesiod's story.

The bottom line of Greek mythology is that everything happens for a reason. Why? Because a god is benevolent or unhappy or just doing their job.

1.1.2.2 Thales' Science and His Successors

GREEK RESEARCH PROGRAM #1 : Thales ushers in the first Greek Research Program, that the world is made of some fundamental substance that behaves according to natural laws.

Thales was the first that we know of to take a different approach. He's best known for asking what is the underlying, common structure of the universe, what Aristotle called on his behalf, the First Cause.[6] Thales reasoned that all of our universe depended on a single substance, and for him that substance is: water. After all, without water or moisture, things perish. Water is in the air and condenses and wets surfaces. It evaporates and reappears, sometimes revealing (creating?) soil underneath. Nothing lives without water and when things die, they become dry. So as a single substance acting as the basis of all things, it's not too bad. This

[6] Aristotle uses that word. But Aristotle was fond of Aristotle's philosophy and his reliance on "Cause" and "Substance" in his own work, motivated his description of the Presocratics' work. Those words were not available to the early Presocratics.

description of the world is **materialistic** and **monist** (the view that there is one underlying substance).

This concept is the first of three novel features of Presocratic proto-scientific thinking.

1. Thales suggested that humans could understand how the world works—what causes the events and things that we experience—and that the world is made of fundamental stuff. Further, rules were postulated—laws of nature, so to speak—that govern how that stuff operates. The world needn't be a mystery.
2. Their "how" commitment searches for naturalistic reasons for events and existence. The previous "why" commitment was satisfied that "a god did it." For the "how" answers, the gods aren't involved. For example, the early Greeks inherited an ancient idea that the Earth is a flat disk with a dome of sky overhead, surrounded by a river (the Ocean or *Okeanos*) and the whole thing is held up by Atlas as a punishment handed out by Zeus. Thales agreed with the geographical part of this cosmology that the disk floats on water but earthquakes happen when the water sloshes. A wildly wrong explanation, but completely naturalistic. Poseidon is not involved.
3. Finally, the Presocratics jousted with one another: an idea or a research program from one, might be incorporated in another's account. Or, an idea or research program of one might be a focus of criticism resulting in an alternative account.

> This is not yet science, but science can't happen without at least these three commitments: **we can know about a rules-based universe**, **"how" cannot depend on the supernatural**, and **competition and collaboration are essential** to carry a project forward. All of this is new. And now familiar.

Others who came after Thales adopted the same "research program" hypothesizing and defending an underlying substance for the world. Thales' "A" students, **Anaximander** (ca −610 to −545) and **Anaximenes** (ca −570 to −525) asked that question and answered it in different ways, but with the same basic motivation. Each of them had their own underlying substance idea.

Anaximander gave us one of the first maps, perhaps the sundial, and a full cosmology including a cylindrical Earth floating at the center of the universe. He watched the stars go around us and concluded that the Earth can't be falling. . . so it must be balanced at the center of the cosmos.

> Here, is our first reasoned theory of MOTION, in particular MOTION BY THE EARTH. He concluded that there is no motion for the Earth, but for a reason. Because of symmetry and balance.

Anaximenes went a step further and realized that what's important is *process*—things turn into other things. Cycles happen. Lawlike behavior is evident. Neither

Anaximander nor Anaximenes went along with Thales' contention that water could be the sole source of stuff—how can water be the source of its opposite, fire? That's not the point, though! They rejected his specifics, but bought into the project: While Anaximander chose something etherial and not itself one of the substances (the spooky "Apeiron"), Anaximenes chose air as the fundamental substance, but he had a scheme whereby air's various guises could account for the actual things we experience.

By this point, proto-scientific practice is pretty much up and running. They were naturalists, materialists, and the first Empiricists—using their powers of observation to study their world and attempt to explain it without recourse to a deity or a dogma.

1.1.3 ACT III: Pythagoreans in the West

It must be exhausting being a philosopher in your day job while also moonlighting as a deity and yet **Pythagoras of Samos** (ca −582 to −497) seemed to function as both, or so his followers asserted. Yes, that Pythagoras: of the triangle, although it's probably not what you think. What Pythagoras taught and what evolved out of the long Pythagorean school is difficult to parse today so it's not fair to attribute all of "Pythagoreanism" to that one person. The ideas that are attributed to him originated in Italy but evolved considerably becoming a dispersed movement that spread throughout the Hellenic world and beyond to the Renaissance hundreds of years later. Indeed by Plato's time, Pythagoras was already an enigma. As we'll see, he probably learned about him through Philolaus of Croton and Archytas of Tarentum, two acknowledged second generation Pythagoreans and mathematicians in their own right. So we have a nearly mythical figure: In the near-term there was Pythagoras, "so-called Pythagoreans" (as Aristotle called them), and Pythagoreanism. . . the seed-philosophy of mathematics that has lasted in some form to the present day. I'll mostly use the plural "them" rather that the singular, "him." "Pythagoras" is essentially the name of a movement and a culture and no more reliably an individual.

His biographical details are from Roman-era writers and enthusiasts and it's difficult to know what's believable. He grew up in the Aegean island of Samos and reportedly met the elderly Thales, and maybe studied with both Anaximander and Anaximenes. So suggested Heraclitus, from whom we do have actual written (critical) fragments. He may have traveled around the Aegean with his merchant-marine father and probably on his own lived in Egypt and maybe Babylon for at least two decades, absorbing language, philosophy, and mathematics. So, a well-traveled, probably comfortable young intellectual. The politics of Samos became tenuous and in spite of the fact that he'd established a following of students, at the age of 40, he relocated to the large Greek city of Croton in the "instep" of the boot of Italy. Some accounts suggest that he was accompanied by a number of loyal followers—the Pied Piper of Samos?—but most suggest that he moved by himself.

In Italy he again established a following of reputedly as many as 600 (some say thousands) men and women in Italy and actually wielded some civic influence in Croton, serving as both an advisor and unwelcome busybody. He eventually founded a school that was to last 300 years, twice as long as my own Michigan State University has been around.[7] The ideas generated from that time evolved and the border between the man and the movement is impossible to demarcate today.

This unusual school also functioned as a mystical, essentially religious cult. Its members were regimented as to how to dress, what they could eat, what they may believe... and what secrets they must keep. They loved secrets. Pythagoras was its head and was by legend, supreme. He taught reincarnation and himself thought he remembered past lives.

Music was the basic discovery moment that established the underlying belief-system: that numbers and mathematics are a fundamental fabric of the universe. Although they were not in competition with the Ionians, reliance only on a substance-based first principle wasn't sufficient for them. Not given the discoveries that they believed they'd made about mathematics and the world:

> "All things have form, all things are form; and all forms can be defined by numbers." Pythagoras

> "The Pythagorean ... having been brought up in the study of mathematics, thought that things are numbers ... and that the whole cosmos is a scale and a number." Aristotle

1.1.3.1 The Most Durable Discovery in History

GREEK RESEARCH PROGRAM #2 : Pythagoras ushers in the second Greek Research Program, that the world is mathematical. Or even that the world is mathematics.

Pythagoras left no writings, but stories/fables/tales reported by dozens of others abound. He claimed (or it was claimed for him) to have discovered integer relationships among the strings of a lyre[8] and the pleasant chords it could make. The lyre was probably a 7-string variety although he reportedly built a one-stringed tool ("kanon" or "monochord") to study its behavior (although that story

[7] But both his and mine are mere babes, as compared with Oxford University, the University of Paris, or the Academy of Plato.

[8] and the tones from cups filled with different amounts of water which were noted for their pleasing sounds

is disputed). A quick taste of what the Pythagoreans left for us (and for Plato!):

When you pluck a string, clamped at the ends, you cause the string to vibrate with a fundamental frequency related to the length (and tension—think, a guitar). Call that the "ground note." (A Pythagorean scale is different from how a piano is tuned, but I'll use piano as my analogy.) A piano's middle C is a natural ground note and has a frequency of 261 Hertz (Hz, cycles per second). Pressing the lyre string at a half-way point and then plucking one of the two halves will cause the ground note to be repeated, but an octave higher. (On the piano, C above middle C is a frequency of 522 Hz, twice 261 Hz.) Pressing a lyre string at 2/3 of the length and plucking the long remaining string, causes the fifth above the ground to sound (for the ground of middle C, that would be G, or 392 Hz, 3/2 of middle C's frequency) and pressing 3/4 of the length, a fourth above that (A above middle C at 348 Hz, 4/3 times that of middle C's frequency).

Play those intervals on a lyre or chords on a modern piano and your ears will be happy. These are pleasant-sounding combinations while other combinations are not so sweet—we say dissonant. To the Pythagoreans, the difference between pleasant and dissonant was in the integer ratios of the string lengths—the *numbers themselves* were important.[9]

> To them, this revealed an **intimate link between numbers and the world**: integer ratios 2/1, 3/2, and 4/3 $\rightarrow$ to specific lyre string lengths $\rightarrow$ to pleasing your ear (your soul). This relationship made the numbers 1, 2, 3, and 4 very special to them.

Lyres had been around for millennia, so surely this particular discovery was not news. But what Pythagoreans did was new. They elevated numbers to a significance that's *beyond just counting*. They **invented the concept of number itself**: from 2 oranges to the abstract concept of "2." This direct connection between a few integer numbers, their ratios, and special numbers with important meanings[10] influenced all that's "scientific" up to the present day. This discovery is one of a few discontinuous events in the history of science. To them and many to the present day, mathematics is embedded in the physical world's tissue: A brand new commitment.

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[9] It's a matter of current physiological research to understand why some combinations of tones are pleasing and others are dissonant.

[10] Notwithstanding "42" as the numerical explanation of everything in *Hitchhiker's Guide to the Galaxy*

When it comes to Pythagoreans, who did what, when is murky. In the lower timeline of Figure 1.2 between Pythagoras and Plato you'll see **Philolaus of Croton** (ca -470 to -385) who was the first Pythagorean to write about their program, although only fragments and references from others remain. A lot of what Plato and Aristotle knew probably originated from his writings. (Plato only mentions "Pythagoras" and "Pythagorean" once each, but Aristotle was more expansive.) Plilolaus was a scholar in his own right and it's hard to discern what ideas were his and what came from Pythagoras himself, or even in Pythagoras' lifetime. What Plato and Aristotle knew of Pythagoreanism probably came from Philolaus or Archytas, another Pythagorean known well to Plato[a] Highly readable accounts are Ferguson (2008) and Lloyd (1970).

[a] And, what we know of Philolaus might have come from the Pythagorean, Hippasus. The most unlucky Pythagorean. He is remembered as having constructed bronze disks who's thicknesses matched the lyre string ratios. When struck they would then create the same pleasing sounds as strings. He's also historically the poor guy who found the non-integer problem with the Pythagorean Theorem. Stay tuned.

This connection between integers and one's soul seemed to have been just the beginning. They also connected numbers with shapes and so geometry and by extension, to space itself. Keep them in mind: 1, 2, 3, and 4.

What can you do with them? Well, you can add them: $1+2+3+4=10$ which is not such a complicated thing, but these are special numbers after all and so their combinations must be special as well: "10" was important.

You can also make patterns with numbers—and a highly useful definition of modern mathematics (especially in physics)—is that it's the process of finding patterns. Figure 1.3 shows examples of Pythagorean patterns with integer numbers.

Figure 1.3 (a) starts with one stone, and adds the first odd number, 3,[11] arranged around it turning $1+3$ into 4, *but it also laying them out as a pattern in space.* Numbers = geometry for the first time. This is a "square number" which follows the rule (in modern notation) of $1+3=4=2^2$. We can expand this into more square numbers and the next one is in Figure 1.3 (b) which shows that $1+3+5=9=3^2$. One can also take two stones and add the next even number around them in (c), say three above and one to the right, to get a "rectangular number."

Especially important is the arrangement shown in Figure 1.3 (d). Remember, $1,2,3,4$ are special. Lay out four stones, then layer three on top, then two, and finally one. You've now made a special triangle—-the tetraktys ("fourness")—with 4 stones on each of three sides. So it's an equilateral triangle and all four of the important numbers are contained in it... adding to 10. Maybe they liked bowling.[12]

[11] The number 1 was not a number for them: numbers meant a plurality. One is not "odd" nor is it "even." It's unique.

[12] There is a fable that a Pythagorean became ill at an inn while traveling but had no money to compensate the owner for his care while convalescing. The traveler told the owner to hang an image of the tetraktys and other Pythagorean travelers would compensate him far beyond his original costs.

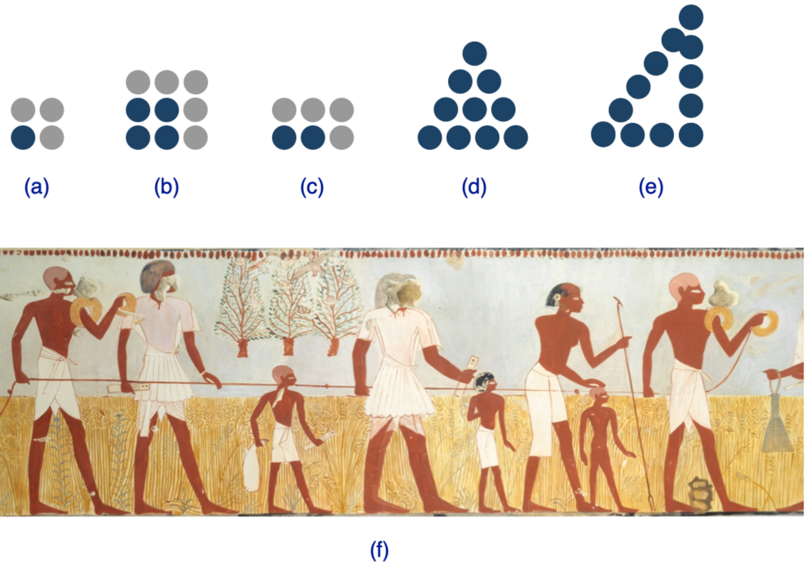

Figure 1.3: Dots represent stones that they would have used to signify numbers—precisely like the dots on dice. The image (f) is from the Tomb of Menna showing Egyptian workers getting ready to do surveying with a knotted rope. See the text for a description.

There's another connection between numbers and geometry—again, connected with the physical world. "1" was a special number, neither odd nor even (for them) and plays the role of a beginning. The source. A single isolated point is the starting point (no pun intended) for everything. "2" represents a line, which starts with a point and is constructed of points. "3" represents a triangle which delineates a flat plane and is constructed of lines, and "4" represents a tetrahedron, a three-dimensional solid constructed of triangles. That's it. Three dimensions to our physical space is all there is and so "4" represents completion and its encoding in the tetraktys (count the stones in any direction in the tetraktys and you'll count 1, 2, 3, and the base, 4) and that relationship with "10" tied it all together for them. (Of course today multidimensional spaces are a mathematical walk in the park. We know that our physical world consists of at least four dimensions. So stopping at "4" was premature!) There's more. "5" is special as it's the sum of the first even and first odd number. "6" is special since it's both the sum of the first three numbers and simultaneously, the product of the first three numbers. And so it goes.

Notice that there's another triangular pattern in (e). If you count the spaces between stones, you'll find that they delineate $3-4-5$ which is a familiar triangle to some of you, but a familiar triangle to thousands of years of Egyptian builders. This triad of numbers has practical value as it's a sure-fire way to make a right angle. Take a length of rope and tie 12 knots equally spaced from end to end (like the stones in the figure). Then have a worker hold one end, another hold

And they did. So it goes.

the third knot, and a third 4 more knots along. If the other end is then given to the first worker. The only way that making each of the three segments taunt is for there to be a right angle between the 3 and 4 knot segments. There are other such triads that make a right angle in this way, for example $6 - 8 - 10$. The ancient Babylonians and Egyptians knew of many of them and used them in surveying and building without realizing that this was an important thing. Figure 1.3 (f) is from the Tomb of Menna showing a knotted rope for surveying. As you know from high school, Pythagoreans figured out what this means in an abstract way.

There was a mystical quality to numbers and numerology was a thing and so the numbers also had special meanings for things beyond just "quantity." For example, 5 is the sum of the first even and odd numbers $2 + 3$ and since 2 symbolized female and 3 male, then 5 symbolized marriage. The first even number is 2 and squared is 4 and so that first square number, 4 symbolized *justice*. Likewise, the first odd number is 3 and its square is 9 and so it also symbolized *justice*. (Even today, we refer to a "square deal" as a proper deal.)

In fact, 10 was such an important number that in one version of Pythagoras' cosmology we have another early moment of MOTION BY THE EARTH. The Earth and all of the other celestial objects moved around something called the "central fire." This actually comes from Philolaus:

> "The first thing fitted together, the one in the center of the sphere, is called the hearth." Philolaus

The bodies are, from the inside-out, Earth, Moon, Sun, Venus, Mercury, Mars, Jupiter, Saturn, and the celestial sphere, but. . . wait. That adds up to 9. It must be 10 in order to be right, so they added the "counter-Earth" who's orbital mechanics are such to be perfectly positioned to block our view of the central fire since we don't see it.

> ". . . inasmuch as ten seemed to be the perfect number and to embrace the whole nature of numbers, they asserted that the number of bodies moving through the heavens were ten, and when only nine were visible, for the reason just stated they postulated the counter-earth as the tenth." Aristotle

That's a very modern interpretation of the use of mathematics in physics. You postulate the importance of a principle ("10 is magic"), you create a model of the universe (or some small part of it) built within the model, and then using the basic rules of the model (like arithmetic or something fancier) tweak it while still committing to the model. Here the counter-earth was such a tweak. That's actually how physicists work within models until they become unwieldy or are ruled out by experiment. I'll have more to say about a modern day view of Pythagoreanism *More of the Greek's Story* in Section 1.2.2. It's a matter of much discussion (a polite way of saying, "argument") today. It gets worse when we add Plato to the mix.

> The connection of music and integers led Pythagoras to assert that the regular harmonies of the cosmos were everywhere. The planets and stars all move and emit tones that ordinary humans can't hear since it forms a background to everyday noise. But Pythagoras? Ah, he's different. He's the only human who can hear the Harmonies of the Spheres. Being a deity has its privileges.

You're wondering about that theorem, I know you are. Look at Figure 1.4 and relive high school for a moment. Notice that Figure 1.4 (b) is the knot/stones-version of the Egyptian right-angle trick.

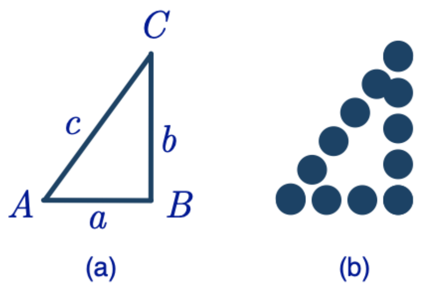

Figure 1.4: In (a) is a modern-day version of drawing a triangle, while in (b) is the same thing but with stones or knots delineating distances. The length c is the hypothenuse.

Maybe you remember the little song for a right-angled triangle: "…the square of the hypotenuse is equal to the sum of squares of the other two sides."

Or less lyrically,

$$c^2 = a^2 + b^2.$$

> There's no evidence that Pythagoras first proved this, and in fact, plenty of evidence that it was long known before him. (There are now about a hundred different proofs of the "Pythagorean Theorem.") The Egyptians had a real estate problem to solve: the Nile overflowed its banks every year and the fertile crop land alongside of it would be covered with water. That meant a problem: once the water receded, whose land was whose? Out of a need, geometry for Egyptians was a necessity. This was another job for the practical $3^2 + 4^2 = 5^2$. But the Babylonians were the champs. Not only did they keep accounting records, they did so in a base-60 number system...which must be 6 times better than our base-10 system, right? We've fragments that showed that they had worked out things like $119^2 + 120^2 = 169^2$ which admittedly doesn't come up every day.

There is a Pythagorean-Theorem story that tells you much of what you need to know about his cult. Remember, integers were the thing and so we feel sorry for the poor guy (historically, maybe Hippasus) who noted that a triangle with legs of 1 would have an hypotenuse that's Pythagorean-impossible since $1^2 + 1^2 =$

$(\sqrt{2})^2$. This $\sqrt{2} = 1.4142135624\ldots$[13] never ends—the definition of an "irrational number"—it goes on forever and so decidedly not one of the mandated integers. Since he'd found a non-integer, for his trouble, as the story goes, he was thrown overboard from a ship in order that his little discovery not be revealed to the other cult members. Maybe this happened.

In the end, as sometimes happens with cults, Pythagoras' welcome in Croton wore out. His house was burned and he escaped, only to die in his escape... or not. We don't know. But what he and his colleagues created lived far beyond them.

1.1.4 ACT IV: The Eleatics in the West

What happened next unsettled the young enterprise of philosophy and, after Plato and Aristotle, initiated millennia of philosophical controversy. We saw that the Ionians relied on their senses and took it for granted that events in the world changed in time. But you and I have both learned that our senses can be tricky and not always accurate. And, even if we see/hear/feel accurately, the targets of our perception themselves change. So if that's the case, then what about our "scientific" observations? Can we trust our senses to gather accurate impressions of our surroundings and base our theories on those impressions? This investigation traditionally pits two Presocratics against one another, the "Riddler" of Philosophy, **Heraclitus of Ephesus** (ca −540 to −480) and the first "Lawyer" of Philosophy, **Parmenides of Elea** (ca −514 to −450). The former was an Ionian from the big city of Ephesus, not far from Miletus. The latter, was from the colony of Elea in southern Italy. Pythagoras' territory.

Heraclitus was a loner, while Parmenides evolved a school of philosophy called the "Eleatics." You might not have heard of that, but you may recognize one of Parmenides' significant followers: Zeno... of Achilles and the Tortoise fame. The Heraclitus (by himself) and Parmenides and his followers took up the subject of change. Heraclitus was decidedly on the side of, sure, things change. But he took it in an abstract direction. On the other side, Parmenides concluded that change is an illusion. He even *proved* that change is an illusion. At first glance, that seems strange, but his novel method of philosophizing was persuasive and as a consequence he created two branches of philosophy. And in the course of digging into the problematic nature of Change, set off a huge argument over centuries. Obviously, this is prior to any kind of physics-like analysis of MOTION!

GREEK RESEARCH PROGRAM #3a : The Problem: Tension between Change versus Permanence begins with Heraclitus and Paremenides.

[13]"dot dot dot," ... is mathematics-speak for "never ends."

1.1.4.1 The Riddler

Although we know few details of Heraclitus' life, he was apparently prominent in Ephesus. His father was said to have been an aristocrat, but Ionia was under Persian control during his life and suggestions that Heraclitus might consider a political life might be hard to picture. He wasn't a people-person. He would have been a child when Anaximenes died but he was critical of the Milesians and scathing in his criticism of his contemporary, Pythagoras. About 100 fragments of Heraclitus' work remain showing that his style was... unusual. He wrote very short tweets which have puzzled and delighted readers for thousands of years.

He was a monist as well: fire was his fundamental substance. And as interpreted by one of his aphorisms, he had a cosmology,

> "This world-order [*kosmos*], the same of all, no god nor man did create, but it ever was and is and will be: everliving fire, kindling in measures and being quenched in measures." Heraclitus

This is the first time that the word "cosmos" appears in Greek philosophy and he's clearly insisting that the cosmos always was the case and always will be the case. That's interesting since Plato deliberately labeled him inaccurately as naively saying that "everything changes" and that nothing is permanent.

You and I think of MOTION ON THE EARTH as moving from one place to another during some time, right? Remember, the Greeks were just beginning to do this analysis and moving from place to place was not their primary concern. Change by itself was and Heraclitus was the first to abstract *any* change as basically a form of motion, seeming to assert that universal "flux" was an important feature in the world.

> "It is not possible to step twice into the same river." Heraclitus

This is a famous paraphrase of a translation of his most famous of three "river aphorisms," The idea is that the river is always flowing and if you step into "the river" once, and then step into it a second time, it's a different river. So two rivers sort of functioning at the same time. It's a little different from this one:[14]

> "As the same thing in us are living and dead, waking and sleeping, young and old. For these things having changed around are those, and those in turn having changed around are these." Heraclitus

A young person is connected to their older self through the changes that they undergo. A is different from B, but linked because *A changes into B*. But, living and dead? This is a deep idea and seems to suggest that A and its opposite, B, are

[14] While the most famous Heraclitus aphorism, there are at least three versions of it and some dispute as to its overall authenticity.

actually the same thing. In fact Change here has a job: it's a sort of glue that links together different things or different aspects of a thing. So apparent opposites are connected and so everything in the world is connected. One.

Plato used Heraclitus as a punching bag. He said look what Heraclitus gives us: logical contradictions! Plato had an agenda. Aristotle was a little more forgiving and we'll see how Aristotle codified and categorized change, which will explicitly include our notion of loco-motion. But it seems that he had to go through Heraclitus to get there.

It's easy to be amused by Heraclitus' words, and for millennia, that's been a sport and I have more for you in *More of the Greek's Story* in Section 1.2.1 below.

1.1.4.2 Nothing Gets Done: The Parmenides Problem

Parmenides took the extreme, opposite position, probably writing after Heraclitus. His argumentation is tightly logical so much so that it's possible to be swayed by the apparent inevitability of his arguments. If you can penetrate the denseness of it. I'll call his oddly persuasive but troubling conclusions the **Parmenides Problem**. It will seem to us like the Parmenides Problem will not go away.

He is the first in a long line of philosophers of both Metaphysics (the philosophy of the nature of being) and Epistemology (the philosophy of knowledge). He wrote a single book in verse (and according to Aristotle, not very well). It's a narrative story about his meeting with a goddess and how she teaches him about two kinds of knowledge.

There is the "first path" to knowledge: knowledge that is true by necessity. This "Way of Truth" is confined to your reasoning, not your senses. The second path to knowledge, that of perception, is "habit" and from "your heedless eye." This "Way of Seeming" is needed in order to get along in the world, but you can't trust it because you can be fooled. For that reason the "Way of Seeming" can't tell you what is true. So:

> True means permanent. Change means not true.

Remember our own experiences: our senses can fool us and the objects of our perceptions can evolve between observations. What can you trust in the world if not your eyes? So he got rid of both issues. Truth can only refer to permanent things.

Accepting his premises, his logic seems oddly persuasive. In a nutshell, which could be on a T-shirt, I can sum up Parmenides in his two words (read it carefully... if nobody's around maybe even read this out loud): "**It is**." It's punchy. He also then reasons that "**It is** and it cannot, **not be**." *It cannot... **not be***. If something **is**, it can't be **not-is** at the same time. Further, if something **exists**, then **it is**.

Consequently, if it **doesn't exist**, then it is **not-is**. So knowing what **is**, is to know what **exists**. So far, so good. Something can't exist and not exist simultaneously. (Can you see how this is against Heraclitus, who seemed to welcome *A* and not-*A* simultaneously?)

He goes further. If something exists (it **is**) then also it could never have been different in the past, nor will it be different in the future. If it came into existence as **is**, then before that event it must have been: **not-is**. If it changes into something else in the future, then it goes from being **is** to then being **not**-is. How can something at one time be **not-is** and at another time be **is**? That can't happen! So if something **is**, it's always been **is**. In some sense, then the past and the present are one. Whew. Are you with me?

He's staked out clever ground in two new ways: His approach so logical that it launched philosophical analysis as an appropriate way to make arguments. And, he's defined what it is to be real: what's real must be true and therefore, it must be unchanging. And the landscape of truth is in your head. Where you reason.

Parmenides' sidekicks ran with this. Zeno took his arguments to the extreme and that's our connection with MOTION. Maybe you remember the story of how Achilles couldn't beat a tortoise in a race?

> This is one of 10 of "Zeno's Paradoxes," **The Achilles**. Achilles, being the fastest human, is to race a tortoise, maybe the slowest animal, so he gives the tortoise a head start, halfway to the finish line. They both start but poor Achilles is faced with an impossible task. In order to traverse half of the distance to the tortoise's staring point, he has to traverse half of that half. Then half again of *that* half. In fact he needs to travel through an infinite number of paths, which is impossible. So not only can he not beat at tortoise in a race, he can't move at all (and neither can the tortoise)! There are other three other paradoxes on motion (The Dichotomy, The Arrow and The Stadium), all designed to support Parmenidean conclusions about motion.

Zeno gets this from Parmenides and since the reasoning seemed to be impenetrable, all of those races that you've seen with your lyin'eyes were apparently fooling you. I touch on two others of Zeno's Paradoxes in *More of the Greek's Story* in Section 1.2.3 below.

> We've now encountered examples of significant philosophical or scientific commitments. Sides were beginning to be drawn in natural philosophy that continue to this day: Can knowledge about the world be gained by thinking? Or must knowledge come from observation. The former is called Rationalism and the latter, Empiricism and to this day physicists argue about this. Clearly Pythagoras is in the first camp and so was Parmenides—distrust of the senses disqualified observation as a source of truth. And, geometrical argument seems like a good example of what must be true. The Ionians pioneered the second camp gleaning knowledge and theories about the universe by looking and hypothesizing from their observations.

Finally, the void. The vacuum. A state of actual nothing! By now you can imagine what Parmenides thinks of such an idea: it's impossible since it's the state of **non-being**. Another Eleatic, Melissus took this to the ultimate conclusion without the need of Zeno-like paradoxes. Just logic: anything that **is** cannot move since it would need a place to move to— it would need an open space where **nothing is** in order to relocate. But a place where **nothing is**... is nothing. But nothing can't be the case so there is no motion. Another MOTION problem.

Parmenides was the first to seriously question what can be known and by what means. Your senses deceive you all the time. You can't depend on your observations for truth. But at the same time, your rational, logical thought—an argument assembled before Aristotle invented the actual rules of logic—is dependable. He then laid out a dispassionate argument that leaves one wondering what in the world is wrong with it.

> How do we get around this? In order to do science, or frankly, to live, one has to be able to hold an idea in your head that's tentative, hypothetical. Something less than demonstrably true. In fact, a scientist never deals in truth, but in degrees of trust of a theory. But Parmenides was worried about Truth with a capital "T" and so he can't abide an idea that is not true and so his philosophy was sterile.

Well, this is embarrassing. My project here is an account of MOTION and now we've just encountered what seems to be a persuasive argument that MOTION is impossible. That's not progress, is it.

The Parmenides Problem is an important stepping-off point for Plato.

1.1.5 ACT V: Antidotes to Parmenides?

Parmenides' arguments were unsettling. The notion of a tightly logical argument was brand new, and yet even if its conclusions seemed nonsensical, you've got to struggle to find holes in his reasoning. But that didn't stop four intrepid souls. We still call them "Presocratics" but really they were "Co-socratics" since they all lived around the time of Plato's mentor. They're our last stop before Plato.

GREEK RESEARCH PROGRAM #3b : Attempts at solutions: Back to Monism for solutions to the Parmenides Problem?

1.1.5.1 Empedocles and Anaxagoras

One philosophical god was apparently not enough. **Empedocles of Sicily** (–494 to maybe –434) was another self-appointed deity. He was a contemporary to the Ionian, **Anaxagoras of Ionia** (–500 to maybe –428) who had a similar solution to the Parmenides Problem. Both took the position that the world is made of multiple entities and that *those* entities are what's permanent, but their *combinations* are multitude and accommodate change. In some ways, a modern approach.

Empedocles was a character. He dressed in a purple robe, with wreaths around his neck. He claimed to have performed miracles, raising folks from the dead, curing illness, and so on. He claimed to have been reincarnated from previous lives as a bird, a fish, a girl, a bush. . . His brand was very Pythagorean he worked in that same part of the Greek confederacy as the still functioning Pythagorean society, so there must have been some infuence. He famously wore bronze-soled shoes everywhere. They figure into his legendary ascendence at the end. He was supposed to have leaped into the active volcano at Etna and disappeared but one of those distinctive shoes was left behind casting doubt on that last miracle.

We only have fragments from Empedocles who wrote in verse, as seemed to be the custom in the west. It is from him that we get the familiar **Earth, Air, Fire, and Water** as basic elements (he called them "roots"). We will see that Plato and Aristotle took hold of this idea and ran with it all the way to Galileo's time. These four roots accommodate change by mixing with one another driven by two opposing forces, "Love" and "Strife." Again, a simplistic but modern-sounding notion of fundamental forces acting on the basic constituents of matter.

This is inspired. The roots are indivisible and have always existed, as have the two "forces" of Love (an attractive force) and Strife (a repulsive force). He also agreed that no-thing can come from nothing. So, check the Parmenides permanence and not-nothing boxes. But he accommodates our senses, while warning of their fragility. What we observe is that things in our world are different from one another and that there are many of them. Some rocks are hard and some rocks are brittle. They're both rocks, so how do we build our observed rocks with only four roots? His contribution was that everything we observe is constructed of varying *proportions of the roots*. All rocks might be made of the same combinations of the roots, but a hard rock would have more of the Earth root than the brittle rock. With infinitely mixing proportions of the four roots, you can make the variety of the world. Sounds a little like a proto-chemistry.

Empedocles insisted that there was no purpose to the universe and that we're all subject to chance, postulating that we actually live in an undulating, repetitive cycle of a spherical universe in which Love and Strife compete for dominance.

His contemporary, Anaxagoras was from the other side of the West-East divide. He was an Ionian who ended up in Athens, establishing the first of a long string of Athenian philosophers. His arrival came during the classical period when the architecture; sculpture; literature; and yes, philosophy that we think of when we think "Greek" began.

Rather than only four substances, Anaxagoras presumes as many elements as there are things. Things... are themselves infinitely divisible. How do you acquire hair and bones? Well you eat foods that contain elements of... hair and bones. Everything is in everything. He insisted that the senses give us a window or a picture into aspects of reality that are not directly observable, but nonetheless, existing. Again, another modern idea from one of our "Co-socratics."

Notice that neither of our two characters explicitly address the issue of locomotion. This is a confusion that Aristotle promulgates, as we'll see. "Change" *per se* is broader than a thing moving from one place at one time to another place at a later time. So as you'll see in *More of the Greek's Story* Section 1.2.3 while Zeno works on that problem, he starts with the presumption that change is not possible and so locomotion is logically not possible and hence the paradoxes that try to persuade you that motion is not possible. Our next two "Co-socratics" do find a way to explain locomotion which again, Aristotle rejects out of hand.

1.1.5.2 Atoms

I'll bet that you first learned the origin of the word "atom" in elementary school. "*Atomon*" is Greek for indivisible and the origin of that idea was again, the anxious need to find a way around the Parmenides Problem. You probably also learned that the inventor of atomism was **Democritus of Abdera** (about –445 to –370), originally from a region that's closer to Macedonia than it is to Athens, so a northerner. Here are three interesting things about Democritus. First, we classify him as a Presocratic, but that's really a misnomer. He's a "Post-socratic," younger than Socrates by more than 20 years. Secondly, he didn't invent the idea of atoms. He inherited it from **Leucippus of Miletus** (about –480 to –420). Finally, Plato doesn't mention him! He apparently burned Democritus' books. Aristotle knew him very well, maybe because of their shared northern roots.

Obviously, the idea of atoms is an idea with legs, albeit with ups and downs over the next two millennia, usually, unwelcome and only accepted when Einstein found two ways to demonstrate that there are indeed invisible chunks of matter. (That's a story that's not our current Einstein focus, but a large part of his miraculous 1905 year.)

However, the atoms (typically a mixture of Leucippus and Democritus' contributions) of classical Greece and our idea of atoms are very different. First, there are an infinite number of Greek atoms of all possible shapes. Some have hooks and can attach to others (think velcro), while some pairs have shapes that fit together. They move around and bounce off of one another, or they cling to one another forming compounds that eventually become the substances that we're familiar with. We know of them because of the sensible qualities that they bring to object we can deal with using... our senses. For example, things that taste sweet are composed of smooth atoms while things that are acidic are composed of sharp-edged, angular atoms.

How is this an antidote for the Parmenides Problem? First, the atoms are permanent but second they are constantly in motion and all change is due to their arrangements, and re-arrangements.

But the real way in which this works is that both atomists insist that what's real are atoms... *and the void*. The void is the place where moving things can go *to*. So locomotion is possible. There. That does it for Parmenides. So, the atomists are happy to make room (so to speak) for MOTION ON THE EARTH.

> The void is an unpopular idea, and to this day we continually redefine what the vacuum is (or isn't). Our current understanding, again, my scientific playground, is that there is no place in the universe where there is nothing. The vacuum is full, but it's a quantum mechanical fullness that has no connection to any ideas before about 1950.

But, as I said, Plato ignored this singular, logical conclusion to the Parmenides Problem, which seems a cowardly way of dealing with an idea. As we'll see, Aristotle could not abide the void so he's no atomist either.

There's one more interesting fact about this pair's ideas and that's an idea that Plato would embrace, but with only partial credit to the right people. Everyday objects are not real things, and the attributes that we ascribe to visible, touchable, tasty, smelly, and loud objects of our sensible world are all based on convention. Democritus wrote:

> "By convention sweet and by convention bitter, by convention hot, by convention cold, by convention color; but in reality atoms and void."

Even though we can't see atoms, we know they're there because our minds tell us about what we can't see. A reality that's beyond our senses. Now this is a very modern idea and also a very Plato-idea and we'll see it emerge in a slightly different guise when we talk about Galileo and how he invented physics when he used this notion—now labeled "Platonic," but could be labeled Democritus-ian.

1.1.6 What's Important For Our Project

Our project is about motion and light. Does it make any sense to speak of either of them without numbers? Motion implies speed, immediately bringing to mind numbers: miles per hour, for example. Light involves brightness and color... qualities that we can describe using words, but they're a stand-in for actual numbers as well: you'd evaluate a lightbulb's brightness by "lumens" which is a number and "red" is a name for the frequency of light.

This is so much a part of our thinking now, that it almost seems trivial to mention it. Wouldn't it seem odd to think in any other way for almost everything, from cooking to taking a pain reliever to deciding when to buy new tires? Attaching numbers to the physical world is a gift of the Presocratics and in particular, the Pythagoreans. Trivial or not, before the Pythagoreans, numbers as more than just counting would have been a foreign concept, after them, well, numbers are *in* everything.

But their gifts were generous beyond just this. Lets quickly summarize what the Pre-, Co-, and Post-socratics have brought to the scientific table.

The invention of the scientific commitments that we use today came from them:

1. They eliminated the supernatural as an acceptable argument for why things in the world happen. We can know about the physical world.
2. They conceived of the notion that the universe is made of naturalistic stuff: the water, aperion, air first-guesses, to more intricate and even modern-sounding permanent entities that go together in proportions to build the stuff we experience.
   (a) They toyed with the idea that these entities had to obey rules that allowed for their interactions, and in some cases, motions.
3. They invented the notion that mathematics is tied both to geometry and to things in the world, essentially birthing modern mathematics. We literally have no other way to describe and predict the properties and behavior of the physics world.

They committed themselves to the idea that truth allowed for no compromise: truth is eternal, unchanging, and in conflict with our senses. That created all manner of consternation from what I've called the Parmenides Problem. It's also wholly incompatible with modern science, so this was a dead-end... except the struggle to come to grips with the Parmenides Problem led to Plato and the atomists. That's not unusual: a bad idea can often lead to good ideas in the quest to kill the bad idea! We'll see that over and over in this book.

4. Some Greeks realized that learning about the universe involved seeing, touching, and hearing what the universe of things does. But our senses are unreliable and so couldn't reliably deliver truth, if "truth" meant "permanent" and "never changing." Since they questioned the reliability of sense data

as a source of knowledge by demanding that things that are True must be unchanging, they squarely set up the problem that Change itself is problematic. They should have questioned that all-or-nothing definition of truth! Taking a page from their high school geometry class, mathematics become a pretty good model of what is constant and true. But we only can deal with geometry at the level necessary through reason. So: don't look at the world, *think* about the world. That's what I've called the Parmenides Problem: is change really an illusion?

5. Reactions to the Parmenides Problem led to at least two directions: primary substances mixed in proportion, Earth, Water, Air, and Fire or atoms.

And, proto-science, and now science as we know it, is a social activity.

6. They argued. One philosopher added to or reacted to what another said. This created the necessary social structure and behavior necessary to support the scientific enterprise.

We're now ready for Plato.

1.2 More of the Presocratic Greeks' Stories

1.2.1 Tweeting With Heraclitus

Heraclitus is challenging because he's tough to analyze and because the available material is... pithy. The general view is that he really did write in these short aphorisms and that they aren't somehow surviving snippets of something larger.

The most famous of them, that tends to support his historical brand that "everything changes" is the River Analogy. The most famous version is due to Plato's rendition which he wrote in *Cratylus*:

> "Heraclitus, I believe, says that all things pass and nothing stays, and comparing existing things to the flow of a river, he says you could not step twice into the same river." Plato

But there are actually three versions of the river tweet:

> "On those stepping into rivers staying the same other and other waters flow." *Cleanthes*, a Greek Stoic from two centuries after Heraclitus' life and almost a contemporary of Plato

> "Into the same rivers we step and do not step, we are and are not." *Heraclitus Homericus*, a commentator from 500 years after Heraclitus' life

> "It is not possible to step twice into the same river according to Heraclitus, or to come into contact twice with a mortal being in the same state." Plutarch, from the Renaissance

The first is probably the most likely and doesn't contradict the more popular version. However, this story illustrates the difficulty, once again, of the detective work involved in assigning credit (or blame) to the Presocratics.

I mentioned that he wasn't a people-person, probably unsuited for political leadership (notice the disdain for his Italian contemporary, Pythagoras):

> "One is worth ten thousand to me, if he is the best."
>
> "Eyes and ears are poor witnesses to people if they have uncultured souls."
>
> "War is the mother of everything."
>
> "The best of men choose one thing in preference to all else, immortal glory in preference to mortal good; whereas the masses simply glut themselves like cattle."
>
> "It is not good for men to get all that they wish to get."
>
> "What sense or thought do they have? They follow the popular singers, and they take the crowd as their teacher."
>
> "Learning many things does not teach understanding. Else it would have taught Hesiod and Pythagoras, as well as Xenophanes and Hecataeus."
>
> "Poor witnesses for men are the eyes and ears of those who have barbarian souls."
>
> "The adult citizens of Ephesus should hang themselves, every one, and leave the city to children, since they have banished Hermodorus, a man pre-eminent among them, saying, Let no one stand out among us; or let him stand out elsewhere among others."

His unity of opposites appears in multiple places:

> "Sea is the purest and most polluted water: for fish drinkable and healthy, for men undrinkable and harmful."
>
> "Collections: wholes and not wholes; brought together, pulled apart; sung in unison, sung in conflict; from all things one and from one all things."
>
> "Every pair of contraries is somewhere coinstantiated; and every object coinstantiates at least one pair of contraries."
>
> "Good and ill are one."

But, he's also inspirational:

> "Nature loves to hide."
>
> "Sound thinking is the greatest virtue and wisdom: to speak the truth and to act on the basis of an understanding of the nature of things."
>
> "Abundance of knowledge does not teach men to be wise."
>
> "This world-order [kosmos], the same of all, no god nor man did create, but it ever was and is and will be: everliving fire, kindling in measures and being quenched in measures."
>
> "The character of man is his guardian spirit."

> "The sun is new every day."

. . . and amusing:

> "And they pray to these images, as if one were to talk with a man's house, knowing not what gods or heroes are."
>
> "Souls smell in Hell."
>
> "Every beast is driven to the pasture with blows."
>
> "Asses would rather have straw than gold."

1.2.2 Modern Day Pythagoreans

Want to liven a party? Raise the following question:

1. Is mathematics invented? Or,
2. Is mathematics discovered?

That is, are the theories, proofs, and concepts of mathematics the creation of human thought, or are they "out there" waiting to be revealed by thinking about them? Platonists (and by my reading, not Pythagoreans) would rally around #2.[15] Pythagoreanism would argue that the concepts, numerical relationships, and equations of mathematics are embedded inside of physical reality and that we discover them by investigating the world.

Okay, now if you want to rejuvenate your now yawning party-goers, narrow the question to:

3. Is mathematics invented in order to explain the physical universe? Or,
4. Is mathematics discovered to be already "in" the physical universe?

Number 3 suggests that mathematics is only a tool —a language—to describe the universe. It's a lucky break that we've invented it and that it seems to do pretty well. Perhaps another tool might have worked? For example, a musical score for guitar could be represented by standard musical notation. But it can also be represented by chord diagrams. The use of mathematics doesn't come with a guarantee that it's an essential part of reality. Many of us would subscribe to #3.

Number 4 suggests that the discovery of mathematical relationships and their match to what we observe in the universe represents an uncovering of its fundamental mathematical fabric. Here, Pythagoreans do find a place: their discovery was that #4 is how it goes. Numbers (and in modern language, equations and patterns) are *in* physical objects.

[15] Want to start an argument? Try to defend any definition of what Pythagoreanism is. (You can also spice up the conversation by trying to defend what Platonism is, which is the next chapter.) I'm not a philosopher, but I do have a sense of how my interpretation of these two ideas fits my experience in modern physics research.

I think most rough and ready physicists would lean towards #3, but not everyone. I'm close to #4, but in a practical and not spooky way. (Some of my contemporaries are okay with spookiness when it comes to math and reality.) So maybe I'm not #3 or #4... maybe #3.5?

We owe a debt to Pythagoreans and while their application of "number" to the world is primitive, there are vestiges of their discovery that make science (and modern life) possible.

Here is my rendition of Modern Pythagoreanism (MPy):

MPy_1 The physical universe is uniquely characterized by mathematics.
MPy_2 There are functions, particular numbers, and ratios that are important features in the physical universe.They are accessible to experiment and mathematical, logical, and physical reasoning.
MPy_3 Integers play an important role in the physical universe.
MPy_4 Patterns, symmetries, and geometry play structural and causal roles in the universe.
MPy_5 The mathematical relationships and values may be characterized approximately and computationally modeled algorithmically.

MPy_1 Mathematics describes the universe There is this nagging feeling about math being pretty special. Before the advent of Pythagoreanism, we saw that the Ionian approach to parting ways with deities was to ascribe a fundamental "stuff" as the basis of all physical things. Now, we don't depend only on that. We use math.

Take the weather. Before Pythagoreanism took hold, numbers meant "one apple," "two apples," and so on. Counting and nothing more. Before Pythagoras, I think that describing the weather using numbers might have seemed as strange as saying that the weather is "happy." While the ancient Pythagoreans didn't use numbers in most of the ways that we do, they wouldn't be surprised for us to be comfortable with a mathematical interpretation of the weather. We have learned to completely describe the weather using them and convey more information than with words. I just looked at the weather in Pythagoras' modern Crotone in Italy and it's not happy: it's 22° C (79° F), with a relative humidity of 76% and since the dew point is 71°, that's uncomfortable. The barometric pressure is 1016 mb and rising and with a cloud cover of only 11%, and so visibility is 10 miles. This short narrative puts a picture in your mind of the weather conditions that words would do much less efficiently or accurately. I could take those numbers and recreate exactly those conditions in a lab. These numbers are a logical and rational account for us and that's due to our Pythagorean inheritance.

Aristotle criticized Pythagoreans for assigning numbers as a *cause* of things and few of us would go there, but one can see where they might think so. I live in Michigan and if I want to know what the weather is going to be tomorrow, I'll simply look to see what the weather in Chicago is today. Does Chicago weather cause Michigan weather? Not strictly. But there is an "if this, then that" predictive relationship that I can use to anticipate tomorrow's day.

MIT cosmologist, Max Tegmark has taken this idea that mathematics is in a reality of its own— he holds a very strong #4 commitment and would say that the numbers in our story aren't just *in* the weather, they *are the weather*. That is, if there's a one-to-one correspondence between a number and my interpretation of what the number means, then they're the same.

A taste from his controversial book, regarding the electric field:

> "If you can thus pair up every entity in our external physical reality with a corresponding one in a mathematical structure ('This electric-field strength here in physical space corresponds to this number in the mathematical structure,' for example), *then our external physical reality meets the definition of being a mathematical structure* (emphasis, mine)—indeed, that same mathematical structure. Tegmark (2014), page 280

That he's under attack suggests that physicists do have strong opinions about #3 versus #4, as much as they'd probably outwardly profess disinterest.

> "So the bottom line is that if you believe in an external reality independent of humans, then you must also believe that our physical reality is a mathematical structure. Nothing else has a baggage-free description. In other words, we all live in a gigantic mathematical object—one that's more elaborate than a dodecahedron, and probably also more complex than objects with intimidating names such as Calabi-Yau manifolds, tensor bundles and Hilbert spaces, which appear in today's most advanced physics theories. *Everything in our world is purely mathematical—including you.* (my emphasis)" Tegmark (2014), page 260

Or, in his technical publication Tegmark (1998),

> "Physical existence is equivalent to mathematical existence."

I've heard him ask what is a tree. To most it's a barky, green, leafy structure with roots and a hardness and so on. To him it's a collection of electrons and quarks and reflecting and absorbing light. In turn, the electrons are "-1, 1/2, 1, and 0.511." That is, the properties of trees are the collection of the properties of electrons and electrons are uniquely described as a negative charge of -1 unit, a spin of 1/2, a lepton number of 1, and a mass of 0.511 MeV/c^2. Protons, neutrons, and quarks... and the light that's absorbed and emitted are also described completely and uniquely by a different set of numbers.

Now the labels that the numbers have are entirely human-defined. But no matter how an alien species might define the unit of electric charge, the electron (and proton) have ± 1 of it. So, to him what is a tree is defined by what are the properties of a tree, which are entirely defined by a small set of numbers.

Tegmark is not alone, but his is a very small club.

***MPy*2 There are special numbers** While I'd not be prepared to say that marriage is "5" and when justice is done, that "9" is involved, there are special numbers that our universe seems to have latched onto that both explain what we observe, and were some of these numbers different, we would not be here. I just referred to one such special number, the charge of an electron or a proton.

Many numbers in nature play a role that designates unique properties of substances or processes that substances undergo. There are static properties of matter which have conventionally-defined, critical numeric values. So for example, if the mass of the electron was slightly different, then our chemistry would not happen. If the mass of the proton were slightly different, then the formation of hydrogen atoms in the early universe would have occurred at a different temperature and our early universe would not have formed galaxies. Water freezes at a particular temperature. What the number is depends on a conventional scale (° C or ° F), but that there is a definitive event and that it can be quantified by a unique number of degrees makes it special. If that freezing point of water were slightly different, then the geological history of the Earth would have been different. Relations of one property to another are important, dimensionless ratios that don't depend on a scale convention. Aliens would find that a neutron is 1.1 times more massive than a proton and that tiny amount guarantees that our protons don't all decay away leaving no atoms.

Inherent in a Pythagorean view of the physical universe is that the "numbers are in the thing" and that we can poke at nature with experiments and extract the mathematical essence that's embedded inside. Just like Pythagoras did... before anyone else.

***MPy*3 Integers matter** Remember, the original Pythagoreans focused on whole numbers. So do we. Prior to 1912, integers were not particularly special in representing nature but with the advent of quantum mechanics that changed and integers took center stage. Atoms and so our fundamental chemical reality, are based on electron orbitals surrounding a nucleus of protons and neutrons. The orbitals (where electrons in an atom are located) are separated from one another by their distance from the nucleus and that distance is characterized by integer "quantum numbers." The "principle quantum number" runs from $1, 2, 3, ...$ and the relationship of radius of the second orbital as compared to the first orbital is 4, the third orbital relative to the first is 9. So, nature here prefers squares.[16] Furthermore the complex organization of the periodic table is all integer relations. First, the number of total electrons that an atom with principle quantum number n has is $2n^2$. So for $n = 1$ there can be an atom with 1 electron (hydrogen) and an atom with 2 electrons (helium). For an atom with $n = 2$, there can be as many as $2 \times 4 = 8$ electrons. How the electrons are distributed within multi-electron atoms is further defined by other integer relationships. Integers now abound, driven by

[16] Without this ordering, chemistry would be impossible and we wouldn't exist!

the functional nature of quantum mechanics. "Magic numbers" in nuclear physics. "Isospin," "strangeness," and other quantum numbers in particle physics and so on. Figure 1.5 shows the fundamental quark building blocks out of which all nuclei are constructed.

***MPy*$_4$ Patterns and symmetries play important roles** Atomic theory is just one of the many ways that nature makes integers do a lot of work. The same thing is true in the application of the mathematics of symmetry (the mathematical discipline of symmetry is called Group Theory) to physics—integers are important there also. The favored Pythagorean equilateral triangles are precisely the way that the Group Theoretical organization in the quark model arranges the up, down, and strange quarks in a symmetry-inspired, triangular mathematical pattern. The vertices of that quark-triangle are defined as precisely specified rational numbers. That the magnet on your refrigerator holds up that picture of your mom is because at room temperature a symmetry inherent in the quantum mechanics of iron lattices has been broken—the formula that describes the magnet actually becomes more symmetrical in its variables when it's hot and the magnetism goes away. If you were to heat up your kitchen, at 560° C (1000° F), your magnet's magnetic properties would instantly disappear, mom's picture would fall to the floor, and the magnet would become a bland hunk of metal, albeit bright red. Of course that would be the least of your worries.

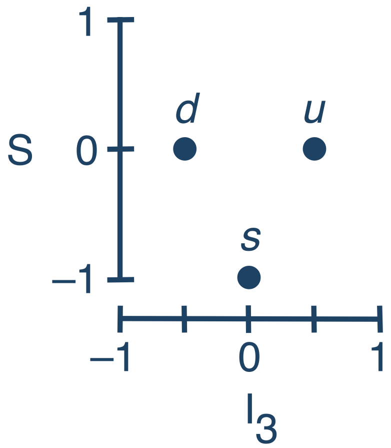

Figure 1.5: The toolbox of quark ingredients that form all nuclei. An up quark has numbers $S = 0, I_3 = 1/2$, the down quark has numbers, $S = 0, I_3 = -1/2$, and the strange quark has numbers, $S = -1, I_3 = 0$. They go together so that, for example, a proton is the quark combination of uud.

***MPy*$_5$ Approximation is appropriate** A modern approach to complex physical phenomena is that either closed mathematical description is not possible in principle, or that it's difficult to practically model a process with a computer. *Approximation* is a modern accomodation to Pythagoreanism and would not have been welcome even a century ago as acceptable. A famous example is Newton's gravitational law. It's highly precise and we use it to design spacecraft trajectories through the solar system. As long as you're calculating the force between two objects, you're good. The formula works. But as soon as you add a third object, it is mathematically impossible to solve the equation—a "closed mathematical description" can't work. Approximations are required. And we still land people on the Moon and probes on Mars.

> It's not possible to test a "theory" in my opinion, rather we create models using theories as frameworks and test them. Models are our connection to the physical world and by their nature are approximations. Remember that temperature that I found in Crotone, Italy? I read from my phone that it was 22° C, but the actual temperature at some point in the city could have been 22.5° C, or 21.9995° C. Inherent to measurement and probing the physical world is that precision is a matter of experimental design and scientific goals. (Nobody needs to know the weather to multiple decimal places and so weather apps will not report it that precisely.) I think that's consistent with Pythagoras' lyre. Overtones might be different for different strums. My ears might not be precisely sensitive as your ears. Your lyre might be slightly more resonant than mine. But absolute precision is not required to have the same personally pleasant sensation, which was the point, after all.

1.2.2.1 Unreasonable?

Generally, we physicists don't generally lack in confidence. So in the interest of full-disclosure, here's a complete capitulation, a sort of "I dunno" response to why math and physics are so linked up:

Ask Mr Google to search just for the words "unreasonable effectiveness" and stand back. In less than a second, you'll be treated to a list of 12 million references to the Nobel Laureate, Eugene Wigner's 1960 article, *The unreasonable effectiveness of mathematics in the natural sciences.* It's actually a written version of a lecture he gave at NYU and it's among the most famous documents in physics. It's so ubiquitous, that Wiley Publishing is pleased for you to download it for free.

In that same confessional vein, here's a word that you won't find physicists using: "miracle." The last paragraph in Wigner's article states:

> Let me end on a more cheerful note. The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning.

Can you get a sense of how the preceding nine pages went? "A more cheerful note"? "Miracle," for heavens' sake?

There's a straight line from Pythagoras (and Pythagoreans... remember) to Plato and Platonism and to physics! But we don't understand this "unreasonableness," and sometimes it is kind of uncomfortable. Gloves come off when physicists and astronomers argue about multiverses, string theories, and measurement theory in quantum mechanics.

So, by now maybe you're a little more aware of the possibility that we may *all be a little bit Pythagorean*. At least in my *MPy* sense. Over and over we learn this.

1.2.3 Zeno and His Paradoxes

Parmenides had a following and his most devoted, and enthusiastic partner was the younger **Zeno of Elea** (ca –490 to ca –430). What he did was mess with everyone's mind about simple, common-sense experiences. He's remembered primarily for 10 paradoxes, two of which about motion I'll remind you of here as the most famous. He wants to show you that what you think you know, you don't, that common sense deceives. (Like in Quantum Mechanics and Relativity, where common sense left the building a long time ago.) I'll do them in reverse order. (By the way, how do we know of his arguments? Plato, again, in a dialog where Socrates deals with the young Zeno, playing himself. And Aristotle, who goes after Zeno.)

The "Dichotomy": the racetrack. This is the famous one. In order to run the 100 meter dash, you've got to get to 50 meters. In order to get to 50 meters, you've got to get to 25 meters. See where I'm going (or maybe *not going*)? According to Zeno, there are an infinite number of distances that have to be traversed in order to move in space at all. So you can't get to 100 meters, in fact, you can't move at all. MOTION ON THE EARTH is impossible.

Now this has been dissected for centuries. Just ask Mr Google about "Zeno" and you'll see 36 million hits. The push-back begins with Aristotle, who argued persuasively, but in the end, inconclusively, that you can move through an infinite number of spaces if the time intervals become shorter and shorter while you do it. Aristotle hated infinity, so this must have been hard for him. But this presumes that Zeno was suggesting that the motion would take an infinite amount of time, but maybe it's because he was trying to cram an infinite number of steps into a finite period of time. So Aristotle's argument is not general enough.

The modern solution requires an understanding of how speed relates to time and space, a very modern set of ideas that are the heart of Relativity. Briefly here (I'll show you a complete explanation in Appendix A.1).

Speed is the change in a distance with respect to a time, or $v = \frac{x}{t}$. So distance is

$$x = v \times t.$$

If the speed is constant—which is how Zeno characterizes the situation, then any increment of distance is proportional to an increment in time. Make the distance smaller, and the time proportionally follows in step. So traveling $1/2$ the distance, then $1/4$ of the distance and then $1/8$ of the distance, you accomplish that in $1/2, 1/4, 1/8$... of the total time. Preserving the constant speed.

The Paradox of Infinite Divisibility. This paradox is the jumping-off point to an entirely different way of dealing with Heraclitus and Parmenides: If an object is made of parts, then one should be able to start cutting... into two parts, then four parts, and so on. At some point you reach some end: 1) If after an infinite number of slices, you find nothing... then the object was made of nothing—a **not-is**. 2) If after a finite number of slices, you find something... but it has zero size, then the object was made of something that has no size. Another kind of **not-is**. 3) If after a finite number of slicings, you find something that has finite size, like an element? Well, you're just not done slicing!

> This is a modern thing as we are perfectly content to imagine that quarks that make of the proton and neutron have no size,[a] likewise the electron. But we have a field description of elementary particles and the forces among them, so we have a quantum mechanical push-back against Zeno here. But prior to the 20th century, a physics solution was not possible.
>
> [a] Yes. Our word "particle" creates an image of a little billiard ball, doesn't it? In actuality, the size of quantum mechanical objects is so ill-defined as to have little meaning outside of an agreed-upon criteria involving waves.

You can see how this works. Zeno was apparently clever enough to waste the pixels on your computer screen in 36 million hits... all in service to the Parmenides two arguments: **Nothing changes** and **knowledge from perception cannot lead to truth**.

Appendices

Appendix A

Greeks Technical Appendix

A.1 Zeno's Paradox

A.2 Proof of Pythagoras' Theorem

A.3 Socrates' Geometrical Problem

Sources

Ferguson, Kitty (2008), *Pythagoras: His Lives and the Legacy of a Rational Universe*. Walker Publishing Company, Inc, New York.

Lloyd, G. E. R. (1970), *Early Greek Science Thales to Aristotle*. Chatto & Windus, London.

Russell, Bertrand (1905), *The History of Western Philosophy*. Simon & Schuster, New York, New York.

Tegmark, Max (1998), "Is "the theory of everything" merely the ultimate ensemble theory?" *Annals Phys*, 270, 1–51.

Tegmark, Max (2014), *Our Mathematical Universe*. Knopf Doubleday Publishing Group, New York.