“Superfluous”: The Stories of Einstein’s Special Relativity

Draft

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Chapter 2

Can’t Live With ’Em Or Without ’Em : Plato and Aristotle

“The safest general characterization of the European philosophical tradition is that it consists of a series of footnotes to Plato.”

- A.N Whitehead (1861-1947), *mathematician and philosopher*

Bert and Ernie, Kirk and Spock, Mantle and Maris, Venus and Serena, Abbott and Costello...Plato and Aristotle. One can’t have one without the other and, just like the other pairs in that list, these last two are deep subjects. My need for Plato and Aristotle’s contributions to the study of MOTION are for two ideas: following Pythagorean inspiration, Plato and his collaborators built the first spherical working model of MOTION BY THE EARTH and Aristotle expanded on it. They were both wrong.

And, while Plato didn’t concern himself with MOTION ON THE EARTH (except in an almost impenetrable portion of his last book), Aristotle was all over MOTION ON THE EARTH and invented its systematic study, informing—and infecting—science for 2000 years. It took until the 17th century before we could be all over with Aristotle. His models of MOTION ON THE EARTH and MOTION BY THE EARTH became Medieval and Renaissance Church dogma, but are wrong in almost every respect. By pushing back scientists learned what was better and why.

So why is it that Plato’s shadow hangs around while Aristotle’s importance for physics disappeared more than 400 years ago? We still talk about Platonic worldviews in physics, but nobody talks about Aristotelian–anything. In many ways my field of particle physics is relentlessly Platonic (but don't tell anyone that I said that). Plato put important questions in play that remain troubling: What can we know? How do we know when we're right? And, most importantly, what is the role of mathematics in the fabric of the universe?

It was the worst-kept secret sneak attack in history. Everyone knew that the Persians were coming as under King Darius' son Xerxes the Great's command, the invading infantry slowly marched along in parallel to the Persian navy counter-clockwise around the inside of the Aegean basin, subjugating the Ionians along the way. Anaximenes lived under that locally-sourced, Persian rule that drove Pythagoras to Italy. About 100 years before Socrates' execution following a 10 year advance in −480 the battle was joined with an amassed Persian force of at 150,000 soldiers and 600 warships. Athens was evacuated and the Persians destroyed the city.

The Greek confederation organized itself: the wounded Athens mounted the naval campaign and Sparta, the foot soldier command. What followed was a series of military maneuvers, still studied today. Spartan heroism of King Leonidas with 300 Spartan troops and a total of 9,000 allied soldiers met and slaughtered the Persians at the pass at Thermopylae. The movie and the comic book series *300* might jog your memory Snyder (2006). While this was going on, the Athenian navy engaged and overwhelmingly defeated the much larger Persian naval force. Finally during the summer of −479, the Persians were defeated in a decisive land battle. Yet, war continued in one form or another for thirty more years until the Persians fled the Aegean leaving behind a Sparta with a greatly enhanced reputation. Proud Athens rebuilt after that disaster in −480 and under Pericles' leadership — throughout the decades of extended conflict, began its 75 year Golden Age when everything you think of as Greek in culture, art, architecture, and philosophy was intentionally created.

Ironically, even though Sparta could be credited as having been the major military force in the Greeks' victory, its isolated and belligerent nature simply did not equip it to lead during peacetime. In contrast, while Athens had been destroyed, its nature was to rebuild stronger, to politically organize, and to lead. All while doing what Greeks did best: fighting.

While the Golden Age was unrolling, Athens simultaneously managed to battle with: Sparta −465; Corinth and Sparta −459; Samos −440; Corinth again −433; Potidaea −433; Mageria −433; Sparta again −431 (Socrates was active as a soldier during this period), Sparta 1, Athens 0; Syracuse and Sparta −415, Sparta 2, Athens 0 ; Sparta now allied with Persia −414, Sparta 3, Athens 0. Game, Set, Match.

After that third war with Sparta,[1] Athens surrendered to Spartan general Lysander in −404. Plato was 23 years old and Socrates had five years to live.

Athens badly handled their unfortunate overreach and eventual defeat and in the final stages of the war they managed to: expel their leading general. execute six other military leaders and flip from autocracy to democracy and back to autocracy. Socrates was on the autocracy side and it was the democrats who condemned him to drink the hemlock in −399.

Athens' subjugation by Sparta after the two Peloponnesian Wars was tumultuous — governance of the city jerked back and forth between oligarchs and democrats. In the same way that the Golden Age of Classical Greece emerged during war with the Persians, in the midst of the city's internal chaos, western philosophy began and was followed quickly by the first systematic attempts to understand MOTION BY THE EARTH and MOTION ON THE EARTH by our two lead actors. Yet the catalyst to all of this progress was interested in neither. Socrates' persistent question was: how to live a virtuous life, not how do things move. As his talented acolyte, Plato adopted the older man's voice and wrote truly engaging tales, but expressed his own ideas and, while his program was ostensibly one of ethics, the Socrates/Plato approach opened a new front in the battle with the Parmenides Problem which resonates in modern physics today. And, as so often happens in philosophy (and physics), the next productive steps were in opposition, launched by Aristotle, one of the most remarkable intellects in all of Western history and whose voice is probably from lecture notes and not intended as his legacy literature. Yet in physics: Plato endured and Aristotle is gone.

2.1 Act V A Little Bit of Plato

Plato (−429 to −348) is actually a nickname, suggesting someone of broad shoulders or perhaps a wrestler. The name on his driver's license would have been Aristockes and his aristocratic family had been influential for generations. Plato was no democrat and grew up during the Peloponnesian War (−431 to −405) and the subsequent subjugation of Athens by the victorious Spartans. In many ways Plato's idea of the correct form of government was clearly informed by the collectivism and brutality of the Spartan way.

> One of the signature events of his life was the story of his attempt to help form a government in Syracuse where he somehow got the idea that he could turn the tyrant Dionysius into a philosopher-king, since in Plato's opinion leaders should be philosophers. That got him imprisoned and even sold into slavery for a while (or so the story goes), until he was ransomed. He actually tried two more times, which brings to mind one's questionable mental state as per Einstein's observation much later about repeating the same mistake over and over and expecting a different outcome..

[1] who actually allied with Persia!

His life's direction was formed when he, like many young men in the newly democratic Athens, started to associate with **Socrates** (−470 to −399) who, after his (apparently distinguished) service as a foot soldier in the war, took philosophy on an entirely different course from investigating the nature of reality to how best to live a satisfactory life. We learn in school about Socrates' self-administered execution at the hands of the democratic Athenian politics—one of the reasons that Plato was distrustful of democracy. It wasn't his first choice of punishment—it was traditional to give the convicted options, and Socrates suggested that his value to the city warranted that he be given free food for life. That was turned down and eventually death by poisoning was prescribed.

Plato's 35 books are all in the dialogue form, conversations between Socrates and a variety of fictional and real persons. Unlike Aristotle's largely academic writing, these books are literature and valued for their style and lyricism. Plato himself is only mentioned twice and he never speaks directly. The assumption is that he's speaking through his mentor, Socrates and that the ideas are his, and not meant to be those of the older man. (One book, *The Apologies*, might have been more personally Socrates as in that volume he defends himself against his accusers.) So the ideas are Plato's and the books comprise his philosophy as it evolves over his productive, long life. Almost all of his work follows a general theme, and what he seems to struggle with is what I've called the Parmenides Problem. Plato wants to contrast what we experience in our everyday world—objects (physical things) and ideas (like virtue, justice, beauty, what's good)—with analog, abstract ideas that are the source of the variety of physical things and the imperfect values we associate with more aspirational ideas.

It might be reasonable to view the Socrates of Plato's dialogs as a literary invention, but he was known to broader Athens and even parodied in the *Clouds*, a vicious comedy by Aristophanes and figured in other writers' accounts, including in dialog form. But the world now knows of Socrates through Plato and he figures into every one of Plato's dialogues as "that guy" who irritates everyone, although in the later dialogues his role diminishes. His job is to ask simple-seeming questions (the Socratic Method) of an assembled group of friends (or foes), often about an ethical matter. What's temperance? What is virtue? What is justice? The course of these sorts of innocent sounding conversations is repeated: the folks being questioned are maneuvered into impossible rhetorical cul-de-sacs, shown to be incapable of any kind of logical thinking, and more often than not, shown to not know things that they should know. Meanwhile, Socrates rarely says what he thinks, in fact, he usually hides behind the assertion that he doesn't know either, but at least he knows that he doesn't know. Superior to a fault. These questions also often segue into something more than they seem, and many of them move to more weighty topics like how *do* you know what you know. That is, they form the beginning of serious Epistemology, one of the foundational philosophical disciplines.

Plato's output was large and I'll choose only a few topics that inform our scientific project. Unlike almost all of the previously considered Greek philosophers, we have complete writings in his chosen dialogue format. He famously started *The Academy*, a school that lasted more than 700 years whose star pupil was Aristotle, whom we will consider below. Bertrand Russell (in his Literature Nobel Prize *A History of Western Philosophy*) sums up what we're about to dive into appropriately:

> "Aristotle's metaphysics, roughly speaking, may be described as Plato diluted by common sense... He is difficult because Plato and common sense do not mix easily." Russell (1946), *A History of Western Philosophy*

My focused concern is with two aspects of his philosophy and then his physics and they're related. I'll leave his modeling in astronomy to the next chapter when I will consider all of the Greek astronomy at once, but I'll consider his overall approach to astronomy here. Of concern then (and now) are Plato's Epistemology—what does it mean to know something (from the *Meno* and *Phaedo*), his Metaphysics—what is the nature of reality (from *Phaedo*, *Parmenides*, and *Republic*), and his physics (from *Republic*, *Timaeus* and Book X of the *Laws*).

2.1.1 What Is True Knowledge?

Plato was deeply influenced by our Parmenides Problem and took this on with a study of the broader question of what actually constitutes true knowledge. He thought deeply about this and his conclusions became grist for philosophical mills for the next 2500 years.[2] He decided that there are two hallmarks to knowing: that knowledge should be infallible and that it should be "of something that is." Typical was the exchange between Socrates and the 16 year old Theaetetus in the dialogue by that name. Socrates teases out of the boy his ideas of four kinds of knowledge, and demolishes every one of them. First up, what do we learn by *perception* as a source of knowledge? That's dispatched by Socrates, it's infallible (since your internal evaluation of a perception is true to you), but perception is incapable of demonstrating that the objects of perception actually exist. So it fails on the second hallmark. Next, is *belief* as a source of knowledge? That results in a blistering dissertation on subjectivity. And the same thing happens with "true belief." They are both fallible, so failing on the first hallmark. Strike three. And finally, what about *belief with a reason to hold that belief,* what in the context of *Theaetetus* is sometimes called "true belief plus and account" or, "Justified True Belief"? This is sometimes incorrectly described as Plato's own theory of knowledge, but Socrates makes hash of JTB and leaves the question in an unsatisfying state. Let's look at a couple of examples.

J+T+B was considered to be among the best efforts into nearly the present day and relies on the three aspects memorialized in its name. The B: one can't claim knowledge about something you don't believe. (It's Tuesday and I know that, but I

[2] I'm grateful to philosopher, Professor Harold Brown for important discussions on this complex topic in Platonic philosophy.

believe it's Monday certainly doesn't qualify as knowledge of Monday.) The T: the fact must be true (if the fact is not true, then you cannot be said to have knowledge of it.) The J: whatever you claim about the fact, you need to be able to justify it.

Consider this claim: It is 3 o'clock, I believe it's 3 o'clock, because I looked at my watch and see that time displayed. B, T, and J are all in play and this seems a reasonable example of knowledge.

> But there are holes and weaknesses. What about instead of that J, how about J2: It is 3 o'clock, I believe it's 3 o'clock, because 3 is my favorite number. I'm right, since it really is 3 o'clock but that justification is silly and certainly doesn't qualify as knowledge of the time. How about this: It is 3 o'clock, I believe it's 3 o'clock, because I looked at my watch and see that time displayed. But...I didn't know that my watch was broken and had stopped at precisely 3 o'clock. So it was just luck that my reading corresponded to the right time. So that's hard to accept as knowledge. In fact, it was only in 1963 that Edmund Gettier apparently found counterexamples to JTB which are now called "Gettier Cases."

Clearly Justification is the rub and many efforts have tried to turn J+T+B in to J+T+B+X…where X is some thing added to take care of the Gettier Cases. It's an ongoing problem. For scientific claims of knowledge, sometimes Justification weaknesses turn on problems with observation and even the senses so we're right back to the Parmenides Problem.

Plato had an answer and it turns out to be more than a theory of knowledge, but also a theory of what's real: fixing epistemological problems resulting in metaphysical commitments.

True knowledge for Plato can only come from permanent, unchanging things. Thanks, Parmenides. If something is true, it must be so forever, which means that it was never not true, nor will it ever become not true. He falls squarely in the Being camp, as opposed to the Becoming camp.

> Notice how this demand of permanence as the qualifying feature of true knowledge is an **unquestioned commitment**. There's no room for degrees of knowing which is not scientific, nor does it really function in everyday life. We all know things with varying levels of trust and this is especially true in science where not being able to question some assertion is actually now the very definition of "unscientific." I think that their insistence on permanence is a function of their being impressed with geometry and the fact that it was very early days in the brand new field of epistemology.

Plato differed from ardent Eleatics like Parmenides by insisting that knowledge is indeed possible, but there's a catch.

He proposed after *Theaetetus* that there are two worlds:

- The world of the Forms.
- The world of the senses.

2.1.2 The Forms

Plato's theory of the Forms is one of the most difficult ideas in philosophy but comprehending it is critical for an understanding of his projects, but also for appreciating physics. He gives abstract concepts an existence of their own and a job to do with a consequence that sort of grates on you.

Take high school (please): if you ever took a geometry class you were given a set of pieces out of which you could create new pieces with just a ruler and a compass. These pieces include things like points with no extent and lines with no thickness. You manipulated and proved theorems about perfect triangles and perfect circles. Let's focus on that last one.

Think of all of the "circular" things that you come in contact with in your everyday life. Coins, dials on appliances, buttons on your shirt, a camera lens, a cookie, maybe a rendition of something circular in an image or on a screen. You know that none of these circles are the circles of your geometry class. But all of these circular things share their property of circularity. They may woefully miss in the perfection of that high school circle in your mind—but through thinking about it, you know that your Oreo is circular, almost.

Plato would say that that unique abstract circle actually exists as a "Form." That there's a kind of reality—a realm— that's different from the reality that you think of when you drop that circular plate in the kitchen. That realm is where the Forms exist.

That high school geometry-circle is such an abstract notion. But you can grasp that reality, you can apply it, engineers can use it, and you recognize it when you see it. But try an experiment: construct the best circular thing that you can and measure its diameter in a hundred points around the center at micron precision— while your rendition may be a good one, it's not that abstract high school one, is it. The Form of a circle is aspirational but can't be studied by measuring regular life circular things, rather it can only be brought to life through your intellect. The Form of a circle always been there (circles were not born) and that realm is outside of space and time. Can you get on board with abstract things being real?

Maybe Plato's assignment of "real" to mathematical abstractions is a little less odd than at first glance. But he went further than geometry and you might have experience with non-mathematical abstractions. Here's one: "We hold these truths to be self-evident, that all men are created equal..." What is a self-evident truth? If it's a "truth" then questioning it is a waste of effort, it's permanent in a Parmenides sort of way. If an idea is self-evident, then in some sense it's always been there, imprinted in us, while apparently accessible, but at the same time, distant.

You can't hold such a truth in your hand and you know it's not universal in our everyday life since, if "all men are created equal" is untestable since the ones we know are tall, some are smart, and yes, some are disadvantaged. That they're "equal" is an abstraction—again, an aspirational idea of perfection—that we can hold in our minds but we know won't be realized in "our world." But a nation of 300 million swears by that truth.

What about realities outside of our plate-dropping reality? If one is a Christian, then you've been brought up to believe in such a reality—heaven (and hell) are outside of our everyday lives.

When I go to a furniture store I see hundreds of sofas. They're all different, but they all share... a "sofa-ness." They're all participating (sharing) in the Form of the Sofa which I can (only) know of in my mind. It's a perfect sofa. The Parmenides Problem is dealt with in a brand new way: there is a world of Being and a world of Becoming and they are connected, but in a hierarchical way. And, it's not just living room furniture that has Forms. There is a Form for everything: even Justice, Virtue, Beauty, and the Good... the latter of which is somehow a super Form.

2.1.3 The Republic

Plato's contribution to science is not any particular theory or practice, but as Lloyd (Lloyd (1970)) suggests it is more his philosophy of science that we value. This is laid out most explicitly in *Republic*, probably his most famous book, ostensibly a treatise on politics and good governance. It's here where he describes how a city should be ruled—definately not by popular election, but by the training of a special category of people bred and educated in order to be rulers, the philosopher-kings, the guardians. Their lives would be scripted from early ages, living communally, and essentially the pool of potential candidates for leadership. Their educations would be scripted as well, culminating in intensive study of mathematics. The goal is for them to be completely comfortable with the most abstract concepts, including Justice and what's Good and mathematics is a primary route to that appreciation. *Republic* includes a few analogies to try to get Plato's point across. Two are relevant for physics.

Analogy of the Divided Line.

Along with the Allegory of the Cave, the "Analogy of the Divided Line" is important for Plato and I think important for physics—as Galileo and modern physics will eventually enlighten for us. A rendition of the Divided Line is in Figure 2.1. What we can know is a hierarchy, from muddled to perfectly clear and divides into two broad "realms," one representing our *Becoming* world—the one we occupy in everyday life, and the other representing the *Being* world, outside of space and time and only recognized through thought.

The Becoming realm is broken into two levels of which the objects of the first, and

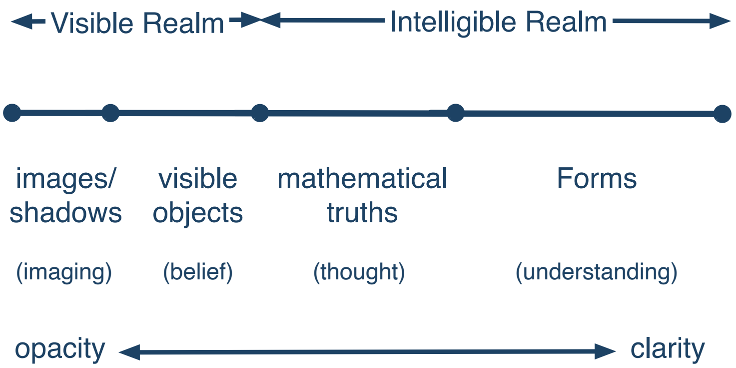

Figure 2.1: The line represents a kind of knowledge-hierarchy, from unclear to perfectly clear.

lowest segment are shadows and illusions of objects in our experience. The shaky knowledge we have about them are mere illusion and dreams. The objects of the second stage are actual ordinary, everyday objects themselves, and the knowledge we have about them are opinion and belief gleaned through our (untrustworthy) senses. Taken together these two stages constitute our knowledge of our everyday world, where things change: the Becoming or Visible realm in which we use our senses and dreams to navigate.

The Being realm is likewise divided into two more sophisticated segments which are only accessible through thought and reason. The first is knowledge gained by mathematics and hypotheticals (think high school geometry) about which we have knowledge through reasoning. And finally, the highest segment of the Being realm is of the Forms, the pinnacle of clarity, "beyond hypothesis" which is aspirational, not easily realizable.

Earlier I opined that degrees of knowing is a more modern way of thinking and the Divided Line actually sneaks up on just that. As we'll see when we study Galileo, there is a realm of the universe which is very hard to observe (on Earth) but which is our goal when we theorize about nature. So I'm not quite willing to pass this off as silly, while at the same time I don't agree with the realm of the Forms as an ethereal parallel universe that we cannot access but through rationality. Stay tuned.

Allegory of the Cave.

He famously tries to work out more of this progression in the *Republic* with the "Allegory of the Cave" and in the *Meno* with the idea of" Reminiscence." The Cave is perhaps familiar. Prisoners in a dark cave are shackled to the ground facing a wall. They can only look straight ahead and what they see are shadows of objects and puppets that are held in front of a fire behind them so that they project on the wall. If they see a sofa on the wall, it's because the Form of the sofa, which is behind them and out of sight, is projected as a shadow of the real Sofa in front of the fire.

Now, if one of the prisoners escapes her bonds and looks around she'll see the fire and the contrived circumstances. The light from the fire would hurt and she'd want to go back to her former spot. But if she were dragged out of the cave and into the sun, she's blinded but slowly she'd look around her and realize that there are actual things in the world and not just shadows. Notice that in the Allegory, she's moving from left to right in the Divided Line in Figure 2.1? She ventures back into the cave and tries to describe that true reality to her still captured colleagues. But in the dark she'd not see well and the prisoners would not allow her to persuade them to follow her into the sun since it apparently takes away one's sight. Plato even worries that the prisoners might kill the one who escaped.

Obviously, Plato is describing the daunting project that he's taken on as the enlightened prisoner trying to explain what's Real and True to everyday people who don't want to accept it. The similarities to Neo's trip out of the realm of perceptions and into the realm of the real is not an accident as the movie *The Matrix* is full of philosophical allegories, and the Cave is one of them.

What we can know of the world of the Forms is true knowledge. That's an aspiration of philosophy. What we can know of the world of appearances is simply opinion. The Forms inspired many in the centuries to follow, from Neo-Platonic Christian images to modern science. We'll come back to them in Galileo where finally, properly characterizing MOTION begins. By the way, Plato despised art. A painting of a mountain as nothing but an imitation (the painting) of an imitation (a sensible, actual example mountain) of the form of Mountain, which is the only real thing.

2.1.4 Mathematics For Plato from Republic

Plato's experience in Italy wasn't limited to a failed experiment in his theory of governance, but began as a deliberate project to study with Pythagoreans. Now, Pythagoras had been gone for a century by that point, but two schools grew up around his legacy. The *acusmatici* viewed themselves as the guarantors of Pythagoras' the man's legacy as a complete system. Not only his mathematics, but the other aspects of the Brotherhood were preserved and defended without expansion or elaboration. On the other side were the *mathematici* who bought into reverence for the man, but intentionally expanded the mathematics to new areas of research, an unwelcome sin in the eyes of the *acusmatici* who eventually died out.

Recall that Plato and Aristotle probably learned most of Pythagoreanism from Philolaus, but Plato's mathematical inclinations came from a contemporary, one of the mathematici that Plato befriended and learned from, **Archytas of Tarentum** (ca −420 to −355) who is one of the title characters in Chapter 3.2.2. The other title character in the next chapter is **Eudoxus of Cnidus** (−408 to −355), a student of Archytas and the most significant mathematician before Archimedes. Both influenced Plato and Aristotle's cosmology, and that subject kicked off two millennia of modeling and eventually dogma. The mathematics required in the guardians' education came from Architas, arithmetic, geometry, astronomy, and harmonics. Plato didn't fully agree and added a fifth subject, solid geometry.

This is very much physics as we'll see. Maybe you can begin to understand Plato's elevation of mathematics—in the Greek life of his day, geometry and proportions—to the point of his famous sign above the door, "Let no one who is not a geometer enter." (Well, that sign only crops up in the 4th century AD, so it's probably a myth.) Geometry is venerated by Plato and all who follow for centuries.

This is hit directly in *Republic* where Socrates extracts from Glaucon[3] the reasoning behind requiring astronomy for guardian training. As usual, Socrates/Plato starts out with a theme which in the course of explaining it, evolves into a matter of serious philosophical interest. Glaucon tries to guess at why astronomy is important. Maybe because it's useful for recognizing seasons, or timing agricultural events. Practical things. That doesn't go over well and so he tries again: maybe astronomy is "good for the soul"... that looking at they sky takes us away from looking at everyday things. Again, not productive for Socrates. Here's where geometry comes in and where Plato earns an uncertain reputation for suggesting that armchair astronomy is the only way to go: doing astronomy without ever looking at the stars. Here's how I interpret this:

Take out a ruler and the sharpest pencil you have and carefully draw the most precise triangle you can create and now get the best protractor you can buy and try to verify that the interior angles of that triangle all add up to 180°. No matter how careful you are, you'll fail to perfectly measure 180.000...°. In fact, Socrates/Plato would tell you to not bother since studying an everyday triangle won't help. The perfect 180° is in your head and its truth is one of reasoning and geometrical proof.

Socrates/Plato suggest that the same is true for astronomy.

> "We shall therefore treat astronomy, like geometry, as setting us problems for solution", I said, "and ignore the visible heavens, if we want to make a genuine study of the subject and use it to convert the mind's natural intelligence to a useful purpose." Socrates/Plato, *Republic*.

He says that you can look at the stars but discerning their actual motions cannot be done by measuring the apparent, but flawed motions of the imperfect stars and planets. You can only understand their motions by reasoning. He's sometimes accused of arguing for "armchair astronomy" without looking up! Like the triangle, you might get hints from the world of Becoming, but only through reasoning can you learn what the stars and planets do in the perfect world of Being.

> Here is **another unquestioned commitment** by Plato. That the stars and planets would necessarily execute perfect motion is an assumption. Again, this is the very earliest days of astronomy and philosophy and it's built on a variety of prejudices.

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[3]Possibly, Plato's older half-brother's name.

Plato's "Doctrine of Reminiscence" is another idea that comes from the Forms. In the *Meno* Socrates demonstrates that a slave boy actually knows geometrical proofs without knowing that he knows them! By asking questions, in his Socrates-way. In the *Meno* the protagonist, Meno (a real, young aristocrat) asks Socrates if Virtue can be taught and of course Socrates begins by asking the young man to define what Virtue is and then dismembers his multiple attempts at an answer. The scene degenerates into Meno now becoming frazzled and paralyzed as the discussion evolves. As often happens more than the problem at hand emerges, including what's called "Meno's Paradox": the realization that if you know something, you don't need to ask about it but if you don't know it, then you don't know enough to ask. Of course this all leaves everyone unsatisfied. (It's surprising to me that anyone ever wanted to talk to Socrates.)

The discussion turns to a religious view that the soul has always existed and will exist after we die and that the soul knows all that there is to know before and after and therefore, we already know everything... we've just forgotten it. He then proceeds to demonstrate this idea by asking a slave boy the geometrical proof of how to double the area of a square. By asking him questions, he actually induces the boy to give the right answer. (You can see the answer by following Socrates' questions in Appendix A.3.

> Now anyone who has ever faced a problem or derivation in mathematics (or mathematical physics) has probably come to the feeling when you reach the end that the solution was somehow there all along, that in the process of the derivation, you were revealing something that was always the case. That you kind of remembered it. Now this is the basic characteristic of Deductive Reasoning. It doesn't lead to anything new, but kind of reinforces—-or recalls—something that was already in the premises. This isn't what Socrates was aiming at, but I know I've had that feeling and I can understand why Plato chose a geometric proof to bring it out. What Plato was really after was the fact that the Form of a geometric proof was there all along, in that special realm all the time.

2.1.4.1 The Soul

The "Soul" is a very Greek idea which functions at multiple levels for Plato, in one dialogue, he assigns three separate jobs to the Soul. For our purposes, he's impressed with the idea that some things are inanimate — like a rock — and that somethings appear to be animate. The very word "animate" gives you a sense of what he thought might be the distinguishing feature between two kinds of objects: can they move on their own? So in some ways, this is a question of MOTION ON THE EARTH. He found it useful to ascribe to all things that can move of their own accord — he would speak of "self-motion" — as imbued with Soul. It's not only humans, but birds, flowers, even planets which appear to be able to execute locomotion on their own that enjoy their very own Soul. We'll see that this idea actually figures into some of his astronomy, so in a backdoor sort of way... this is an example of MOTION BY THE EARTH! It is this very talented Soul that moves objects, but persists before and after death and the glimpse of which we get when we do a mathematical deduction, as Socrates demonstrates with the slave boy.

2.1.5 Timaeus

Boy, the medievals must have been confused about Plato. The sole Latin translation of his works was just one: *Timeaus*. It's notoriously difficult, convoluted, and ripe for repackaging by the "neo-Platonists" up to Augustine. In this difficult late dialogue, *Timaeus* the title character, a fictional Greek statesman and scientist from southern Italy (ah, as we'll see, surely a Pythagorean), who when asked by Socrates at yet another get-together, tells the origins story of the universe. A sort of Greek Carl Sagan. Timaeus is less a dialogue than a monologue and it covers a lot of ground without Socrates being his usual, obnoxious self. Obviously, Plato had a lot on his mind in this book.

Plato was so enamored of mathematics that through Timaeus' voice, he builds what he calls a "likely story" of cosmology by mixing geometricized ideas of the atomists with a relentlessly Pythagorean numerology (that he learned directly from Archytas?), a major focus in Chapter 3.2.2.

The universe was assembled (not created) through the actions of a spooky "craftsman," the Craftsman[4] who builds everything—animals, planets, stars—from a plan following eternal ideas—surely, the Forms.

The dialog begins with Socrates counting, "One, two, three,..." a portending of the strange, mystical use of numbers as the Craftsman does his job. I'll reserve the astronomy part of Timaeus for Chapter 3 and make reference here to only those parts of the dialogue that overlap with our project. That leaves most of *Timaeus* untouched.

The story begins with fables about Athens of 9000 years previously, among which are its war with Atlantis and the idea that Earth is periodically destroyed, erasing memories for everyone... but somehow, not the Egyptians. This leads to a discussion of how the universe began. Timaeus asks (with Parmenides looking over his shoulder?):

> What is that which always is and has no becoming, and what is that which is always becoming and never is? That which is apprehended by intelligence and reason is always in the same state, but that which is conceived by opinion with the help of sensation and without reason is always in a process of becoming and perishing and never really is. Now everything that becomes or is created must of necessity be created by some cause, for without a cause nothing can be created" Plato, *Timaeus*

[4] In Greek, the "Demiurge"

Of course, this is a reference to the Forms ("always is and has no becoming") in contrast to particulars and everyday things ("process of becoming and perishing and never really is"). In sympathy to Parmenides' poem, Timaeus also tells about both kinds of knowledge. This is his stepping off point to the fact that the universe has "become" and so was not always around which implies a creation act or a cause, or in any case, a creator. That's the Craftsman's job who follows a plan which is an aspirational blueprint.

The universe isn't created out of nothingness (more Parmenides?) but rather the Craftsman works with the material at hand using the Forms as a blueprint and fashions it into an Earth-centric ("geocentric") model, which we'll talk about in the next section. Plato leaves the impression that the Craftsman does the best that he can — a best-effort universe! There is a difficult overall purposefulness and expectation that the Craftsman is ". . . .greatest and best and fairest and most perfect." This is the best possible world.

Suffice it to say that the Sun, Moon, and planets all take their familiar places according to a mathematical (even musical—Pythagoras, again) format and that Time itself is created along with the planets. In fact the motions of those most-nearly-perfect celestial bodies is the cause of time. The ancients told the days, months, and years by the motions of the Sun, planets, and stars and so it's maybe not a surprise that Time and those objects have a causal relationship to one another.

The Craftsman isn't omnipotent and is restricted to Empedocles' four elements — the materials at hand. The *Timaeus* outlines the way in which Fire, Water, Air, and Earth go together (again, in proportion) by assigning them solid shapes: Fire is made of tetrahedrons, air is made of octahedrons, water is made of icosahedrons, and finally Earth is made of cubes. The solids themselves are made of two kinds of constituent triangles; the isosceles and scalene triangles. The former is what results from cutting a square into two parts along diagonals and the latter is a triangle in which the hypothenuse is twice the length of the shortest side. Two scalene triangles, side by side, attaching the long sides. . . makes an isosceles triangle. So the atoms (in a modern sense) of the four elements are made of two elementary, triangular constituents (like modern atoms are made of electrons and nuclei): tetrahedrons (4 faces of equilateral triangles), octahedrons (8 faces of equilateral triangles), icosahedrons(20 faces of equilateral triangles), and cubes (12 equilateral faces).

Water then, could be broken down into fire and air as a isocahedron can be decomposed into two octahedrons of air and one tetrahedron of fire. In fact, that water evaporates can be modeled in his scheme by noting that two water solids can geometrically be reduced to five air solids. He's used up 4 of the 5 known three dimensional solid forms, historically (but inaccurately) called the Platonic Solids. So, having bought into a theory, he did what any theoretical physicist would do. If the solids are fundamental and only 4 of them seem to immediately come to good use, then there must above a job for the fifth shape, the dodecahedron, and he assigned that to some measure of the universe itself as it has so many faces, it's close to being a sphere?

So, Plato appears to be very much a Pythagorean. The world is geometry—pure, abstract form. But he's just getting started as his Pythagoreanism knows no bounds as we'll see when we consider his cosmology in Chapter 3!

> Platonism is not just confined to philosophy or mathematics. The Medici family in Renaissance Florence was instrumental in reacquiring Greek philosophical texts from the Byzantine empire by importing Greek-speaking academics. They set up a school dedicated to Greek philosophy and a school for the children of the court. One of those children was a ward of Lorenzo and would have learned of this approach to the world. So when Michelangelo later noted, "I saw the angel in the marble and carved until I set him free" he was expressing a very Platonic idea that he absorbed as a young ward of Lorenzo di Piero de' Medici, modestly, Lorenzo the Magnificent.

2.1.6 Platonic Legacy

We've covered a lot, but only a little of the large subject that is Plato. I view the history of physics as ebbing and flowing between Plato's and Aristotle's influence and out of that I have concluded that our recognizable scientific discipline—my life's work—didn't happen until the history of physics swerved in the direction toward Plato and away from Aristotle. So our discussion of the Forms and how the mathematical picture is illuminated by his conclusion that there are two sorts of reality is necessary in order to tell the whole story of MOTION. There is one negative legacy that's more complicated than it's normally presented: the idea of "Saving the Phenomenon," or "Appearances." This is the statement that is used to assign this idea to him:

> "This was the method I adopted: I first assumed some principle, which I judged to be the strongest, and then I affirmed as true whatever seemed to agree with this, whether relating to the cause or to anything else; and that which disagreed I regarded as untrue." Plato, *Pheado*

It's more complicated and people still argue about it. I suspect that there are four reasons that this seems to lead to that direction:

1. Aristotle seems to be critical of that way of thinking (see his statement from *On the Heavens* below on page 2.2)
2. There's the "armchair astronomy" admonition by Socrates in *Republic*, described above.
3. There's the fact that his student/colleague Eudoxus takes on the task of describing the motion of celestial bodies using only circles. This will be discussed in the next chapter.
4. And there's this quotation from *Phaedo*.

The person most responsible for making this direct connection to Plato was the neoPlatonist, Simplicius, who flourished in the 6th century (CE) (He reported that Plato proposed the problem of finding "by the supposition of what uniform, circular, and ordered motions the appearances of planetary movements could be saved.")

In any case, this methodology had legs. Can you see how *un*scientific this is? First create the theory, and then interpret the facts only to support the theory. This is especially the case in his astronomy.

His positive legacy is, as I've hinted, critical and abstract. His ideas were reformulated a number of times and Neo-Platonism was a pre-medieval version that eventually found its way into Catholic Church doctrine, much through Augustine, only to be reassessed centuries later.

What can't be overstated is the influence that Plato had on our project of describing the universe using mathematics: "Platonism" is an enduring feature of fundamental physics. Johannes Kepler in the 15th century was among the first truly Platonic (or even Pythagorean) scientists and as I joked earlier, my particle physics subdiscipline is very Platonic.

Notice that MOTION has not been a feature of my discussion of Plato. In part, we think of Plato's ideas about motion as focused on astronomical topics, which we'll cover later in this chapter. But also his ideas as expressed in *Timaeus* (and to some extent in the *Laws*) are so esoteric as to be mostly unintelligible. There we learn that the Soul is responsible in part for "self-motion." It's all very unsatisfying.

"Unsatisfying" is a good stepping-off point as we consider Aristotle and his huge negative impact on physics. For someone so wrong, it's ironic that we can't ignore him.

2.2 Act VI A Little Bit of Aristotle

> "Aristotle is a Foal. When a foal has had enough milk, it's known that it kicks its mother." ascribed to Plato

While Plato's practical impact on physics was limited to abstract and esoteric notions, not so with **Aristotle of Stagira** (-384 to -322) an even bigger subject. He was a systems builder with practicality and abstraction as joint projects. The extent of his intellectual reach was incredible and not only did he further philosophical ideas, he invented whole fields of science and philosophy.

He was born in Stagira, near Macedonia north of Greece and was connected to Macedonian royalty as the son of the king's physician. He emigrated to Greece to study at Plato's Academy at the age of 17... and then stayed for almost 20 years. While he was in residence, probably beginning his writing, the Macedonian King Philip II began his conquest of northern Greek cities, including Athens... which came under his control through concession, and only limited conflict.

When Plato died in −348, Aristotle went to Assus in the northwestern area of modern-day Turkey, married, and began (or continued) an impressive series of biological, marine biological, and zoological researches which he wrote about in *The History of Animals* and *On the Parts of Animals*. He was a details-person and described animals and insects with minute detail through dissection and description, beginning the classification exercise that established the whole science of biology for centuries. He classified more than 500 different species into genus and species forming categories of likeness and habit of mammals, fish, reptiles, and insects. It was here that he established his insistence on observation as the source of knowledge, an evolution away from Plato that was obviously severe. Think of his approach as like taking a deck of cards that's all swirled together on a table, and ordering the them all by identifying and sorting for like features—suit, color, and number. That kind of organization came naturally to Aristotle, it's very modern, and it seems to have first been apparent to him as a scientific practice.

His range was remarkable, covering: Law, physical science, psychology, natural science, philosophy, logic, ethics, and the arts. Words that we have from him include: energy, dynamic, induction, demonstration, substance, attribute, essence, property, accident, category, topic, proposition, universal... His metaphysics informed the development of his science and confused the awakening Western world from about 1100 to 1600. And, everything was a part of his system, and so abandoning one piece that might not make sense would bring the whole system down. It was a philosophical game of Jenga. In particular, his astronomy—and especially his physics didn't make sense—and we'll see that the Medievals knew that it didn't make sense. But selectively adjusting it seemed impossible.

One positive thing, if only his followers had preserved it: we have Aristotle to thank for dampening enthusiasm for the unwelcome Platonic idea of "Saving the Phenomena":

> "...speaking of phenomena, they say things that do not agree with the phenomena...They are so fond of their first principles that they seem to behave like those who defend theses in dialectical arguments; for they accept any consequence, thinking they have true principles—as though principles should not be judged by their consequences..." Aristotle, *On the Heavens*

We have three Aristotelian issues to consider for our narrow project which together only sample a small sliver of his whole universe: what is real, how does change happen, and his physical science.

2.2.1 Aristotle and What's Real and What's Knowledge?

Unlike Plato, Aristotle rejected the idea of a super-sensible realm housing the ethereal Forms. He had a different job for the Form that linked it with actual substance, here on Earth, closer to our idea of the form of a physical object. His focus—which was refreshing after the Parmenides Problem and now the Plato Problem—was on *individual things* which we learn about through a personal experience with the world, not through some intellectual abstraction. What's real for him are *particular objects*.

> "If we did not perceive anything we would not learn or understand anything." Aristotle, *On the Soul*

Like I said, refreshing.

Substance—stuff—and Form work together to make the world. The oft-used metaphor of a house is instructive. In order to make a house you need stuff—wood, nails, and so on—and a plan, an organizing principle. Substance and Form. An individual thing is then matter which has been given a form and you can't separate them. An individual thing must have both.

For Aristotle, perceived facts are the necessary ingredients for knowledge. We organize them in our memories, looking for commonalities and differences. We categorize our facts into bins of like and unlike with relationships among them. We have an individual perception of things, collect facts, ruminate on them by comparing in our memory with our internal database, and categorize. This is classical Empiricism, as opposed to Plato's classical Rationalism. So far, so good. (Think about that deck of cards, now abstracted as a philosophical goal.)

2.2.2 Change and Cause

But we still can't get away from the Parmenides Problem and Aristotle also did battle with change and permanence. Let's race through how he thought about change and how it functioned in his physics.

For him, Change relieves a... tension. An actual thing, what **is**, has within it a potentiality to become something new. As long as it's not in that newer state—it's "deprived" of it—and it is obligated to go there. Inevitably. So everything is also in a Hericlitean flux, but in a very particular and interesting way. In sympathy, perhaps, with Parmenides, in order for something to change into something else, it had to **be** in the first place and taking that all the way back, takes him into an abstract place where there needed to have been an original Unmoved Mover. We'll not follow that line of thought.

What's important about change for Aristotle, which fits into his bigger system, is that in order to acquire knowledge of something that changes means you can identify the Cause of change. Because: all change must be caused and what can be caused comes from within a the set of Aristotelian "Categories" (of being). The ten Categories is a complicated idea and we'll skim. They are: substance, quality, quantity, relation, time, place, position, state, activity, and passivity — his complete set of predicates that can be assigned in a statement. For example, what can you say about Galileo:

- Galileo was human (substance)

- Galileo was smart (quality)
- Galileo was 5 feet tall (quantity)
- Galileo was older than Kepler (relation)
- Galileo lived during the 16th and 17th centuries (time)
- Galileo lived in Florence (place)
- Galileo sometimes sat at his desk (position)
- Galileo sometimes wore shoes (state)
- Galileo sometimes wrote with a pen (activity)
- Galileo was sometimes ill (passivity)

A particular substance must be all of these things in order to be a thing. In order to exist. Like I said, you have to be impressed with Aristotle's ability to take a complex topic and break it into its constituents. Remember, he invented Logic.

> " Nature is a principle of motion and change, and it is the subject of our inquiry. We must therefore see that we understand what motion is; for if it were unknown, nature too would be unknown." Aristotle, *Physics*, III, 1, 200b12-14"

Substances have "motions" but not the kind you're thinking of. They're very Greek motions and can be quite abstract. For Aristotle, motion is anything that goes *to* something. In this change a substance remains a substance, but Form adjusts, characterizing the natural evolution of a state in which a goal is not achieved into a state in which a goal is achieved. And that idea of a "goal" is very important and in part, where Aristotle's physics goes astray. So the substance of a seed changes as it evolves into a flower. But the form of the seed and its various guises changes. Stay with me.

Motions can be of any of the Categories of being, but usually are among just three of them:

- change of quality
- change of quantity
- change of place

For example:

- Galileo changed from a boy to a man. That's a change of quality.
- Galileo changed from a person who weighed 50 pounds to a person who weighed 150 pounds. That's a change of quantity.
- Galileo moved from Padua to Florence. That's a change of place.

That last one, a change of place, is our modern idea of "motion" which he called "locomotion." But for him, locomotion is no more fundamental than any other kind of motion and that's very Greek. But, again, he's thought deeply and by accident, all three kinds of motion have examples today. Not just *loco*motion:

- Modern Change of Place: We tend to think of locomotion as the only one of his categories to apply to change in physics: objects moving from this place to

that place, during some time.
- Modern Change of Quality: A phase transition like water boiling or freezing could be considered a change of quality.
- Modern Change of Quantity: Aristotle could not have imagined a nuclear or particle decay from one thing into three different things, like the decay of a neutron into a proton, electron, and neutrino.

As for goals, it's easiest to think of the nature of something and that involves potentiality and deprivation. An acorn becomes a oak tree. An acorn does not become a Galileo, so it has within it the potential only to be an oak from the beginning. That inevitability also is universal and directed and that even becomes an argument against infinity since there is no such thing as unrealized or unconstrained potential.

> "It is not what has nothing outside that is infinite, but what always has something outside it." Aristotle, *Physics*

Now we know what properties a thing must have in order to exist and we know what kinds of change can happen. Again, to have knowledge of a change one must understand the causes: in fact, four causes. They are the material cause, the efficient cause, the formal cause, and the final cause.

Take a statue:

- The material cause of the statue is the marble.
- The efficient cause of the statue is the action of the sculptor.
- The formal cause of the statue is the plan in the mind of the sculptor.
- The final cause of the statue is the purpose for which the statue was made.

There is sometimes a discussion about whether these function as causation or explanation. Are they the four "becauses"? In any case, the last one of them is problematic for physics as the notion that everything moves for a purpose (that "goal" again) doesn't work in modern terms. This is called "teleological." (One can imagine an argument for Aristotle that there is some teleological logic to how plants and animals "move" from one kind to another... seeds to plants, kittens to cats, and so on.) Of the four (and there's a lot more detail in Aristotle than just enumerating them), Efficient Cause comes the closest to a modern physics cause. That's splitting hairs!

2.2.3 Aristotle's Physics

Aristotle inherited his ontology (the philosophy of being) from his teacher, who inherited it from Empedocles. That is the four elements of earth, air, fire, and water are supplemented by one more, "quintessence" which is outside of the earth-bound region of the universe. Like the reactions to Parmenides, Aristotle envisions "stuff" as mixtures of the four elements. But he goes further than just classification, as their makeup, Causes, and Categories all feed into his explanation for the sort of motion that we think of. So understanding locomotion is intimately tied to the entirety of the Aristotelean system.

With respect to our MOTION, he was very much an empiricist and locomotion in particular fits his overall philosophy. Watch a high kick of a soccer ball or a towering home run in baseball or a shot in the shot-put. The projectile will race to the top of its trajectory and then appear to fall steeper and faster than its rise. Drop a feather and a crumpled up piece of paper and a metal key. Will they hit the ground at the same time?

In each of these everyday examples it seems like the heavier object will hit the ground first. That fits his philosophy, or maybe his philosophy grew from watching things fall since the heavier an object is, the more deprived it is of its most natural place: the Earth. So any object seeks its place by virtue of the amount of earthiness it has in its composition. Heaviness is an attribute and the natural motion associated with heaviness is down, toward the center of the Earth. Lightness is also an attribute for Aristotle (for us, that's just less heaviness). Natural motion for a Light object is up, toward the sky. Below the orbit of the Moon, objects have two kinds of natural motion:

- Natural locomotion for heavy objects is down and natural motion for light objects is up. These Earth-bound motions — MOTION ON THE EARTH — both follow straight lines toward their preferred places. So firey things want to be at the edge of the Moon's orbit and earthy things want to be at the center of the universe (the Earth).

But the planets and cosmic objects don't move in straight lines and have no apparent pushing force, so they must be composed of different stuff from Earth, Water, Air, or Fire and have a different sort of natural motion:

- Cosmic objects are made of "quintessence" and have circular natural motion.

Like all motions, Earth-bound objects away from their natural places are deprived and realization of their potential is to . . . go there. Celestial objects move naturally in circles. To fulfill their essence.

There is another kind of locomotion which is un-natural, dubbed "violent," and for Aristotle what causes violent motion must be a contact force. So throwing a ball is violent and unnatural, since it's not directed down. When the ball is in contact with your hand, you're making it move. When it leaves your hand? Well, here Aristotle had trouble and everyone knew it. The contortions that he went through to explain projectiles are pretty contrived. But he was wedded to his system and in spite of his scorn for Plato's Saving the Phenomenon, he seems all about that here.

When the ball leaves your hand, it doesn't immediately head towards the center of the Earth. The medium of the air is critical in two ways:

1. The motion of the hand is (somehow) transferred to the air which (somehow)

successively creates forces in steps... air moves the projectile, then another segment of air moves the projectile... and so on until the ability of the air to perform that critical contact-force job is used up. Somehow the forces of air meet some dissipative force... of the air(!), and it stops.

2. Then the object falls directly to the ground because the air stops it.

The air both moves it and stops it. Also, the projectile doesn't share both unnatural, forced motion and a component of natural, downward motion. There's a lot not to like about this, and nobody did. Even probably including Aristotle given his complicated explanation. Figure 2.2 is a 16th century depiction of Aristotle's projectile paths: straight line up, then straight line down.

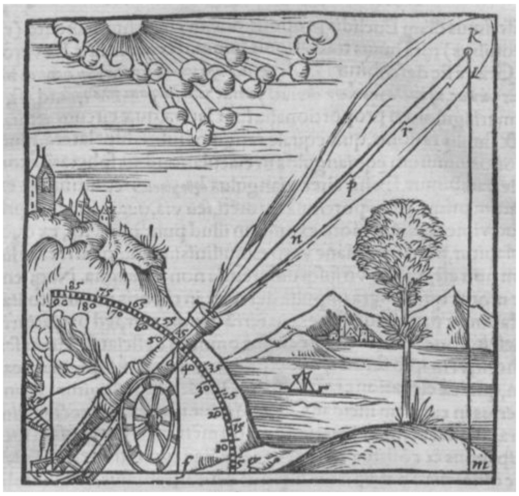

Figure 2.2: A drawing of Aristotlean projectile motion in a mathamatics text by Daniel Santbech in 1561.

Now he's not entirely consistent in his descriptions. In his Physics, he says:

> "Again, as it is, things thrown continue to move, though that which impelled them is no longer in contact with them, either because of"mutual replacement" as some say, or because the air which has been thrust forward thrusts them with a movement quicker than the motion by which the object thrown is carried to its proper place." Aristotle, *Physics*, IV, 8

Later in Book VIII he says:

> "Therefore, we must say that the original mover gives the power of being a mover... to air... naturally adapted for imparting and undergoing motion... The motion ceases when the motive force produced in one member of the consecutive series [of forces imparted by the air] is at each stage less, and it finally ceases when one member no longer causes the next member to be a mover but only causes it to be in motion. The motion of these last two—of the one as mover and of the other as moved—must cease simultaneously, and with this the whole motion ceases.

The first quote seems to make reference to an idea that's in *Thaeatus* called antiperistasis, in which Plato tries to explain respiration, suction, and falling bodies as displacing the air and back-filling it to avoid a vacuum. This either evolved too, or was also a suggestion by Aristotle that the air in front of a ball rushed around to the back and pushed the ball forward. I know. It makes no sense. Everyone knew it and the Medievals were very critical and modified the ideas.

Aristotle didn't know algebra, but we can most easily summarize his points with some simple proportions. The mathematical symbol for "proportional to" is $\sim$.

He would describe the motion of a projectile with these ideas:

- Heavier objects (made of more earth than other elements and so highly deprived of its natural place) would fall faster than light objects: $t \sim \frac{1}{W}$ where W is the weight, a stand-in for earthiness.
- Heavier objects would then fall faster than light objects —have a higher velocity. He had some sense of the resistance of air and so the velocity is like $v \sim \frac{W}{R}$ where R is some measure of the resistance that air or water or some medium asserts on the falling object.
- This leads to a convenient conclusion. If there is no resistance, then $R = 0$ and the speed that if falls would become infinite. But nothing can be infinite in Aristotle's philosophy, so there is no vacuum allowed. . . no medium with zero resistance.
- And finally, for violent motion, which requires an external force in contact with the object, $v \sim \frac{F}{R}$. No force, no speed. More force, more speed.

Each of the bullets describe exactly what you and I experience every day in a sport with a ball or just life. So Aristotle is clearly a champion Empiricist. The above is what we all experience.

There's more. If linear motion is the only natural motion then the Earth must be stationary otherwise, we'd would feel the effects of some tangential wind-force rotating the Earth. And we don't, so the Earth does not rotate. Orbits are for quintescent objects which are outside of the Moon's orbit, as centered on the Earth. So the Earth cannot be in orbit itself by definition.

2.2.4 Summary of Aristotle and Locomotion

So to sum up the first real study of MOTION. . . ever.

1. MOTION ON THE EARTH is of two types:

    1. Natural motions are toward or away from the center of the Earth according to the degree of heaviness (among the four elements, Earth would dominate the others) or lightness (among the four elements, fire would dominate the others) that compose their substance. Natural motions are in straight lines. They represent the fulfillment of an object's potential.

   2. Unnatural, or violent motions are those which are not natural. They all require that an external force is applied throughout whatever trajectory a body experiences. Take away the force, and the motion would cease. These motions can be of any shape.

2. And MOTION BY THE EARTH?

   1. It's zero. The Earth is stationary because no forces can be detected that would be required to make it move. And, motion on the Earth doesn't suggest that the Earth is moving. Throw a ball up and it doesn't fall behind you, as he suggested would be the case if the Earth were moving. So he has an explanation as to why it must be stationary, but not a prediction. He's justifying his contention.

3. And celestial MOTION?

   1. It's circular. Objects outside of the Moon's orbit are of an entirely different substance that what we experience: quintessence. Why? Since if they were of the same material that that of and on the Earth, its natural motion would be in straight lines.[5]

Aristotle's theories of MOTION BY THE EARTH and MOTION ON THE EARTH are relentlessly empirical: they are theories of what we all observe in our everyday lives. His theories of motion are also wrong.

2.3 Plato and Aristotle, Today

2.3.1 Modern Day Platonists

> "I imagine that whenever the mind perceives a mathematical idea, it makes contact with Plato's world of mathematical concepts. . . When mathematicians communicate, this is made possible by each one having a direct route to truth, the consciousness of each being in a position to perceive mathematical truths directly, through this process of 'seeing."'
> Roger Penrose (1931-), theoretical physicist, Nobel Laureate

It's unlikely that anyone would wonder about the application of Aristotelianism into the physics of MOTION ON THE EARTH nor MOTION BY THE EARTH, but thousands of pages of writing (and links) have been devoted to the application of Platonism into modern physics, and especially in mathematics. Recall my party-question in the previous chapter: Is mathematics discovered or invented? Many mathematicians and physicists have concluded that it's discovered and that's the bumper-sticker version of modern Platonism: suitable for the 21st century.

In this "Plato and Aristotle, Today" second I'll describe a more modern version of Platonism that might function in physics in two different aspects which I'll call "The

[5] some circular reasoning there, no pun intended

Platonic Process in Physics" and "The Platonic Reality in Physics." It's all about an evolved notion of the Forms.

2.3.2 The Platonic Process in Physics

The Forms were by far the Platonic idea with the most impact in science and mathematics. His premise is that reality consists, not of only everyday stuff (that's the Ionian "monist" position that all of reality is made of matter) but that there is an additional reality-realm which consists of non-material entities outside of space and time. This is the premise of the movie *The Matrix* in which Morpheus gives Neo the choice of two pills: if he takes the blue pill, he's choosing to continue to live his life in an artificial but comfortable world in which we don't examine what's true and happily accept opinion as knowledge. If he takes the red pill, he's chosen the more difficult path: to live in the truth. The references to the *Allegory of the Cave* are obvious, but it's also the old biblical story of eating from the Tree of Knowledge.

Paying homage to Morpheus' red and blue pills, let's call our everyday, physical world, the **Blue World** (BW) and the ethereal, more truthful world, the **Red World** (RW...in order to help us remember, think of it also as the "Real World."). And let me try to suggest that to be a modern physicist is to be partly a Platonist, working as if the simultaneous existence of both the BW and the RW is the best way to make progress.

Plato's classical RW is where the Forms reside in which they had two broad characteristics:

1. Forms exist in the RW which are permanent, outside of space and time, and represent the essences of all things and ideas. All objects in the BW — objects we would call physical objects — "participate" in the Forms. My example was sofas: a BW sofa, the one we sit on, participates in the more essential, perfect RW Form of the Sofa. Sofa-ness is what our living room furniture participates in and in some sense the Form of the sofa is *in* our BW sofa. Plato also goes beyond BW objects to include lofty notions like Justice, Beauty, the Good, and so-on. And, properties: my wife's car is red and so is that apple. So "red" exists as an independent property of RW red things and so to a classic Platonist red things are participating in the Form of "red."
2. The RW contains the only true things and so acquiring Truth (with a capital "T") means somehow realizing the Forms in their natural, unusual habitat uniquely through our intellect.

So Plato's is both a story about ontology (the philosophy of what exists) and epistemology (the philosophy of what we can know).

The heated debates of the last 50 years about Platonism are largely about mathematics. In this literature it's not hard to find challenges like whether Sherlock Holmes is real, maybe the not-quite-real reality of fiction. Or whether the reality of a tree is different from the reality of $\sqrt{2}$. I think it's fair to generalize that there are broadly three schools of thought in the Philosophy of Mathematics that can be labeled as:

- Intuitionalism, where mathematics is just the product of mental activity and a mathematical entity is constructed by the mind and lives solely in the mind. This is also sometimes called "structuralism" or "constructivism."
- Formalism, is probably the most popular camp in which there is no truth-value assigned to any mathematical property or entity. It's all just the study of logical consequences... dubbed "if-thenism." There's no commitment to anything beyond manipulating marks on paper according to the rules of the game.
- Platonism suggests that mathematics is the study of abstract entities that do have an existence as objective as science is objective. So the question is: do abstract things exist?

I'm intrigued with a particular strand of Platonism that's due to **Willard Quine** (1908- 2000) in the 1950s through 1990's, and **Hilary Putnam** (1926-2016), who later found common cause with Quine. Together, their ideas are called the "Quine–Putnam indispensability argument" and a country-physicist (like myself) version of this is: science (read "physics") works and since mathematics is indispensable in physics, mathematical entities in the RW should enjoy the same level of reality as the objects of experiment in the BW. Why? Because physics works and it works this way. Quine and Putnam say that both the BW (physical reality) and the RW (mathematical reality) must go together or the standard idea of progress in physics falters.

> "[talk of" mathematical entities is indispensable for science... therefore we should accept such talk... [which] commits us to accepting the existence of the mathematical entities in question." Putnam (1971), *Philosophy of Logic*.

Quine called himself a "reluctant Platonist" and I think that physics has joined that club. And as I'll show, Galileo was the charter member and he showed us all how to make progress in unraveling MOTION BY THE EARTH and MOTION ON THE EARTH once the club's Platonism was embraced.

I've had the misfortune... or fortune... of doing physics research for half a century after a masters degree in the philosophy of science. That means that I've never been able to avoid standing back and looking at what I do and what my colleagues do and categorizing and analyzing process, what counts as a valid argument, what counts as a valid scientific question, and what counts as an acceptable answer.

I've concluded that we are relentlessly *both* Platonic and Pythagorean. We can't make progress nor explain the incredible success we've enjoyed without this joint PP connection. I'll talk about one of the early great Greek mathematicians in the next chapter, the Pythagorean, Archytas. His relationship with Plato is fascinating and he was on the receiving end of criticism from Plato over his research into music.

The Pythagorean fascination with the music-mathematics connection could have been due to Archytas; his teacher Philolous; or he inherited it from Pythagoras himself. In any case, Archytas' research on tones and the intricate mathematical arithmetic and mean values between tones, was sophisticated and beyond the bare bones description I outlined in the first chapter.

Archytas was an expert in the music of his time and explicitly studied the sounds *as he heard them* and Plato attacked, insisting that only abstract harmonies should be investigated using only intellect (not the ears). That's the difference between Pythagoreanism and Platonism. The former deals in the BW ("heard harmonies") while the latter shuns the BW and only deals in the RW ("abstract" harmonies). Archytas was Pythagorean because of his commitment to the BW of actual musical sounds.

What's a law of physics? It's really a statement about an abstraction. When we say that a projectile—a thrown ball, for example—travels in the path of a parabola, we assert that that's an important statement about how MOTION ON THE EARTH works. Look in any physics textbook in the last century and any student will come away with the conclusion that projectiles' paths are parabolic. But that's not true in our BW! Nobody—until we went to the Moon—has ever seen perfect parabolic projectile motion. All sorts of things get in the way: air resistance; deformation of the ball's surface; turbulence caused by a ball's rough surface; and variations of air density, temperature, and humidity in the path of the ball's trajectory. Each contributes to changing the shape of a trajectory from parabolic to something else. And yet, what do we consider the *actual theory* of projectiles? It's the basic set of kinematical equations that result in parabolas. Said differently: every projectile's path is different. There aren't as many laws of physics as their are thrown baseballs, there is only one law of physics.

How do we know that these law-like equations represent the general motion of thrown objects? We test predictions and that requires both P's: Platonism and Pythagoreanism. Here's a sketch of how I view the process:

1. I write down a model of motion that's in a perfect environment, let's call that theory, T which is a ball's motion in a RW. For all practical purposes, T exists for me and by writing it down in mathematical notation and solving the equations I'm working within a realm that's separate from the BW. Earlier, I referenced an oft-repeated question about Sherlock Holmes' existence. One version of modern Platonism suggests that fictional characters *do* exist, within a *possible* reality. Maybe my equation-writing qualifies as such. Additionally, to counter an argument that it's all between my ears only, I can communicate T in the form of those equations to another person. It's not in my head. It's public. My marks on the paper describe a possible world of abstraction that we share.

2. The question is: is T the right theory? That requires a test which is where

Pythagoras comes in.

3. Testing requires that I make a prediction of how the motion would manifest itself. That is, I need to provide predicted numbers (positions, times, speeds, etc.) to compare to experiments which result in measured numbers. I do that in two steps. T alone will not match the BW motion because of all of the "real world" effects that make the path deviate from perfect.

   - I must first ask myself what instrument I will use to measure motion in the BW. That tells me how precise my extrapolation from the RW to the BW must be. If my goal is to determine whether the ball comes down by Thanksgiving, the only time-measuring device I need is a calendar. If the goal is to understand its position every millisecond, then I need a more precise instrument.
   - Then I model the motion beginning with the RW mathematical theory, incrementally adding in the real world affects that would cause my BW measurement to deviate from the perfect RW model itself. If my measurement is crude, then perhaps air resistance is sufficient and I'll stop calculating after adding that effect to my original equations. If my measurement capability is refined, then I might add in additional perturbing effects. I only add as much as the measurement can detect.

4. The end result is a set of numbers which I predict will describe the motion of the ball in physical space and time and BW conditions.
5. If it matches well (within a precision appropriate to my measurement uncertainties), I'll declare that T is a good theory, even though by itself it doesn't do the job without the Pythagorean steps of computing numbers and matching numbers.[6]

I assert that I go from a **Platonic prescription to a Pythagorean application** when I make and check a prediction of a theory. I can't limit myself to the Platonic realm —its demand of perfection would be unsatisfactory. That paper–RW–reality is not our BW–reality. In the same sense, I can't start with the Pythagorean approach because there's no underlying theory on which to build the prediction.

Every single law or theory of physics follows this process and every one requires both P's.

This is not a philosophy book, but because the seminal mid-course discovery by Galileo is the adaptation of a philosophical idea, I wanted to set the stage. Throughout the book I'll take you through stories of discovery in which it will be evident that this Platonic-Pythagorean push and pull is the way to make progress.

[6]What if it doesn't match the measurement? Proponents of a naive view of Karl Popper's theory would demand that T be thrown out. Actual scientists would probably modify T...or more likely modify the real-world effects that were applied to T to model the BW effects. This, as you can imagine, is a philosophical argument. But not with physicists.

From Aristotle to Einstein, will be a steady accumulation of successes through this PP commitment. We don't even think about it any more. It's the nature of the profession.

One of the things that disturbs people about the Forms' existence—-and even Plato is self-critical in his dialogue *Parmenides*, is how does a RW idea get transferred to the BW by a human? Is there a third "mental world" in between the RW and the BW? I've argued that the intermediate stage is in part writing down the mathematics. It's concrete (doesn't disappear when I'm not looking), it's descriptive of a possible perfect world, and it's a language that is not personal but can be communicated and shared with others. But there's a more direct answer to this question of the RW–BW connection. Nature seems to have done it for us.

While the Process is my first commitment to Platonism. There's more. Go back to the *Analogy of the Divided Line* in Figure 2.1 on page 13. I'm about to argue that the separation between "mathematical truths" and "Forms" in the "Intelligible Realm" has been erased in modern physics.

2.3.3 The Platonic Reality in Physics

What I've described is process. But there's also an "ontology" —- a commitment, or maybe realization of what is actually real in physics that's thoroughly Platonic but in a 21st century way. There are many such realizations in "modern physics" but let me just note two.

2.3.3.1 Their Own Forms

There is no sofa that's identical to its form. Even two sofas designed and constructed in the same manufacturing facility will not be identical. Patterns on one will be slightly altered from the other. Tolerances on color or fabric structure or leg shape cannot be perfect. A BW sofa is not identical to it's RW Form.

The 20th century upended this very Platonic notion but Plato might have been intrigued with the result.

A molecule of hemoglobin in your blood contains 10,000 atoms of hydrogen, oxygen, nitrogen, and iron. Each of these atoms have protons, neutrons, and electrons. Isn't it remarkable that each of the many thousands of electrons in that single hemoglobin molecule are identical to one another?

Isn't it even more remarkable that each of those electrons in my blood is absolutely identical to an electron in an atom of hydrogen in the outer edges of the Andromeda Galaxy? Or to every electron that was flying around the early universe before atoms formed at 370,000 years after the big bang when the very first atoms of hydrogen were formed. (I might note that every hydrogen atom in your hemoglobin was in fact formed in the big bang.)

- **Elementary particles—electrons, quarks, neutrinos—are their own Forms.**
- **That is, $\mathbf{BW}(\mathbf{electrons}) \equiv \mathbf{RW}(\mathbf{electrons})$.**

Recall from the last chapter, I noted that an electron is "-1, $1/2$, 1, and 0.511." These numbers stand for very specific defining properties of what it means to be an electron: anything with electrical charge of -1, spin of $1/2$, lepton number of 1, and mass of 0.511 is an electron.[7] If Plato's craftsman was creating electrons from the material he had available, he would have created: "-1, $1/2$, 1, and 0.511."

So the distinction between Forms and the objects in the BW that participate in the Forms evaporates as soon as we begin to deal with elementary particles. That is, when we begin to confront the universe as it is composed in the BW. But the idea of the Form still holds rather perfectly for elementary particles and that mysterious connection between RW and BW from the last section, disappears. So a commitment to reductionism—the idea that the whole of reality is built from the small parts and that one law builds on another from the simplest to the most complex—is a commitment to the idea that *elementary particles are their own Platonic Forms*.

2.3.3.2 Are Wavefunctions BW Or RW Or Not Real At All?

Want some serious Plato? Quantum Mechanics is the theory of the very small: atoms, electrons, nuclei, elementary particles, and quantum fields. Atoms and all of chemistry is precisely determined by a single relation called the Schrödinger Equation which can be solved to determine the "state" of an atom, for example, and the spectra that result from electrons promoted to high energy levels and then fall to low ones with the emission of unique colors of light. Quantum mechanics is exquisitely precise and its predictions match experimental results to mind-boggling precision. It works better than any theory ever invented.

But Quantum Mechanics comes with a very strange commitment to a reality that we cannot see, hear, touch, or measure. Elementary particles are mathematically manipulated by calculating the evolution of the spooky entity called the wave function, ψ. The wave function seems to me to be the very definition of a RW-existent, mathematical entity. Essential to the physics, but existing outside of space and time (on paper only)—the very Quine-Putnam idea.

We describe the results of an experiment involving elementary particles by first preparing them in a beam or an atom and then mathematically evolving them in time using the functional form of the Schrödinger Equation that takes $\psi(t_1)$ at this time and tells you precisely where $\psi(t_2)$ will be at any time in the future. This works perfectly. Every time. But here's the rub: ψ is intrinsically undetectable. The only connection that ψ has with the BW and our measuring apparatus is the *probability*

[7] An alien culture might have different names for these four properties, but there would be only four and there would still be ratios and dimensionless properties that electrons would posses, which would grow out of *their* particular names or units.

that a particle will be here. . . or there. . . .or over there. . . or on the Moon. That comes from ψ^2.

Let me repeat: in a strictly determined fashion, we can calculate the value of ψ at any time or place in the future, but to connect with a measurement, we can only predict probabilities, no certainties are allowed. Ever. We cannot get from the equations of Quantum Mechanics to a measurement in the BW without passing through a RW Platonic manipulation of the mathematical entity, ψ.

This famously bothered Einstein, arguably the inventor of Quantum Mechanics! But if you ever needed a definition of a mathematical entity that behaves as if it has a reality in some realm, the wavefunction, ψ, is the poster child for exactly that. To make the most precise predictions which always have always matched the most precisely determined observables, we must work wholly inside of a very strange mathematical RW which indispensably (in that Quine-Putnam sense) is very real.

I'd like to suggest that these are the important modern-day influences of and reliance on an evolved Platonism (and Pythagoreanism) for physics.

2.3.4 Aristotle's Legacy in Physics and Engineering

This section is a little more detailed than normal, but the payoff is large! Aristotle left us a legacy which instantly became an active research project for ancient and medieval philosophers and eventually, present day philosophers, mathematicians, engineers, and scientists! He created a tool that guarantees how to properly analyze and judge conclusions reached through argument: Formal Logic. Read the next seven pages in detail for the whole story, skim them for a taste, or jump to the punch-line on page 43.

In everyday life, we all make arguments but have you ever thought about what makes you successful in defending your case? The facts need to be on your side but your stated reasoning should also be "logical." We all have a sense of what "logical" means, but it's surprisingly nuanced. Consider the following reasoning:

- Squirrels with superpowers can fly
- Rocky the Squirrel has superpowers
- Therefore, Rocky the Squirrel can fly.

This doesn't make sense because the first two sentences—the "premises"— are nonsense. And yet *it's a perfectly valid argument*! Appreciating the difference between a *valid* argument and a *true* argument leads us to Aristotle's amazing discovery that the rules of valid reasoning are due entirely to an argument's structure and arrangements of the sentences, not the specifics of the content. Your and my lives are now governed by Aristotle's invention of Formal Logic, his most important, lasting contribution.

Obviously, the distinction between *validity* and *truth* can be easy to spot. But the distinction between valid and invalid argument can be subtle. Think about these two arguments:

Table 2.1: How to not reason logically.

A	B
Those who take the vaccine stay well. Those who take the vaccine are smart. Those who are smart stay well.	Those who take the vaccine stay well. Those who are smart take the vaccine. Those who are smart stay well.

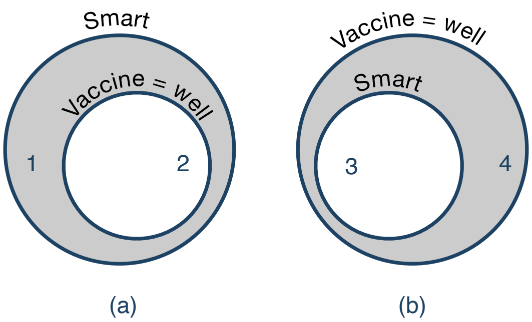

Figure 2.3: A diagrammatic way to show that argument A in Table 2.1 is invalid and that the conclusion of argument B is valid.

The argument in column A is invalid, not because the premises are ludicrous, but because of the form of the terms in the sentences. Read it very carefully with an eye on Figure 2.3. Notice how the righthand and lefthand circles are different (not really Venn diagrams, but a cousin, called Euler Diagrams). The first premise in argument A is that if you take the vaccine you're going to be well. So in the lefthand diagram, everyone who took the vaccine is in region 2. The second premise in argument A says that those who took the vaccine are smart, but it doesn't rule out the logical possibility that some smart people didn't take the vaccine—region 1. So the conclusion, that if you're smart, you're well does not hold.

Argument B says things slightly differently. Again, smart=well. But then the second premise says that if you're smart, you took the vaccine, so all of the smart people are in region 2 and, they're vaccinated. That, of course leaves the possibility that there are people who took the vaccine, but aren't smart, region 4. That's good! But not the argument which leads to a valid conclusion: Those who are smart stay well (and because of the first premise, they also took the vaccine).

2.3.4.1 Greatest gift

Aristotle's greatest gift to us was his invention of Formal Logic which is a rigorous way to judge the validity of arguments. For example, he could tell you that the argument in column **A** is not valid and why and tell you how to construct arguments like column **B** which *are* logically valid. Every time. And sometimes surprisingly, independent of the actual subject-matter of the argument.

> Officially, Formal Logic is the field that studies reasoning and the various ways that conclusions can legitimately be drawn from premises.

This new-born subject is covered in a number of his books, including: *Categories*, *On Interpretation*, *Prior Analytics*, *Posterior Analytics*, *Topics*, and *On Sophistical Refutations* which collectively, were much later dubbed "*Organon*" which means "instrument" which suggest by that time, Logic was viewed as just a tool, as opposed to a part of philosophy. Now it's firmly the philosophical camp and even an important part of an entire branch of mathematics called Discrete Mathematics.

Logic became a research program almost as soon as he wrote it down (or lectured on it) and two millennia worth of people—to this day—study logical formalism, expanding it into new directions. It's studied by every student of physics and engineering in forms directly evolved from Aristotle.

2.3.4.2 Deduction and Induction

Broadly, there are two kinds of logic which you use every day. The first works according to strict rules which I think of it as the *algebra of reasoning* and you'll see why in a bit. Reason according to those rules, and you will reach correct conclusions. This is **Deductive Logic.**

The second kind of logic is less certain since it's not rule-bound and it delivers conclusions which can seem persuasive but aren't certain. This is **Inductive Logic**. From this point, when I refer to "logic" I'll mean deductive logic.

Among things that are obvious to us (and to everyday Greeks), Aristotle seemed to intuit as requiring bottom-up attention. He tightly defined terms and "obvious" ideas, dissected arguments finding rules along the way, and set down what it means to be clear with exquisite precision. Look at these two statements:

- All squirrels are brown.
- No squirrels are brown

1) Can these both be true at the same time? Of course not and this obvious idea has a name: *the law of contradiction.* Aristotle needed to be precise and actually provided multiple "proofs" to demonstrate this principle.

2) One of these must be true... there's nothing in-between, which is called the *law of the excluded middle*.

   > "...there cannot be an intermediate between contradictories, but of one subject we must either affirm or deny any one predicate" Aristotle, *Metaphysics*.

Centuries of ink have been spilled over precisely understanding the implications of law of the excluded middle and how to symbolically state it unequivocally. But here's the first hint of our modern debt to him: his logic is two-valued, either true or false with no in-between. Hmm. Binary: True and false...one's and zero's.[8]

Last one:

- A squirrel is a squirrel.

This is called *the law of identity* and Aristotle didn't invent it and it sounds like Parmenides: "What **is**, **is**." These three ideas, collected together by him, are often called the Rules of Thought and were believed to be the bedrock for all of Logic. (That this was disputed in the 20th century shows that Logic is still a living-breathing subject.) Nobody ever thought this way before — so clearly—-and in Aristotle's patented approach to system-building, he lays it all out out exhaustively. As a master system-builder, he was the right man for the job.

His unique invention was to create an *algebra of language*. Here is a seminal moment in history, from the first book of his *Prior Analytics* (focus on the last sentences):

> "First then take a universal negative with the terms A and B. If no B is A, neither can any A be B. For if some A (say C) were B, it would not be true that no B is A; for C is a B. But if every B is A then some A is B. For if no A were B, then no B could be A. But we assumed that every B is A. Similarly too, if the premiss is particular. For if some B is A, then some of the As must be B. For if none were, then no B would be A. But if some B is not A, there is no necessity that some of the As should not be B; e.g. **let B stand for animal and A for man. Not every animal is a man; but every man is an animal.**" Aristotle, *Prior Analytics*.

I don't blame you if you get bogged down quickly in this quote. Look at the sentences that I've highlighted: he's using variables A and B, to stand for particular things, here in his example, A = man and B = animal. So his first sentence says for this particular case, "If no animal is a man, neither can any man be an animal." Instead of men and animals, you can plug in anything you want for A and B. It's the form of the argument, not the contents that determine whether the argument is valid.

Introducing variables as a placeholder for the subjects and objects in a statement is a seminal moment in the history of mathematics.

Amazing. Out of this, your mobile phone was born.

There are many different forms of arguments and for Aristotle, the **Syllogism** is just one of them. It's an argument written in a structure in which there are three

[8] Things didn't stop there. Now there is a multi-valued logic with degrees of truth and falsity with many engineering applications. "Fuzzy Logic" is a legitimate decision-making tool in transportation control systems, earthquake prediction, even home appliance efficiency.

sentences with a subject and a predicate[9]: two premises and a conclusion and inside those sentences are three "terms."

Here is one of the syllogistic forms:[10]

- premise 1: If all A are B
- premise 2: and if all C are A
- conclusion: then, all C are B

> There are actually 256 possible argument-combinations of subjects and predicates and 24 were thought to yield valid deductions. Maybe you can see why studying Logic became a matter of intense research following Aristotle's death and into the first 100 years of both Arab and Western philosophers. There was lots of work to do.

Let's make a syllogistic argument about squirrels. I'll define C = squirrels, A = the group of all animals in trees, and B = brown animals. One kind of syllogism would have the form:

- All mammals in trees (A) are brown animals (B)
- and if all squirrels (C) are mammals in trees (A)
- then, all squirrels (C) are brown animals (B).

Before I moved to Michigan, the only squirrels I'd ever seen where brown. Now my yard is full of black squirrels. They're everywhere. Yet, my argument above seems to prove that squirrels are brown. So what went wrong?

My "Squirrels with superpowers" shined a bright light on the premises: they have to be legitimate. In scientific arguments, premises might be ... hypotheses, in which case a deductive argument describes a way to test those ideas. Aristotle was well-aware of induction, deduction, and how they might go together.

Back to my squirrels proof. I reasoned inductively:

- (As a child) There's a brown squirrel
- (As an adult... many times) There goes another brown squirrel
- Wow... more brown squirrels and no other ones
- What is it with all of the brown squirrels?
- Gosh, all squirrels must be brown! (which was my premise)

Until I moved to Michigan. All it took to ruin my theory about squirrels was the observation of one black squirrel, much less an entire herd of them. Squirrels are

[9] since his Categories are predicates, these topics were a part of his overall system

[10] Before 500 CE, Aristotle's original form was used:

- If A, then B
- If B, then C
- So, A is C

not only brown, they're black. My proof founders on a false premise: "All mammals in trees (A) are brown animals (B)."

By the way, Sherlock Holmes is reputedly the Master of Deduction. Well, sorry. That's not true. If you look at his stories you'll see very, very few examples of deductive reasoning. He's the Master of Induction![11]

2.3.4.3 Your phone

Theophrastus (−371 to −287) was a favorite student of Aristotle's who led the Lyceum for 37 years after his teacher's death. Aristotle even willed him the guardianship of his children...and his library. While a devoted student, Theophrastus went beyond his teacher and expanded and modified some basic Aristotelian notions—extending a concept of motion to all 10 of the Categories, for example. He also moved the study of botany forward and worked extensively in Logic. Theodor Geisel (Dr. Seuss) used "Theophrastus" as a pen name.

He is probably the one who extended the form of argumentation into a new direction with the invention of "propositional logic" in which there are two items, rather than three of a syllogism. This is where the modern engineering action is. One form of such a proposition is called "Modus Ponens" (Latin for "method of affirming") which is an offshoot of the classical syllogism and is one of four possible "rules of inference." Modus Ponens goes like this:

- If A (the antecedent) is true, then B (the consequence) is true
- A is true
- Therefore, B is true.

Here, each line is a proposition (there can be more than two) with the first two being "premises" and the last, the "conclusion." The first sentence is a proposition which is conditional: the antecedent implies the consequence and it's "affirmed" if the next statement is true. B here is the consequence of A. Here's a concise way to present this:

- $A \rightarrow B$
- A
- $\therefore B$

The $\rightarrow$ symbol means "implies" and is associated with an "If...Then" kind of statement. The $\therefore$ symbol means "therefore." It doesn't seem like much, but it's powerful and misunderstanding (or misusing) it is the source of many logical fallacies. Table 2.2 shows an example:

[11] Or more appropriately, the Master of Abduction. Look it up.

Table 2.2: A typical logical fallacy involving public health.

A valid argument	A fallacy
• If a reactor leaks radiation (A), people nearby will get cancer (B). • The reactor leaks radiation (A). • Therefore, people nearby will get cancer. (B)	• If a reactor leaks radiation (A), people nearby will get cancer (B). • People nearby got cancer (B). • Therefore, the reactor leaks radiation (A).

The argument on the left is an example of Modus Ponens, while the argument on the right is a classic fallacy known as "Affirming the Consequent," a regularly exploited tool for those intentionally making invalid claims. Especially those who dispute public health strategies. Look at how the two columns are different. Remember, that in the proposition, B is the consequence of the antecedent, A and not the other way around. In the second row of the fallacious argument, the antecedent and consequence are reversed as compared with the valid argument. The fallacy is that people can get cancer from other causes than the proposition states.

Let's make a plan to picnic outdoors which requires us to keep an eye on the weather since if it's raining the ground would be wet and of course we wouldn't have a picnic if the ground is wet. We'd actually use Modus Ponens in our thought process and reason among ourselves:

- If it's raining, then the ground is wet
- It is raining
- and so the ground is wet.

Let's build a table—a picnic table (sorry)—that takes each line in the argument and makes it a column in a table. We could then ask a set of questions: Is it raining (Yes), is the ground wet (Yes)...was the proposition confirmed? Yes.

Table 2.3: The picnic is cancelled because:

If A, then B	it's raining?	it's wet?	A	B	If A is true and B is true, then:
If it's raining, then the ground is wet	Y	Y	T	T	T

There are actually four complete ways in which the antecedent and consequence could appear:

- rain? Yes or No
- wet? Yes or No

So what about: suppose the ground is not wet (wet = F) then can it be raining? Well...no (rain = F). So if wet = F and rain = T, then the proposition would not be true since rain should imply wet. We can build up these four conditions into what is called Truth Table, which was invented in the early 20th century as an analyzing tool. Table 2.4 describes the complete story:

Table 2.4: All of the logical possibilities for two pieces of a conditional premise: raining and wetness. Here's a picnic table (sorry):

If A, then B	it's raining?	it's wet?	A	B	If A is true and B is true, then:
If it's raining, then the ground is wet	Y	Y	T	T	T
If it's raining, then the ground is not wet	Y	N	T	F	F
If it's not raining, then the ground is wet	N	Y	F	T	T
If it's not raining, then the ground is not wet	N	N	F	F	T

Sometimes these are hard to unravel. The first two lines are pretty obvious. It's asserted that when it rains that the ground is wet, so the second line is obviously false. The proposition requires "wet" with rain. The last line is pretty clear also. No rain, let's picnic since it will not be wet. The third one requires some thought. What does the if statement say about the ground if it's not raining? Nothing. You could be wet for other reasons so this does not falsify the proposition, so it's not F...and in a two-valued logic, the only alternative to F is T. Go lie down before we go on because it's about to get interesting and relevant.

Before getting to the punchline, let me make a couple of points:

- The $\rightarrow$ or if...then argument is one of six "connectives," all of which have truth tables like above. They are negation, conjunction ("AND"), disjunction ("OR"), conditional (that's the $\rightarrow$ conjuctive), biconditional, and exclusive OR.
- The Modus Ponens argument got its Latin name from the Medievals who seriously studied Logic. They identified it as one of four "Rules of Inference" which we use today: MP, Modus Tollens, Hypothetical Syllogism, and Disjunctive Syllogism.
- The Hypothetical Syllogism is just one form of the "regular" syllogism of our squirrel proof above. In fact, it can actually be proved to be the combination of two Modus Ponens arguments, one for A $\rightarrow$ B and the other for B $\rightarrow$ C. There's debate about whether Aristotle might have recognized his syllogism to have been an "hypothetical" in this sense with a deeper structure.
- In Appendix B I've gone into some more detail logic gates as they're used in digital circuit design.

There are a handful of seminal discoveries about Logic that extend to our modern reliance on it. **Gottfried Wilhelm Leibniz** (1646–1716) refined binary arithmetic. In 1854, **George Boole** (1815–1864) invented the algebra of two-valued logic...how to combine multiple conjuctives into meaningful outcomes which can only be T or F, 1 or 0. In 1921 in his dense and very terse *Tractatus Logico-Philosophicus*, **Ludwig Wittgenstein** (1889–1951) invented the Truth Table, which can be used in logical proofs and complicated logical solutions to multi-variable inputs. Finally, in 1938 **Claude Shannon** (1916–2001) realized that Boole's algebra could be realized in electronic, "on-off" circuits. This was realized in the 1940's with vacuum tubes and then in the 1960's with transistors.

Notice that the picnic table can be thought of as a little machine: you input the four T-F possibilities in pairs for rain and wet and out comes the truth value of the proposition. Figure 2.4 is a cartoon of such a machine.

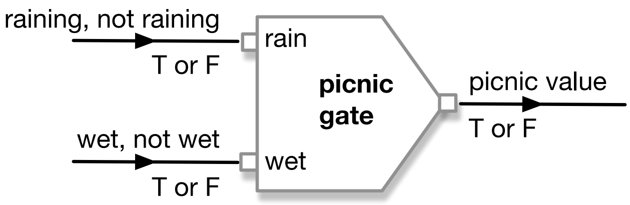

Figure 2.4: A fake "picnic gate" machine that does the work of Table 2.4

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The image in this figure is maybe suggestive of digital component representations which are called "gates." There are electronic gates for eight functions, which are a practical expansion of the conjunctives mentioned above. Think about that. The whole of our digital world can be made with these eight gate functions.

What I wanted to show you is that your entire life now is based the ancient Greek Logic research program. For example, the 2022 iPhone 14 has 18 billion transistors in it and every one of them speaks through Aristotle to get their individual jobs done—or I should say their collective jobs done, since their language is forming and evaluating billions of logical two-term arguments in the same spirit as our raining-wet table.

2.3.4.4 The Punch Line:

Let's review what just happened:

We've found that Aristotle made a simple but profound discovery, namely that one could take a sentence, like "Fire engines are red or yellow" and turn it into essentially a mathematical statement, like "A are B or C" and then draw general conclusions about the combinations of general statements that don't involve the details. That sentence involving A, B, and C could also be a representation of the sentence, "All squirrels are either black or brown." This allowed him to then create a system of rules that could guarantee the validity of arguments, which, after all, are combinations of sentences.

The first kind of argument is now called the "categorical syllogism," and involves three variables and, like fire engines and squirrels, can be specific or more usefully, general, like:

All men are mortal.	A are B
Socrates is a man.	C is A
Therefore, Socrates is mortal	therefore, C is B

This evolved quickly into a rules guaranteeing validity of conclusions from a different form of argument involving two variables (an "hypothetical syllogism"):

If all men are mortal, then Socrates is a mortal	If A, then B.
All men are mortal	A is true.
Therefore, Socrates is mortal	therefore, B is true.

In fact there are variety of valid forms for each sort of argument but what's interesting in the second sort is that the truth value of arguments involving two variables can actually be created using electronic circuits using tables ("truth tables") of the different logical outcomes of the truth or falsity of the premises in an hypothetical syllogism. This was realized in 1938, built into vacuum tube circuits in the 1940's, and transistor digital electronics in the 1960's.

The first digital computers relied on thousands of vacuum tubes and filled whole rooms with hot, clunky racks of tubes and wires—your phone has 10s of thousands of times more processing power than these first early 1950s computers. When the transistor became commercially viable in the 1960s the digital world came alive.

In the spirit of overview, Figure 2.5 shows two transistor arrangements and their modern "gate" symbol—please don't worry about the details! Just for flavor. (a) is the layout for a common transistor package that does the job of the logical gate symbol shown in (b). It's the NOR operation. A comes in, and NOT–A comes out. (c) is another transistor layout that has two inputs and produces the logical OR combination, and (d) is the logical gate symbol for performing that operation. Finally, (e) is the digital gate solution for the Conditional argument from Table 2.4—it's a real-life engineering representation of the fake "picnic gate" in Figure 2.4.

With binary arithmetic, gates can be combined to do arithmetic functions, logical functions, and importantly, storage of bits. Digital memory consists of four so-called NAND gates, and so four transistors and is the basic cell of a computer 1-bit memory. It's a clever implementation of an input bit—to be stored—and an enable bit—which allows the output to change or not change.

All of these—and more—transistor components are actually imprinted in tiny silicon wafers in which a single transistor package might be only 20 nanometers in size. With the logical functions and the manufacturing techniques of today, my

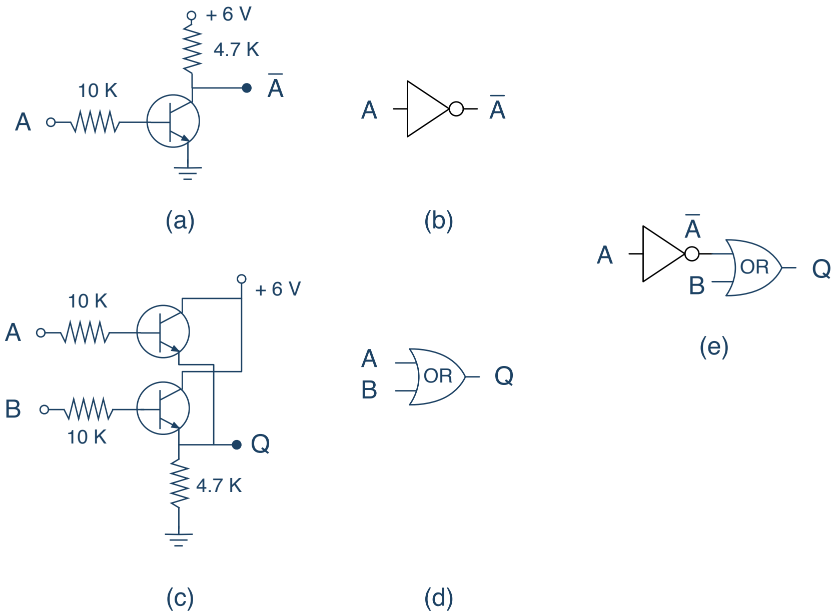


Figure 2.5: (a) and (c) are the transistor-equivalents of the two logic gates, NOR and OR in (b) and (d). The little circuit to evaluate rain causing wetness...or not...is shown in (e).

current Apple Watch has 32GB of random access memory (RAM) and so it can manage 32,000,000,000 Bytes of information, which is 25,6000,000,000 bits and so 102,400,000,000 individual transistors are inside my watch, just for the memory! The CPU and control circuitry would add millions of additional imprinted transistors and their gate-equivalents. All on my wrist. All speaking "Aristotle."

Obviously, the 2500 year path from Classical Athens to kitten videos on YouTube is full of breakthroughs and smart ideas. But it all started with Aristotle.

Chapter 3

The Most Important Mathematician You've Never Heard Of : Eudoxus and Greek Astronomy

"His [Aristarchus'] hypotheses are that the fixed stars and the Sun remain unmoved, that the Earth revolves about the Sun on the circumference of a circle, the Sun lying in the middle of the orbit, and that the sphere of fixed stars, situated about the same center as the Sun, is so great that the circle in which he supposes the Earth to revolve bears such a proportion to the distance of the fixed stars as the center of the sphere bears to its surface."

- Archemedes, *The Sand Reckoner*

I'll bet that many of you have seen the solar system arrangement as imagined by Copernicus with the Sun in the center and all of the planets, including Earth, obediently orbiting it in perfect circles. While we tend to think of our solar system as Copernicus' (surprises await in Chapter **??**), the roughly 500 years since is a short era relative to the competing view of his time. He was challenging the ancient, and universally-held idea, that it's the stationary Earth that's in the center of the universe. Fascination with that picture as is prevalent in many decorated medieval manuscripts through the centuries. One of the earliest is from a poem by the Greek poet, Aratus called *Phaenomena*, named after a book of the stars and constellations by the Greek mathematician, Eudoxus, who created that 2000 year old "geocentric" model of the universe—one in which the Sun, Moon, planets, and stars all orbit around the stationary Earth. Figure 3.1 is from a 10th century version of *Phaenomena* from the British Museum.

I took some pains in the last chapter to underscore that mod-

Figure 3.1: Aratus the poet lived about a century after Eudoxus (and hence, Aristotle) and turned his astronomy book into a poem. Later, Cicero translated it and this 10th century manuscript is an illustrated copy of that work. https://sarahjbiggs.typepad.com/.a/6a013488b5399e970c01bb07c8696d970d-pi

els of motion on the Earth belong in Aristotle's corner. He was beholden to few for his physics and really invented the dynamics of motion. But while we tend to ascribe that geocentric model of the universe to him as well, he borrowed it lock stock and barrel from Eudoxus and Plato. We'll develop that in this chapter and go beyond them.

The Greek world—indeed, the whole world—was radically and violently altered by Alexander the Great and during the period between Aristotle and Cleopatra, astronomy become an experimental and quantitive science. The culmination of Greek astronomy came after Greek-everything became Roman-everything and just before the Roman Empire began its decline. One last Greek, in our long string of Greek philosophers, mathematicians, and scientists remained and we'll close our chapter with Ptolemy's calculating-model of the heavens.

A game that many scientists play is to trace their scientific lineage back for centuries—their major professor's professor and so on. I followed mine back through centuries and found that I descended from Copernicus! I'd like to think I've made him proud.

Sometimes it turns out that someone's student ends up in the history books. But not many students actually take over the known world by force!

After nearly 40 years of the Peloponnesian War , Sparta had to govern. Not just Athens, but all of the vast Athenian empire which had been loosely held together by agreement, but in which Sparta had to govern through force. They felt relentless pressure from everywhere—especially from Thebes, Athenians, and the emboldened northern Macedonian threat. Remember that "tumultuous" is the signature skill of political Greeks: during the time of Plato and Aristotle, if you randomly threw four colors of paint at the wall, you'd pretty much replicate a map of the four different allied and opposing regional Greek and Macedonian states.

When Plato died, encouraged by the Macedonian King Philip II, Aristotle relocated, first to the northern Aegean and then to Macedonia, acceding to a royal summons to teach his 13 year old son, Alexander. There he set up a school, taught Alexander (and perhaps the future general, Ptolemy) for three years, and then stayed for seven more before returning to Athens where he started the Lyceum. By this time the teenaged Alexander was already on the battlefield and with his father, had occupied the entirety of the Peloponnese. So Athens was once again ruled by outsiders—connect to Aristotle.

After Philip II was assassinated,[1] Alexander, soon to be "The Great," ascended to the throne and began his brutal lightening-fast, nine year conquest of three continents of the entire known western world: modern Turkey, the middle east, Egypt, and Arabia all the way across Afghanistan to India, leaving military oversight over Athens and the rest of Greece. While he stayed in touch with Aristotle, sending him samples from all over Asia, his teacher became distant, put off by Alexander's adaptation of Persian customs and persona.

Alexander died in Babylon in -323 under suspicious circumstances and Ironically, within a year Aristotle himself died at the age of 63 of natural causes at his mother's family estate outside of Athens. Athens had turned on him and his Macedonian connections were dangerous. Impiety was charged, a death sentence issued, and he fled uttering his famous remark about the city not sinning against philosophy for a second time. In his absence, the Lyceum stayed active under new management for another century.

Meanwhile, in the "Partition of Babylon" Alexander's senior commanders divided up the sprawling kingdom among a dozen generals and aides and then did what came naturally: they fought among themselves for 40 years. In the end, three kingdoms and a dizzying array of city-states were established: Macedonia and

[1] Assassination, murder, and betrayal were a family hobby.

Greece, Seleucia (roughly modern-day Iraq), and Egypt.

Hundreds of thousands of Greeks migrated into the newly organized territories establishing an international Greek-ness of culture, arts, and philosophy which was the beginning of the **Hellenistic Age.** During the final two centuries, BCE, "Greek" ceased to be the province of only a loosely-constructed, turbulent nation but rather characterized the entire, western world. Of the two dozen cities that Alexander created or conquered named for himself, the "Alexandria" that mattered most to him, and to us for physics and history, was the Egyptian Alexandria, a new port city on the Mediterranean.

Egypt became unusually secure under Alexander's former body guard and general, **Ptolemy I Soter** (-367 to 282) who eventually fashioned himself, "Pharaoh." He adopted Egyptian customs, including that of rulers marrying their siblings, and was an intellectual of sorts, creating the first state-supported national laboratory and library. The "Alexandrian Museum" was a national facility devoted to research and among its first recruits was the mathematician, Euclid who then wrote *Elements*, the most-read book in history, besides the Bible. For 2500 years, from Copernicus to Thomas Jefferson, mastering *Elements* was the route to mathematical literacy. Ptolemy found it rough-going and asked for an easier way to learn it, but was told by the author that "...there is no Royal Road to geometry," a sentiment still relevant today. For centuries the Museum was home for scores of Greek scholars, all supported by the dozen Ptolemy's from the Ist to the final, Cleopatra.

The Library of Alexandria probably contained all of the known—and now mostly lost—manuscripts of all of the philosophers, poets, playwrights, and doctors we've discussed in the previous chapters. There was a hunger for knowledge of all sorts and Ptolemy and agents of his library director searched every ship that docked, stealing or copying any books on board. They rented or stole manuscripts from all of the major cities for copying. The Library and Museum suffered through a couple of wars, including during Marc Antony's escape, and probably survived until the conquest of the Muslims in the 7th century CE.

Among the scores of Alexandrian scientists are the astronomers whom will figure into this part of our story: Eratosthenes of Cyrene, Aristarchus of Samos, Eratosthenes, Hipparchus, and especially Claudius Ptolemaeus. Essentially, only Heraclides of Athens and Apollonius of Perga played major roles outside of Alexandria. The Ptolemy dynasty lasted 300 years until the legendary feud involving "the" Cleopatra (a common name for female Ptolemy-family successors) and Marc Antony and Julius Caesar. The Library and Museum lasted into the first five centuries CE until it was eclipsed by great Muslim libraries in Babylon, Cairo, and Cordoba in Spain. Astronomy is our concern in this chapter.

3.1 A Little Bit of Eudoxus

Recall that Philolaus was the source of Plato and Aristotle's knowledge of Pythagoreanism—for example, the "Pythagorean" cosmology came through him or probably originated from him. Was he a student of Pythagoras? The dates of their overlaps almost work out to imagine that relationship, but it's controversial. He's certainly the closest we get to the great man so it's not far-fetched to continue the teacher → student theme that began this chapter: Pythagoras → Philolaus → Archytas → Eudoxus. Lunar craters are named after each which is not the normal teacher-student legacy.

Eudoxus of Cnidus (circa −408 to around −355) was the son of a physician and became one himself, but we know of him as a gifted mathematician and astronomer. As we'll see, astronomy and medicine were connected through astrology and mathematics and astronomy have always been kin, so these seemingly disparate skills go together. Cnidus was a city founded by Sparta on the southern Aegean coast of modern Turkey and was where he started... and finished, between which times he traveled all over the Aegean to study and teach. As a young man he went to Tarentum to study mathematics with the pre-eminent Pythagorean mathematician (and much more) **Archytas of Tarentum** (−428 to −347). Let's learn a little bit about Archytas in Figure Box 3.2 on page 52. After you've read about him, return to this point ☟ and continue reading about his student, Eudoxus.

After his mathematics instruction, he went to Sicily to study medicine, then by the age of 23 he went to Athens and stayed briefly (and apparently, unhappily) with Plato's Academy (rooming 7 miles away, so a long commute to lectures). After less than a year, he was back on the road to home in order to raise funds...so that he could travel even further to Egypt and with what we'd call a scholarship, studied astronomy there for 16 months, before leaving for the northern modern Turkish Black Sea coast and the Greek colony of Cyzicus. By this point he's lecturing on his own and established a popular school and an observatory. With data from his observing in the north and from Egypt, he published his first book, *Phaenomena*, which was a compendium of star locations and *On Speeds*, of their motions.

FIGURE BOX 3.2

The figure on the left is a famous engraving (by an unknown artist...maybe late 18th century) suggesting an ancient sentiment due to Archytas, a friend and competitor of Plato. Among the most famous arguments in cosmology is whether the universe is infinite or finite in size and Archytas had the first of many similar inspirations that the universe cannot be finite: He did a thought experiment, imagining traveling to its presumed edge and attempting to thrust his stick beyond that limit. If he could extend it, then, well, that's not the edge...and so he'd have to go further, repeating the experiment without end. This is a good example of the kind of intuitive cleverness that seemed to be built into this great Greek mathematician, politician, and military leader. The very model of a modern major—Pythagorean— general. As I noted in the last chapter, Archytas was a committed Pythagorean and a mathematician of great skill. But he also he was a civic leader and an elected military general, in spite of Tarentum law, re-elected seven times because he never lost a battle. (Did I mention that Greeks fought constantly?) When he did step down, the army started losing.

Archytas was reported to be an even-tempered, cultured man who led Tarentum through a period of democracy and that Aristotle apparently wrote more (lost) books about Archytas than any other person. There is some evidence that he wrote a book on mechanics and that he enjoyed making mechanical toys for children—very un-Plato-like in spirit.

His mathematical skills were legendary and he solved an old problem with mystical roots: Apollo sent a plague to the city of Delos and a delegation was sent to Delphi to learn from the Oracle how to rid themselves of the pestilence. The instructions were to take their cubical altar to Apollo...and build a new one with double its volume. This is called the problem of "duplicating the cube" (also called the Delian Problem) and it required cleverness on Archytas' part, beyond just geometry, which caused Plato to disparage his effort. Archytas contributed to many branches of mathematics and Euclid's *Elements* includes some of his' proofs.

All in all, Archytas was the most accomplished Pythagorean of all and in the spirit of the opening to this chapter, we're indebted to him for his products, but also one of his students. The most accomplished of all Greek mathematicians before Archemides, Eudoxus, from whom 2000 years of cosmology originated. Now go back to page 51 and pick up where you left off.

Around −368, during his 30s, he moved his school to Athens, by which time Plato is 60 years old and Aristotle has left for Macedonia. His arrival in Athens may have had a diplomatic purpose since, of course, a war broke out between Athens and Persian forces which oversaw much of the Ionian colonies. An over-extended

Athenian army enjoyed a short-lived victory and then faced the threat of a full-on conflict with Persia and there's reason to believe that Eudoxus' relocation to Athens helped to calm the situation. It was here, as the legend goes, that Eudoxus was challenged by Plato to form a geometrical model of the heavens. The legend is unlikely the case as by this point, Eudoxus was the mathematical champion of the Greek-speaking world and more likely to issue challenges, than accept them. As we'll see below his model was born and in various guises, persisted until Galileo, Kepler, and Newton.

His mathematical skill was by that point unprecedented. As a student of Archytas, he was well aware of the Pythagorean horror of irrational numbers (recall Hippasus's reputed unfortunate end by discovering that the Pythagorean Theorem as applied to triangles of length 1 for the legs leads to the hypothenuse of length $\sqrt{2}$). Archytas also tutored Plato, as you'll recall, so their mathematics had common origins. But Plato's skill was no match for Eudoxus'. He nearly perfected the mathematics of proportion, demonstrating that lengths corresponding to irrational values could be compared to those of rational values. He also (perhaps invented) the "method of exhaustion" in which the area of a circle, for example, could be approximated to any degree of precision by adding up the areas of triangles positioned side by side with their apexes at the center. The more triangles, the better the area derivation. This is of course a precursor to taking a limit in calculus and was expanded on by Archemedes. He also showed that in comparing the area of one circle (a curved shape) to another, was related to the square of their radii (linear quantities) and extended that to spheres. Eudoxus' work was memorialized in a number of Euclid's *Elements*.

It's Eudoxus' astronomy and cosmology that are our concern here and so let's work up to that with a review of the problems that everyone in antiquity faced when trying to describe what we observer from Earth and then work through Plato's ideas that formed an almost linear line of inspiration: from Pythagoreans, to Plato, and to Eudoxus.

3.2 A Little Bit of Greek Astronomy

Greek cosmology came in fits and starts and I alluded to some of it in Chapter 1. The biggest export was Aristotle's model of the cosmos with its Earth-centered ("geocentric") description of MOTION BY THE EARTH. It became popularized, petrified, and deified when it was officially incorporated into Church dogma after the work of Thomas Aquinas in the late 13th century. So from that point until the Baroque era, Aristotle reigned supreme. He was revolutionary and inventive in so many areas, so it's amusing that his cosmology model had the longest run and that it was almost entirely due to Plato.

3.2.1 What We Know Now

If you go outside after dark and watch the sky throughout a few hours, you'd probably reach same conclusions as Greeks and all of antiquity:

1. We seem to be stationary (you don't feel like you're moving).
2. Through the night the whole panorama of stars seems to revolve east to west around an axis that points to through the Earth's north pole. It's as if we're at the center of a vast rotating sphere with the stars painted on the inside. The axis of this sphere — called the "Celestial Sphere"— of stars goes through the rotational axis of the Earth, so the Earth's equator is in line with the equator of the Celestial Sphere.
    (a) However, the times that stars begin to appear on the eastern horizon changes each night by four minutes early out of 24 solar hours, which is called "heliacal rising." This time advances through the year and the "ascendency" of stars in the east became milestones on a "calendar" that people could use to predict when events to happen. For example, when each year the bright star Sirius in the constellation Canis Major appears in the eastern sky just before dawn, Egyptians they knew that the Nile's flooding was coming.
3. There are a few brighter objects (the planets) which each independently execute similar east-west motions through an individual night. They have some puzzling features:
    (a) The planets (and the Sun and Moon) appear to make their way across the sky in a "lane" or a strip about 14° wide. The constellations of the zodiac are distributed around the sphere within that strip of the sky.[2]
    (b) The center of that lane (called the "ecliptic") is inclined by 23.5° to the Earth's equator.
    (c) The planets' brightness changes during a year.
    (d) The star patterns seen behind the planets change from night to night.
    (e) When watched night after night, some of the planets visibly move backwards against the constellations behind them. This is called "retrograde motion" and is described below.
3. From the northern hemisphere, there is a fuzzy broad stripe of stars across the southern sky.
4. The Moon has its own independent motion against the background stars which shows dramatic phases on its surface of light and dark through a month.
5. During the day, the Sun also has its own east-west motion similar to the nighttime objects. If we could see the constellations behind the Sun (during the day!), they would be different through the year.
6. The seasons are not the same length—spring and summer are longer than fall and winter.

[2]There are 13 zodiac signs, but that's inconvenient for astrologers so they ignore one.

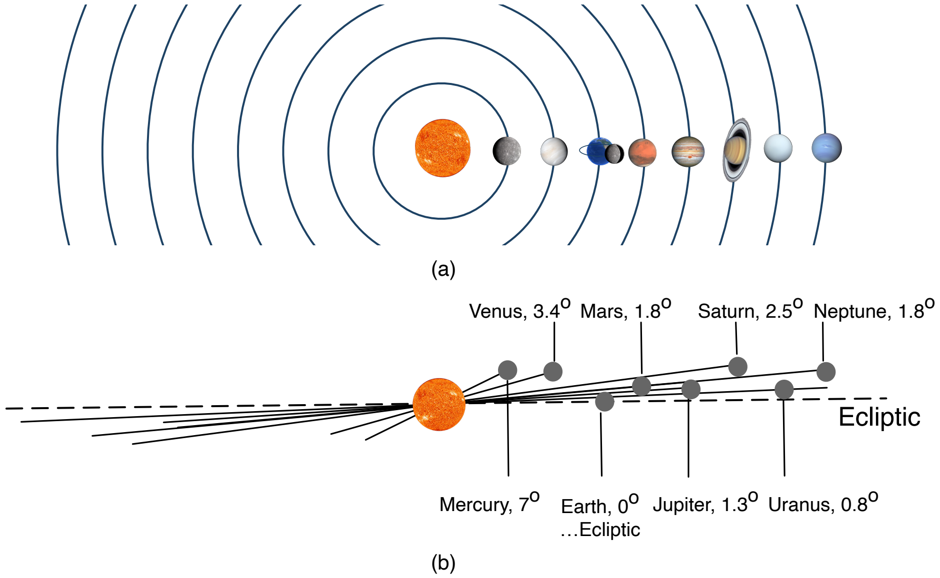


Figure 3.3: A sketch of the solar system. These sketches are not in scale and the lining-up of the planets is artificially arranged for simplicity. (a) is a top-down view showing the ordering: Mercury, Venus, Earth (with its moon), Mars, Jupiter, Saturn, Uranus, and Neptune. (b) is a side view with the various orbital planes of the planets as compared with that of the Earth which we define as the ecliptic plane. This defines the 14° (twice the 7° inclination of Mercury) strip of the sky in which all of the planets, Sun, and Moon move as they pass overhead as viewed on Earth.

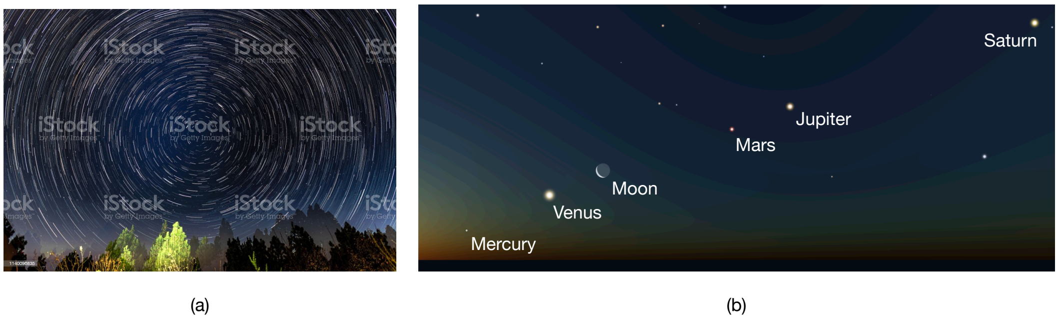


Figure 3.4: The figure on the left is a time-lapse photograph of the north celestial sphere (essentially pointing the camera at Polaris, the North Star). One can clearly see that the stars in the sky all follow circular tracks around that pole position. The figure on the right shows the eastern sky just before dawn in East Lansing, Michigan on June 24, 2022 in which five of the planets plus the Moon and hint of the rising Sun are all in-line along the ecliptic. (timeanddate.com)

Other ancient observing cultures collected data but didn't try to interpret. The Greeks, however, needed a materialistic model that explained all of these seemingly unconnected motions and patterns and that urge to account for what they saw grew with time, culminating with Aristotle and Ptolemy.

Let's set the record straight from our more advanced vantage point: every one of the above points is explained overall by a Sun-centered solar system in which the Earth and other planets orbit. From the outside–in, we know that our solar system is built of seven planets (Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune). We know that their orbits are not circular, but slightly elliptical, with the Sun at a focal point. This explains why the seasons are different durations since one side of the long path of an ellipse is closer to the focus than the other side. Figure 3.3 (a) shows an overhead sketch of the orbits of the planets. (This is grossly not to scale, circular for simplicity, and their co-alignment is just for display). Below that is an edge-view sketch of the individual planets' orbital plane inclinations. We take Earth's orbital plane to define the ecliptic (0°) so relative to that, Mercury's orbit is the most inclined at 7° from the ecliptic. All of the other planets' orbits are within that 14° band of $\pm 7^\circ$.

If you were to stand in the middle of your room and spin around you would see the a door, a window; a chair; a picture; your sofa; and, as you got all the way around, that original door after you've made a complete revolution. This is exactly what happens when we look up at the sky and see the stars appear to move around us. In 24 hours, it repeats. Because the Earth rotates in 24 hours, not the sphere of stars.

Figure 3.4 (a) shows a time-lapse photograph of the sky, looking at the north star, Polaris showing the coherent rotation of all of the stars described in item 2. above. In (b), an image of the eastern sky on June 24, 2022 just before dawn from East Lansing, Michigan. Notice that the sun is just peeking over the horizon and Mercury, Venus, the Moon, Mars, Jupiter, and Saturn are all nearly in a line along the ecliptic.

If your friend walked by you very, very slowly while you were spinning, you'd see them, first against the picture then as you spun all the way around a second time, then against the sofa, and then after another spin, against then the door. This is like how we see the planets, which have their own motions around the Sun along with Earth's. If viewed night after night (one revolution of 24 hours), the planets will line up against different background stars each night.That's because the planet does have its own (slow) motion and because each night we look at each planet from a *different point* on Earth's orbit—relative to what goes on in the solar system, the background stars don't

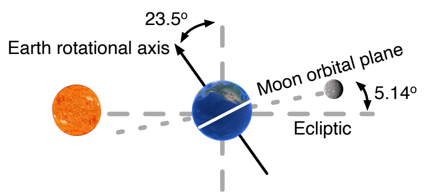


Figure 3.5: The inclination of the Earth's spinning is oriented away from being perpendicular to the ecliptic in which the Earth's orbit is fixed. Also, the orbital plane of the Moon's orbit around the Earth is slightly inclined relative to the ecliptic as well.

move. But Earth and the planets move relative to one another in successive nights and their positions can appear to migrate *backwards* during some nights...and then reverse course and those positions commence in the direction. Figure 3.6 is a sketch and explanation of how that happens.

The Earth's axis of rotation is not perpendicular to the ecliptic plane, but is tilted by 24.5°. That explains why the ecliptic equator is inclined by that amount from the Earth's (and hence, the Celestial Sphere's) equator. The whole Earth actually wobbles over centuries like a top and so the Greeks would not have seen the axis of rotation to point at Polaris. That's just an accident of our epoch.

We don't have solar and lunar eclipses every month! That's because, like the planets, the Moon's plane of its orbit around the Earth is inclined by 5.14° and so while predictable, these eclipses are rare. Both of these last two phenomena are illustrated in Figure 3.5.

Finally, that broad fuzzy strip of light that's seen in the southern sky in a dark night is us looking through the Milky Way galaxy on edge. Our position in our galaxy is in an outer arm.

Without modern astronomical techniques, the Greeks (and everyone for the next 2000 years) had their hands full trying to explain what they saw. Let's see how what they came up with and why it was so hard to construct and why it lasted so long.

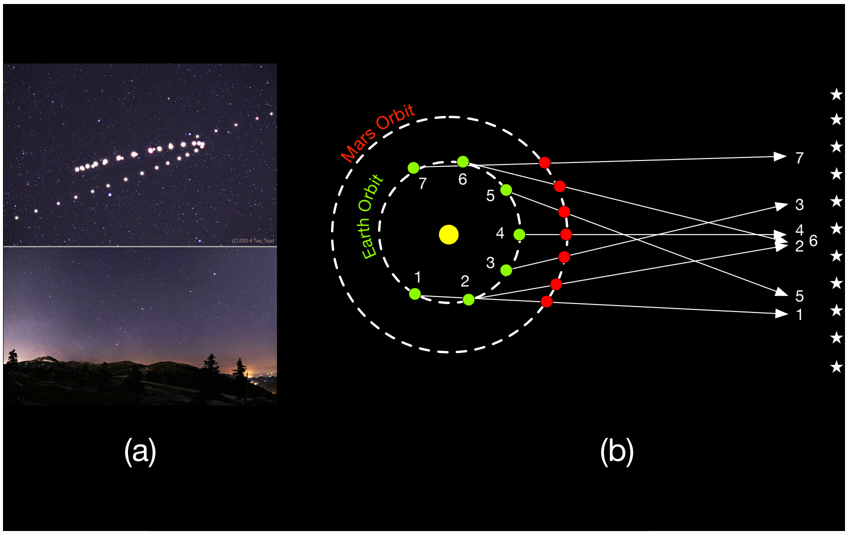


Figure 3.6: Retrograde motion by Mars. In (a) the sky in Turkey shows a photograph of Mars from December 5, 2013 in the upper right hand corner and then an overlayed photograph taken every five or six nights until August 8, 2014. The looping behavior in the middle is the retrograde motion. (b) shows how this happens (see the text for an explanation) https://twanight.org/gallery/tracing-the-red-planet/?preview=true

3.2.2 Greek Astronomy, Presocratics

Let me remind you of the Presocratic cosmologies that I touched on in Chapter **??**. As I emphasized, what we know of the Presocratics is 2nd, 3rd, 4th... hand, but Aristotle was the first to systematically chronicle their approach to astronomy and cosmology.

Attributed to Thales was a picture of a flat Earth floating on water, surrounded by a river. He inherited this story from even more ancient Egyptian ideas but used it to his advantage in order to explain how earthquakes happen. So for him, MOTION BY THE EARTH was nil. In fact in Homer's *Iliad*, the description of the shield Achilles used in his famous battle with Hector at Troy contains a number of astronomical allusions, including the Oceanus river around its rim and reference to a "starry," solid dome held up by pillars.

His student Anaximander made a brilliant leap to an entirely different picture in which the Earth (a squat cylinder for him) is not held up by anything, but at the same time it's not moving because, he said, there's no particular reason to move since all around it is the same:

> "The earth is aloft, not dominated by anything; it remains in place because of the similar distance from all points [of celestial circumference]." attributed to Anaximander

This is either reasoning by a symmetry argument or reasoning by reference to balanced forces, or both. Either way, it's very much a mathematical inference.

> "Even if we knew nothing else concerning its author, this alone would guarantee him a place among the creators of a rational science of the natural world." (
> citekahn1994).

So for Anaximander, MOTION BY THE EARTH is zero as well.

In fact, most of the Presocratics believed that the Earth is flat, including many of those who came after Parmenides.

Parmenides had a number of original ideas about the heavens. He was the first to conceive of the whole universe as being spherical and finite.

> "... like the mass of a well-rounded sphere, from one middle, equal in every respect." Parmenides

He was apparently the first Greek to note that the Moon must be spherical and was even poetic about it:

> "[the moon is a body] shining by night, wandering around earth with borrowed light..." Parmenides

"Borrowed light" is a nice phrase. If the Moon "borrows" its light from the Sun and doesn't emit it on its own, then the shape of the phases of the Moon lead to the spherical shape conclusion. Thales is sometimes credited with this observation, but it's not mentioned in any of the commentaries on him from before Parmenides. It was traditional to credit Parmenides with extrapolating from a spherical Moon to declaring that the Earth, too, is spherical. But that's not authenticated and Pythagoreans' claim to a spherical Earth is perhaps more likely. After the Eleatics, Anaxagoras and Empedocles both assume that the Moon reflects the Sun's light and even formed a model for eclipses.

Ironic, isn't it that Parmenides can perhaps be credited with a scientific discovery— one that requires observation— when we tend to think of him as anti-scientific (which is quite a feat given that science really hadn't been born yet).

The Pythagorean team extrapolated their fondness for the number 10 into their cosmology through Philolaus and and we learn of their cosmology largely through Aristotle, and his students. In the haze that followed over centuries, the spherical shape of the Earth was attributed to Pythagoras, but that too is sometimes disputed. What's not in dispute is that he (they!) did take the Earth to be spherical and his (their!) model was the first in which the Earth moves.

There was a first version of his cosmology in which the Earth is at the center of the universe and inside of the Earth is a "central fire" or "'Hestia," in homage to the immobile goddess of the hearth. But that morphed into the cosmology that I described in Chapter **??** with the central fire now a central object of its own with the Earth —and the counter-earth— plus all of the celestial objects revolving around it. Figure 3.7 (a) shows a rendering of the Pythagorean idea.

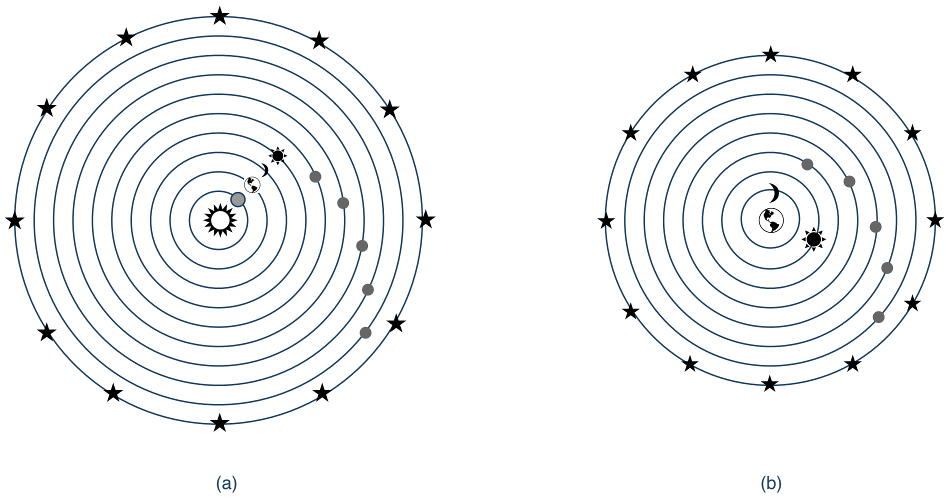


Figure 3.7: (a) shows the Pythagorean system and (b), the Platonic system as described in the *Republic*.

The idea of a cosmology and based on *regular, circular motions* and a model in which MOTION BY THE EARTH is not zero, is due first to the Pythagoreans who in effect, kicked off two millennia of astronomical and physics research. Circles, everywhere. And, as hinted above, the idea of a spherical Earth probably was another conclusion of Pythagoreans. Circles, as the perfect shape, everywhere you look.

3.2.3 Plato's Two Models

Recall that in the *Republic* proper education of the Guardians included astronomy which led to the controversial idea that Socrates/Plato denounced observational astronomy for a contemplative exercise determined to visualize and understand the true motions of the celestial objects. His first idea was borrowed from Pythagoras in broad ways, and actually serves as maybe a beta-version of the final Platonic model described in more detail, and with more realism in the *Timaeus*. Which is full-on Pythagorean.

Students from philosophy to political science study the *Republic* and are often surprised by its ending: the *Myth of Er*. Socrates is trying to motivate why someone should live a good life and relates a cosmic carrot-and-stick story, not unfamiliar to other religious admonitions. Er is a soldier who was killed on the battlefield and does what all deceased do... they go to a place where their lives are evaluated, not by St. Peter at the Pearly Gates, but by four judges in an ante-room with four doors, two up and two down. Er is told to wait and watch what happens as he's got a job to do: after 12 days he's to be resurrected from the dead, dramatically it seems on his own pyre before it's lit.

> "Once upon a time he died in war; and on the **tenth day**, when the corpses, already decayed, were picked up, he was picked up in a good state of preservation. Having been brought home, he was about to be buried on the twelfth day; as he was lying on the pyre, he came back to life, and, come back to life, he told what he saw in the other world."
> Plato, *Republic*

(Why 10 days? Well, some Pythagoreanism is maybe showing?) Er watches good deceased people sent upstairs, and good people coming downstairs to go back into the world as Pythagorean reincarnation is a part of the whole cycle of life. Their spirits remain intact, but are housed in different bodies (remember the remembrance from the *Meno*). The other two doors are for those who's past lives are unsavory. Some are fined and punished and sent back to the world to try again, but some remain in the underworld which is a less than pleasant place.

Er mingles with some of the lucky ones and they observe a spectacle— a pillar of light that extends to the heavens which when they look closer, appears to be what we'd call today an umbrella structure. Plato describes a spindle and whorl used for spinning wool. Figure 3.8 (a) shows a Roman woman spinning wool with the weighted whorl at the bottom which spins as she works. Figure 3.8 (b) is the umbrella-like structure (the whorl upside down) that Socrates describes:

> "Its shape was that of (whorls) in our world, but... it was as if in one

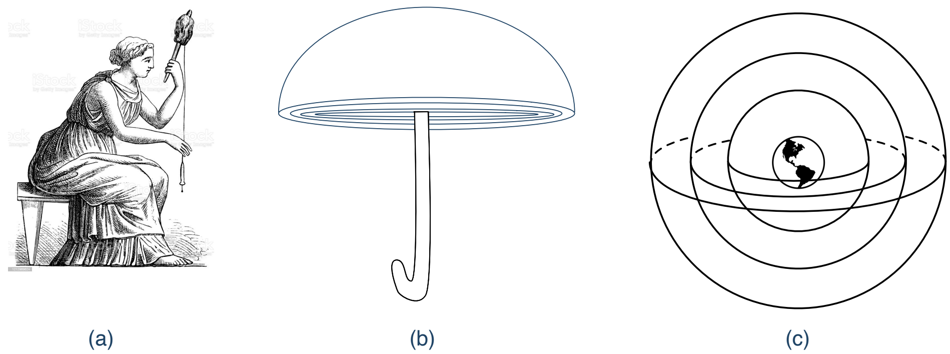


Figure 3.8: The figure in (a) is a Roman sketch of a woman spinning wool using a spindle and whorl, which is the weight at the bottom with a hook. The image in (b) is Plato's description of the whorl actually hollowed out with nested layers of whirl-shaped half-spheres. The image in (c) is the cosmos that the onion-layered whorl represents. I've not included all eight of the spheres in my cartoon.

> great whorl, hollow and scooped out, there lay enclosed, right through, another like it but smaller, fitting into it as containers that fit into one another, and in like matter another. . . There were **eight of the whorls** in all, lying within one another. . . " Plato, *Republic*

The eight "containers" are hinted at in my sketch in Figure 3.8 (b) and the whole is abstracted as nested spheres in Figure 3.8 (c). He also tells you how they move and the sounds that they emit as a Siren sits on each sphere and sings a tone. The nested whorls actually have a structure that's tied to the musical scale—again, homage to the Harmony of the Spheres that only Pythagoras could hear.[3]

This is Plato's (and the world's) first three dimensional cosmological model and it's shown in Figure 3.7 in the style that we normally see ancient cosmic players.

Figure 3.7 (a) shows the now familiar Pythagorean cosmology with the central fire in the center, with the counter-earth, then Earth, Moon, Sun, and the outer five planets, and finally the outer region with all of the stars. Figure 3.7 (b) is Plato's rendition along these same lines. Now Earth is in the center (a "geocentric" system has the Earth as the central object), with concentric spheres of the Moon, Sun, the outer planets, and again, the stars on the furthest shell, which Socrates says is "speckled." So, Plato's first cosmology has MOTION BY THE EARTH as zero.

Plato must have realized that the Er model wasn't sufficient. For example, everything is on one plane, so the motions of the planets, Sun, and Moon through the night sky in the ecliptic would have the same rates as the stars themselves and the ecliptic and celestial equators would have been the same. His attempted to fix this led to the very complex solution from the *Timaeus*.

[3] I've shown the spacings as equal, but the plan was different from that.

The *Timaeus* is another story, literally and figuratively. We discussed the material and almost chemical aspects of the use that the Craftsman made of the four solid geometrical pieces that make up our "stuff" but Socrates teases out of Timaeus—the Pythagorean—a geocentric picture that's more sophisticated than that planar version from the Republic. Now Plato becomes mathematical, in a very Pythagorean way.

The spheres now are living beings, moving at speeds dependent again on their distance. But numbers are the instructions to the Craftsman:

> "And he began the division in this way. First he took **one portion** from the whole, and next a **portion double of this**; the **third half as much again as the second**, and **three times the first**; the **fourth double of the second**; the **fifth three times the third**; the **sixth eight times the first**; and the **seventh twenty-seven times the first**. Next, he went on to fill up both the double and the triple intervals, cutting off yet more parts from the original mixture and placing them between the terms, so that within each interval there were two means, the one (harmonic) exceeding the one extreme and being exceeded by the other by the same fraction of the extremes, the other (arithmetic) exceeding the one extreme by the same number whereby it was exceeded by the other." Plato, **Republic**

Okay the numbers seem arbitrary. But there's an algorithm:

- one portion of the whole: ○, 1
- double of this: ○○, 2
- half as much again: ○ ○ ○, 3
- double of the second: ○ ○ ○○, 4
- three times the third: ○ ○ ○ ○ ○ ○ ○ ○ ○, 9
- eight times the first: ○ ○ ○ ○ ○ ○ ○ ○, 8
- twenty-seven times the first: ○, 27

Now manipulate:

- The first four are the famous 1,2,3,4 and since they're the special numbers, they have a job to do:
  - Square each of the first numbers—remember, 1 is not a number— (Greeks knew how to multiply): and you get 4 and 9.
  - Cube those same first two important numbers: and you get 8 and 27.

So all of the numbers in that excerpt are some manipulation of the numbers 2 and 3. Remember, that the Pythagoreans only had use for three dimensions, so Plato stops his squaring-calculation with 3. So collecting all of the numbers, but now into even and odd strings (remember, 1 is neither even nor odd for Pythagoreans and apparently also, for Plato):

- 1, 2, 4, 8
- 1, 3, 9, 27

Then, Timaeus says that if you take the number strings and fill in the arithmetic and harmonic means between each of the numbers, you actually construct the intervals of the diatonic musical scale. More Music of the Spheres. Whew. Wait until we get to Kepler.

Then the Craftsman did some rearranging.

> "This whole fabric, then, he split lengthwise into two halves; and making the two cross one another at their centers in the form of the letter X, he bent each round into a circle and joined it up, making each meet itself and the other at a point opposite to that where they had been brought into contact." Plato, *Republic*

Figure 3.10 shows what this is really about. He's created two strips (of something) that go around the Earth—think of it as taking a ribbon and bending it around to make a circle. And, then do it to a second ribbon and attach them at two opposite points which are each labeled "X." He inclines the strips so that one is higher than the other. These are the celestial equator in a sphere, the axis of which goes through the North Celestial Pole and the other is equator of the ecliptic that's another sphere with an axis going through the Ecliptic Pole.

Figure 3.9 is a silly attempt to explain this. Figure 3.9 (a) is a person playing with a hula hoop, perfectly aligned so that the axis of the toy's rotational plane points through our person's head. Figure 3.9 (b) shows just how good at this they are: two hoops rotating, the original, and another that somehow our friend manages to get to rotate at an angle relative to the first one. Some serious hip-action would be required for this. This is Plato's idea: two rotating hoops.

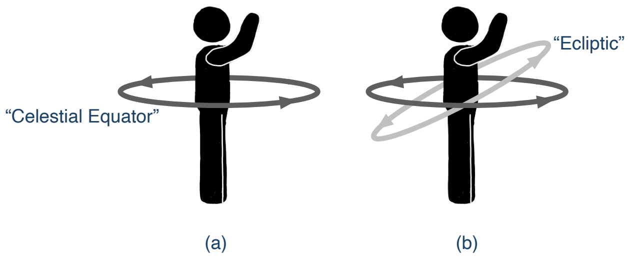


Figure 3.9: Pretty good hula hoops chops.

> "He then comprehended them in the motion that is carried round uniformly in the same place, and made the one the outer, the other the inner circle. The outer movement he named the movement of the Same; the inner, the movement of the Different. The movement of the Same he caused to revolve to the right by way of the side; the movement of the Different to the left by way of the diagonal." Plato, *Republic*

The strip that Plato called the "Same" is the equator of a sphere that doesn't change, the furthest most out. That's the celestial sphere and its axis I've called the NCP (north celestial pole). The other strip is the equator of another sphere that Plato called the "Different" and makes an "X" where it crosses in two places with the Same. Inside of this strip, he's broken it into segments which differ in radii from the center according to the six spaces between the pairs of: 1, 2, 3, 4, 8, 9, and 27. Of course, the Different is the ecliptic and the individual bands in which the planets reside are at those radial spacings are the locations of the Moon, Sun, Mercury, Venus, Mars, Jupiter, and Saturn.

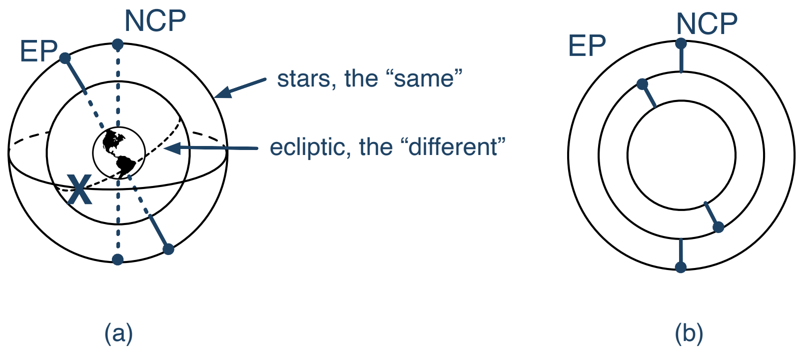


Figure 3.10: (a) shows the two spheres with their equators. One the Celestial Sphere (carrying the stars around the Earth each night, so an axis centered on the North Pole of the Earth) and the other is the ecliptic (in which the planets reside as they appear to go around the Earth) with the pole of that sphere, the North Ecliptic Pole. (b) takes away the three-dimensional view and will be a useful sketch for these kinds of constructions in what follows.

Of course, this is a little mad. But it's Plato being an empiricist when he doesn't really want to admit it! He had to fix the Er model in order to fit the apparent nature of the passage of the planets, Moon, and Sun in a plane that's different from the rotation observed by the stars.

This still leaves a lot unlearned about our list of what we know. But don't worry, Plato had a collaborator, his former student, Eudoxus, the most significant mathematician before Archimedes.

3.2.4 Eudoxus' Model

By the time Eudoxus had returned to the Academy, he was surely familiar with the *Republic* and probably *Timeaus*. The timing of his work with those books isn't clear, but the consensus is that it probably came after those late Platonic stories. Plato clearly struggled with his vision of what the true cosmology should be and how to account for even the most rudimentary celestial appearances. Once he'd fixed the ecliptic path for the Sun and planets in their individual circular bands, he still needed to explain retrograde motion. And he knew it:

> "...as for the dances of these and how they relate to each other, the **backward-cycles and forward-progressions** of the circles to each

> other... to speak without visual representations of these same would be a **vain effort**." Plato, *Timaeus*

So, he gets it... but without a solution, let's give up and move on ("vain effort") he's giving-up. Plato was out of his depth but Eudoxus was ready. He came up with a brilliantly complex model and while it's not known what Plato thought of it, it's clear how Aristotle reacted: he adopted it and made it his. The model is intricate, so let's go to the box and work out the inner workings of the idea and then skip to the end. Look at Figure Box 3.12 on page 67. After you've read the material in that Box, return to this point ☟ and continue reading.

Figure Box 3.12 forms the tool-kit that Eudoxus used to construct a full model of all of the planets. Instead of the planets carried in bands around the Earth, they ride on the equators of spheres. That's a clever idea because the spherical shape makes it possible to connect the motion of one planet to the other celestial shapes. Bands would not allow that. The two spheres show in the box aren't enough to replicate the observations. The inner two form the minimal number of moving parts that will be unique to every planet and they need to be each embedded inside of two other spheres, one for the ecliptic whose equator include the rough paths of the planets and the other is the celestial sphere which includes the motions of the stars around the Earth every nearly 24 hours. The punch-line is in Figure 3.11. Let's take it slow:

(a) is a slightly different rendering of Figure 3.12 (b)
(b) is an abstraction of (a) taking out some of the lines that suggest a solid sphere, for clarity
(c) includes the sphere of the ecliptic (EP for Ecliptic pole is shown) with axis of rotation CC′. Notice that it's attached to the outer sphere of Eudoxus' tool-kit pair.
(d) includes the sphere of the outer stars, the celestial sphere (NCP for the North Celestial Pole is shown) and the ecliptic shere is attached to it.

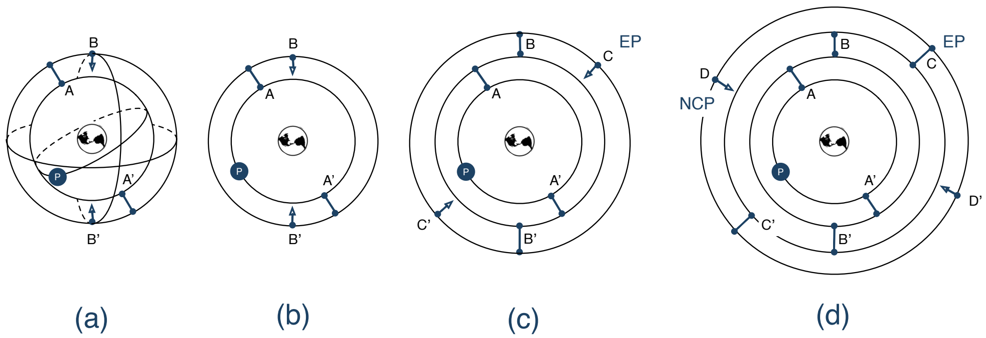


Figure 3.11: CAPTION

Now all of these separate motions are coupled... and that's just for one planet! By tuning the inner two sphere rotation speeds and the inclination of the inner axes, the motions of the planet can be made to do the figure-eight dance at just the right time of year and with the right elongation in the sky.

Each planet requires four spheres and the Sun and Moon require three each, plus the Celestial sphere, or 27 spheres to do the job. This was a mammoth intellectual puzzle that Eudoxus solved with those relatively simple pieces of interlocking spheres.

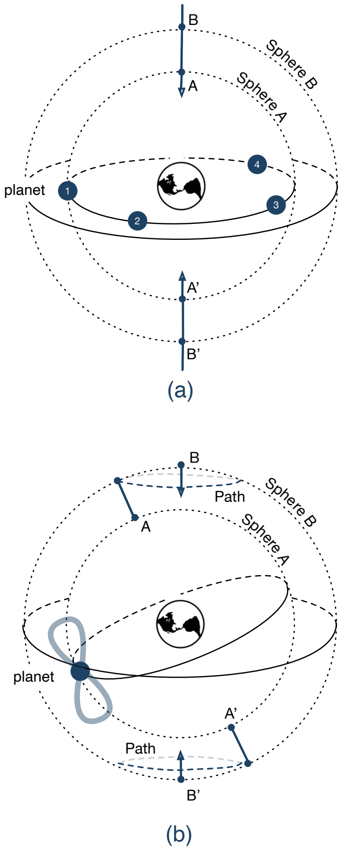
It still didn't quite do the job as well as it might and in the best tradition of what Thomas Kuhn would have called "Normal Science," someone tried to make it better without starting over. **Callippus of Cyzicus** (−370 to −300) was a student of Plato's and worked with Aristotle and he worried about details like the fact that the seasons are not the same lengths (which we know is because of the Earth's elliptical orbit). So this is getting into the details, but trying to make it more accurate. Calliopus added two additional spheres for the Sun and Moon and one each for Mercury, Venus, and Mars for a total of seven more. So now the model has 34 spheres.

Was it all just an exercise in geometry?

3.2.5 Aristotle's Model

By now we know that Aristotle insisted that natural motion was straight and since he wanted to account for the celestial objects which obviously don't move in straight lines, then those objects must be special and made of different stuff: quintessence. The hierarchy is such that if all of the motions of all substances were separated into their individual natures, then all of the earthy material would condense at the center of the universe. That sphere would be surrounded by a thick sphere of water. That would be surrounded by a sphere of air and then fire. So a spherical sandwich of the four terrestrial elements filling up the whole volume below the Moon, the "sub-lunar realm." Only outside of that larger sphere was the realm of the celestial objects, individually made of quintessence.

FIGURE BOX 3.12



The model that Eudoxus created is an impressive bit of geometry mixed with inspired imagination. It's the famous "nested spheres" model that made it all the way to the Baroque as an explanation for the odd motions of the planets. In a very modern way, it's full of parameters that could be tweaked to make it fit the observations...some of which he made himself at the observatory he created in his school before he returned to Athens.

Imagine taking two hoops, one of which is slightly smaller than the other and is attached inside the larger one across their mutual diameters. Figure 3.12 (a) shows this with a "planet" attached to the equator of the inside hoop. Now if we spin that hoop around its axis AA' the planet will follow a circle from position 1 through 2, 3, 4 and so on. This spinning essentially defines a sphere, Sphere A, here centered on the Earth. If the two hoops are attached, and if the outer hoop spins around its axis, BB', creating the surface of Sphere B, then the motion of the planet will be the sum of the two speeds at the hoop pair equators. So if the outer hoop spins at the same rate as the inner hoop, but in the opposite direction, then the planet would appear to the Earth to remain stationary at position 1.

Now imagine that the axis of the inner hoop is attached at a point offset on the surface of the Sphere B as shown in Figure 3.12 (b). Now when Sphere B spins, it takes the AA' axis of Sphere A around with it tracing the path shown. In addition, if Sphere B spins while its following that path independently, the motion is a complicated figure eight pattern as shown. Eudoxus figured this out and named the shape a "hippopede" which is "horse fetter" in Greek. (A fetter is like a chain.) Now there are many variables at work which would alter the shape of the hippopede: the speeds of the two spheres and the angle at which AA' axis of Sphere A is inclined to the BB" axis of Sphere B.

Now go back to page 65 and pick up where you left off.

So Aristotle's universe is of finite volume in space all the way to the outermost starry sphere, like that of the Pythagoreans. Furthermore, it's always been there. So, a finite event in space, but an infinite extent in time. But Aristotle did not like the Pythagorean picture in which the Earth revolves around a central point and his argument — shared by others — was a scientific argument.

3.2.5.1 Critique of the Moving Earth Cosmology

Look at a point across your room with one eye closed and put your finger in front of you and notice what's behind it on a wall or distant surface. Imagine a line extending from your eye, through your finger, and ending on the far wall. Now switch eyes and notice that the what's behind seems to have moved. Again, a line from eye-finger-wall, but a different line from the other eye. These two lines are at crossed directions as can be clearly seen if you open and close each alternate eye successively. The background will appear to jump from side to side relative to your finger. This is called "parallax" and it's because your eyes are inches apart enough that the lines of sight from each are slightly different: that is, those lines are at different angles.

If the Earth is orbiting a center, and not itself the center, then at one point of the year a particular star would appear as a line at a particular angle. Set up a structure that points at that star. Then six months later, when the Earth is on the other side of that center, look for that same star and it will be at a completely different angle relative to the structure you erected when you first looked. "Stellar parallax" or "annual parallax" is the name of this phenomenon.

Nobody observed stellar parallax and so there were really only two explanations. Either the Earth doesn't move around a center of revolution, or the stars are so far away that parallax is not visible. Nobody was prepared to imagine a universe that big, and so the conclusion was: a stationary Earth. It took until the 19th century to actually observe stellar parallax because the universe really is that big. This is one of two reasons why for Aristotle— and nearly everyone who followed for nearly 2000 years afterwards, MOTION BY THE EARTH was zero. The second reason was physics, which we'll come to below.

3.2.5.2 Details of Aristotle's Model

Ever the mechanist, Aristotle wanted a mechanical model of the heavens and wasn't content to just live with a mathematical description. He took over the Eudoxus/Callippus spheres and worried about how to actually imagine real material objects.

But Aristotle was about "how" do they move?

3.2.5.3 The Physics of Aristotle's Cosmology

Remember that Aristotle's philosophy is a tightly-woven and highly correlated set of principles and that his physics is inflexible as an important component of the whole. Perhaps he was led to the spherical Earth hypothesis out of the observation that in a lunar eclipse (when the Earth is perfectly between the Sun and the Moon and blocks our night-time view of it) the Earth's shadow creeps across the Moon's face and is always a convex, circular shape. But he was always in awe of his own ideas and his ideas about natural motions for earthy, watery, airy, and fiery objects and that seems to have been his primary commitment and since all earthy material would be drawn to the center of the universe uniformly, the result would be a spherical Earth volume.

The second reason for a stationary Earth, besides the lack of parallax, was his physics of unnatural and natural motions. What one observes if a rock is thrown directly overhead, is that it comes right back down. One could imagine two kinds of MOTION BY THE EARTH, one rotational around its axis and the other in a path around a fixed point (either a rather arbitrary point like in Pythagoras'/Philolaus' model or around the Sun itself). Heraclides of Pontus, a member of the Academy and maybe its interim director when Plato went to Italy the second time, proposed that the Earth did sit at the center of the universe and rotated on its axis once a day to explain the observed motions of the stars.

If the latter then throwing a rock up would leave it behind.

3.3 More of Plato's, Aristotle's, and Ancient Astronomy's Stories

3.3.0.1 Modern Day Platonists

> Linking numbers and geometrical patterns to features of the world is another modern concept. The model that we trust for the fundamental constituents of matter is the Quark Model. It is based on a complex idea of mathematics and produces geometrical patterns similar to the dots above, the location of which designates properties of the quarks.

Our best description of the universe at the quantum level is described by a mathematical function called the "wave function." This is a solution to a basic equation that describes its behavior in space and time and when squared, gives the probability that a microscopic entity (like and electron) will be at a particular place at a particular time. It works very, very well this wave function idea. It's also completely undetectable. It's certainly "in" the objects of nature.

Yes, the wavefunction (and its relativistic cousin, the relativistic quantum field) that describes the world, from chemistry to the Large Hadron Collider, is precisely calculable, but completely off-limits to any detection. It gives you probabilities, not certainty. Now, if that's not an example of mathematical fabric of the universe, then I don't know what is! The wavefunction's "reality" is that it's just a mathematical function!

3.3.0.2 Indebtedness to Eudoxus

Appendices

Appendix A

Greeks Technical Appendix

A.1 Zeno's Paradox

A.2 Proof of Pythagoras' Theorem

A.3 Socrates' Geometrical Problem

Appendix B

Plato–Aristotle Technical Appendix

B.1 Digital Gates

One more bit of insight makes really complicated electronic digital design possible and came from the very strange, yet enormously influential philosopher **Ludwig Wittgenstein** (1889-1951) who invented the concept of the "truth table," which we've already used in Table 2.4. It's an orderly setup of all possible starting places (for two valued propositions) and their results when various operations are applied. Let's look at a three. True now is the bit 1 and False is the bit 0:

- The NOT operation: If I have an A then NOT–A creates the opposite of A. If we work in the zeros and ones world, then if A=1, then NOT–A = 0. The symbol for NOT is usually so if A = 1, then A = 0. (The symbol is the common notation used by logicians. Engineers and physicists would write $\overline{A}$ to represent the result of NOT–A.)
- The AND operation: This is between two states of, say, our A and B. In order for A AND B to be true, both A and B must be true—1— themselves. Otherwise, A AND B is false, or 0. The symbol for AND is $\wedge$ So A AND B = A $\wedge$ B.
- The OR operation: This is the combination that says A OR B is true if either A = 1 or B = 1 and false otherwise. The symbol for OR is $\vee$.

There are 5 other logical combinations. Table B.1 shows the truth table for AND and for OR. In the first set, the AND process, I've stuck to our T and F language, but the rest uses the zeros and ones language of engineering and binary arithmetic.

Table B.1: Truth tables for the AND and OR functions plus the construction of Modus Ponens. The **symbol for AND is** $\wedge$, the **symbol for OR is** $\vee$, and the **symbol for NOT (negate) is** . Notice that (A) $\vee$ B is a construction out of AND and NOT of the conditional that's the first premise of Modus Ponens.

AND			OR			Combined function				=
A	B	A $\wedge$ B	A	B	A $\vee$ B	A	B	A	(A) $\vee$ B	If A then B
T	T	T	1	1	1	1	1	0	1	= 1
T	F	F	1	0	1	1	0	0	0	= 0
F	T	F	0	1	1	0	1	1	1	= 1
F	F	F	0	0	0	0	0	1	1	= 1

Let's look at the first line so that you get the idea.

For AND:

- A is T and B is T and the AND of two T's is itself a T.

For OR:

- A= 1 and B = 1 and the OR of 1 $\vee$ 1 is 1.

Then the combination:

- repeating the A and B conditions from the first and second columns A= 1 and B = 1.
- taking the NOT of A, takes 1 into 0.
- combining that with the B in an OR results in A $\vee$ B = 0 $\vee$ 1 = 1

The last column shows that this is the same as the first line result of our picnic decision making in Table 2.4. The rest of Table B.1 builds that combination for all possible A and B states, first by negating A and then combining that by "ORing" it with B. The last column shows the original "If A then B" premise that we worked out about raining and wetness. They formula and our reasoning lead to identical conclusions.

Appendix C

Greek Astronomy Technical Appendix

C.1 The Greek Sky

C.1.0.1 Observation #1: stars' motion

Figure 3.4 (a) shows a time-lapse photo of the northern sky from the northern hemisphere showing that every single star seems to be rotating around a single point in the sky around the Earth's north pole—now the star Polaris— every 24 hours. What we know of course is that this apparent motion is not of the stars—which look to be stationary on human timescales— but is due to the Earth's daily rotation exposing different parts of the star fields all around us. That the Earth also moves around the Sun means that the stars will emerge each night in a slightly different place than the previous night. So, the Earth's rotation and its orbit around the Sun explains observation #1.

> That we see Polaris at nearly the axis of rotation (it's less than a degree away), is an accident! The Greeks observed this circular motion but Polaris wouldn't have been at that point because like a spinning top, the Earth's axis of rotation precesses in a circle, pointing at different parts of the sky over millennia. It will continue to do so, and the pole star will appear to slowly move away from our familiar Polaris-center-point, returning to it in 25,900 years.

C.1.0.2 Observation #2: planets' motions

Our local neighborhood is the solar system in which we're the third planet from the Sun. FigureFigure C.1 is a cartoon of that system and while the scales are arbitrary, there is one thing that's accurate about this figure: the planets are all in a plane. This has to do with their original formation out of the dust and elements that were swirling around the Sun almost five billion years ago. They gravitationally clumped together, slowly forming the eventual eight planets. This common plane of the planets' orbits (and the Sun) is called the "ecliptic." If the Earth's rotational axis were perpendicular to the plane of its orbit, then we'd see all of the planets, the Sun, and the Moon staying within a lane that would be above the Earth's equator. But the Earth's axis is slight tilted by 23.5° so that plane appears at an angle relative to the Earth's rotational axis.

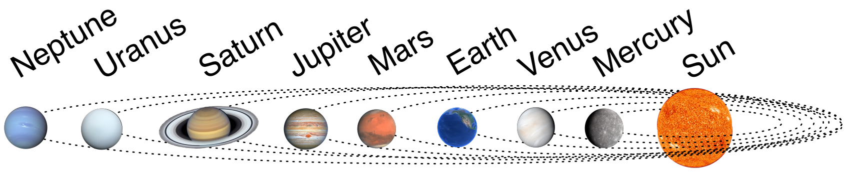


Figure C.1: Our solar system, dramatically not to scale: the Sun is enormous compared with the individual planets, they are much further from it than the picture suggests, and the size differences among them are significant. The eliptical orbits of the planets is greatly exagerated.

There are three spheres that orient the Earth to the solar system and stars. A sphere centered on the Earth's rotational axis is the "Celestial Sphere." When the Earth rotates on that axis it looks to us like the stars are all rotating. Because the Earth is inclined by that small angle, the Sun, Moon, and planets are all in that lane and form the equator of another sphere, the "Ecliptic Sphere," the axis of which is inclined by 23.5° from the Celestial Sphere (Earth's rotational) axis. Finally, unless you live on the equator, the plane that you see as your horizon is related to your longitude and your "zenith" is directly overhead and your horizon is the equator of that personal sphere.

Figure Box C.3 summarizes much of what I just described about the three spheres on page 80. After you've read the material in that Box, return to this point ☞ and continue reading.

The paths that the planets take around the Sun are not circles, but they're ellipses in which the Sun is not at the center, but at one of the "focii" meaning that the planets' speeds around the Sun are not constant. When they're closest to the Sun, they are going faster than when they're the furthest from the Sun. (We'll understand that in Chapter **??**).

Figure 3.4 (b) is a photograph of the southern sky in which Jupiter and Venus are visible. Notice that the horizon is still a little bit bright and so the Sun is just setting. Venus and Mercury are close to the Sun and the Greeks noted that uniquely they seem to cling to it as it rises and sets. The Greeks could observe the Sun, the Moon, and the closest five planets: seven celestial objects plus the background stars.

The planets also have varying brightnesses during different periods of the nights which is especially the case for Venus which shows phases like the Moon, as it passes in front of, and then around the back of the Sun as we view it.

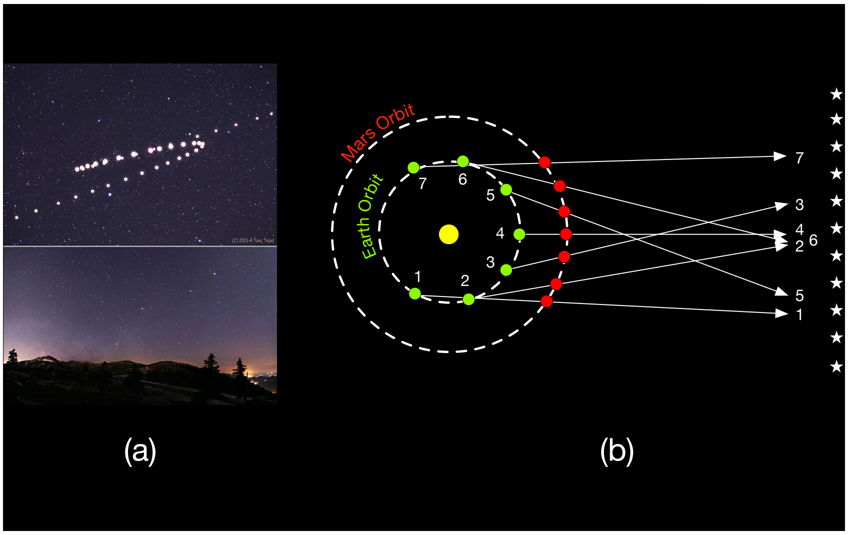
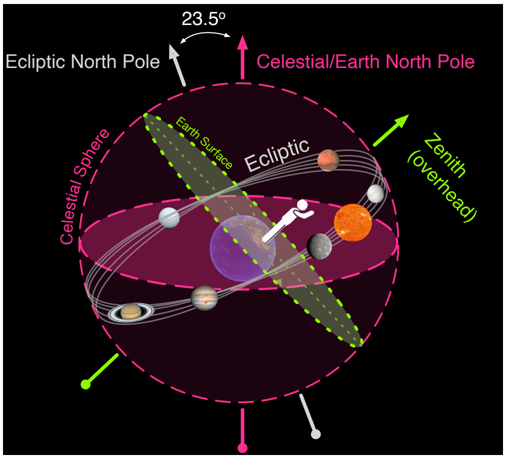


Figure C.2: Retrograde motion by Mars. In (a) the sky in Turkey shows a photograph of Mars from December 5, 2013 in the upper right hand corner and then an overlayed photograph taken every five or six nights until August 8, 2014. The looping behavior in the middle is the retrograde motion. (b) shows how this happens (see the text for an explanation) https://twanight.org/gallery/tracing-the-red-planet/?preview=true

But there's a very odd dance that the planets execute called "retrograde motion" depicted in Figure 3.6. Suppose you took a wide-view snapshot of, say Mars each night at, say midnight, noting where it is in relation to the (nearly fixed) background stars. You'd see that each night at midnight it's moved a little bit east against the background stars.[1] At some point in your nightly observation the planet would slow its nightly rate and turn around and start moving west against the stars. . . and then reverse its course again and continue. Figure 3.6 (a) shows exactly this situation, overlaying multiple nightly pictures of Mars over Turkey.

[1]Note, this does not mean that Mars moves east during a single night, but that if you looked at the same time in successive nights.

FIGURE BOX C.3



This image shows the Earth and a person standing in the Northern Hemisphere. The outside dashed sphere is the "Celestial Sphere" with its "Celestial North Pole" axis going through the Earth's geographic North Pole, which is of course the axis of Earth's daily rotation. Their observed horizon in all directions appears to him as a plane, with the "Zenith" directly overhead. Notice that if the observer were at the Earth's equator, then the Zenith would be perpendicular to the axis of rotation by the Earth. If they were at the North Pole, then the Zenith would point along the Earth's rotational axis. In East Lansing, Michigan my latitude is 42.7325° N and so the angle between my northern horizon and the celestial/Earth pole is 42.732° and the zenith is 47.268° from the celestial/Earth pole.

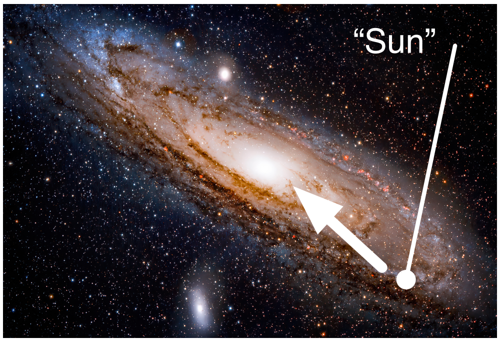
The lineup of planets appears around the Earth in the ecliptic plane which is inclined to the Earth's equator by 23.5°, that angle of tilt that the Earth has relative to the plane of its orbit around the Sun. The ecliptic's North Ecliptic axis is inclined to the Celestial Sphere's axis as shown by that same 23.5° angle. The paths of the planets are not exactly arranged, but the ecliptic's equator forms a lane in which the planets are all contained as they pass overhead.

Now go back to page 78 and pick up where you left off.

In Figure 3.6 (b) provides the explanation. Here, a cartoon of the Sun (yellow circle in the center) and orbits of the Earth and Mars are drawn not to scale (this could be any planet outside of our orbit). The numbers refer to hypothetical nights' of observing Mars (the outer red circles) from Earth (the inner green circles). The arrows refer to an Earth observer each night sighting Mars in reference to the distant star background. So night #1 points low in the field, night #2 points towards the middle, #3 a bit higher, and then #4 Mars *appears* to go backwards against the stars. That backwards motion continues until night #6 when it's back to its original trajectory. So a heliocentric ("Sun-centered") model of the solar system explains observation #2.

C.1.0.3 Observation #3: that fuzzy stripe

Our Sun is about four and a half billion years old and is one of 100 thousand million stars in the Milky Way galaxy, which is itself more than 13 billion years old. It's not much but it's home—in fact, it's a relatively average galaxy. Figure **??** (a) is a photograph of the Andromeda galaxy (not ours!), which is the largest galaxy near us and similar in structure to and a little bigger than the Milky Way. It's about 2.5×10^6



(a) (b)

Figure C.4: The figure on the left is a photograph of the Andromeda galaxy, which is very similar to our own Milky Way galaxy. Pretending that this is the Milky Way, the approximate location of our solar system is shown in one of the spiral arms. The arrow is our line of sight to the center of our galaxy. The right figure shows what the Milky Way looks like from Earth as we look along that arrow in (a) toward the galatic center in the constellation Sagitarius. We're looking from the edge to the center through all of the stars between us and the galactic center.

light-years away from us so the image shows what Andromeda looked like almost three million years ago.[2] The Milky Way shape is broad, with arms of stars in a spirals outward, hence its classification as a "barred spiral galaxy." It has a diameter of about 87,000 light-years across and a thickness of roughly 1,000 light-years, so its flat like a pancake. Our Sun is about 25,000 light-years from the center in one of the spiral arms and we're rotating around the center of the galaxy at a speed of about 800,000 kilometers per hour (~500,000 mph) and the arrow in Figure **??** (a) suggest the relative position of our Sun, using Andromeda as a stand-in for the Milky Way.

Figure **??** (b) is a photograph of the southern sky showing what it looks like to look through the edge of the Milky Way—through the pancake—towards its center in the constellation Sagittarius (where there is an enormous black hole with a mass of about 4 million suns.) The arrow in Figure **??** (a) shows our line-of-sight towards that center. So our location in our Milky Way galaxy explains Observation 3.

"'=latex "'

[2]Each galaxy is moving through the local cluster of galaxies...toward one another. They will collide in about four and a half billion years from now and likely form a merged galaxy, but their trillion stars are so widely dispersed that the likelihood that any two stars will collide is negligible. Their two central black holes will possibly collide and emit substantial gravitational radiation. Simulations suggest that the solar system might be ejected, but probably in-tact. Eventually, all of the galaxies in the local group will merge. Not to worry.

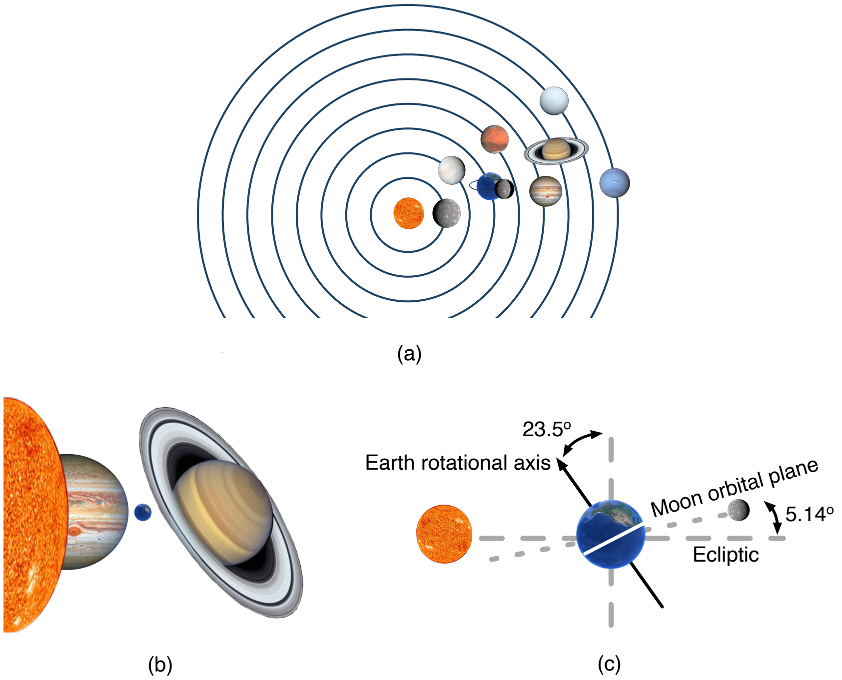


Figure C.5: CAPTION

C.1.0.4 Observation 4: The Moon

If we were above the Earth's orbital plane (the ecliptic) and positioned ourselves above the Earth's north pole we would see it rotating counter clockwise, once around every 24 hours. We'd also see the Moon 384, 399 km (238, 855 miles) from Earth's center orbiting in the same direction, always presenting the same face to Earth, and taking 27.322 days to complete an orbit. [The Moon is also rotating on its axis, but in sych (it's "tidally locked") so even though it rotates once around per orbit, we still see the same side.] If we looked the Earth-Moon system from the side (in the ecliptic) with the Sun to our left and Earth to our right, we'd see that the Moon's orbital plane is about 5° inclined relative to the ecliptic and the Sun-Earth line, so that sometimes the shadow that it casts from the Sun is above or below the Earth so we don't have a solar eclipse every month.

What does this dance look like from an observer on the Earth? The Sun always illuminates half of the Moon's surface, but because its rotational period is different from Earth's the Moon's appearance to us changes in a unique pattern which we call the "phases." If the Moon is between Earth and the Sun, during our night, we're looking away from the Moon and not see it: that's the "new moon." At that position it's at its closest position to the Sun relative to us but when we revolve back around and see the Sun during our day we'd also see the Moon for a few days even in the sunlight.

Each night, then the Moon's surface would be more and more visible (Waxing Crescent, First Quarter, and Waxing Gibbous) until we see the whole Moon's surface when it's on the other side of Earth from the Sun and the Full Moon brightly reflecting the Sun's light back to us when we're looking during our night-time. Then each night the Moon would slightly start showing the next phases (Waning Gibbous, Last Quarter, and Waning Crescent) until it's back between Earth and the Sun. Notice that when the Moon is in one of its slimmest slivers of Sun reflection, sometimes we can see the rest of the Moon's surface very dimly showing. That's reflection of light from the Earth bouncing off the Moon, back to us. . . "Earthshine." This complicated and somewhat random arrangement of Moon-Earth explains Observation 4.

C.1.0.5 Observation 5: The Sun

It sure looks like the Sun "rises" in the east and "sets" in the west and of course we speak that way. But we know of course that this apparent motion is due to the fact that the Earth is rotating toward the sun and illuminating our surface and then away leaving us dark. But the Sun's appearance is a little more nuanced than that. That 23.5° inclination of the Earth's rotational axis relative to the ecliptic plane means that as we move around the Sun in our "year" at one point the Earth's axis is pointed *away* from the Sun (our winter) and six months later, it's pointed towards the Sun (our summer). Ironically, that winter configuration is when the Earth is physically closest to the Sun and the summer configuration, it's further away. The warmth that the Earth's surface experiences is a matter of how dense the Sun's rays are when they illuminate the ground—when the axis is tilted toward the Sun, the light-intensity per unit area is highest. The tilt of our axis also explains why the Sunrise seems to happen at a different point on the eastern horizon every morning and why the Sun's position at noon—when it's the highest in the sky—changes every day. Again, when it's summer, the Sun is up higher and stays up longer, and in the winter it's the opposite.

Since the summer is when the Earth's axis is pointed towards the Sun, but is further away, our summer is longer than our winter. Because of the elliptical path, when the Sun is closest (winter), it's moving faster than when it's further away, it's slower. That the seasons are not the same length was confusing to anyone trying to explain the sky.

How high the Sun appears at noon depends on where you're watching from the Earth's surface. When it's at its highest point during the year, it's the summer solstice and directly above the Tropic of Cancer, which happens on June 20th, and the day that it is the lowest is December 21st, called the winter solstice when the Sun is directly above the Tropic of Capricorn. The sunset on the summer solstice is 23.5° above due west and at the winter solstice, it's 23.5° below due west. Every day between those extremes, the Sun takes a slightly different path in the sky during the day within the lane of the ecliptic. When the Sun appears to rise exactly at due east, or set due west we call that the spring (or vernal) equinox on March 21st and fall (or autumnal) equinox on September 23rd. On those days, the Sun is directly above our equator.

Sources

Lloyd, G. E. R. (1970), *Early Greek Science Thales to Aristotle*. Chatto & Windus, London.

Putnam, Hilary (1971), *Philosophy of Logic*. Harper, New York.

Russell, Bertrand (1946), *The History of Western Philosophy*. Simon & Schuster, New York, New York.

Snyder, Z. (2006), "300." Studio: Burbank, California: Warner Bros. Pictures. produced: Nunnari, G. and Canton, M.