## hi

## Day 15, 12.03.2019

Einstein's Special Theory of Relativity, 4.5

# 19 days until opening day 

Kiss week

## housekeeping

Gotta come to class
question about anything?
I'll make a movie for you:
Madame Curie movie - we have a quorumintavor
right now: looks like Monday, March 18
I'll remind you in FB to confirm that date
Section 2 folks:
Project has begun in phases:


Document 1: software, introduction, tutorial: due March 22
Document 2: your individual dataset and project instructions: due Final Exam
https://qstbb.pa.msu.edu/storage/QS\&BB2019/Homework Projects/
honors project 2019/Minervalnstructions1 2019.pages.pdf
MasteringAstronomy, finally after 3 emails and phone calls:
Course ID: MABROCK41459; free code: WSSPCT-BLIDA-INANE-TOGUE-RIGOT-UNRWA check it! let me know if it is now working...

March 2019




1. All laws of physics - mechanical and electromagnetic - are identical in comoving inertial frames.
taking Galileo seriously, and then adding Maxwell
2. The speed of light is the same for all inertial observers.

## a fact of nature

taking Maxwell seriously

## the airport, going fast

some time interval


## never get there



## mass

\&
energy
equivalent.

## An object in its own rest frame:

## has mass:

has inherent energy: $\quad E_{m}=m c^{2}$
speak of the "energy of mass" and the "mass of energy"

## An object in an Away frame, u:

has mass for Home frame: $\quad m_{R}=\gamma m$
has energy for Home frame: $\quad E_{T}=m \gamma c^{2}$
total energy: $\quad E_{T}=E_{m}+K$

## Newton vs Einstein




## a useful

## invariant

fun fact...just with a little algebra...

$$
\begin{aligned}
E_{m} & =m c^{2} \\
E_{T} & =m \gamma c^{2} \\
p & =m \gamma v
\end{aligned}
$$

and an important formal linkage

$$
E_{T}^{2}=\left(m c^{2}\right)^{2}+(p c)^{2}
$$

$$
E_{m}^{2}=E_{T}^{2}-p^{2} c^{2}
$$

$$
m^{2} c^{4}=E_{m}^{2}=E_{T}^{2}-p^{2} c^{2}
$$

no velocity dependences, just a number...

"Energy-momentum relation"...

$$
E_{T}^{2}=\left(m c^{2}\right)^{2}+(p c)^{2}
$$

## Energy/momentum relations:

the mass of an object in its own frame
"relativistic mass"... $m_{R}=m \gamma$
the mass of a moving object
"Energy"... $E_{T}=m \gamma c^{2}$
the total Energy of a moving object

$$
\text { "rest Energy"... } E=m c^{2}
$$

Kinetic Energy... $K=m c^{2}(\gamma-1)$

$$
\begin{aligned}
& \text { the energy } \\
& \text { due to motion }
\end{aligned}
$$

```
the mass-energy
of an object in its
own frame
```

Relativistic momentum... $p=m \gamma u$

$$
\begin{aligned}
& \text { momentum for } \\
& \text { each component } \\
& \text { of space }
\end{aligned}
$$

Energy-momentum relation... $E_{T}^{2}=\left(m c^{2}\right)^{2}+(p c)^{2}$

> an alternative, useful expression

## You might want to remember this:



energy of motion...Kinetic Energy<br>+ energy of mass... Rest Energy<br>Total energy of an object

there aren't any other kinds of energy

## a

## collision

## from

earlier

## where

## mechanical

energy was not conserved.

and they stick together

But we certainly would have said: $\quad m_{1}+m_{2}=M_{12}$

Now...energy conservation is different:

$$
E_{(\text {before })}=E_{(\text {after })}
$$

$$
\begin{aligned}
{\left[E_{(\text {Object 1) }}\right]+\left[E_{(\text {Object 2) }}\right] } & =\left[E_{(\text {Object 12) }}\right] \\
E_{m(1)}+K_{1}+E_{m(2)}+K_{2} & =E_{m 12}+K / 12
\end{aligned}
$$

$E_{m(1)} / \mathrm{c}^{2}+K_{l} / \mathrm{c}^{2}+E_{m(2)} / \mathrm{c}^{2}+K_{2} / \mathrm{c}^{2}=M_{12}$ thing!

## this is how

we can take two protons, crash them together, and produce 2 "top quarks"...

## each of which has the mass of 170 protons

## particle colliding beam



Use head-on collisions to make objects moretmassive than protons. Make Two things that each have $M$ (thing) $=3.5 \bullet M_{p}$


## a hydrogen atom, take 2

 weighs less than the components of a hydrogen atom
## so it can't fall apart into its components

 where is that "missing mass"?in the energy of the Electric Field,

```
speak of the "energy of mass"
    and the "mass of energy"
```



## isn't anything constant?

glad you asked
jargon alert: invariant
refers to:
something that is unchanging under some transformation
entomology: not-variant
example:
the spacetime interval

## Can't we agree on anything?

## Is EVERYTHING RELATIVE?

no.

The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies Cheir strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an

H. . Minderatic independent reality.

## ISP220: Quarks Spacetime, and the Big Bang

space and time are mixed together and together become a single entity

## "coordinate systems"'

can mean "reference frames"

## lengths are invariant

 as viewed from all coordinate systems

$$
\begin{aligned}
& L^{2}=x_{1}^{2}+y_{1}^{2} \\
& L^{2}=x_{2}^{2}+y_{2}^{2} \\
& L^{2}=x_{3}^{2}+y_{3}^{2}
\end{aligned}
$$

## lengths are invariant

 as viewed from all coordinate systems

$$
\begin{aligned}
& L^{2}=x_{1}^{2}+y_{1}^{2} \\
& L^{2}=x_{2}^{2}+y_{2}^{2} \\
& L^{2}=x_{3}^{2}+y_{3}^{2}
\end{aligned}
$$

## all

## coordinate systems in

 space will place that point on the circle.

$$
L^{2}=x^{2}+y^{2}=x^{\prime 2}+y^{\prime 2}=x^{\prime \prime 2}+y^{\prime \prime 2}=x^{\prime \prime \prime 2}+y^{\prime \prime \prime 2}
$$

Let's call this: the invariant curve

# What about SPACETIME? 

what's constant? What's a Spacetime "Length"?
Try the same approach for spacetime as for space:
Euclidean
construct the Invariant Curve for spacetime

## spacetime regions

collapse three space dimensions into 1
collapse one time dimension into...um... 1

## 2 dimensional screen

substitutes for a 4 dimensional screen


# a trial invariant curve 

for the airport

For two observers in two rest-frames:
suppose invariant spacetime curve is like:



# make sense 

right?

## How about a hospital?

## we ${ }^{\prime} r e$

 afterthe invariant curve for spacetime guess that it's a circle like "regular" geometry!

Now, OA are event intervals, not just space-lengths


If spacetime's invariant curve is a circle...then

- if $A$ is the event in one frame,
- then $A$ is another viewpoint from another frame

But...so is A okay in a third frame. Uh oh.

## the

## invariant

"length"
in

## spacetime

## "the interval," $s$

## Remember:

The invariant curve for space is the equation of a circle:

$$
L^{2}=x^{2}+y^{2}=x^{\prime 2}+y^{\prime 2}=x^{\prime \prime 2}+y^{\prime \prime 2}=x^{\prime \prime \prime 2}+y^{\prime \prime \prime 2}
$$

Minkowski's discovery was that the invariant curve for spacetime is

$$
s^{2}=c^{2} t_{H}^{2}-x_{H}^{2}
$$

## the equation of a hyperbola

This is the spacetime "length" that all inertial observers would agree on.
"s" is the "spacetime interval"





there is no frame in Benjamin's future in which "crying" comes before birth


$s^{2}=\left(c t_{\mathrm{H}}\right)^{2}=\left(c t_{\mathrm{A}}\right)^{2}-x_{\mathrm{A}}^{2}$
$s^{2}=\left(c t_{\mathrm{H}}\right)^{2}=3^{2}=9$
$s^{2}=\left(c t_{\mathrm{A}}\right)^{2}-x_{\mathrm{A}}^{2}=4.2^{2}-2.9^{2} \sim 9$



## causality is preserved

in Minkowski spacetime

## a useful

fun fact...just with a little algebra...

## invariant

| $E_{m}=m c^{2}$ | $m^{2} c^{4}=E_{m}^{2}=E_{T}^{2}-p^{2} c^{2}$ |
| :--- | :--- |
| $E_{T}=m \gamma c^{2}$ |  |
| $p=m \gamma v$ |  |
| and an |  |
| important |  |
| formal linkage |  |

another invariant...independent of the frame, just like:

kinship:
$t$ and $E$
$x$ and $p$

# three things are always, always constant 

 speed of light: c spacetime interval: sinvariant mass: $\mathrm{mc}^{2}$

Einstein preferred "Invariant Theory" to "Relativity"

## Cousin Quantities!

- Space and time are not separate entities, but linked as spacetime
- Electric and magnetic fields are not separate entities, but linked as electromagnetism
- Energy and momentum are not separate entities, but linked as 4-momentum


## so, how was this all received?

According to Einstein's sister,
...he anticipated a large reaction with much criticism
What he got at first was silence.
oh, a nice note from Max Planck asking for some clarification
then a seminar by Planck in Berlin which touched on Relativity...

- only then... a little professional attention, to "Prof. Einstein, University of Bern"

The first paper published on Relativity by not-Einstein:
also by Planck, who derived the relativistic momentum relation, $p=\gamma m v$

The 1908 Minkowski lecture, in which he worked out completely in modern form the mathematics of relativity and the spacetime view got people's attention

What about experiment?

## so, how was this received?

According to E's sister: not well
"Icy silence followed the publication"
Max Planck
editorial board member of Annalen der Physik
"immediately aroused my lively attention"
gave lecture at Berlin, published himself (1906), and began correspondence:
""Relativtheorie"
sent his assistant, Max Laue to visit in 1907
Laue published 8 papers on relativity himself in the next 4 year:
Einstein was not Dr Einstein until 1905 at University of Zurich

promoted from 3rd class technical expert to 2 nd class

## not everyone liked Relativity

## Antisemitism was in the fabric of European life

"As remarkable as Einstein's papers are...it still seems to me that something almost unhealthy lies in this unconstruable and impossible to visualize dogma. An Englishman would hardly have given us this theory. It might be here too, as in the case of Cohn, the abstract conceptual character of the Semite expresses itself." Arnold Sommerfeld

## incredibly proliforic

1906,07
17 papers published
played in a string quartet weekly
good father to 3 year old
commissioned to write a review
had no library to do a literature search, "...closed during my free time."

Privatdozent position at Bern? A story ensues

## the first experimental confirmation

## New experiments were done,

## and by 1910, the results were:



Kaufmann lost again...
Max Planck corrected his analysis

These results are from 1910 for three experiments, and the curve is the special relativity prediction

From this point on relativity has become a part of everyday scientific and engineering life


