hi

Day 13, 26.02.2019

Einstein’s Special Theory of Relativity, 3

30 days until opening day

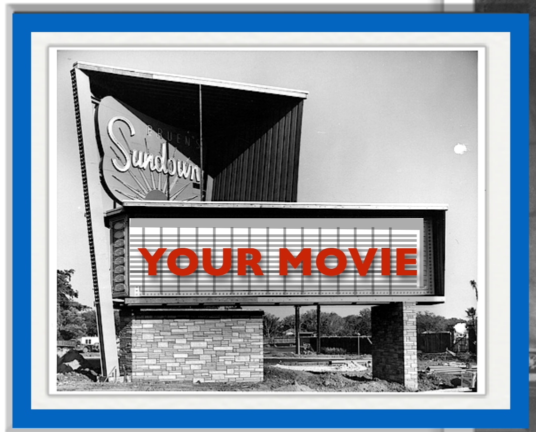
Dirty Dancing week

housekeeping

Gotta come to class

question about anything?

I'll make a movie for you:



Please remember – especially true starting now:

need to take hand-written notes

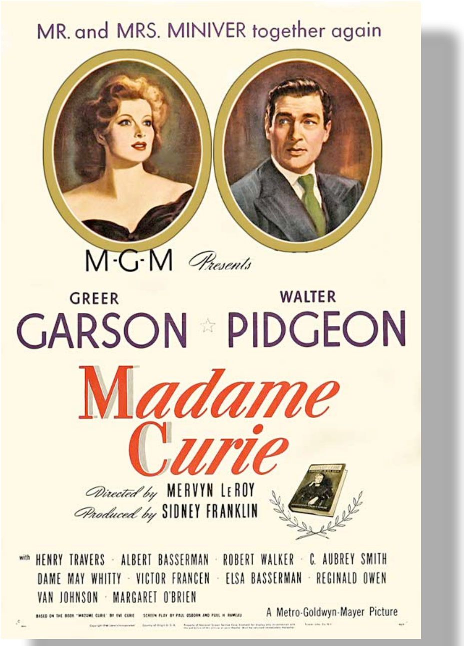
No computers or phones are allowed.

Midterm…closes tonight

Madame Curie movie - we have a quorum in favor

I've posted another FB poll targeting the 2 weeks after break

right now: looks like Monday, March 18



Next readings:

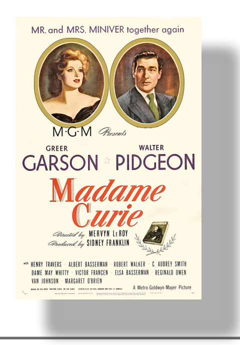
Cosmic Perspective (aka "CP") in MasteringAstronomy

likewise HW 6

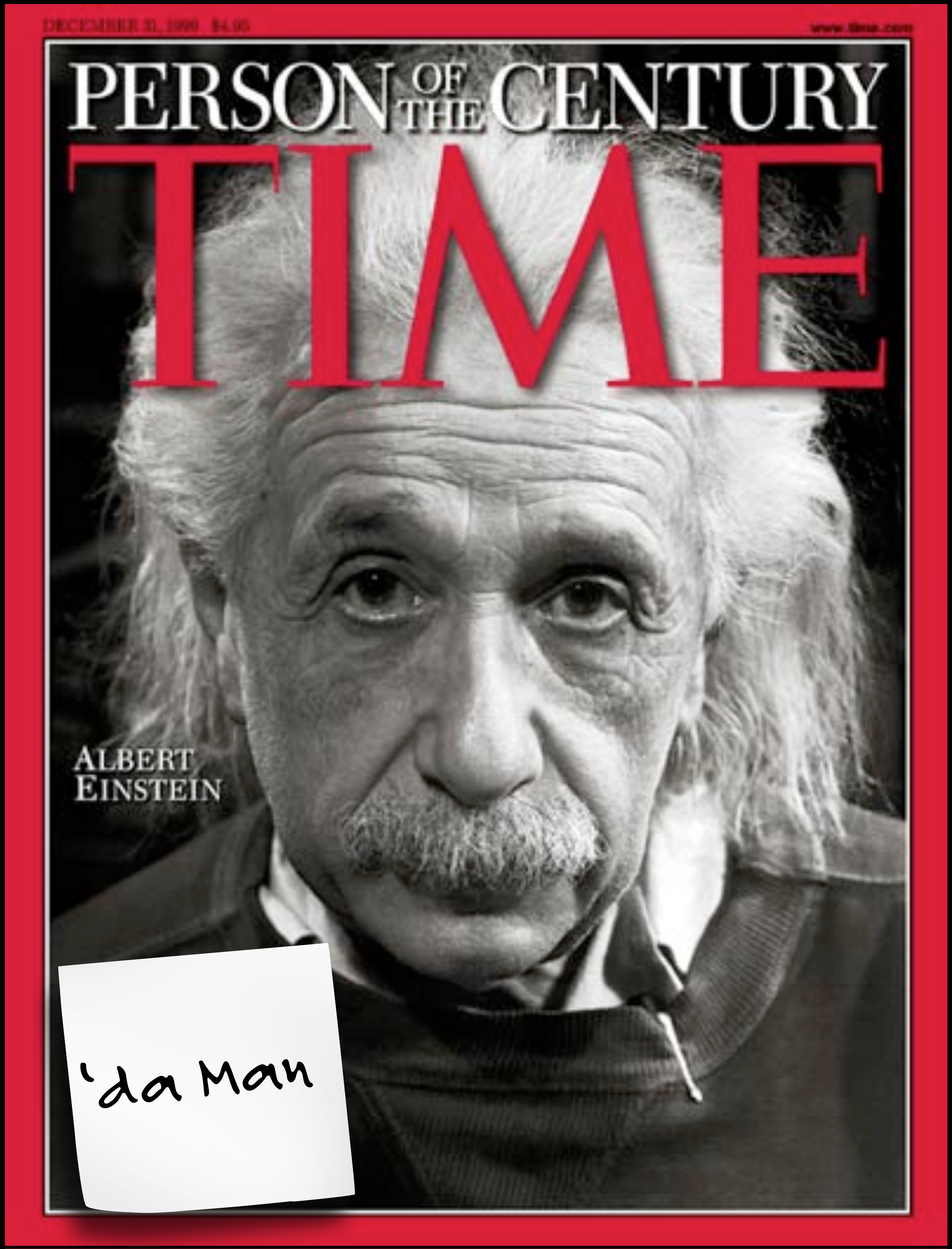
February 2019

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
27	28	29 yadda yadda yadda	30	31	1	2 HW2
3	4	5 lessons 10,11,12	6	7 lesson 13	8 HW2 due	9 HW3
10	11	12 lecture	13	14 lecture	15 HW3 due	16 HW4
17	18	19 lecture	20	21 lecture	22 HW4 due	23 HW5
24 ←	25 midterm	26 → lecture	27	28 lecture	1 HW5 due	2

March 2019

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
24	25	26	27	28	1	2
3	4 ← spring break	5 spring break	6 spring break	7 spring break	8 spring break →	9 HW6
10	11	12 lecture	13	14 lecture	15 HW6 due	16 HW7
17	18 	19 lecture	20	21 lecture	22 HW7 due	23 HW8
24	25	26 lecture	27	28 lecture	29 HW8 due	30 HW9
31	1	2 lecture	3	4 lecture	5 HW9 due	6

KEEP
CALM
AND
LET'S
REVIEW


PERSON OF THE CENTURY
TIME
ALBERT EINSTEIN
'da Man

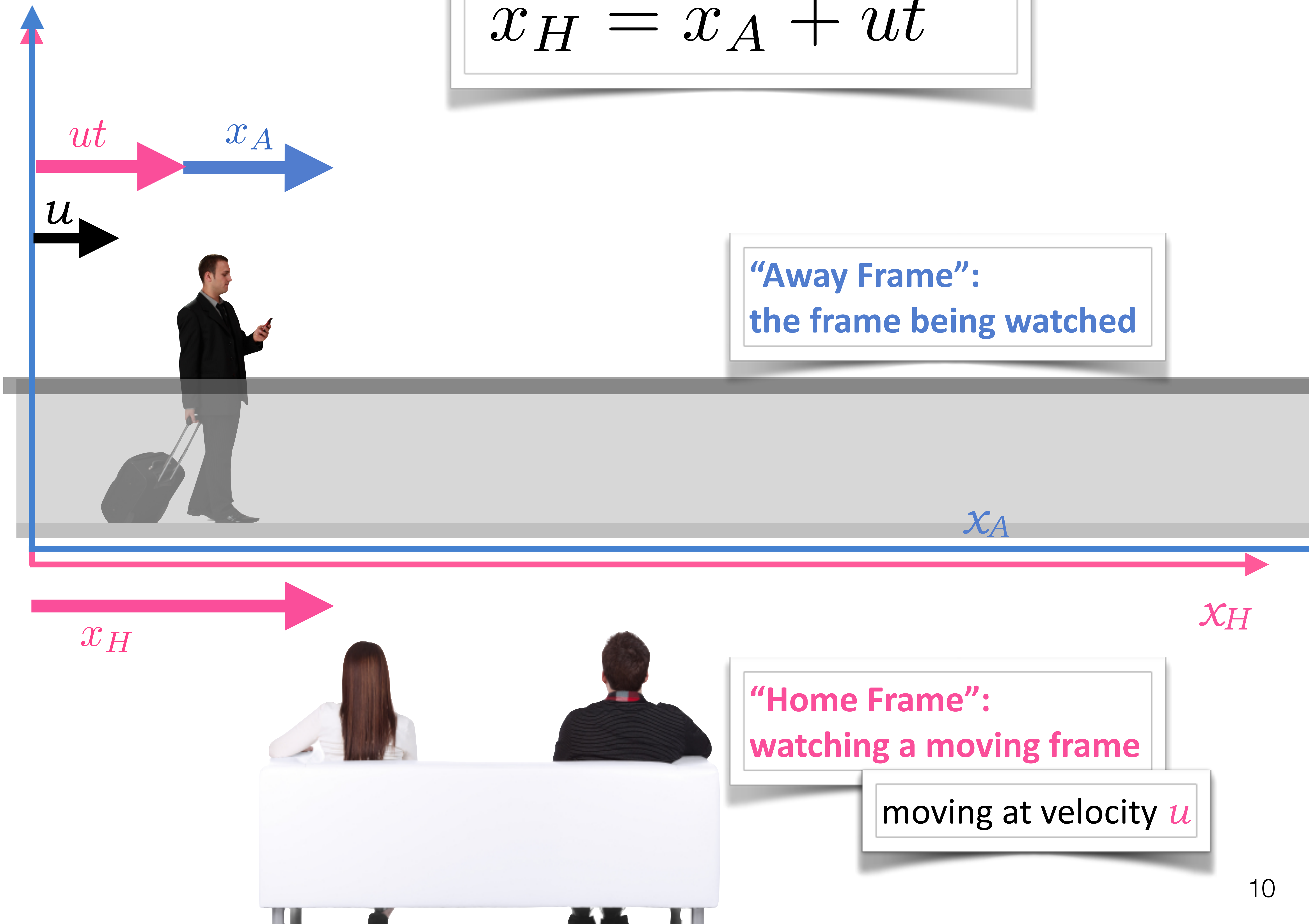
Special Relativity

frames of reference

the airport

"Galilean Transformation"

$$x_H = x_A + ut$$



"Away Frame":
the frame being watched

"Home Frame":
watching a moving frame

moving at velocity u

jargon alert:

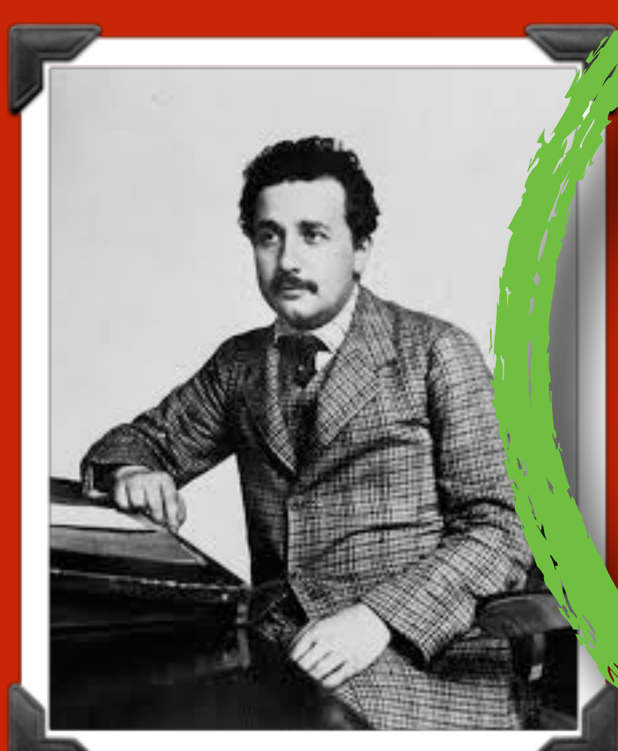
Inertial Frame of Reference

refers to:	a Frame of Reference moving at a constant, linear velocity
entomology:	from Newton's First Law idea
example:	a spaceship at constant speed

2 Postulates:

"inertial frame":

constant velocity

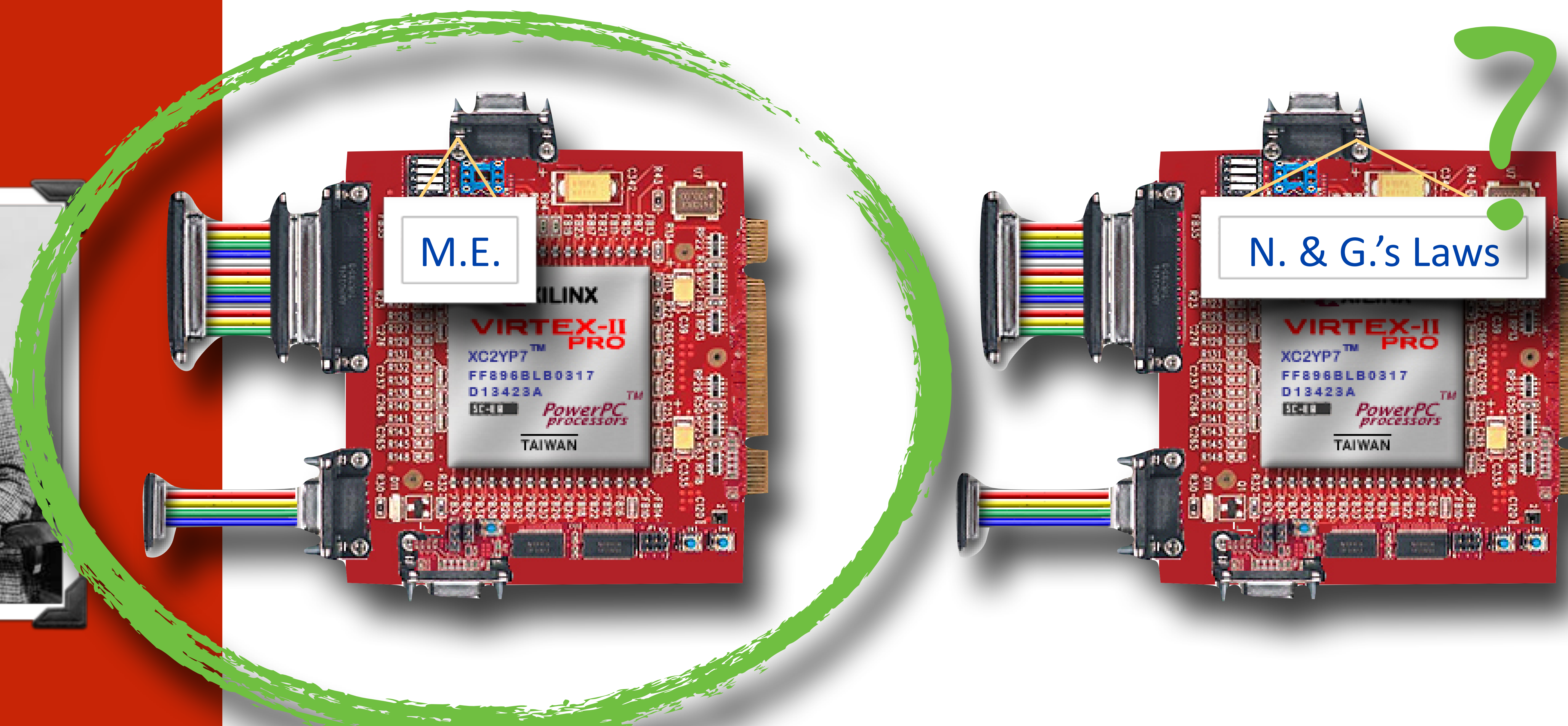


1. All laws of physics – mechanical and electromagnetic – are identical in co-moving inertial frames.

taking Galileo seriously, and then adding Maxwell

2. The speed of light is the same for all inertial observers.

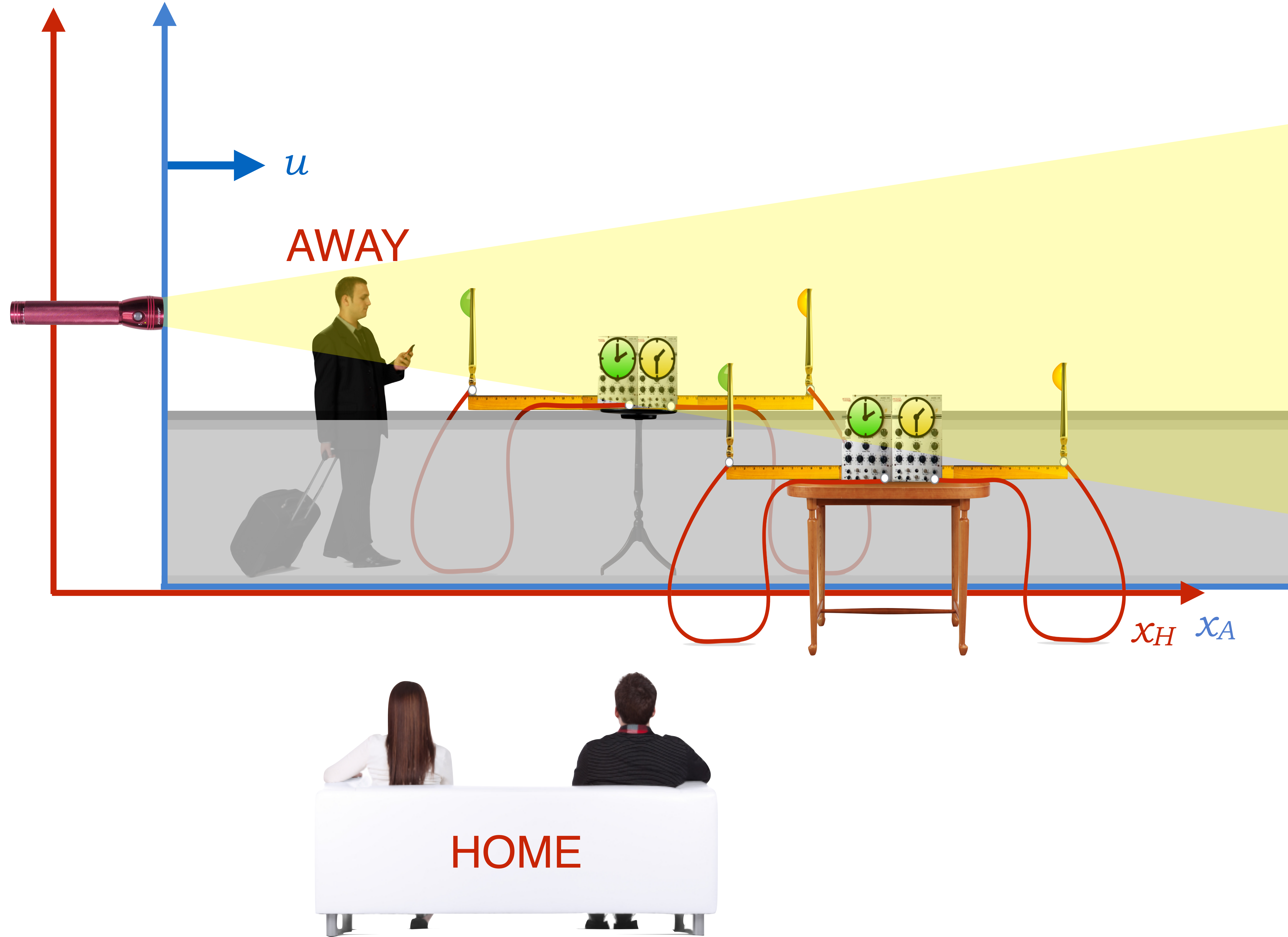
taking Maxwell seriously



think about how weird

the second postulate is.

c is a constant…always.


u
AWAY
x_H
x_A
HOME

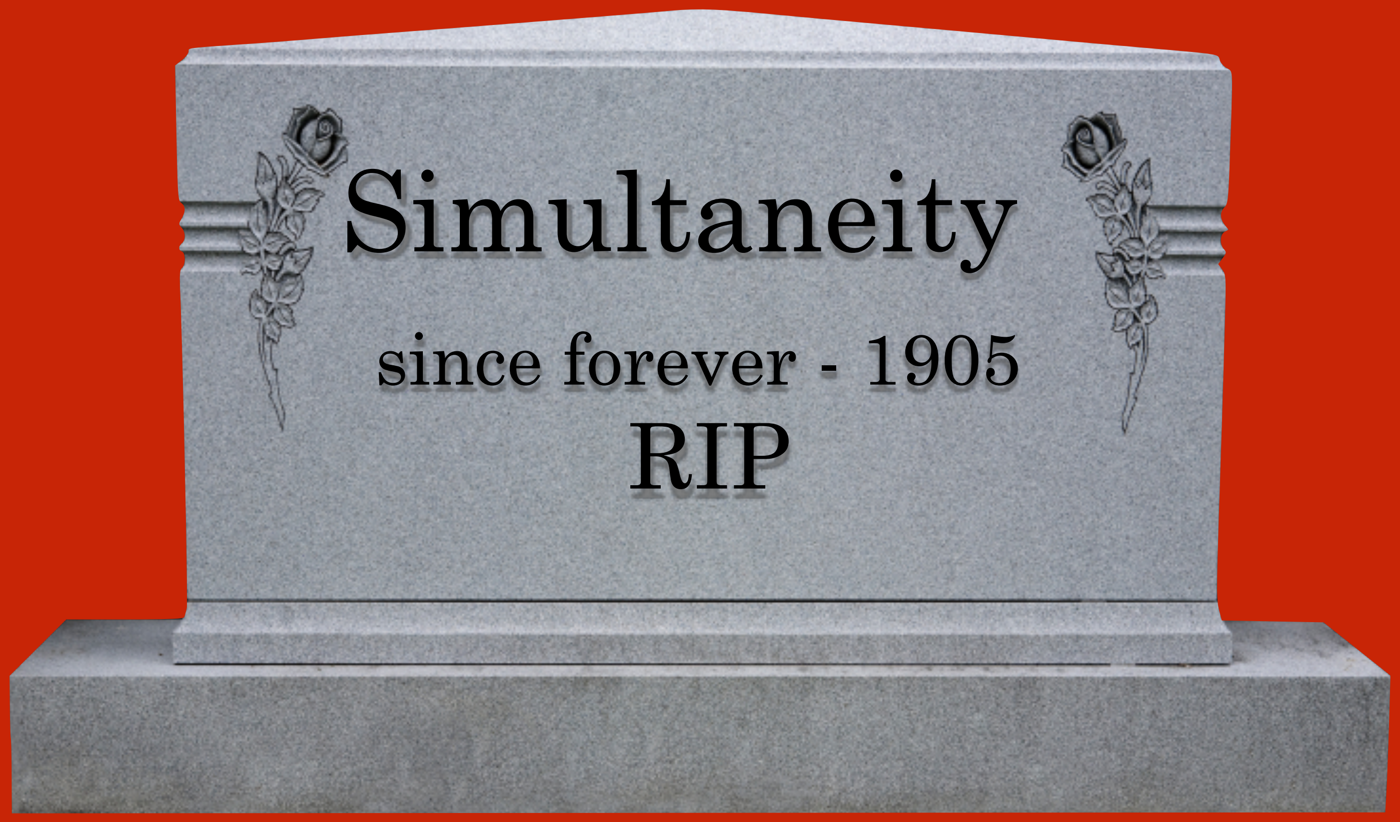
a consequence

of the second postulate:

if two events are simultaneous in one frame

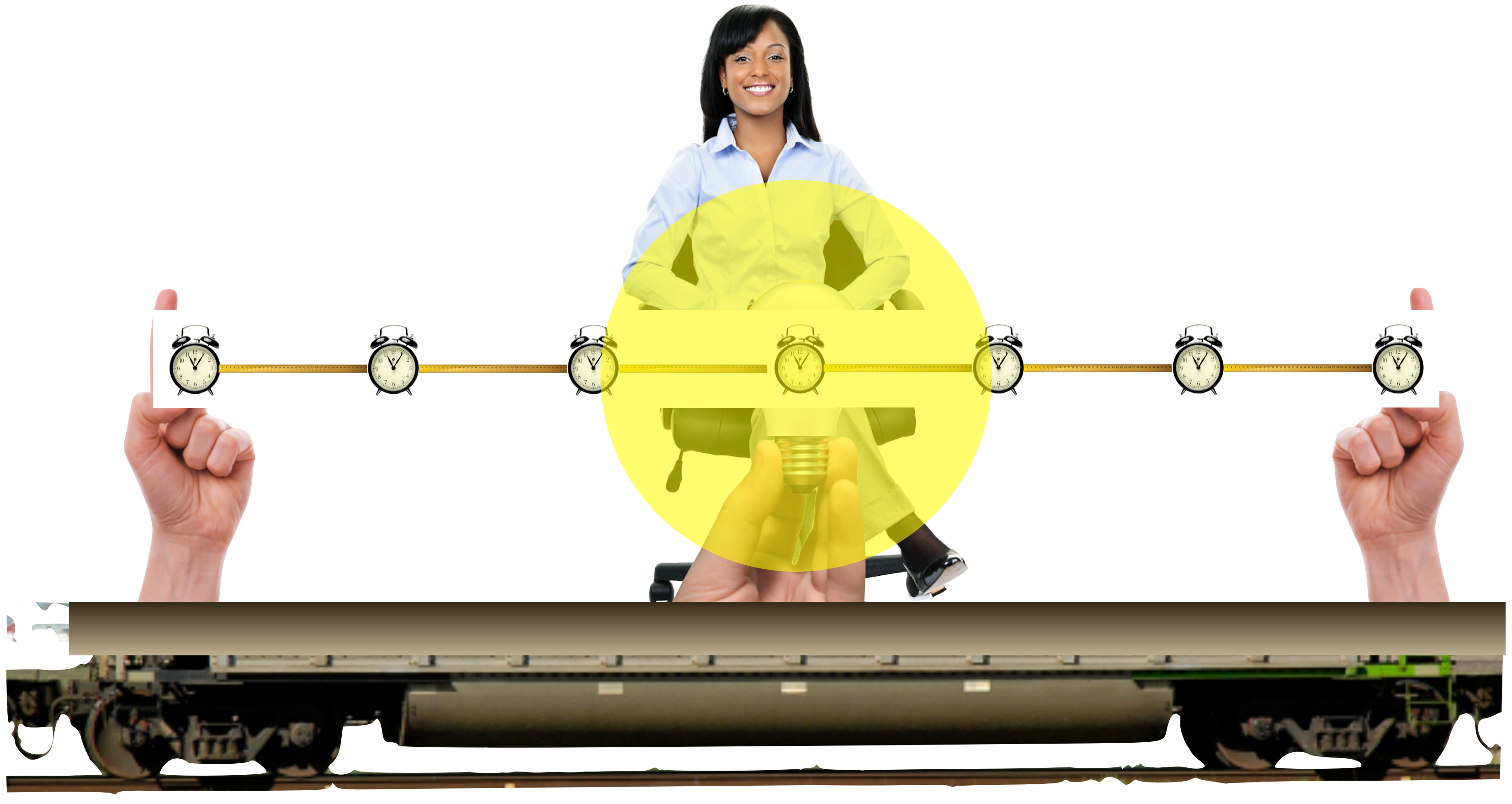
they are not for a co-moving inertial frame

There is no such thing as the *concept of* simultaneous events



synchronize clocks

imagine a series of rulers and clocks



to synchronize clocks...a simultaneity operation

set a clock simultaneous with the arrival of a test beam

she believes

that she's the center of the universe

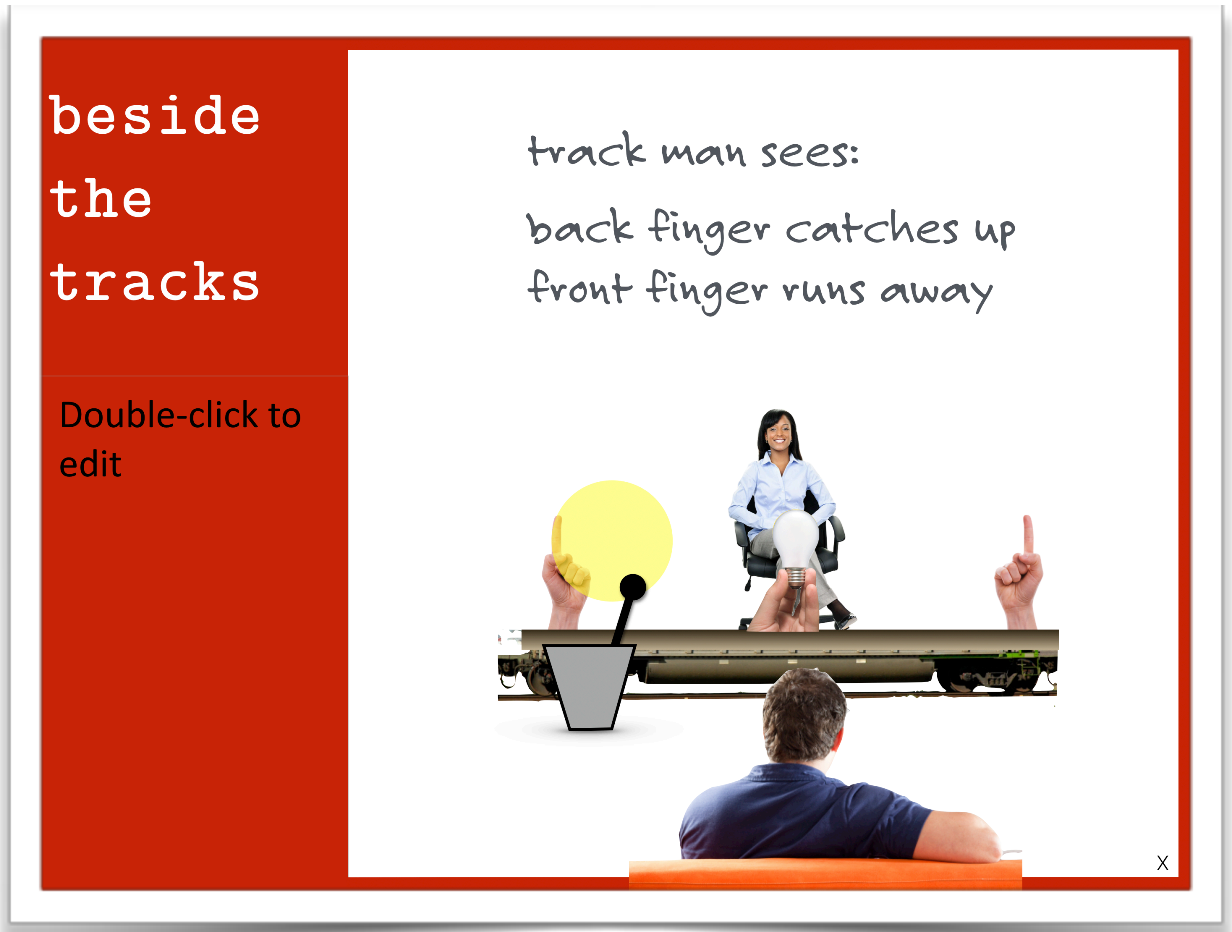
residing in Newton's Absolute Space system

Where the Ether is and where Maxwell's equations were thought to work

all motion: referred to her clocks and rulers

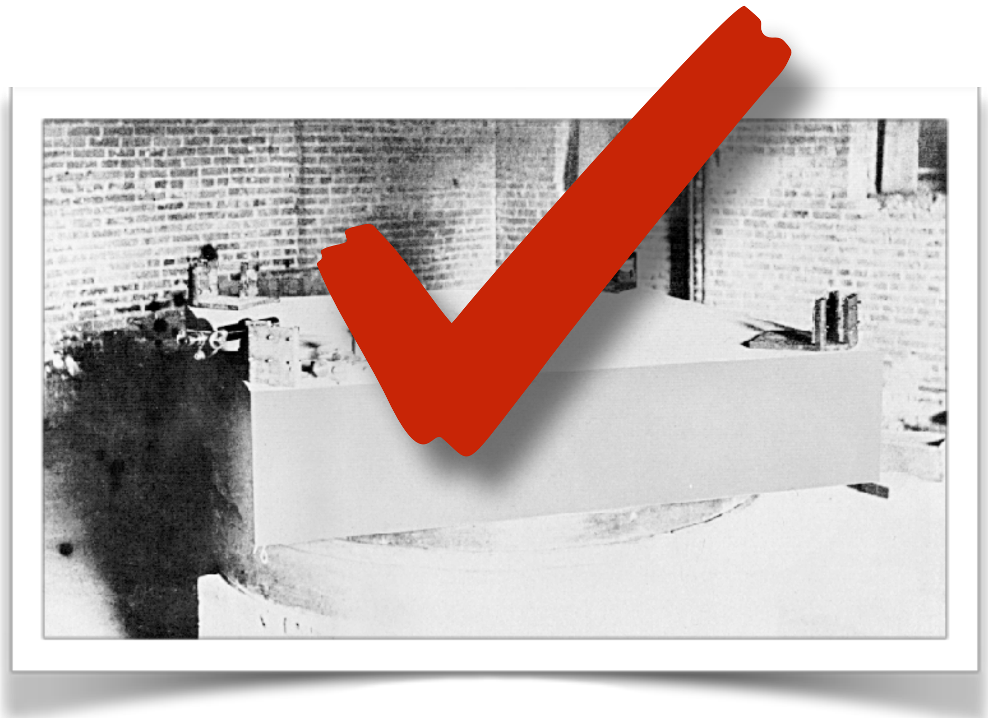
to translate one frame's measurements to hers?

you can't!



You'd have to synchronize clocks!

An absolute space system is useless!



26 yo Einstein:

"The introduction of a "luminiferous ether" will prove to be superfluous inasmuch as the view here to be developed will not require an 'absolutely stationary space' provided with special properties…"

not only

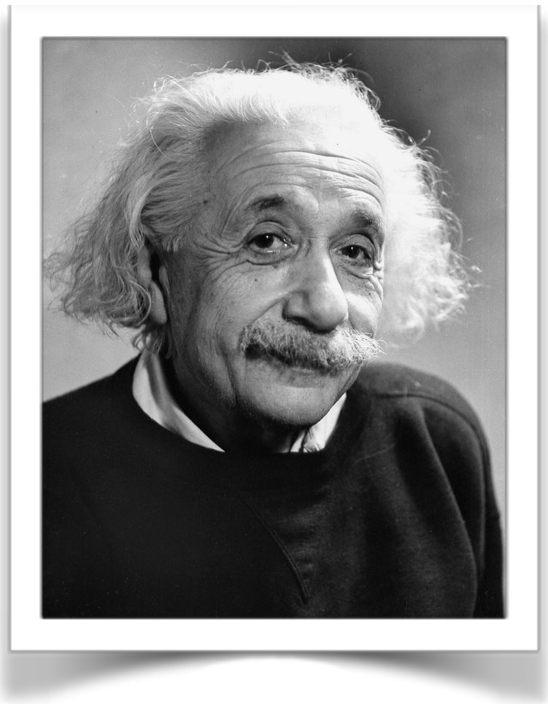
can you not detect inertial motion with any experiment

you can't synchronize clocks between inertial systems

So:

the ether is a barren, useless concept

new criterion for physical reality:



If it can't be measured it can't be real

The ether can't be measured, so it cannot be real...

2. “Causality” requires care

Two observers disagree about when events happen

the same time? at different times?

Suppose the hospital order is: first I’m born, then I cry

would a moving observer observe that first I cry, then I’m born?

there are consequences to the Second Postulate

we made a light clock

and followed the mathematics story

$$t_H = \frac{t_A}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

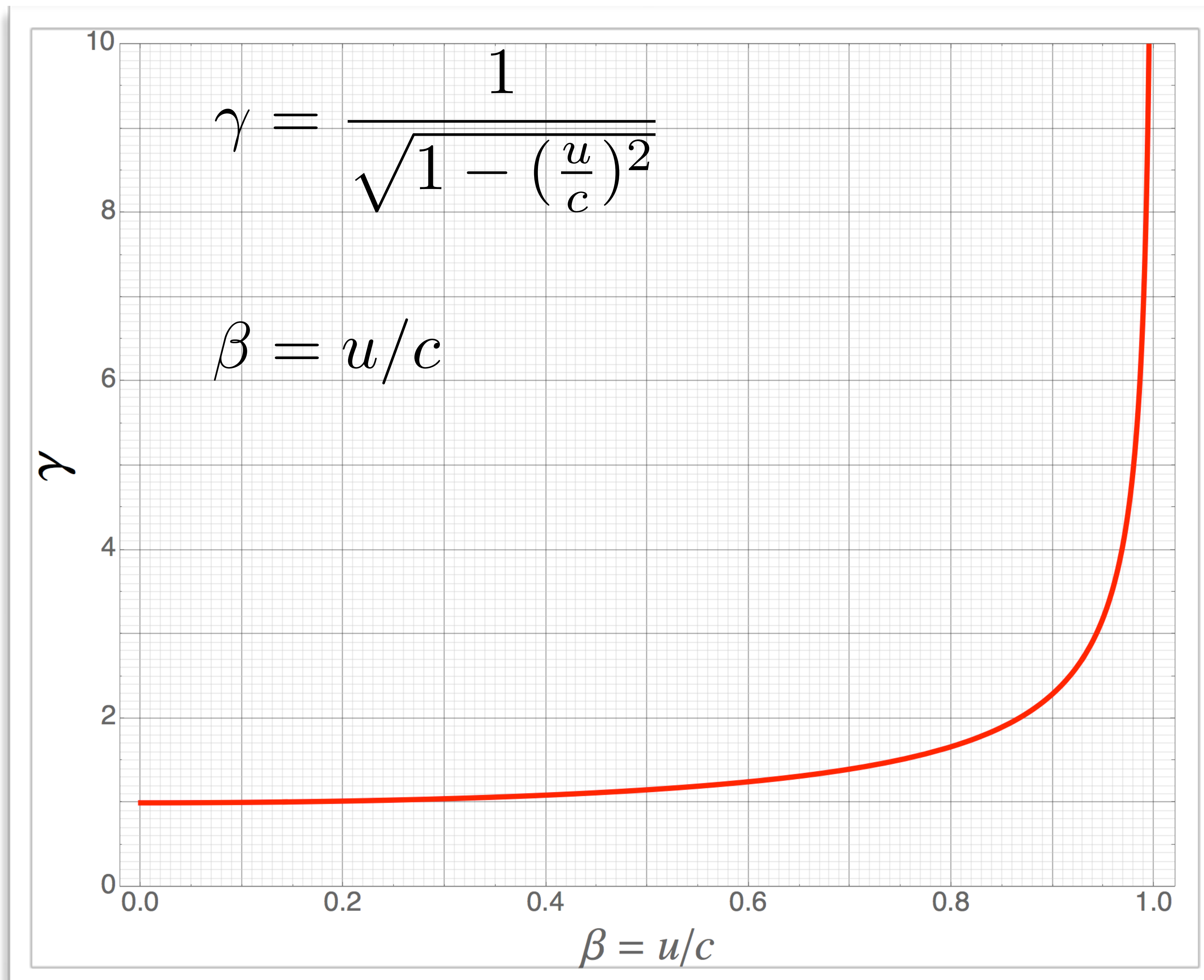
Moving clocks appear to run slower as seen by a relatively stationary observer

$$t_H = \frac{t_A}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$t_H = \gamma t_A$$

time dilation

the second of 3 strange things about space and time

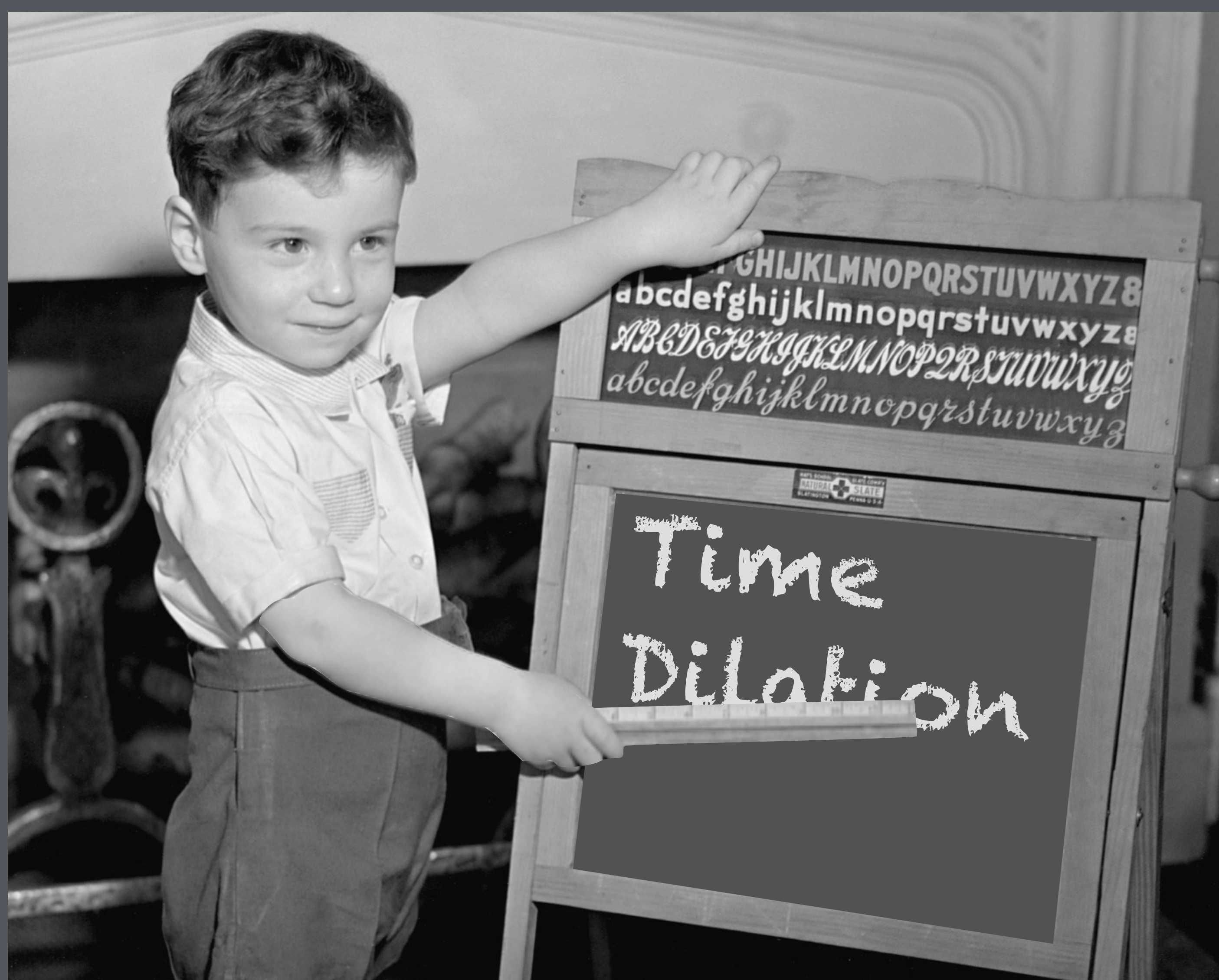


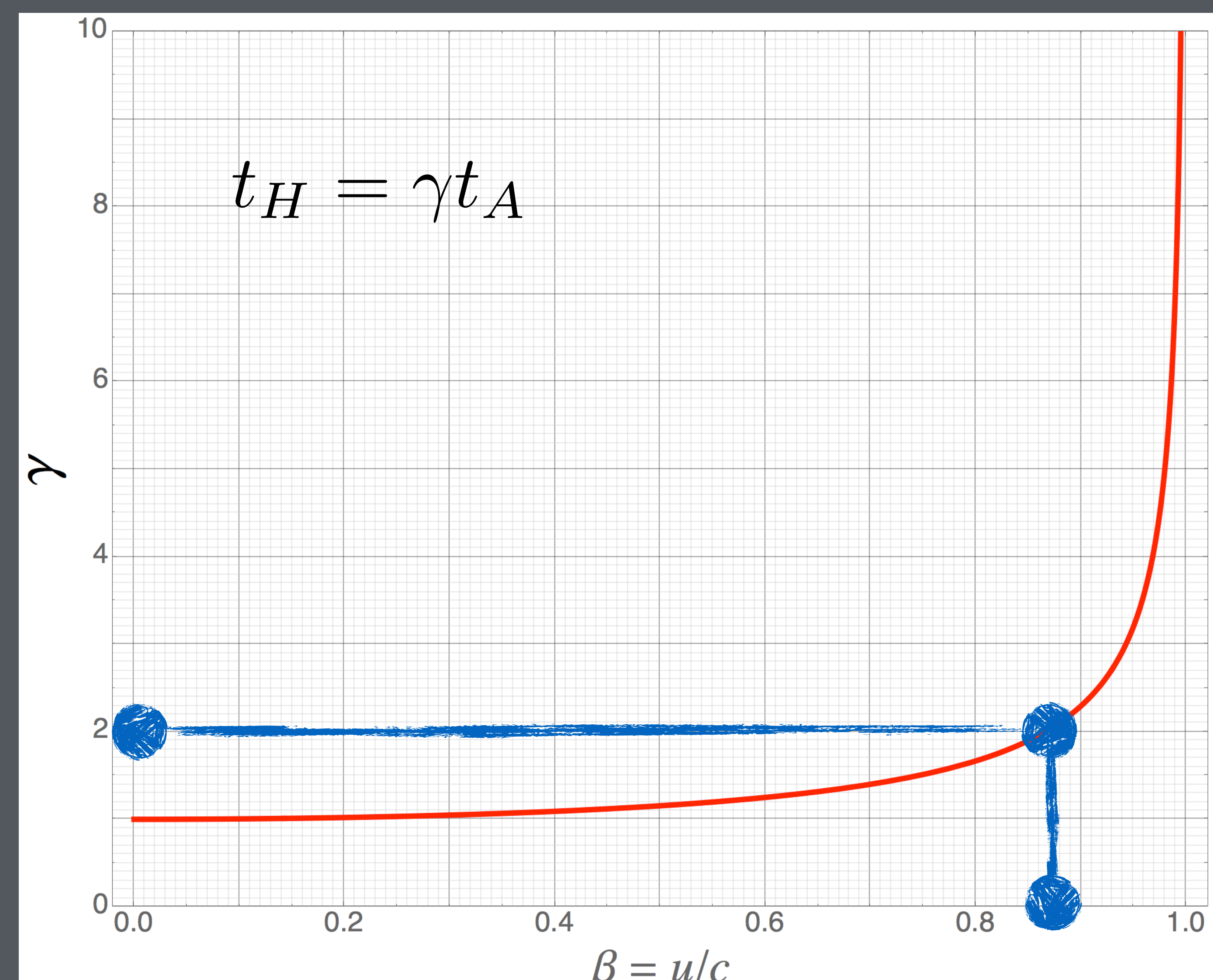
You have a clock and I have a clock and they are identical. I observe yours is in an inertial frame of reference moving past my frame of reference.

I also observe that 1 hour on your clock seems to take 2 hours on my clock.

Yours appears to be slower or faster than mine?

How fast is your frame moving relative to mine?





this works for any clocks

actual clocks

atomic transitions

elementary particle lifetimes

biological clocks

remember what's constant...

The speed of light, c. ...a speed.

$$c = \frac{\text{distance interval}}{\text{time interval}}$$

If clocks are messed with Δt depends on the frame...

and the velocity of light is constant,...

Doesn't it stand to reason that lengths are also messed with...

ΔL depends on the frame...?

...shorter as viewed from the home frame:

$$L_H = \frac{L_A}{\gamma}$$

a length in the away frame will seem...

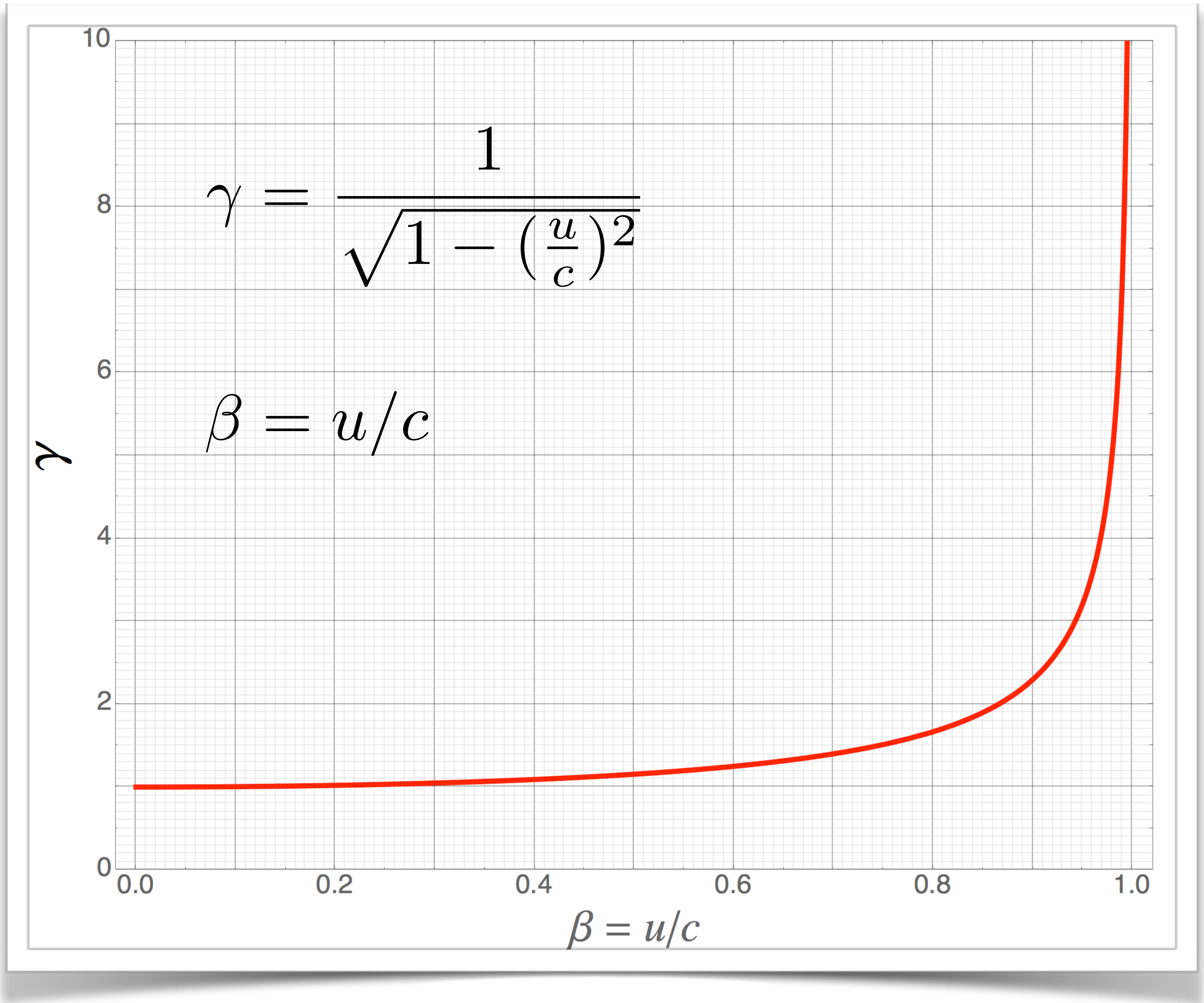
$\gamma > 1$

Moving lengths appear shorter to a relatively stationary observer

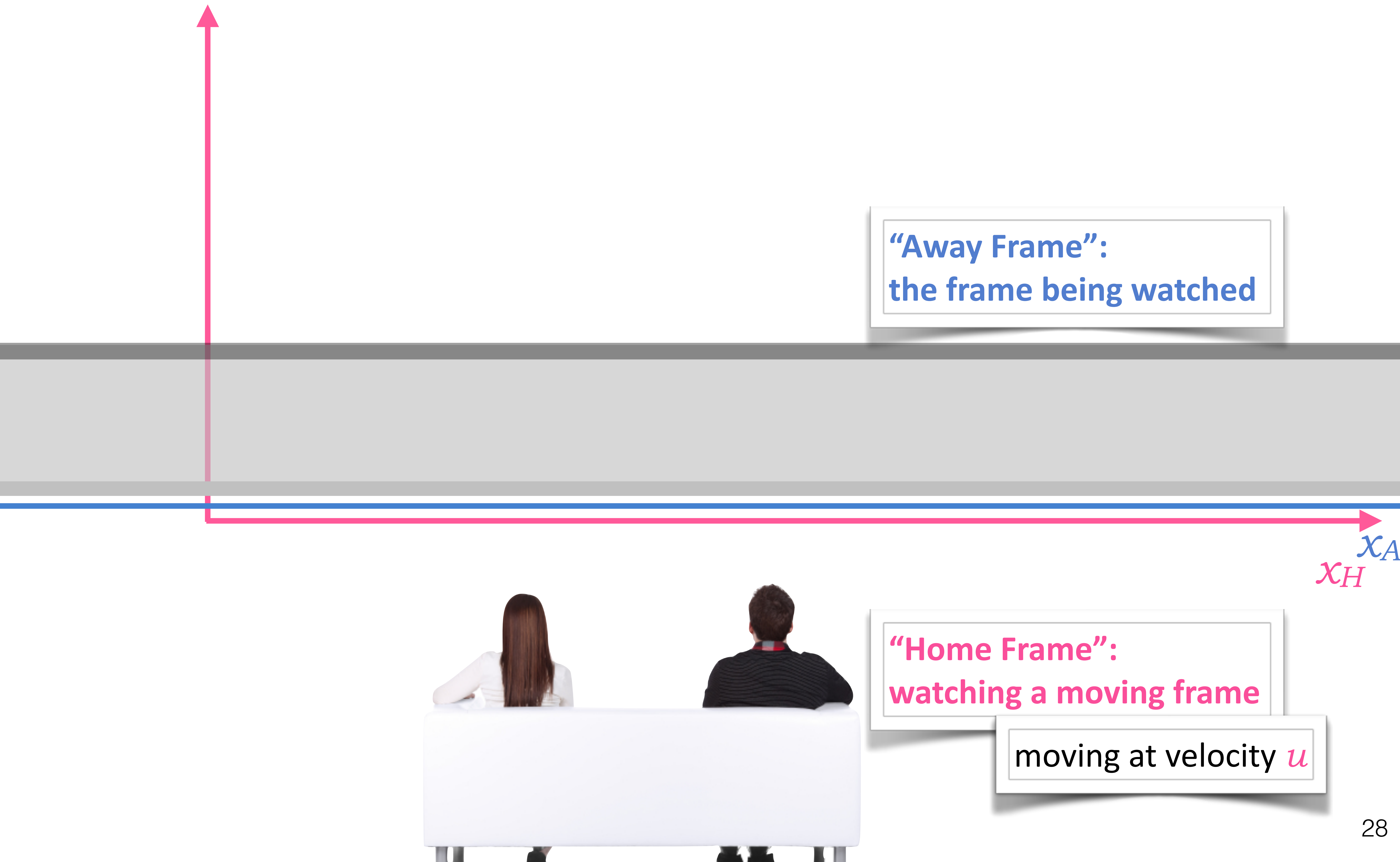
length contraction

the third of 3 strange things about space and time

$$L_H = \frac{L_A}{\gamma}$$



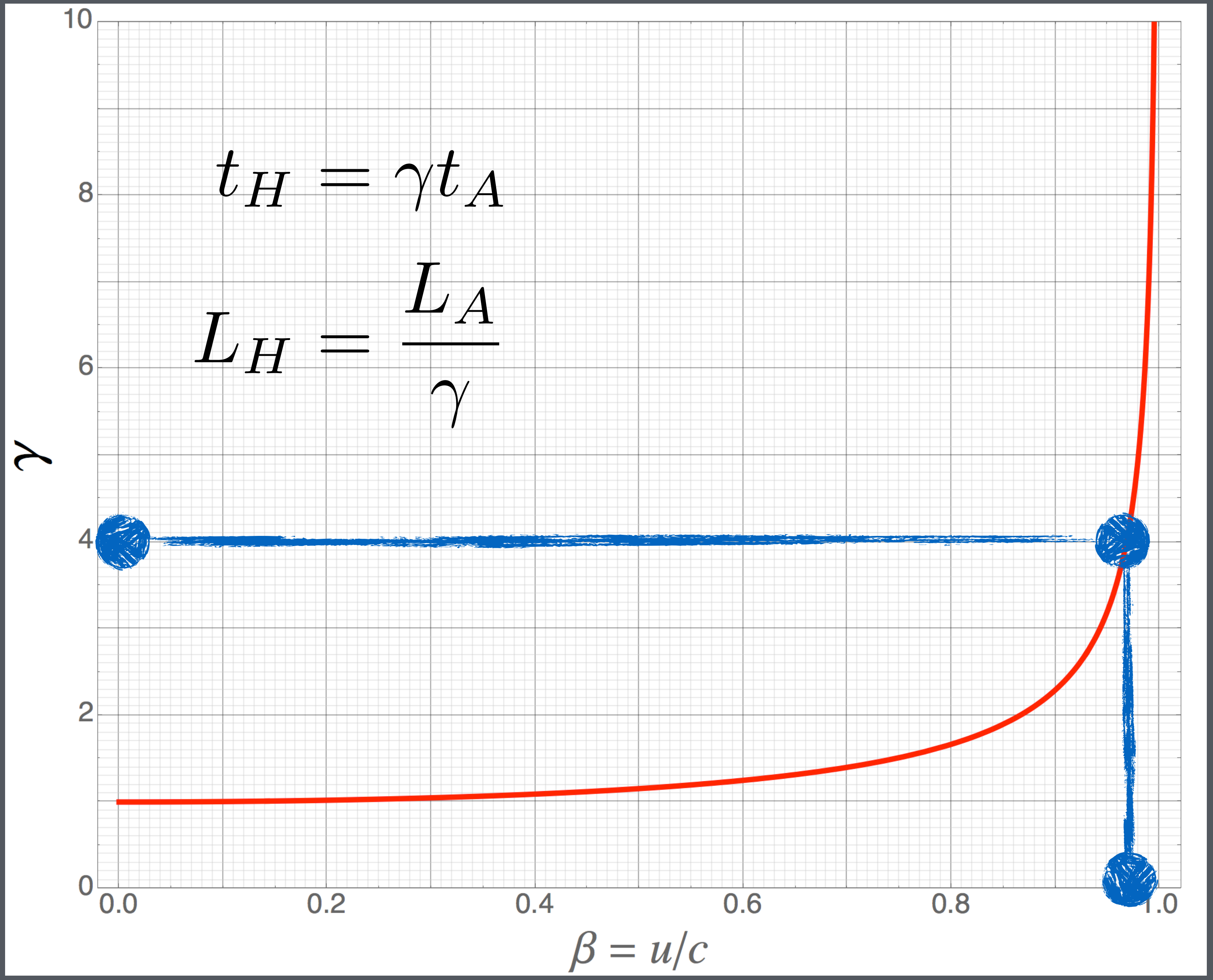
the airport



“Away Frame”:
the frame being watched
x_A
x_H
“Home Frame”:
watching a moving frame
moving at velocity u

what's he see?

How fast must a meter stick be traveling relative to you in order for its length to appear to be 25 cm as measured by you?



collecting these two consequences

of the two simple postulates

"Time Dilation":

$$t_H = \gamma t_A$$

Moving clocks appear to run slower as seen by a relatively stationary observer

"Length Contraction":

$$L_H = \frac{L_A}{\gamma}$$

Moving lengths appear shorter to a relatively stationary observer

Let your fingers do this...and show me: 1.

at the top of your board, write the equation for γ

what value does γ approach as $u << c$?

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

u still finite but

u/c tiny so u^2/c^2 super-tiny!

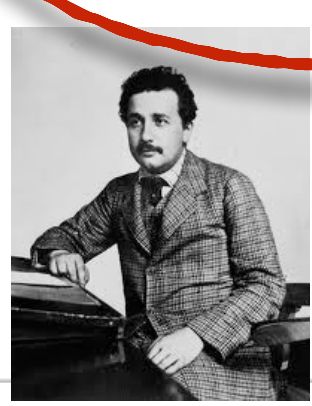
so $1 - u^2/c^2$ very close to 1

so $\gamma \simeq 1$

$$x_H = \gamma(x_A + ut_A)$$

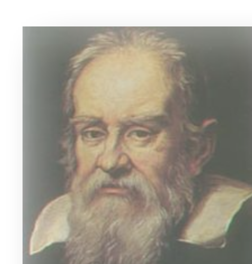
$$t_H = \gamma(t_A + \frac{u}{c^2}x_A)$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{u}{c}\right)^2}}$$



$$x_H = x_A + ut$$

$$t_H = t_A = t$$



Let your fingers do this...and show me, 2.

NOW write the equation for $x_H \, and \, t_H$

what value it look like if $u << c$?

$$x_H = \gamma(x_A + ut)$$

$$x_H \rightarrow 1 \cdot x_A + ut \quad \text{for } u^2/c^2 <<<< 1$$

$$t_H = \gamma(t_A + \frac{u}{c^2} x_A)$$

$u/c \simeq$ small $\quad u/c^2$ really small

$$t_H \simeq 1(t_A + 0)$$

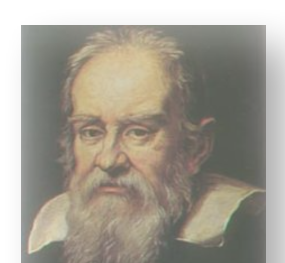
$$x_H = \gamma(x_A + ut_A)$$

$$t_H = \gamma(t_A + \frac{u}{c^2} x_A)$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{u}{c}\right)^2}}$$

$$x_H = x_A + ut$$

$$t_H = t_A = t$$

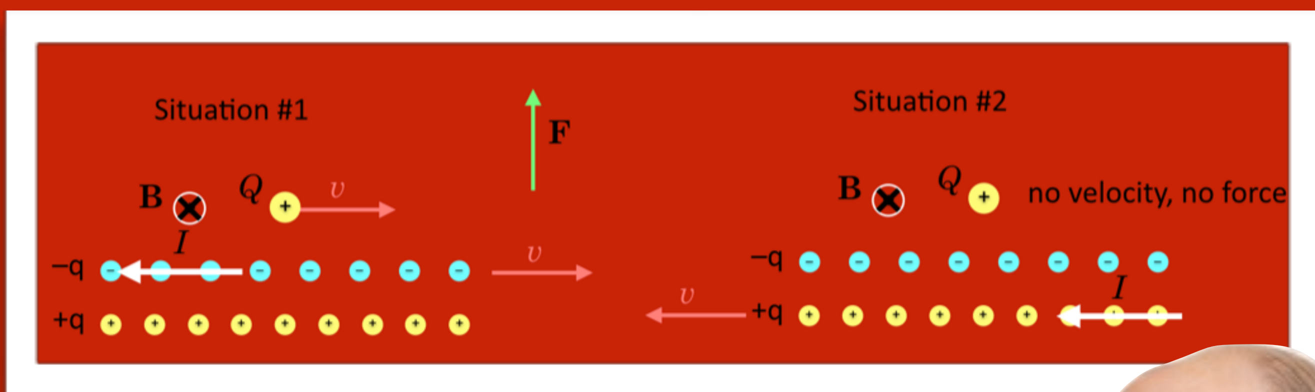


where he started:

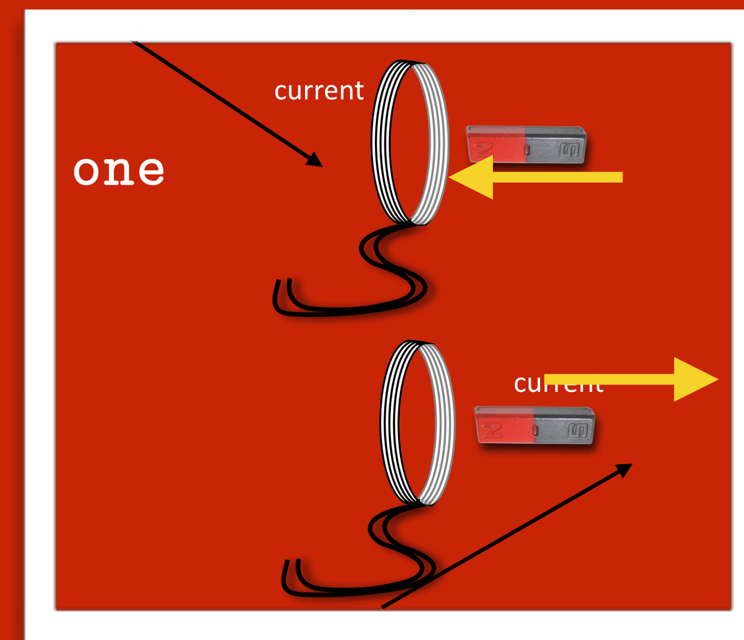
Maxwell's Equations

and apparent anomalies

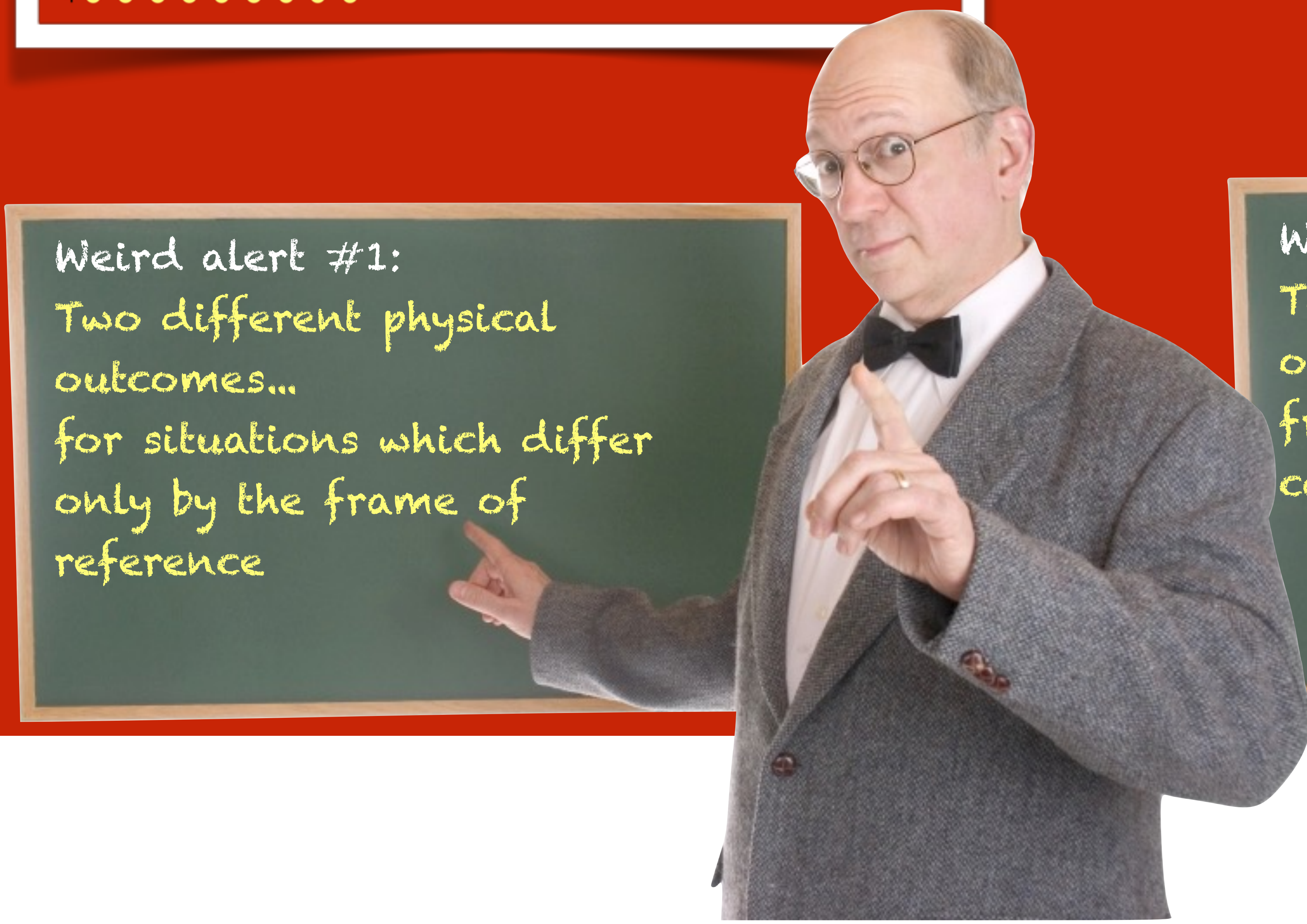
remember?



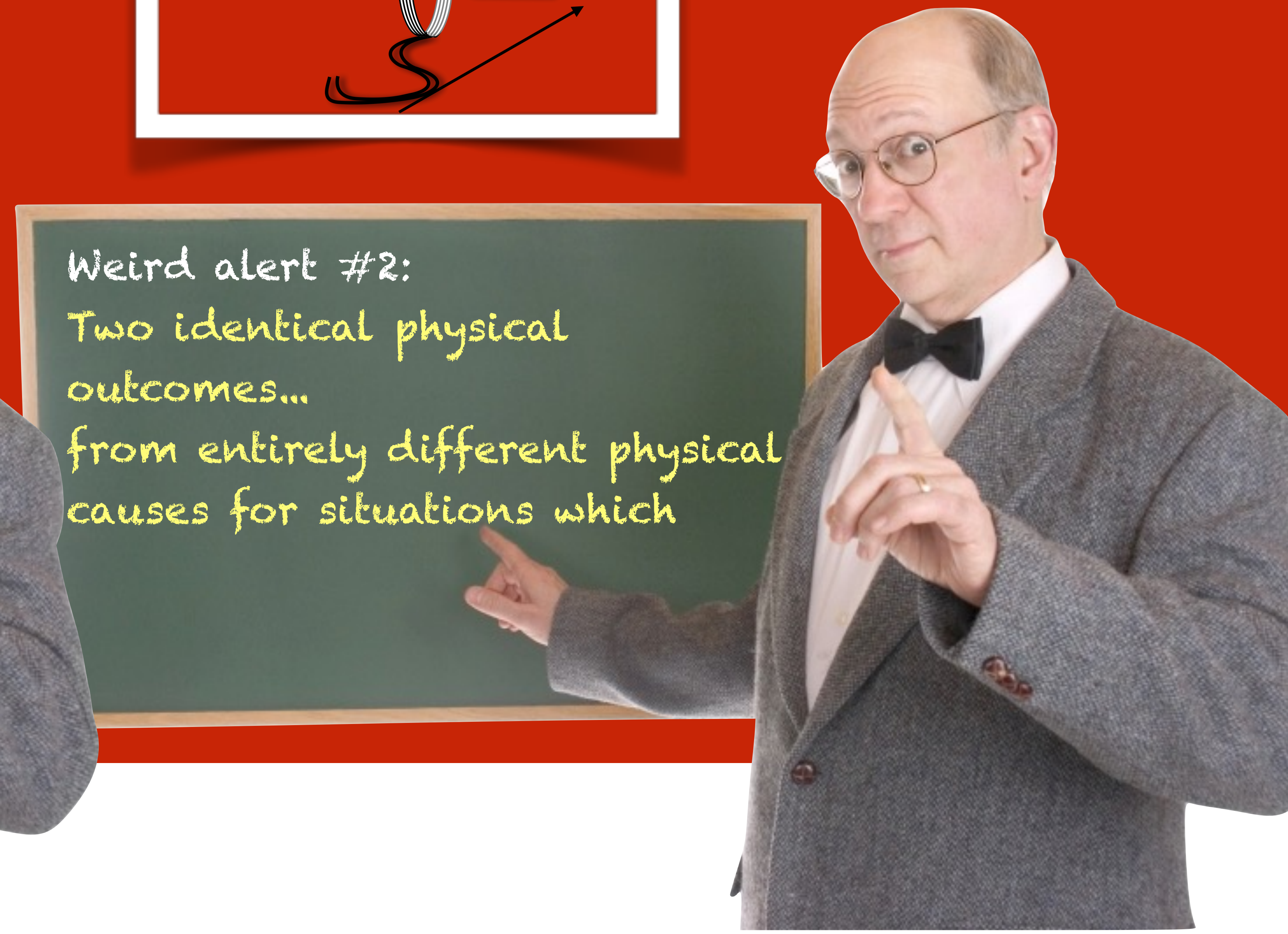
Situation #1
F
B
Q
v
I
−q
+q
Situation #2
no velocity, no force



current
one

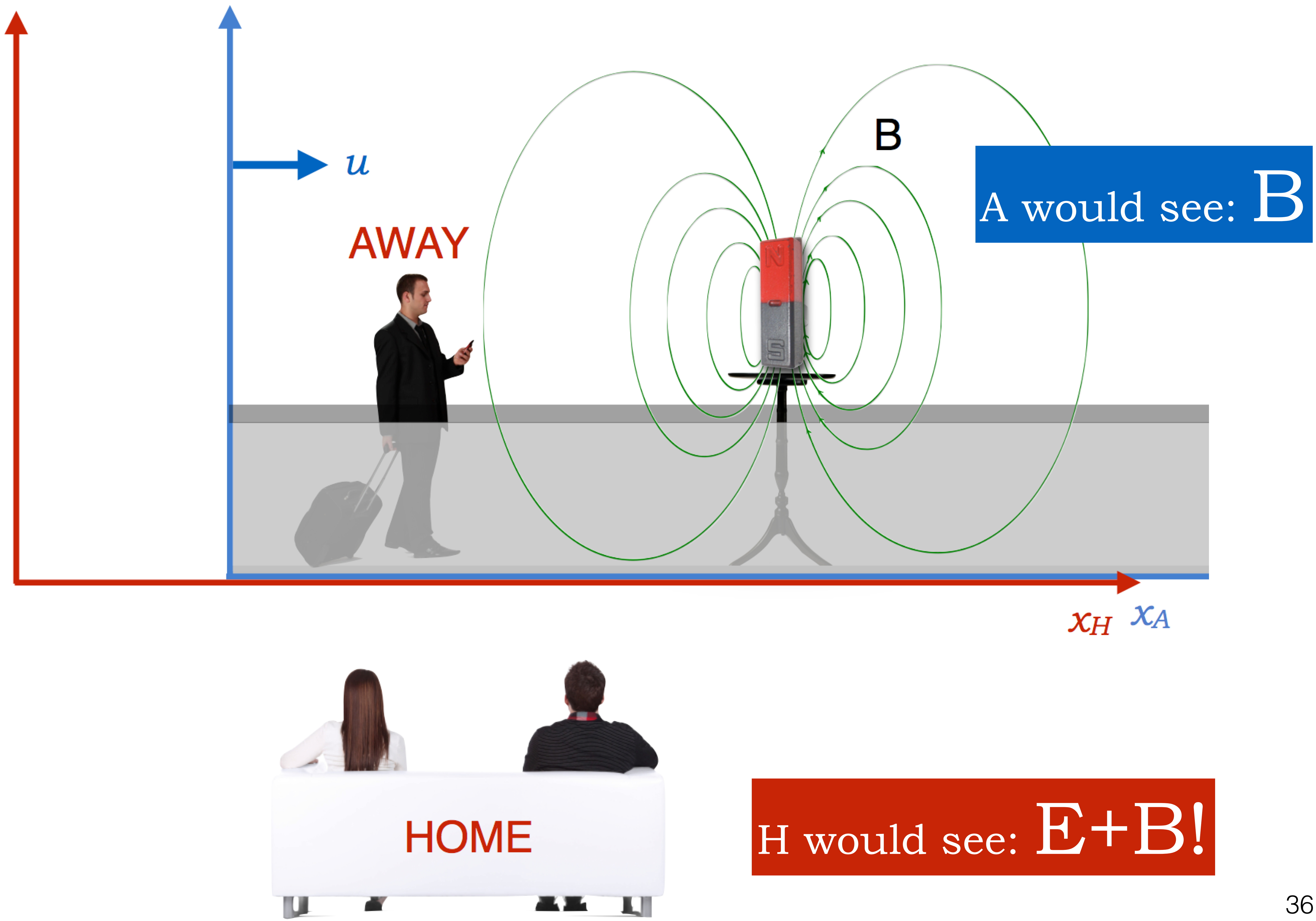


Weird alert #1:
Two different physical outcomes...
for situations which differ only by the frame of reference



Weird alert #2:
Two identical physical outcomes...
from entirely different physical causes for situations which

back to the airport



so the original problems are solved by:

the Lorentz transformations in *x* and *t*

actually **mix** electric and magnetic fields

so

A **magnetic field** in one frame

is a **mixture of magnetic and electric fields** in another frame

An **electric field** in one frame

is a **mixture of electric and magnetic fields** in another frame so the original problems are solved by:

E and B are two manifestations of one thing:

the Electromagnetic Field

the Lorentz transformations in *x* and *t*

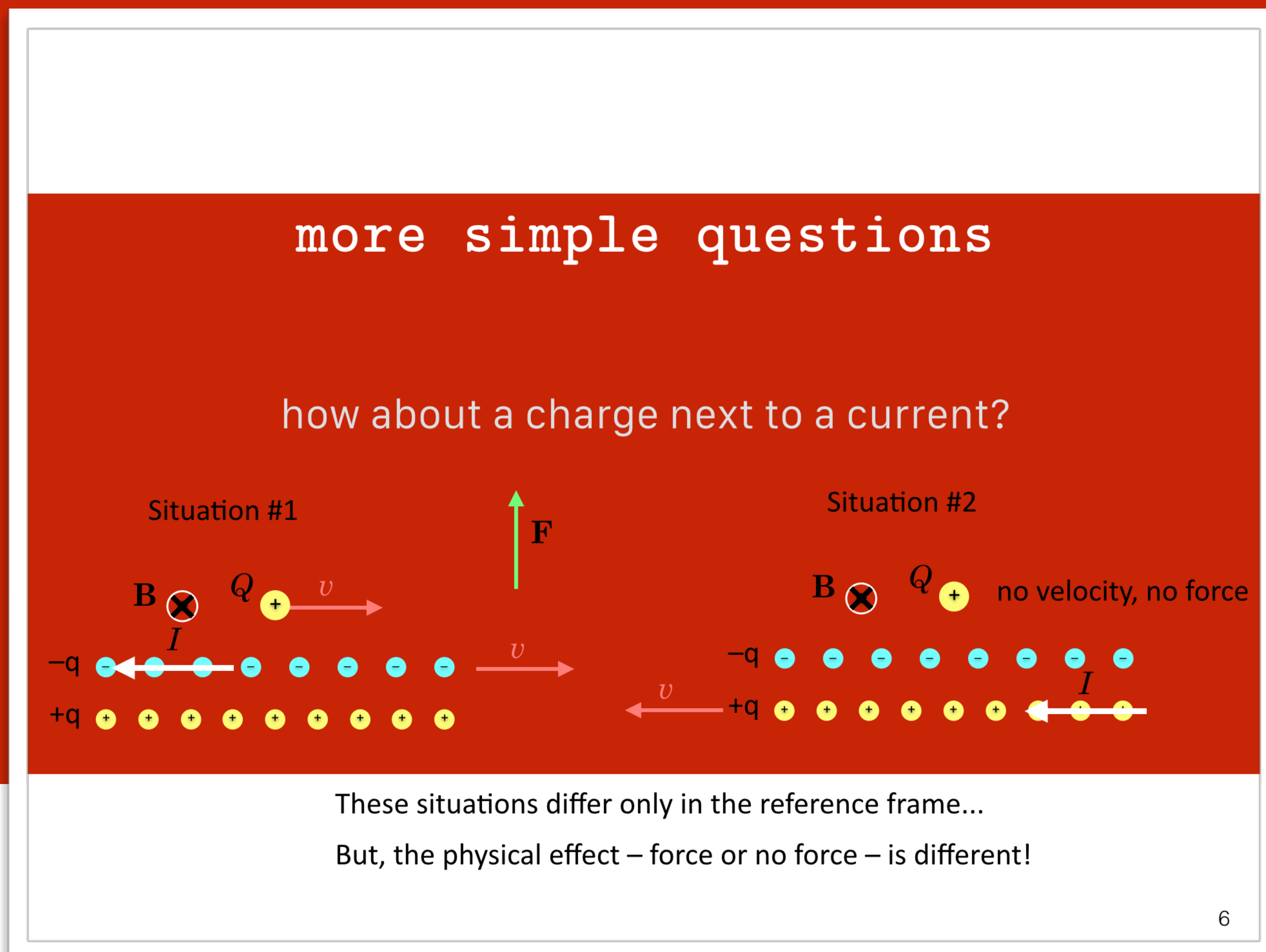
mix electric and magnetic fields

is a mixture of magnetic and electric fields in another frame

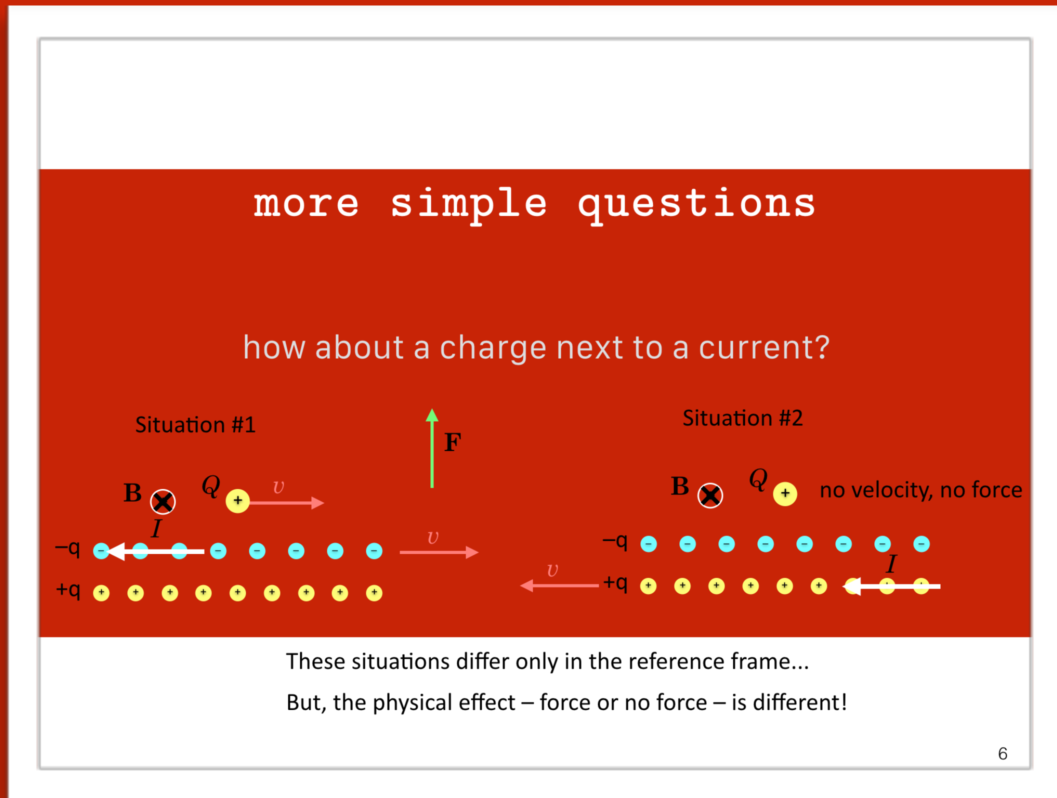
An electric field in one frame

is a mixture of electric and magnetic fields in another frame

remember:

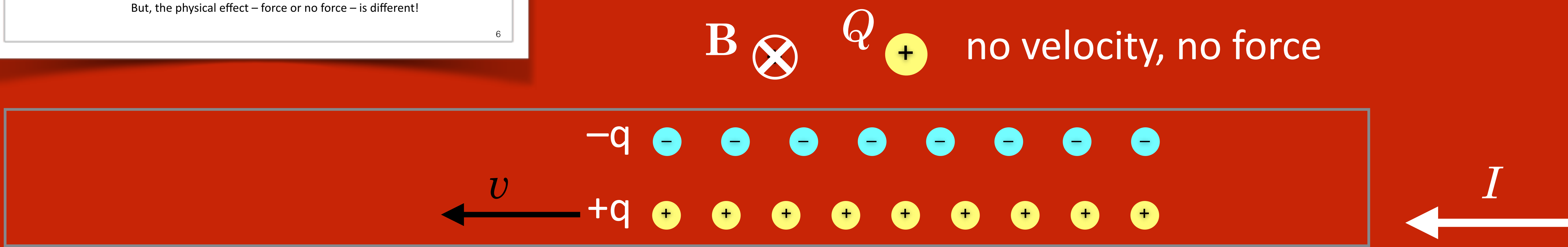

more simple questions
how about a charge next to a current?
Situation #1
F
B
Q
v
I
−q
+q
v
Situation #2
B
Q
no velocity, no force
−q
I
v
+q
These situations differ only in the reference frame...
But, the physical effect – force or no force – is different!
6

remember?



Situation #2

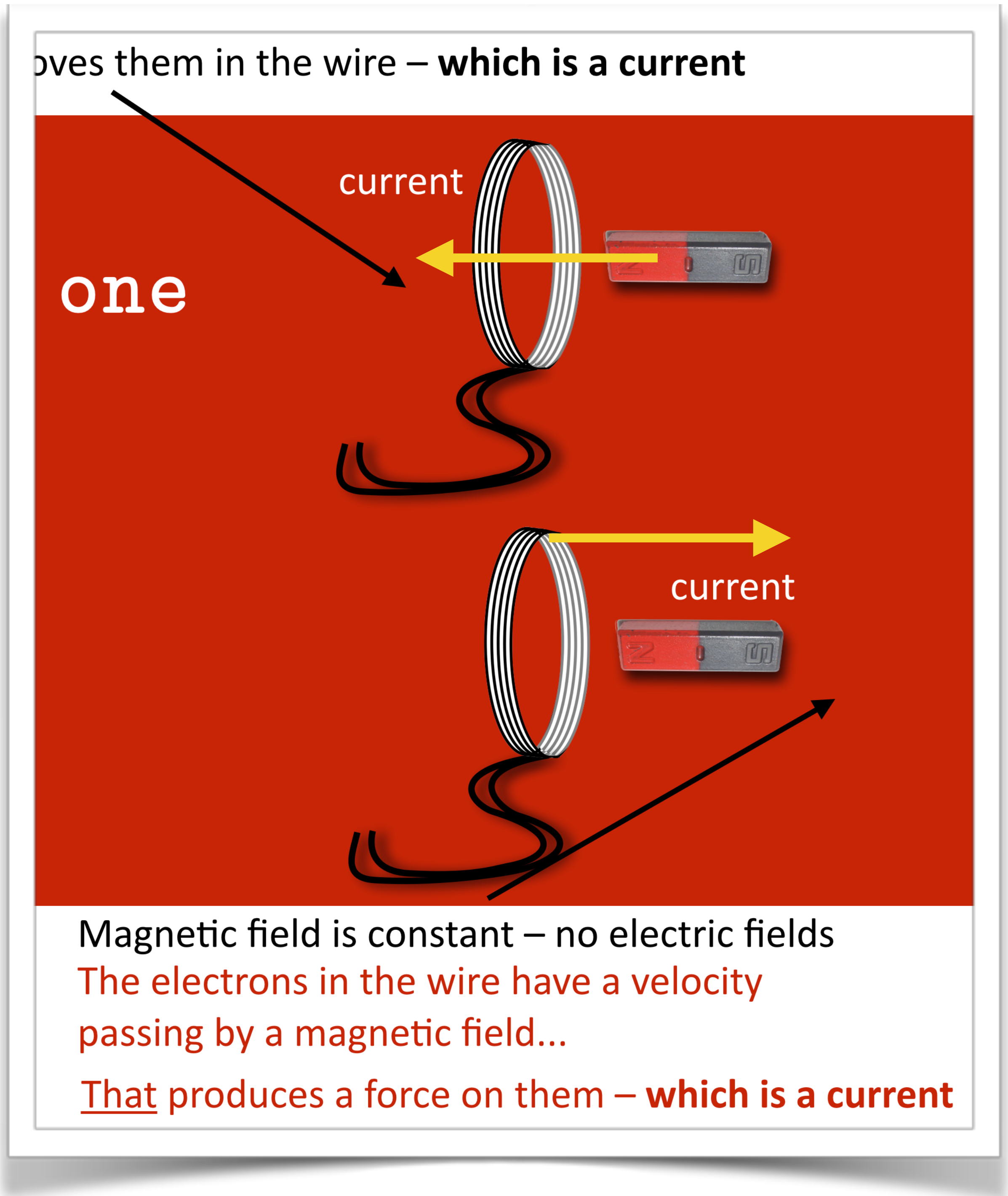
but there should have been a force!



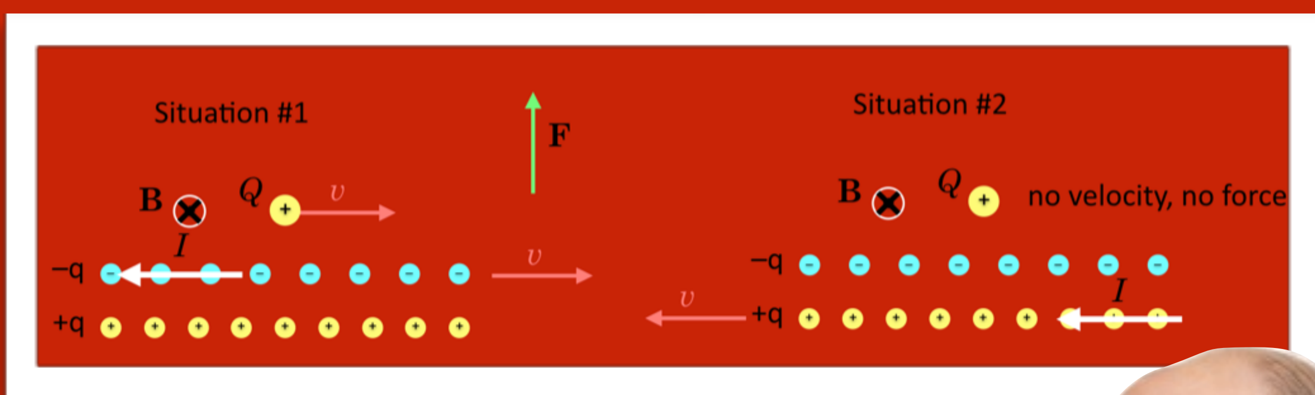
and the coil?

yup. right observation all along.

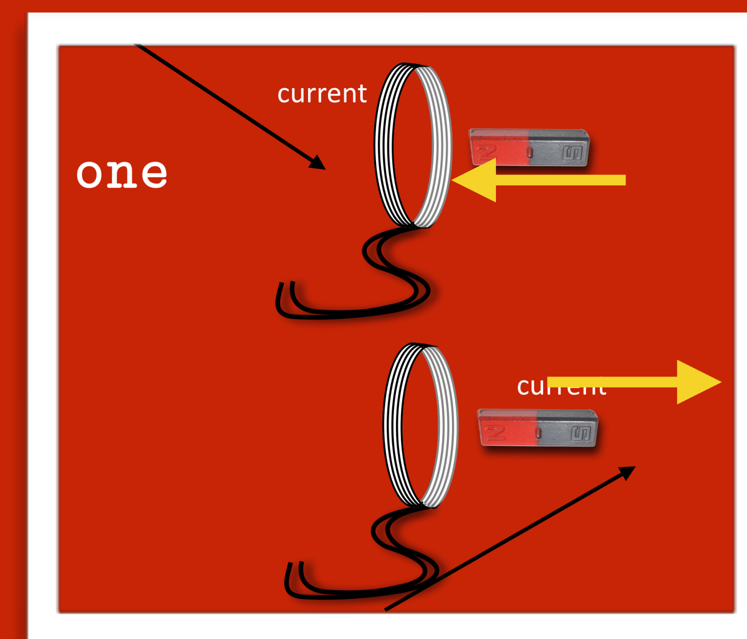
Electric field or magnetic field, depending on the relative frames



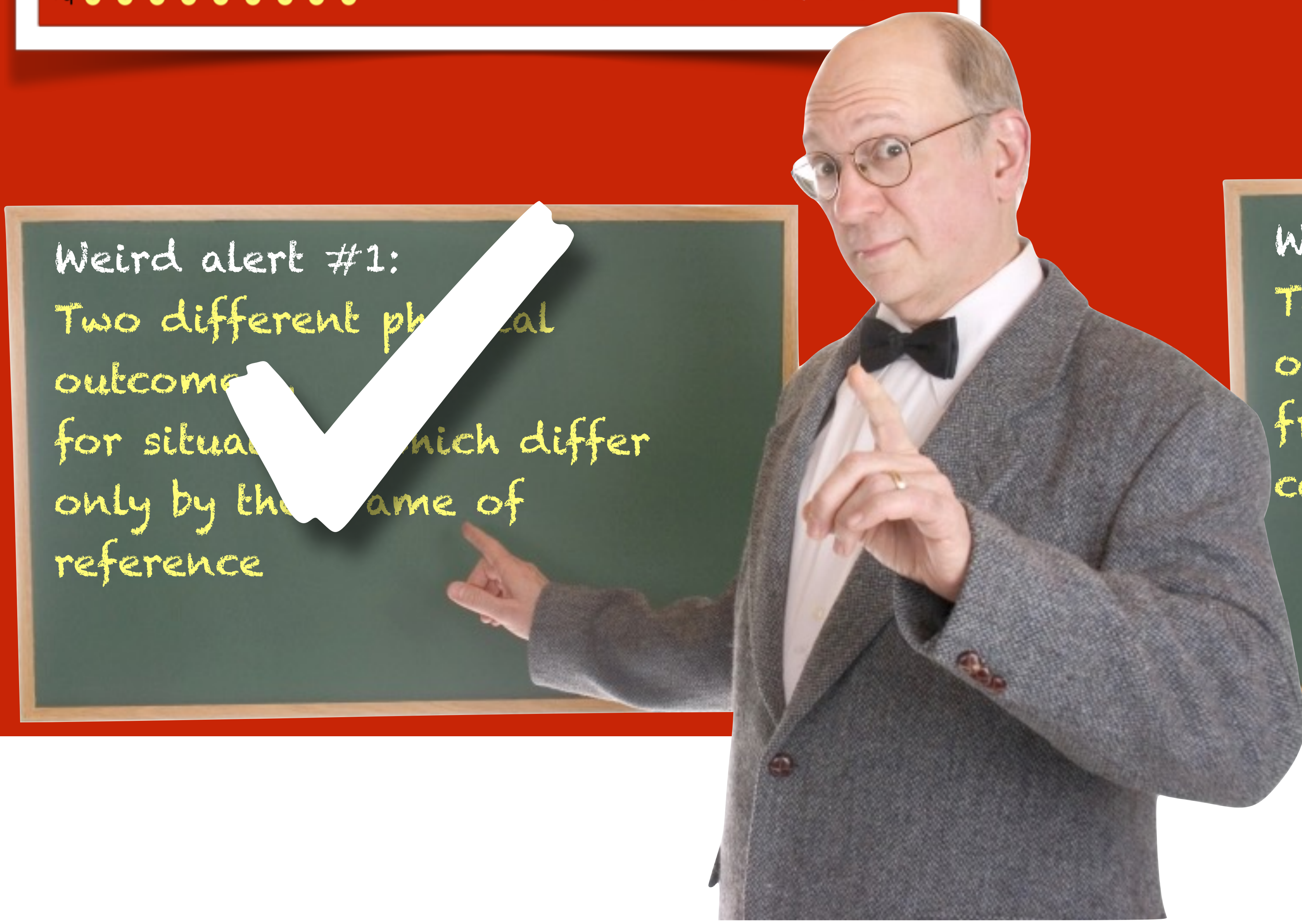
remember?



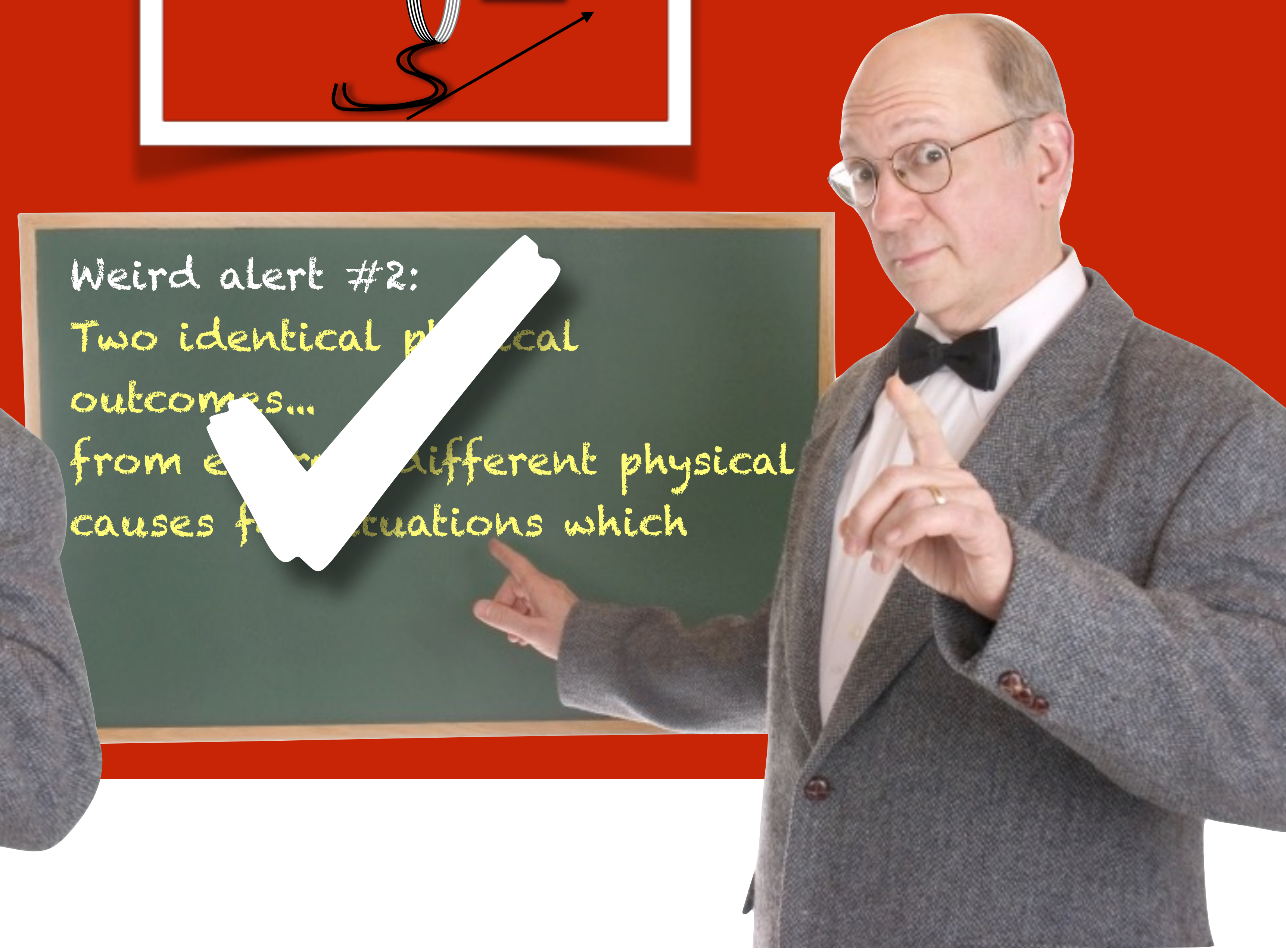
Situation #1
F
B
Q
v
I
−q
+q
Situation #2
no velocity, no force



current
one



Weird alert #1:
Two different ph al
outcome
for situa hich differ
only by th ame of
reference



Weird alert #2:
Two identical p cal
outcom s...
from e ifferent physical
causes f tuations which

Principle of Relativity

2 Postulates:

1. **All laws of physics – mechanical and electromagnetic – are identical in co-moving inertial frames.**

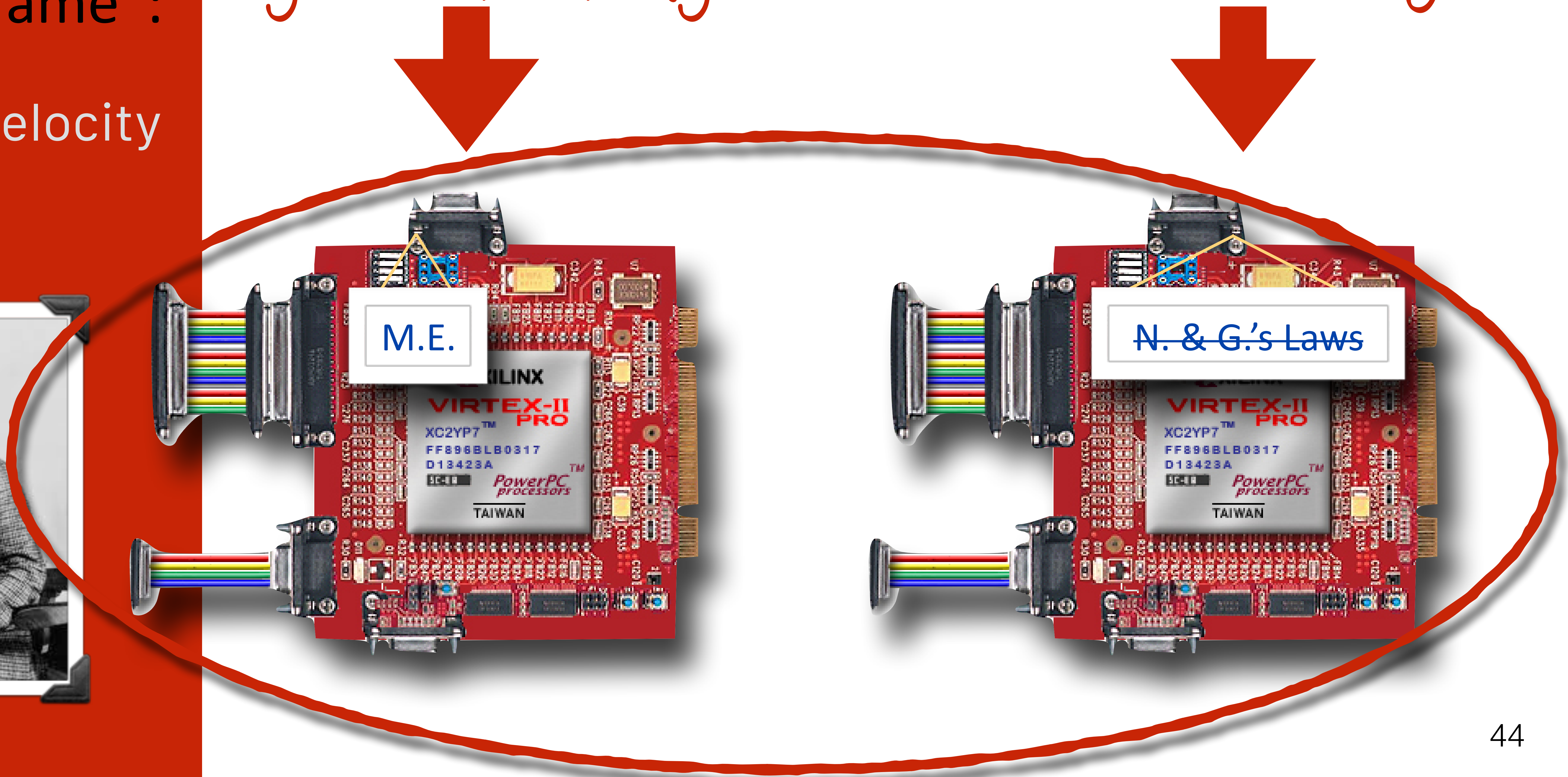
2. **The speed of light is the same for all inertial observers.**

"inertial frame":

constant velocity

good all along!

had to change!



`is Relativity`

the case?

relatives

this is an electron, e: 

this is a cousin of an electron...the “muon,” μ: 

they are exactly alike except that

$m(\mu)$ = 209 x $m(e)$

and in about 1.5 microseconds:

muon’s lives are short and sweet

‘‘muons’’: μ

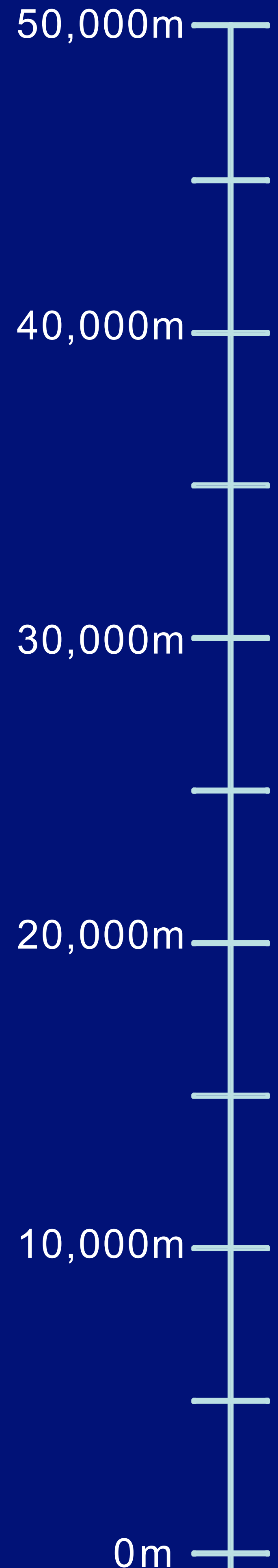
are unstable particles which are easily made in an accelerator lab and shown to have a half life of 1.56 μs…

1.56×10^{-6} seconds

Under Newton’s view, even if the muon goes at the speed of light, then it lives for only $(3 \times 10^{8}) \times (1.5 \times 10^{-6}$ *seconds*$) = 450$ *m*

stand-up cosmic

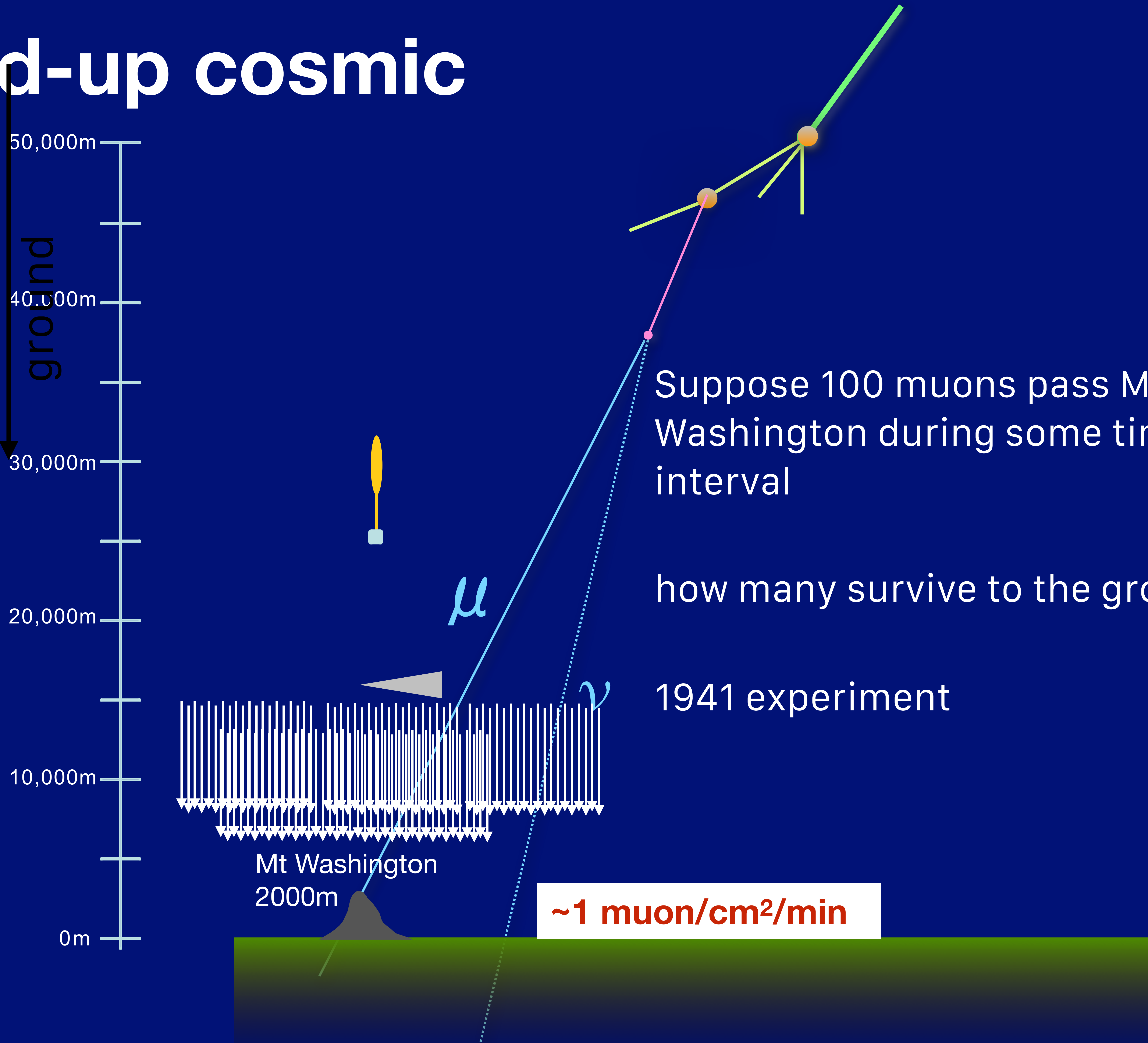
~20 particles/cm/s



Mount Washington Observatory
New Hampshire

D. H. Frisch and J. H. Smith, "Measurement of the Relativistic Time Dilation Using μ-Mesons," American Journal of Physics, 31 (5): 342–355, 1963).

stand-up cosmic

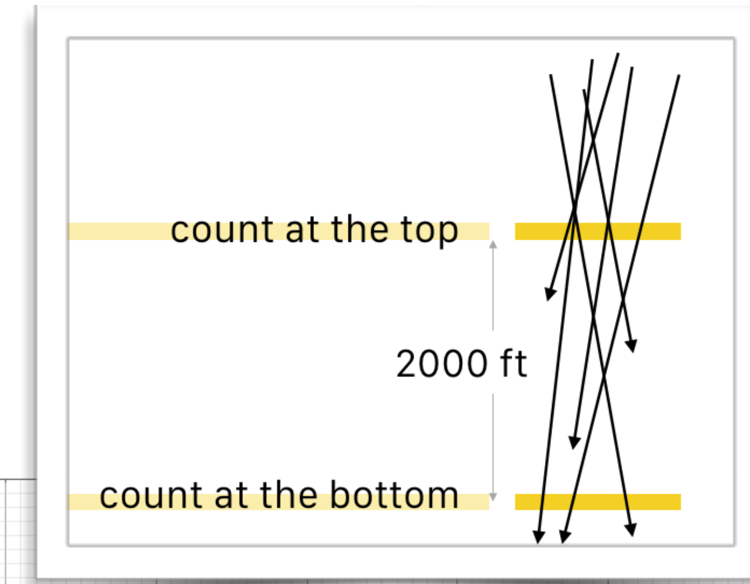


Suppose 100 muons pass Mt Washington during some time interval

how many survive to the ground?

1941 experiment

home and away

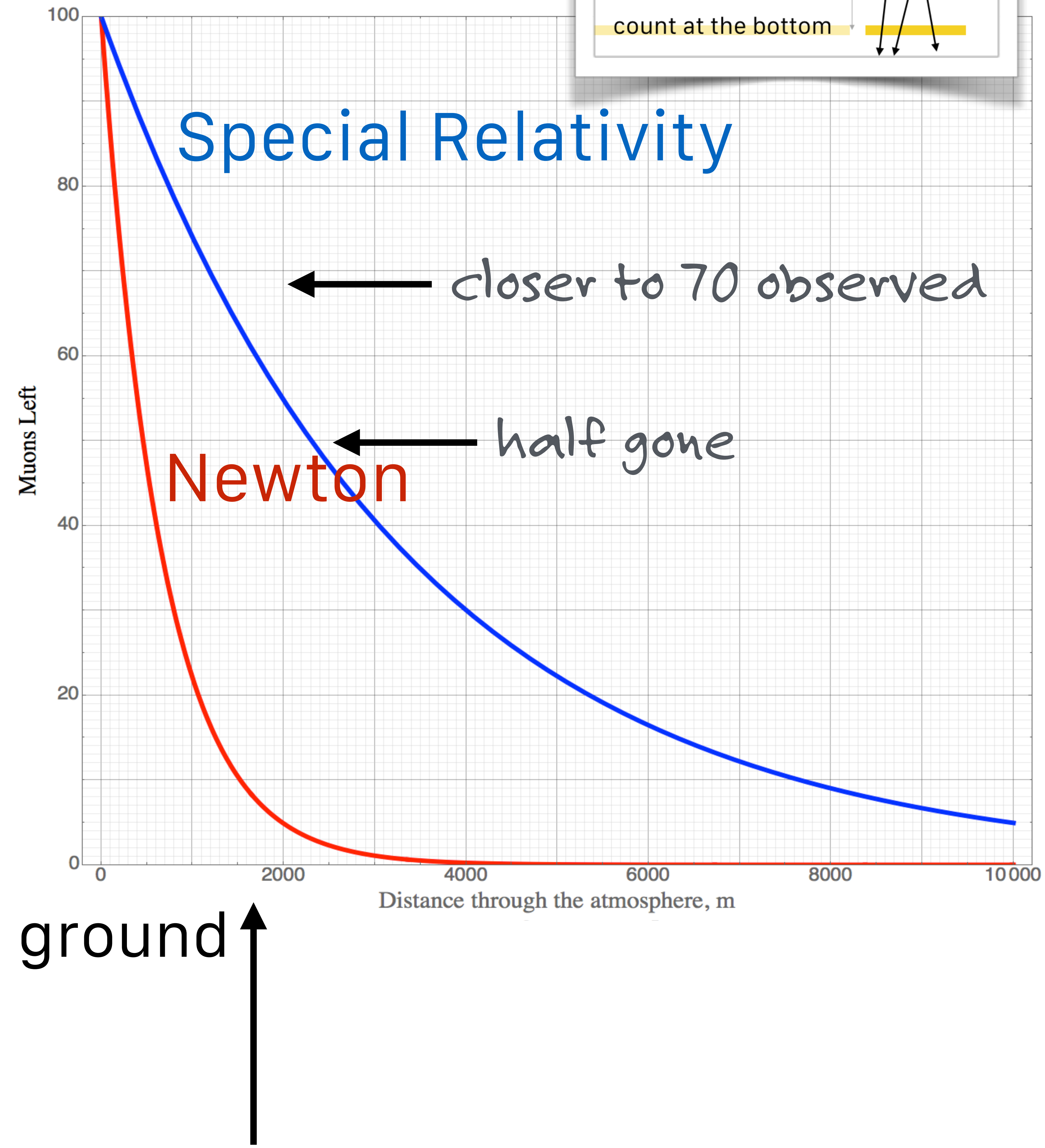


But relativity says:

for the muon moving with $\beta = 0.99$

as observed by the mountain, its clock appears to slowed to

γ x 1.6 microseconds



how can it decay and not decay?

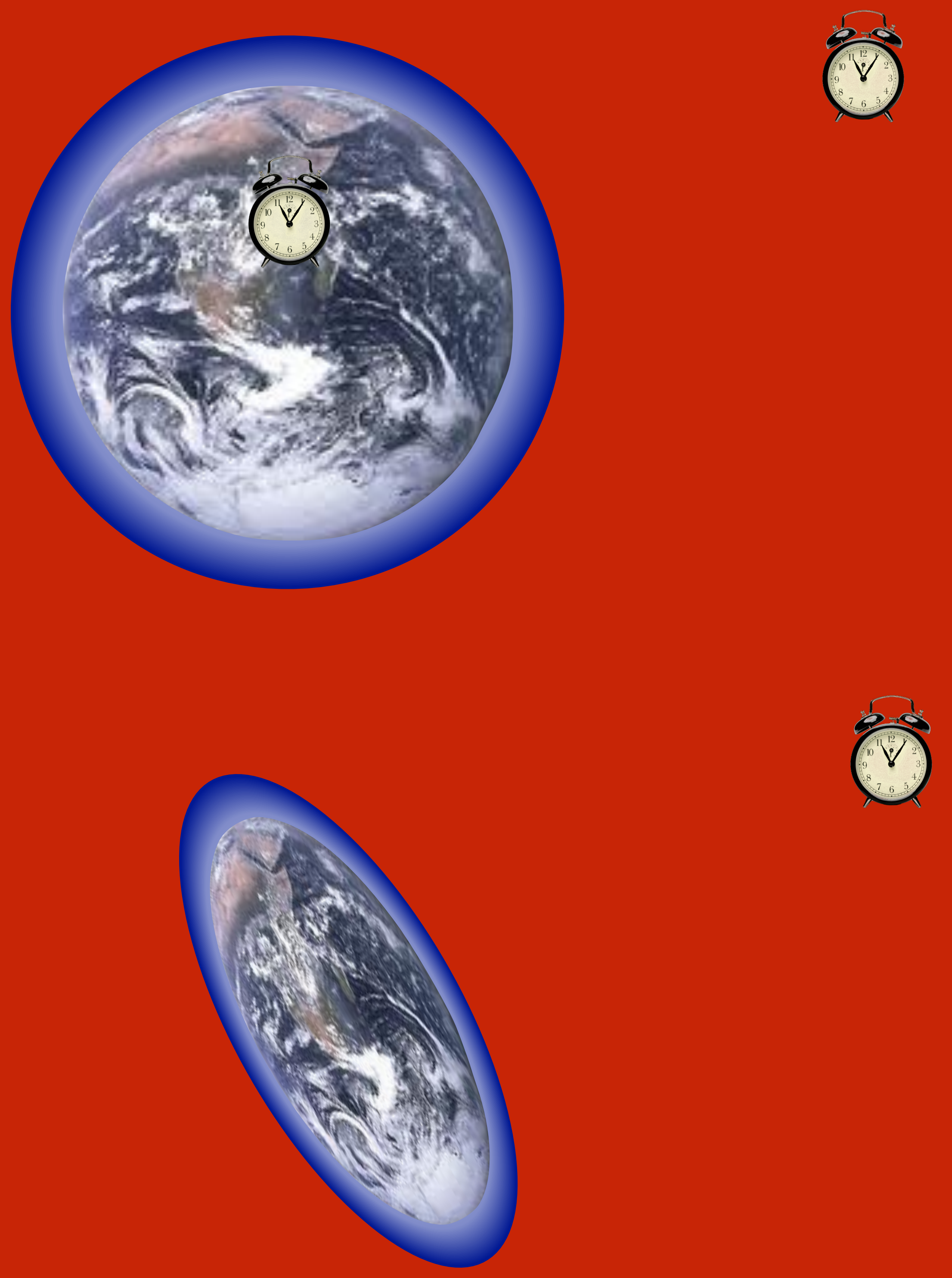
reciprocity

while it decays in 1.5μs in its rest frame…

it sees the atmosphere coming toward it at nearly c

which, to the muon, is Length Contracted

shorter by the same factor that the lifetimes differed



This has been measured many times:

an atomic clock was carefully carried around the world in 1972 and carefully calibrated and compared with ground-based clocks

J. Hafele and R. Keating

There are a number of corrections: accelerations, decelerations, the rotation of the orbit, the fact that the earth is not inertial - but relativity was absolutely correct

Predicted Effect	Flying East	Flying West
GTR (Gravitation)	+ 144 ± 14 ns	+ 179 ± 18 ns
STR (Velocity)	- 184 ± 18 ns	+ 96 ± 18 ns
Total	- 40 ± 23 ns	+ 275 ± 21 ns
measured:	- 59 ± 10 ns	+273 ± 7 ns

redone twice more in airplanes
and rockets/satellites

We trust S… Relativity

we no longer do exp… irm or disconfirm it

it’… ow

Principle of Relativity

2 Postulates:

"inertial frame":

1. All laws of physics — mechanical and electromagnetic — are identical in co-moving inertial frames.

taking Galileo seriously, and then adding Maxwell

2. The speed of light is the same for all inertial observers.

taking Maxwell seriously

a fact of nature

combine speeds!

Galileo, nope.

Einstein, yup.

the airport, redux

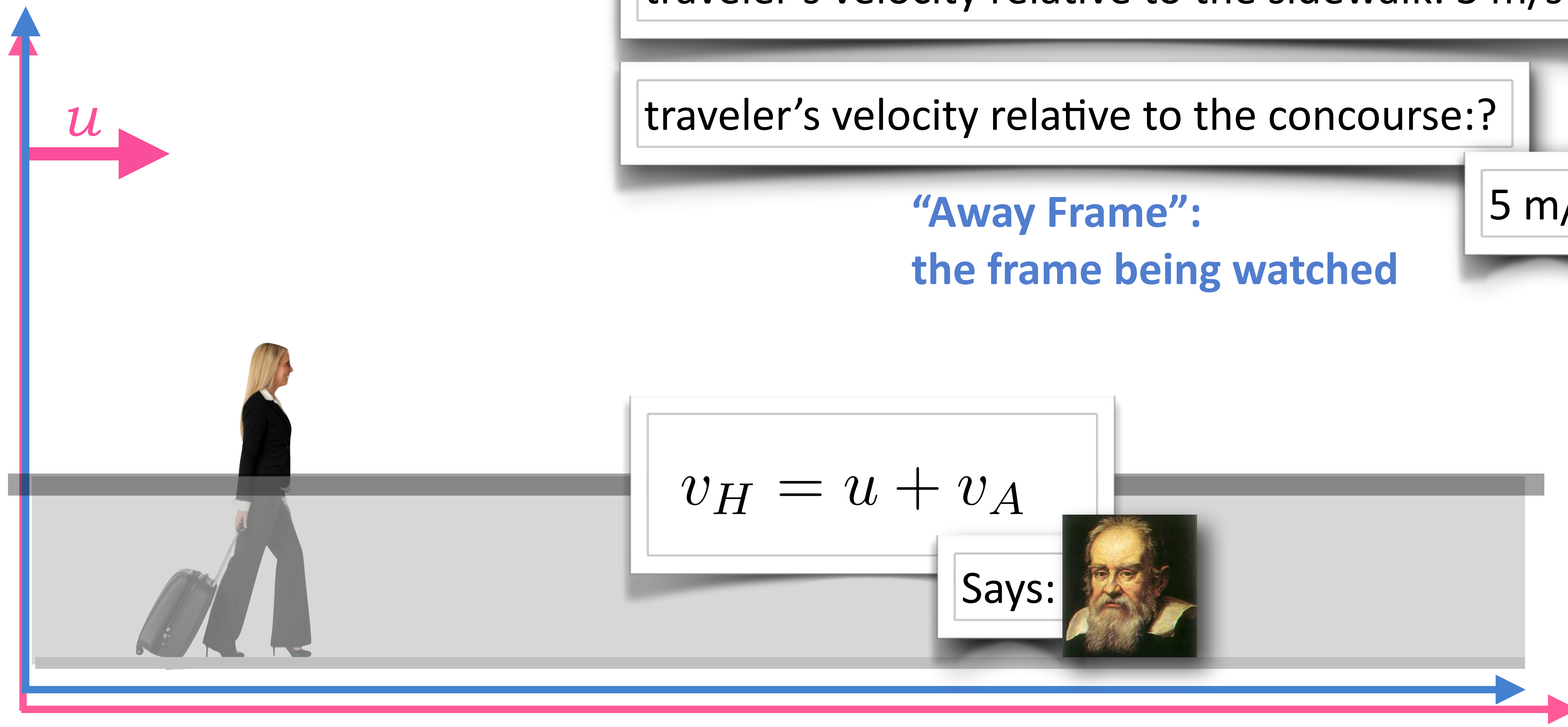
sidewalk velocity: 2 m/s

traveler’s velocity relative to the sidewalk: 3 m/s

traveler’s velocity relative to the concourse:?

5 m/s

“Away Frame”:
the frame being watched

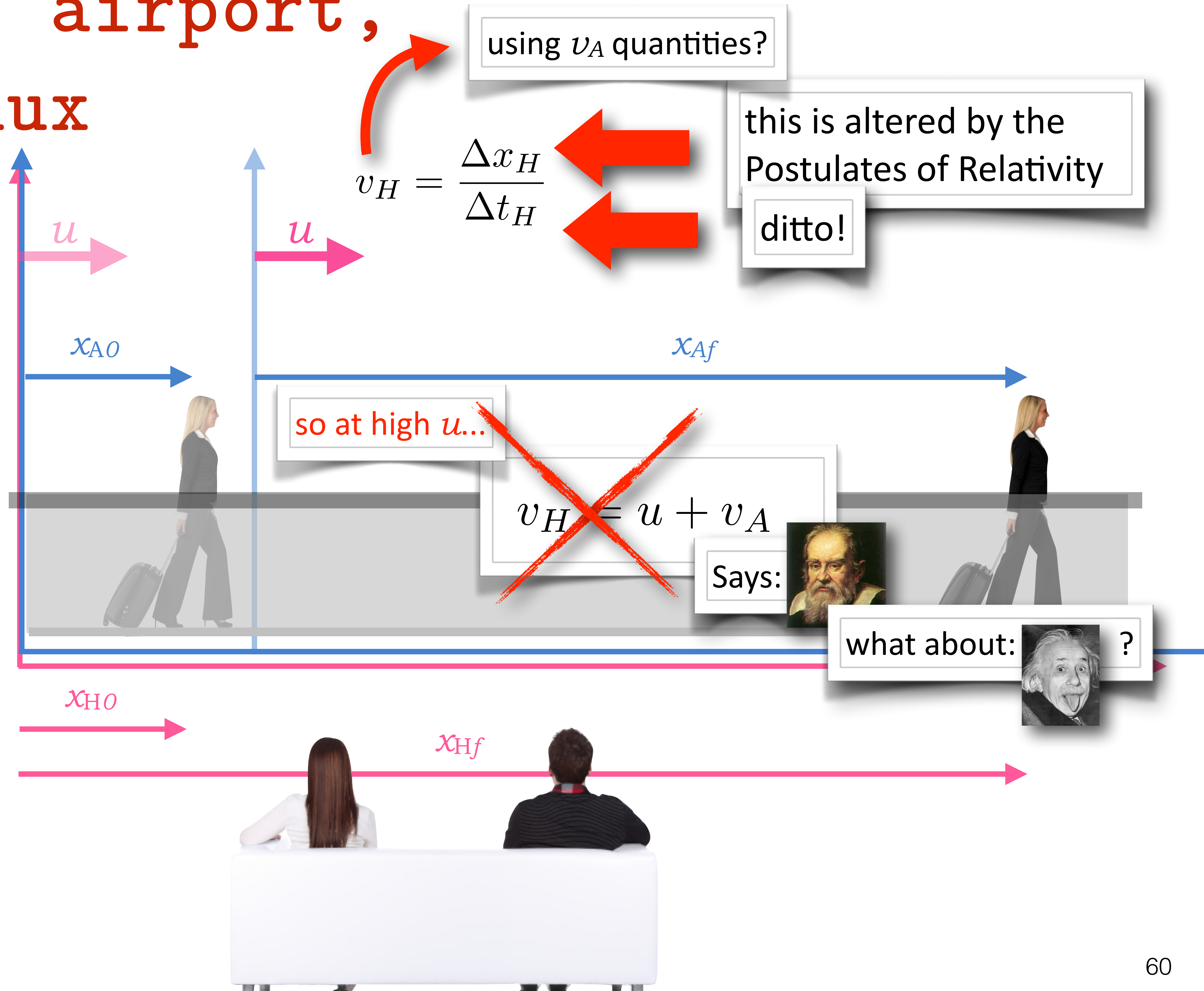


$$v_H = u + v_A$$

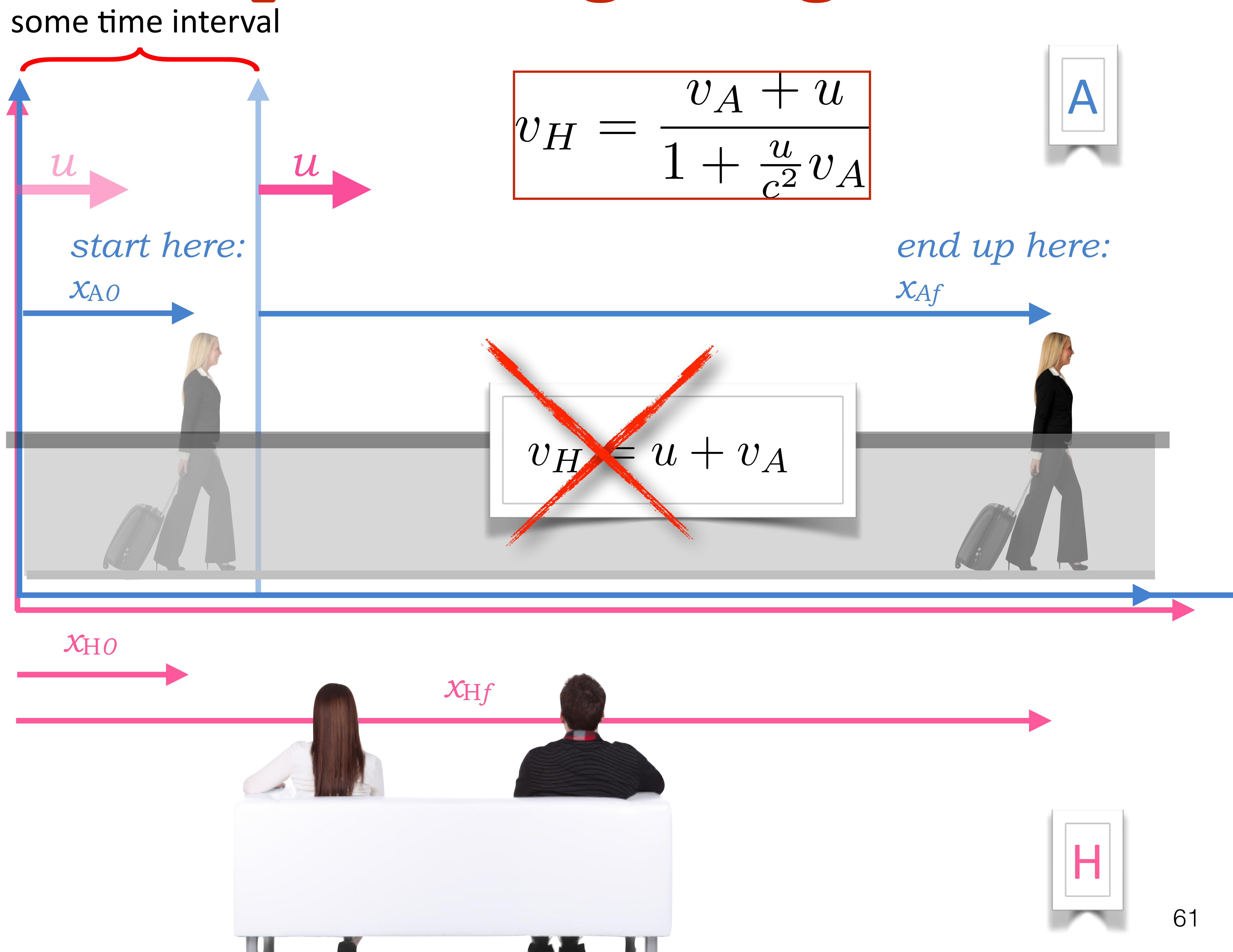
Says:

“Home Frame”:
watching a moving frame

the airport, redux



the airport, going fast



write it down.

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

relativistic velocity transformation

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

Look at this formula carefully...

Suppose u/c is very small...like normal life.

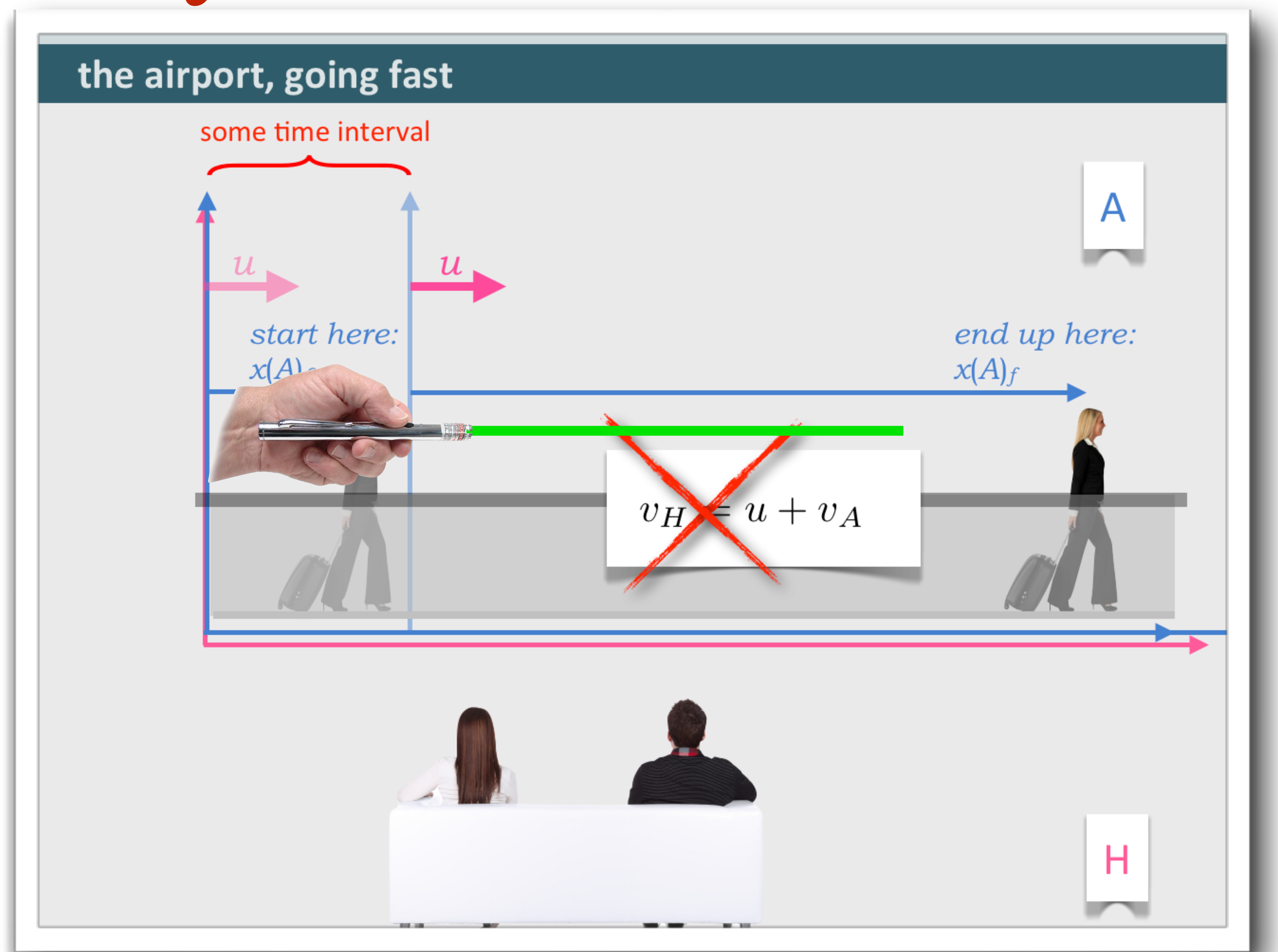
work it out

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

$\frac{u}{c^2}$ <<1...so

$$v_H \rightarrow u + v_A$$

and the old-time, non-relativistic airport sidewalk formula emerges



Suppose it's not a traveler, but light.

$v_A = c$ **work it out**

$$v_H = \frac{c + u}{1 + \frac{u}{c^2} c} = \frac{c + u}{(c + u)} c = c$$

The Second Postulate is preserved!

nothing

can accelerate to a speed faster than that of light

be careful

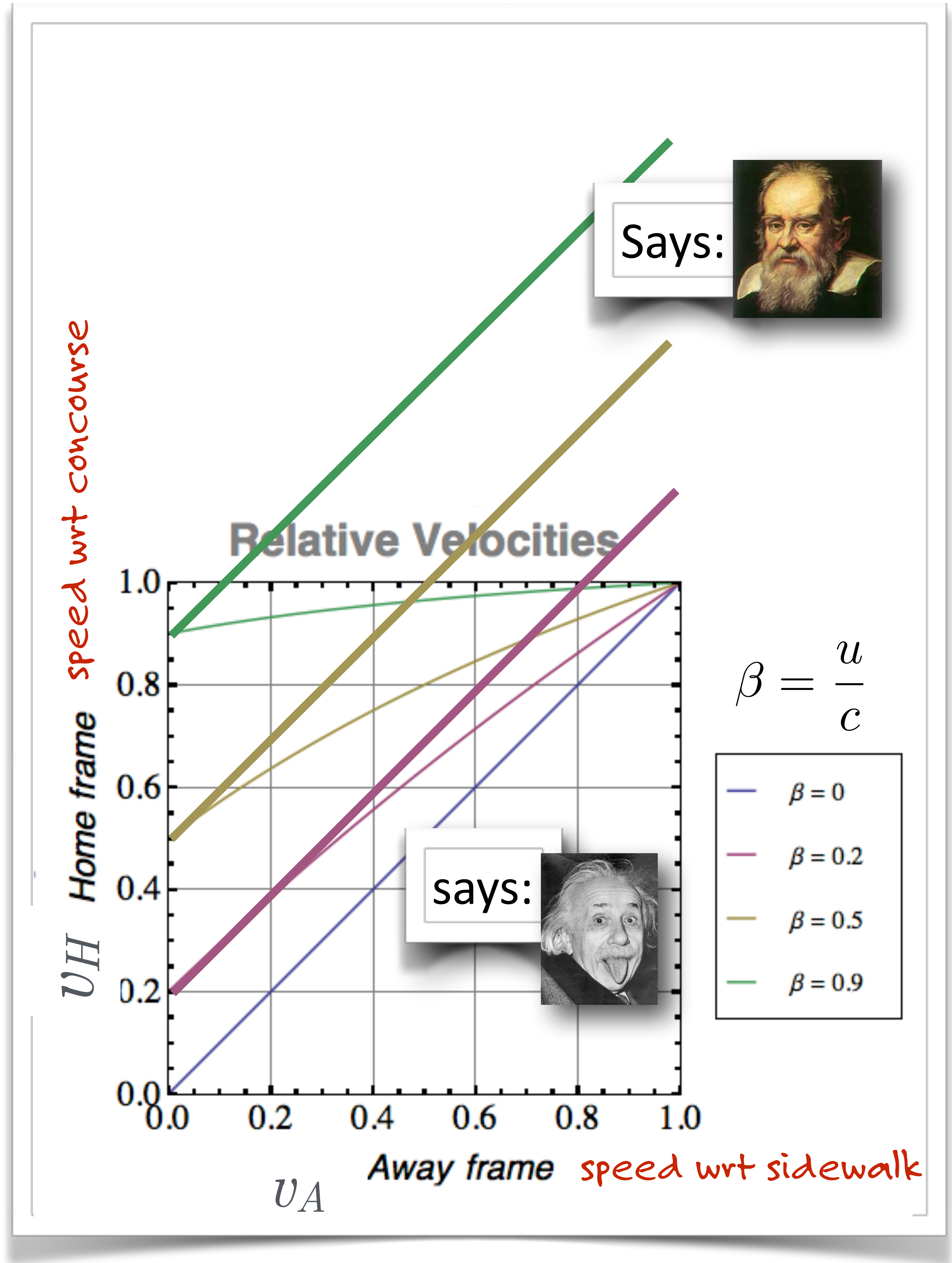
There are 3 velocities going on here.

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

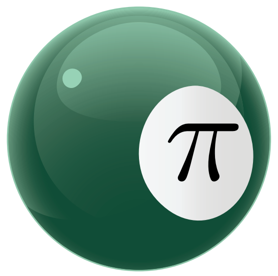
u is the frame velocity
...same, A relative to H or H relative to A (sidewalk)

v_A is the velocity (traveler)
of something measured relative to the A frame

v_H is the velocity (traveler)
of something measured relative to the H frame



another particle

the “pion,” π 

not like the muon or electron

mass $m(\pi)$ = 1.4 x $m(\mu)$

unstable

a pion decays into a muon

the pion travels at $u = 0.5\mathrm{c}$ in the lab (H)

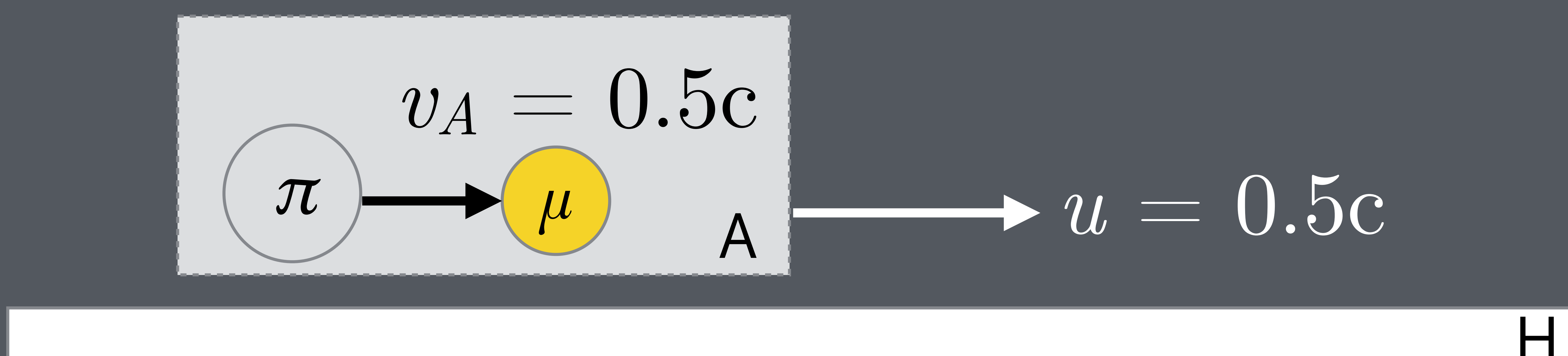
the muon travels right at $v_\mathrm{A} = 0.5\mathrm{c}$ in the pion's rest frame

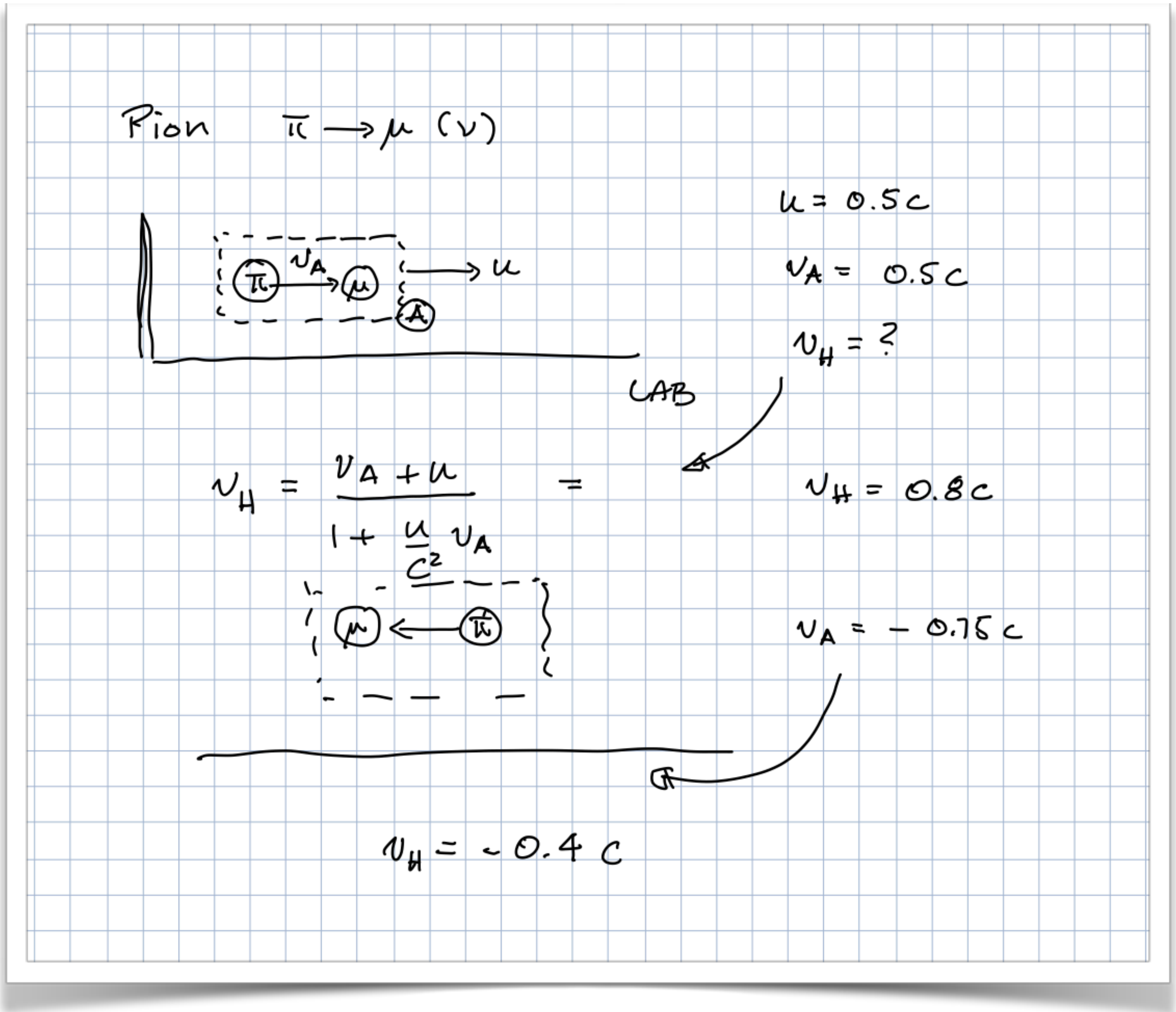
What is the speed of the muon in the lab?

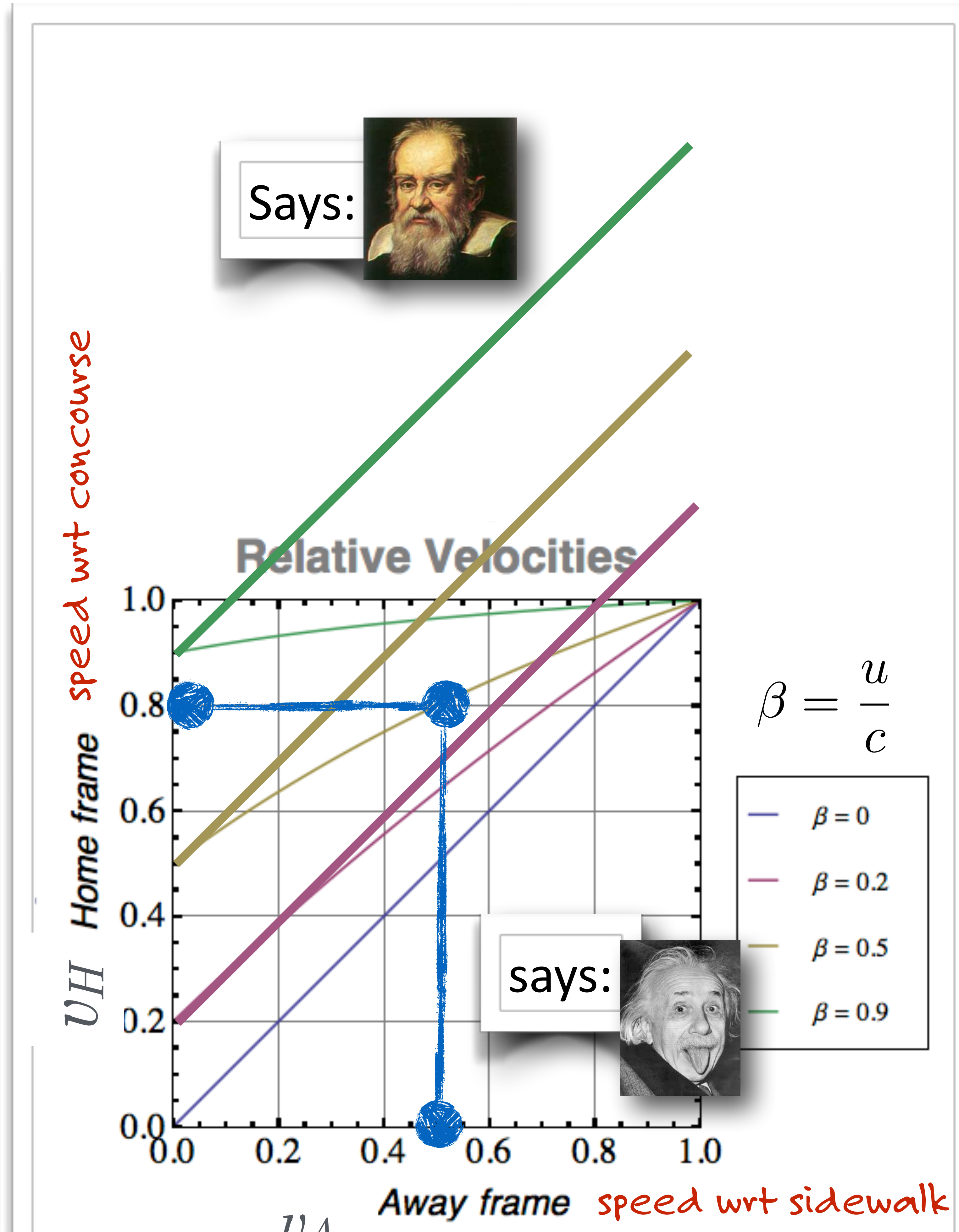
How far does it travel in the lab before decaying?

What is the speed if muon travels left at $v_\mathrm{A} = -\,0.5\mathrm{c}$ in the pion's rest frame?

What if the muon travels left at $v_\mathrm{A} = -\,0.75\mathrm{c}$ in the pion's rest frame?




Pion π → μ (ν)
u = 0.5c
v_A = 0.5c
v_H = ?
LAB
$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$ =
v_H = 0.8c
v_A = − 0.75c
v_H = − 0.4 c


Says:
says:
Relative Velocities
speed wrt concourse
Home frame
v_H
Away frame
speed wrt sidewalk
v_A
$\beta = \frac{u}{c}$
β = 0
β = 0.2
β = 0.5
β = 0.9

Energy

push on something

constant force to create a constant acceleration of

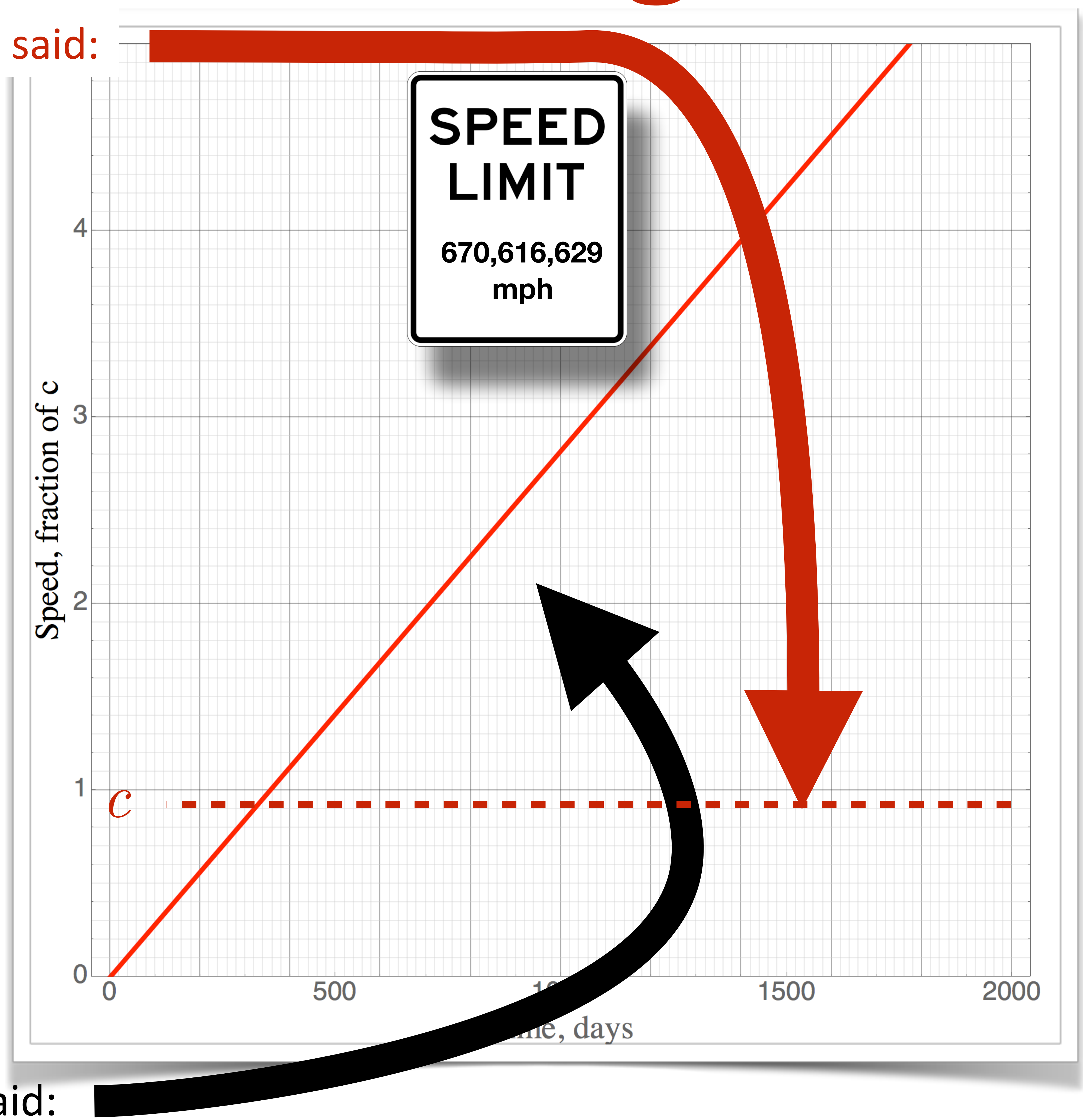
$1g$

Galileo/Newton said

speed increases:

$$v = gt$$

Einstein said:

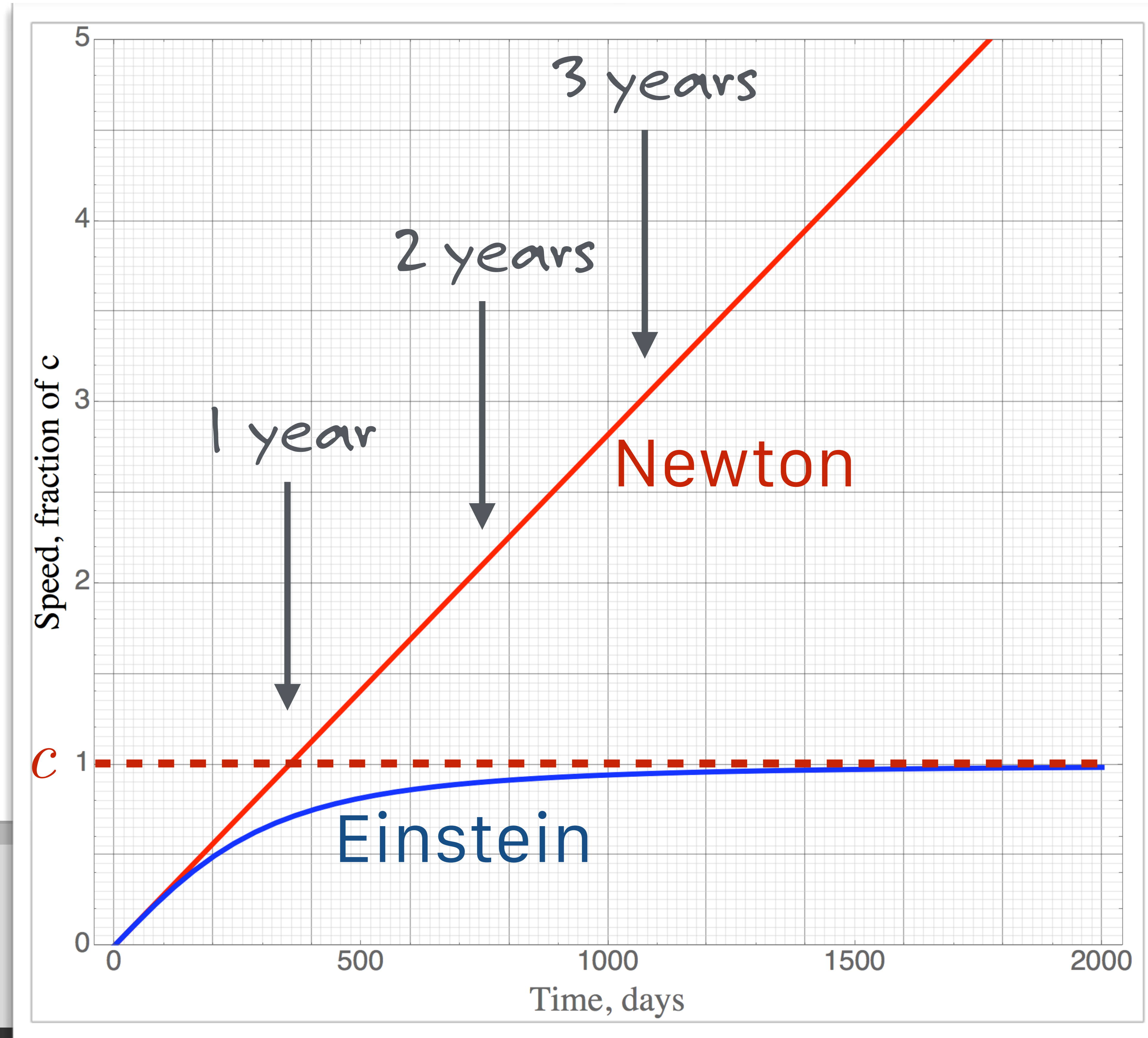


Newton said:

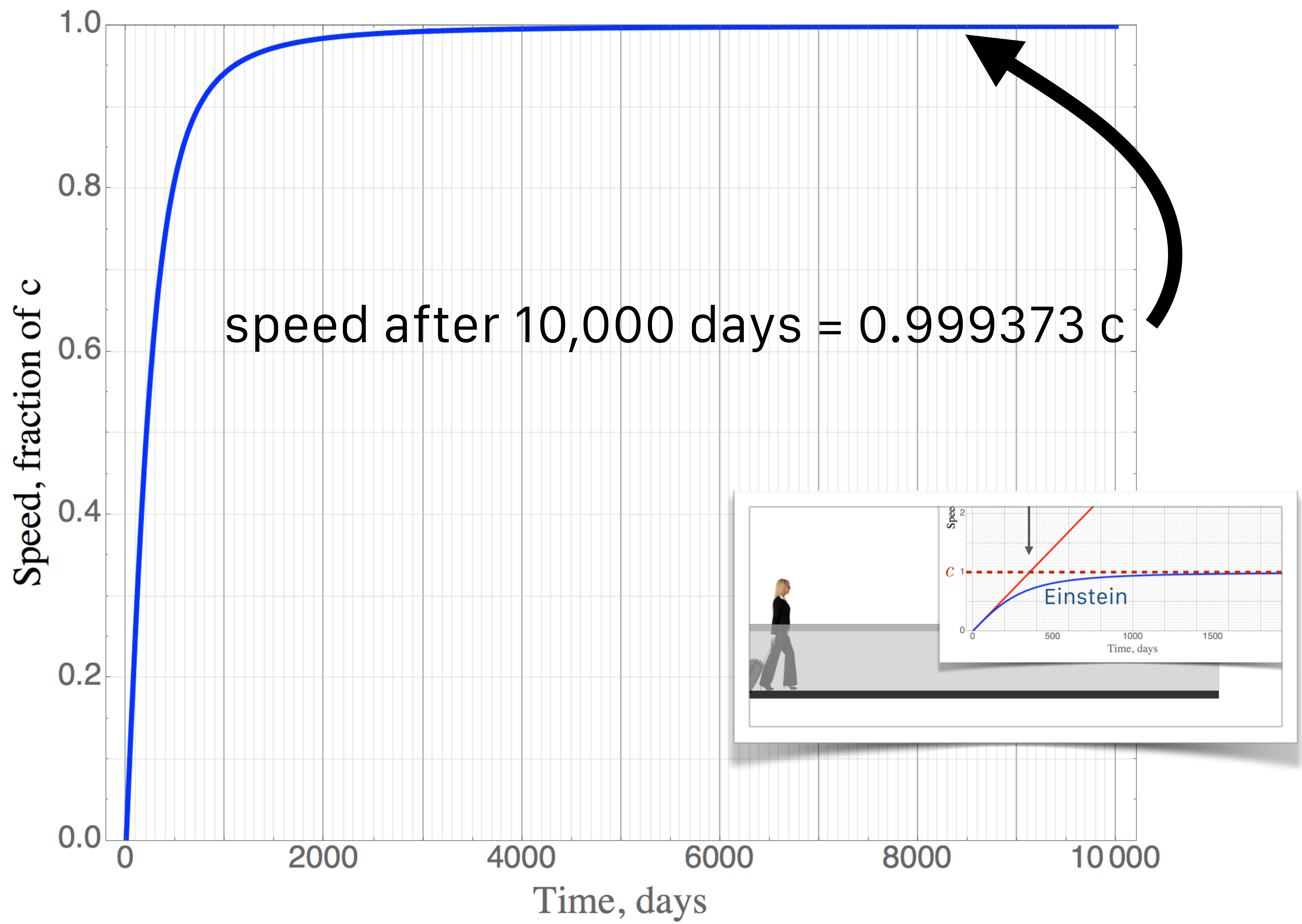
traveler transformation, 1g acceleration

speed not linear

in no frame can she be observed to go above c



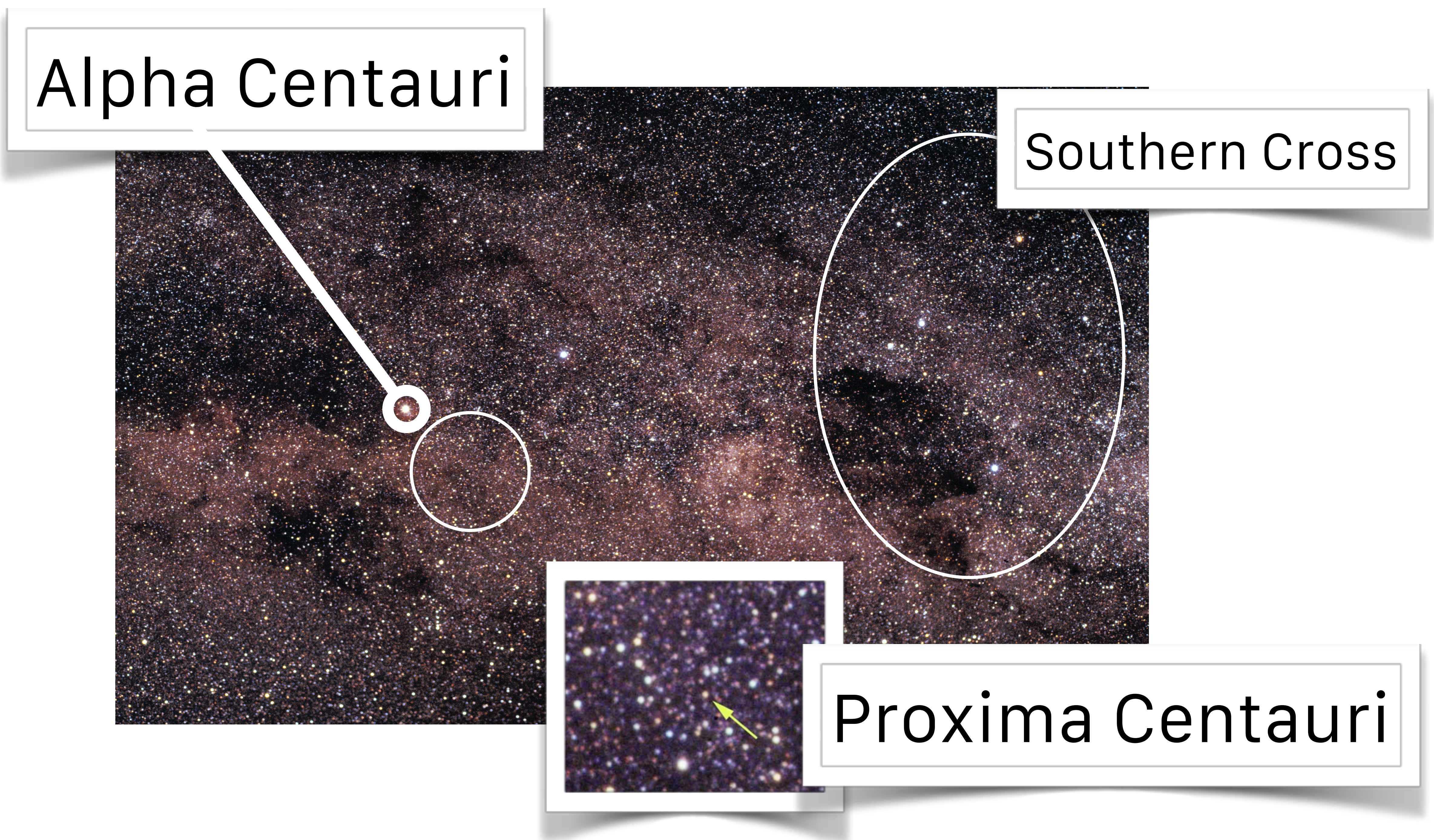
never get there

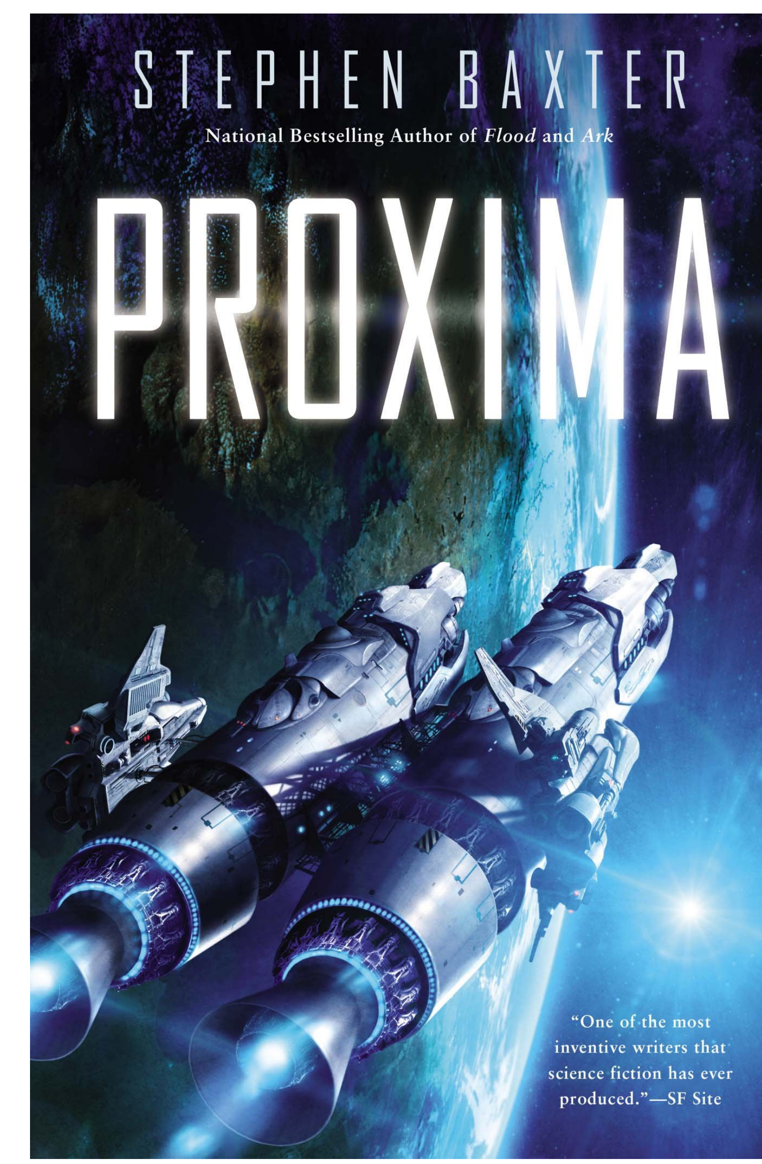


BTW

nearly every science fiction story ever

Closest star to Earth: Proxima Centauri: 4.23 light years



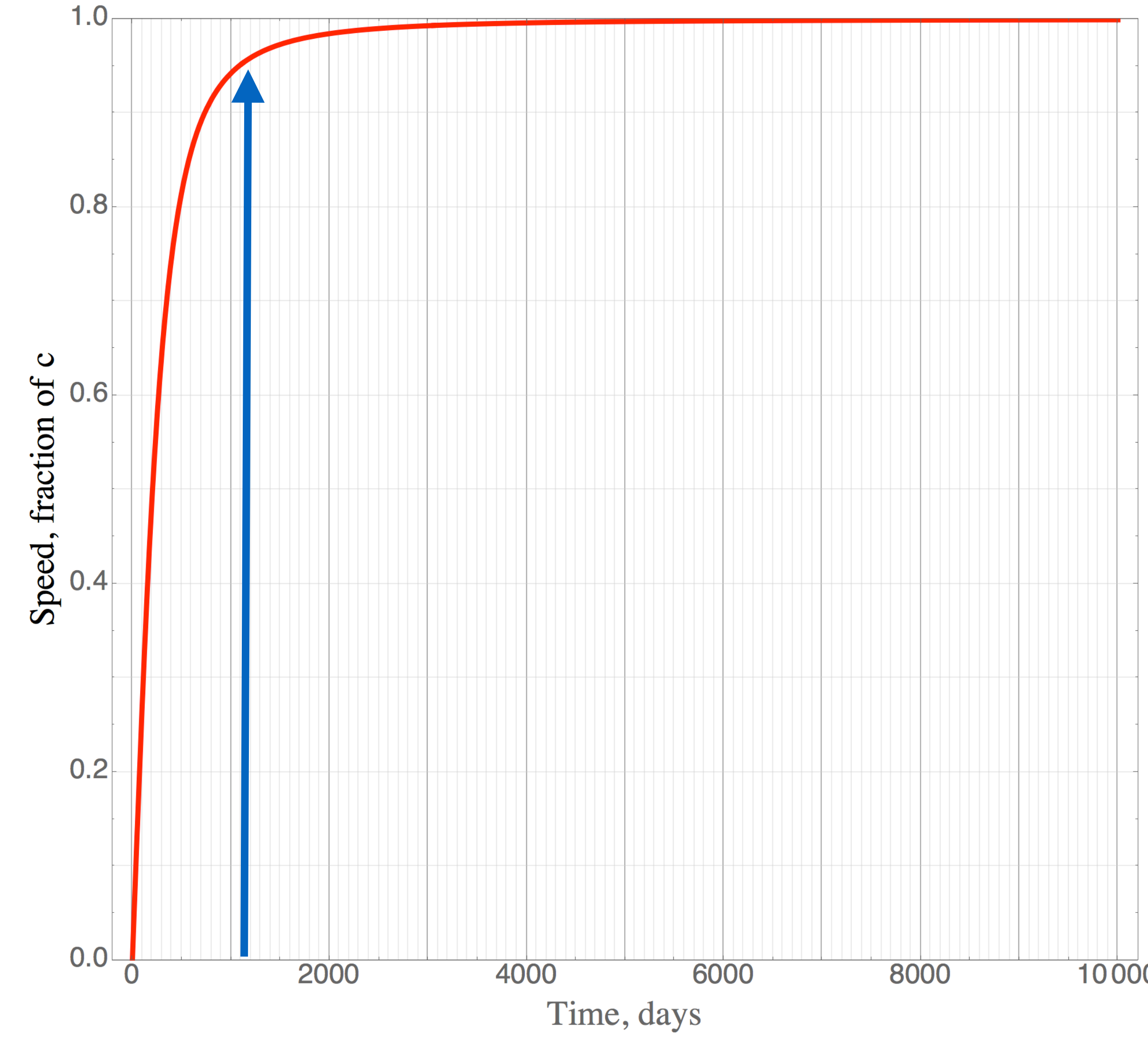


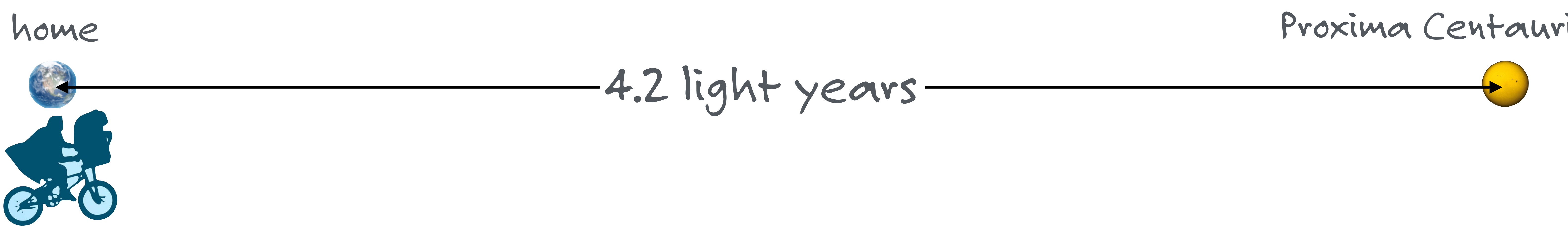
let's go

accelerate at 1g for 2 light years

cruise for 0.2 light years

decelerate at -1g for 2 light years



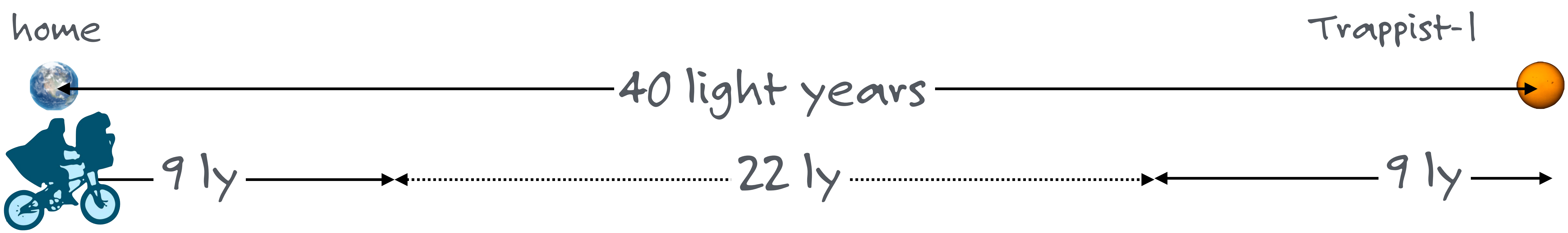


acceleration time, relative to Earth	2.8 years
top speed, relative to Earth	0.9453 c
acceleration time, relative to ship	1.7295 years
whole trip time, relative to Earth	5.8695 years
whole trip time, relative to ship	3.5428 years

let's go there

40 light years away…star is "Trappist-1" which is a dwarf

How about traveling there? Again, assume 1g acceleration

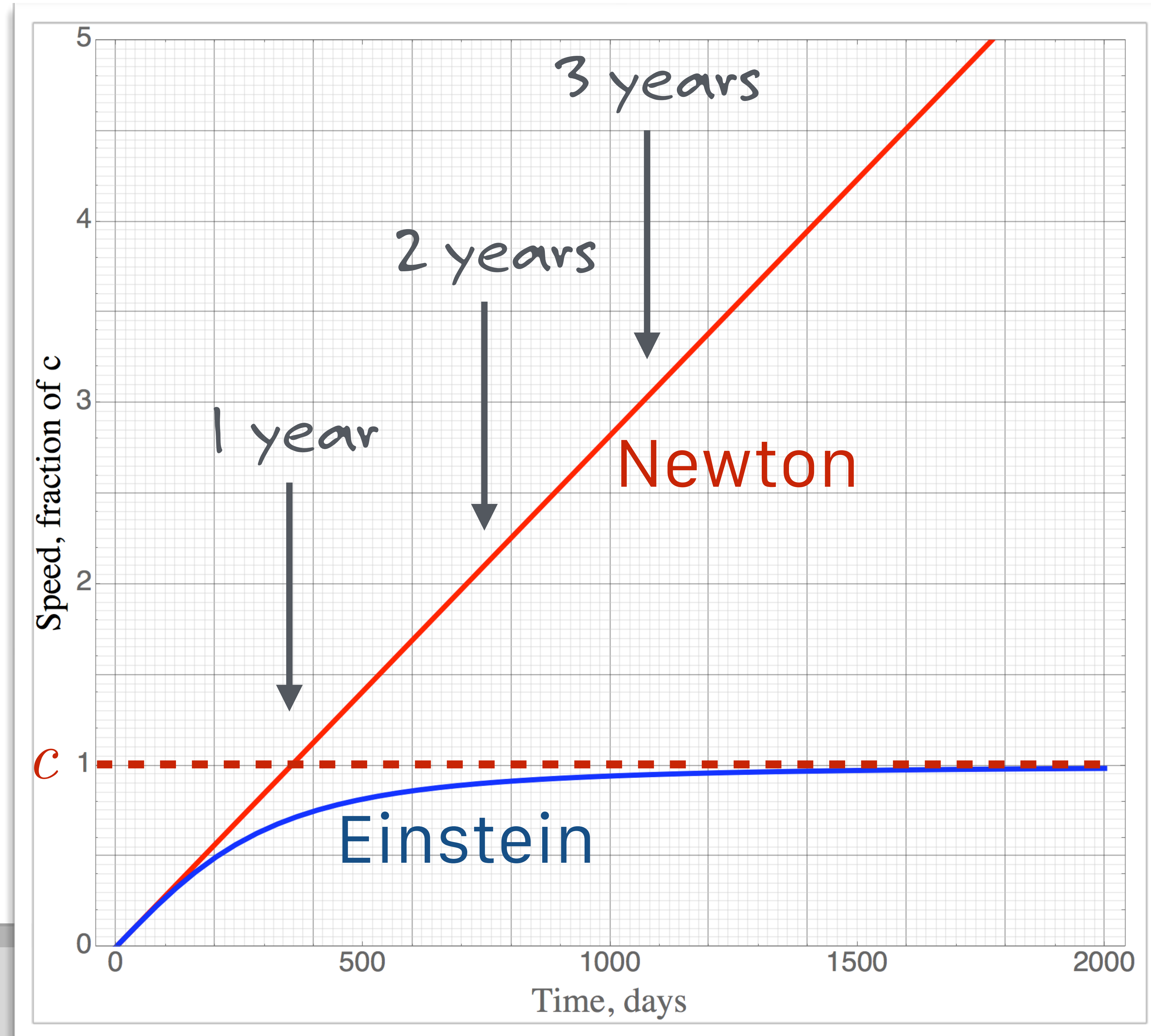


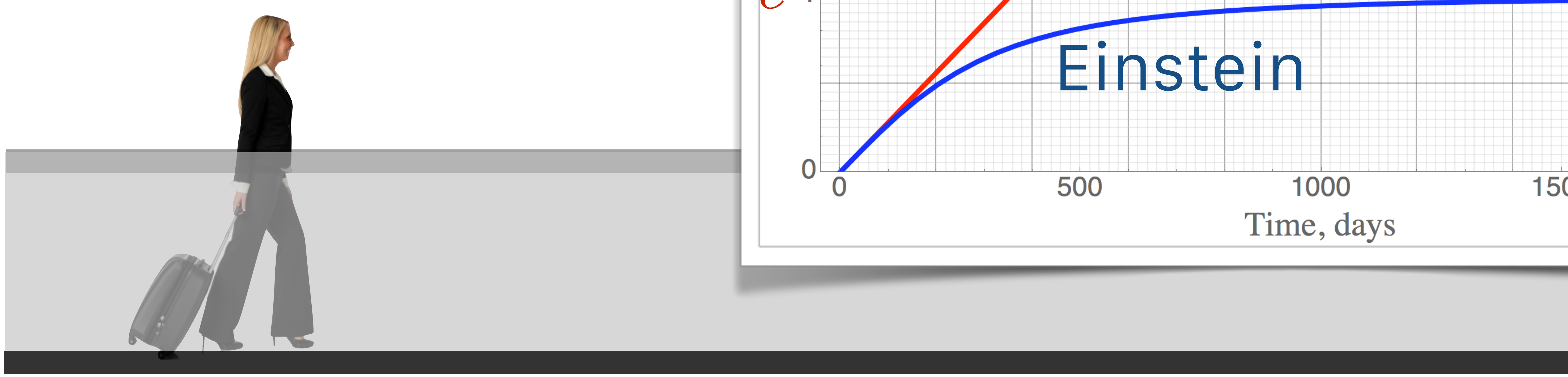
acceleration time, relative to Earth	9.9 years
top speed, relative to Earth	0.9953 c
acceleration time, relative to ship	2.97 years
whole trip time, relative to Earth	41.9 years
whole trip time, relative to ship	8 years

traveler transformation, 1g acceleration

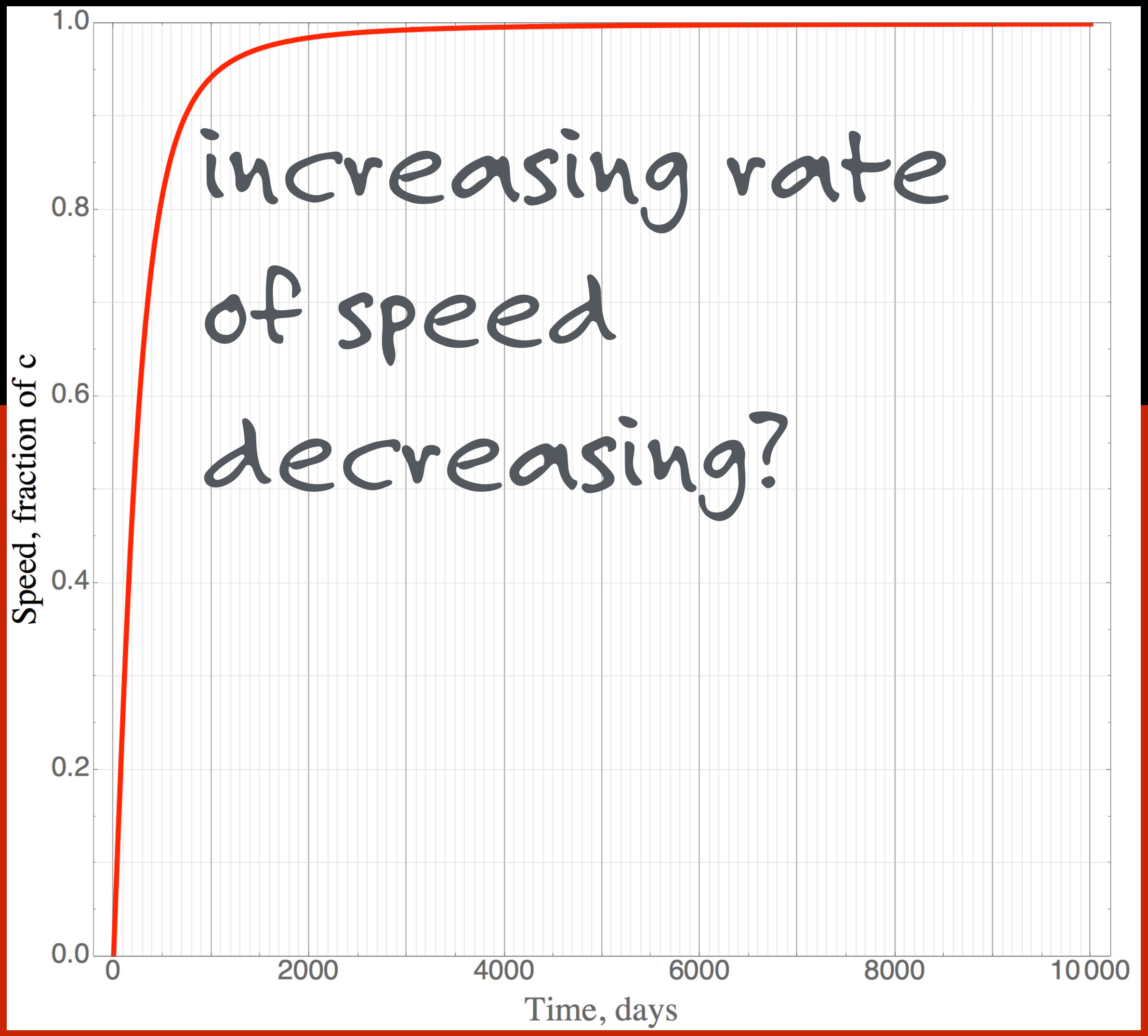
speed not linear

in no frame can she be observed to go above c





doesn’t this look like a reluctance to being accelerated?



Well.

What quantity is a measure of the reluctance to being accelerated?

Inertia.

If this reluctance increases...inertia seems to increase

and…what's the measure of a body's inertia?

mass

classical dynamical quantities

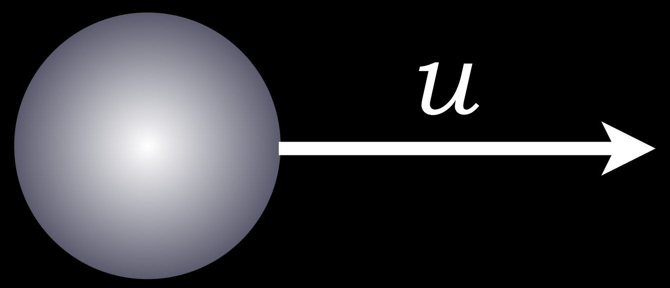
momentum, $p = mv$

Kinetic Energy, $K = 1/2mv^2$

and force $F = ma$

These have to change!

New, relativistic quantities reduce to these when u/c is very small



Momentum in relativity

got to be different from Newton

$$p_H = m\frac{\Delta x_H}{\Delta t_H}$$

want to preserve the idea of momentum conservation

Relativistic Momentum: $p = m\gamma u$

relativity and energy

through the back door…

there’s a “real” derivation, but too much mathematics

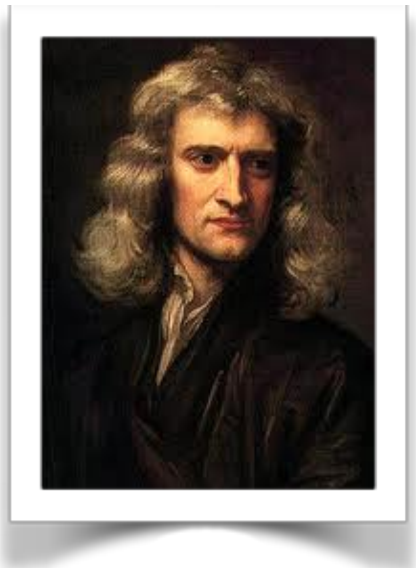
quick aside

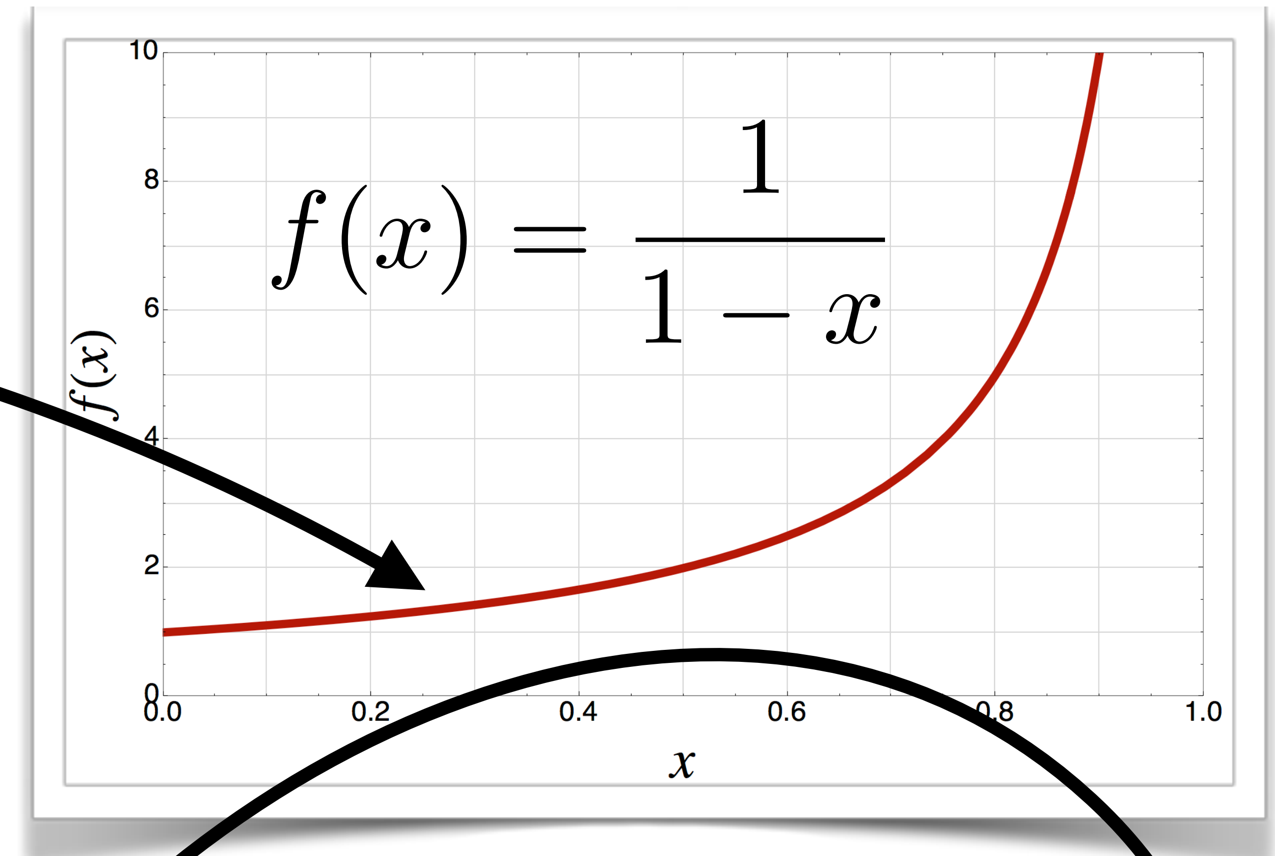
approximating functions

see manuscript math refresher chapter

somewhere in your life: the Binomial Series

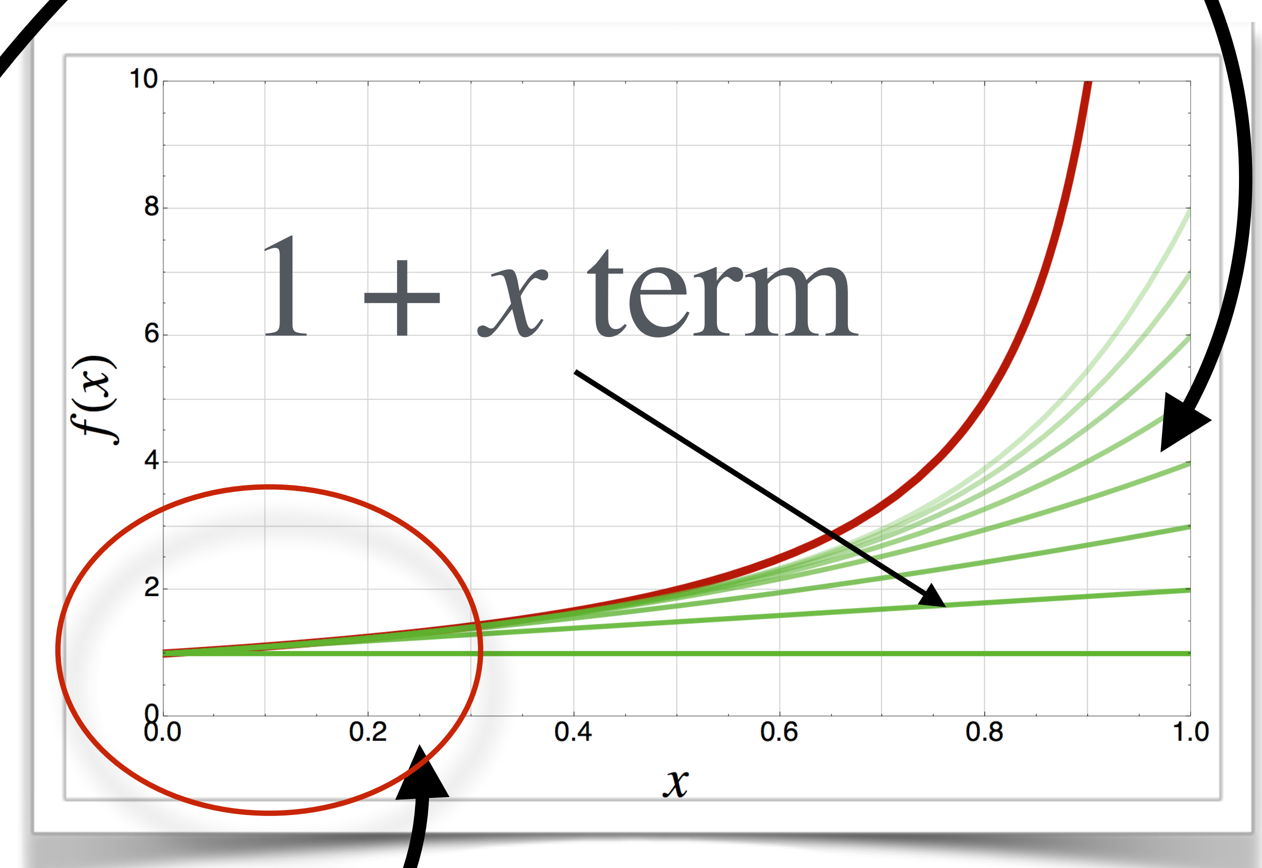
Binomial Series...useful to approximate functions.





$$f(x) = \frac{1}{1-x}$$

$$f(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + O\left(x^{11}\right)$$



Suppose that x << 1, then the function could be approximated by a couple of terms...

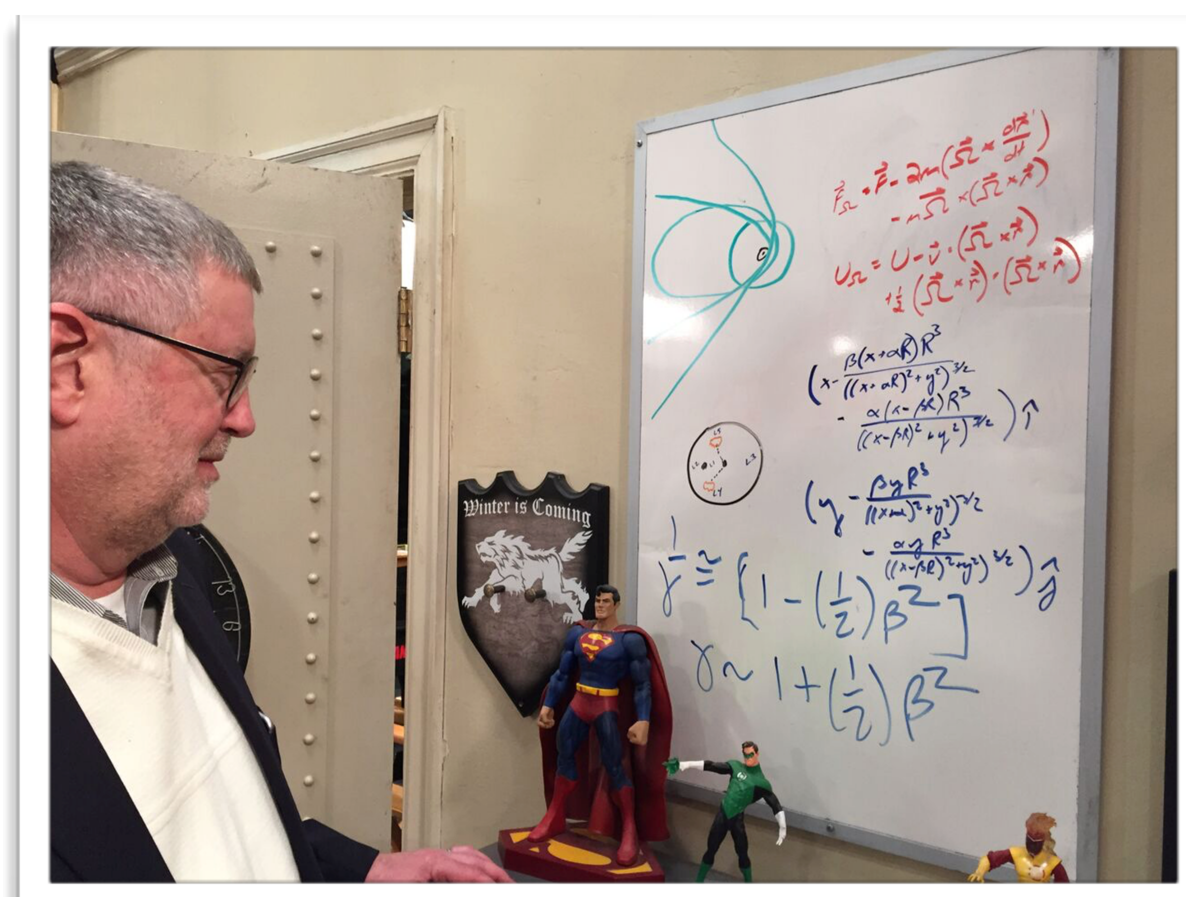
$$f(x) = \frac{1}{1-x} = 1 + x$$

what equation comes to mind?

when you're on the spot?

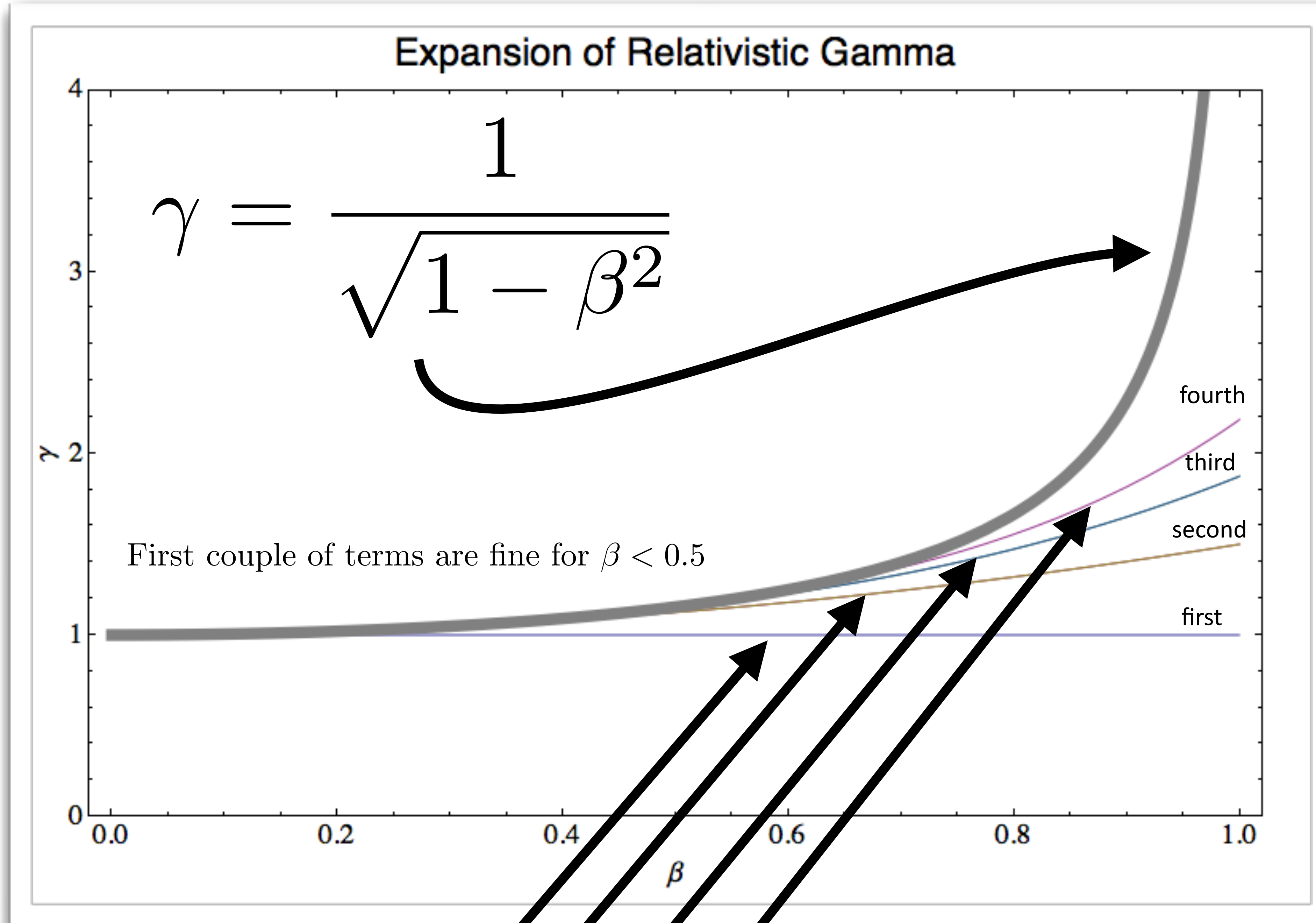
Why the binomial expansion of the relativistic gamma function, of course. Because, Relativity.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \sim 1 + \frac{\beta^2}{2} + \frac{3\,\beta^4}{8} + \frac{5\,\beta^6}{16} + \frac{35\,\beta^8}{128} + \frac{63\,\beta^{10}}{256} + \frac{231\,\beta^{12}}{1024} + \frac{429\,\beta^{14}}{2048} + O[\beta]^{15}$$

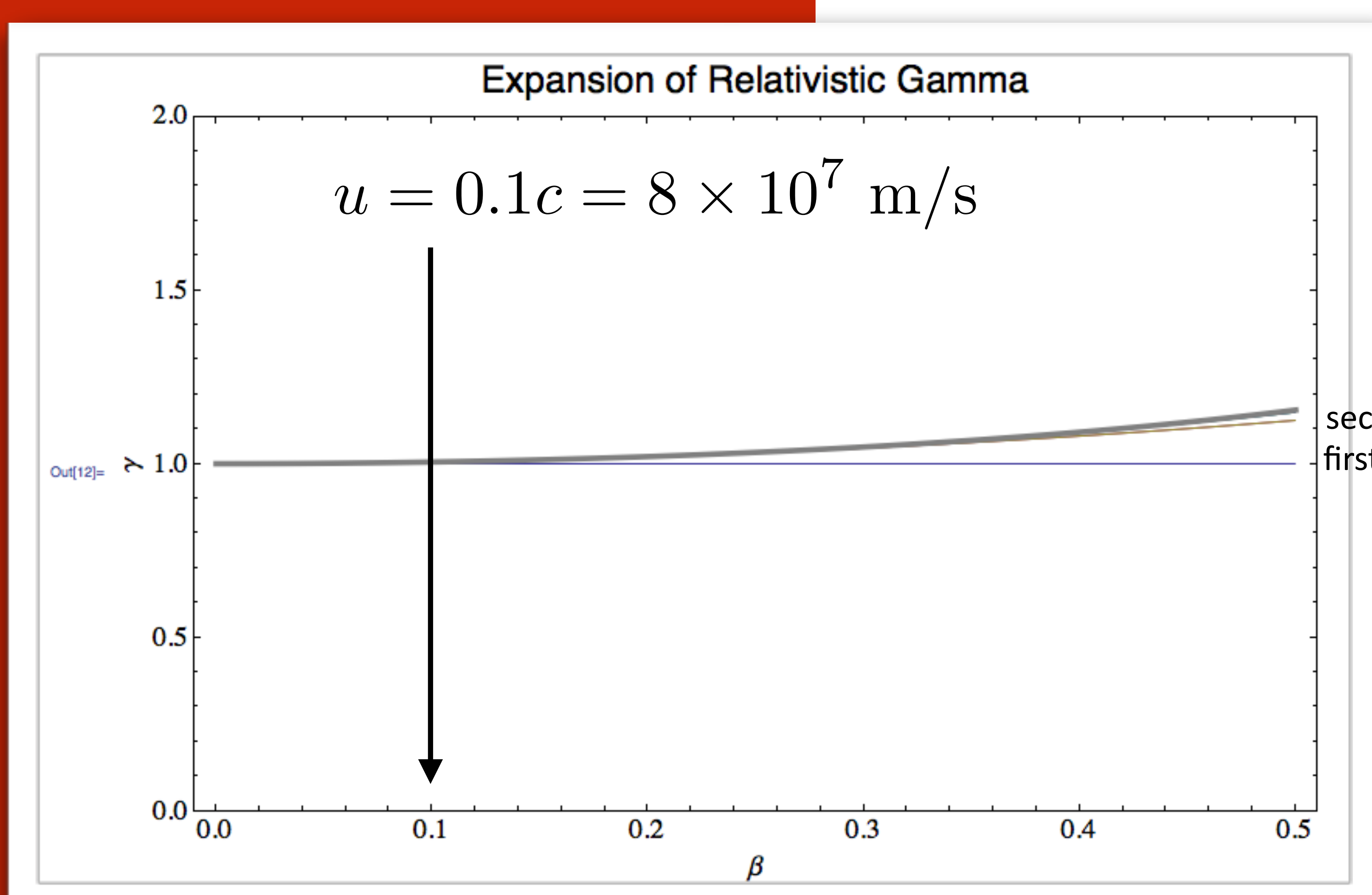


how well?

look at 8 terms in the expansion



$$\gamma \sim 1 + \frac{\beta^2}{2} + \frac{3\,\beta^4}{8} + \frac{5\,\beta^6}{16} + \frac{35\,\beta^8}{128} + \frac{63\,\beta^{10}}{256} + \frac{231\,\beta^{12}}{1024} + \frac{429\,\beta^{14}}{2048} + O[\beta]^{15}$$



Season 9, Episode 12

The Sales Call Sublimation, January 7, 2016

so let’s use this and look for familiar things

slow moving objects but not completely classical

$$\gamma \sim 1 + \frac{\beta^2}{2} + \frac{3\beta^4}{8} + \frac{5\beta^6}{16} + \frac{35\beta^8}{128} + \frac{63\beta^{10}}{256} + \frac{231\beta^{12}}{1024} + \frac{429\beta^{14}}{2048} + O[\beta]^{15}$$

sing along

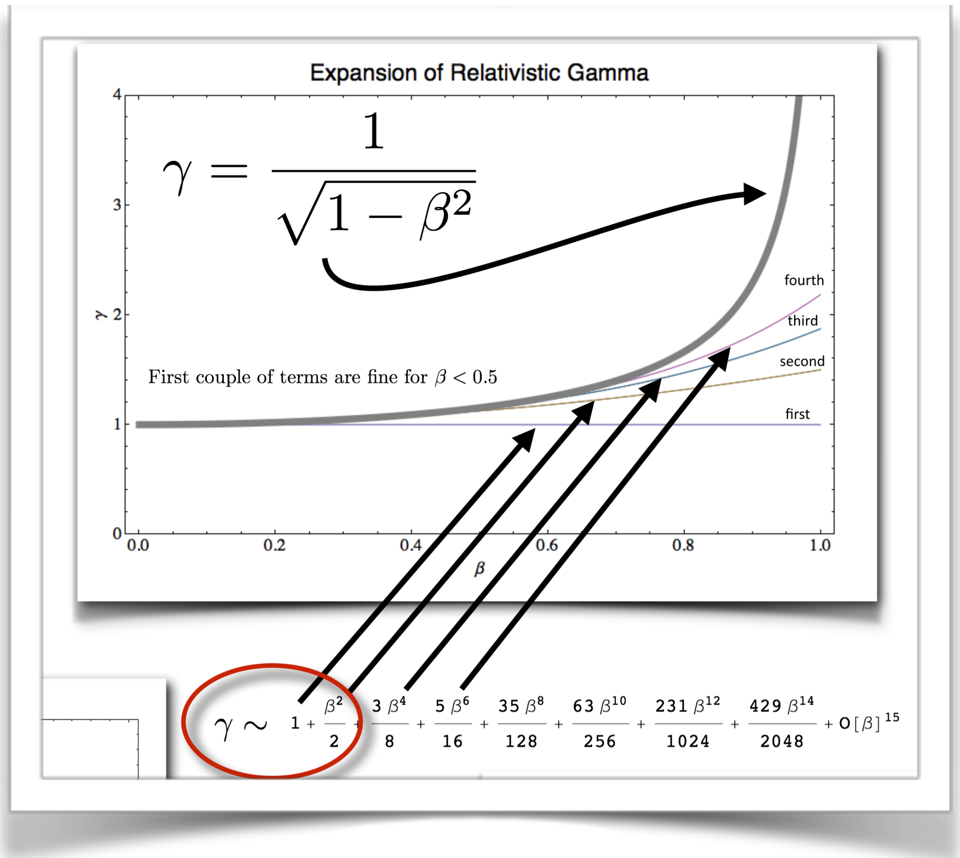
for β small:

now copy the approximate forms, but insert $\beta = u/c$

$$\gamma \sim 1 + \left(\frac{1}{2}\right)\beta^2$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

now, write along with me:



sing along

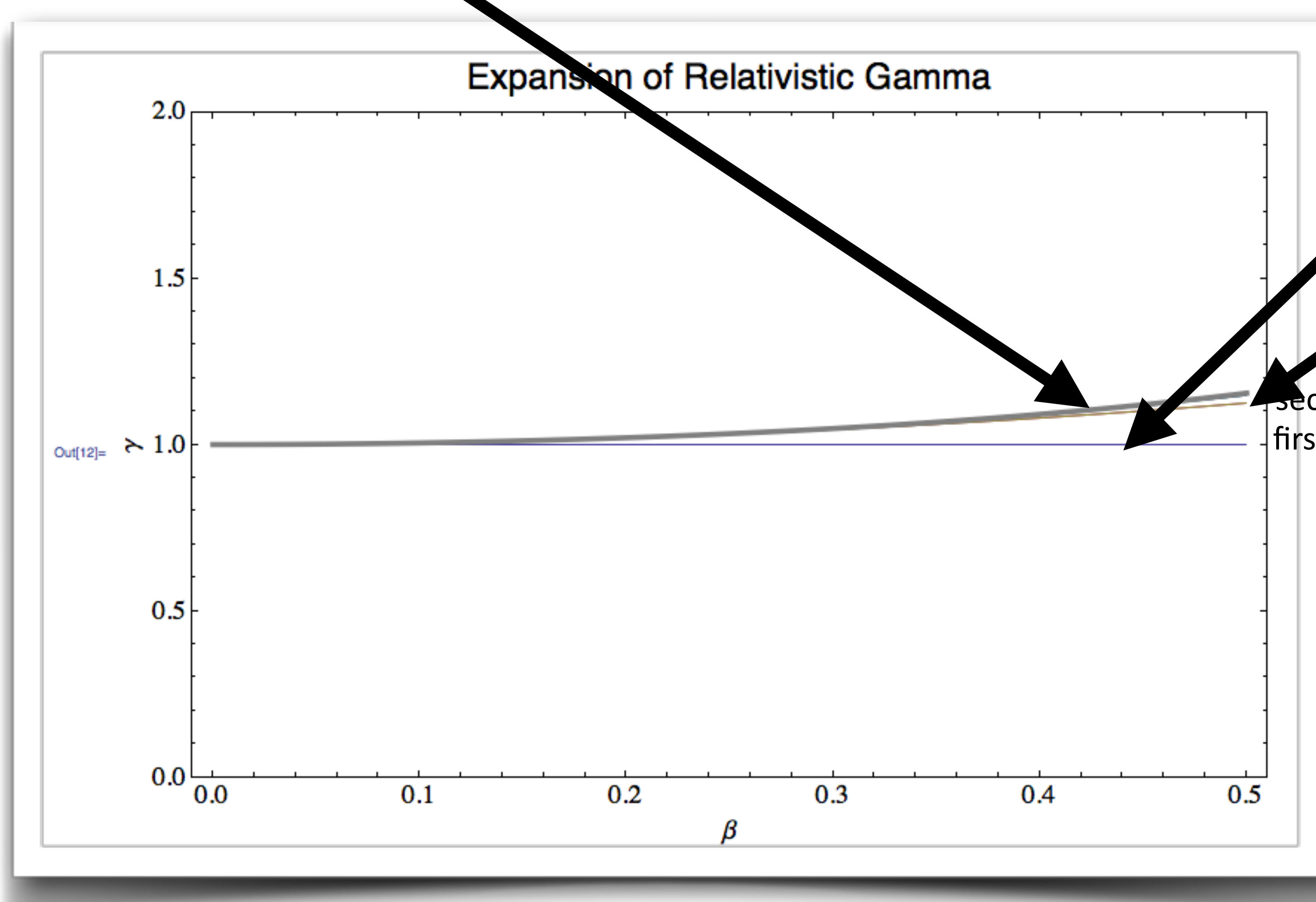
now copy the approximate forms, but insert $\beta = u/c$

for β small:

$$\gamma \sim 1 + \left(\frac{1}{2}\right)\beta^2$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\gamma \sim 1 + \left(\frac{1}{2}\right)\frac{u^2}{c^2}$$



BTW: escape velocity = 11,000 m/s

$$\beta = \frac{v}{c}$$

$$\beta = \frac{11,000}{3 \times 10^8} = \frac{1.1 \times 10^4}{3 \times 10^8}$$