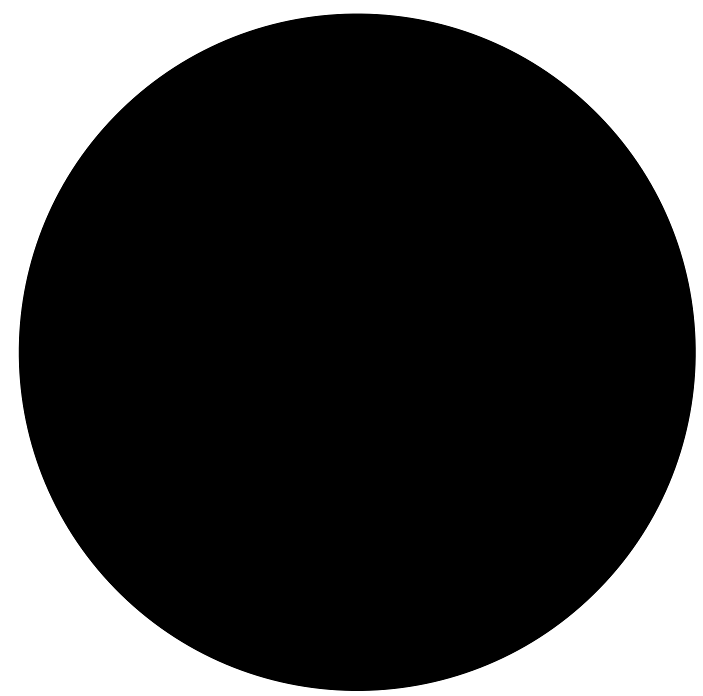
hi

Day 22, 04.09.2019

Quantum Mechanics 5

NCAA final four is only 360 days away!

Black Sabbath week



EHT: 0900 tomorrow

housekeeping

Friday!

Starting grading book reviews. Some tips for the second one:

1. "novels" are fiction. these books are not novels!
2. Please, please put the book title and author in the heading of your report
3. Many of you did not "reserve" your book, so I had to constantly edit the Googledoc.
4. Treat it like a university paper. Make it look decent, please? That means formatting, a title, your name, the date...
5. and...proofread.

Grades to date: Projects, quizzes, notes in a pdf in the slides area

the rest of your grades are in LON-CAPA or MasteringPhysics

The "redshift homework problem."

this week

April 2019

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
31	1	2 lecture	3	4 lecture	5 HW9 due	6 HW10
7	8	9 lecture	10	11 lecture	12 project day 2 HW10 due	13 HW11
14	15	16 lecture	17	18 lecture	19 HW11 due	20 HW12
21	22	23 lecture	24	25 lecture	26 Honors data upload HW12 due	27
28 2nd Midterm	29	30	1	2	3 FINAL EXAM 07:30	4

KEEP
CALM
AND
LET'S
REVIEW

some particles

spin is a defining quality of an electron

electron

symbol: e

charge: $-1e$

mass: $m_e = 9.0 \times 10^{-31}$ kg ~ 0.0005 p

spin: 1/2

category: fermion, lepton

particle: **photon, γ**

symbol:	γ
charge:	0
mass:	0
spin:	1
category:	an intermediate vector boson, a messenger particle

some jargon

jargon alert:

beta particles,

β (old name for an electron)

refers to: the emission of an electron in the decay of some nuclei - **beta decay**

entomology: alpha, beta,...

example: Carbon-14 → Nitrogen-14 + e

jargon alert:

alpha particles, α

(old name for a Helium nucleus)

refers to: the emission of a Helium nucleus in decay of some nuclei - **alpha decay**

entomology: alpha, beta,...

example: Uranium-238 → Thorium-234 + e

some numbers

constant of nature:

Planck's Constant, h

value: $h = 6.62606896(33)\times 10^{-34}$ J-sec

units: Energy - time

usage: everything at atomic and smaller sizes

some relations

relation alert: **Planck's Law**

refers to:

$$E = hf$$

Energy of radiation comes in a discrete amount for each frequency

example: photoelectric effect

relation alert: **Bohr Model**

refers to:

$$E_n = -\frac{\mathcal{E}_1}{n^2} \qquad r_n = a_0 n^2$$

Energy levels and "orbits" of atomic electrons are "quantized"

example: Hydrogen spectra, esp. Balmer Formula relation alert: **deBroglie relation**

refers to:

$$p = \frac{h}{\lambda}$$

wavelength tied directly to momentum

example:

electron diffraction

Compton scattering

relation alert:

Heisenberg Uncertainty Relation

refers to: $\Delta x \Delta p \geq h$ & $\Delta t \Delta E \geq h$

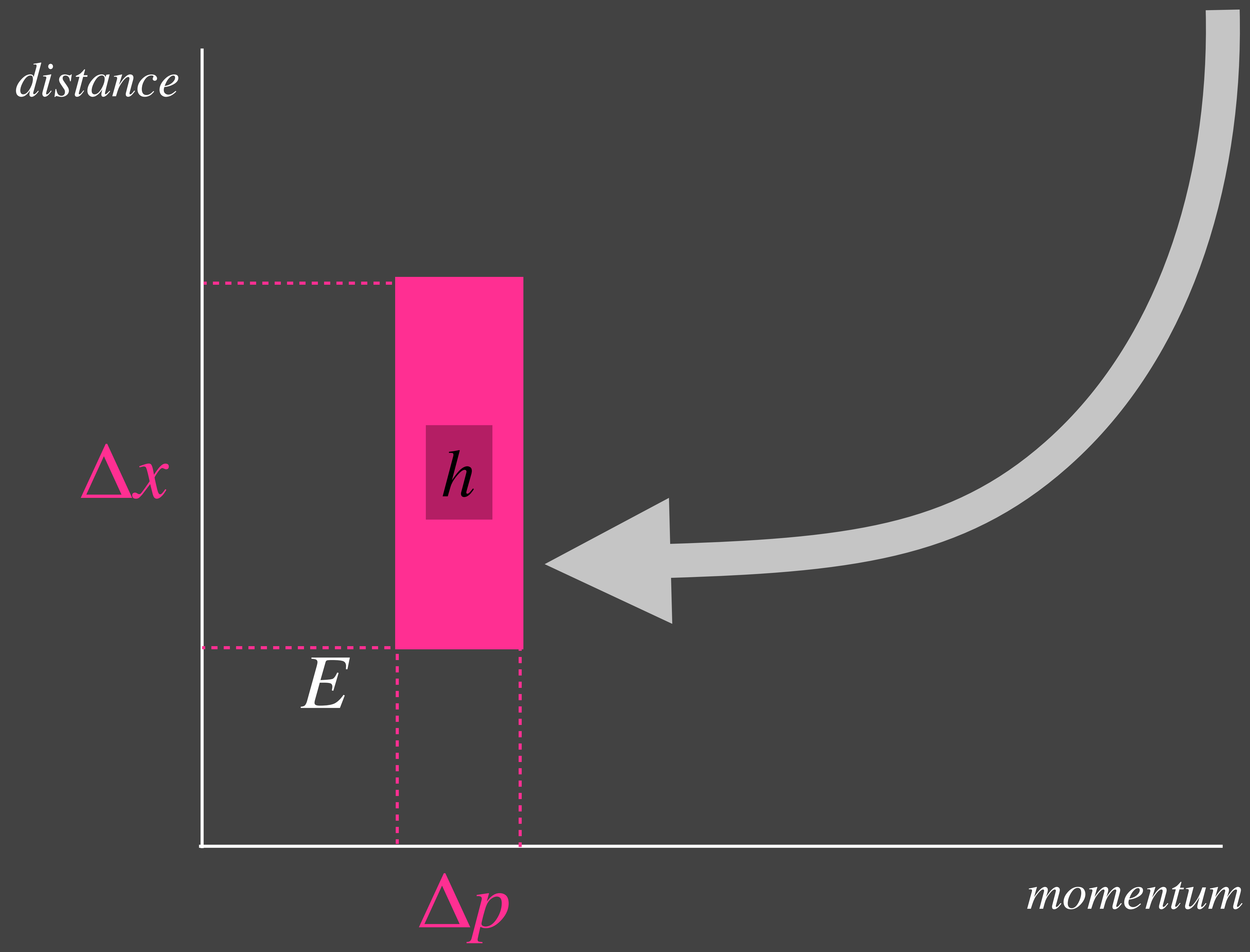
an inherent property of Nature

example: objects to not possess precise position and precise velocity at the same time.

uncertainty principle

one. more.
time.

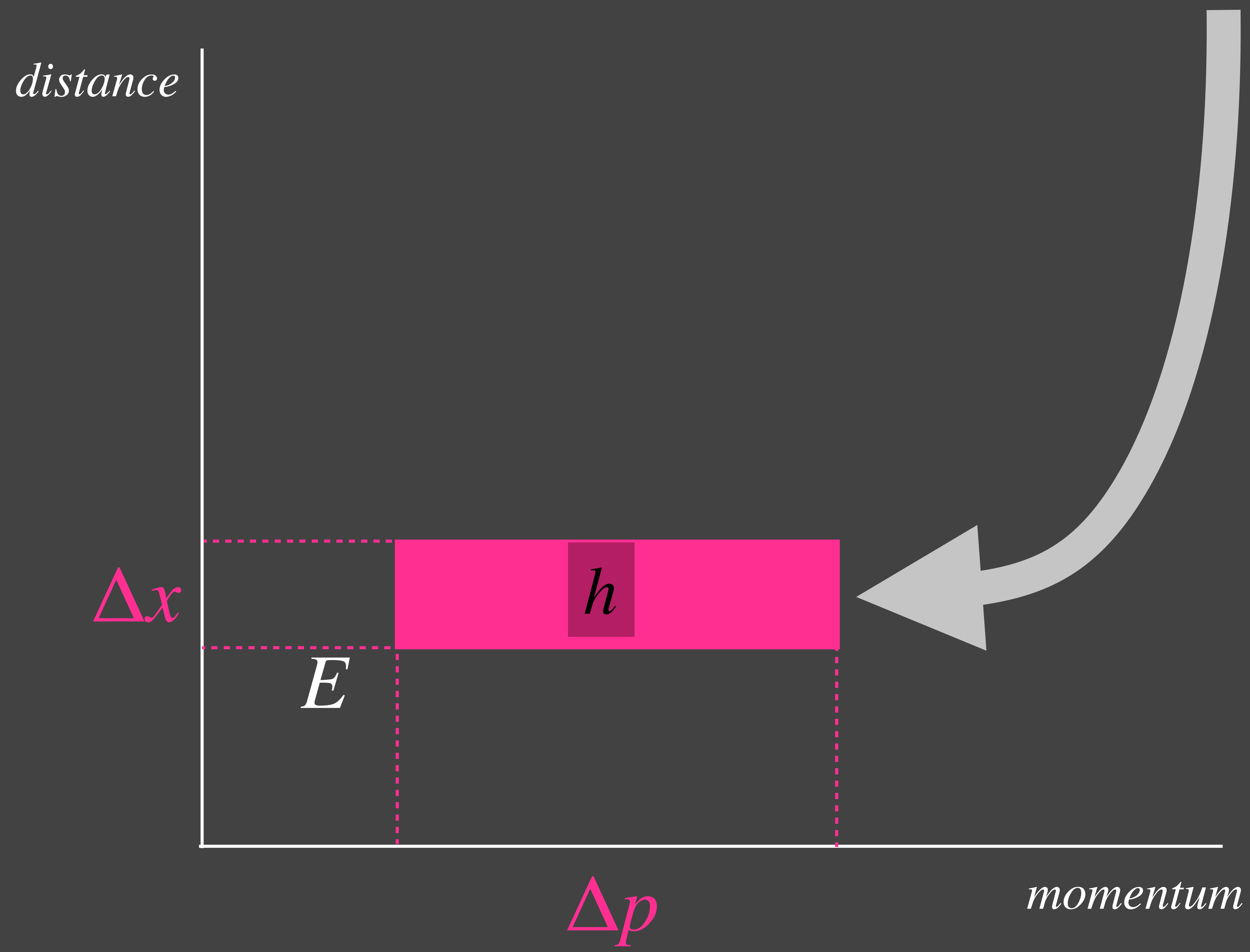
$$\Delta x \, \Delta p \sim h$$



uncertainty principle

one. more. time.

$$\Delta x \, \Delta p \sim h$$



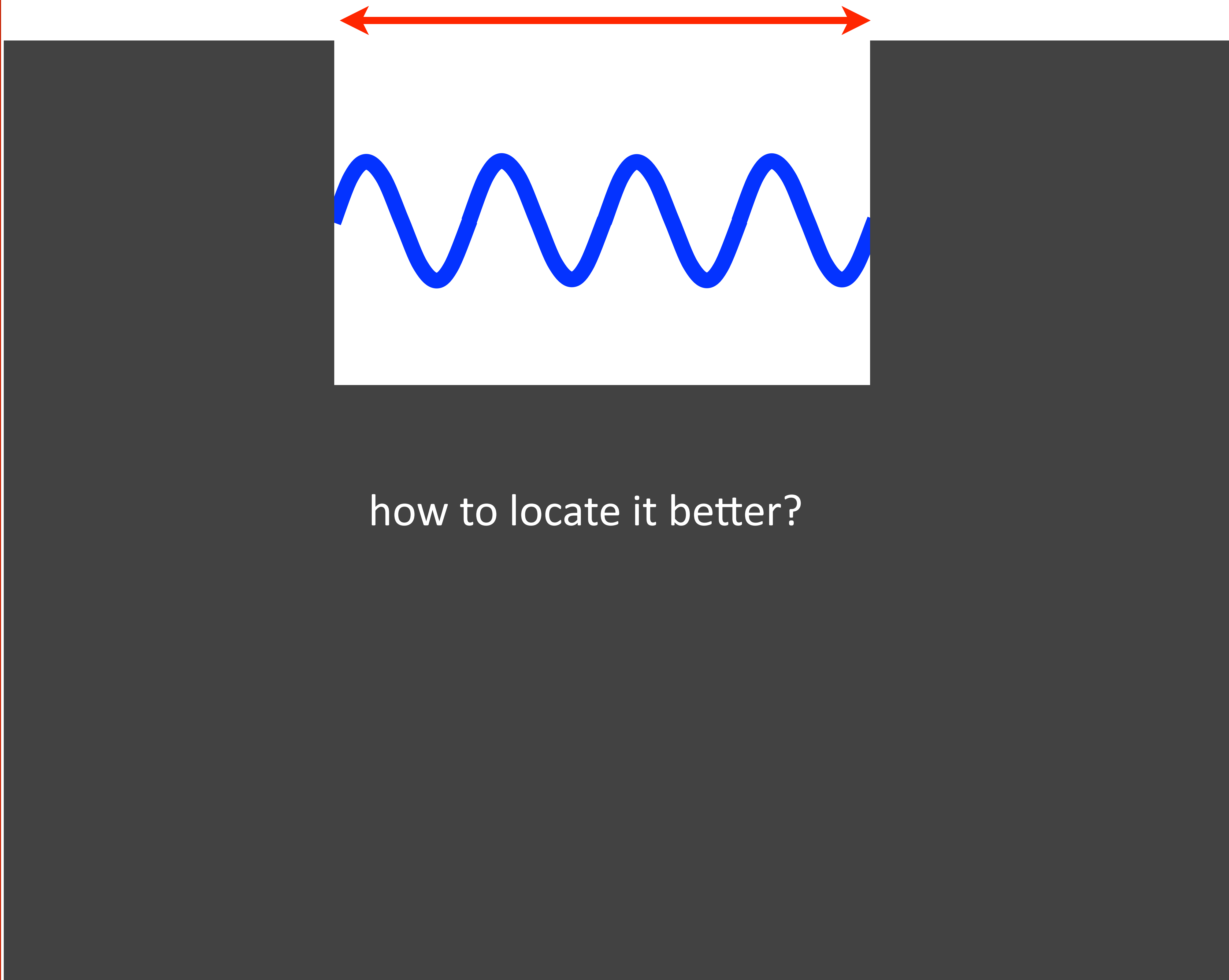
suppose we trap

an electron

$$p = \frac{h}{\lambda}$$

Where's the electron?

somewhere here:



how to locate it better?

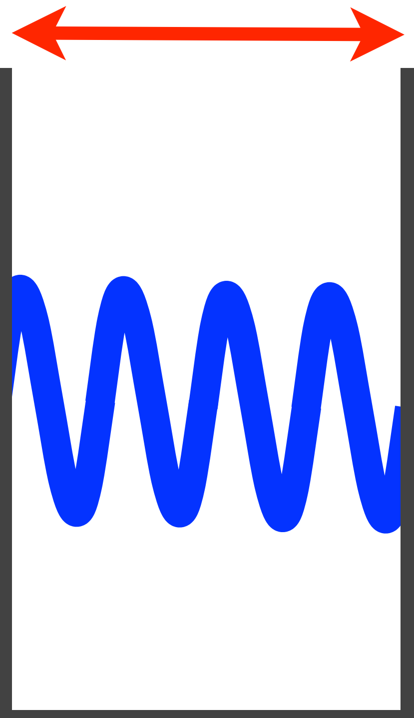
suppose we trap

an electron

$$p = \frac{h}{\lambda}$$

Where's the electron?

somewhere here:



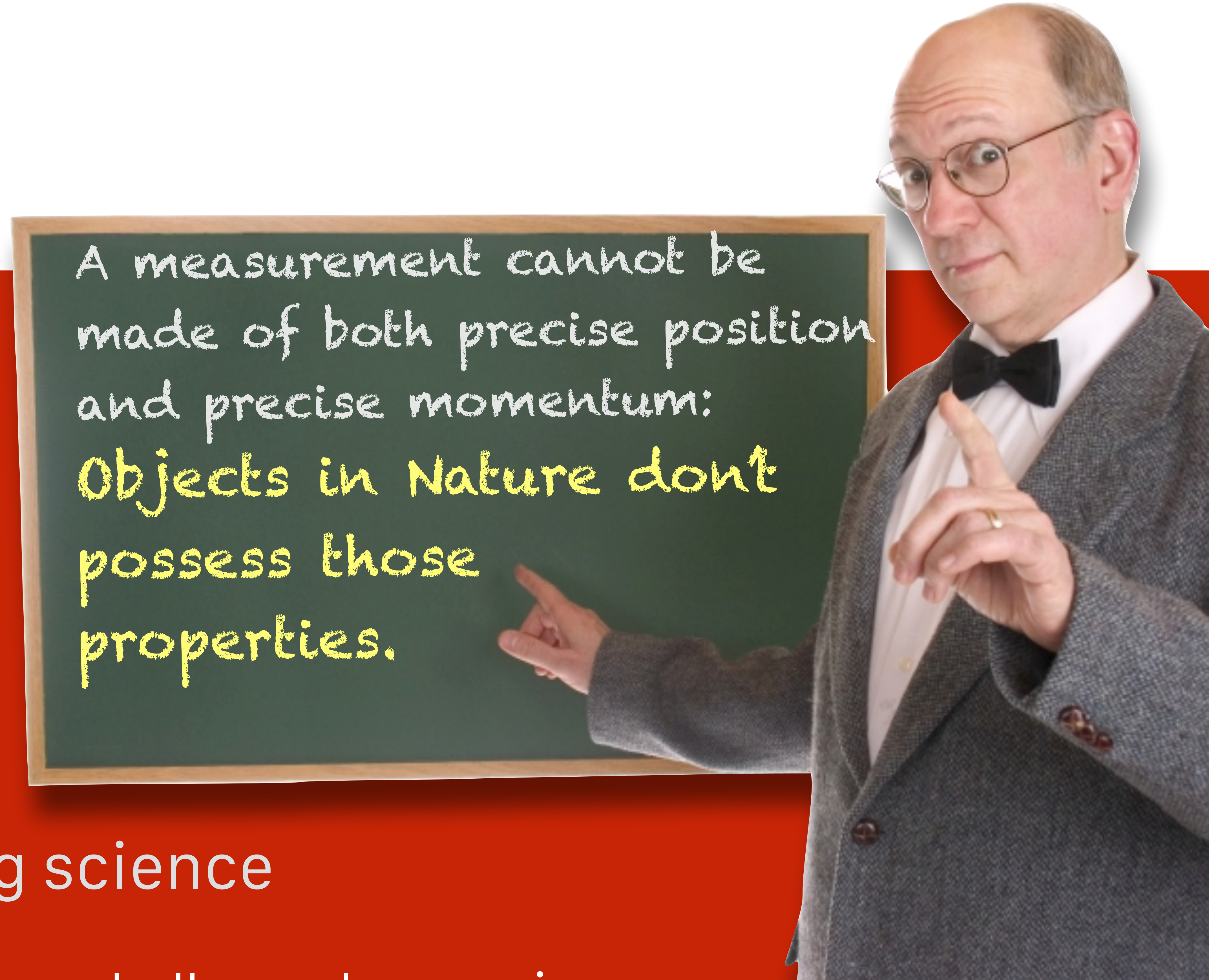
make the trap smaller

The wavelength is shorter...
So the momentum is higher!

an inevitable trade-off

in order to make the location more precise

you pay the price that its **speed is higher** a new way



Of thinking and doing science

we lose another classical, unchallenged scenario some of

the weirdness

of quantum mechanics

relation alert: **wavefunction**

refers to: ψ

"state" of a quantum object

example: what "waves" in quantum mechanics, but is directly unobservable relation alert: **quantum probability**

refers to: $|\psi|^2$

what's measurable in quantum mechanics

example: hydrogen "probability clouds"

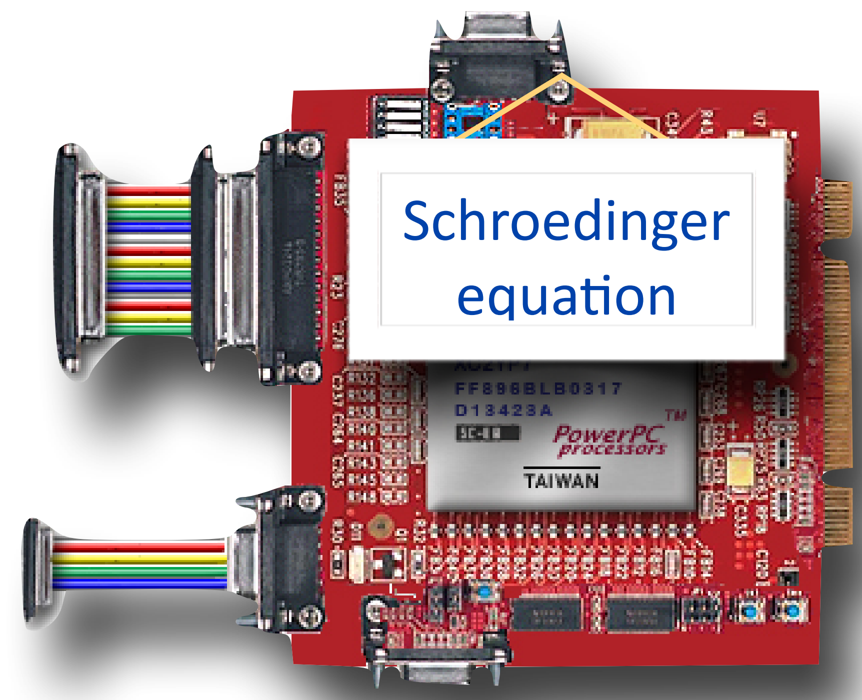
the “state” of electrons in hydrogen

an electron and proton

coupled by the Coulomb’s Force?

Coulomb’s Law

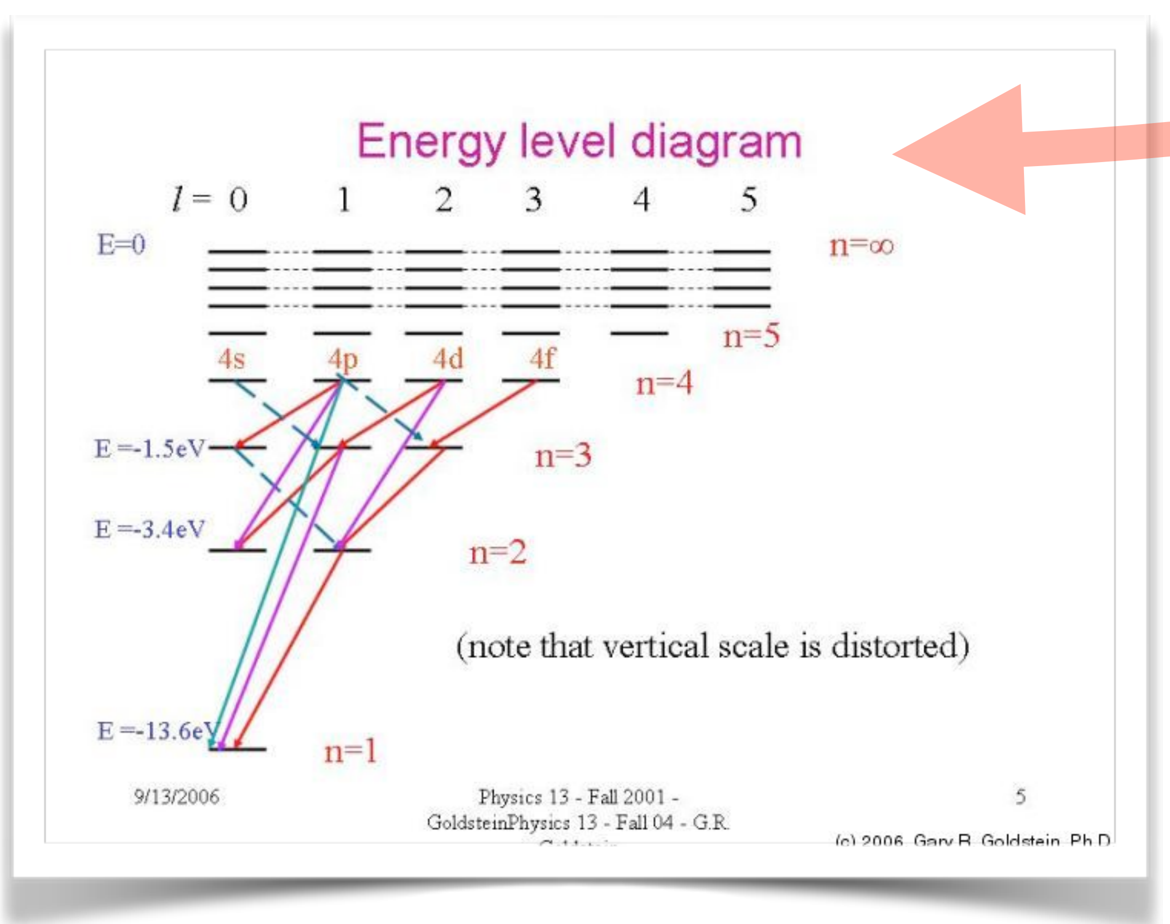
initial state at x_0, y_0, t_0

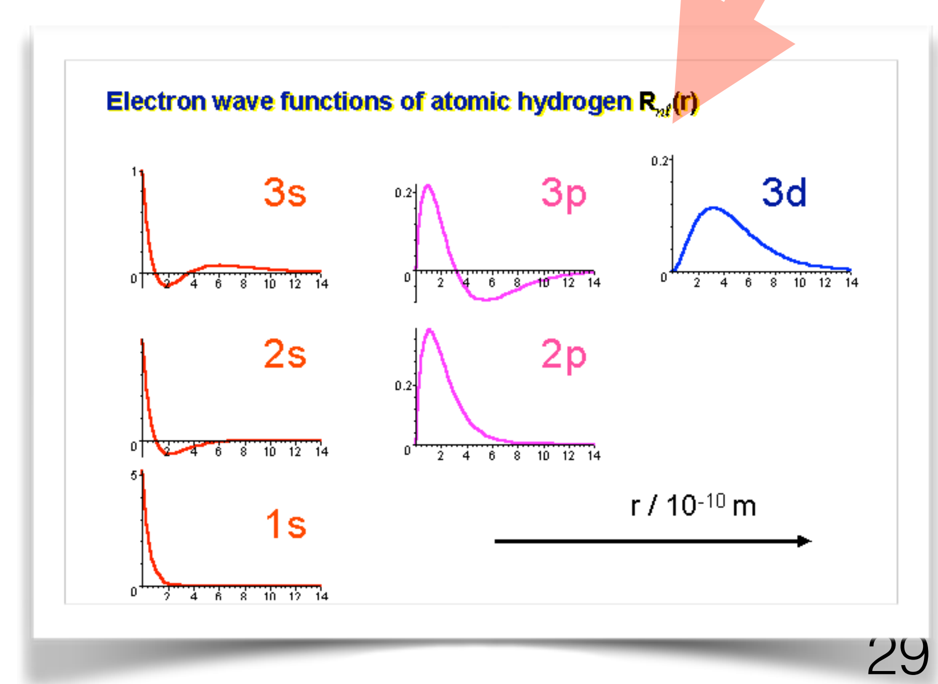


$\psi_{nl}(x,t)$

& $E_{n,l}$

at any time, all over the volume

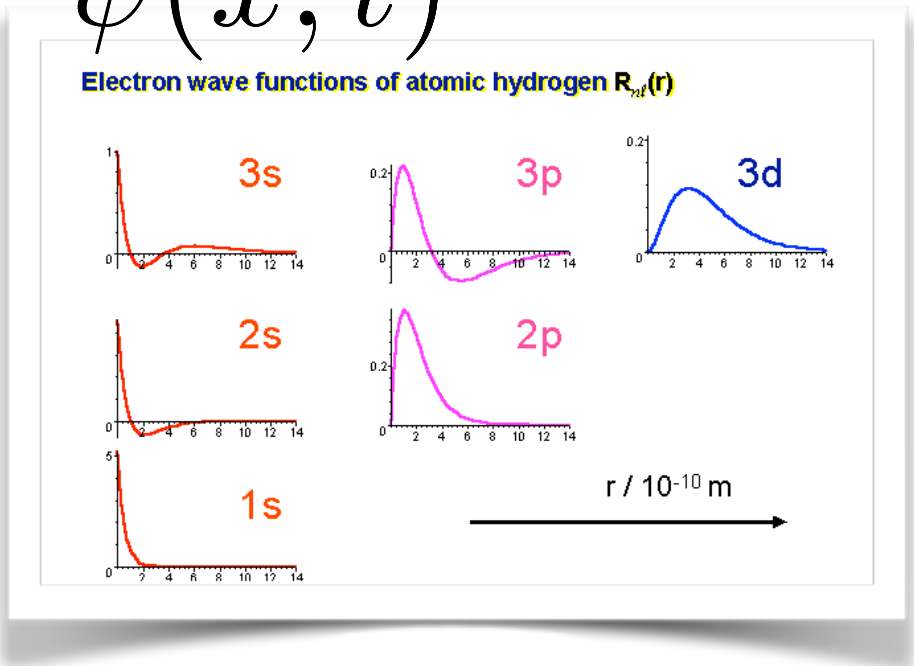


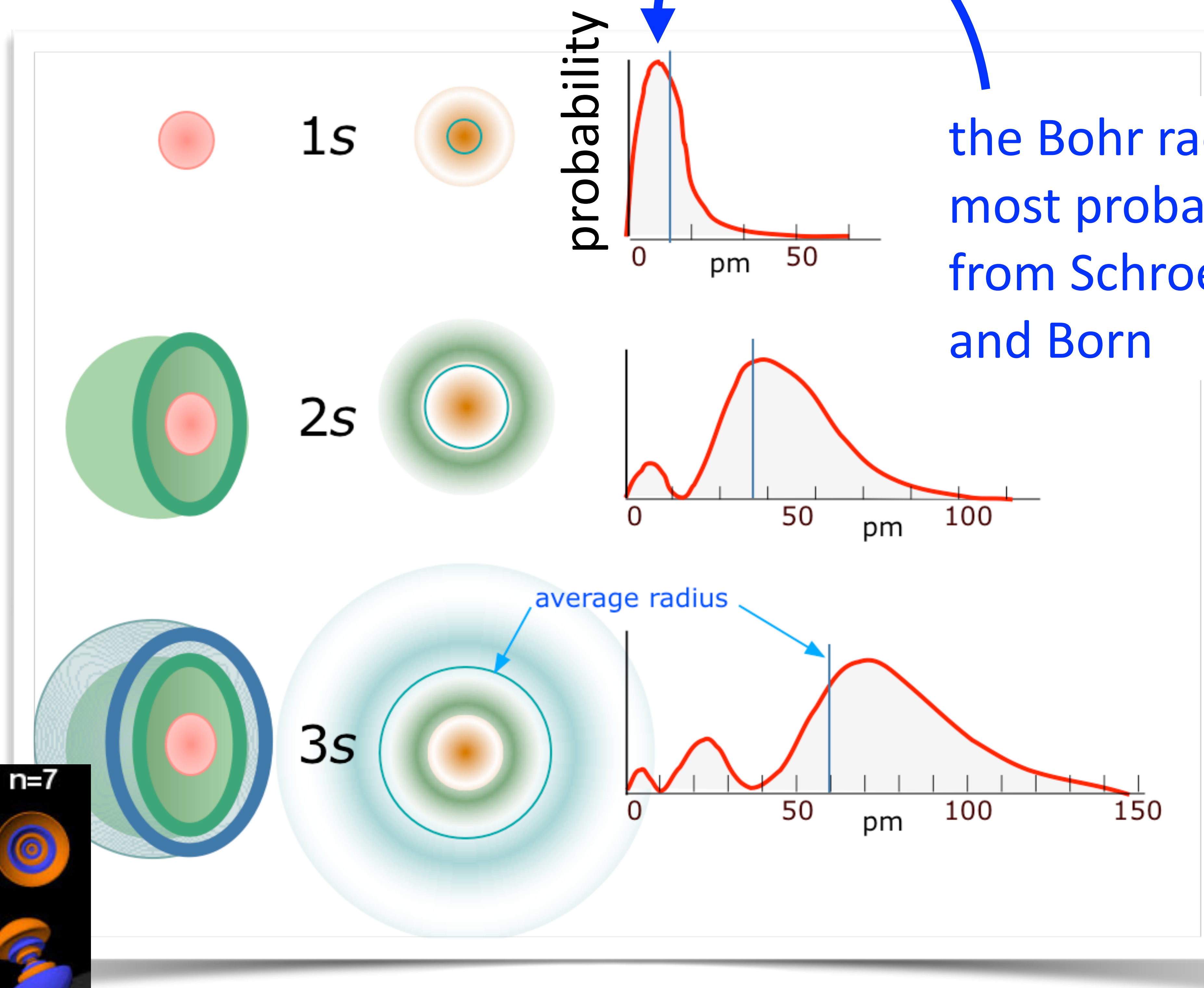


slice through the probability density of Hydrogen

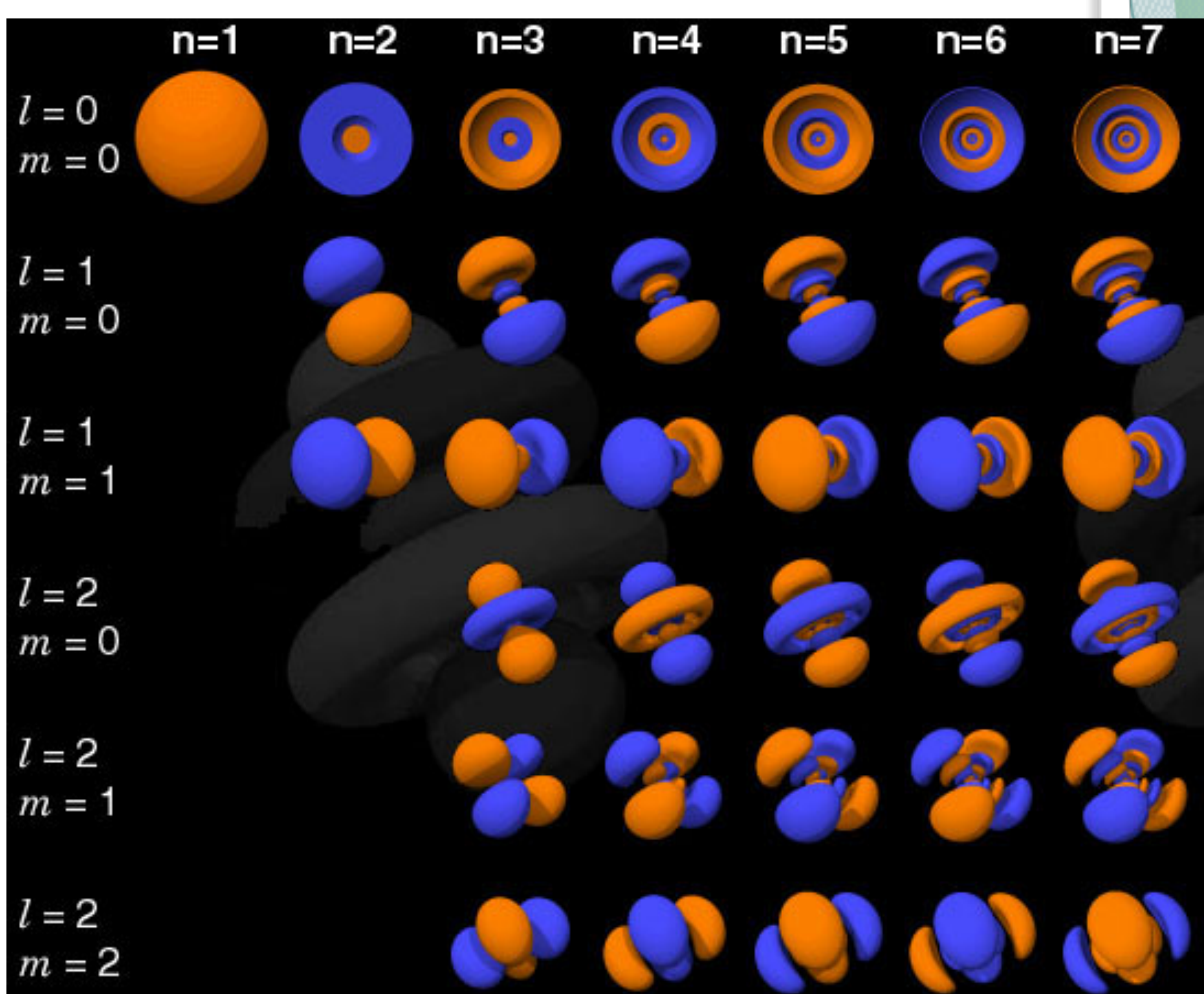
Square these:

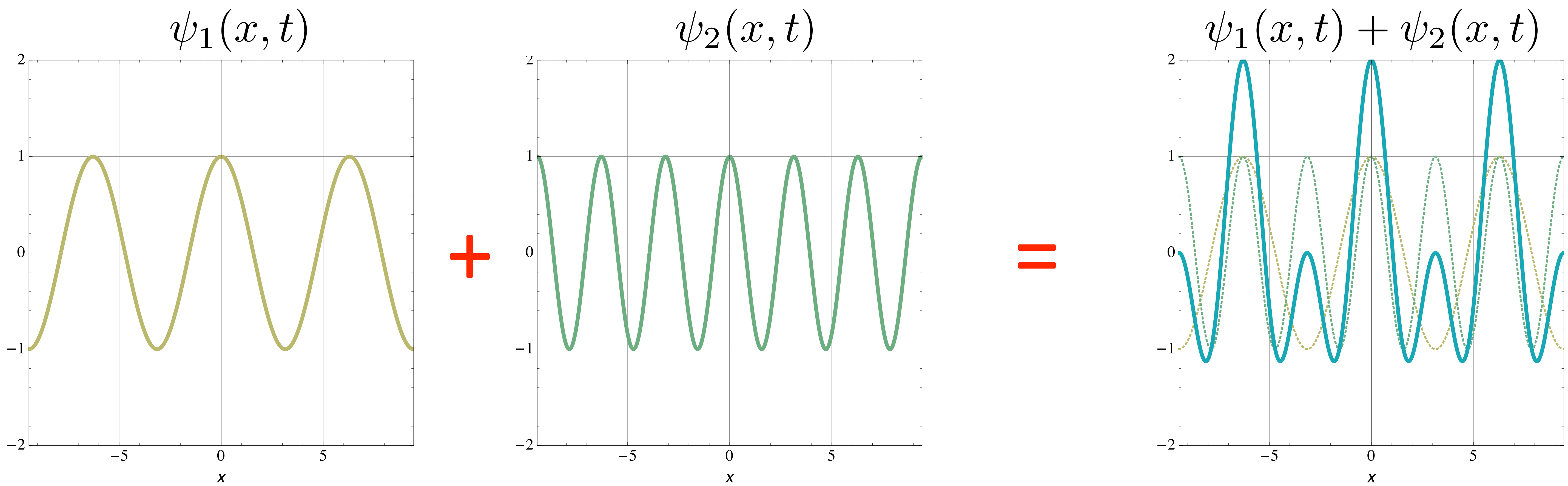
$$\psi(x,t)$$



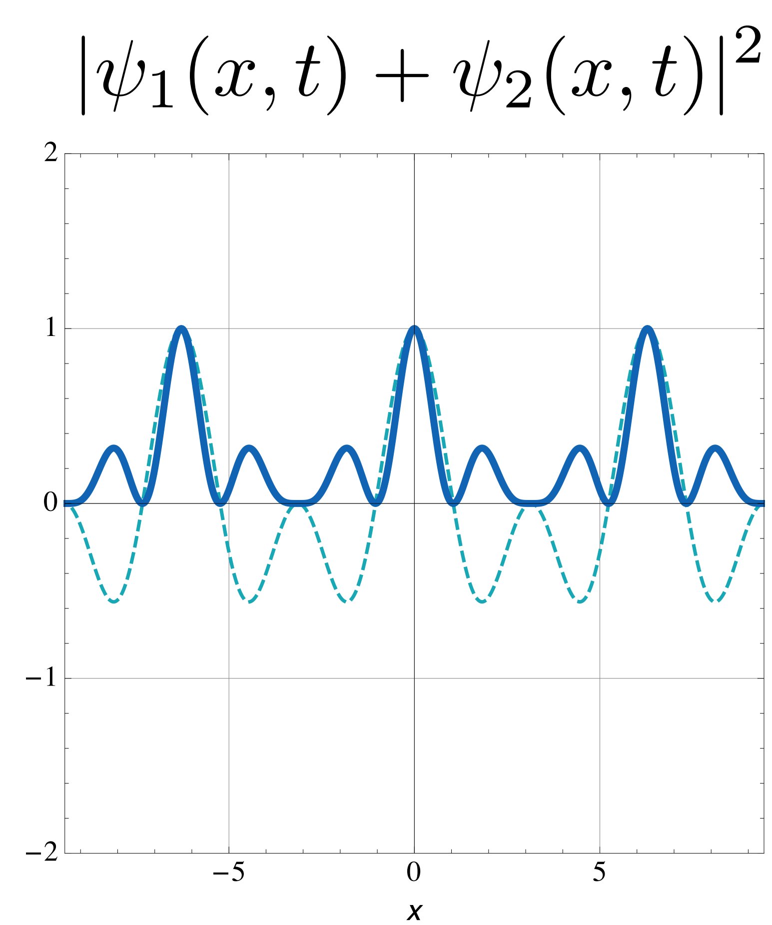


the Bohr radius - the most probable radius from Schroedinger and Born





But, remember that what's real about the quantum fields is the square: $|\psi(x,t)|^2$



notice the peaking

(I've changed the heights)

the wave combinations localize the state...with a spread in x

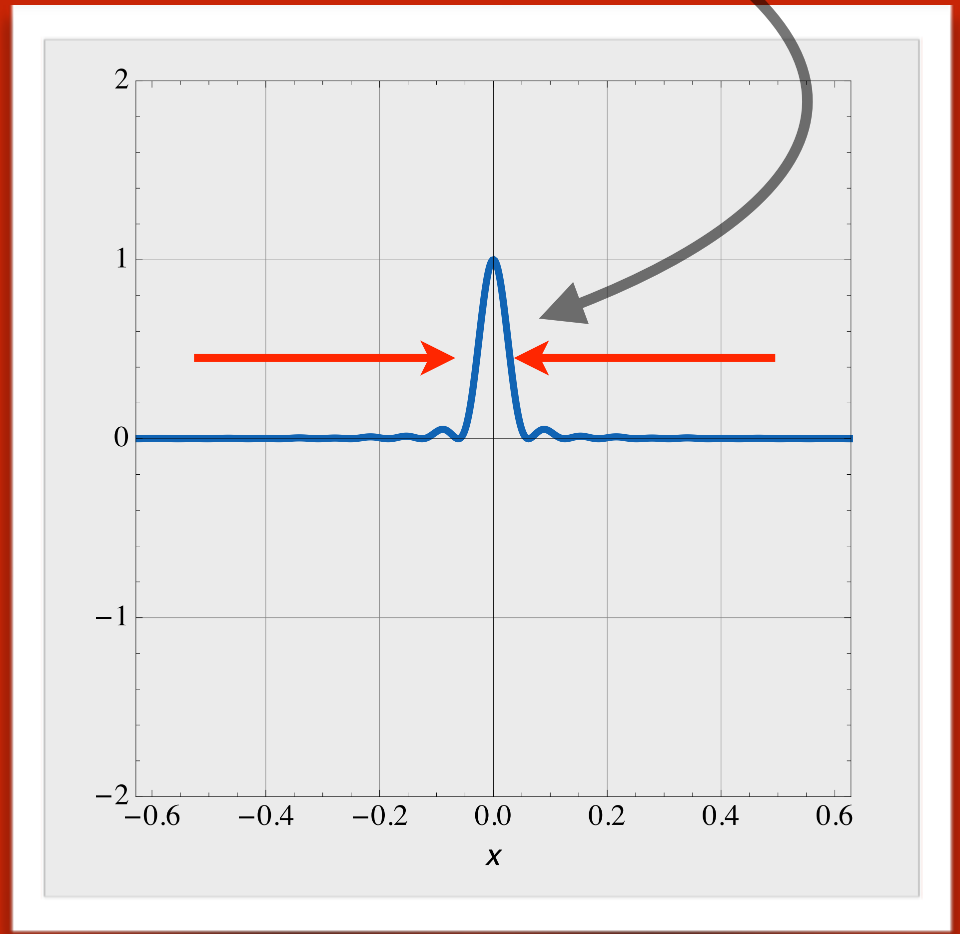
waves of different wavelengths?

different momenta

$$p = \frac{h}{\lambda}$$

Heisenberg Uncertainty Relation at work again

called "wavepackets": a particle interpretation

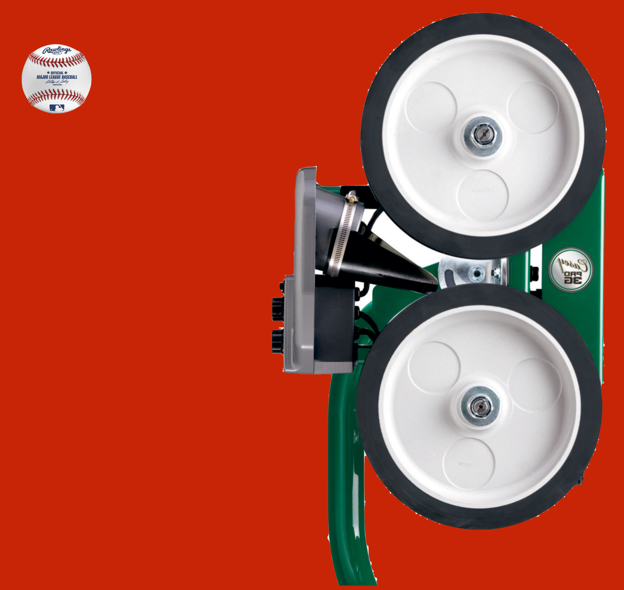


all of the wave combinations means all of the momenta contribute: a spread in p.

Nature’s little joke

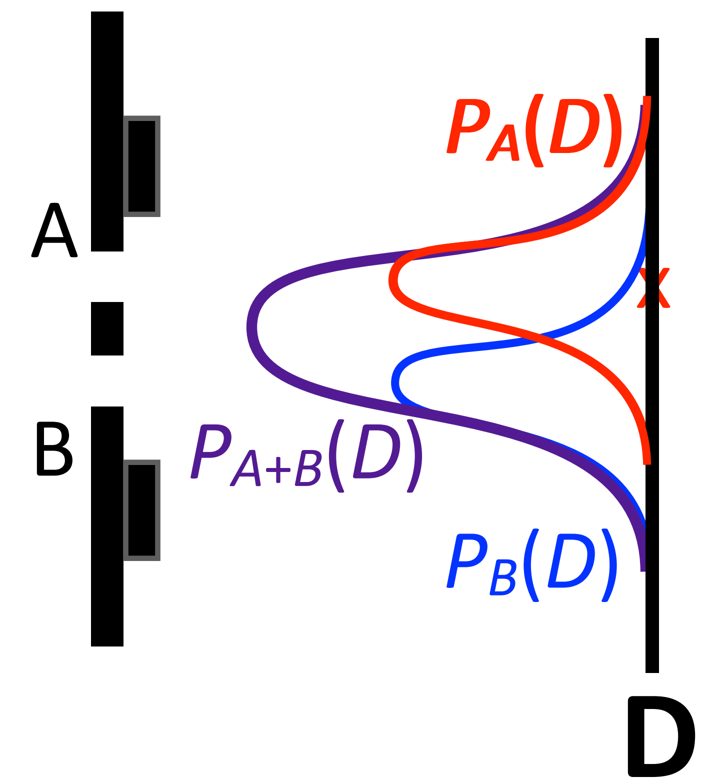
is encapsulated in a famous Feynman-description

a Gedankenexperiment...



two slit experiment

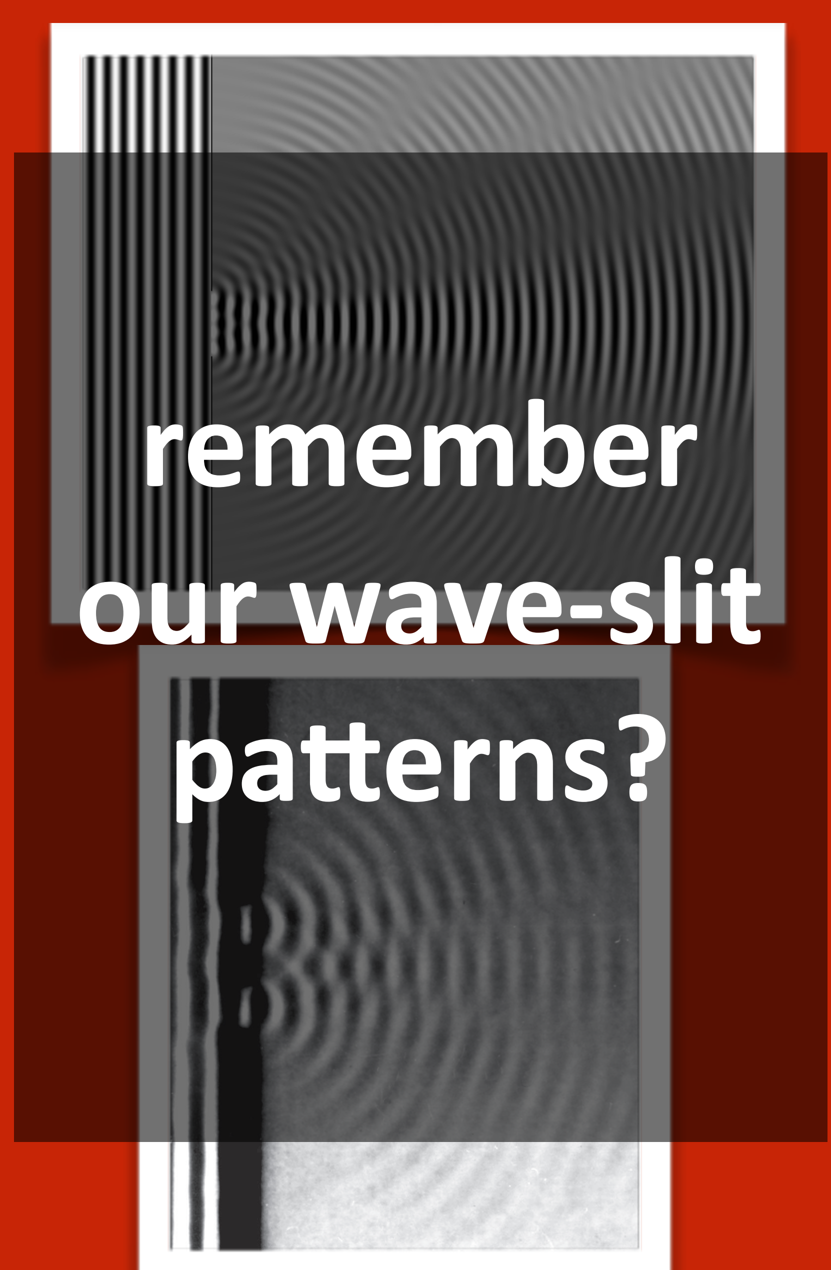
2 + 1 ways

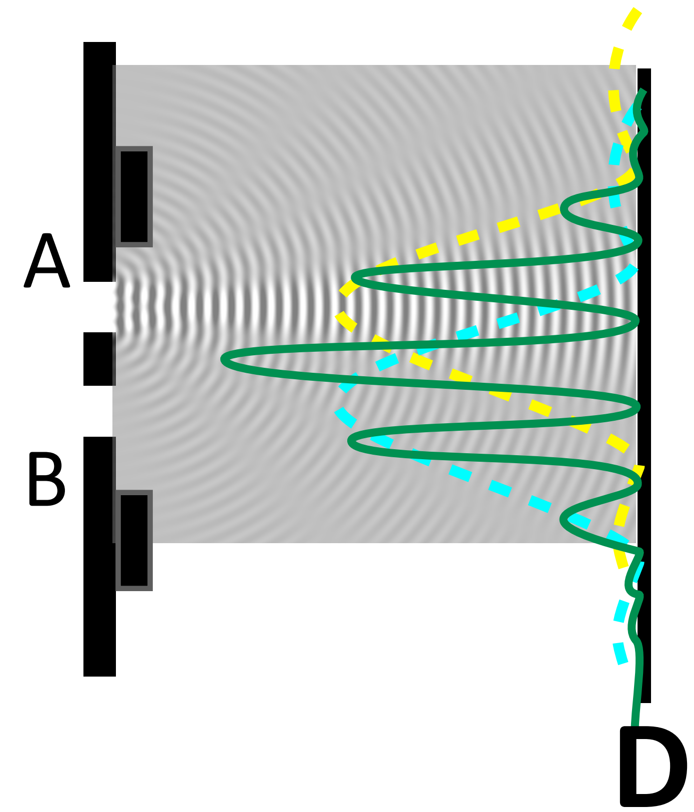


Two slit experiment with classical baseballs

$$P_A(D) + P_B(D) = P_{A+B}(D)$$

Like the "classical" situation of asking what is the probability of getting heads or tails in a coin flip...you'd add 0.5 and 0.5.

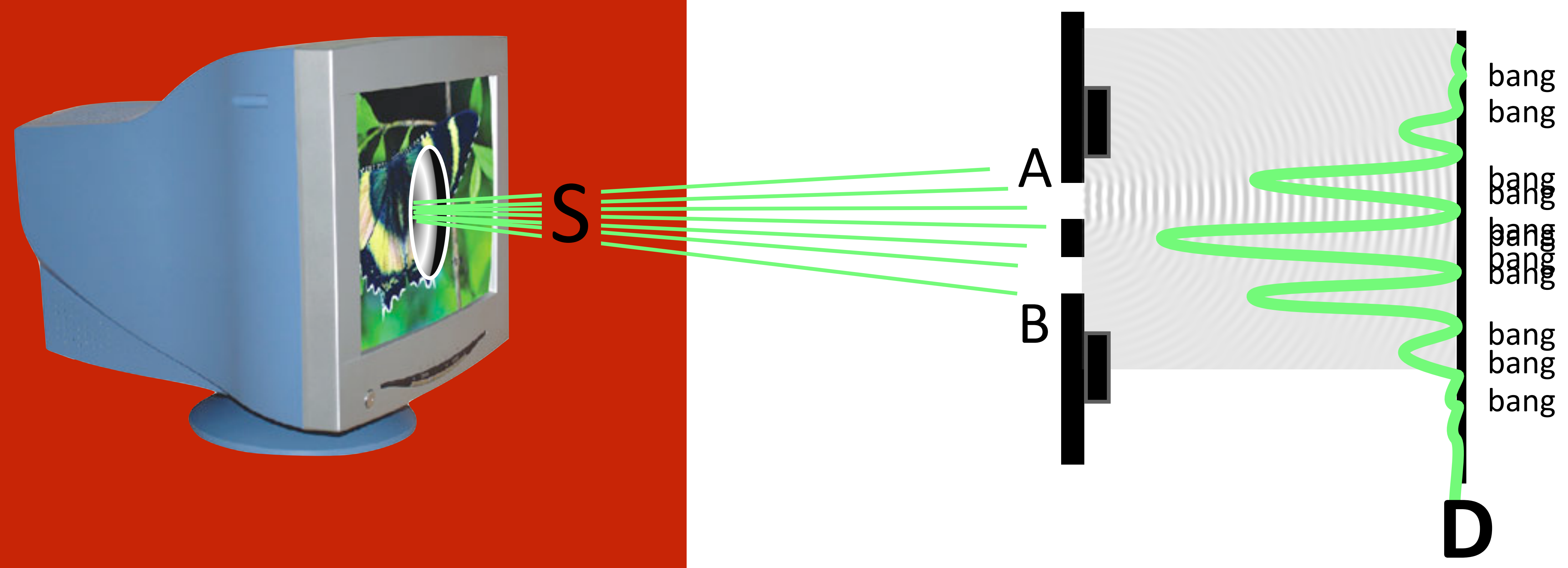




Two slit experiment with waves

$$P_A(D) + P_B(D) \neq P_{A+B}(D)$$

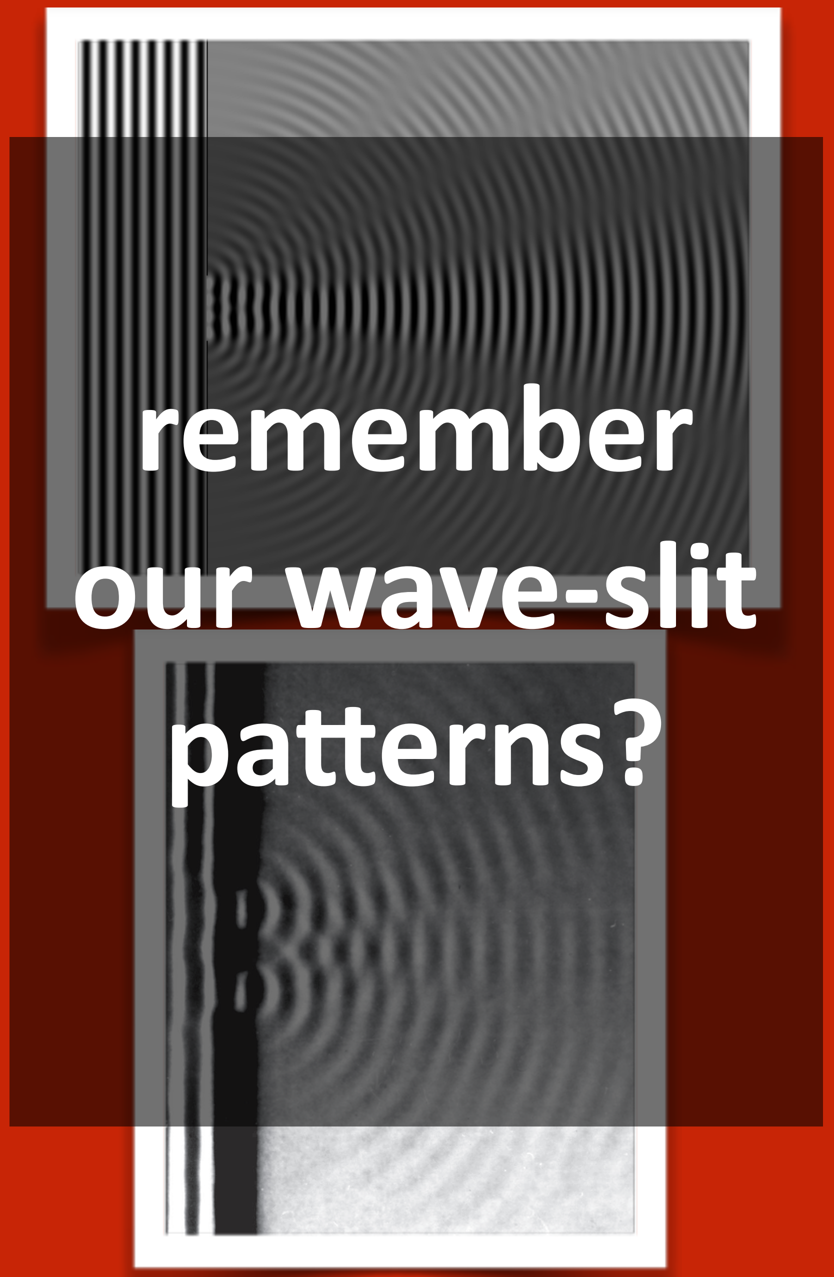
Interference causes the characteristic diffraction pattern



Two slit experiment with **electrons?**

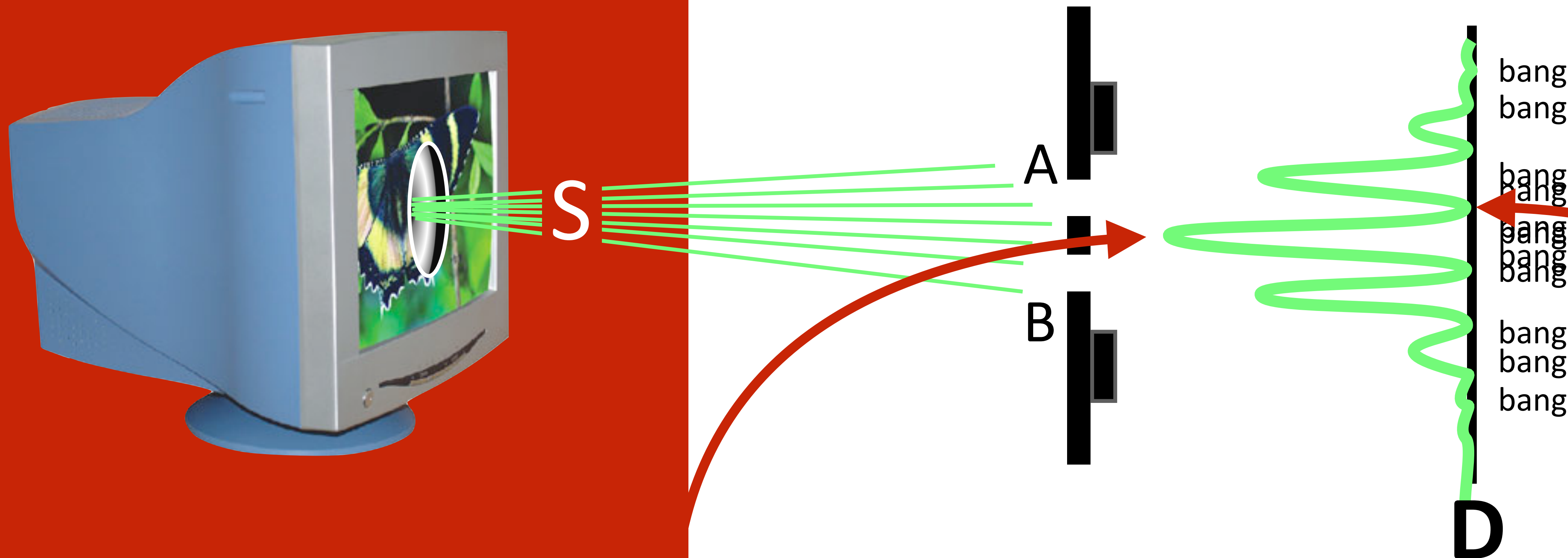
$$P_A(D) + P_B(D) \neq P_{A+B}(D)$$

Interference causes the characteristic diffraction pattern



Same result as for waves.

Maybe not a surprise given what's come before, eh?



probabilities don't add

it's the **quantum fields** that do the wavy-ness!

$$\psi$$

~~$P_A(D) + P_B(D) \neq P_{A+B}(D)$~~

$$P_D = |\psi_A + \psi_B|^2$$

$$P_D = \psi_A^2 + \psi_A^2 + \psi_A \psi_A^*$$

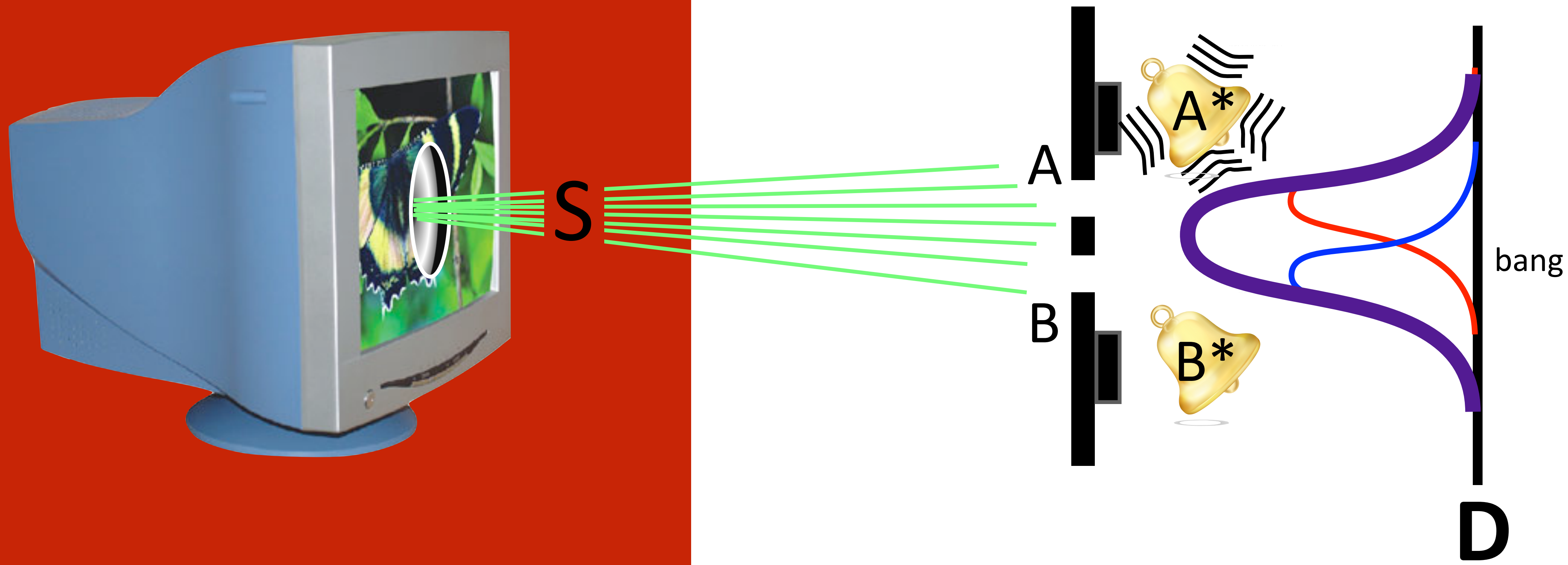
at some points this can be negative, sometimes positive

which gap did any electron come through?

okay...let's trick it

rig an alarm that sounds when an electron goes through a slit.

Hah!



Two slit experiment with electrons and an alarm?

So the sequence “S-A-A*-D occurred.

Every time A* rings - red curve. B* rings, blue curve.

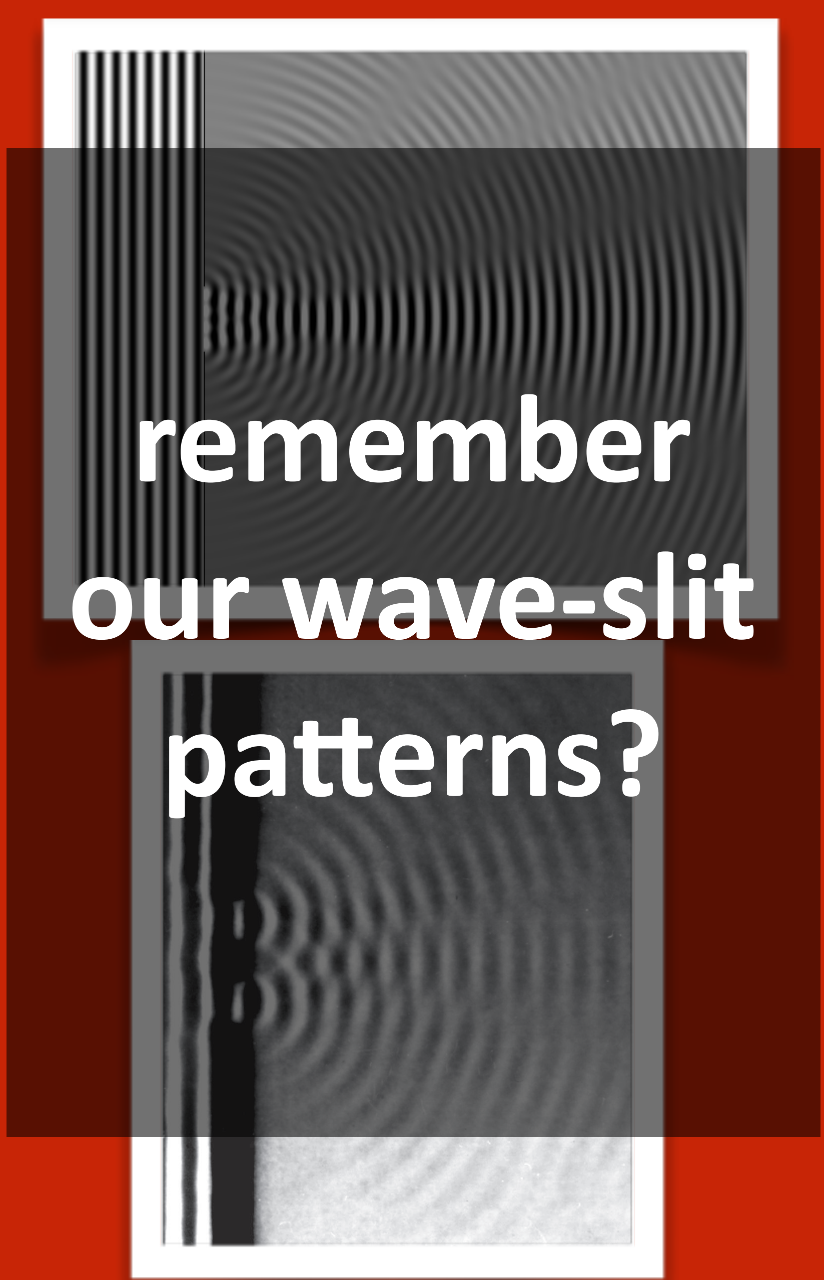
Same result as for baseballs.

Interference has gone away!!

Now: A* is a DISTINGUISHABLE event from B*

We specified the path...

and that changed the reality.



summarize

the classical situations

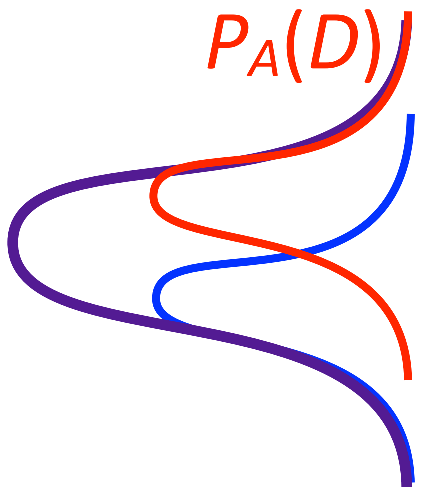
For **macroscopic objects**: outcomes add “normally”:

The result of

whatgoesthroughA and whatgoesthroughB is

the sum of whatgoesthrough(A **or** B)

one or the other



For **waves**: outcomes interfere:

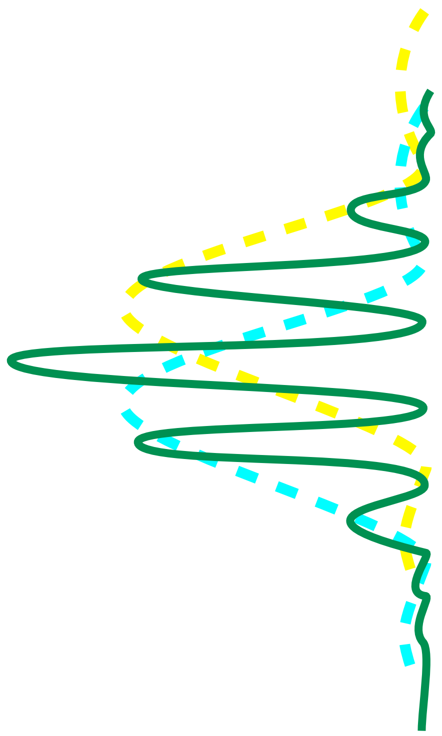
the result of

whatgoesthroughA and whatgoesthroughB is

the interference of whatgoesthrough(A **and** B)

both at the same time

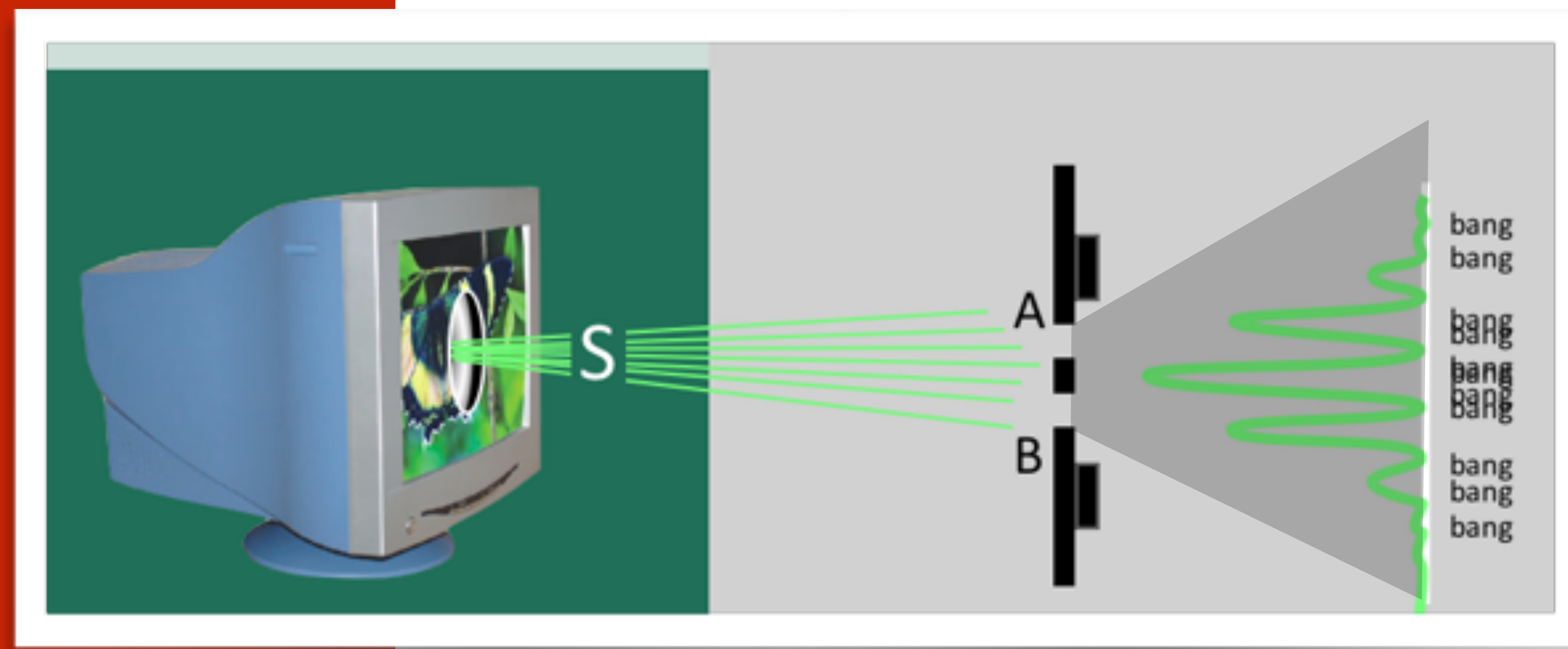
the waves interfere

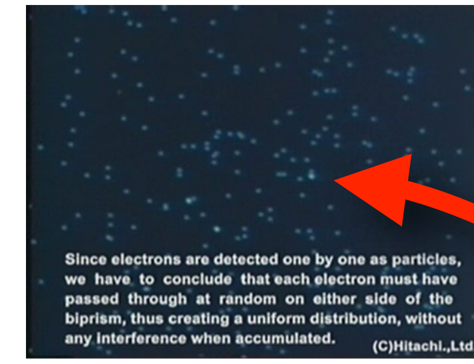


where is the electron?

it's real only when you make a measurement

and your measurement can determine how it's real





The electron is real at the screen.
it's unambiguously...there.
the "bang" is a measurement

what about here?
a nether-region

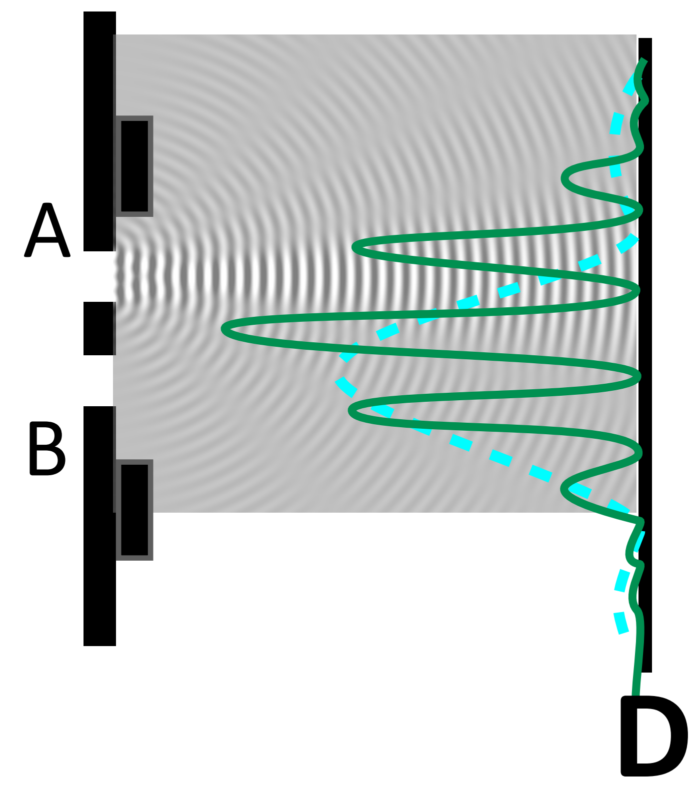
Bohr (and most of us) have to say that an electron:

- goes through both slits
- and is in a "**superposition**" state,
  *here of **both** the state* ψ_A *and the state* ψ_B

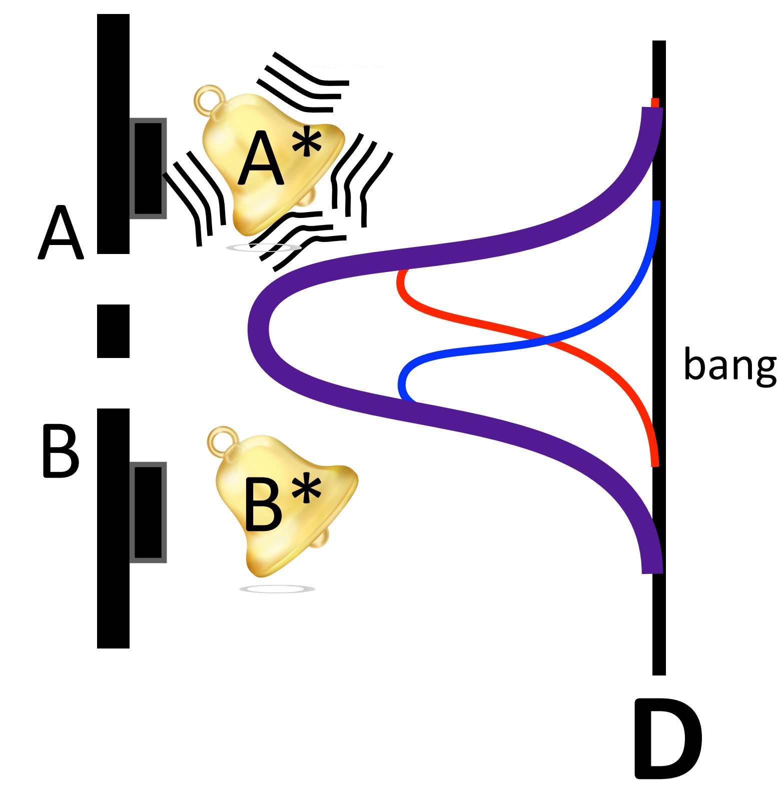
As soon as measurement is made...the superposition goes away and the potentiality becomes the actuality...according to the probabilistic prediction of the Schroedinger Equation.

“delayed choice experiment”

In effect, you’ve determined the nature of the nether region…in the past



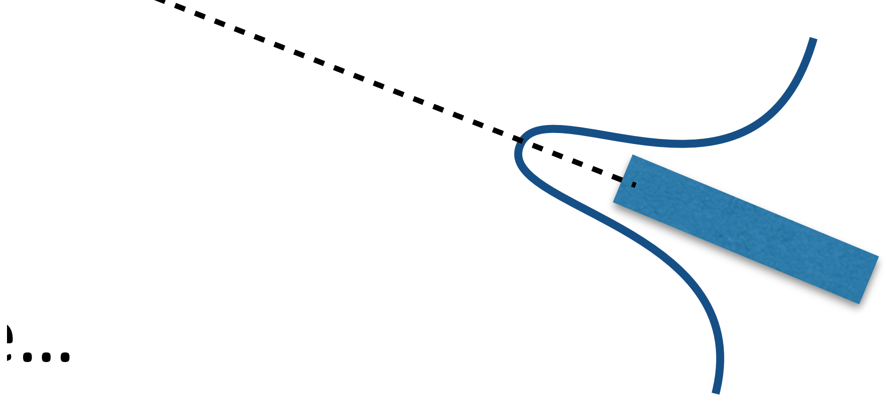
both slits: waves



either slit: particles

determine particle after, then look at screen

top slit: particles



…

particle-like

Bohr, 1928: ‘‘Complementarity Principle’’

quantum entities (all entities!):

exhibit themselves as either particles

or waves

the only way to decide:

make a measurement

Do a particle property measurement? get particles

Do a wave property measurement? get waves

“entanglement”

2 states’ properties are correlated over distance…

Einstein, Rosen, Podolsky “EPR”

spin is conserved: $S_0 = S_1 + S_2$

spin is superposition of 1 & -1

detector that measures any spin:

detector that measures only spin:

Einstein:

“spooky action at a distance”

Nature:

that’s the way it is

what we can say is real

is now very tricky

and not understood.

We know that quantum fields contain all of their potentialities

and a measurement "collapses" them into just one outcome

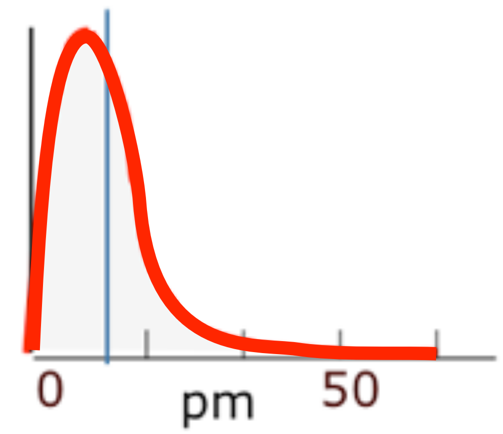
the concept of a "measurement" is totally not understood.

the wavefunctions are everywhere

spread out and overlapping

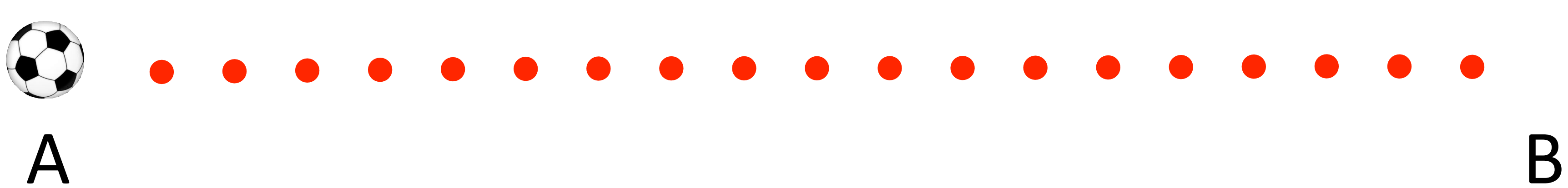
that's how molecules stay together

but...jeez. everywhere.



doesn't go to zero.

There's a probability that the electron in one of your water molecules might spend a brief time at the Louvre



Something big...seems to have a definite trajectory

Something tiny...doesn't.

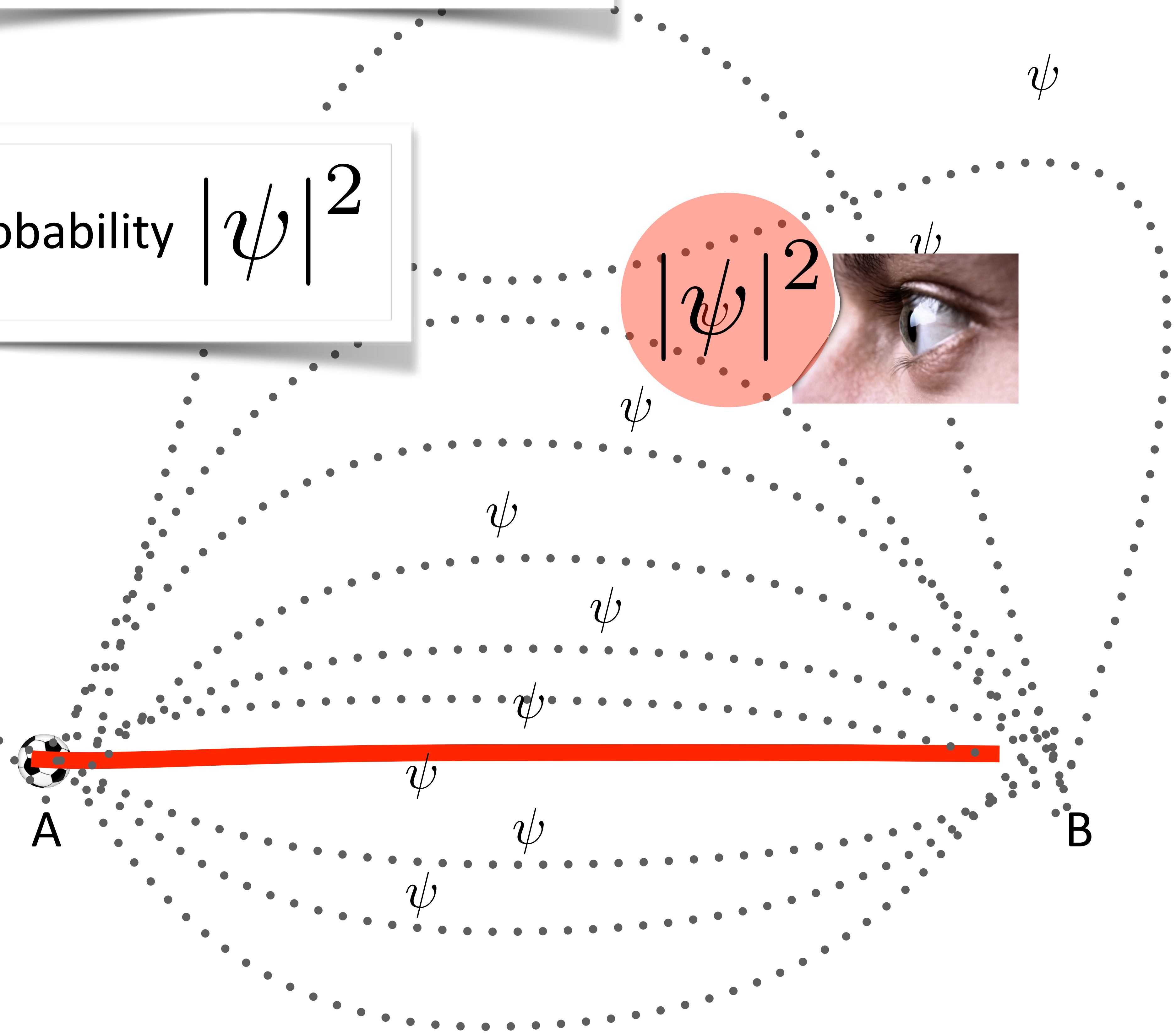
the wavefunctions are everywhere

They're waves, after all.

make a measurement....there

Only then is it real.

the electron is there with probability $|\psi|^2$



Feynman's picture was one of particles: which take all possible paths

We can calculate the wavefunction at any point, very precisely...it's completely deterministic

The trajectory of a big object?

Overwhelmingly probable quantum likelihood: the classical path

so where is a quantum

before it’s measured?

anywhere? everywhere?

yeah.

to take it to an absurd conclusion:
the dreaded Schroedinger's Cat

proposed by Schroedinger as an absurdity in 1935

because he too had become disgusted with this own creation - he switched to biology!

Schroedinger must have been a dog person

Imagine:

a radioactive source,

Geiger counter, and

a glass bottle of a deadly poison

with a cat

in a box,

a weight drops on the glass, breaking it

after the first radioactive decay?

...dead cat.

Schroedinger must have been a dog person

Now imagine that the radioactive nucleus as a **half life of 10 sec**.

so, after 10 s,

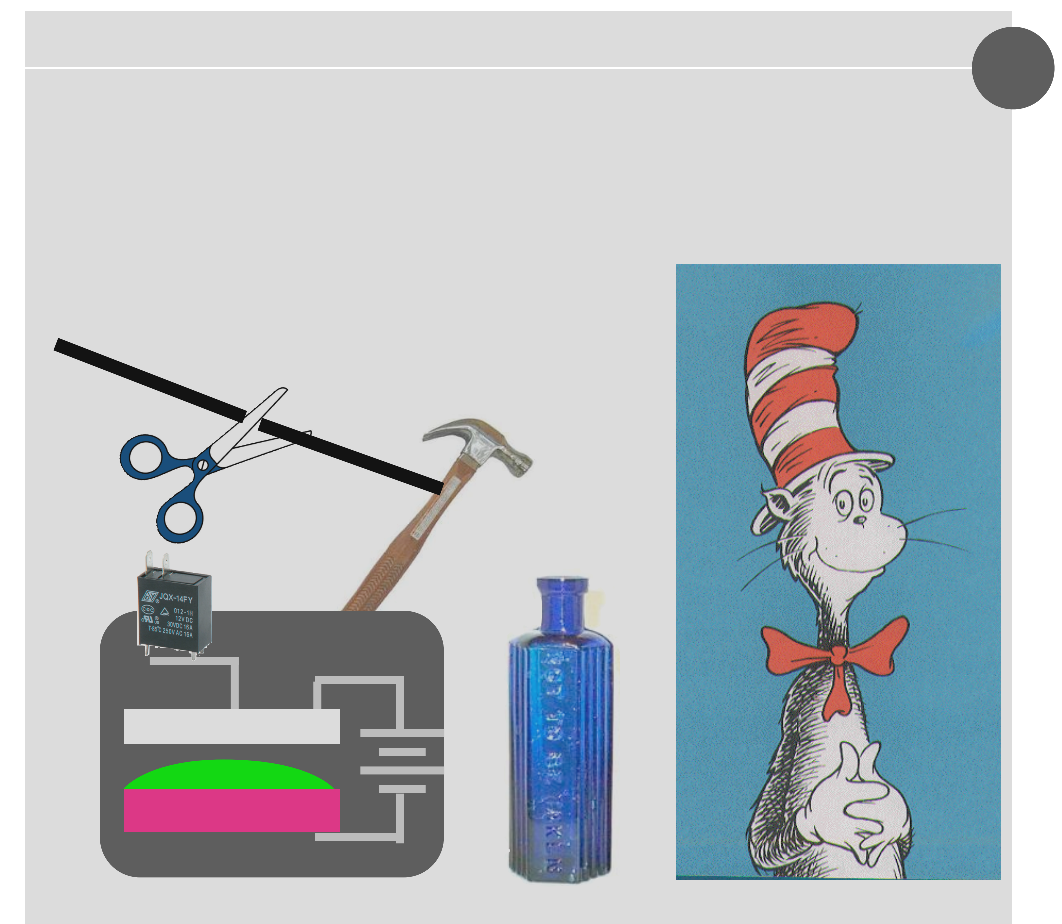
50-50 chance that it has decayed

Set it all up...wait for 10 seconds.

what is the state of the cat?

alive or dead?

or both?



‘‘Copenhagen Interpretation’’

It is meaningless

to speak of reality without a measurement

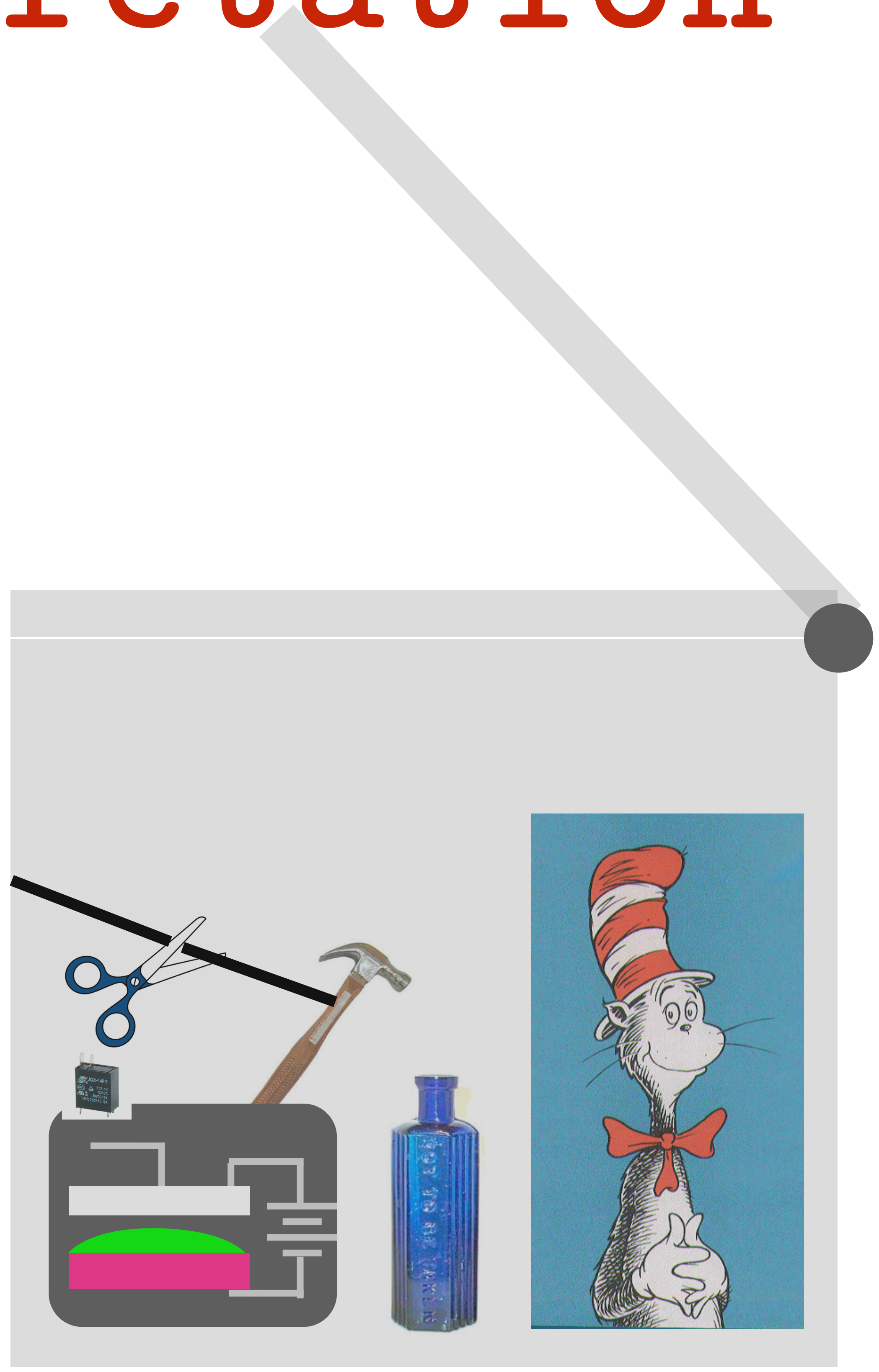
Entities have no definite reality

the cat is neither alive nor dead

or it is both

To know you must open the box

make a measurement

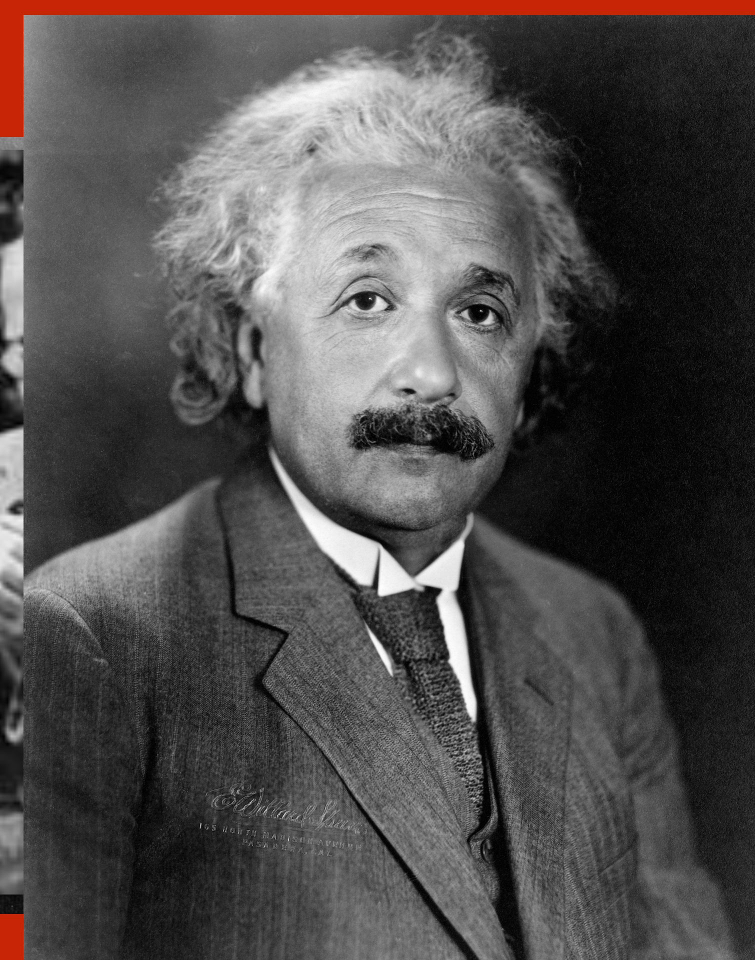


this is how we have to think about it:

before measurement: alive-dead state - **superposition state of both**

after measurement: is **either** alive **or** dead

Einstein and Schroedinger have left the building



here's our house

just before painting
last year

need to pick a color:

my wife says "red"

I say "blue"

SHERWIN-WILLIAMS
quantum paint

I expect it to be:

purple

mixing red and blue

but the quantum mechanical paint

that I paid extra for?

can't "exist" in a superposition, mixed state.

Only one state.

sometimes it's red

but the quantum mechanical paint

that I paid extra for?

sometimes it's blue

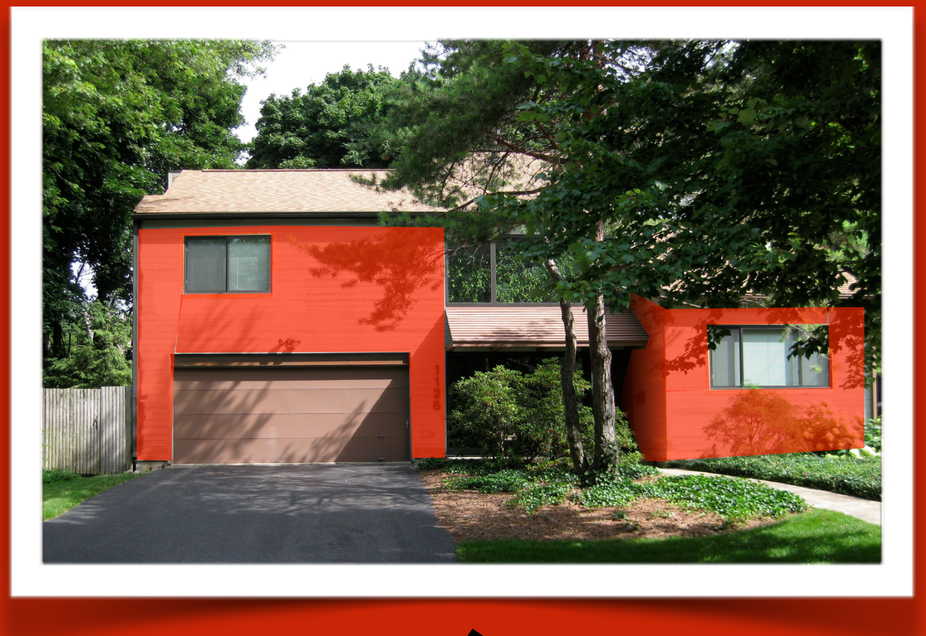
it’s never the mixture

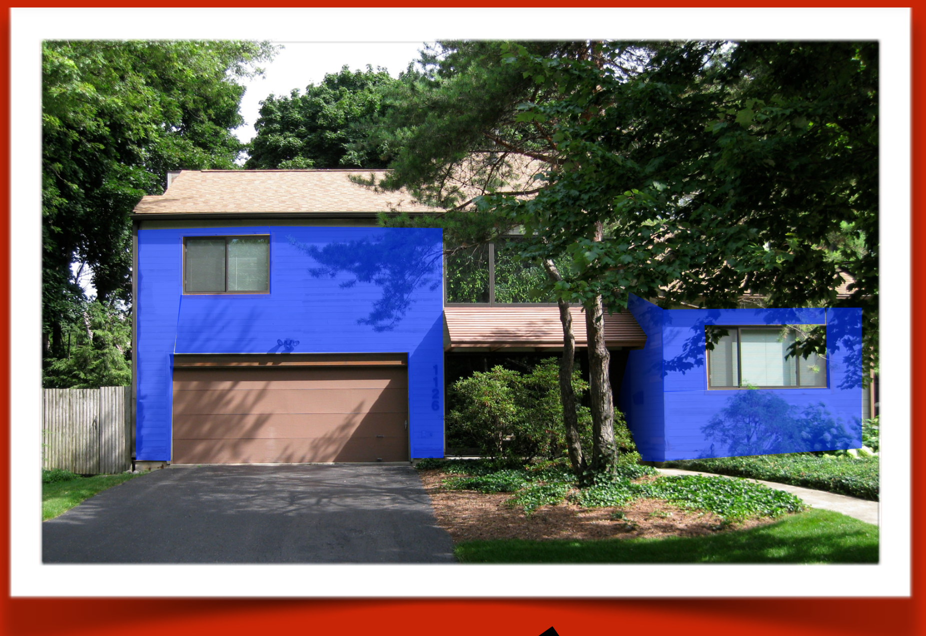
that it potentially might be

one or the other

More red paint?

not redder…just red more often





the cat is either alive or dead, not both.

“

I think I can safely say that nobody understands quantum mechanics.

Richard Feynman

But we can calculate with Quantum Mechanics very, very well.

We’re all highly skilled Quantum *Mechanics*

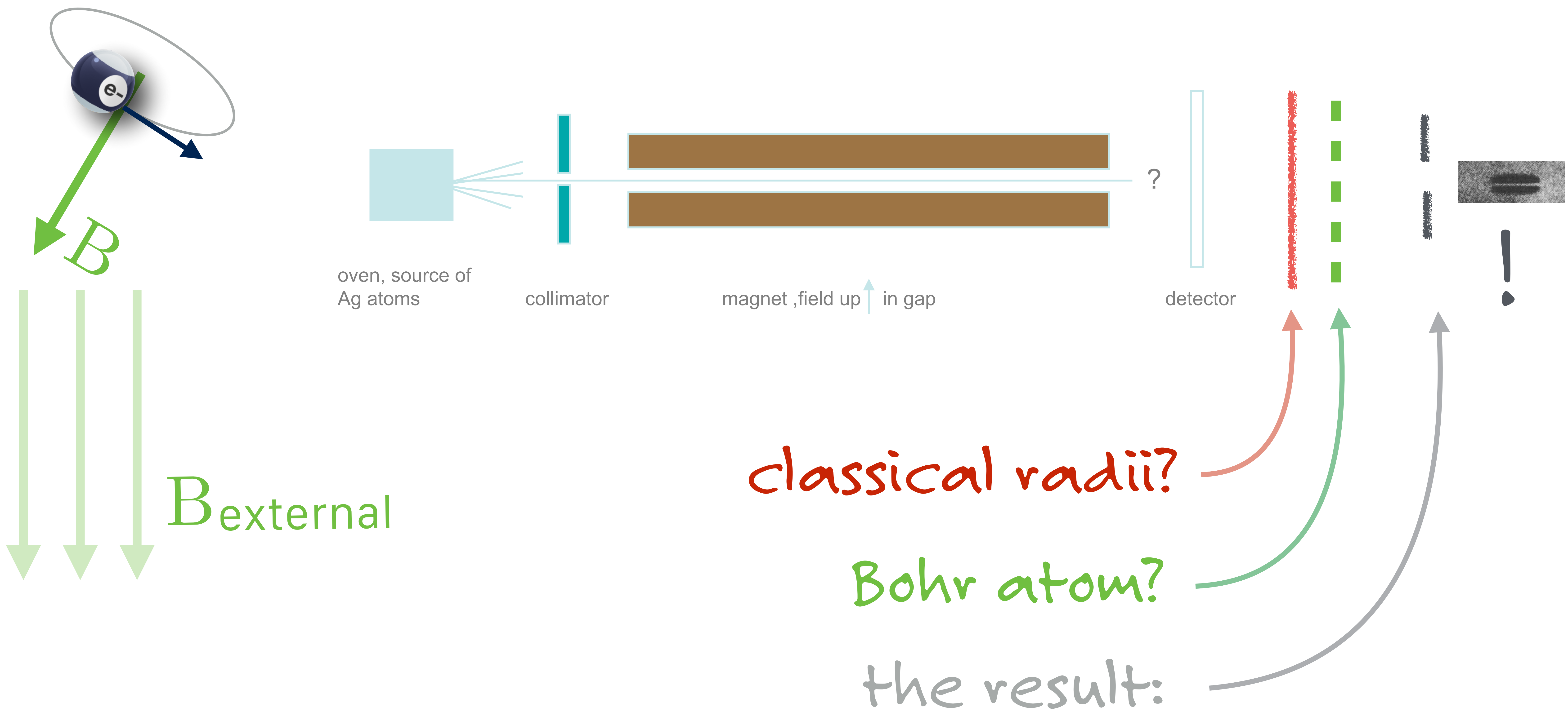
spin

1921

remember circular currents?

act like a magnet...a compass?

Otto Stern and Walter Gerlach. 1921



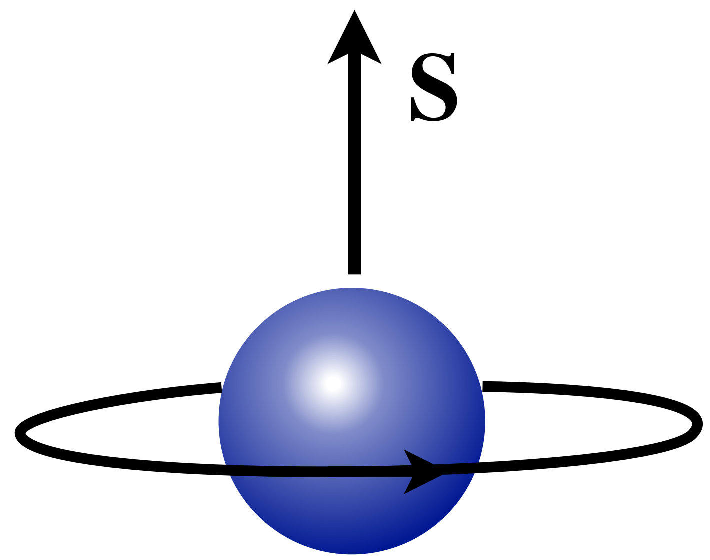
path to Stockholm?

1925: Ralph Kronig - Pauli, Heisenberg: dumb

1925: George Uhlenbeck and Sam Goudsmit - boss...okay, Lorentz: dumb.

they published anyhow...no Prize

The electron **itself** is *like* a spinning charge...

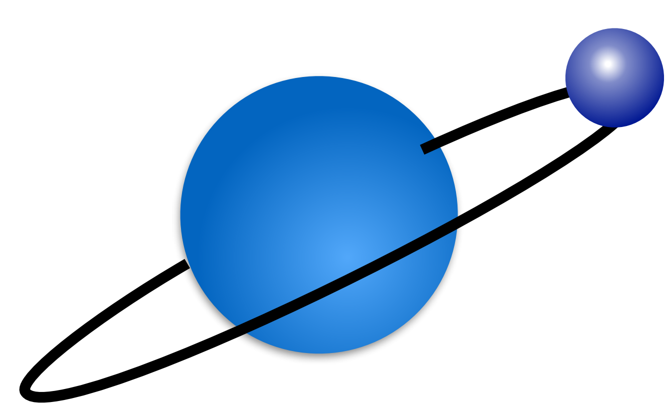


Electrons have an **intrinsic** angular momentum, “S”: “spin”

$$S_z = m_s \frac{h}{2\pi}$$

But, the “spin” can only take on two values:

$m_s = +\frac{1}{2}$ or $m_s = -\frac{1}{2}$



silver has 1 electron outside of closed shells

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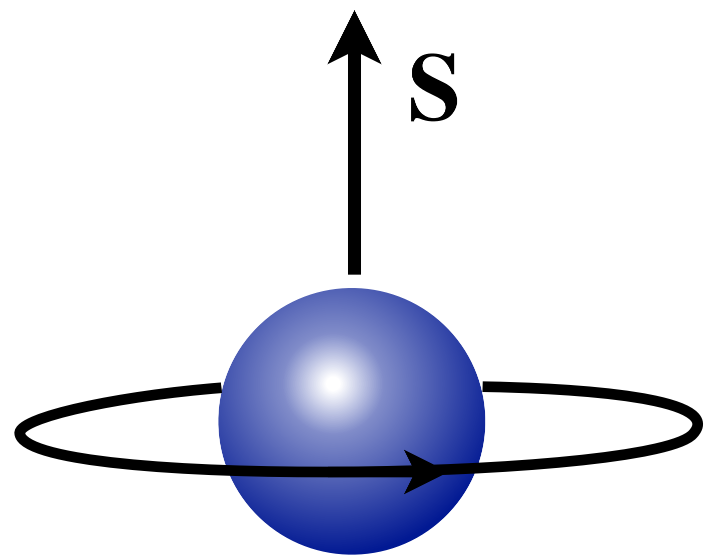

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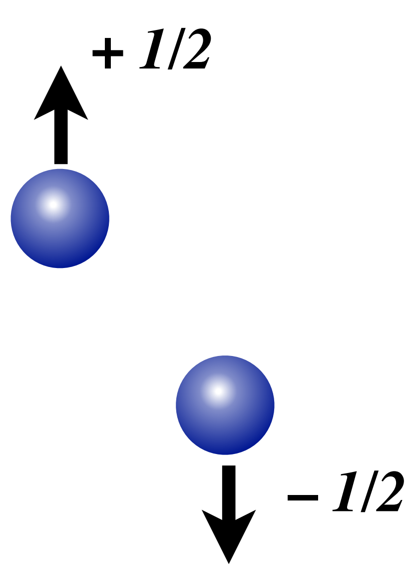
But, the “spin” can only take on two values:

$$m_s = +\frac{1}{2} \quad \text{or} \quad m_s = -\frac{1}{2}$$

We say
“spin, plus 1/2” or “spin up”
and
“spin, minus 1/2” or “spin down”



The electron is NOT

a ball of spinning charge

its outer edges would have to move >> c

This is a quantum mechanical feature with no classical analog

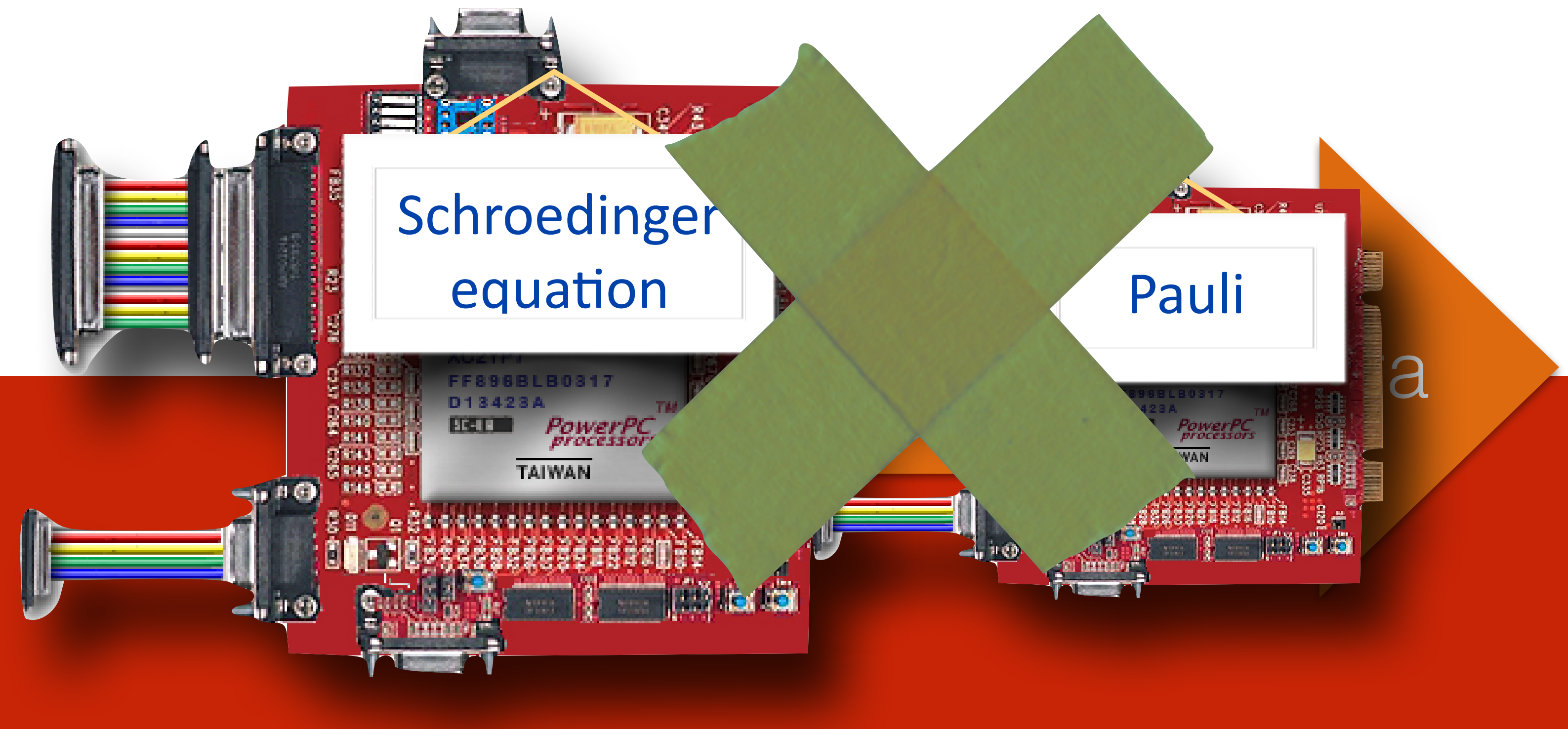
Wolfgang Pauli (1900-1958)

The character in 20th C physics

A wunderkind theoretician - wrote the definitive book on Special and General Relativity at age of 21

Simultaneously the most loved and certainly feared stories abound regarding his blistering criticisms

- *it was he who reviewed a paper by saying that it was so bad, that it wasn't even wrong.*
- *it was he who referred to a young visitor as "So young, and already so unknown."*
- *it was he who characterized Einstein's method as 'E = ma^2, no. E =mb^2, no. E = mc^2...yes.'*
- *To young Emilio Segre' after he had given a talk, "Never, have I heard a talk as awful as yours." *Pause*, then to the person on his other side, "Except when I listened to your inaugural lecture at Zurich."*



Wolfgang Pauli, 1925:

"Pauli Exclusion Principle":

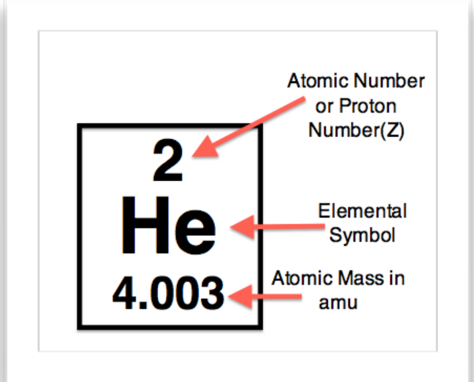
No two electrons can be in the same quantum state

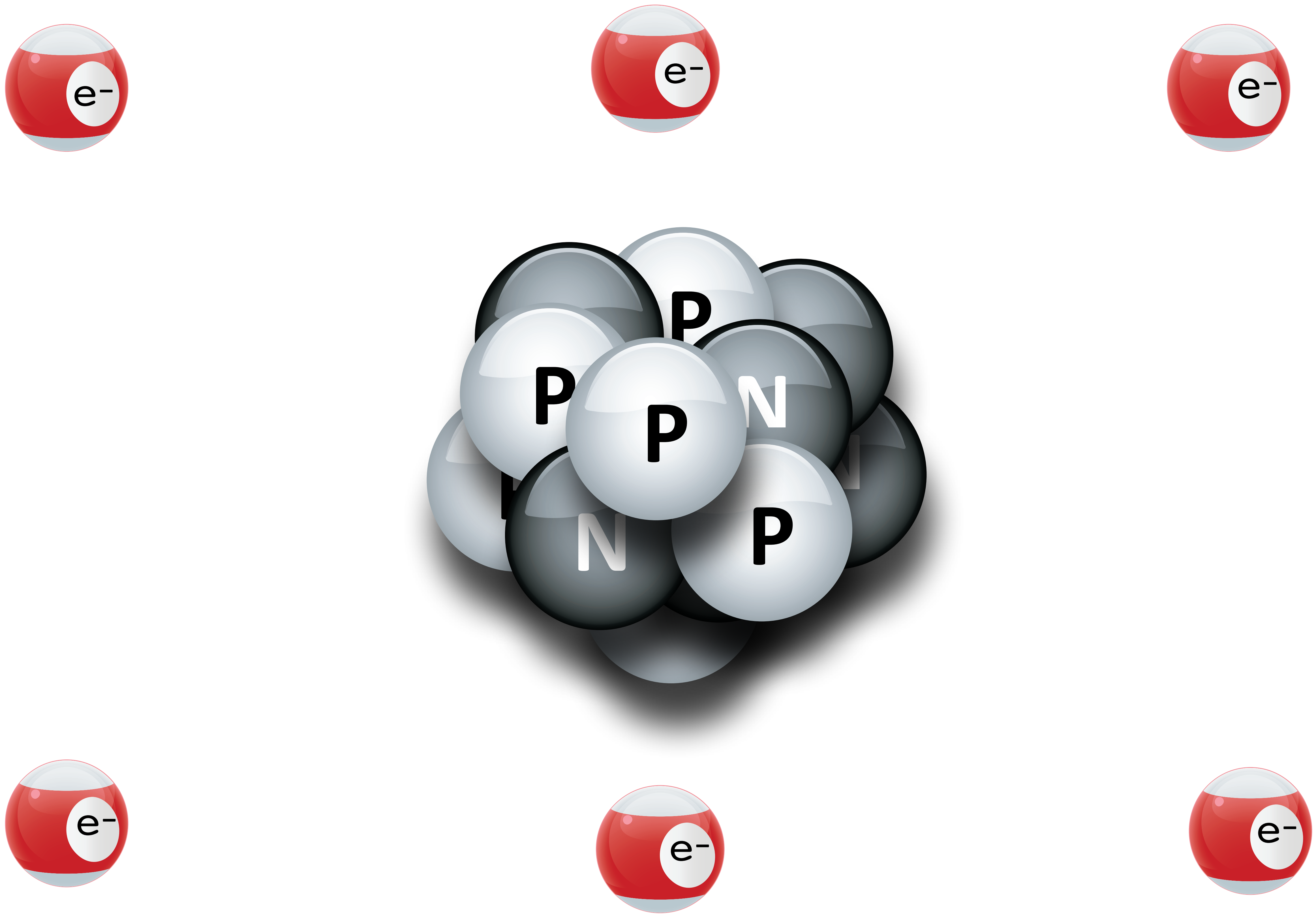
that is, have identical "quantum numbers"

...integers that characterize the atom

Carbon... 6 electrons, 6 protons, 6 neutrons:

$$^{12}_{6}\mathrm{C}$$



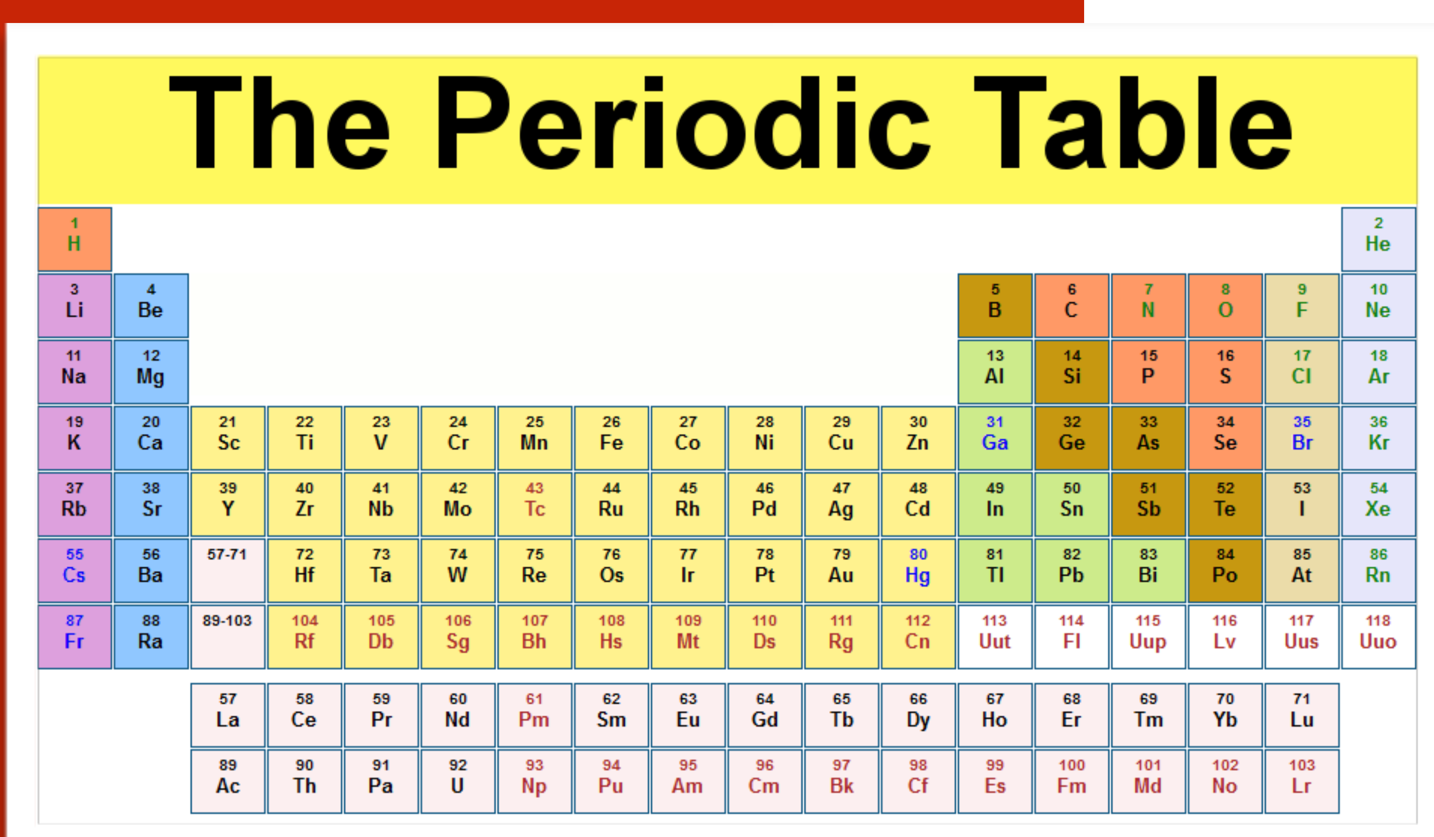


The Pauli Exclusion Principle

Explains it

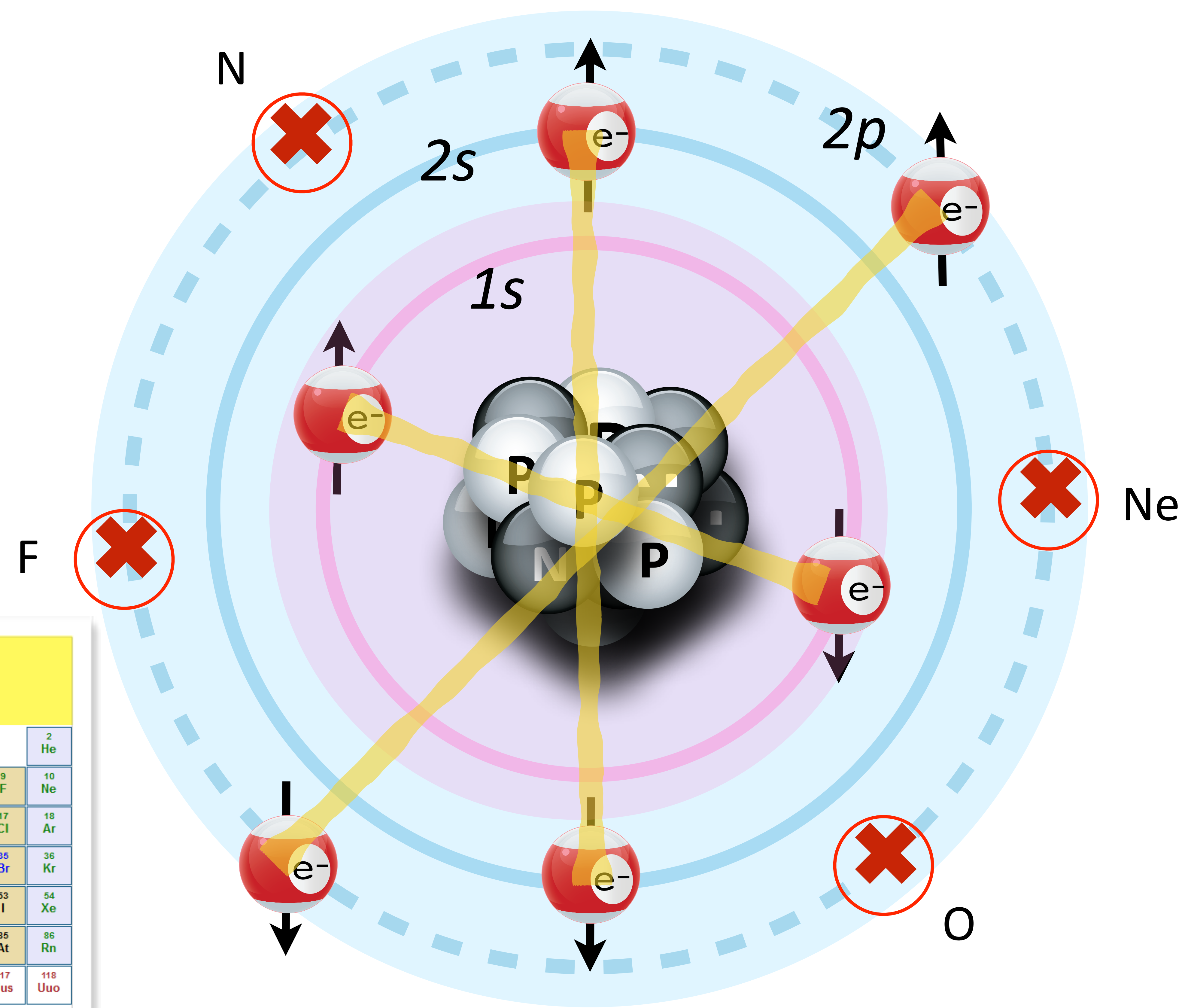
& SPIN is the reason

"1s2 2p2 2p6 3s2 3p6..."



How come Carbon *is* like:

The Pauli Exclusion Principle still works
...since **spin up ≠ spin down, so different quantum states**



The combination of Schroedinger, Pauli, Uhlenbeck and Goudsmit - explained the Periodic Table

the Pauli Effect

the other one:

He was quite proud of the Pauli effect, whereby experimental equipment would unexpectedly break down if he was anywhere within range.

Otto Stern would only converse with him through the closed door of his lab jargon alert: **fermion**

refers to:	any particle with half-integer spin
entomology:	from Fermi's theoretical work on the behavior of large numbers of Fermions
example:	electron, proton, neutron

jargon alert: **boson**

refers to:	any quantum object with integer spin
entomology:	from Satyendra Nath Bose, who worked on the effects of multiple boson aggregates
example:	photon, pion, Higgs Boson

spin is a defining quality of an electron

electron

symbol: e

charge: $-1e$

mass: $m_e = 9.0 \times 10^{-31}$ kg ~ 0.0005 p

spin: 1/2

category: fermion, lepton

again, an inherent angular momentum and a defining property of photons

particle: **photon**

symbol: γ

charge: 0

mass: $m_\gamma = 0$

spin: 1

category: boson, aka Intermediate Vector Boson

Rutherford, 1919

particle: **proton**

symbol:	p
charge:	$+1e$
mass:	$m_p = 1.6726\times10^{-27}$ kg = 1 p
spin:	1/2
category:	fermion, hadron

shifting gears

antimatter

here’s a number:

0

0

zero

the # of successfully combined models of

Quantum Mechanics and Relativity

prior to 1928

remember the relativistic energy relationship

and compare it to the non-relativistic one

Classical

$$E = \frac{1}{2}mv^2$$

$$p = mv$$

$$v = \frac{p}{m}$$

Relativistic

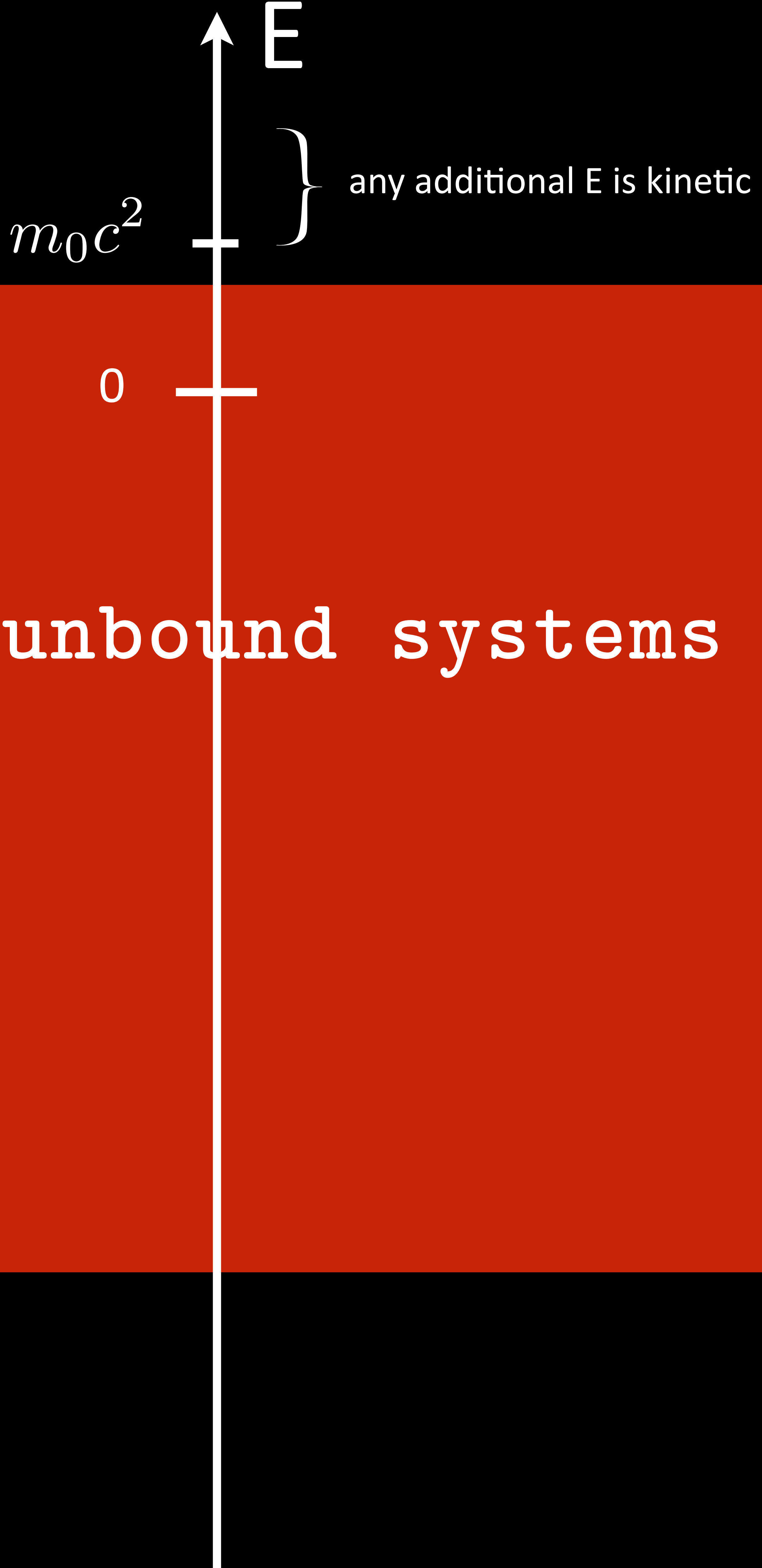
$$E^2 = (m_0c^2)^2 + (pc)^2$$

that square is problematic since it suggests:

$$E = \pm\sqrt{(m_0c^2)^2 + (pc)^2}$$

translated to Schroedinger QM:

negative energies for freely moving electrons



negative energies for unbound systems

a disaster

negative energies for unbound systems

a disaster

negative energies for unbound systems

a disaster

there's no bottom!

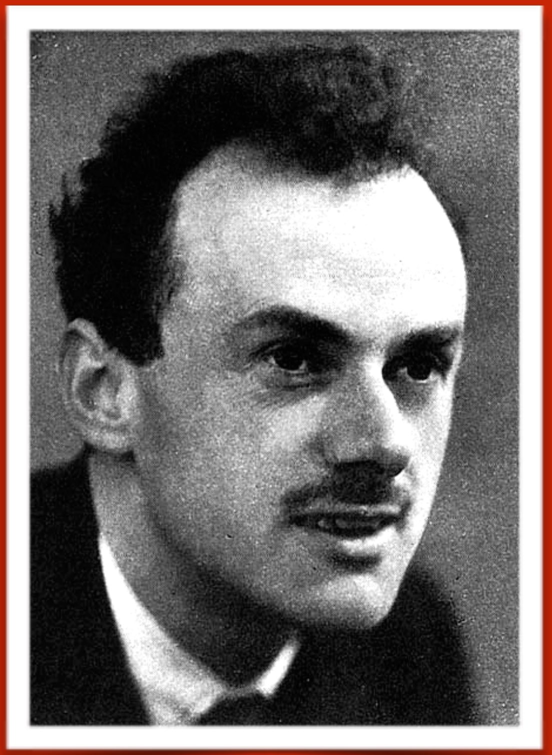
worse!

Quantum Mechanics using Relativity:

required not only negative energies

negative probabilities!

1928



Paul Dirac

1902 – 1984

"At the question period after a Dirac lecture at the University of Toronto, somebody in the audience remarked: "Professor Dirac, I do not understand how you derived the formula on the top left side of the blackboard."
"This is not a question," snapped Dirac, "it is a statement."

hilarious interview with the Wisconsin State Journal from 1929 on the blog.

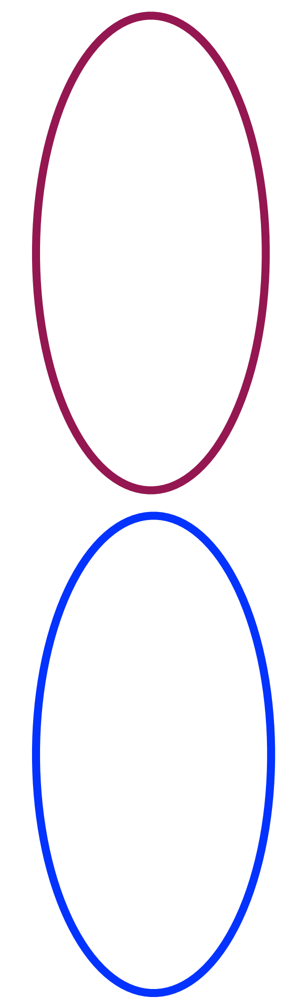
Dirac's Mathematical Imagination

Dirac embraced the negative energy

Solved the negative probability

Dirac set out to find an equation that would solve both problems

Dirac's imagination



The "Dirac Equation" is the correct equation for electrons: Probabilities turn out okay, but required interpretation of negative energies

Dirac's result

required: 4 quantum fields, rather than 1 $\psi_{up}(+E), \psi_{down}(+E)$

2 have positive energy, 2 have negative energy $\psi_{up}(-E), \psi_{down}(-E)$

each pair is related precisely to spin

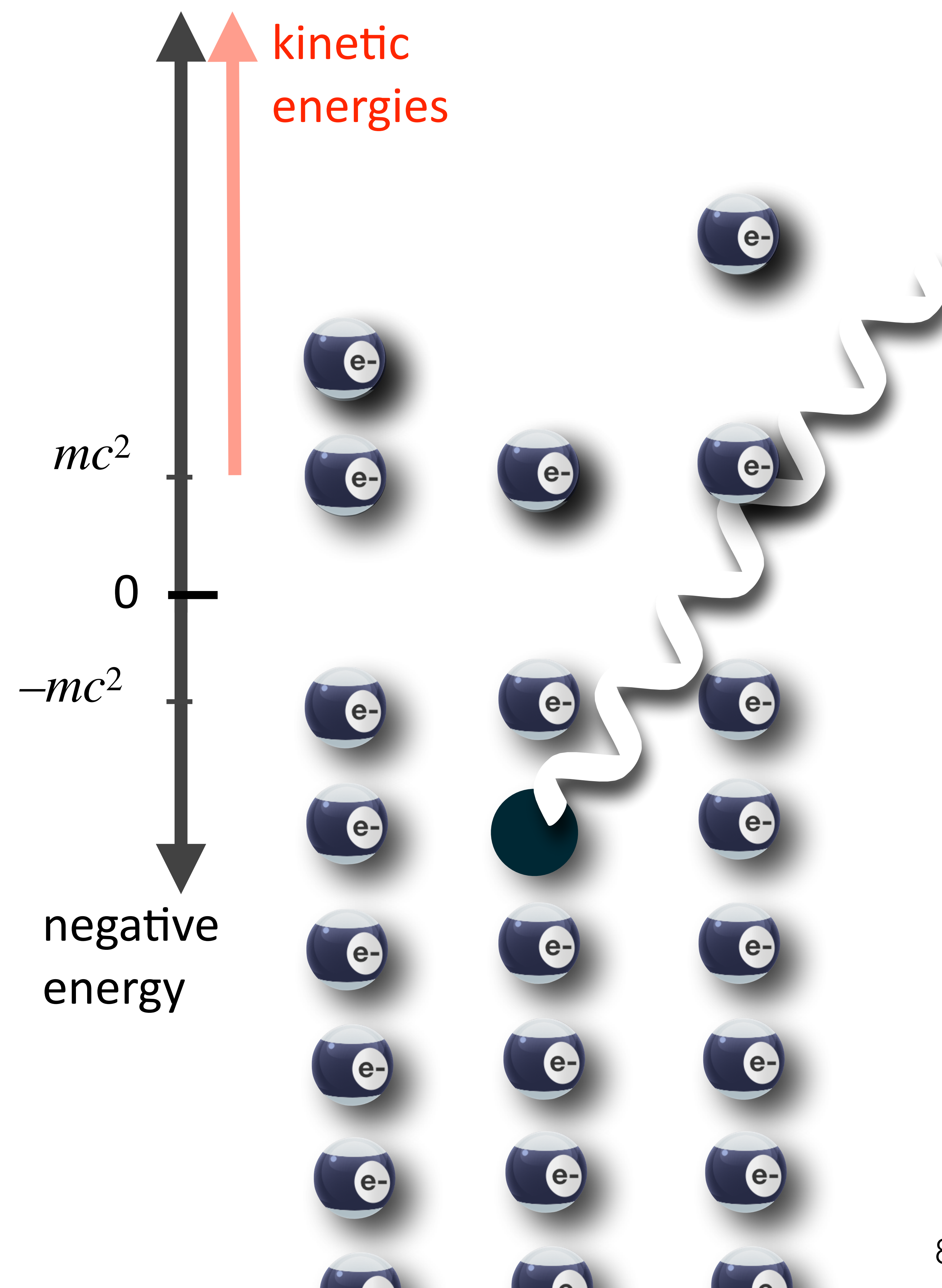
Dirac showed that spin is a wholly relativistic effect

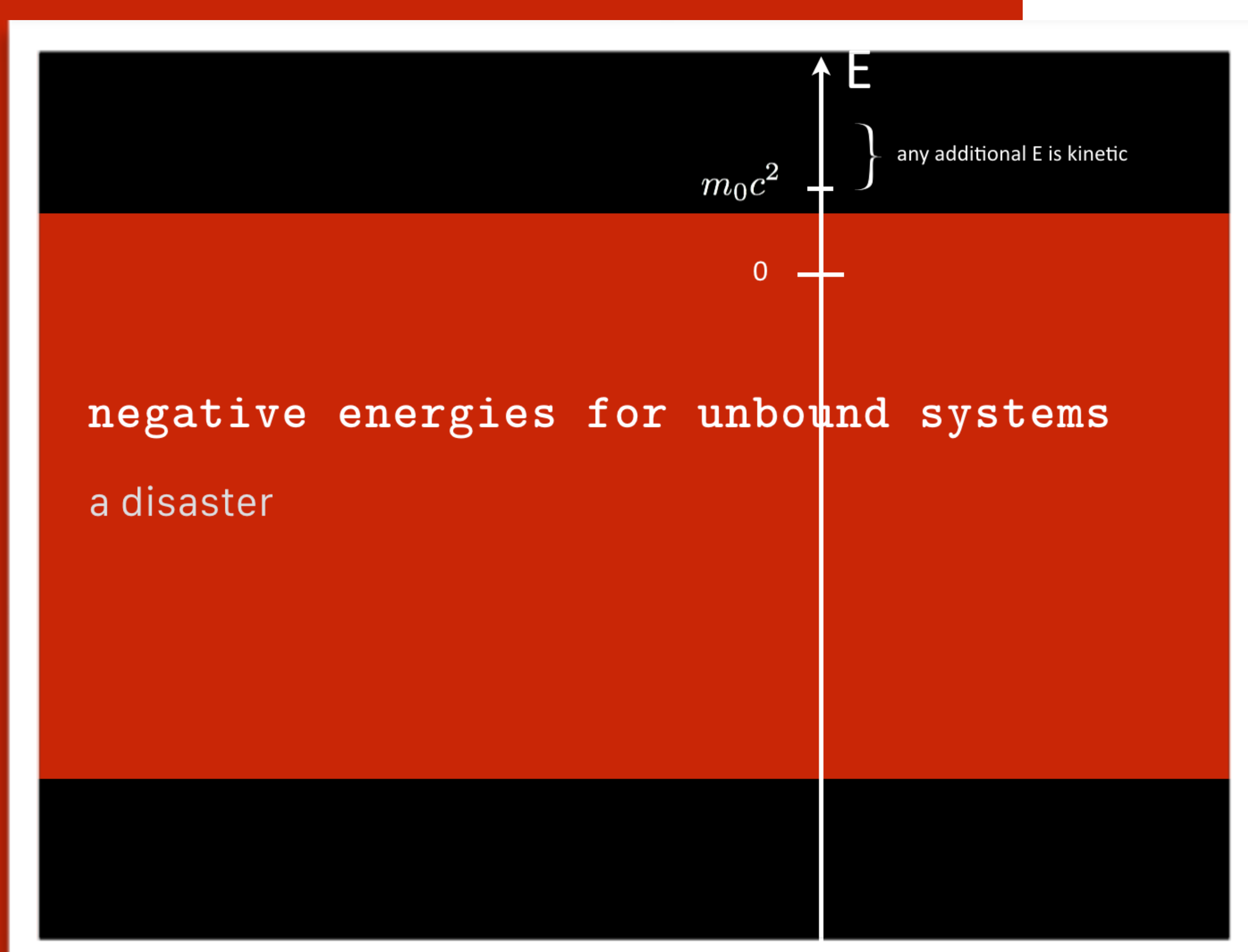
...it just popped out of his equation.

still negative energies?

"solved" it with Pauli's Exclusion Principle

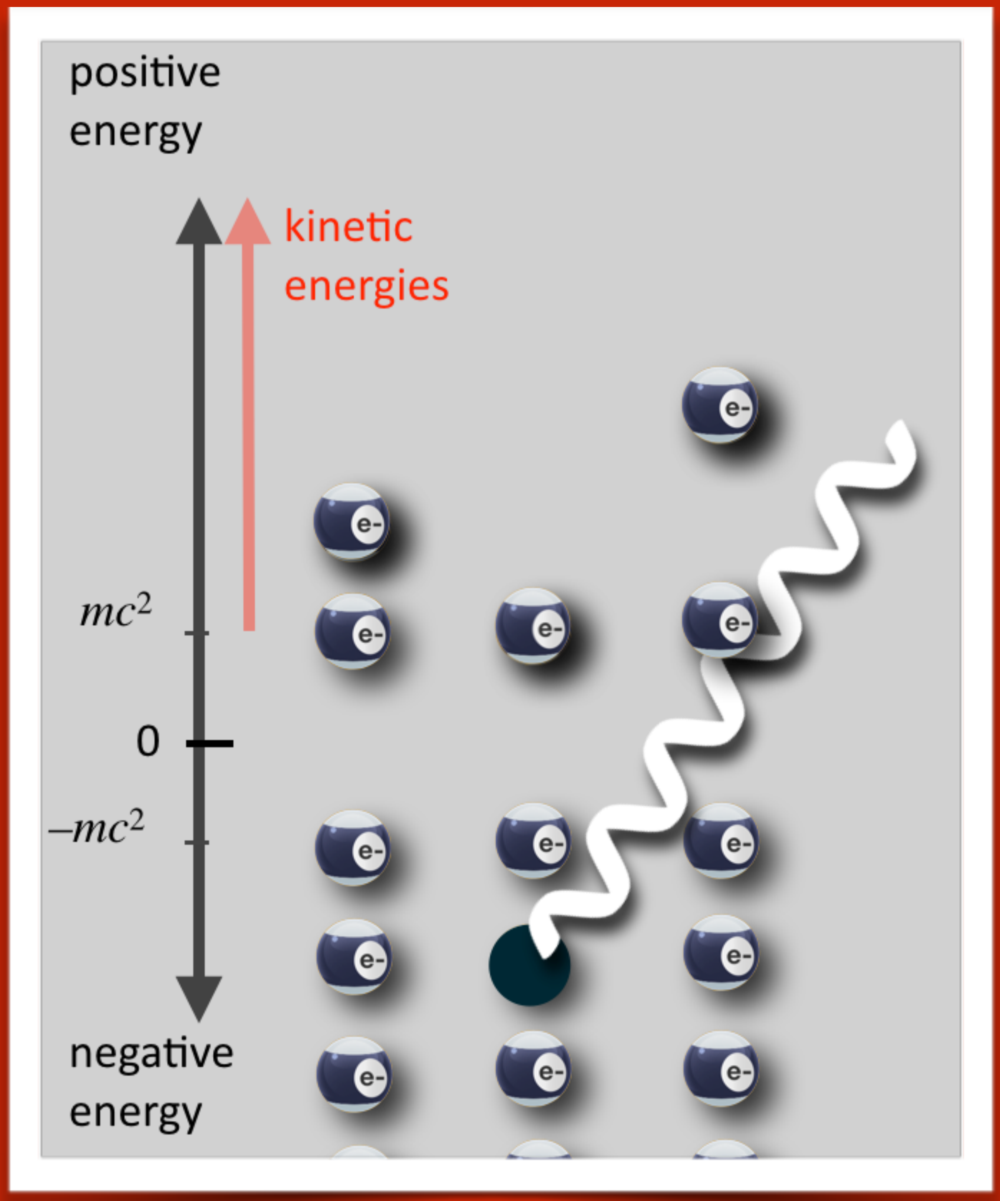
His vacuum is full of negative energy electrons





start with nothing

$E_\gamma > 2\, m_e\, c^2$



NOTHING

+ Energy

Let's talk about Nothing.

Dirac began this discussion

which continues today

in particle physics

and in cosmology