hi

Day 14, 28.02.2019

Einstein’s Special Theory of Relativity, 4

28 days until opening day

Dirty Dancing week

housekeeping

Gotta come to class

- question about <u>anything</u>?
- I'll make a movie for you:

Midterm...closed, average about 85%

Madame Curie movie - we have a quorum in favor

- I've posted another FB poll targeting the 2 weeks after break
- right now: looks like Monday, March 18

Next readings:

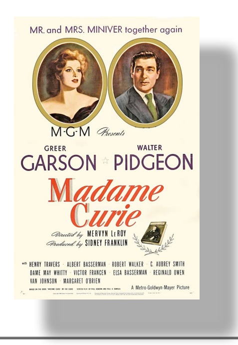
- Cosmic Perspective (aka "CP") in MasteringAstronomy
- likewise HW 6

BTW: I gave everyone credit for the Kepler3rd problem from waaaay back

February 2019

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
27	28	29 yadda yadda yadda	30	31	1	2 HW2
3	4	5 lessons 10,11,12	6	7 lesson 13	8 HW2 due	9 HW3
10	11	12 lecture	13	14 lecture	15 HW3 due	16 HW4
17	18	19 lecture	20	21 lecture	22 HW4 due	23 HW5
24 ←	25 midterm	26 → lecture	27	28 lecture	1 HW5 due	2

March 2019

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
24	25	26	27	28	1	2
3	4 ←	5	6 spring break	7	8 →	9 HW6
10	11	12 lecture	13	14 lecture	15 HW6 due	16 HW7
17	18  ?	19 lecture	20	21 lecture	22 HW7 due	23 HW8
24	25	26 lecture	27	28 lecture	29 HW8 due	30 HW9
31	1	2 lecture	3	4 lecture	5 HW9 due	6

public lecture tonight

The Nobel Prize in Physics 2015

© Nobel Media AB. Photo: A. Mahmoud

Takaaki Kajita

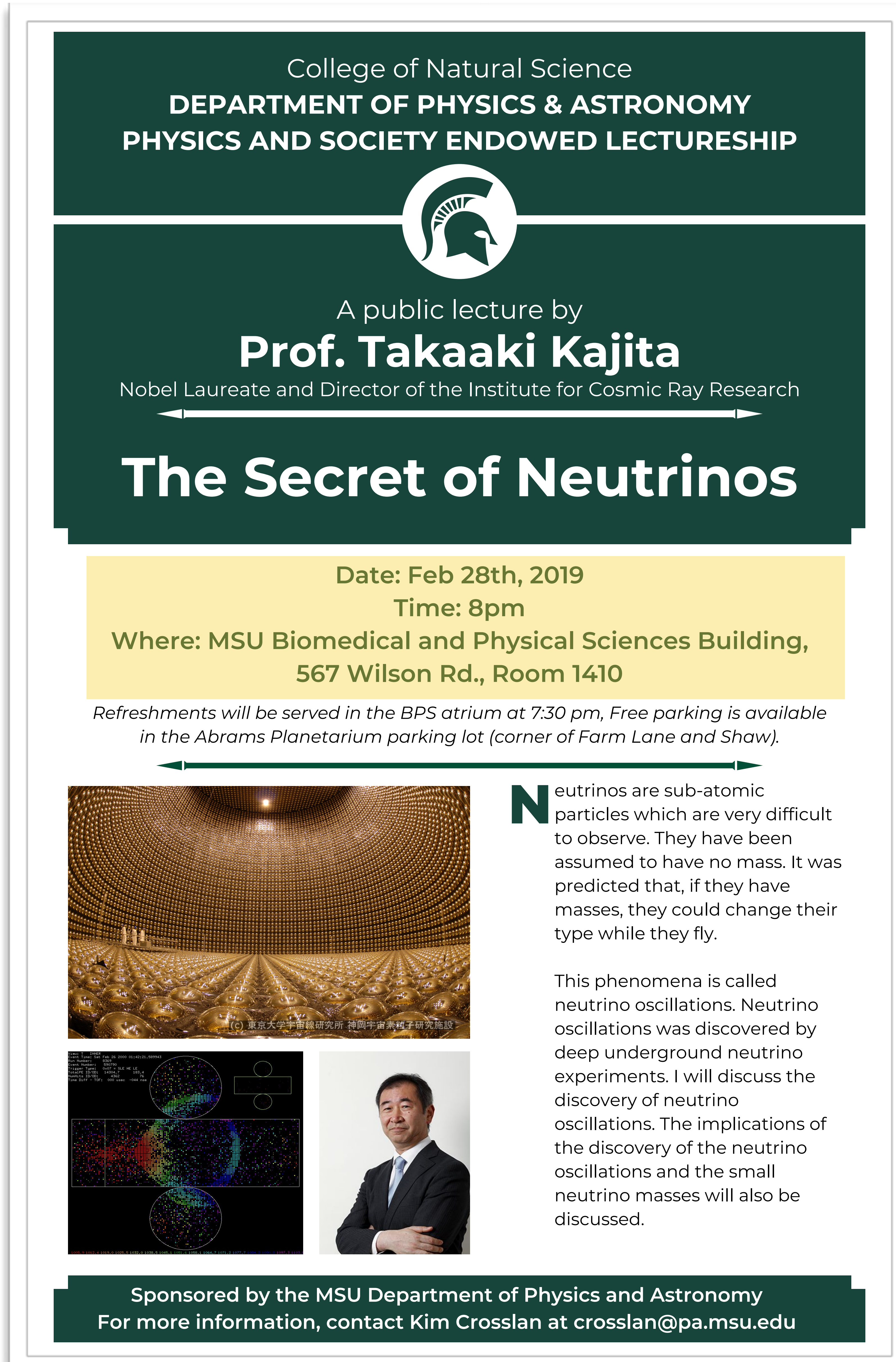
Prize share: 1/2

© Nobel Media AB. Photo: A. Mahmoud

Arthur B. McDonald

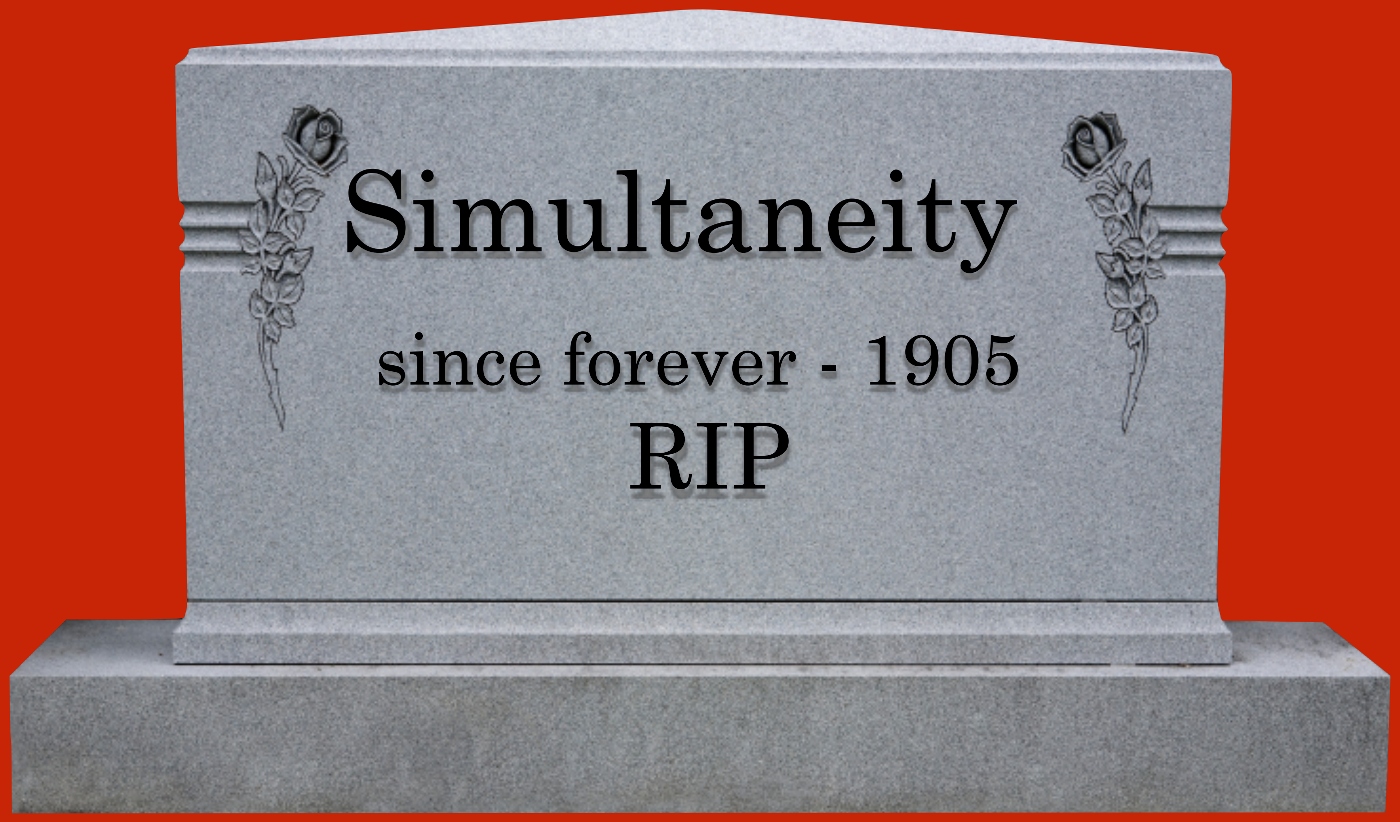
Prize share: 1/2

The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita and Arthur B. McDonald "for the discovery of neutrino oscillations, which shows that neutrinos have mass."



KEEP
CALM
AND
LET’S
REVIEW

There is no such thing as the *concept of* simultaneous events



cannot synchronize clocks between frames

collecting these two consequences

of the two simple postulates

"Time Dilation": $$t_H = \gamma t_A$$

Moving clocks appear to run slower as seen by a relatively stationary observer

"Length Contraction": $$L_H = \frac{L_A}{\gamma}$$

Moving lengths appear shorter to a relatively stationary observer

You can find 2 movies:
timeDilation_reduced.mp4
lengthContraction_reduced.mp4

the original problems are solved:

A **magnetic field** in one frame

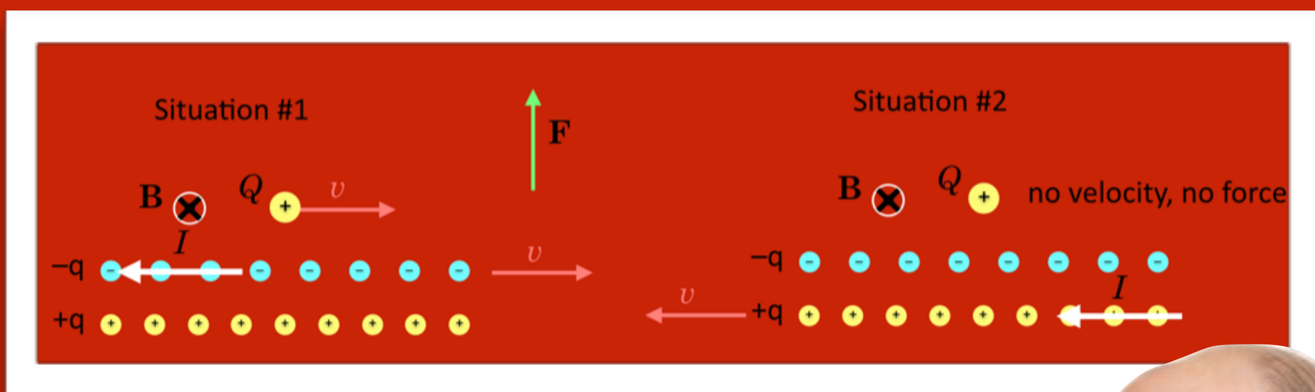
is a **mixture of magnetic and electric fields** in another frame

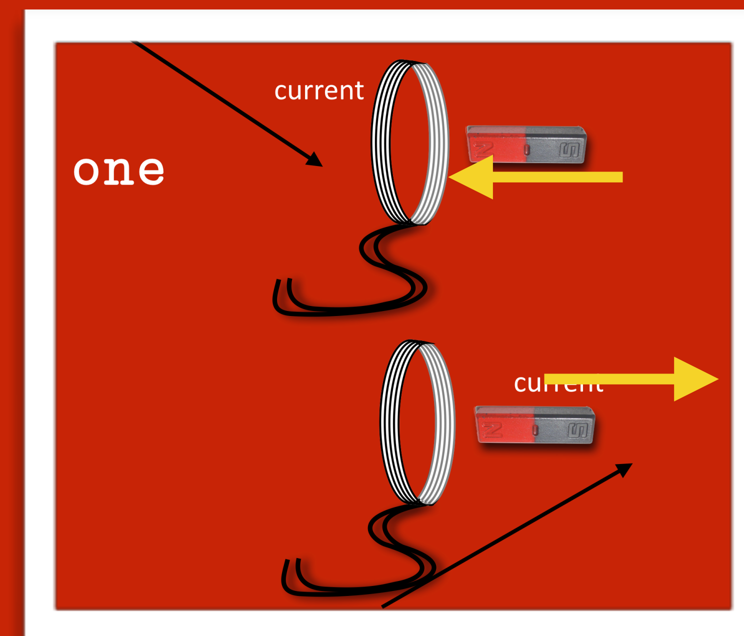
An **electric field** in one frame

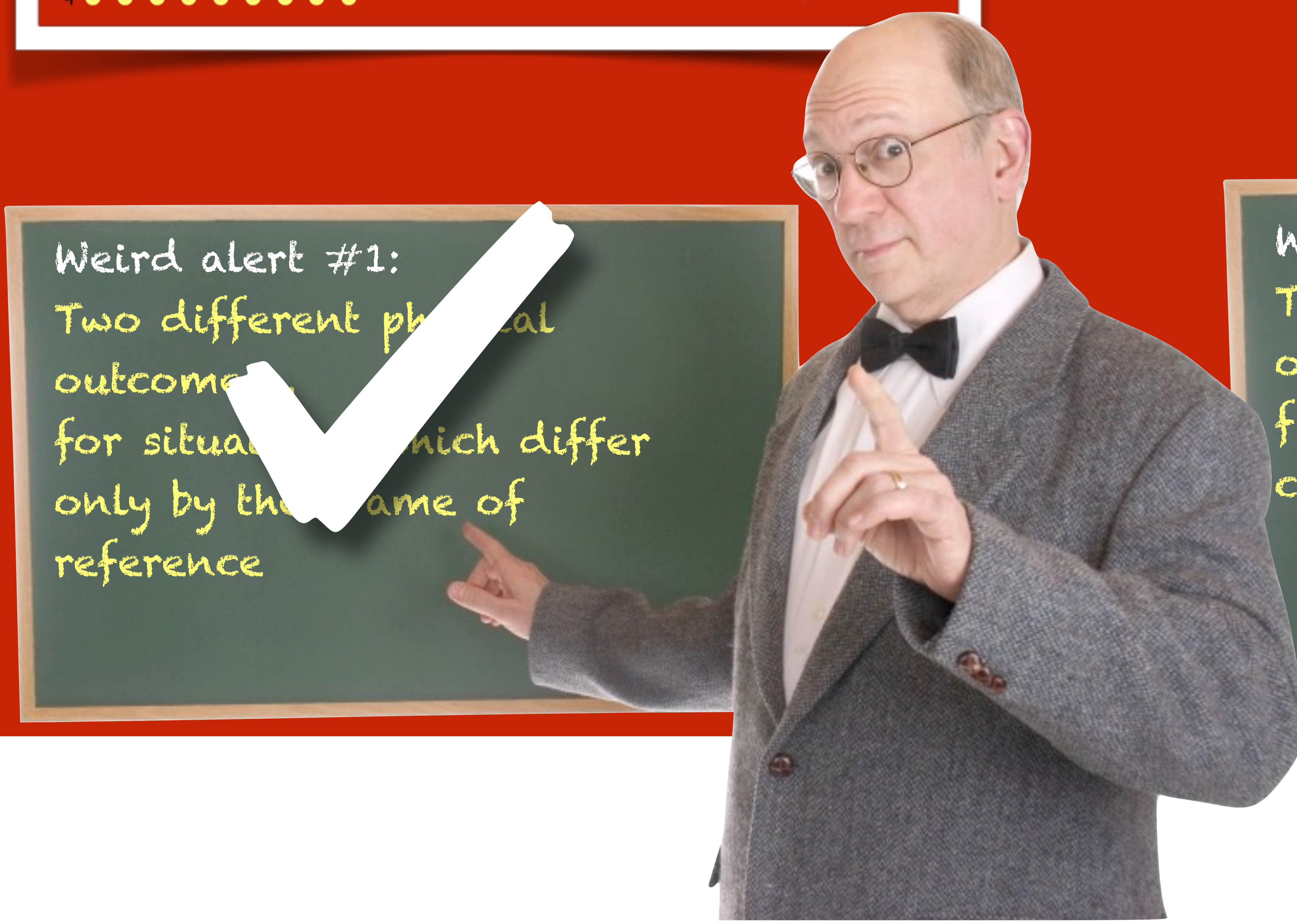
is a **mixture of electric and magnetic fields** in another frame

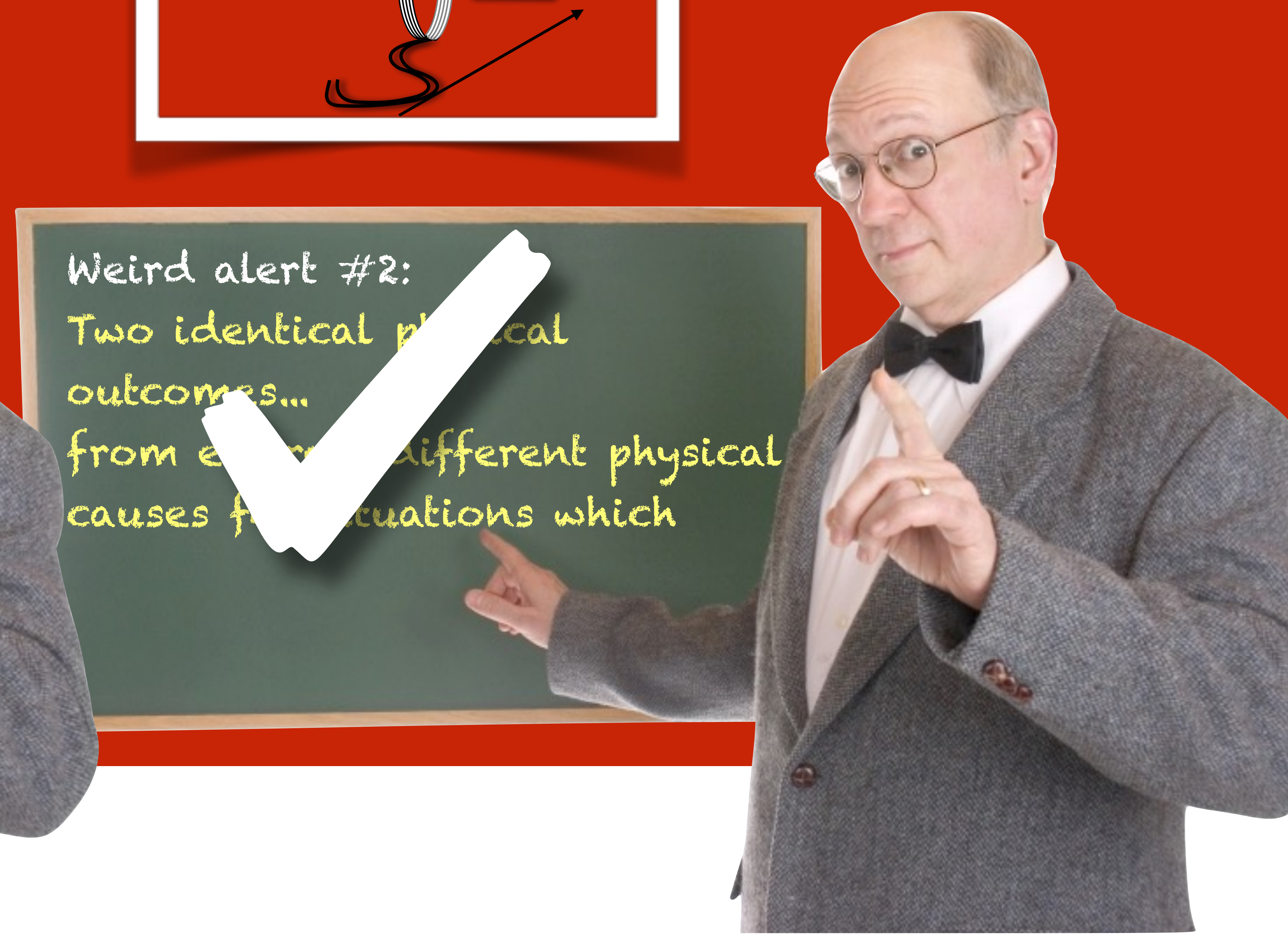
Electromagnetic Fields are the reality

remember?


Situation #1
F
B
Q
v
I
−q
+q
v
Situation #2
B
Q
no velocity, no force
−q
+q
v
I


current
one
current


Weird alert #1:
Two different ph cal
outcome
for situa hich differ
only by th ame of
reference


Weird alert #2:
Two identical p ical
outcomes...
from e different physical
causes f uations which

Principle of Relativity

2 Postulates:

1. All laws of physics – **mechanical and electromagnetic** – are identical in co-moving inertial frames.

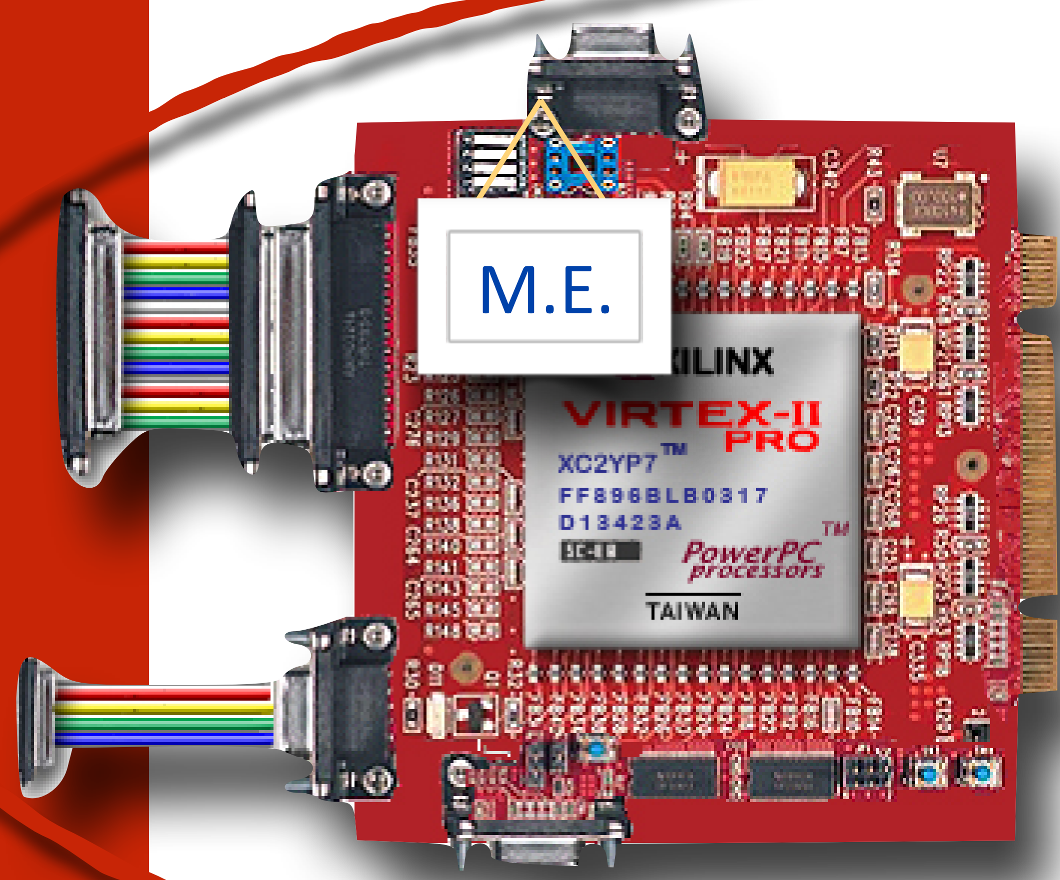
2. The speed of light is the same for all inertial observers.

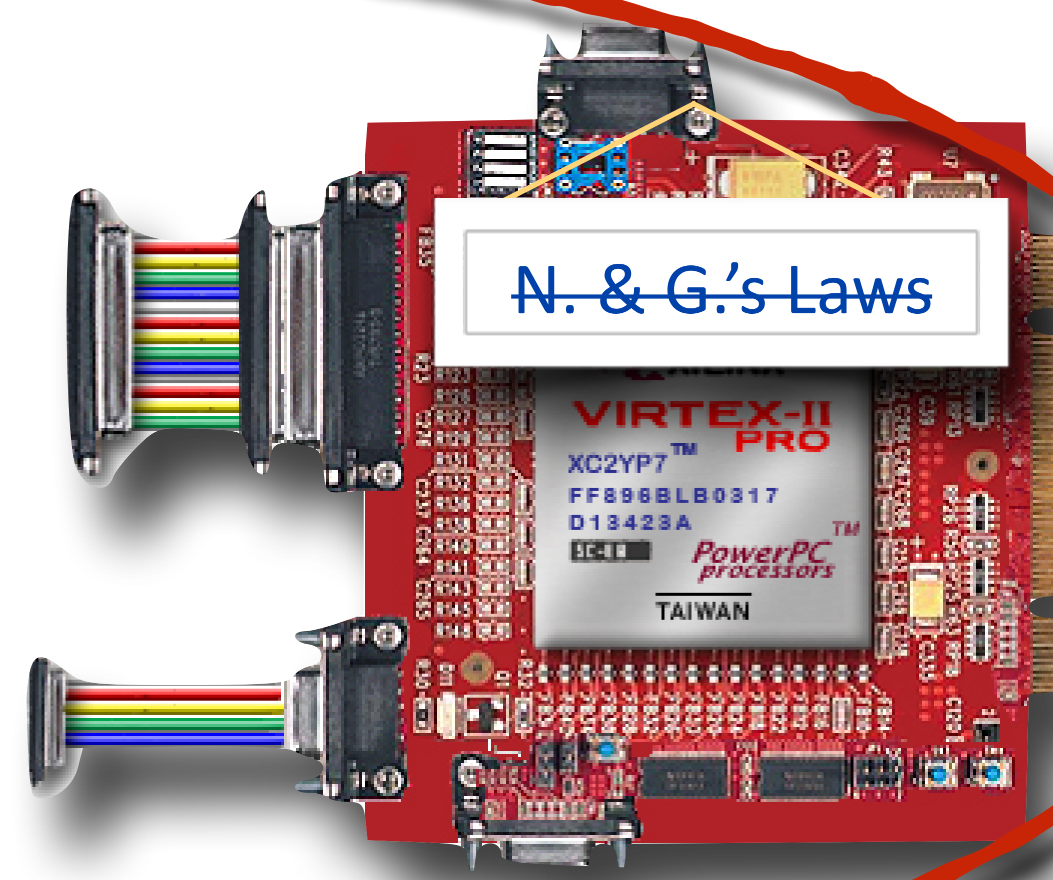
"inertial frame":

constant velocity

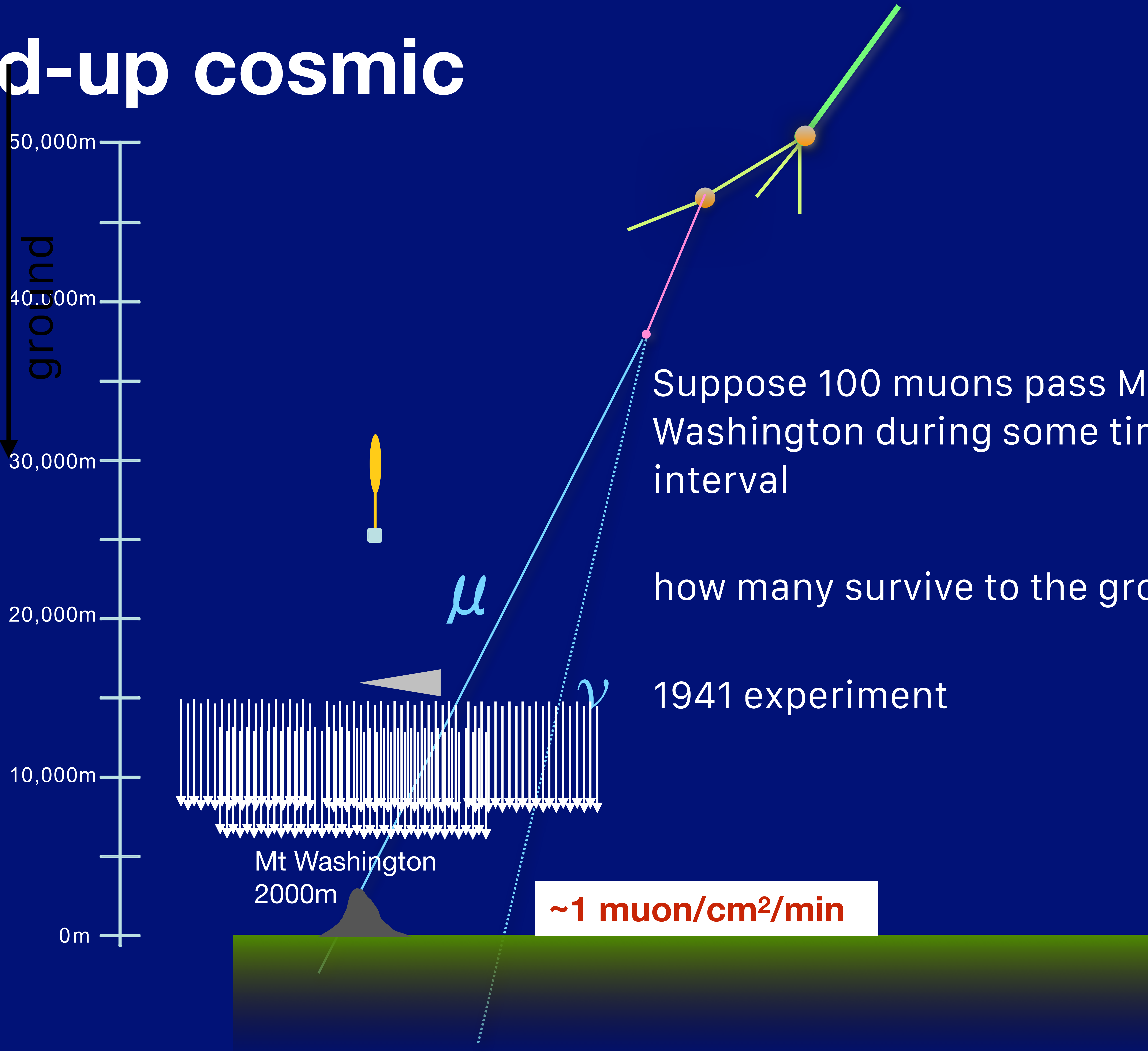
good all along!

had to change!





stand-up cosmic



Suppose 100 muons pass Mt Washington during some time interval

how many survive to the ground?

1941 experiment

decay or not?

the muon lifetime is a clock

which we (Home on earth for us) see running slowly

so it makes it to earth

which the muon (Away in its frame) sees running normally

and yet how can it makes it to earth??

The atmosphere is a ruler

which we (Away for muon) see as 50,000 meters thick

which the muon (Home in its frame) sees shrunk

so it makes it to earth

This has been measured many times:

an atomic clock was carefully carried around the world in 1972 and carefully calibrated and compared with ground-based clocks

J. Hafele and R. Keating

There are a number of corrections: accelerations, decelerations, the rotation of the orbit, the fact that the earth is not inertial - but relativity was absolutely correct

Predicted Effect	Flying East	Flying West
GTR (Gravitation)	+ 144 ± 14 ns	+ 179 ± 18 ns
STR (Velocity)	- 184 ± 18 ns	+ 96 ± 18 ns
Total	- 40 ± 23 ns	+ 275 ± 21 ns
measured:	- 59 ± 10 ns	+273 ± 7 ns

redone twice more in airplanes and rockets/satellites

Principle of Relativity

2 Postulates:

"inertial frame":

1. All laws of physics — mechanical and electromagnetic — are identical in co-moving inertial frames.

taking Galileo seriously, and then adding Maxwell

2. The speed of light is the same for all inertial observers.

taking Maxwell seriously

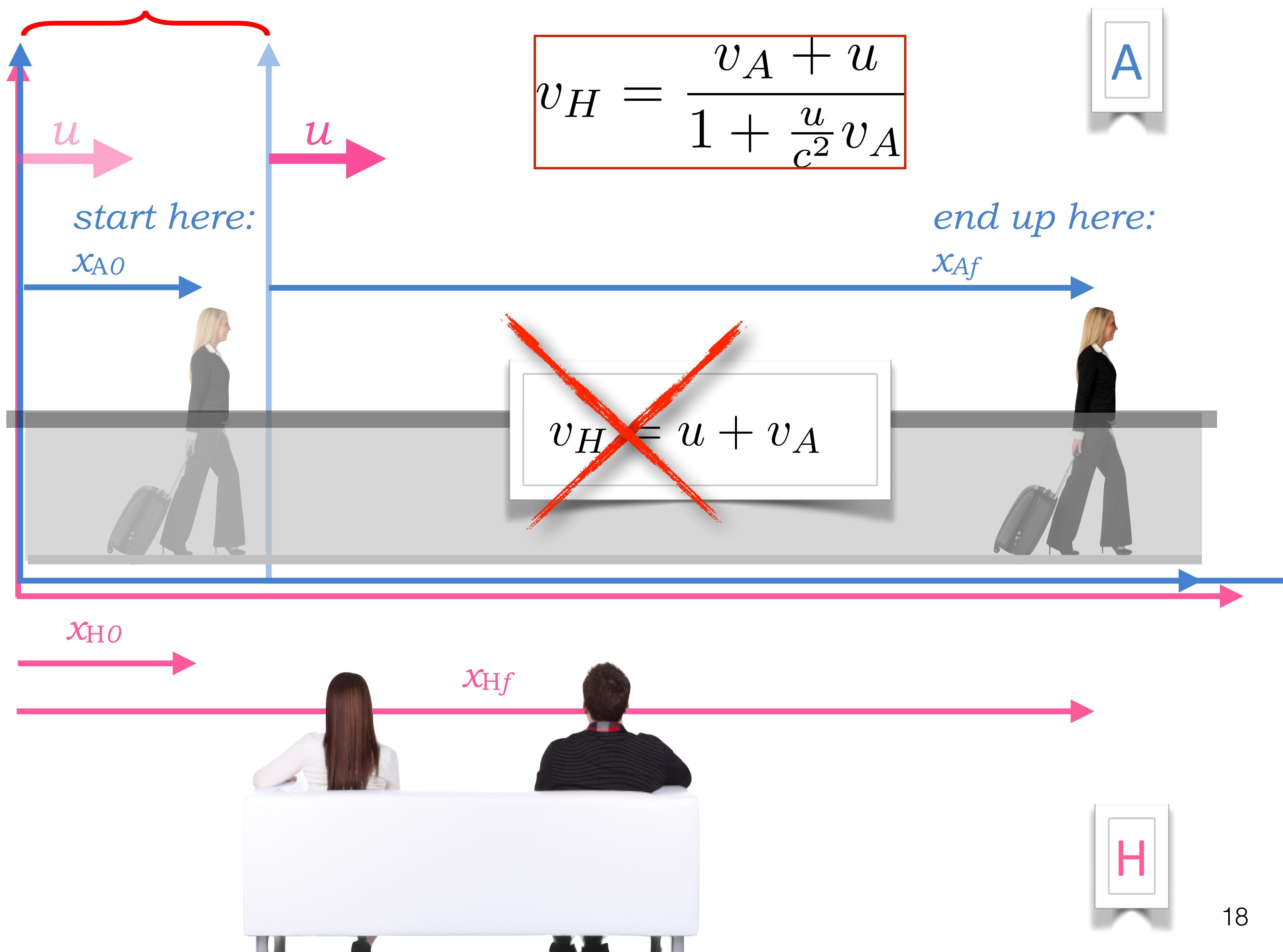
a fact of nature

combine speeds!

Galileo, nope.

Einstein, yup.

the airport, going fast



$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

$$\cancel{v_H = u + v_A}$$

be careful

There are 3 velocities going on here.

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

u is the frame velocity (sidewalk)

...same, A relative to H or H relative to A

v_A is the velocity (traveler walking on sidewalk)

of something measured relative to the A frame

v_H is the velocity (traveler observed walking on sidewalk)

of something measured relative to the H frame

a pion decays into a muon

the pion travels at $u = 0.5c$ in the lab (H)

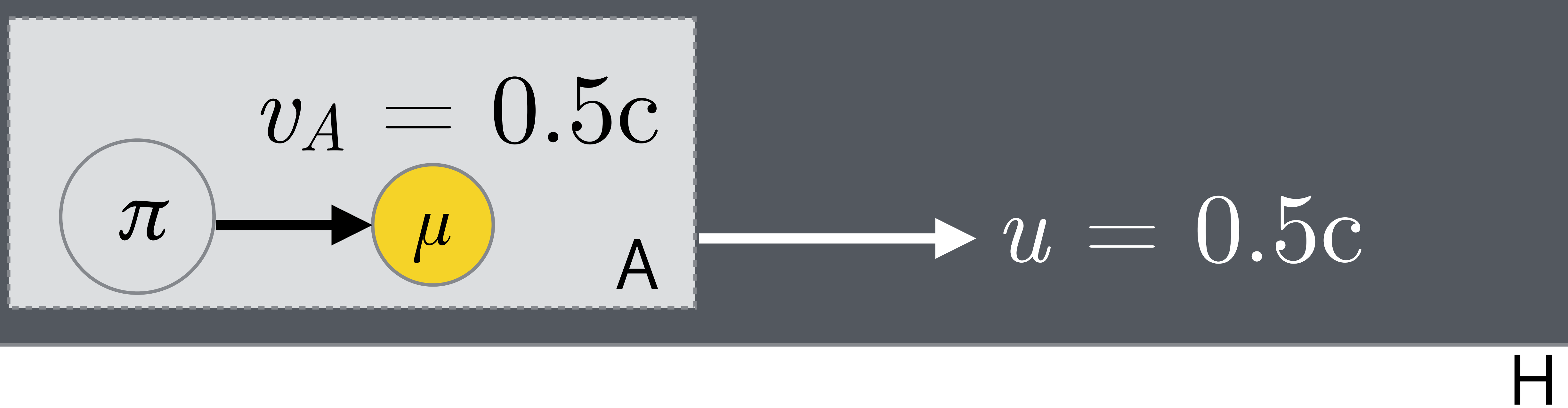
the muon travels right at $v_A = 0.5c$ in the pion's rest frame

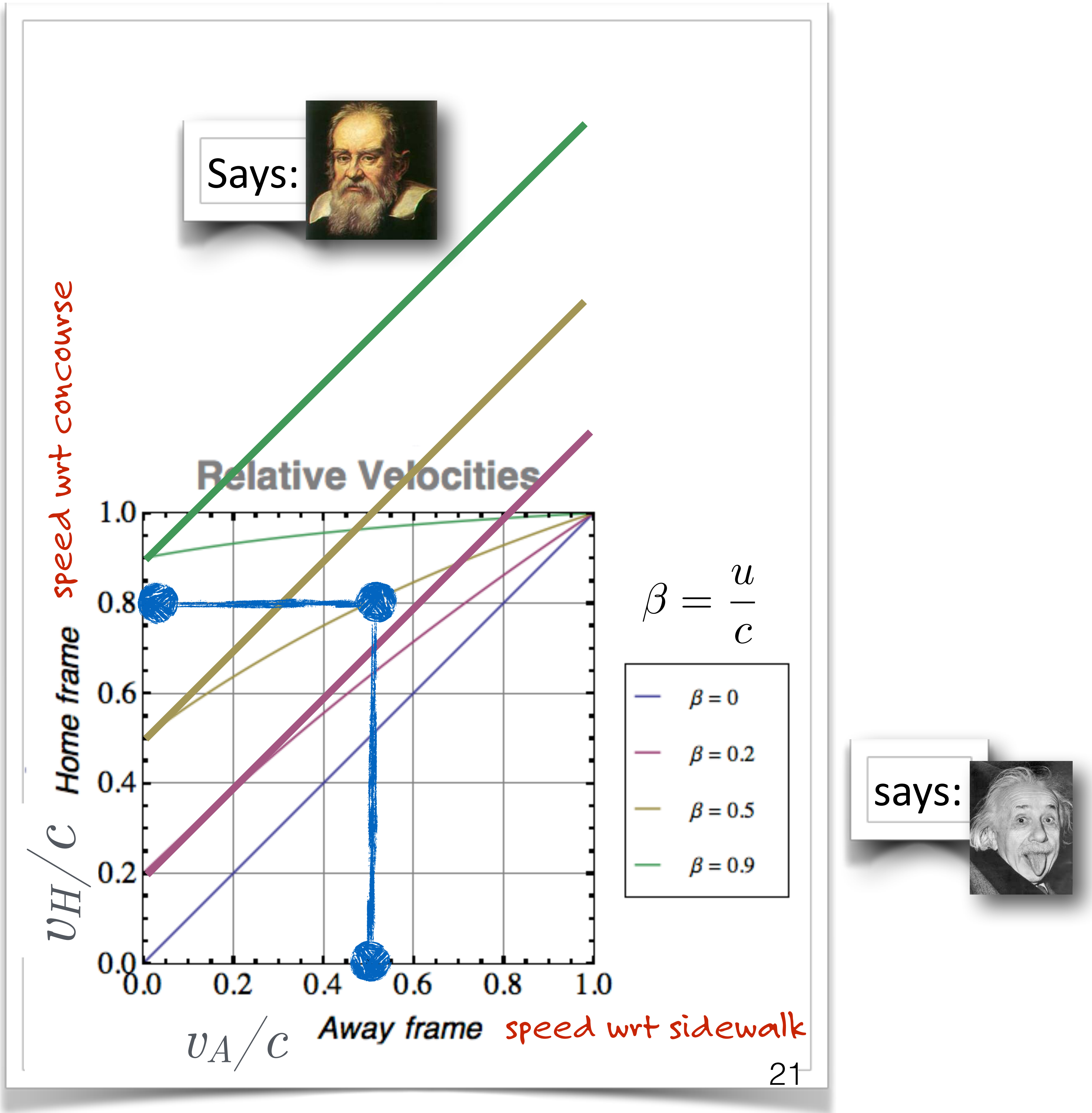
What is the speed of the muon in the lab?

How far does it travel in the lab before decaying?

What is the speed if muon travels left at $v_A = -0.5c$ in the pion's rest frame?

What if the muon travels left at $v_A = -0.75c$ in the pion's rest frame?

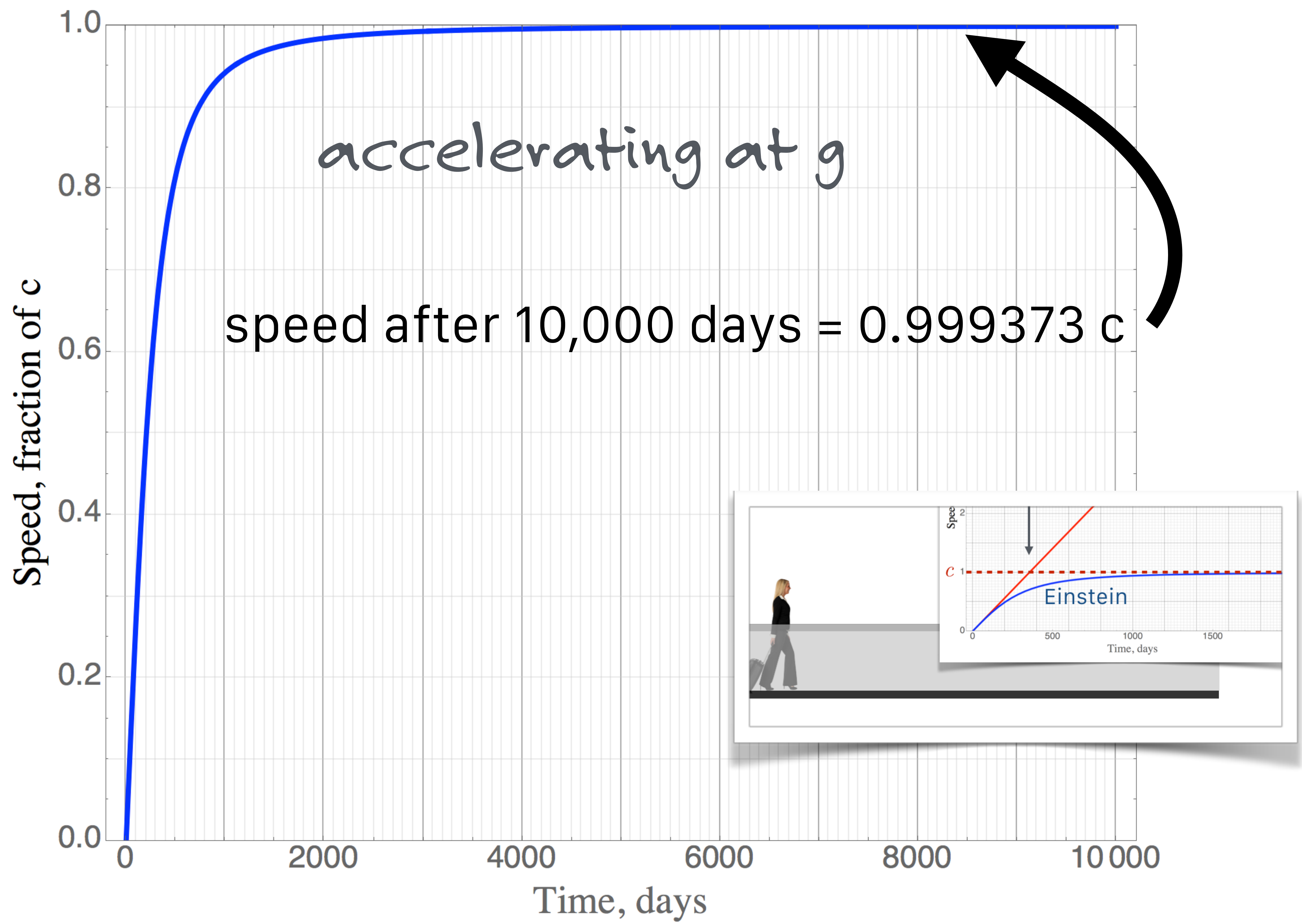




Says:
Relative Velocities
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
v_H/c
Home frame
speed wrt concourse
v_A/c
Away frame
speed wrt sidewalk
$\beta = \frac{u}{c}$
$\beta = 0$
$\beta = 0.2$
$\beta = 0.5$
$\beta = 0.9$
says:
21

Energy

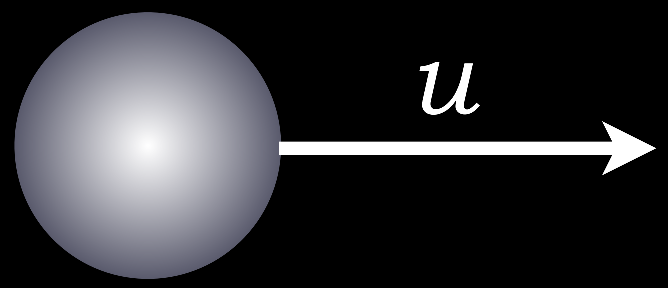
never get there



relativity and energy

through the back door…

there’s a “real” derivation, but too much mathematics



Momentum in relativity

got to be different from Newton

$$p_H = m\frac{\Delta x_H}{\Delta t_H}$$

want to preserve the idea of momentum conservation

Relativistic Momentum: $p = m\gamma u$

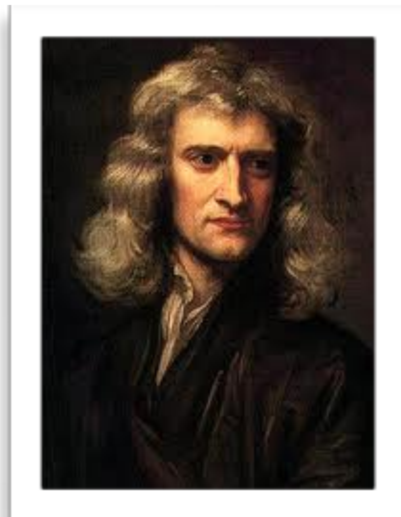
quick aside

approximating functions

see manuscript math refresher chapter

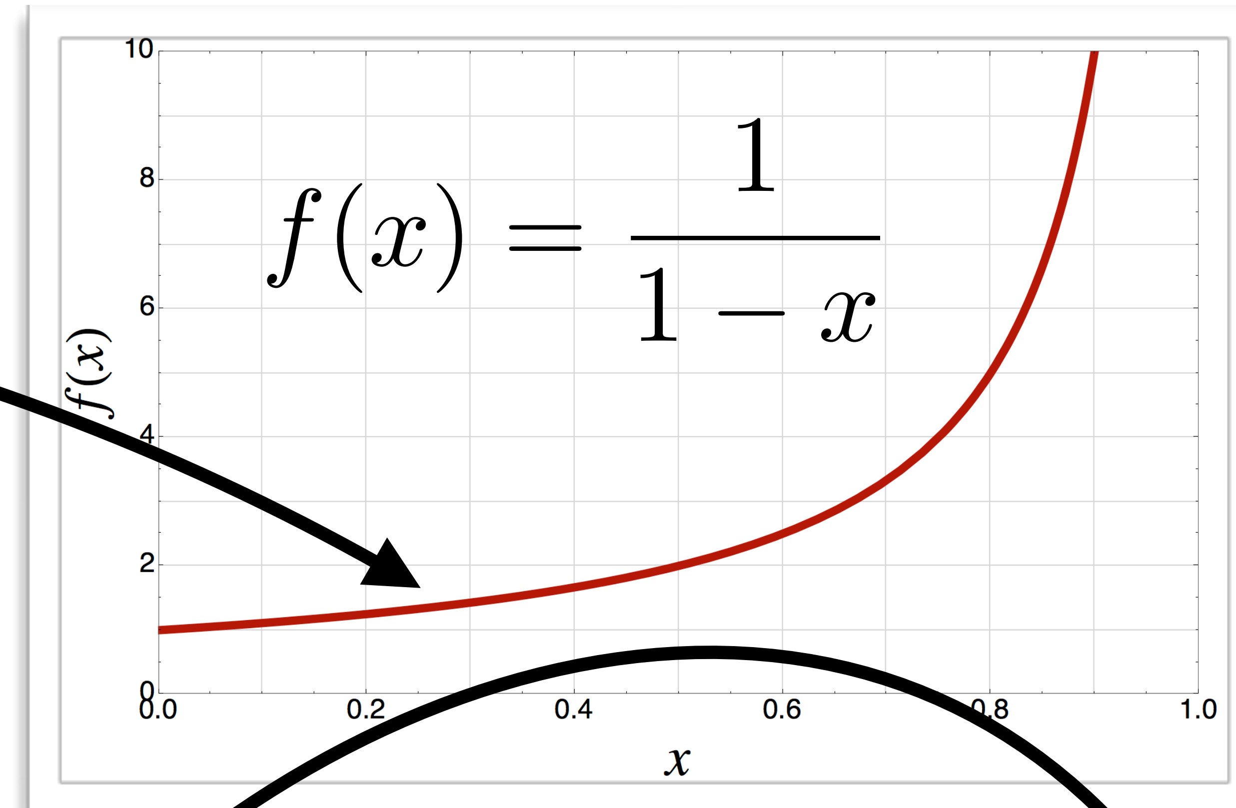
somewhere in your life: the Binomial Series

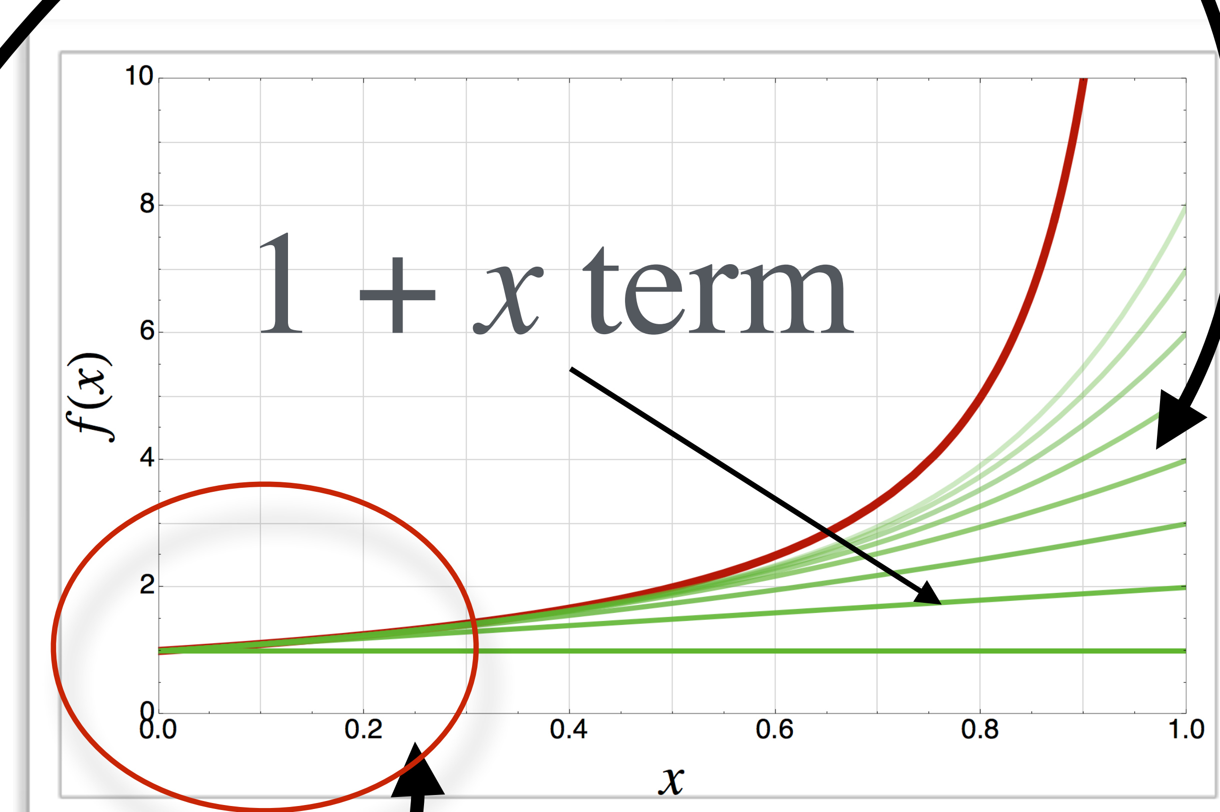
Binomial Series...useful to approximate functions.



$$f(x) = \frac{1}{1-x}$$

$$f(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + O\left(x^{11}\right)$$





Suppose that x << 1, then the function could be approximated by a couple of terms...

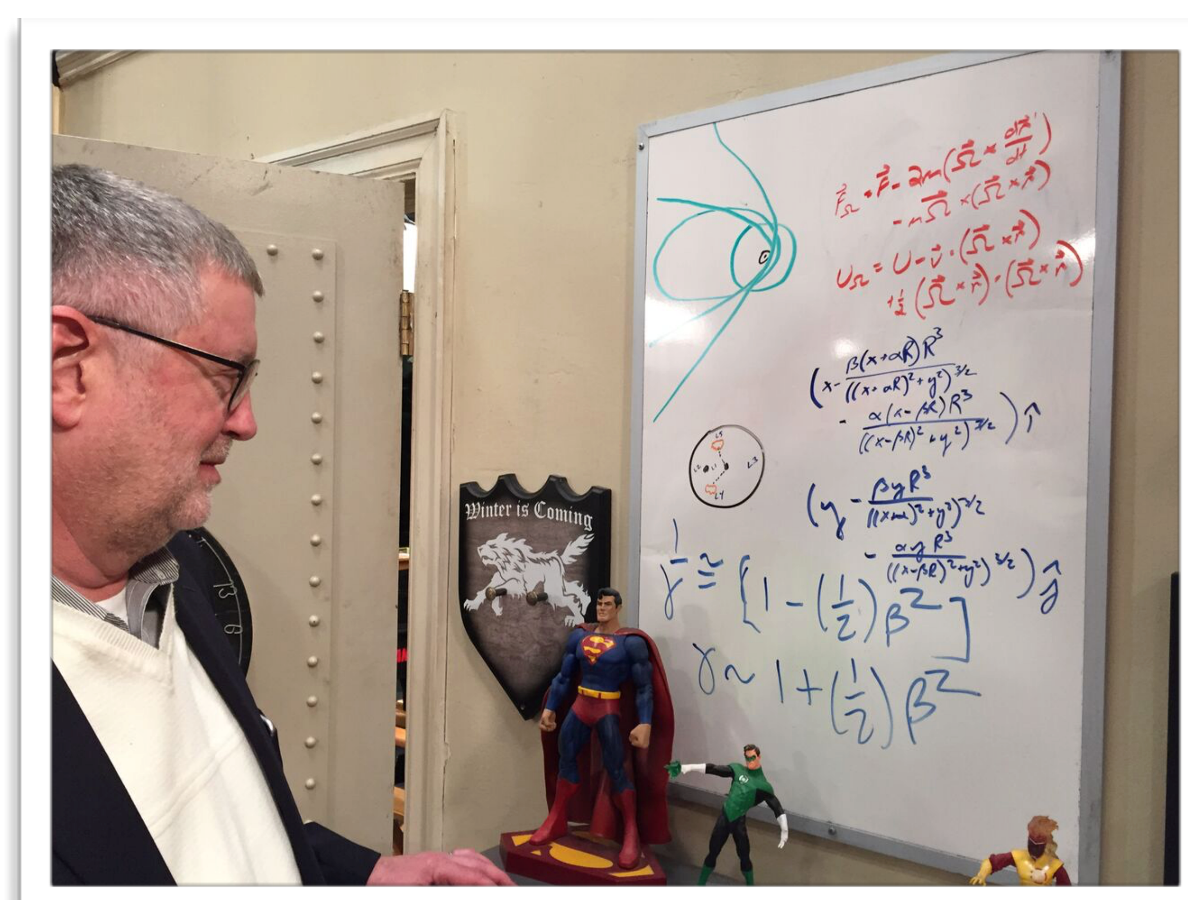
$$f(x) = \frac{1}{1-x} = 1 + x$$

what equation comes to mind?

when you're on the spot?

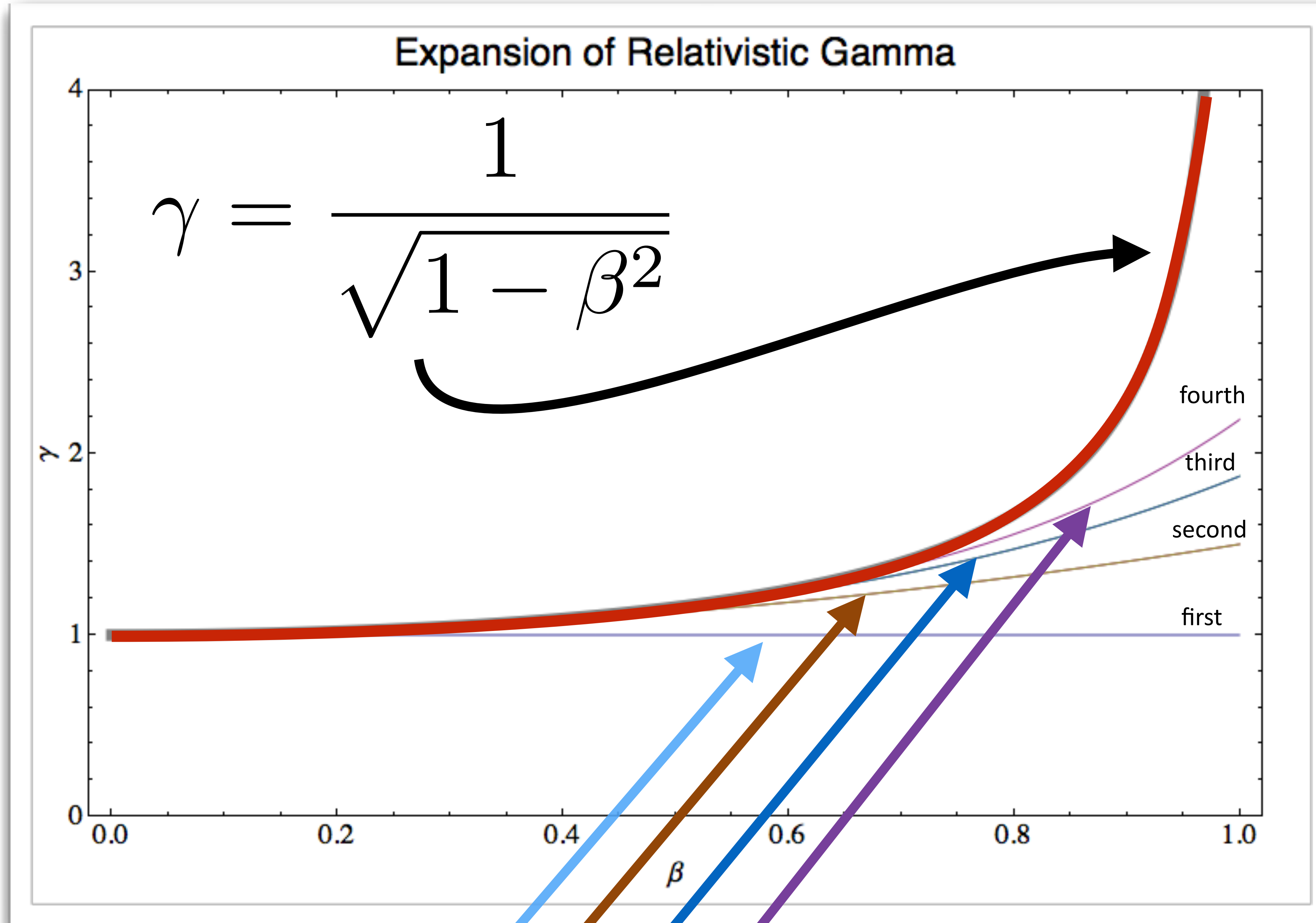
Why the binomial expansion of the relativistic gamma function, of course. Because, Relativity.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \sim 1 + \frac{\beta^2}{2} + \frac{3\,\beta^4}{8} + \frac{5\,\beta^6}{16} + \frac{35\,\beta^8}{128} + \frac{63\,\beta^{10}}{256} + \frac{231\,\beta^{12}}{1024} + \frac{429\,\beta^{14}}{2048} + O[\beta]^{15}$$

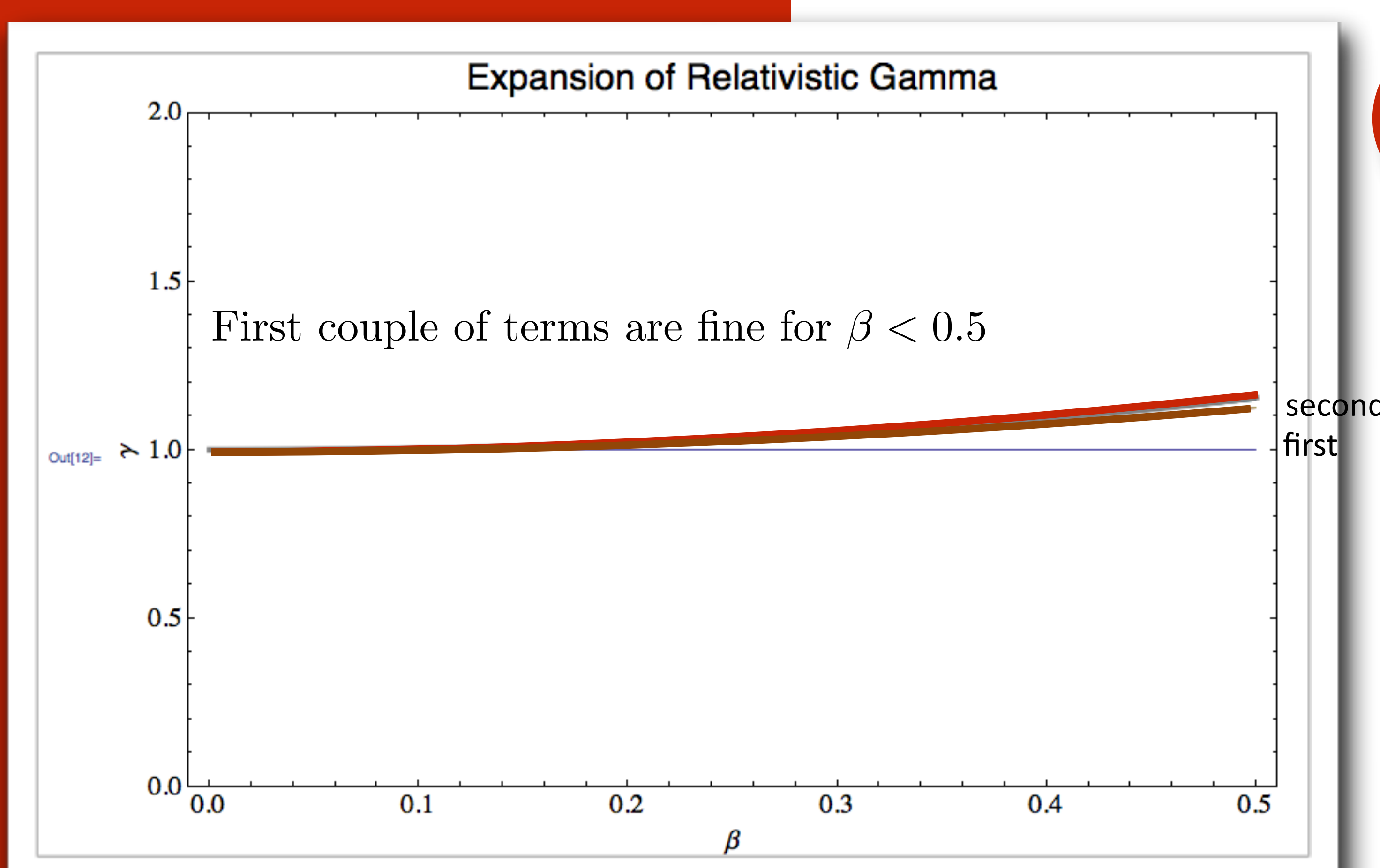


how well?

look at 8 terms in the expansion



$$\gamma \sim 1 + \frac{\beta^2}{2} + \frac{3\beta^4}{8} + \frac{5\beta^6}{16} + \frac{35\beta^8}{128} + \frac{63\beta^{10}}{256} + \frac{231\beta^{12}}{1024} + \frac{429\beta^{14}}{2048} + O[\beta]^{15}$$



BTW: escape velocity = 11,000 m/s

$$\beta = \frac{v}{c}$$

$$\beta = \frac{11,000}{3 \times 10^8} = \frac{1.1 \times 10^4}{3 \times 10^8}$$

Season 9, Episode 12

The Sales Call Sublimation, January 7, 2016

so let's use this and look for familiar things

slow moving objects but not completely classical

$$\gamma \sim 1 + \frac{\beta^2}{2} + \frac{3\beta^4}{8} + \frac{5\beta^6}{16} + \frac{35\beta^8}{128} + \frac{63\beta^{10}}{256} + \frac{231\beta^{12}}{1024} + \frac{429\beta^{14}}{2048} + O[\beta]^{15}$$

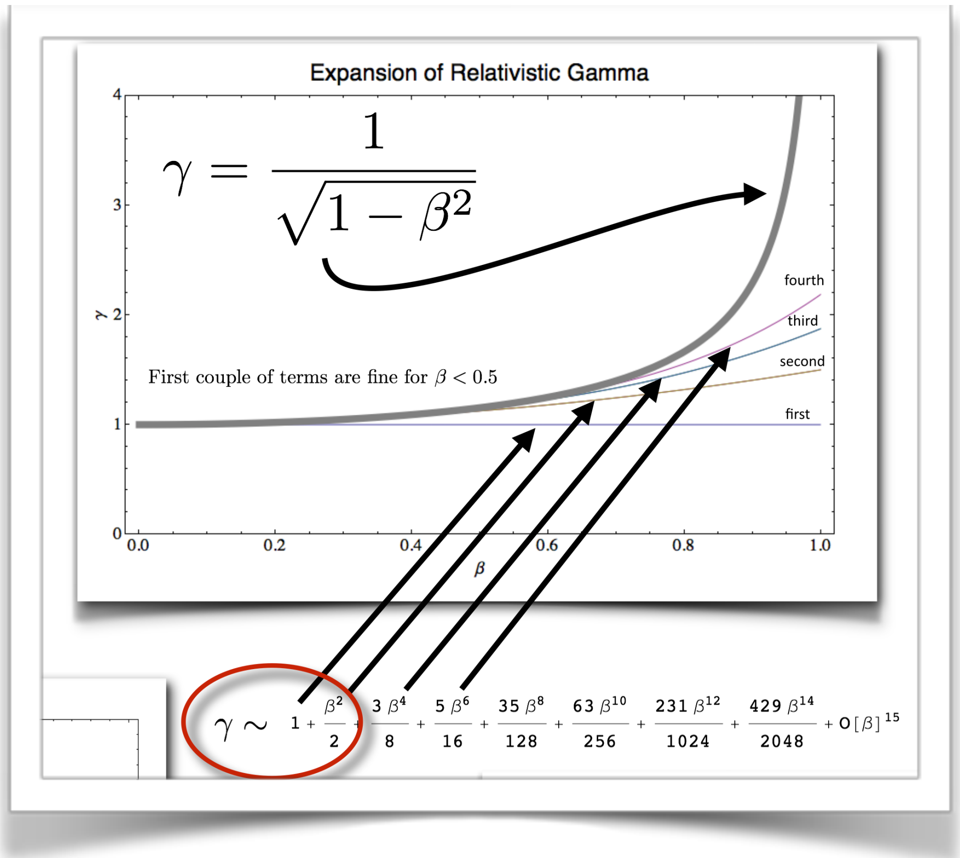
sing along

for β small:

now copy the approximate forms, but insert $\beta = u/c$

$$\gamma \sim 1 + \left(\frac{1}{2}\right)\beta^2$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$



sing along

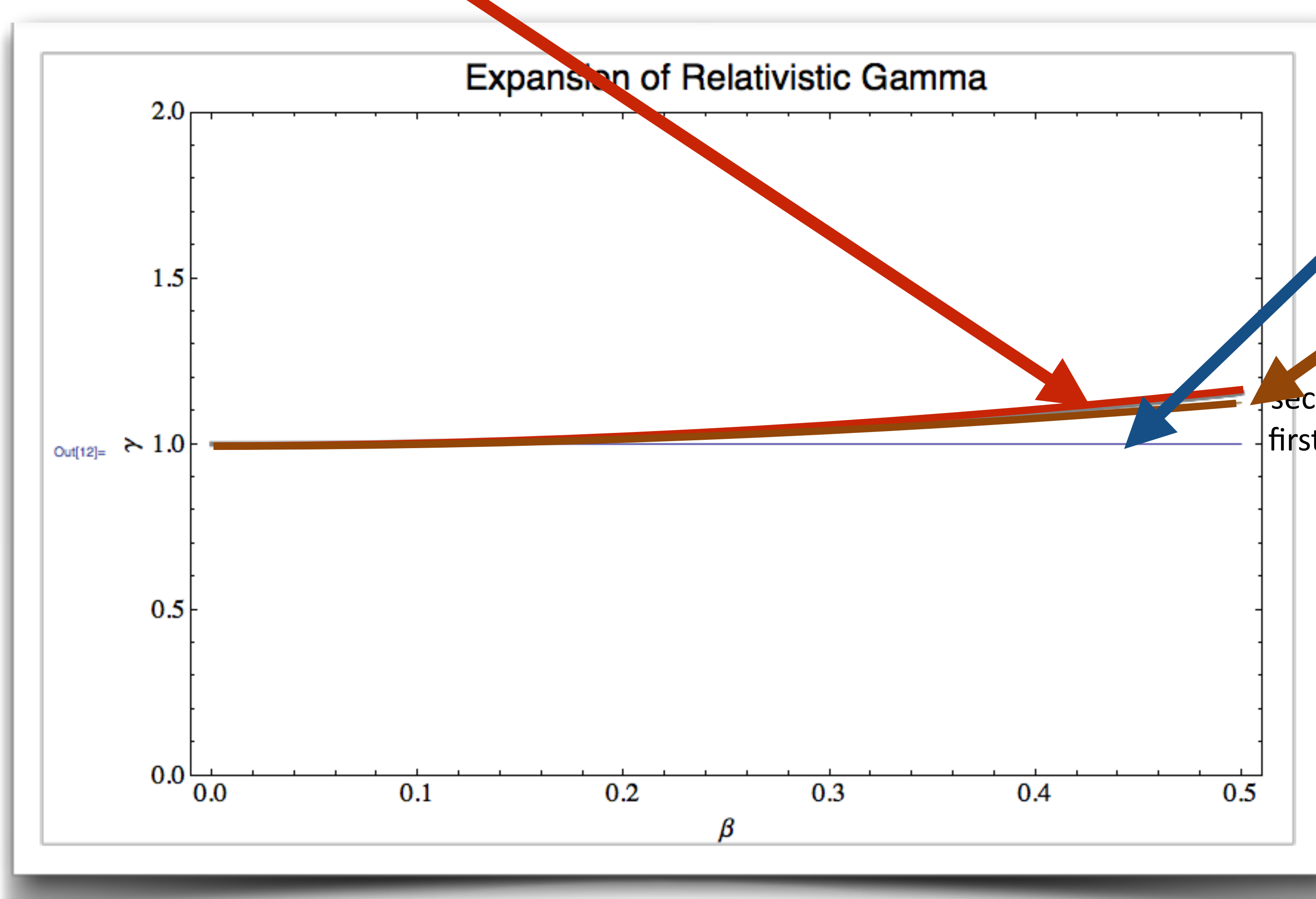
for β small:

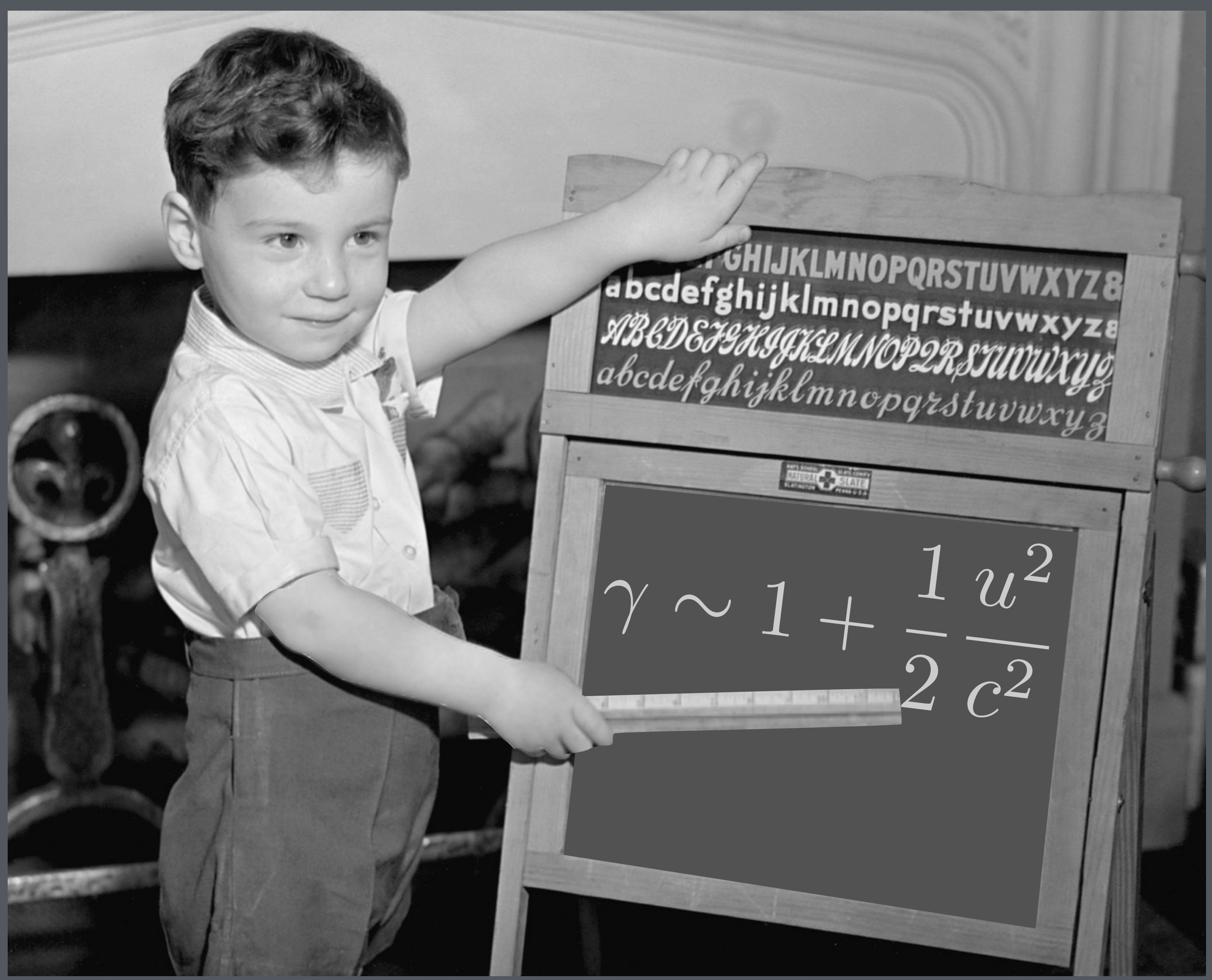
now copy the approximate forms, but insert $\beta = u/c$

$$\gamma \sim 1 + \left(\frac{1}{2}\right)\beta^2$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\boxed{\gamma \sim 1 + \left(\frac{1}{2}\right)\frac{u^2}{c^2}}$$





now let’s play

let's look at the kinetic energy

energy of motion...Kinetic Energy

\+ energy of rest...associated with mass

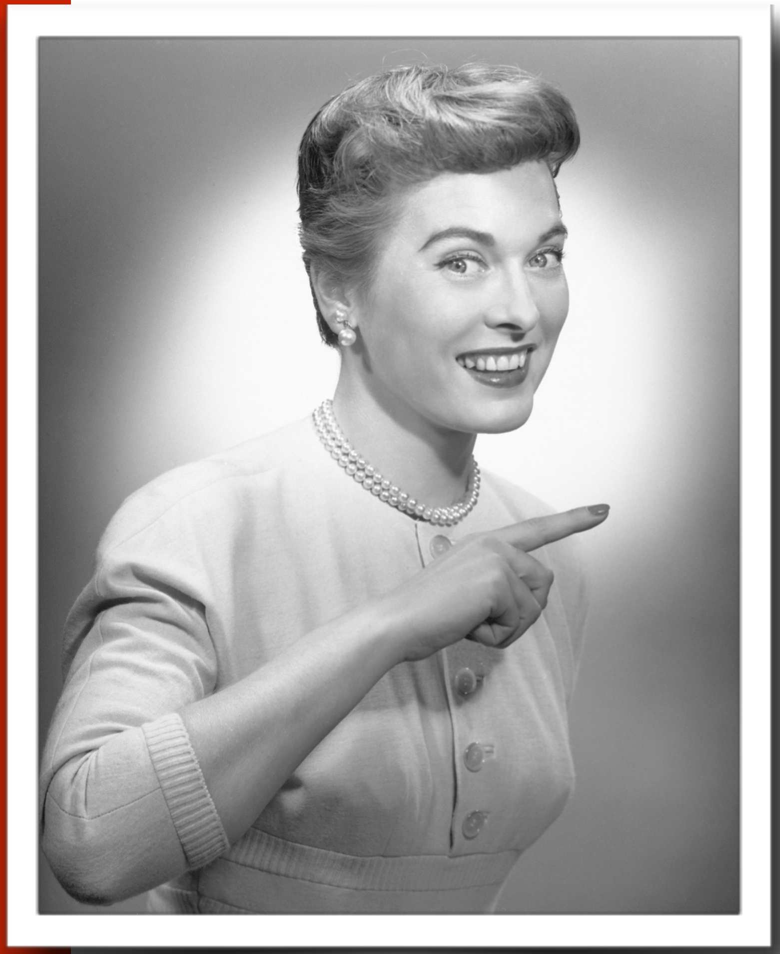
Total energy of an object

mass energy: $E_m = mc^2$

total energy: $E_T = m\gamma c^2$

kinetic energy?

Fully relativistic now



let's look at the kinetic energy

$$E_T = m\gamma c^2$$

$$E_m = mc^2$$

energy of motion...Kinetic Energy

\+ energy of rest...associated with mass

Total energy of an object

$$E_T = E_m + K$$

$$K = E_T - E_m$$

$$K = m\gamma c^2 - mc^2$$

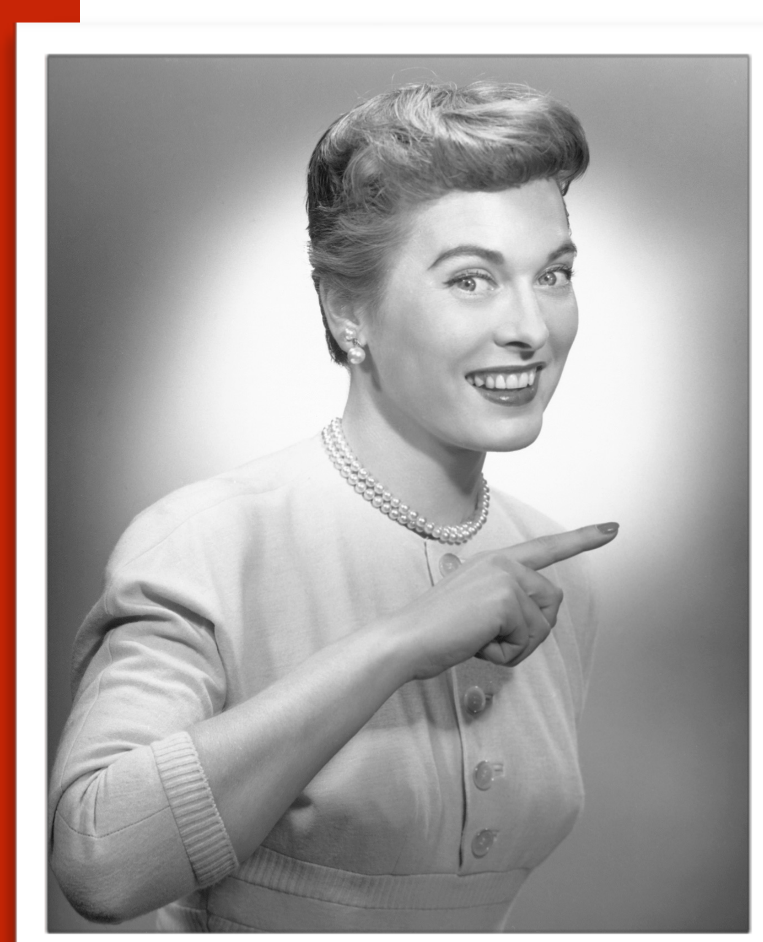
$$K = mc^2(\gamma - 1)$$

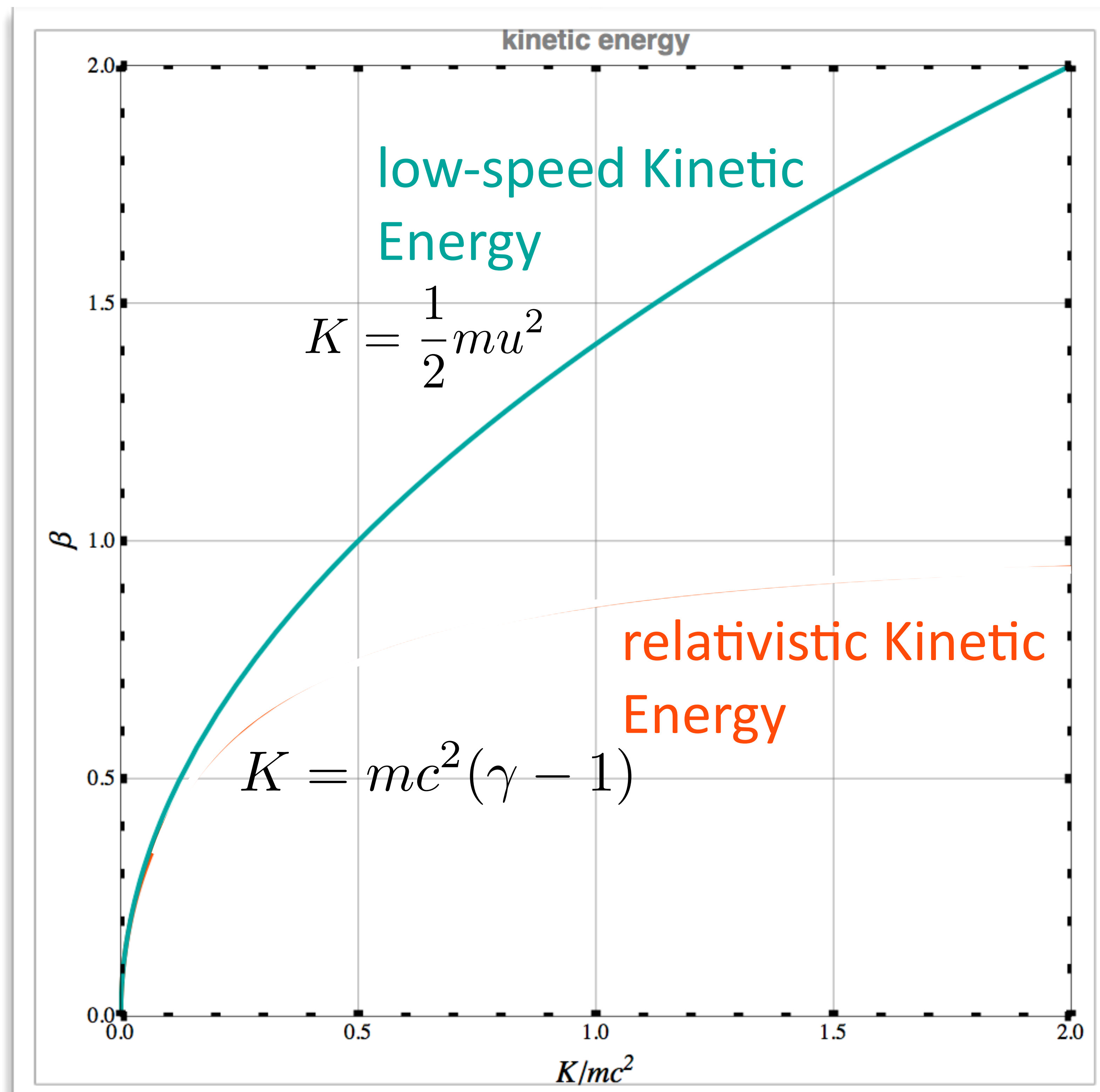
mass energy:

total energy:

kinetic energy?

Fully relativistic now





jargon alert: **rest mass**

refers to: mass of an object in its own rest frame (related to Rest Energy, the mass-energy of an object in its own frame)

entomology: "rest" implying...not moving

example: the rest mass of the electron is 9.109×10^{-31} kg

~~"convert mass into energy"~~

no.

Mass <u>is</u> energy and energy <u>is</u> mass.

$ = € * 0.88 both currency, can both buy stuff

just a conversion factor...

$$E_m = mc^2$$

both energy, can both do work

just a conversion factor...

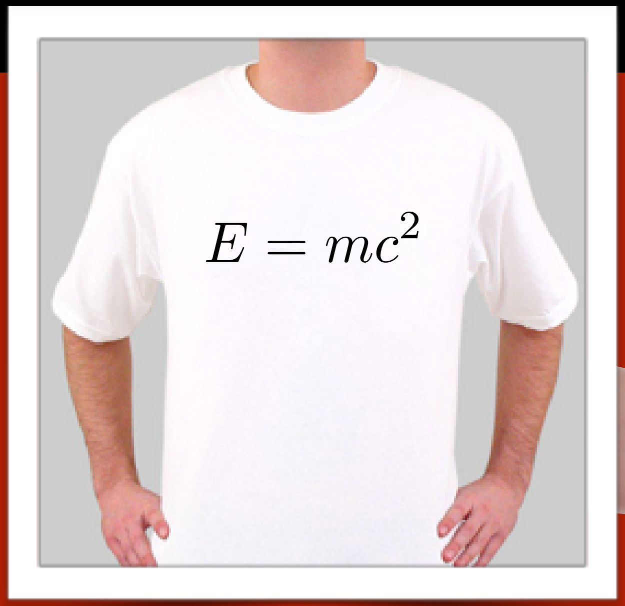
a big conversion factor

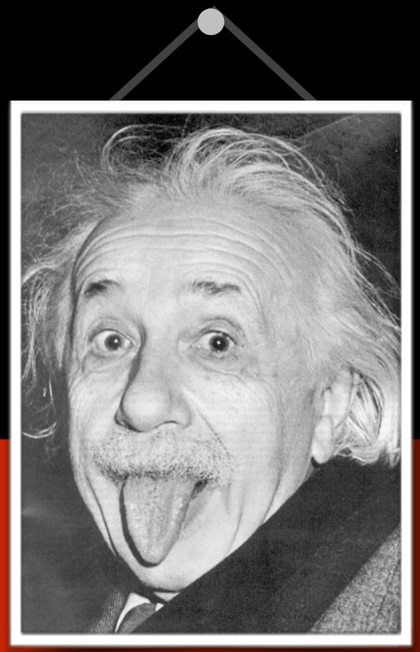
I *could* speak of the energy of mass... and the mass of energy

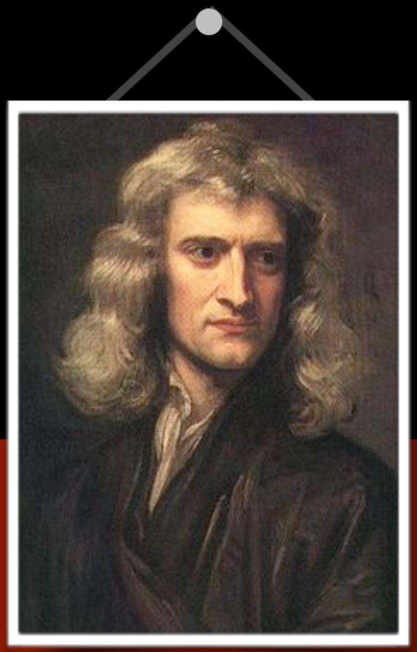
and I will.

Energy

cheat shirt:







lots of pent-up energy in an apple

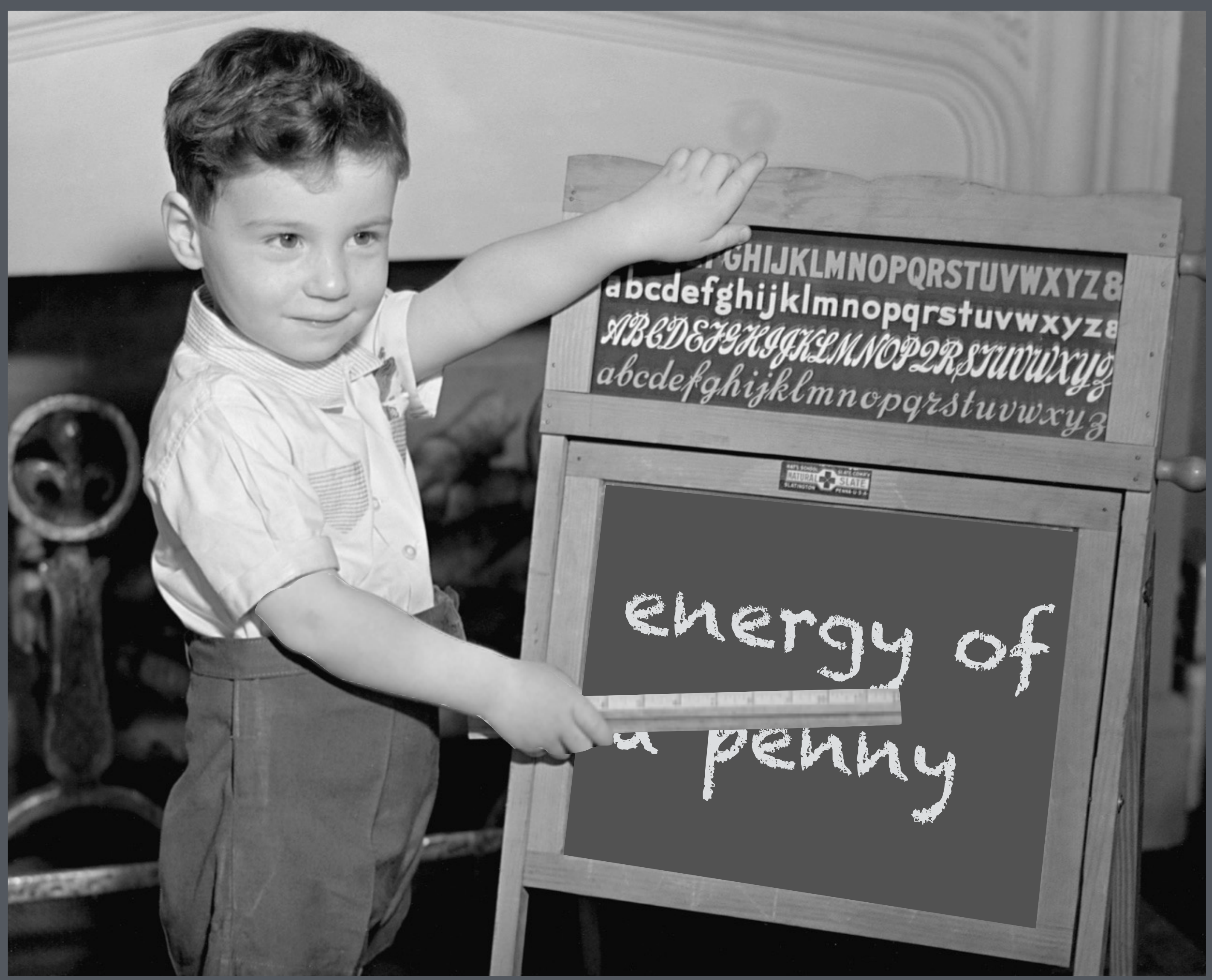
mass of the apple = 100 gm = 0.1 kg

$c^2 = 9 \times 10^{16}\ m^2/s^2$

Motion energy = 1 Joule

$$E_m = mc^2$$

$$= (0.1)(3 \times 10^8)^2$$ = Mass energy = 9,000,000,000,000,000 Joules!



the mass of a penny is

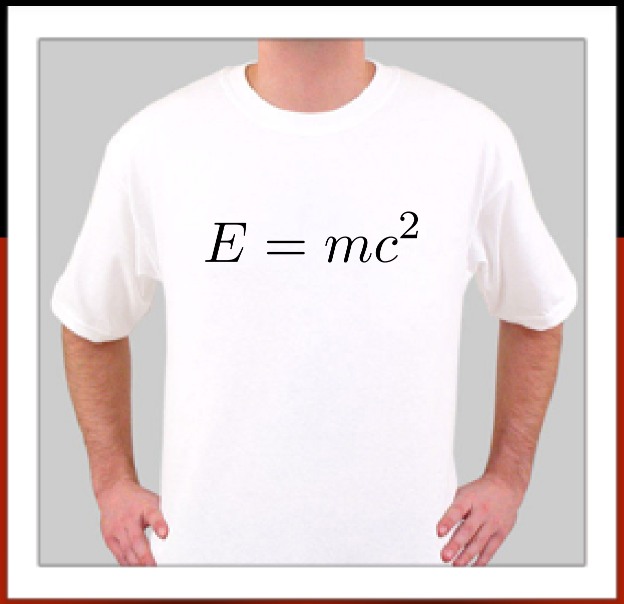
$$3\mathrm{gm} = 3 \times 10^{-3}\mathrm{kg}$$

The speed of light squared is:

$$c^2 = 9 \times 10^{16}\mathrm{m}^2/\mathrm{s}^2$$

How many Joules of energy is trapped in that mass?

that is...what’s the rest energy of a penny?



cheat shirt

the mass of a penny is

$$3\text{gm} = 3 \times 10^{-3}\text{kg}$$

The speed of light squared is: $c^2 = 9 \times 10^{16}\text{m}^2/\text{s}^2$

$$E_m(\text{penny}) = 27 \times 10^{13}\text{J}$$

how much is that?

Aircraft Carrier Nimitz: 91,400 tons at 32 knots:

$$K(\text{Nimitz}) = 1.1 \times 10^{10}\text{J}$$

down this rabbit hole

if an object has mass it has energy

if an object has energy it has mass

look at the total energy

and increase of mass with velocity

$$E_T = m\gamma c^2$$

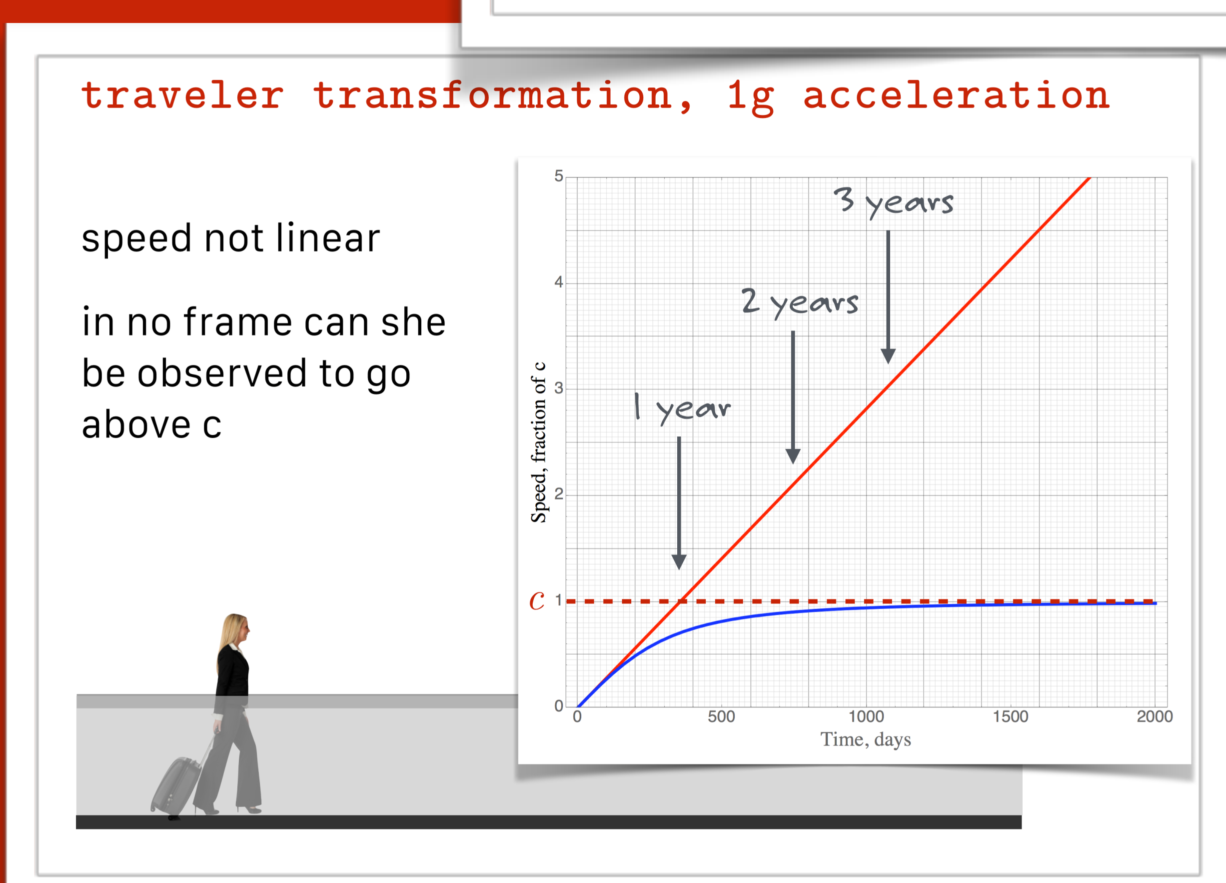
One way to interpret this is to associate gamma and m.

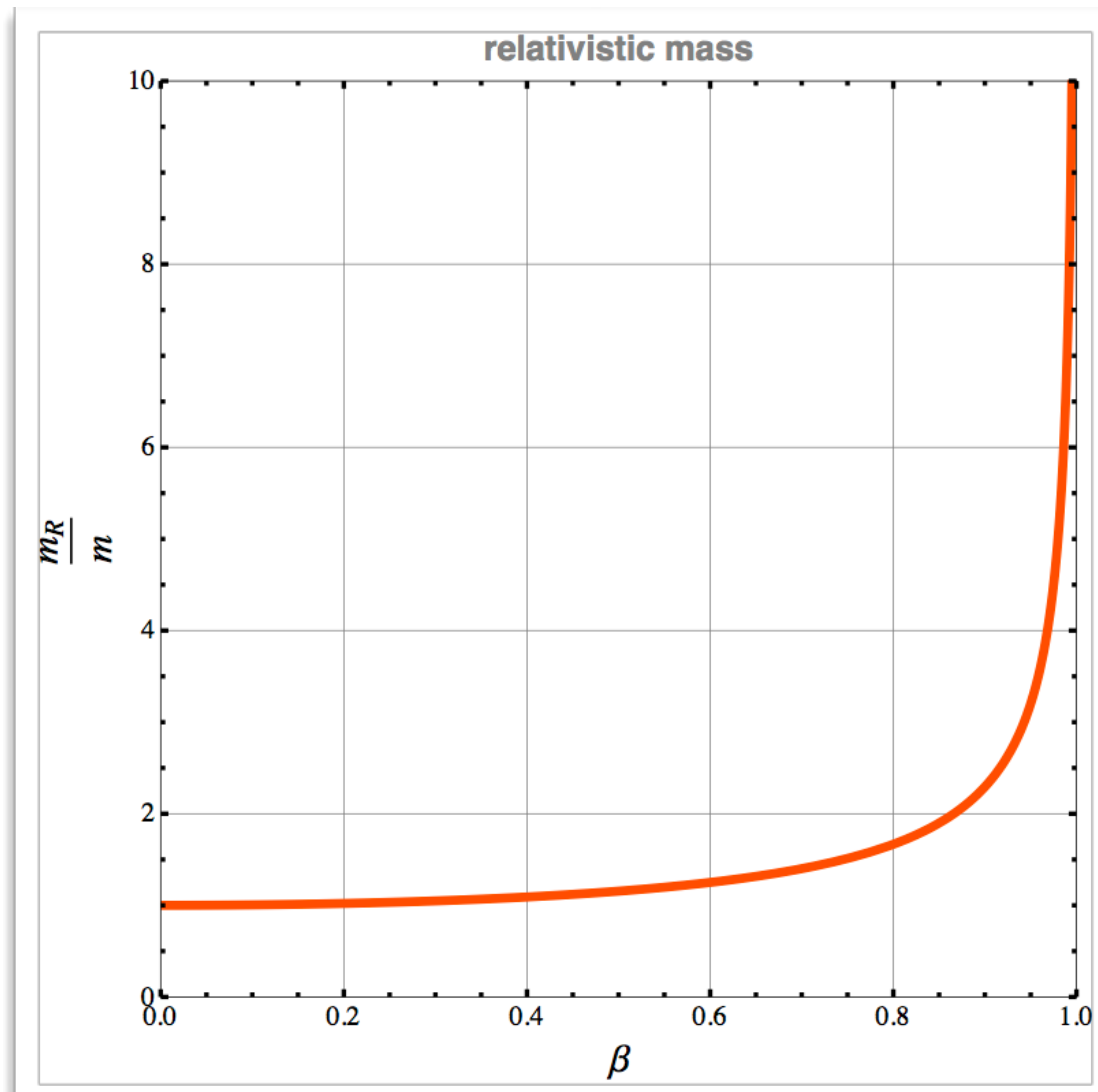
$$E_T = (m\gamma)c^2 \quad \text{so} \quad E_T = m_R c^2$$

...and speak of a speed-dependent "**relativistic mass**."

$$m_R = \gamma m \quad \text{so} \quad \frac{m_R}{m} = \gamma$$

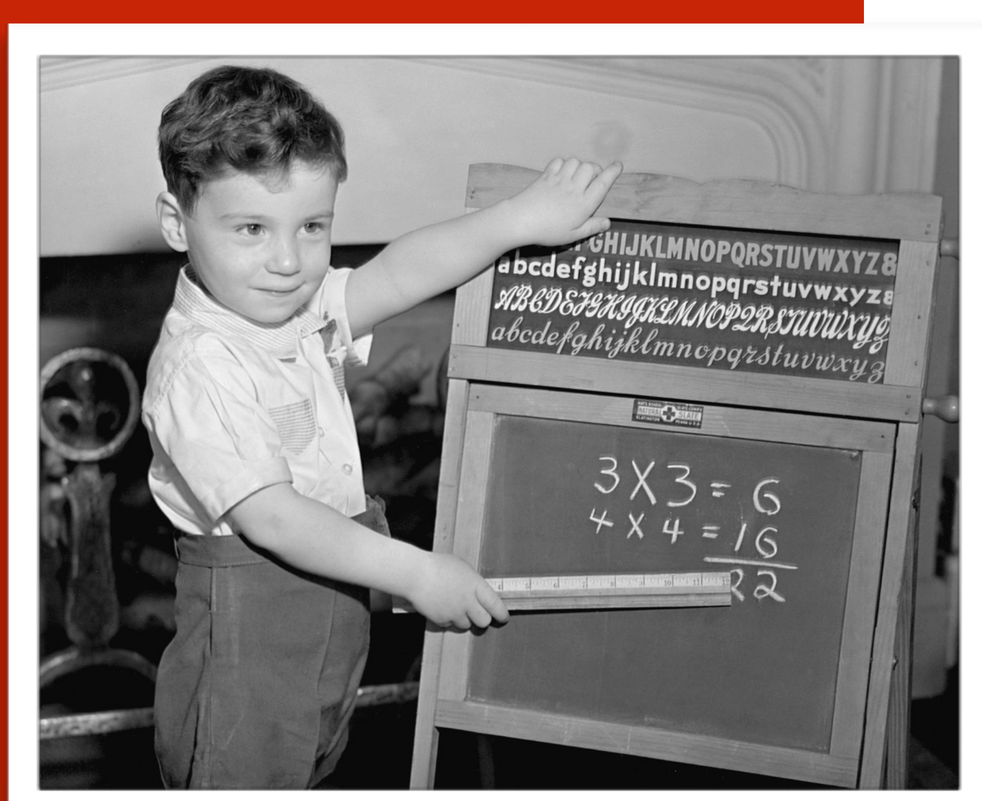
remember the inertia issue?

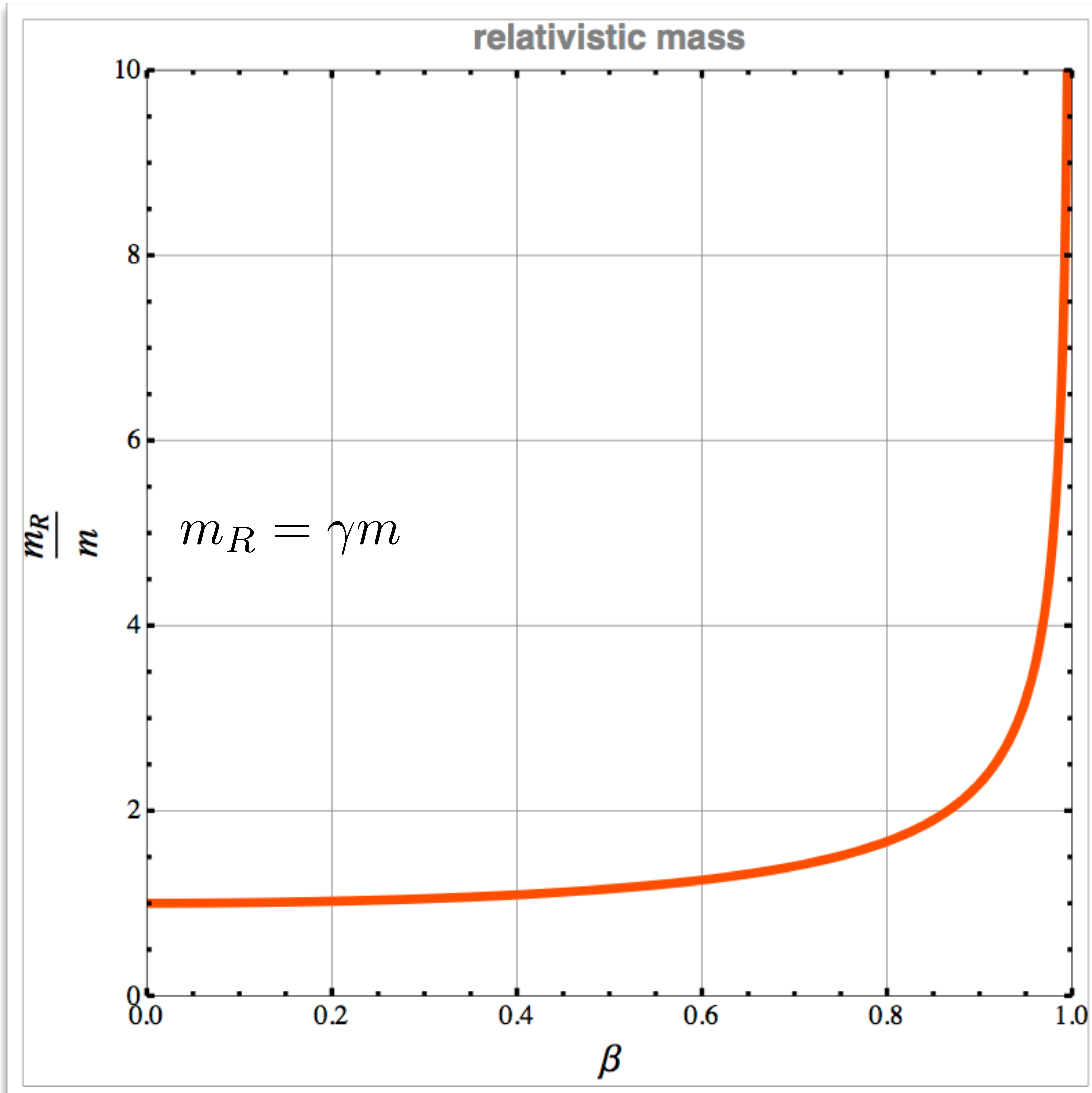




if a proton is going at 0.95% of the speed of light and has mass of 1p

how massive does it appear to be?





from this point on:

if I refer to the rest mass*…I'll say so

otherwise, "mass" is this velocity-dependent quantity

*(This is not how we speak in polite particle physics circles…where "mass" is a constant always.)
But, I think for non-specialists this is more clear.

a useful invariant

and an important formal linkage

$$E_m = mc^2$$

$$E_T = m\gamma c^2$$

$$p = m\gamma v$$

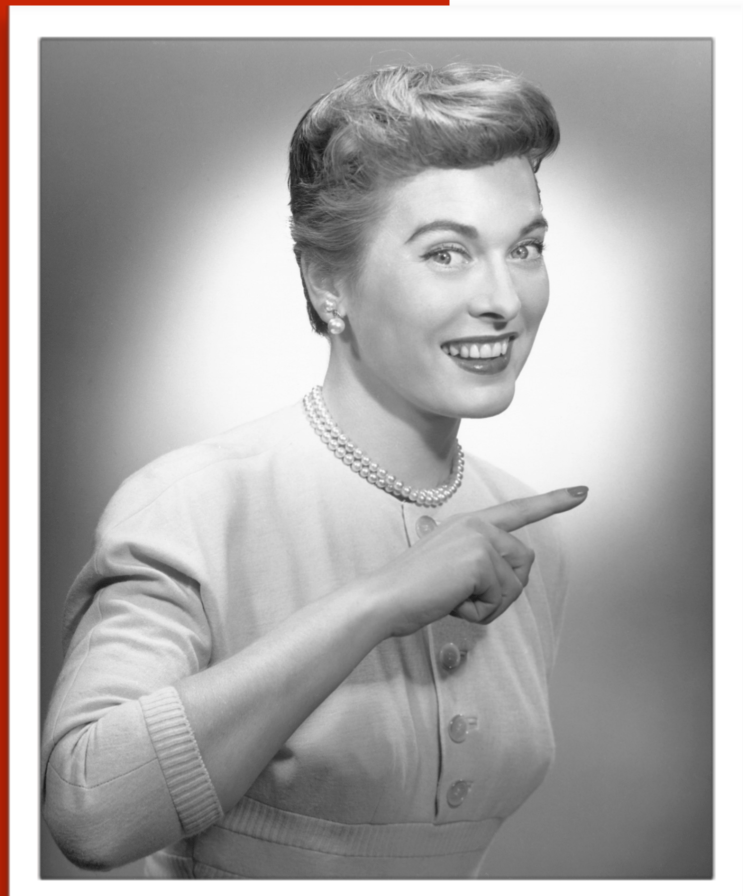
fun fact...just with a little algebra...

$$E_T^2 = (mc^2)^2 + (pc)^2$$

$$E_m^2 = E_T^2 - p^2c^2$$

$$m^2c^4 = E_m^2 = E_T^2 - p^2c^2$$

no velocity dependences, just a number...



"Energy-momentum relation"...

$$\boxed{E_T^2 = (mc^2)^2 + (pc)^2}$$

practical

Energy/momentum relations:

"rest mass"... m

the mass of an object in its own frame

"relativistic mass"... $m_R = m\gamma$

the mass of a moving object

"Energy"... $E_T = m\gamma c^2$

the total Energy of a moving object

"rest Energy"... $E = mc^2$

the mass-energy of an object in its own frame

Kinetic Energy... $K = mc^2(\gamma - 1)$

the energy due to motion

Relativistic momentum... $p = m\gamma u$

momentum for each component of space

Energy-momentum relation... $E_T^2 = \left(mc^2\right)^2 + (pc)^2$

an alternative, useful expression

real electrons

HV transmission lines feed substations?

138,000 V is common (BWL for example)

Assume that arc is at 138,000V, so electrons have that energy

...which would be the Kinetic Energy

You might want to remember this:

energy of motion…**Kinetic Energy**

\+ energy of mass… **Rest Energy**

Total energy of an object

there aren’t any other kinds of energy

completely inelastic collision

1

12

2

a collision from earlier

where mechanical energy was not conserved.

and they stick together

But we certainly would have said: $m_1 + m_2 = M_{12}$

Now...energy conservation is different:

$$E_{(\text{before})} = E_{(after)}$$

$$[E_{(\text{Object 1})}] + [E_{(\text{Object 2})}] = [E_{(\text{Object 12})}]$$

$$\underline{E_{m(1)}} + K_1 + \underline{E_{m(2)}} + K_2 = \underline{E_{m12}} + K_{12}$$

brand new thing!

a collision from earlier

where mechanical energy was not conserved.

completely inelastic collision

1 12 2

and they stick together, and stop

systems' energy of masses + KE's = system's energy of mass + KE

brand new thing!

$$m(1)c^2 + m(2)c^2 + K(1) + K(2) = M(12)c^2 + \cancel{K(12)}$$

$$M(12)c^2 = m(1)c^2 + m(2)c^2 + K(1) + K(2)$$

$$M(12)c^2 > m(1)c^2 + m(2)c^2$$

But now, the mass of the stuck-together system is more than the masses of the projectiles...

The energy of motion has become energy of mass.

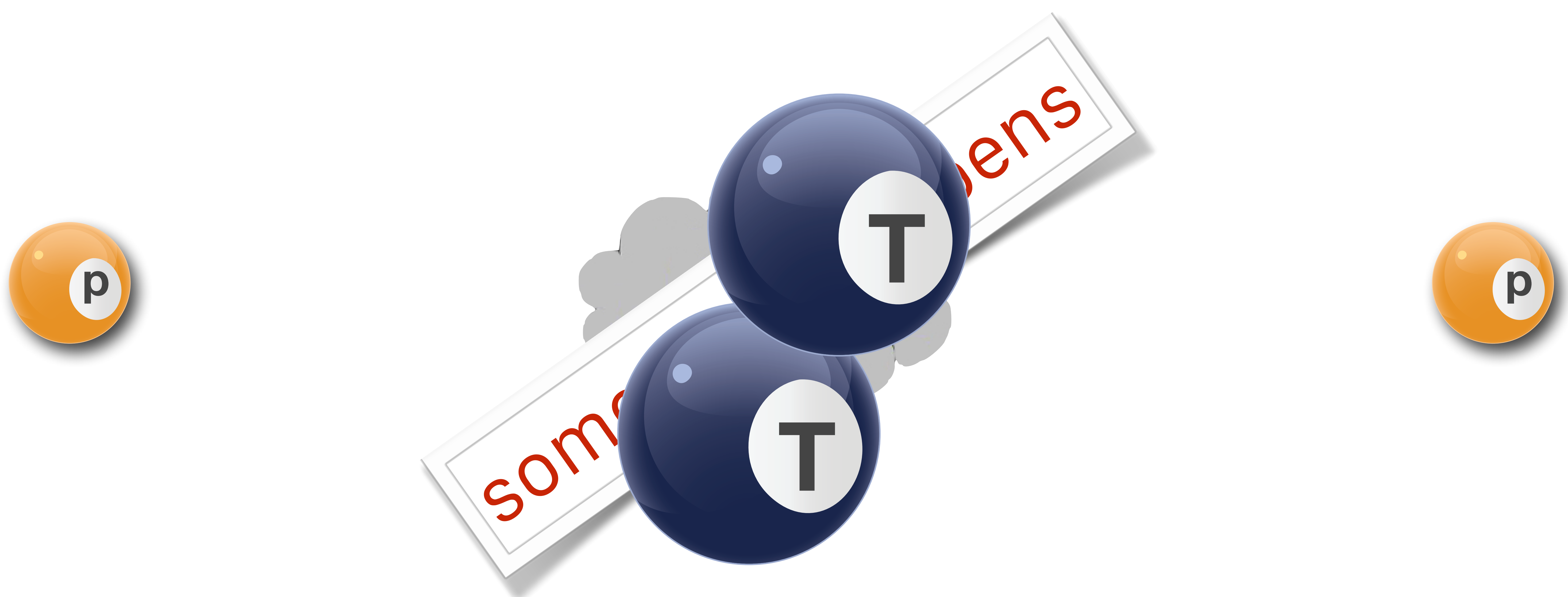
~~But before we certainly would have said:~~

$$\cancel{m_1 + m_2 = M_{12}}$$

this is how

we can take two protons, crash them together, and produce 2 “top quarks”...

each of which has the mass of 170 protons



conserved quantities:

2 of them now:

Momentum Conservation

Energy Conservation:

if both mass-energy ***and*** *kinetic energy are counted*

→ Energy-Momentum conservation

Energy Conservation in a collision:

$$A + B \to C + D$$

$\left[\text{MassEnergy}_0(\text{A}) + \text{KE}_0(\text{A})\right] + \left[\text{MassEnergy}_0(\text{B}) + \text{KE}_0(\text{B})\right] =$

$\left[\text{MassEnergy}(\text{C}) + \text{KE}(\text{C})\right] + \left[\text{MassEnergy}(\text{D}) + \text{KE}(\text{D})\right]$

$$[m(A)c^2 + K(A)] + [m(B)c^2 + K(B)] =$$

$$[m(C)c^2 + K(C)] + [m(D)c^2 + K(D)]$$

particle colliding beam

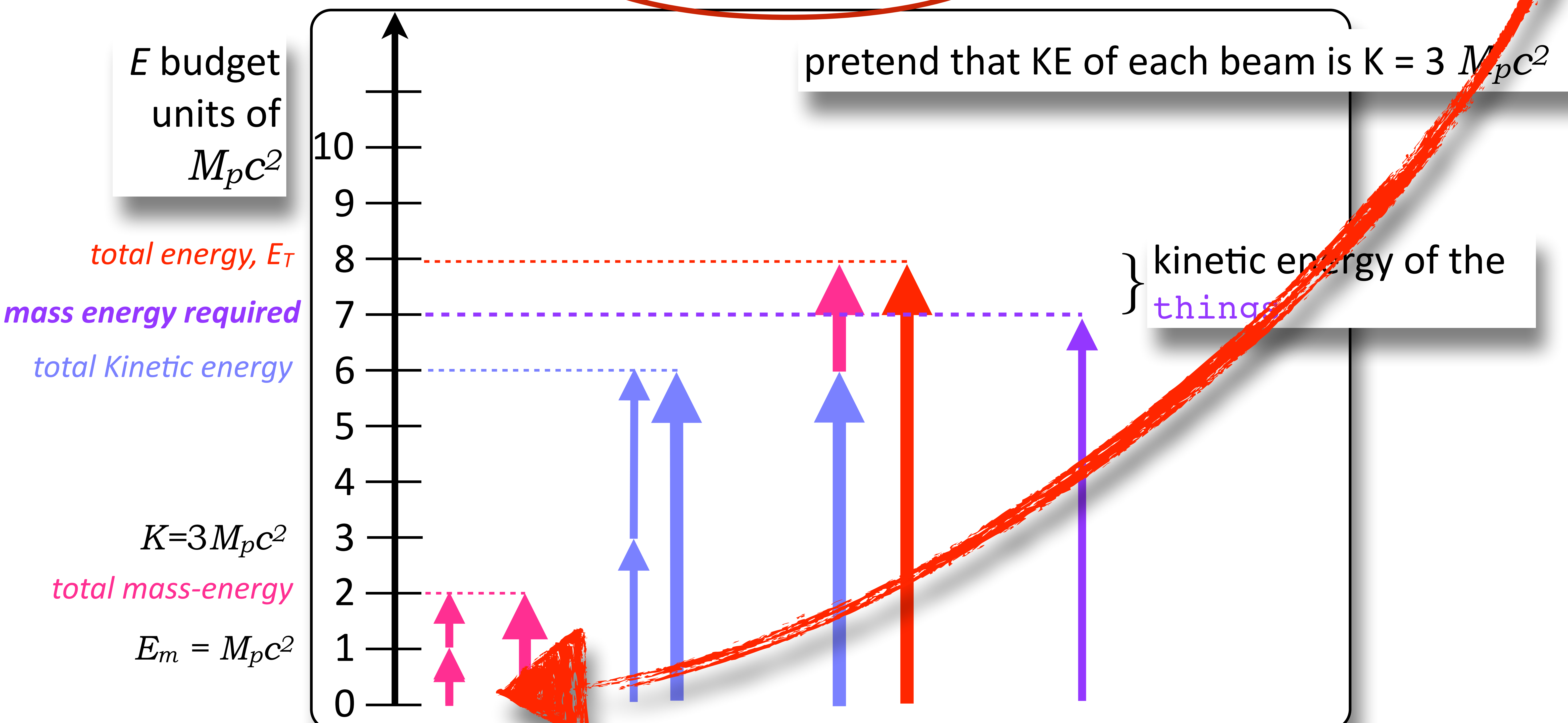
E_T = mass energy + kinetic energy + E_T = mass energy + kinetic energy

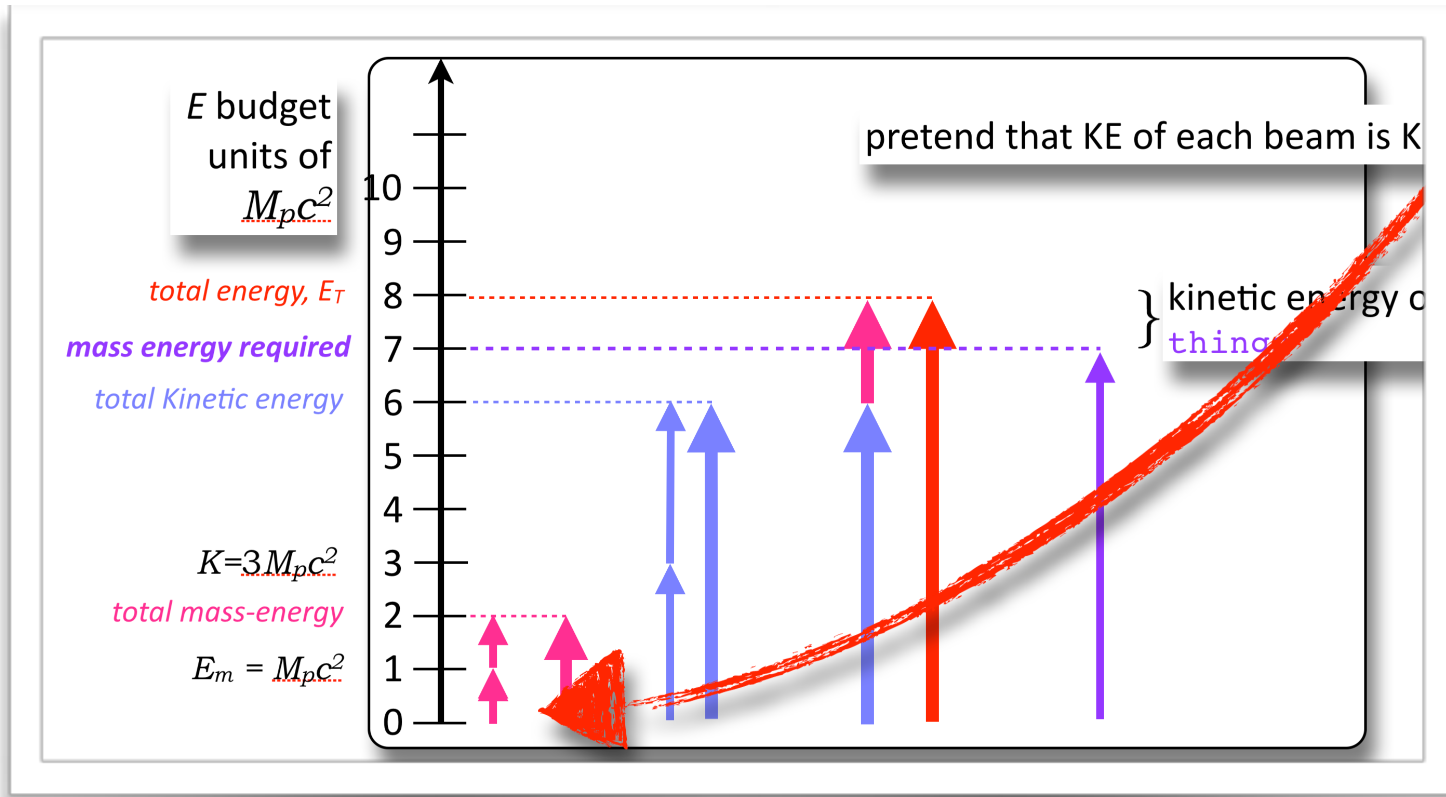
1 proton's mass = M_p

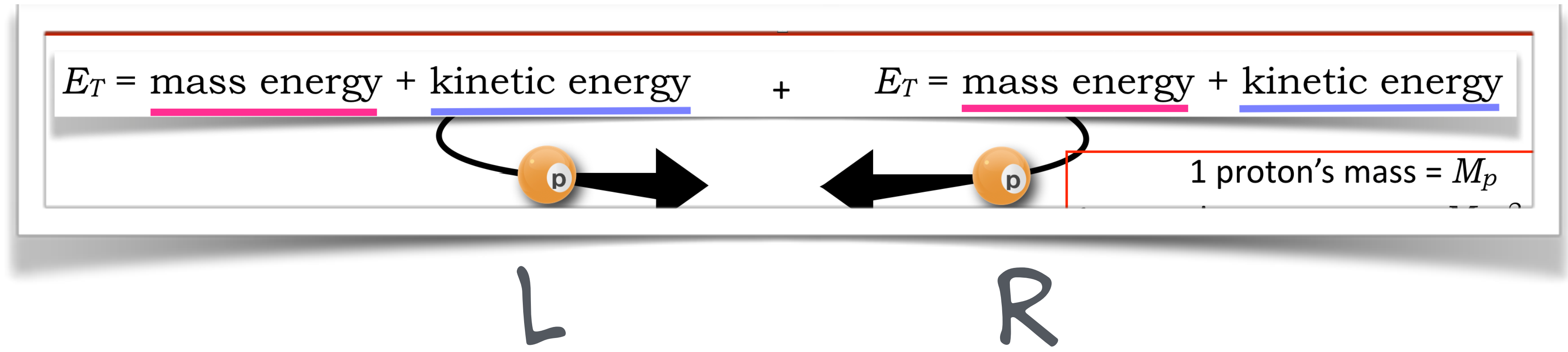
1 proton's mass energy = M_pc^2

Use head-on collisions to make objects more massive than protons.

Make Two `things` that each have $M(\texttt{thing}) = 3.5 \bullet M_p$

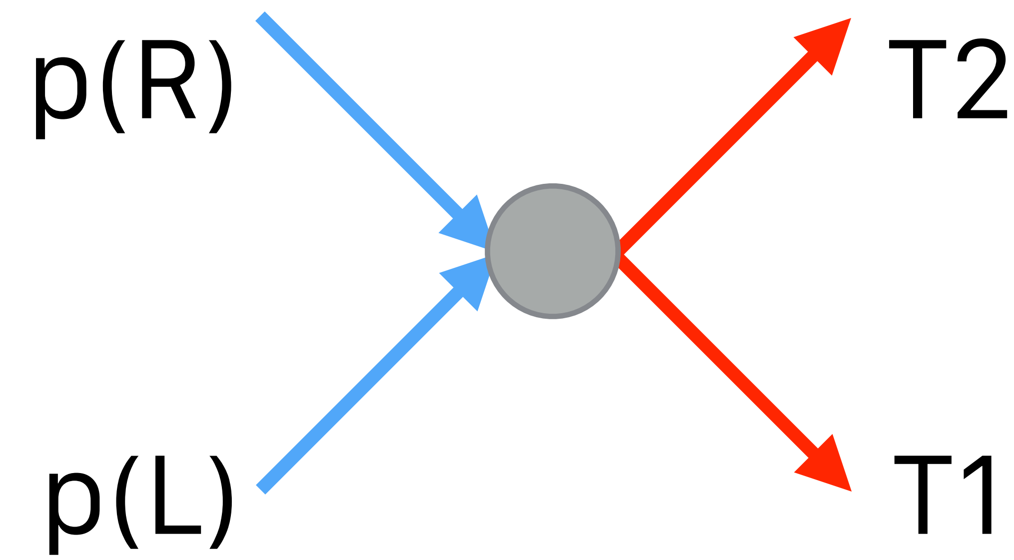






the reaction p(L) + p(R) → T1 + T2

the Feynman Diagram



Energy equation:

$$E_{T,0}(L) + E_{T,0}(R) = E_T(\text{thing1}) + E_T(\text{thing2})$$

1 + 3 + 1 + 3 = 3.5 + 3.5 + ?

8 = 7 + 1

`what about the`

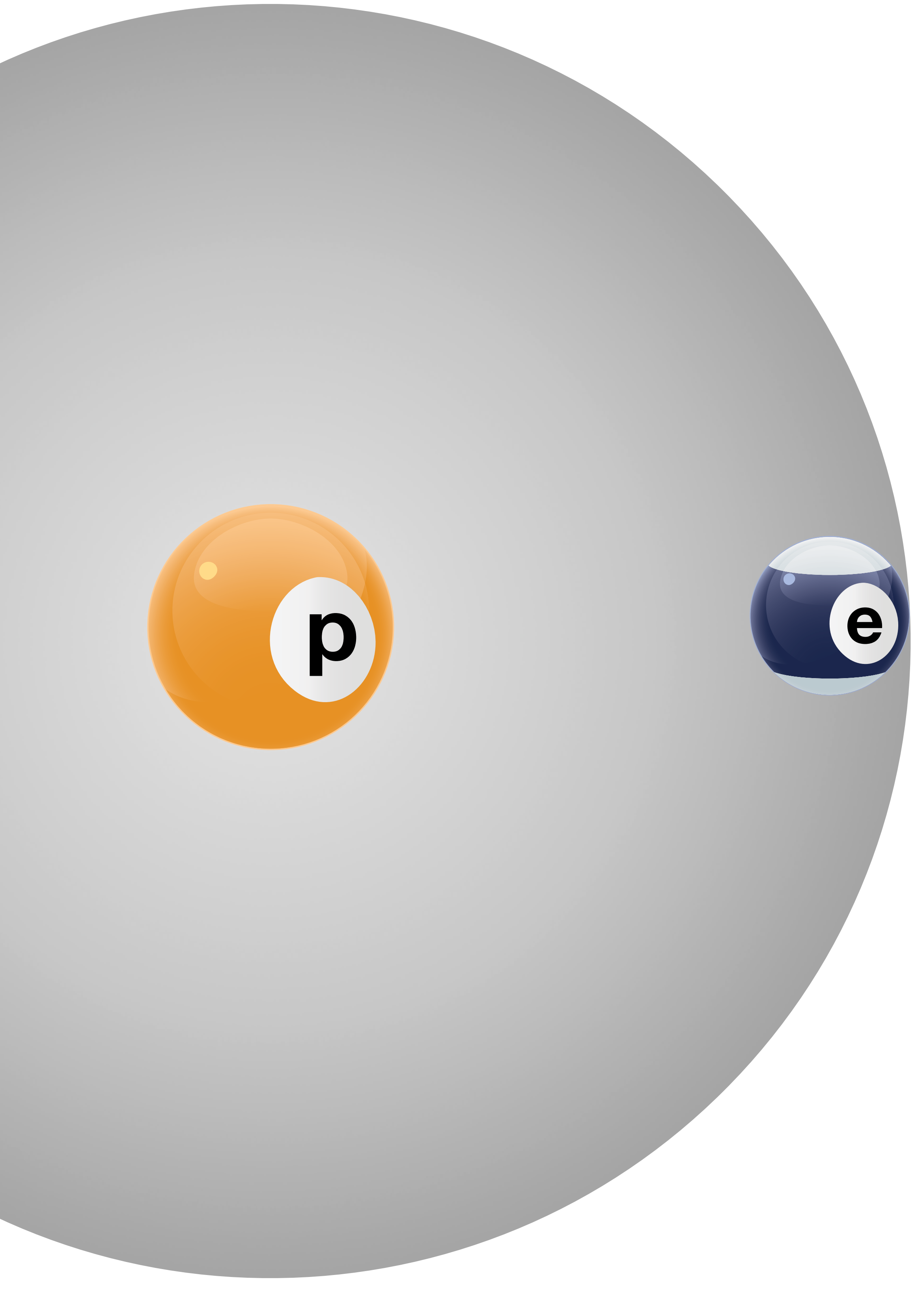
“energy of mass” and “mass of energy” crack?

suppose we have a bound system

What holds the electron to the proton?

Hydrogen Atom

Last week:
the electrostatic force, or the Electric field, right?



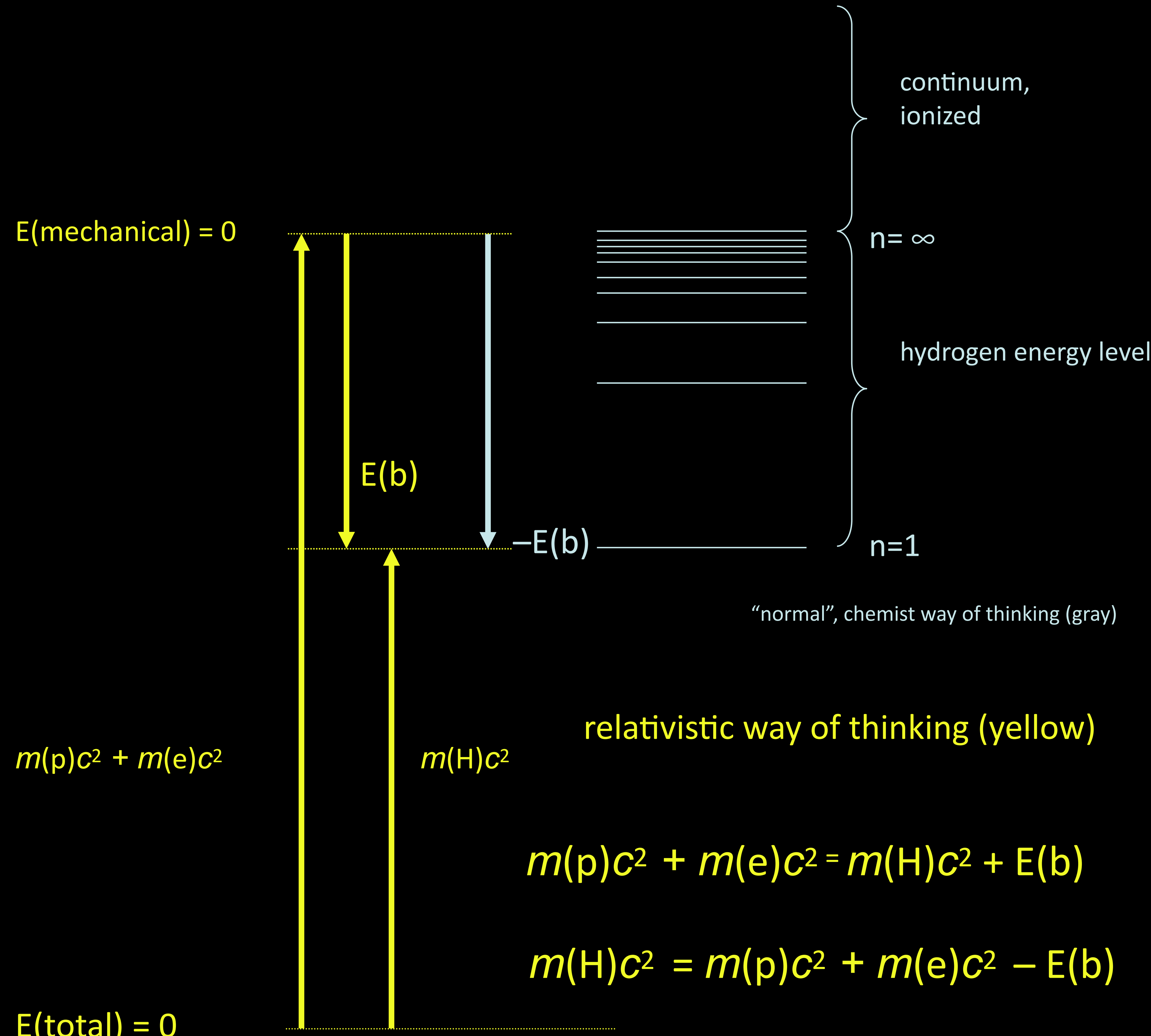
Remember from Chemistry:

What's it take to ionize* Hydrogen?

You must supply 13.6 eV

*make the electron free of the proton's influence

energy diagram for H



$$m(\mathrm{p})c^2 + m(\mathrm{e})c^2 = m(\mathrm{H})c^2 + E(\mathrm{b})$$

$$m(\mathrm{H})c^2 = m(\mathrm{p})c^2 + m(\mathrm{e})c^2 - E(\mathrm{b})$$

The mass of a hydrogen atom is LESS than the sum of $m_p + m_e$
No negative binding energy...just a “mass deficit” in the attraction of the P and e.
The energy is in the field.

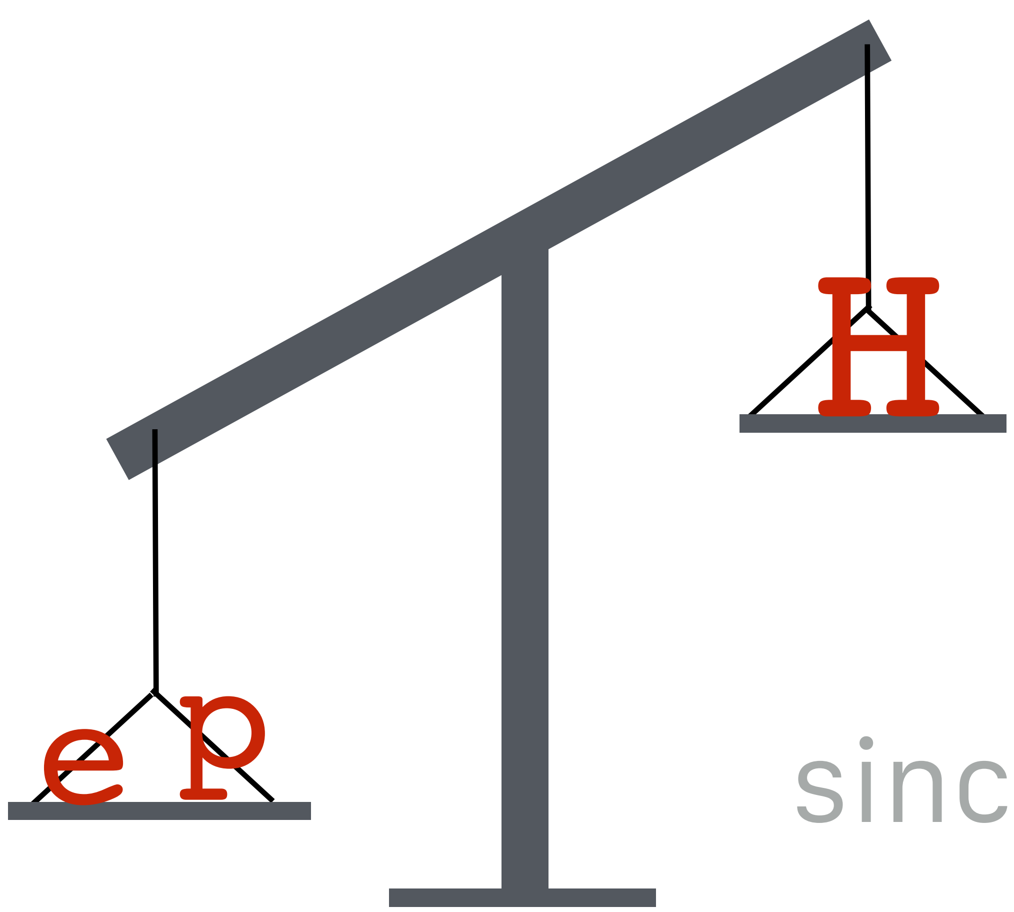
a hydrogen atom, take 2

weighs less than the components of a hydrogen atom

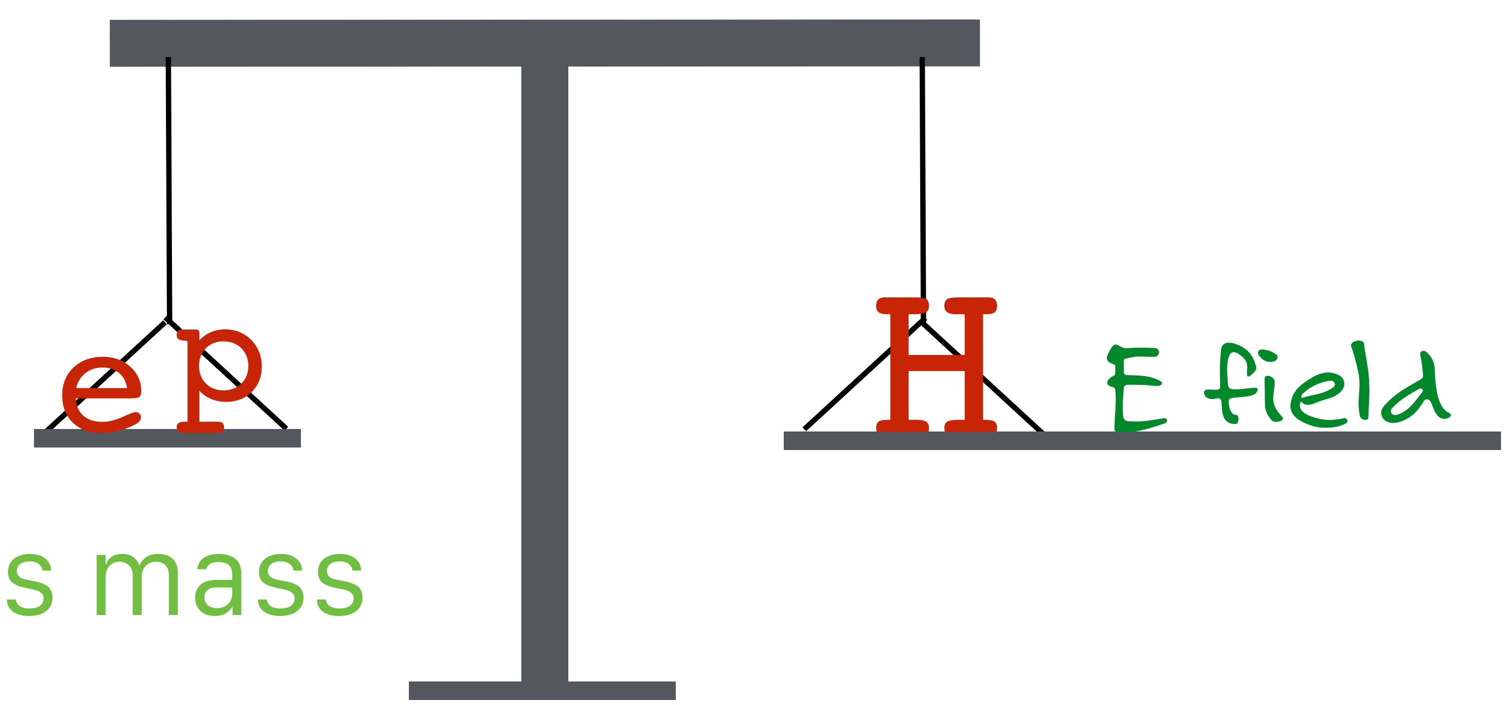
so it can't fall apart into its components

where is that "missing mass"?

in the energy of the Electric Field,



since Energy is mass

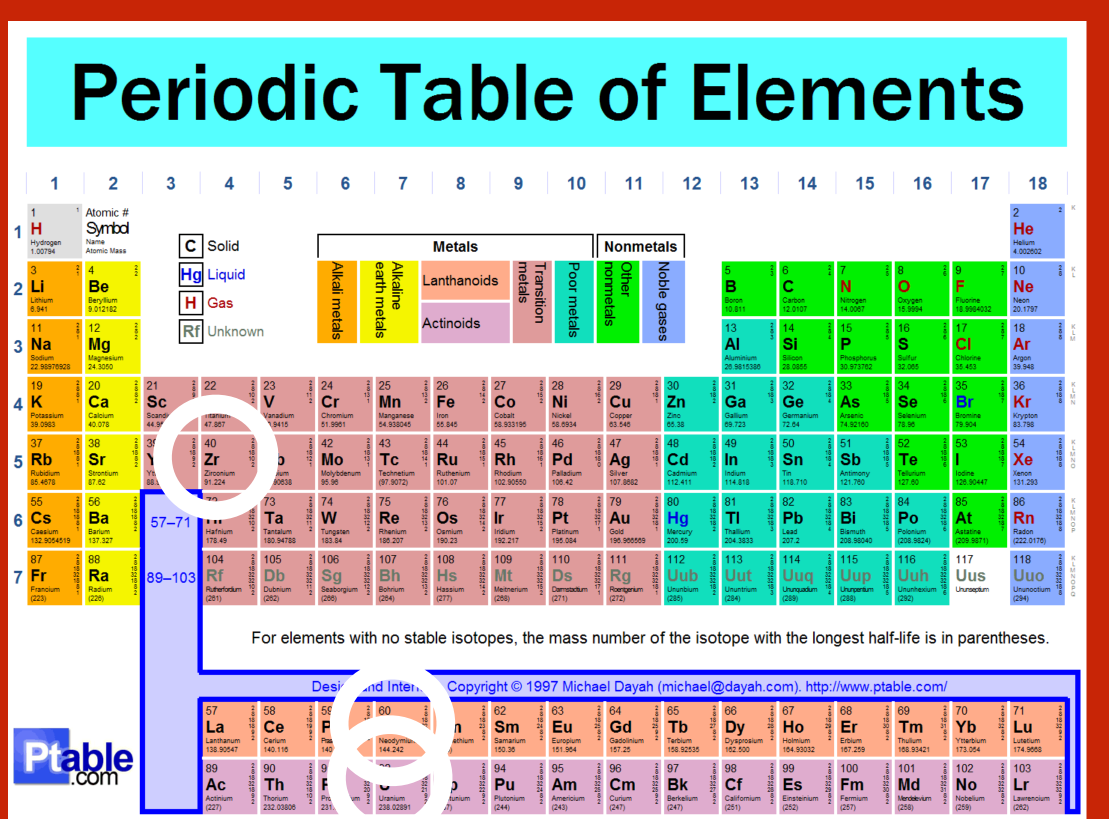


the ‘‘mass deficit’’ in nuclei

is observable and works for good and for ill.

a bound system
like an atom

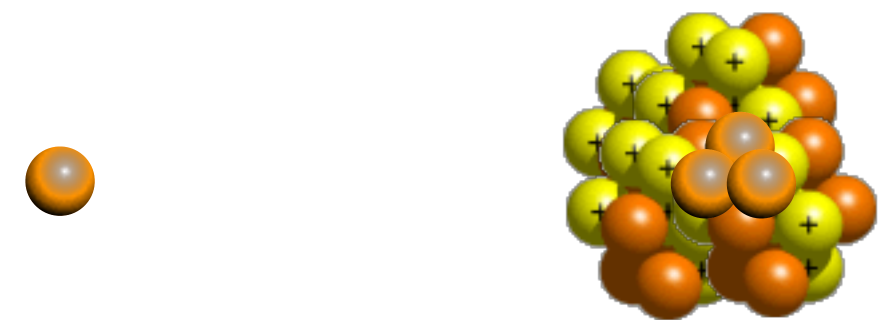
but much stronger!



It happens many ways, here is one:

"Uranium 235" is a big nucleus of 92 protons and 143 neutrons

^{143}Nd



^{90}Zr

the $M(^{235}U) < 143 \times M(\text{neutron}) + 92 \times M(\text{proton})$

so, it's "bound" like Hydrogen

But when a neutron tickles it...
the mass deficit in binding energy is released as K...which becomes heat in nuclear reactors

$M(^{235}U) + M(\text{neutron}) > M(^{143}Nd) + M(^{90}Zr) + 3 \times M(\text{neutron})$

by 200 MeV

1 gm ^{235}U releases 23,000 kW-h

about 25 households' energy needs

looky here...

two things to worry about

$$E_T^2 = \left(mc^2\right)^2 + (pc)^2$$

Energy and momentum are related for massless objects...

$$E = pc$$

What about the negative solution?

$$E_T = \pm\sqrt{(mc^2)^2 + (pc)^2}$$

isn’t anything constant?

glad you asked

jargon alert: **invariant**

refers to:	something that is unchanging under some transformation
entomology:	*not*-variant
example:	the spacetime interval

Can’t we agree on anything?

Is EVERYTHING RELATIVE?

no.

“The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.

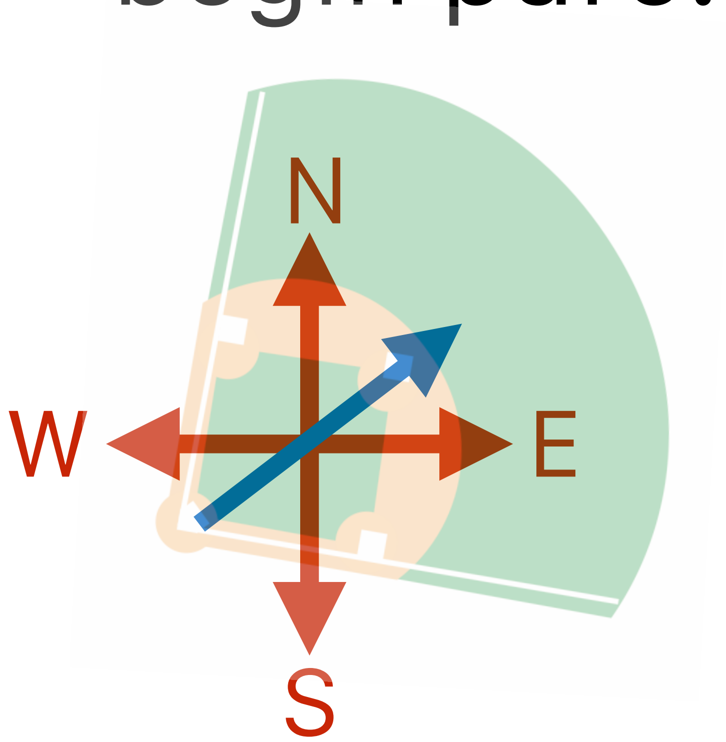
Hermann Minkowski

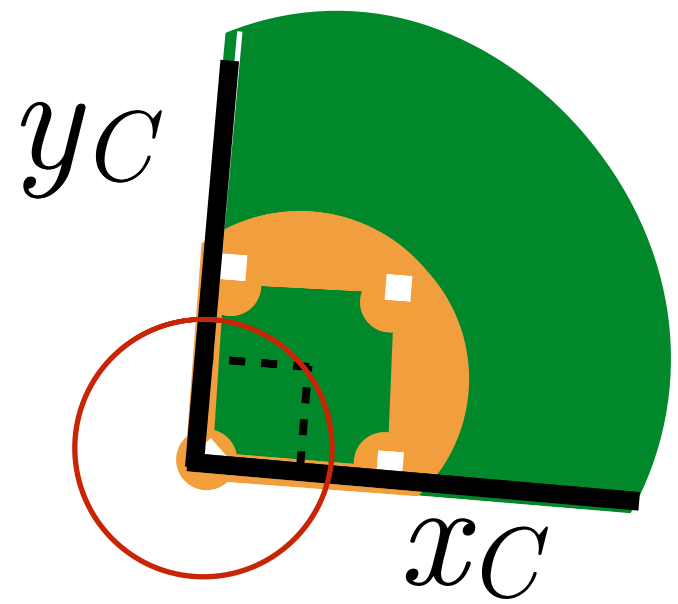
ISP220: Quarks, Spacetime, and the Big Bang

Space and time are mixed together and together become a single entity

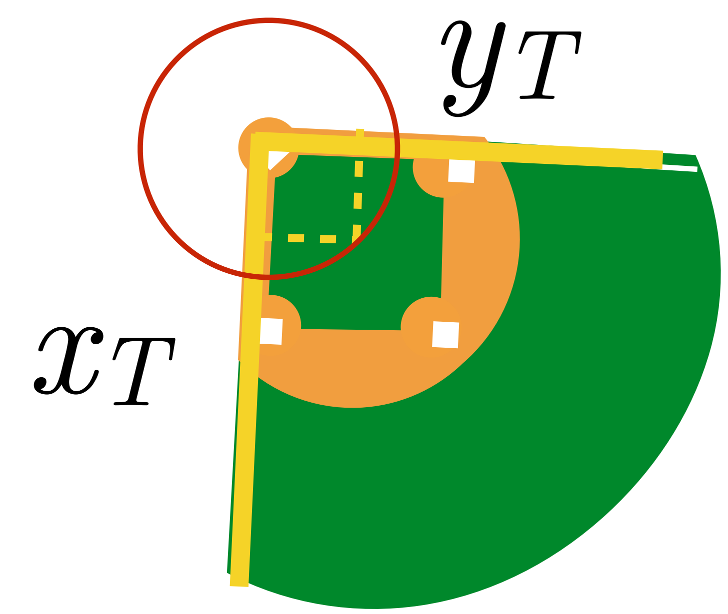
lay out a baseball field

begin pure: rule book says "East, north-East"





Wrigley Field: Cubs



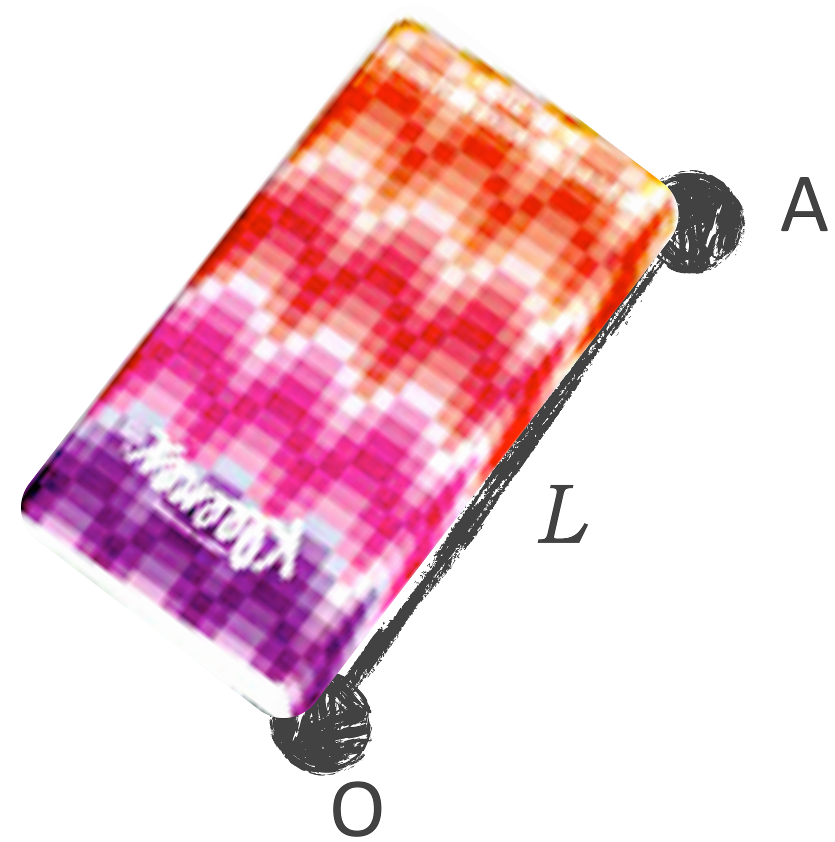
CoAmerica Park, Tigers

Distance to pitcher's rubber at Wrigley? 60 feet, 6 inches

Distance to pitcher's rubber at CoAmerica? 60 feet, 6 inches

$$L_P^2 = x_C^2 + y_C^2 = x_T^2 + y_T^2$$

take your board

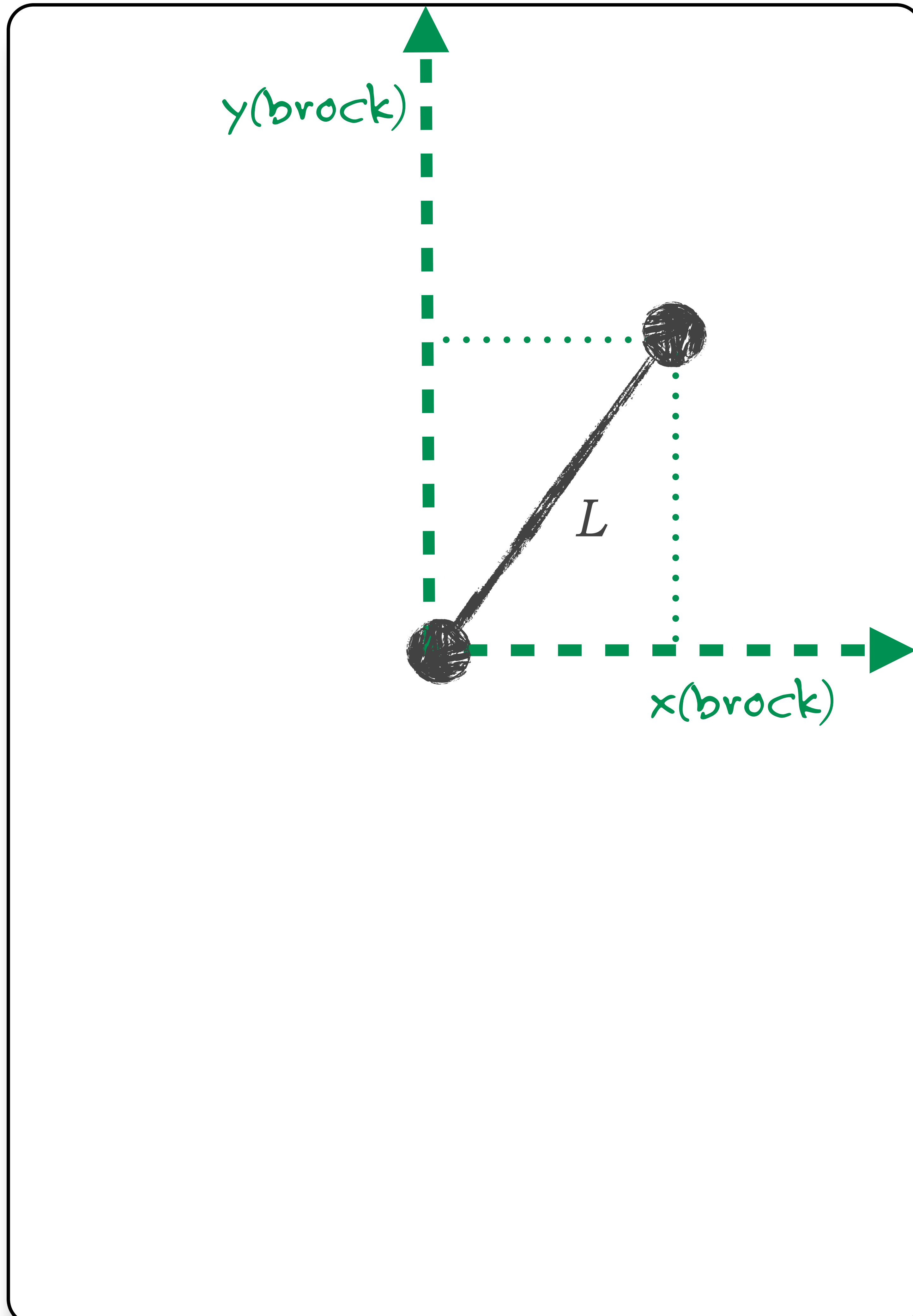


let's say L = 5 cm

take your board in your lap

about the lower dot:

Last name: A-M
draw an x-y set of axes with y pointing towards me



take your board in your lap

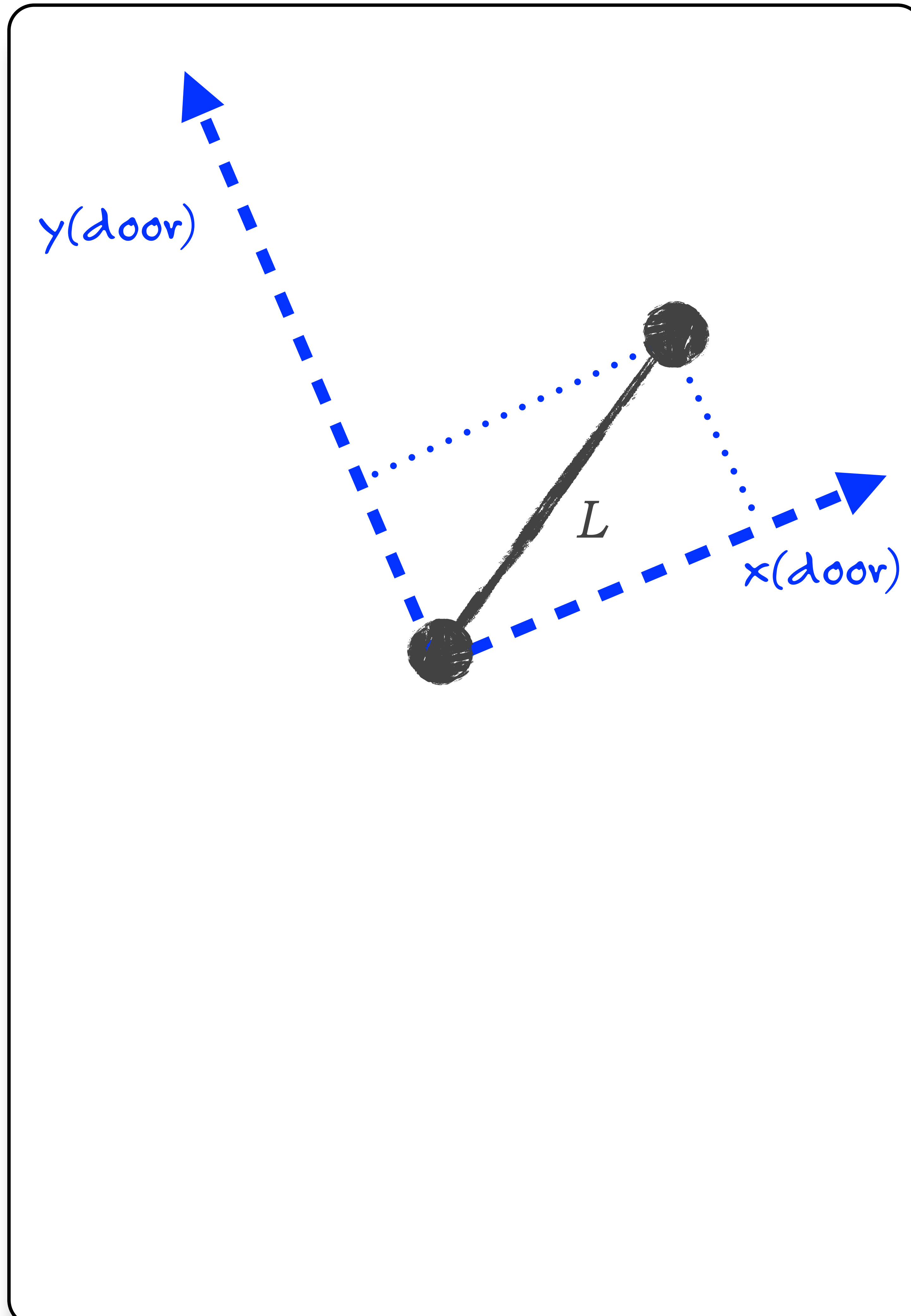
about the lower dot:

Last name: A-M

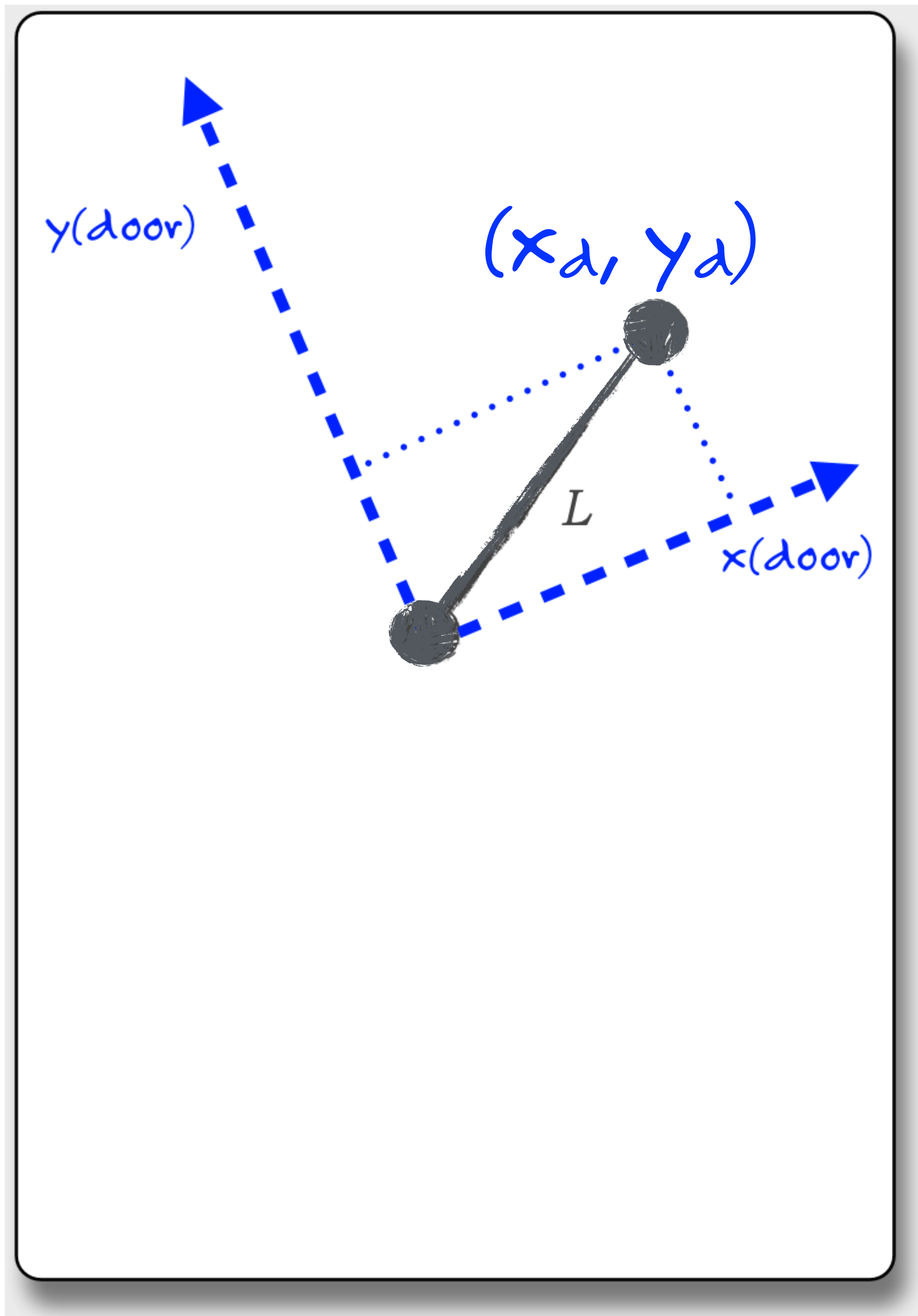
- draw an x-y set of axes with y pointing towards me

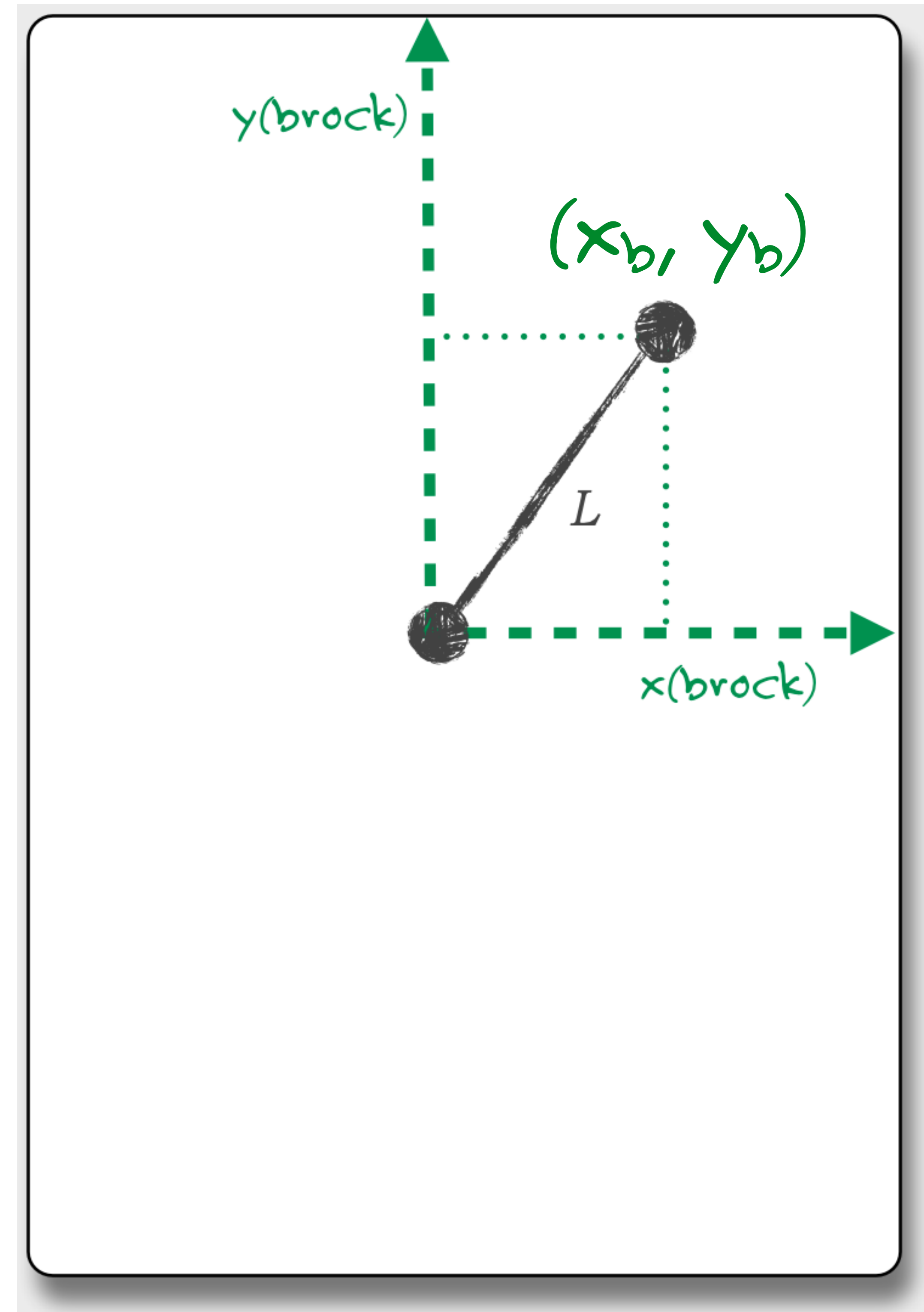
Last name: N-Z

- draw an x-y set of axes with y pointing vaguely towards the outside door



2 different coordinate systems...same line





Would the A-M people report the same x and y values as the N-Z people?

That is, is $x_b = x_d$ and is $y_b = y_d$??

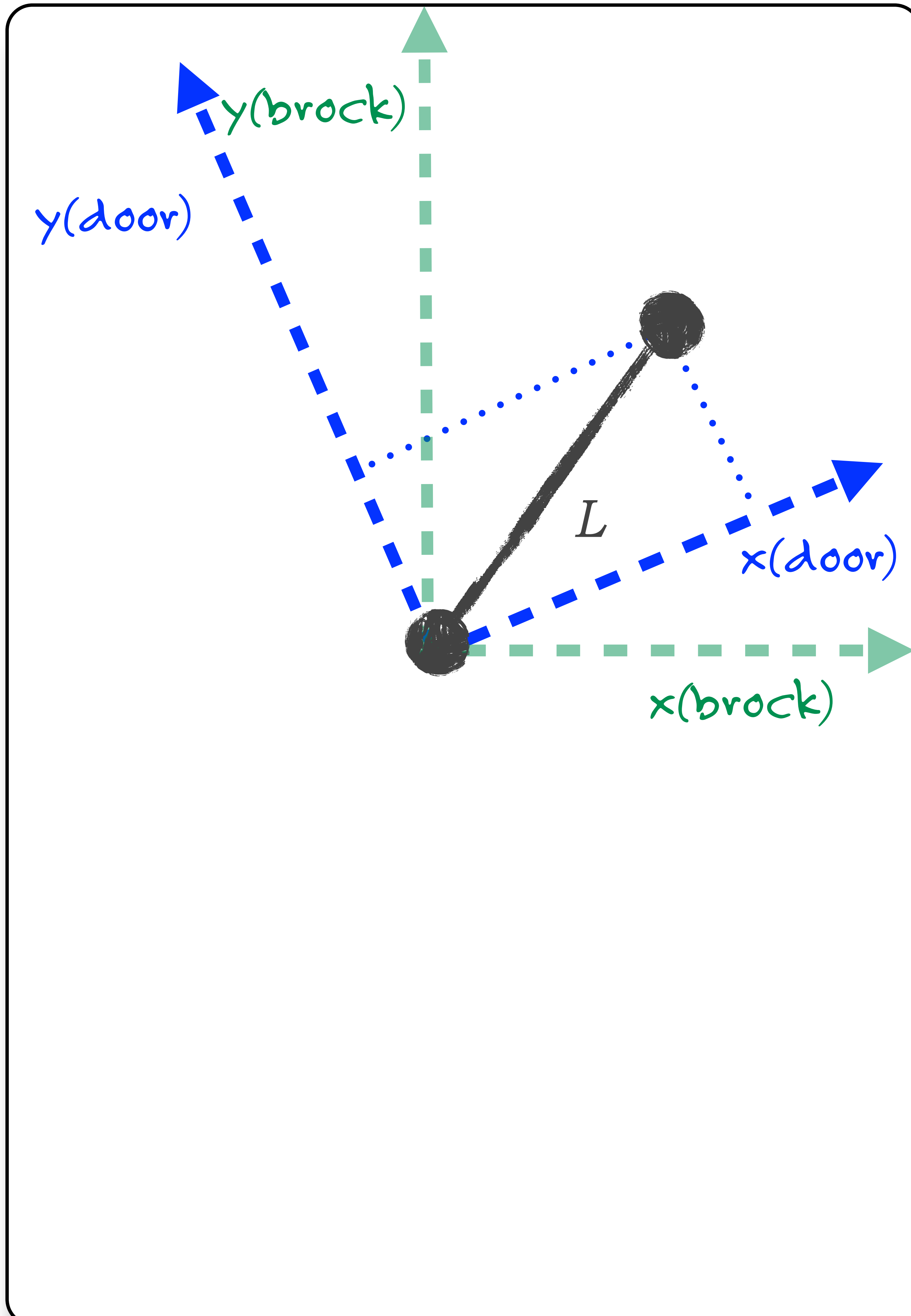
But both would report that the line, L, is 5 cm long.

take your board

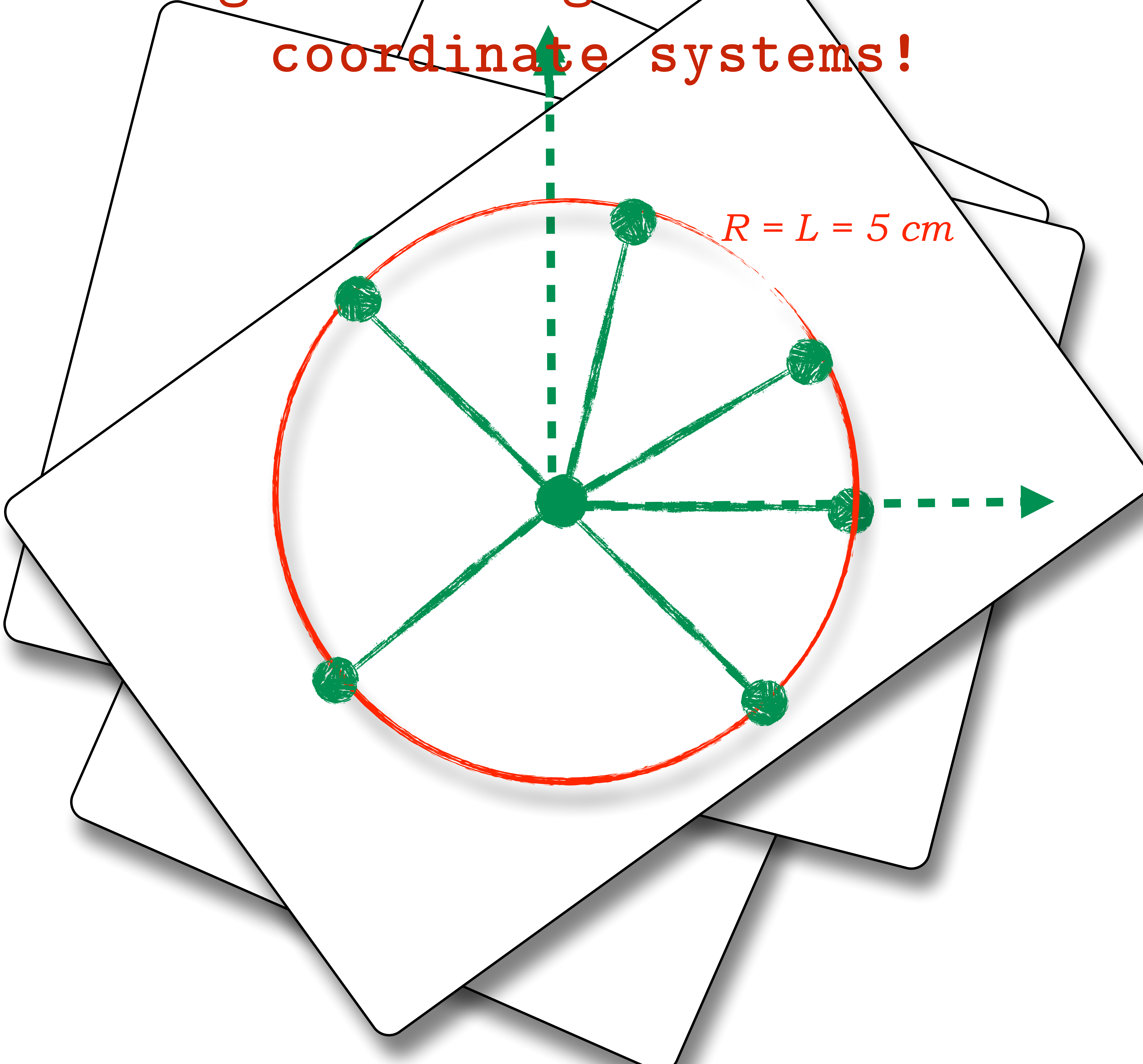
about the lower dot:

Last name: A-M
- draw an x-y set of axes with y pointing towards me

Last name: N-Z
- rotate your board so that your axes align with the A-M axes



do it again and again with different coordinate systems!



in ‘‘regular’’* geometry

Length is “invariant” with respect to coordinate systems

regular space distances preserve lengths

* “Euclidean Geometry”…of flat spaces

in SPACE: regardless of the coordinate system

in 2, 3, 4...any number dimensional space

LENGTH is "invariant" with respect to coordinate system

$$L^2 = \underbrace{x_1^2 + y_1^2 + z_1^2}_{\text{coordinate system “1”}}$$

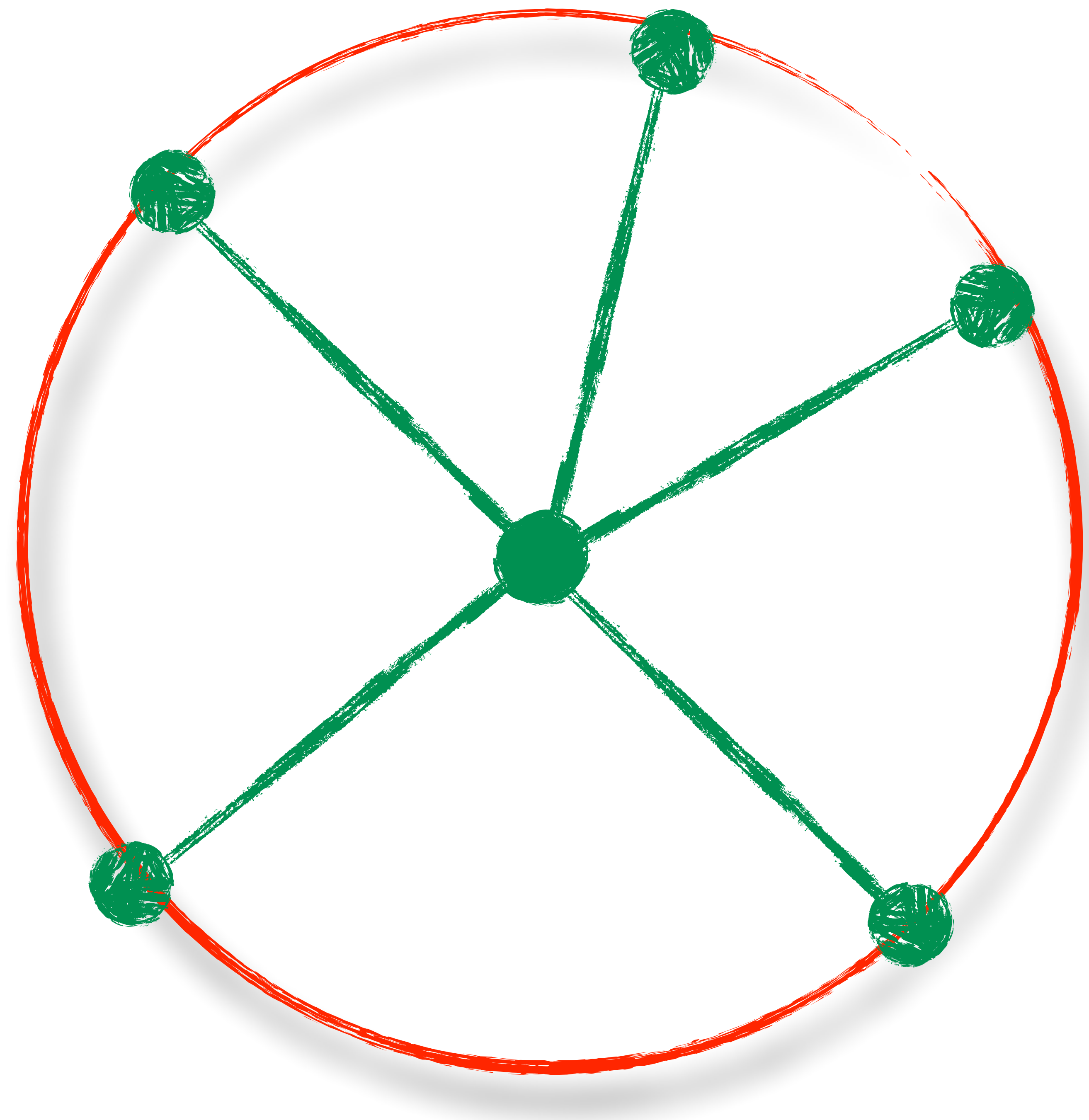
same length

This is a feature of “**Euclidean geometry**”

The “+” signs are the feature.

all coordinate systems in space

will place that point on the circle.



$$L^2 = x^2 + y^2 = x'^2 + y'^2 = x''^2 + y''^2 = x'''^2 + y'''^2$$

Let's call this: **the invariant curve**

What about SPACETIME?

what’s constant? What’s a Spacetime “Length”?

Try the same approach for spacetime as for space:
Euclidean

construct the Invariant Curve for spacetime

‘‘coordinate systems’’

can mean “reference frames”

let's do some superfluous learnedness

"Minkowski Space"
SPACETIME respects a different kind of "length" calculation

A quick aside:

Space and time are not exactly the same thing.

Space is measured in length units - meters.
Time is measured in time units - seconds.

Spacetime units need to be the same:
length is the convention.

Spacetime - *time* is measured with a scale factor that gets the units right: multiply by the speed of light, *c*.

So, a spacetime "distance" of 2 seconds?
represent as 2•*c* meters, or

(2 seconds) • (3 x 10^8 m/s) = 6 x 10^8 m - "lightseconds"?

Rule of thumb: light-length can be visualized as about 1 foot per nanosecond.

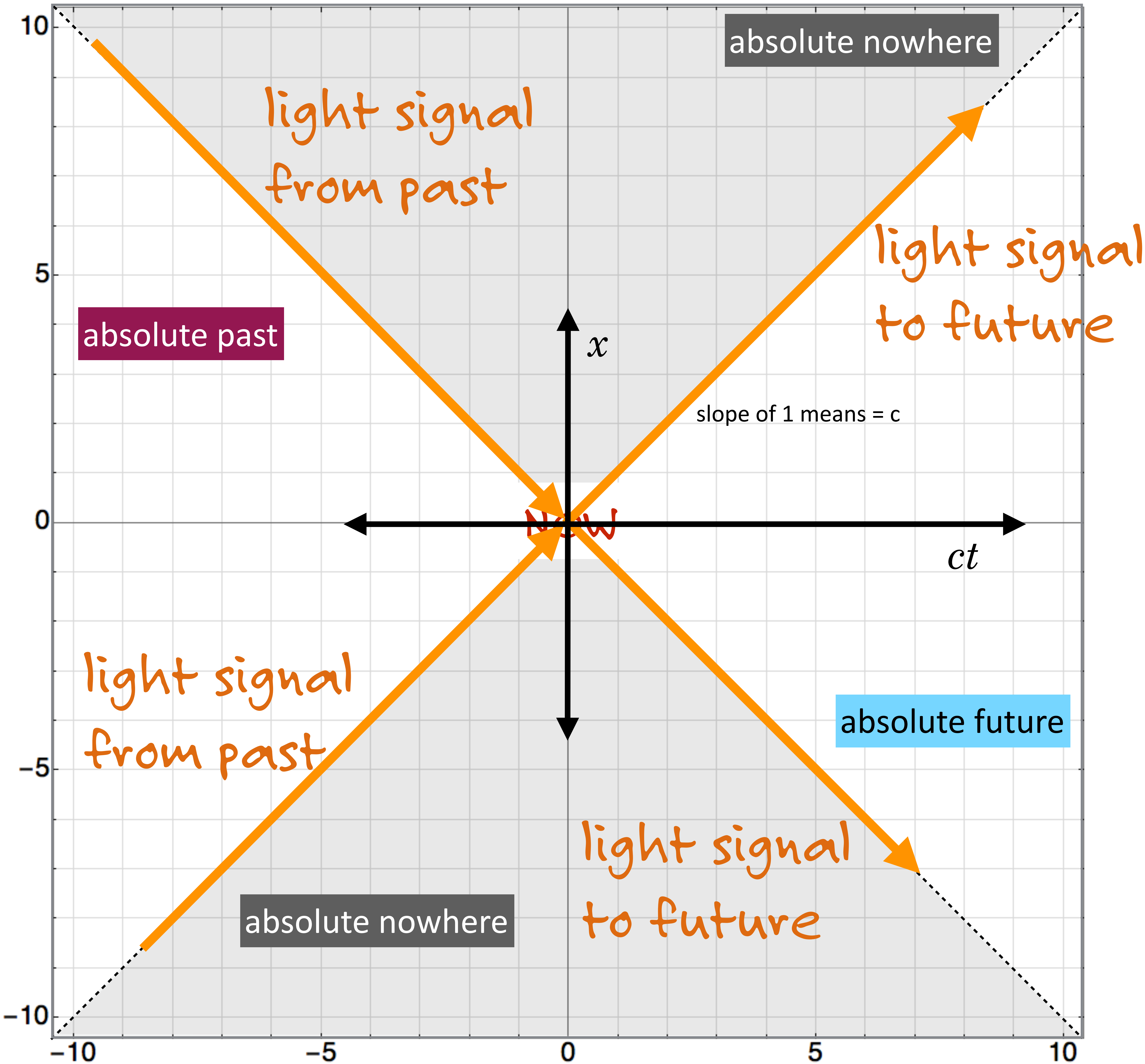
spacetime regions

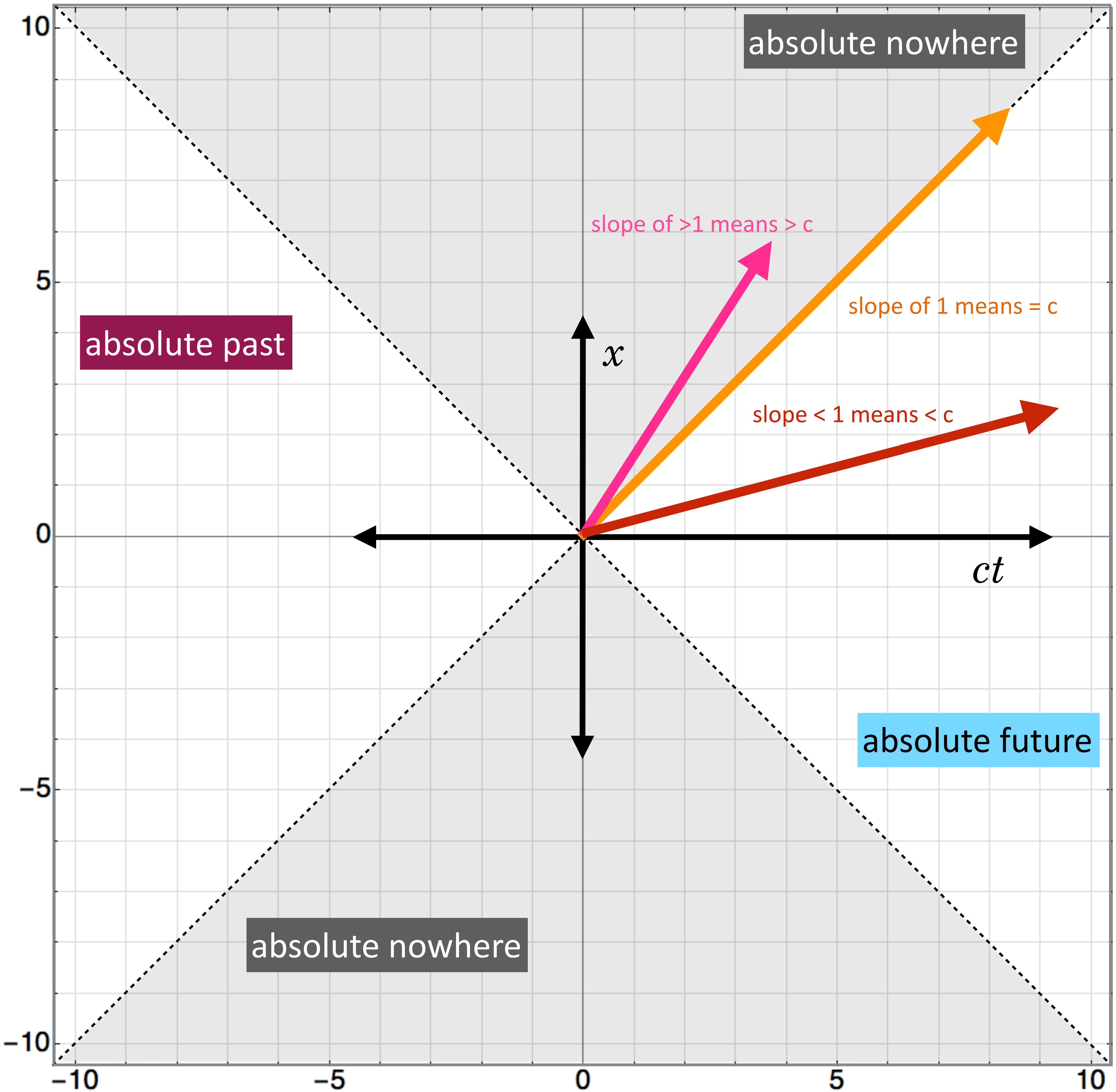
collapse three space dimensions into 1

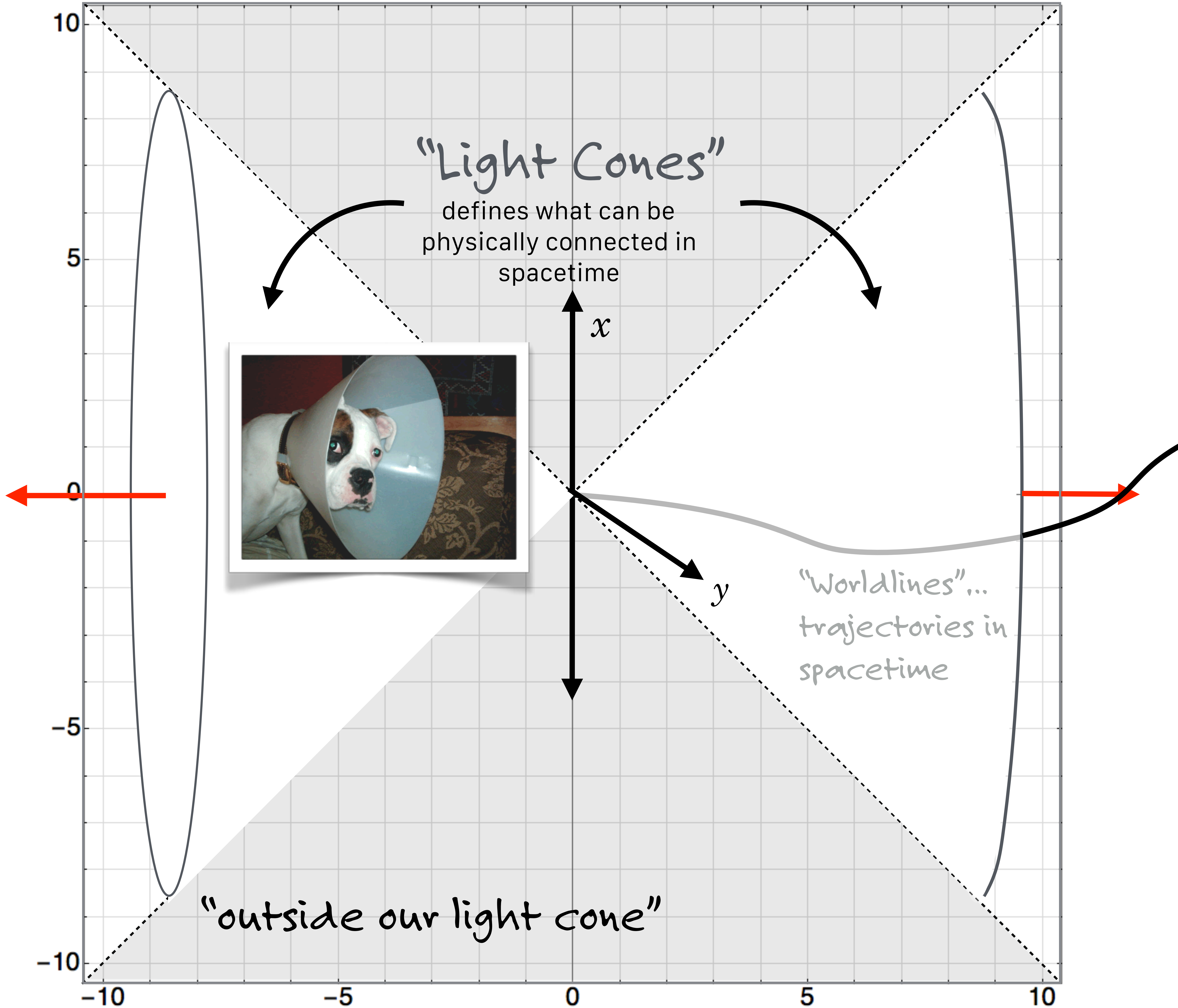
collapse one time dimension into…um…1

2 dimensional screen

substitutes for a 4 dimensional screen


absolute nowhere
light signal
from past
light signal
to future
absolute past
x
slope of 1 means = c
ct
light signal
from past
absolute future
light signal
to future
absolute nowhere
10
5
0
−5
−10
−10
−5
0
5
10


absolute nowhere
slope of >1 means > c
slope of 1 means = c
absolute past
x
slope < 1 means < c
ct
absolute future
absolute nowhere
10
5
0
−5
−10
−10
−5
0
5
10


"Light Cones"
defines what can be physically connected in spacetime
x
y
"Worldlines"... trajectories in spacetime
"outside our light cone"
10
5
0
−5
−10
−10
−5
0
5
10