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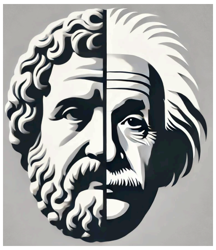
Stories of Motion and Light
From the Greeks to Einstein

CHASING REASONS

THE GREEK PATH TO EARLY SCIENCE

RAYMOND BROCK

Stories of Motion and Light From the Greeks to Einstein: Book I



CHASING REASONS

The Greek Path to Early Science

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Preface:

Why Tell Stories of Motion and Light?

Did the “Einstein” in my title, *Stories of Motion and Light From the Greeks to Einstein* catch your eye? (Let’s nickname it: G2E.) Actually, Albert Einstein only makes his appearance in the last book of the multivolume G2E because this project is about the stories of his virtual, historical collaborators. Over centuries, two successful models of MOTION and LIGHT matured at the close of the 19th century and found themselves at odds. We’ll follow their development and see how the 26 year old, less-than-unknown Albert Einstein cured what was a puzzling inconsistency between them. His solution was written in his spare time in a remarkable paper ushering in one of our most trusted physics models: the Theory of Special Relativity.

> I’ve been a professional particle physicist for half of a century, and I suffer from an unusual affliction that affects my teaching and research. Before I can teach something old or learn something new, I have to know its history. This isn’t an especially efficient way to work(!), but it’s led to a fulfilling pastime and, I suspect, unusual classroom experiences. I’ve become so sure of this approach to teaching physics that I even tell biographical stories in mathematically intense (calculate! calculate!), advanced graduate physics classes— even jaded, exhausted graduate students like to hear them.

For 500 years, my community of physicists, née natural philosophers, has spoken the same mathematical language and shared nearly the same goals. As we *extend our predecessors’ projects* across the decades, it can feel as if we’re connected—almost collaborating, if you will. It’s akin to a conversation when we revisit these models in research or a classroom, and, sure, that sounds a bit romantic. But maybe it’s why physicists and astronomers usually enjoy the pedagogy of teaching so much.

G2E follows almost two dozen virtual colleagues (“From the Greeks to Einstein”), pointing the way toward Special Relativity—colleagues whom I’ve grown to care about. I’m repeatedly amazed at their creativity and their ability to concentrate despite sometimes difficult lives.

> I imagine the orphaned, teenage Aristotle sent alone to Athens to study under Plato, only to find that the older philosopher was in Sicily. I think about Copernicus' powers of concentration, who kept observing the sky and calculating while literally fighting for his life and commanding a besieged bishopric palace in Prussia. Kepler's life would have brought most of us to our knees, but his productivity was remarkable. I'm moved by Newton's loneliness, Galileo's unnecessary feuds, the abuse suffered by Thomas Young, and Michael Faraday's debilitating memory loss. I think about a 19-year-old Albert Michelson boarding the brand new Transcontinental Railroad in San Francisco to plead his Annapolis rejection to President Grant personally. These stories matter to me because they're so human and also because in spite of tough circumstances...*they still got their work done*. These are the kinds of stories I want to tell you.

In the history of ideas, going against the grain requires personal courage and can be an act of loneliness and sometimes personally difficult. It requires taking a step *beyond* your trusted, historical collaborators.

G2E's title is explicitly **Stories** of Motion and Light..., emphasizing my teaching approach: personal and professional accounts of interesting people, their times, their productive ideas, and how they underwent their *push beyond*. If I'm successful, G2E will teach you some physics and astronomy and, at the same time, inspire you like a good biography should.

Einstein?

Albert Einstein is usually imagined to be the very model of a modern major scientist. A brave genius, working entirely alone. By now, you know that's not my take. Inspired by his historical, but virtual colleagues he glued space and time together and calmed a slow-motion, nervous breakdown inside of the world's physics community. He resolved that problem between MOTION and LIGHT.

How we got to that point is the theme of G2E. Starting with the Greeks we follow the parallel storylines of these two very different theoretical clans: The MOTION clan has three theoretical families of MOTION IN THE HEAVENS, MOTION BY THE EARTH, and MOTION ON THE EARTH. The very different LIGHT clan has three members, namely OPTICS, ELECTRICITY, and MAGNETISM. I'll tag these families this way when the appear.

Those six different themes separately developed over centuries, and we'll watch them merge into that conflicting pair: MOTION and LIGHT, reconciled by Einstein. There was only one "Person of the Century," according to *Time* magazine. But my contention is that there might have been *other* qualifying "Persons of *their* centuries..." Idea Revolutionaries.

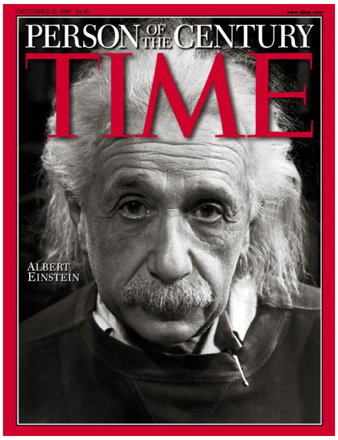


Figure 1: December 31, 1999 cover of *Time* magazine.

Revolutions?

A scientific revolution is a slow-walking event. And in G2E, if I'm to persuade you that my focus on unique individuals is helpful in following the history of ideas, I should be able to identify when a revolution occurred and why. Revolutions don't happen overnight or when someone lays down their pencil. The revolutionary nature of someone's Project[1] reveals itself *only in retrospect*. But outside of the scientific community, the idea of "revolutions" is controversial.

Copernican Revolution?

I'll bet you've heard of the "Copernican Revolution." Both words in that phrase annoy many modern historians. "Copernican" because it singles out an individual as special. "Revolution" because it suggests that there were abrupt, inevitable changes in the flow of intellectual history. Historian of science Steven Shapin is one of the voices of a movement that has recoiled against the idea of THE Scientific Revolution and certainly that a single person might be responsible. In his 1996 *Scientific Revolution*, he begins: "There was no such thing as the Scientific Revolution, and this is a book about it." (Steven Shapin, 1996) This bristled physicist and Nobel Laureate, Steven Weinberg, and in his chapter on Copernicus in the popular *To Explain the World*, (Steven Weinberg, 2015), he chided Shapin with, "There was a scientific revolution, and the rest of this book is about it."

I've seen this up close since my long career has straddled a bonafide revolution stimulated by special individuals, Weinberg among them. So, I've seen a revolution and worked with four creative Nobel Laureate revolutionaries.

Historians are put-off by what's called the "Great Man Theory" of history.[2] And, historians of science are often in that camp. However, we scientists are fully aware

[1]Yes, capital "P" Project which you'll understand in the Prologue.

[2]The "Great Man Theory" of history bristles at the idea, for example, that George Washington was destined to be great—we'll tell stories of cherry trees and absolute thruthfullness and the inevitability

that some of our (historical and current) collogues stand out above the rest of us and I (and maybe Weinberg) are clearly sympathetic to a "Great Scientist Theory" of history.

> Here's an analogy: Every player in the history of the National Basketball Association is a freakishly skilled athlete. But there are some among them, who do amazing things and who have revolutionized the sport. Before Michael Jordan, the best paid players were the centers—for almost 30 years before Jordan, except for two, all MVP awards were to centers. The flow of the game was directed by the centers. Point scoring? Centers. After Michael Jordan, it's now the ball-handlers who control their teams and are highly valued. Even the rules changed (hand-check rules, in particular) to benefit ball-handlers and now the NBA all star ballot doesn't even have "center" as a position—"frontcourt" entries now. Not only did Jordan perform physical feats that caused his teammates —and opponents— to shake their heads in amazement, his Project revolutionized basketball. This "Great Person Theory" is common in sports and the arts...and science.

So I agree with Weinberg. There have been Revolutionary Scientists *and* there have been Scientific Revolutions, and the rest of this series is about them.

I've challenged myself to convince you that there have been revolutionary ideas and people who first had them and I have a tool to guide us: the Project.

of his personal influence on history. Yes, that's sort of silly.

Prologue:

Progress As Projects

I emphasized in my Preface that our work is a collaboration with, and sometimes an evolution from, our virtual predecessors' Projects. Here's my take on how progress happens, on the ground so to speak: Someone completes an interesting Project, perhaps having measured a surprising quantity or conceived of a new model or invented a new mathematical or experimental technique. And if by using those new tools, they solve some old problem or predict novel phenomena, then maybe that's attention-getting. But only when enough other scientists vote with their feet—and their precious time and resources— and adopt those new ideas or tools as inputs to *their* Projects, then, in retrospect, that original Project might be viewed as having been important. Should *the entire community* use those new concepts or tools or even more significantly, adapted a new conceptual framework of the universe? That's a revolution. There is no vote or a mandate, but personal, professional decisions that drive effort towards new Projects.

In trying to reverse-engineer the emergence of innovative ideas in physics for myself and my students, I find myself returning to what *individuals* do. I'm keenly aware that when I choose to spend my limited time and group resources on a project, it's both a commitment and an opportunity-loss for what I decided *not* to work on. So it's a personal decision, and making good choices depends on experience and good scientific taste. For me, the unit of progress in science is what I'll call the Project, which is a lot like how *you* might think of a project.

> There is a more standard, but disappointing "unit of behavior in science" called the "Paradigm" which came from Thomas Kuhn's historic 1962 *The Structure of Scientific Revolutions* (Thomas Kuhn, 1996). When we're working within a paradigm, we're doing what Kuhn called "normal science," which at some point, accumulates contradictions, develops a crisis, a revolution occurs, and a new paradigm begins. Kuhn had trouble clearly explaining what a paradigm was—21 different uses of the word were identified! For example, is it big, leading to historic Revolutions? Or could it be small...lots of paradigms in a scientist's lifetime? It was meant to be a collective worldview, a social thing, which was *also* a problem as it led to accusations of distressing relativism in science.

By the way, in Kuhn's original formulation, the passage of one paradigm to another is not progressive...just different. That was a problem for his model since, for we scientists, good science certainly makes progress, and my working model is designed to show how. I'll be didactic about Projects in my stories.

Simply put, any scientist's Project has inputs and outputs. In order for me to get a Project off the ground, let's think about what I must commit to as starting points. I can think of five categories:

1. **Numbers**. I'll have a set of factual commitments—numbers or parameters—about phenomena that I'll accept.
2. **Theories**. I'll commit to a set of theoretical concepts...accepted views of the world, so to speak.
3. **Techniques**. I'll have a commitment to a set of best-practice mathematical and experimental skills and techniques.
4. **Norms**. I'll inherit and initially commit to a set of community norms and expectations about what Projects are worth exploring.
5. **Curiosity**. This defines my Project's goals. I'll be curious about some actual or imagined phenomenon. Maybe I just want to measure a parameter or do a "what if" theoretical calculation or build an amusing mathematical model. For the duration of my Project, I'll commit to those goals, until they succeed or prove unhelpful.

The Projects in `G2E` were well-designed (after all, "revolutionary") and for each of them we can identify the five commitments. As a pedagogical tool in `G2E`'s historical approach, that's exactly what I'll do:

▻ For my highlighted scientists, I'll unpack their Projects and explicitly enumerate their commitments (#1 through #4) plus what sparked their curiosity (#5). We'll see how their work went from attention-getting to revolutionary.

Watch for full-page tables that summarize the inputs and outputs of each of the six catagories for every essential Project. Yes, I lied, There are actually six:

Science Is Public

That popular Einstein image of the completely isolated genius in productive science doesn't exist. There might very well be completely isolated geniuses, but in their isolation, they didn't influence anyone![3]

So, an essential aspect of doing productive science is doing public science. Some might have had real-time collaborators, or some really did work alone in their rooms, but they all "collaborated" with those who came before them. That's where continuity and progress in science come from: learning from those virtual collaborations and then going beyond them.

[3] We'll see a few who were found to have been on the right track only in retrospect, but they were quiet about it.

So, I have sixth category with two aspects. First, we all learned from others, and I'll try to identify important sources and inspirations for each scientist's Project. Second, any successful Project concludes with a paper, a book, a speech, letters, or a lecture. These are such essential aspects of scientific advancement that I'll call them out. So, a sixth entry among my Projects' commitments:

6. **Influences and Products** I'll have learned from others and I'll have memorialized my conclusions in public products.

Let's begin with the Greeks.

Introduction: CHASING REASONS

The Greek Path to Early Science

It may have once been the case that all roads lead to Rome, but for most of Western philosophy, physical science, and mathematics, modern roads have led *from* Greece. That's because Greek philosophers learned to explain their natural world by demanding reasons, rather than accepting supernatural accounts. Their journey pointed the way toward our Western foundations of modern physics and astronomy: their approach endured.

> Intellectuals in both ancient China and India developed robust natural philosophies. Both cultures' scientists observed the world, documented the world, and engineered the world. And, but for historical accidents, they might have driven eventual Western natural philosophy. Certainly, India made substantial contributions to mathematics that were incorporated into the Western tradition through Islamic mathematicians and astronomers. The Greeks were imaginative and systematic and when their projects were carried forward by Islamic expansion, the rest is...well, history.

In this first book in my series *Stories of Motion and Light From the Greeks to Einstein* my goal is to trace the separate paths the Greeks took to model how things move (the subject of MOTION) and how we see (the subject of LIGHT). It was they who discerned which phenomena deserved careful study and to define what well-formed natural science questions and acceptable answers should look like. Yet despite their early insights, a more modern understanding of MOTION and LIGHT had to wait for Medieval thinkers, since ancient Greeks repeatedly tripped over Aristotle's philosophical authority. Their brightest example of science was Hellenistic Greek Astronomy and sure, Aristotle hung around, but progress in studying the heavens accelerated when his rules were bent, just a little.

MOTION IN THE HEAVENS was a decidedly different problem than both MOTION BY THE EARTH and MOTION ON THE EARTH. The former seemed to consist of almost perfectly repeatable movements while things on the Earth behaved differently, every time. MOTION BY THE EARTH represented mixtures of both problems and

Aristotle's inclusive, huge philosophical system coupled everything together so tightly that progress in astronomy slowed. But over time, his hold on astronomy lessened as Hellenistic Greeks looked up at the night sky...and precisely measured relative positions of stars and planets as a function of time. Then they built the first quantitative models of the cosmos including determinatition of the size and shape of the Earth and the relative distances among the Earth, Moon, and Sun. Seemingly intractable problems (why are the seasons different lengths? why does Mars' motion seem to reverse course?) became subjects of research.

At this point, Aristotle's authority over heavenly motions began to wane, though his influence over earthly motions remained strong. So his influence constrained the modeling of the cosmos until Copernicus, and then Johannes Kepler, freed us of his grip.

In this chapter, we'll trace the progress of problem-solving, from the most abstract Greek thinkers to Claudius Ptolemy—a culturally Greek, but Egyptian-Roman citizen—who arguably holds the record for the longest-lasting scientific influence in history. As a mechanism, his model of the cosmos worked so well that, corrected for modern parameters, it still accurately predicts astronomical events today. Yet, despite this enduring legacy, Ptolemy was the last of the ancient Greek scientists.

Our debt to the Greeks is less profound when it comes to LIGHT. As for ELECTRICITY and MAGNETISM, these phenomena didn't fit neatly into anyone's scientific or philosophical framework, so they gave us our names for the phenomena[4]—but then largely ignored them. The nature of vision— OPTICS—was a speculative subject for thinkers like Democritus, Aristotle, and others. It wasn't until Euclid and Ptolemy recorded geometrical descriptions of how light rays behaved through their systematic calculations and measurements. We needed to wait 1700 years for progress with LIGHT!

We'll begin with the Presocratics in Chapter 1 where it all gets started. Love them, or hate them, the endurance of Plato and Aristotle has been impressive (or oppressive) and in Chapter 2 we pick out the pieces that inform our goals. Geometrical astronomy begins with Plato's colleague, Eudoxus and in Chapter 3 I'll remind us of what they (and we) see every night in motions of the planets and stars and describe the early attempts at modeling those motions. Finally, we'll work our way to the pinnacle, that of Ptolemy's impressive model in Chapter 4.

[4] *ηλεκτρον* for *elektron*, "amber" and *μαγντιξ λιθοξ* for *magnetus lithos*, Magnesian stone.

Chapter 1

It's All Greek To Me : Presocratic Greeks

"We are all Greeks. Our laws, our literature, our religion, our arts have their root in Greece."

- Percy Bysshe Shelley (1792-1822), *poet*

"There is a land called Crete, in the midst of the wine-dark sea, a fair, rich land begirt with water; and therein are many men past counting, and ninety cities."

- Homer, *The Odyssey*

Since this is a book on physics, and since you can only invent something once, I want to tell you how physics started. This is the first of four chapters on Greek philosophy and natural science, and they will be different from the ones that follow as I'll talk about many Greeks rather than focus on a few. In this chapter we'll learn about new habits of mind that evolved two centuries before Plato and drive us still.

About the Greeks' nascent science, I'll ask four questions that will guide our whole project: what is the nature of motion by the Earth? What is the nature of motion on the Earth? What is the nature of the motions of the heavens, and what is the nature of light? You'll know when I'm focused on one of the four because I'll tag the context with: "MOTION" or "LIGHT." Within each, there are more details: MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS as well as MAGNETISM, ELECTRICITY, and OPTICS.

The quotes above are a small sampling of how we modern scientists should look back at the Greeks. In many ways, my field of particle physics is relentlessly Platonic (but don't tell anyone that I said that!). Plato (and, to a lesser extent, Aristotle) continues to challenge us: *What* can we know? And, *how* do we know we know that something is true? And, of course, *how* do things move?

The next chapter will deal with them—but Plato was reacting to the thinkers who came before him, traditionally called "Presocratics," obviously meant to cover those who came before Socrates. Now, "Presocratics" is an all-purpose label that applies to people before Socrates, but also those who were contemporary to Socrates, and even some who where younger than Socrates. For all practical purposes, it essentially means pre-Plato, and this chapter is about the Presocratics.

I can identify four Greek Presocratic Research Programs that still seem modern to me. Each theme was seeded before Plato and Aristotle and then watered and then harvested by them and are:

1. **Is the universe constructed of fundamental building blocks, and might those fundamental entities behave together according to rules?** This is the nature of physics today: my field of particle physics is dedicated to finding and characterizing the fundamental entities that make up everything else. Quarks and Leptons are those entities. But just stockpiling particles is merely stamp-collecting. They have to interact with one another and so the rules are deeply important. We call them the four fundamental forces today.

2. **Is the universe inherently mathematical?** It's long been appreciated that the universe seems to operate according to rules that are mathematical or can be described as mathematical. Discoveries in physics and mathematics have influenced the other. Why that relationship exists isn't understood and is yet so persuasive to some theoretical physicists, that they postulate—still— that the universe is not just mathematical, but *is mathematics*. I'll have a lot to say about this as it underpins not only MOTION and LIGHT but all of modern science.

3. **How can we reconcile permanence with change?** This is a tricky issue and one that bedeviled not only the Greeks but much of philosophy to the present day. Unraveling this tension is intimately connected to theories of knowledge: what can we know, and what can we trust? The permanent part of physics today refers to the various "conservation laws"...the Conservation of Energy, for example. But our

elementary particles move around; they mix together, annihilate, and are born out of the vacuum. All the time. Change and permanence agonized over by the Presocratics and Plato, are firmly a part of our modern story.

4. **How is the Universe structured, and what rules govern its beginning and current state?** "Cosmology" is the Greek word for this study that mashes together their words *cosmos* for "the world" or "universe," and *logos*, the word for "study of" and is now a modern term and a very sophisticated sub-discipline in physics and astronomy. Our Western study of the solar system started with the Greeks, was mangled through Aristotle's authority, quantified by Greeks after Alexander the Great, nurtured by Medieval Arab mathematicians, and solved by Renaissance and Baroque scientists. It took 2000 years to get right.

My first three Research Programs are fleshed out in this and the next chapter and I'll reserve astronomy for Chapters 3 and 4. Greeks reveled in drama, and it was within the turmoil and bloodshed between the Persian Wars and Alexander the Great that Western philosophy and our nascent science had their beginnings. So, we'll picture this as a play in eight acts. The curtain rises...on a catastrophe.

I imagine that it started out like any bright day on the northern coast of Crete. A lazy afternoon in this peaceful paradise.

Then, total darkness.

Without warning, the **loudest sound** ever experienced by humans was followed on the northern horizon by a hint of fire and smoke erupting tens of miles into the previously clear sky. Slowly, the sun dimmed, and then the sky became black as six inches of ash fell all over the island like a dirty rain. In fact, debris fell as far as the whole of modern Turkey, northern Egypt, and the Middle East. Following that sooty deluge, tidal waves fifty feet high engulfed the seaside areas of Crete and destroyed everything for kilometers inland. That terrifying −1650 day...

Wait...Negative years? I'm sorry, but in my head the timeline of history is a number line with positive and negative numbers—years. Sure, it's a number line without a zero, but BC or BCE isn't separated from AD, or CE by a year 0 either. The names are too clumsy and so I prefer almost-straight-up arithmetic to enumerate years since it makes it a breeze to compare one year to another.

...in the capital city of Knossos was the consequence of a massive volcanic eruption on the island of Santorini, about 100 miles to the north. Look at your map application and navigate to 36°23’41.46“ N 25°23’57.55” E. There you’ll see a little Packman-like, backward “C” feature in the Aegean Sea. That’s the scar—the caldera from the “Minoan Eruption”—left behind by the opening act in what might have been the story of us in the West.

Our tragic **Minoa**—modern-day Crete—was a refined culture of master architects, mariners, and traders, an apparently relaxed and leisure-loving people. Their cities didn’t seem to need much fortification—they seemed to be secure among themselves and were rulers of the sea. They were literate and created the first *symbolic*, written language—two of them, actually. Their ancestors were pre-Bronze-Age migrants from the north, seasoned with Egyptian influence from about –3000.

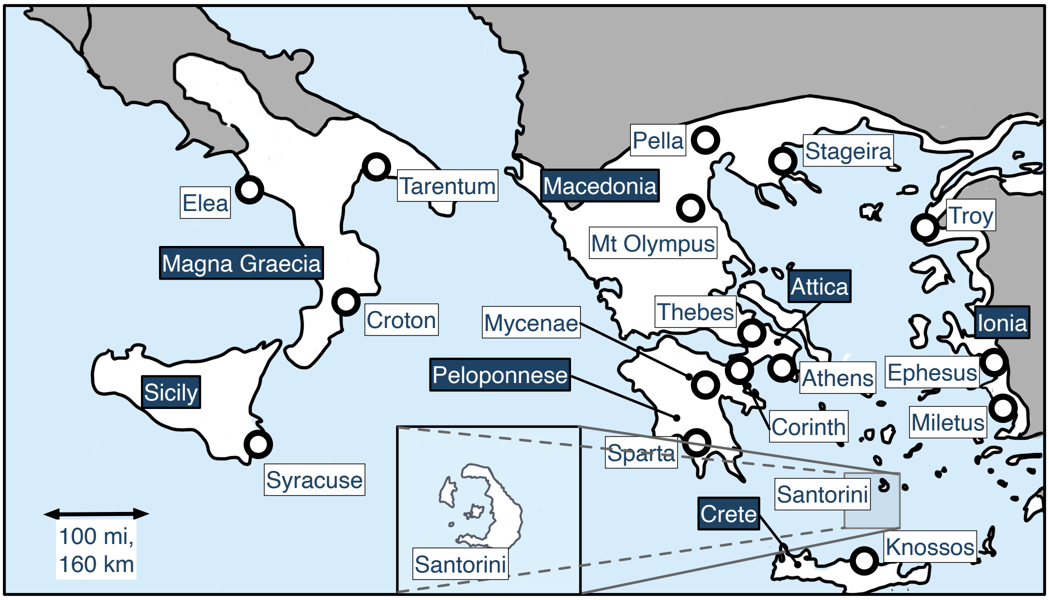
Over the next thousand years, Minoans and Phoenicians became Mediterranean, international sea-going powerhouses trading across its entire breadth. Think about that: 1000 years of prosperity! Trading partners inclusive of hundreds of different cultures. After the volcano, they rebuilt but were never the same and were likely absorbed by a rougher crowd from the Greek mainland. The southern peninsular region where Sparta and Olympia are located is called the **Peloponnese**, while the adjacent northern region where Athens is located, separated by the Isthmus of Corinth, is called **Attica**. The Minoans are our literate ancient scientific ancestors, influencing the Greek culture even though they ceased to exist

I like to think of those long-gone cultured Minoans as the polite part of our Western scientific ancestors—the smart side of the family. But the famously disagreeable Homeric Greeks came from that side of the family that you’d like to hide from your friends.

That “rougher crowd” were the **Mycenaeans** who evolved into the heroic Greeks of Homer’s *Iliad*, made perhaps slightly more civilized by their Minoan acquisition. The centuries following were eventful and then blank: Iron-weapon-wielding northerners created chaos with the Mycenaeans and eventually initiated a multi-century dark age. What emerged around −800 included the still-standing Athens, Sparta, and Corinth; the singing and eventual writing of the Homeric sagas; and an explosive emigrant population prominently on the Aegean islands, western Ionian shores, and the southern boot of Italy. Established by −650, these colonies were active traders, especially in **Miletus** in **Ionia**. Figure 1.1 shows the Greek colonial expanse and details of the immediate Aegean and Italian city-states.

1.1 A Little Bit of The Presocratic Greeks

Around 2800 years ago a proto-science began by people asking modern-sounding questions. We’ll concern ourselves with our scientific parents: the Milesians (in Ionia, on the modern-day west coast of Turkey), who invented the idea of substructure and natural rules; the Pythagoreans (in Italy) who emphasized the fundamental nature of mathematics, the Eleatics (in Italy) who fleshed out the tension between change and permanence, and the Pluralists (in Italy and Ionia), who found a rational



(a)

(b)

Figure 1.1: (a) The white are regions of active Greek language and culture from around the time of Socrates and Plato. The cities listed all figure into our story in this and Chapters 2 and 3. The inset highlights the island of Santorini, the caldera left from the massive "Minoan Eruption" of approximately –1600. It's now a destination for luxury vacations. (b) This is a view of the Mediterranean with white showing the Greek colonies from the same period as in Figure (a), but also superimposed is the political situation following the "Wars of the Diadochi," the successor generals of Alexander the Great's army. For our story, the Kingdom of Ptolemy in Egypt is most relevant and we'll care about the "Hellenistic" period, which is ushered in by the split-up of Alexander's conquests about 30 years after his death (which coincided with Aristotle's death). The cities noted in (b) are important for astronomy during that Hellenistic period in Chapter 4.

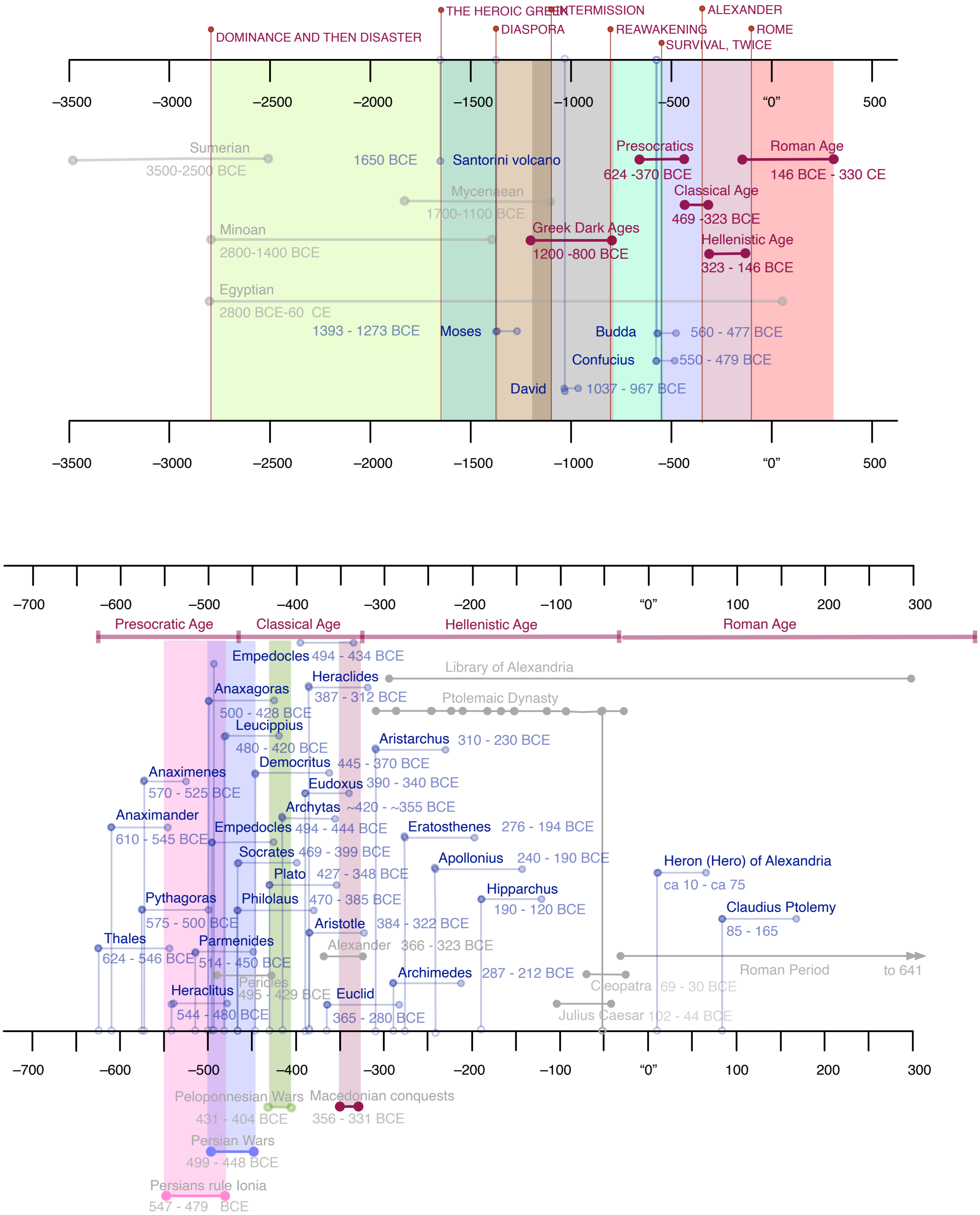


Figure 1.2: On the top, a Mediterranean timeline stretches from pre-biblical times to the end of the Roman empire. The bottom lays out the life spans of all of the Greeks you probably ever heard of...and the overlapping disasters that surrounded their lives.

alternative to the most persuasive and extreme of the Eleatics.

Brief relative (and rare) peace in the Ionian colonies, their positioning in the Mediterranean as a shipping crossroad, and the growth of large city-states led to a period suitable for the growth of a new culture. And this was what emerged: The beginning of Western philosophy. The time of the "**Presocratics**," literally those early philosophers who came before (or overlapped with) Socrates. These folks and their "Post-socratics (?)" asked modern-sounding questions of their surroundings.[1]

The timeline in Figure 1.2 shows roughly three distinct periods with names you might recognize. There are the Presocratics (from about –600 to about –430), the classic philosophers (from about –430 to about –250), and then the Hellenistic philosophers and scientists (from about -250 to +165). Notice that each of these periods overlaps with war: Greeks fighting Persians, Greeks fighting Greeks (after the Persian wars, an over-confident Athens precipitated a dozen conflicts with Corinth and Sparta until the major Peloponnesian war), Macedonians fighting Greeks, and Greeks fighting the rest of the Mediterranean and Middle East. Notice that the whole of Western history since the Magna Carta in 1215 would fit within a tick mark and a half in that top timeline.

1.1.1 ACT I: Is Nature Made From Stuff Governed By Rules?

Thales •Anaximander •Anaximenes •Pythagoras •Philolaus
(Set the context with the timeline in Figure 1.2 on page 22.)

Over my career I've published hundreds of scientific articles. Every publication has a common element: a bibliography with references to dozens or even more than a hundred other scientific works. Science doesn't happen in isolation as we're constantly building on, disputing, or confirming work of other scientists.

Now, take out a piece of blank paper. In many ways, what you are looking at is the bibliography of the first Western philosopher or even proto-scientist: **Thales of Miletus (ca –624 to –547)**. Plato and Aristotle (and neo-Platonic philosophers who came centuries later) tell stories of him, which form a lot of what we know. The fellow who invented history, Herodotus, also is a source.[2] Thales left no first-hand writings, but stories about him abound, and his life put Ionia, the western shore of modern Turkey, "on the map," indeed, on our map in Figure 1.1. An inordinate amount of western philosophy has roots from that strip of the Aegean shore.

Here's one: my favorite *New Yorker* cartoon is a Robert Weber's 1981 image of a professorial-looking, tweedy fellow with a pipe on a NYC street corner asking a cop, "Excuse me, Officer. I'm an academic. Where am I?" That image of us academics didn't originate in a fancy magazine. Plato told the story that Thales was walking along looking at the stars and deep in thought and dropped straight into a well that he didn't see in his path. That embarrassment wasn't enough, as Plato also notes

[1] But the next century would see Ionia ruled by Persian-installed kings and tyrants.

[2] Herodotus was the first to tell about the past by trying to justify his assertions and find reasons for events. He's best known for his detailed history of the Greco-Persian Wars.

that a passing servant girl was on hand to make fun of him in his reduced state.[3] But we also know that he was savvy enough to predict some weather changes and a possible bumper olive crop, so he bought up all of the olive presses in Miletus and made a fortune selling them back.[4]

Maybe that happened. Here's another. Herodotus suggested that Thales studied in Egypt and learned geometry and astronomy sufficiently to predict an eclipse of the Sun on (our dating) May 28, −585, that pretty much stunned everyone, including causing a battle to pause. How did he do that?

Well, he couldn't have. That didn't happen. Available data and predictive capabilities couldn't have allowed anyone to make such a prediction. It's trivial *now* to point back to the line of totality (the swath on Earth that would be dark), which would maybe have indeed been over the historical battle site at that time. But a prediction? No.

Determining the veracity of stories like these is an example of the detective-story-approach to unraveling Thales and the other Presocratics: The eclipse fable suggests that Thales might have been an adult in −585 and thought by Herodotus to be a well-enough respected personage that his "predictions" might have mattered. So this story, while fiction, did contribute to the picture of the man called Thales, his reputation, and his timeline. Thales was a real person.

What's not in dispute is that he initiated, or was a part of, a new way of asking questions *and* a new standard of what constitutes acceptable answers. Nobody thought like him and his immediate successors, and now we all do.

1.1.1.1 The World Before Thales & Co.

Why does it rain? Why are there earthquakes? Why are some people honest and others not? Why did my crop succeed and yours fail? Why is the Earth suspended under the sky? If you're Greek before about −500, there's a god for that. Why are there clouds? Yup, a god for that too. Why does the Sun shine? Another god. I tried to count all of the Greek gods, titans, minor deities, spirits, sea gods, agricultural gods, "rustic" gods, plus health and sleep gods. Oh, plus almost 30 mortals who earned a promotion to god-like eternal life. It's hundreds. There is the varsity team—the 12 gods of Olympus and the 12 Titans. But the god-team bench is really deep.

Greece is prone to earthquakes, ranking fifth or sixth in seismic activity. According to Greek mythology, earthquakes occur when Poseidon, the god of the sea, is upset and strikes his trident on the ground from Olympus. Similarly, rain is believed to be a result of Zeus, the god of the sky and weather, causing trouble with his lightning bolt symbol.

[3] Plato's references to the Presocratics are often to make fun of them.

[4] He was also an astronomer of note and a mathematician with theorems to his credit. An all-around academic.

There's a madness to this, but also a sort of understandable urge to assign every human experience to an outside influence. While Homer's tales include the gods as major actors, it was Homer's contemporary, Hesiod, who thought that the history of the gods needed a rational and believable narrative, and his *Theogony* is basically the story of the world's origin, including the genealogy of the gods. There's also a cosmology in these myths: the gods are themselves born. They've not always been around. And they have lives—outrageous ones.

That's interesting. They could have just "been there," outside of time like in other religions, but Greek myth seemed to require a logical, if not fanciful, structure: Earth (Gaia) and Sky (Ouranos) were the first, and their union is followed by scenes from *Animal House*… no, much worse. Infanticide, incest, fratricide, cannibalism, mutilation, and betrayal follow among the gods and the Titans, and between them and regular humans. Murders are the most light-hearted events in Hesiod's story.

The bottom line of Greek mythology is that everything happens for a reason. Why? Because a god is benevolent or unhappy or just doing their job.

1.1.1.2 Thales' Science and His Successors

GREEK RESEARCH PROGRAM #1 : Thales ushers in the first Greek Research Program, that the world is made of some fundamental substance that behaves according to natural laws.

Thales was the first that we know of to take a different approach. He's best known for asking what is the underlying, common structure of the universe, what Aristotle called on his behalf, the First Cause.[5] Thales reasoned that all of our universe depended on a single substance, and for him, that substance is water. After all, without water or moisture, things perish. Water is in the air and condenses and wets surfaces. It evaporates and reappears, sometimes revealing (creating?) soil underneath. Nothing lives without water, and when things die, they become dry. So, as a single substance acting as the basis of all things, it's not too bad. This description of the world is **materialistic** and **monist** (the view that there is one underlying substance).

This concept is the first of three novel features of Presocratic proto-scientific thinking.

1. Thales suggested that humans could understand how the world works, in-

[5] Aristotle uses that word. But Aristotle was fond of Aristotle's philosophy and his reliance on "Cause" and "Substance" in his own work, motivated his description of the Presocratics' work. Those words were not available to the early Presocratics.

cluding what causes the events and things that we experience. He suggested that the world is made of fundamental stuff guided by rules—laws of nature, so to speak—that govern how that stuff operates. The world needn't be a mystery.

2. Their "how" commitment searches for naturalistic reasons for events and existence. The previous "why" commitment was satisfied that "a god did it." For the "how" answers, the gods aren't involved. For example, the early Greeks inherited an ancient idea that the Earth is a flat disk with a dome of sky overhead, surrounded by a river (the Ocean or *Okeanos*) and the whole thing is held up by Atlas as a punishment handed out by Zeus. Thales agreed with the geographical part of this cosmology that the disk floats on water, but earthquakes happen when the water sloshes. A wildly wrong explanation, but completely naturalistic. Poseidon is not involved.
3. Finally, the Presocratics jousted with one another: an idea or a research program from one, might be incorporated in another's account. Or, an idea or research program might be a focus of criticism, resulting in an alternative account.

▻ This is not yet science, but science can't happen without at least these three commitments: **we can know about a rules-based universe**, **"how" cannot depend on the supernatural**, and **competition and collaboration are essential** to carry a project forward. All of this was new and now familiar.

Others who came after Thales adopted the same "research program" hypothesizing and defending an underlying substance for the world. Thales' Milesian "A" students, **Anaximander (ca –610 to –545)** and **Anaximenes (ca –570 to –525)** asked that question and answered it in different ways, but with the same basic motivation. Each of them had their own underlying substance idea.

Anaximander gave us one of the first maps, perhaps the sundial, and a full cosmology, including a hockey-puck-like cylindrical Earth floating at the center of the universe. He watched the stars go around us and concluded that the Earth can't be falling...so it must be balanced at the center of the cosmos.

▻ Here, is our first reasoned theory of MOTION, in particular MOTION BY THE EARTH. He concluded that the Earth doesn't move, but for a reason: because of symmetry and balance.

Anaximenes went a step further and realized that what's important is *process*—things turn into other things. Cycles happen. Law-like behavior is evident. Neither Anaximander nor Anaximenes went along with Thales' contention that water could be the sole source of stuff—how can water be the source of its opposite, fire? That's not the point, though! They rejected his specifics but bought into the project: While Anaximander chose something ethereal and not itself one of the substances (the spooky "Apeiron"), Anaximenes chose air as the fundamental substance, but he had a scheme whereby air's various guises could account for the actual things we experience.

By this point, proto-scientific practice is pretty much up and running. They were naturalists, materialists, and the first **empiricism**—using their powers of observation to study their world and attempt to explain it without recourse to a deity or a dogma.

1.1.2 ACT II: Pythagoreans in the West

It must be exhausting being a philosopher in your day job while also moonlighting as a deity and yet **Pythagoras of Samos (ca –582 to –497)** seemed to function as both, or so his followers asserted. (Notice on our map in Figure 1.1 that Samos is in northern Ionia.) Yes, that Pythagoras of the triangle, although it's probably not what you think. What Pythagoras taught and what evolved out of the long Pythagorean school is difficult to parse today so it's not fair to attribute all of "**Pythagoreanism**" to that one person. The ideas that are attributed to him originated in Italy but evolved considerably, becoming a dispersed movement that spread throughout the **Hellenic** world and beyond to the Renaissance hundreds of years later. Indeed, by Plato's time, Pythagoras was already an enigma. As we'll see, Plato probably learned about him through Philolaus of Croton and Archytas of Tarentum, two acknowledged second-generation Pythagoreans and mathematicians in their own right. So we have a nearly mythical figure: In the near-term, there was Pythagoras, "so-called Pythagoreans" (as Aristotle called them), and Pythagoreanism…the seed-philosophy of mathematics that has lasted in some form to the present day. I'll mostly use the plural "them" rather than the singular, "him." "Pythagoras" is essentially the name of a movement and a culture and unreliably as a single individual.

His biographical details are from Roman-era writers and enthusiasts, and it's difficult to know what's believable. There's general agreement that he grew up on the Aegean island of Samos and reportedly met the elderly Thales, and maybe studied with both Anaximander and Anaximenes. So suggested Heraclitus, from whom we do have actual written (critical) fragments about Pythagoras. He may have traveled around the Aegean with his merchant-marine father and probably lived in Egypt and maybe Babylon for at least two decades, absorbing language, philosophy, and mathematics. So, a well-traveled, probably comfortable young intellectual. The politics of Samos became tenuous, and in spite of the fact that he'd established a following of students, at the age of 40, he relocated to the large Greek city of **Croton** in the "instep" of the boot of Italy (look at the map in Figure 1.1). Some accounts suggest that he was accompanied by a number of loyal followers—the Pied Piper of Samos?—but most suggest that he moved by himself. In Italy he again established a following of reputedly as many as 600 (some say thousands) men and women in Italy and actually wielded some civic influence in Croton, serving as both an advisor and unwelcome busybody. He eventually founded a school that was to last

300 years, twice as long as my own Michigan State University has been around.[6] The ideas generated from that time evolved and so the border between the man and the movement is impossible to demarcate today.

This unusual school also functioned as a mystical, religious cult. Its members were regimented as to how to dress, what they could eat, what they may believe…and what secrets they must keep. They loved secrets. Pythagoras was its head and was by legend, supreme, teaching about his remembered past lives and reincarnations.

The legendary discovery moment came from thinking deeply about musical tones, which they extrapolated to the proposition that numbers and mathematics are a fundamental fabric of the universe. Although they were not in competition with the Ionians, reliance only on a substance-based first principle wasn't sufficient for them. Rather, they believed that their discoveries in mathematics revealed something fundamental about the world:

> "All things have form, all things are form; and all forms can be defined by numbers." Pythagoras

> "The Pythagorean … having been brought up in the study of mathematics, thought that things are numbers … and that the whole cosmos is a scale and a number." Aristotle *Metaphysics*

1.1.2.1 The Most Durable Discovery in History

GREEK RESEARCH PROGRAM #2 : Pythagoras ushers in the second Greek Research Program, that the world is mathematical. Or even that the world is mathematics.

Pythagoras left no writings, but stories/fables/tales reported by dozens of others abound. He claimed (or it was claimed for him) to have discovered integer relationships among the strings of a lyre[7] and the pleasant chords it could make. The lyre was probably a 7-string variety although he reportedly built a one-stringed tool ("kanon" or "monochord") to study its behavior (although that story is disputed). A quick taste of what the Pythagoreans left for us (and for Plato!):

When you pluck a string clamped at the ends, you cause the string to vibrate with a fundamental frequency related to its length (and tension—think, a guitar). Call that the "ground note." (A Pythagorean scale is different from how a piano is tuned, but I'll use the piano as my analogy.) A piano's middle C is a natural ground note and

[6] But both his and mine are mere babes, as compared with Oxford University, the University of Paris, or the Academy of Plato.

[7] and the tones from cups filled with different amounts of water which were noted for their pleasing sounds

has a frequency of 261 Hertz, which is the **Hz**. Pressing the lyre string at a halfway point and then plucking one of the two halves will cause the ground note to be repeated, but an octave higher. (On the piano, C above middle C is a frequency of 522 Hz, twice 261 Hz.) Pressing a lyre string at 2/3 of the length and plucking the long remaining string, causes the fifth above the ground to sound (for the ground of middle C, that would be G, or 392 Hz, 3/2 of middle C's frequency) and pressing 3/4 of the length, a fourth above that (A above middle C at 348 Hz, 4/3 times that of middle C's frequency).

Play those intervals on a lyre or chords on a modern piano, and your ears will be happy. These are pleasant-sounding combinations, while other combinations are not so sweet—we say dissonant. To the Pythagoreans, the difference between pleasant and dissonant was due to the integer ratios of the string lengths—what was important was not the strings, but the *numbers themselves*.[8]

> This revealed an **intimate link between numbers and the world**: integer ratios 2/1, 3/2, and 4/3 → to specific lyre string lengths → to pleasing your ear (your soul). This relationship made the numbers 1, 2, 3, and 4 very special to them. They concluded that your human well-being is connected to abstract numbers.

Lyres had been around for millennia, so surely this particular discovery was not news. But what Pythagoreans did was new.

▻ They elevated numbers to a significance that's *beyond just counting* by **inventing the concept of number itself**: from 2 oranges to the abstract concept of "2."

This direct connection between a few integer numbers, their ratios, and special numbers with important meanings[9] influenced all that's "scientific" up to the present day: A brand new commitment...to an abstraction.

This connection between integers and one's soul seemed to have been just the beginning. They also connected numbers with shapes, so geometry, and, by extension, space itself. Keep them in mind: 1, 2, 3, and 4.

What can you do with them? Well, you can add them: $1 + 2 + 3 + 4 = 10$ which is not such a complicated thing, but these are special numbers after all and so their combinations must be special as well: "10" was important.

You can also make patterns with numbers—and a highly useful definition of modern mathematics (especially in physics)—is that it's the process of finding patterns. Figure 1.3 shows examples of Pythagorean patterns with integer numbers and an important Egyptian application.

[8]It's a matter of current physiological research to understand why some combinations of tones are pleasing and others are dissonant.

[9]Notwithstanding "42" as the numerical explanation of everything in *Hitchhiker's Guide to the Galaxy*

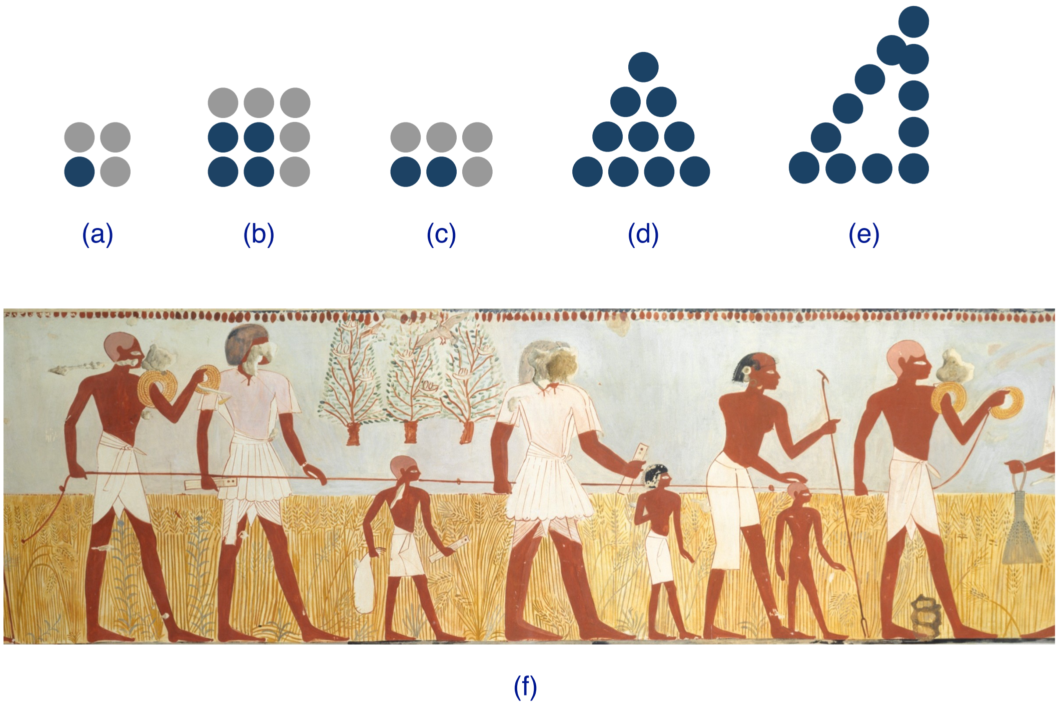


Figure 1.3: Dots represent stones that they would have used to signify numbers—precisely like the dots on dice. The image (f) is from the Tomb of Menna showing Egyptian workers getting ready to do surveying with a knotted rope. See the text for a description.

Figure 1.3 (a) starts with one stone, and adds the first odd number, 3,[10] arranged around it, turning $1 + 3$ into 4, *but it also lays them out as a pattern in space.* Numbers = geometry for the first time. This is a "square number" that follows the rule (in modern notation) of $1 + 3 = 4 = 2^2$. We can expand this into more square numbers, and the next one is in Figure 1.3 (b), which shows that $1 + 3 + 5 = 9 = 3^2$. One can also take two stones and add the next even number around them in (c), say three above and one to the right, to get a "rectangular number."

Especially important is the arrangement shown in Figure 1.3 (d). Remember, $1, 2, 3, 4$ are special. Lay out four stones, then layer three on top, then two, and finally one. You've now made a special triangle—-the tetraktys ("fourness")—with 4 stones on each of three sides. So it's an equilateral triangle and all four of the important numbers are contained in it... adding to 10. Maybe they liked bowling.[11]

There's another connection between numbers and geometry—again, connected with the physical world. "1" was a special number, neither odd nor even (for them), and played the role of a beginning. The source. A single isolated point is the starting point (no pun intended) for everything. "2" represents a line, which starts with a point and is constructed of points. "3" represents a triangle that delineates a flat plane and is constructed of lines, and "4" represents a tetrahedron, a three-dimensional solid constructed of triangles. That's it. Three dimensions to our physical space is all there is, and so "4" represents completion and its encoding in the tetraktys (count the stones in any direction in the tetraktys and you'll count 1, 2, 3, and the base, 4) and that relationship with "10" tied it all together for them. (Of course, today, multidimensional spaces are a mathematical walk in the park. We know that our physical world consists of at least four dimensions. So stopping at "4" was premature!) There's more. "5" is special as it's the sum of the first even and first odd numbers. "6" is special since it's both the sum of the first three numbers and, simultaneously, the product of the first three numbers. And so it goes.

Notice that there's another triangular pattern in Figure 1.3 (e). If you count the spaces between stones, you'll find that they delineate $3 - 4 - 5$, which is a familiar triangle to some of you but a familiar triangle to thousands of years of Egyptian builders. This triad of numbers has practical value as it's a sure-fire way to make a right angle. Take a length of rope and tie 12 knots equally spaced from end to end. Then have a worker hold one end, another hold the third knot, and a third worker grasp the rope 4 more knots along. If the other end is then given to the first worker. The only way to make each of the three segments taunt is for there to be a right angle between the 3 and 4-knot segments. There are other such triads that make a right angle in this way, for example, $6 - 8 - 10$. The ancient Babylonians and Egyptians knew of many of them and used them in surveying and building

[10] The number 1 was not a number for them: numbers meant a plurality. One is not "odd" nor is it "even." It's unique.

[11] There is a fable that a Pythagorean who became ill at an inn while traveling but had no money to compensate the owner for his care while convalescing. The traveler told the owner to hang an image of the tetraktys and other Pythagorean travelers would compensate him far beyond his original costs. And they did.

without realizing that this was an important thing. Figure 1.3 (f) is from the Tomb of Menna, showing a knotted rope for surveying, geometry at work. As you know from high school, Pythagoreans figured out what this means in an abstract way.

There was a mystical quality to numbers, and numerology was a thing, so the numbers also had special meanings for things beyond just "quantity." For example, 5 is the sum of the first even and odd numbers $2 + 3$ and since 2 symbolized female and 3 male, then 5 symbolized marriage. The first even number is 2 and squared is 4, and so that first square number, 4, symbolized *justice*. Likewise, the first odd number is 3, and its square is 9 and so it also symbolized *justice*. (Even today, we refer to a "square deal" as a proper deal.)

In fact, 10 was such an important number that in one version of Pythagoras' cosmology, we have another early moment of MOTION BY THE EARTH. The Earth and all of the other celestial objects moved around something called the "central fire." This actually comes from Philolaus:

> "The first thing fitted together, the one in the center of the sphere, is called the hearth." Philolaus *Fragment 7*

The bodies are, from the inside-out, Earth, Moon, Sun, Venus, Mercury, Mars, Jupiter, Saturn, and the celestial sphere, but. . . wait. That adds up to 9. It must be 10 in order to be right, so they added the "counter-Earth" whose orbital mechanics are such to be perfectly positioned to block our view of the central fire since we don't see it.

> ". . . inasmuch as ten seemed to be the perfect number and to embrace the whole nature of numbers, they asserted that the number of bodies moving through the heavens was ten, and when only nine were visible, for a reason just stated they postulated the counter-earth as the tenth." Aristotle *Metaphysics*

That's a very modern interpretation of the use of mathematics in physics. You postulate the importance of a principle ("10 is magic"), you create a model of the universe (or some small part of it) built within the model, and then use the basic rules of the model (like arithmetic or something fancier) tweak it while still committing to the model. Here, the counter-earth was such a tweak. That's actually how physicists work within models until they become unwieldy or are ruled out by experiment. I'll have more to say about a modern-day view of Pythagoreanism *Presocratic Greeks, Today* in Section 1.2 and their cosmology in Chapter 3. It's a matter of much discussion (a polite way of saying, "argument") today. It gets worse when we add Plato to the mix.

> The connection of music and integers led Pythagoras to assert that the regular harmonies of the cosmos were everywhere. The planets and stars all move and emit tones that ordinary humans can't hear since they form a background to everyday noise. But Pythagoras? Ah, he's different. He's the only human who can hear the Harmonies of the Spheres. Being a deity has its privileges.

You're wondering about that theorem, I know you are. Look at Figure 1.4 and relive high school for a moment. Notice that Figure 1.4 (b) is the knot/stones-version of the Egyptian right-angle trick.

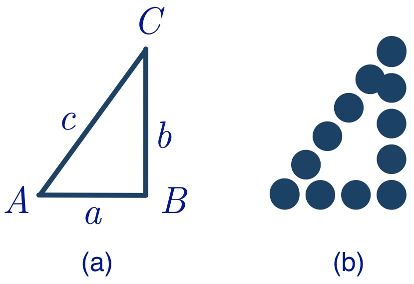


Figure 1.4: In (a) is a modern-day version of drawing a triangle, while in (b) is the same thing but with stones or knots delineating distances. The length c is the hypothenuse.

Maybe you remember the little song for a right-angled triangle: "…the square of the hypotenuse is equal to the sum of squares of the other two sides."

Or less lyrically,

$$c^2 = a^2 + b^2.$$

> There's no evidence that Pythagoras first proved this, and in fact, plenty of evidence that it was long known before him. (There are now about a hundred different proofs of the "Pythagorean Theorem." I offer a couple in the Technical Appendix, A.1.) The Egyptians had a real estate problem to solve: the Nile overflowed its banks every year, and the fertile cropland alongside it would be covered with water. That meant a problem: once the water receded, whose land was whose? Out of a need, geometry for Egyptians was a necessity. This was another job for the practical $3^2 + 4^2 = 5^2$. But the Babylonians were the champs. Not only did they keep accounting records, they did so in a base-60 number system...which must be 6 times better than our base-10 system, right? We've fragments that showed that they had worked out things like $119^2 + 120^2 = 169^2$, which admittedly doesn't come up every day.

There is a Pythagorean-Theorem story that tells you much of what you need to know about his cult. Remember, integers were the thing, and so we feel sorry for the poor guy (historically, maybe Hippasus) who noted that a triangle with legs of 1 would have a hypotenuse that's Pythagorean-impossible since $1^2 + 1^2 = (\sqrt{2})^2$. This $\sqrt{2} = 1.4142135624\ldots$[12] never ends—the definition of an "irrational number"—it goes on forever and so decidedly not one of the mandated integers. Since he'd found a non-integer, for his trouble, as the story goes, he was thrown overboard from a ship in order that his little discovery not be revealed to the other cult members. Maybe this happened.

[12] "dot dot dot," ... is mathematics-speak for "never ends."

In the end, as sometimes occurs with cults, Pythagoras' welcome in Croton wore out. His house was burned, and he escaped, only to die in his escape. . . or not. We don't know. But what he and his colleagues created lived far beyond them.

> When it comes to Pythagoreans, who did what when is murky. In the lower timeline of Figure 1.2 between Pythagoras and Plato, you'll see **Philolaus of Croton (ca –470 to –385** who was the first Pythagorean to write about their program, although only fragments and references from others remain. Much of what Plato and Aristotle knew probably originated from his writings. (Plato only mentions "Pythagoras" and "Pythagorean" once each, but Aristotle was more expansive.) Philolaus was a scholar in his own right and it's hard to discern what ideas were his and what came from Pythagoras himself, or even in Pythagoras' lifetime.[a] Highly readable accounts are Kitty Ferguson, 2008 and G. E. R. Lloyd, 1970.
>
> [a] And, what we know of Philolaus might have come from the Pythagorean, Hippasus. The most unlucky Pythagorean. He is remembered as having constructed bronze disks who's thicknesses matched the lyre string ratios. When struck they would then create the same pleasing sounds as strings. He's also historically the poor guy who found the non-integer problem with the Pythagorean Theorem. Stay tuned.

1.1.3 ACT III: The Eleatics in the West

Heraclitus •Parmenides •Zeno
(Set the context with the timeline in Figure 1.2 on page 22.)

What happened next unsettled the young enterprise of philosophy and, after Plato and Aristotle, initiated millennia of philosophical controversy. We saw that the Ionians relied on their senses and took it for granted that events in the world changed in time. But you and I have both learned that our senses can be tricky and not always accurate. And, even if we see/hear/feel accurately, the targets of our perception themselves change. So if that's the case, then what about our "scientific" observations? Can we trust our senses to gather accurate impressions of our surroundings and base our theories on those impressions? This investigation traditionally pits two Presocratics against one another, the "Riddler" of Philosophy, **Heraclitus of Ephesus (ca –540 to –480)** and the first "Lawyer" of Philosophy, **Parmenides of Elea (ca –514 to –450)**. The former was an Ionian from the big city of Ephesus, not far from Miletus. The latter was from the colony of Elea in southern Italy. Look at our map in Figure 1.1. Elea is Pythagoras' territory.

Heraclitus was a loner, while Parmenides evolved a school of philosophy called the "**Eleatics**." You might not have heard of that, but you may recognize one of Parmenides' significant followers: Zeno. . . of Achilles and the Tortoise fame. Heraclitus (by himself) and Parmenides and his followers took up the subject of change. Heraclitus was decidedly on the side of, sure, things change. But he took it in an abstract direction. On the other side, Parmenides concluded that change is an illusion. He even *proved* that change is an illusion. At first glance, that seems strange, but his novel method of philosophizing was persuasive, and as a consequence, he created two branches of philosophy. And in the course of digging into the problematic nature of Change, set off a huge argument over centuries. Obviously, this is prior to any kind of physics-like analysis of MOTION!

GREEK RESEARCH PROGRAM #3a : The Problem: Tension between Change versus Permanence begins with Heraclitus and Parmenides.

1.1.3.1 The Riddler

Although we know few details of Heraclitus' life, he was apparently prominent in Ephesus. His father was said to have been an aristocrat, but Ionia was under Persian control during his life, and suggestions that Heraclitus might consider a political life might be hard to picture. He wasn't a people person. He would have been a child when Anaximenes died but he was critical of the Milesians and scathing in his criticism of his contemporary, Pythagoras. About 100 fragments of Heraclitus' work remain, showing that his style was. . . unusual. He wrote very short tweets which have puzzled and delighted readers for thousands of years.

He was a monist as well: fire was his fundamental substance. And as interpreted by one of his aphorisms, he had a **cosmology**,

> "This world-order [*kosmos*], the same of all, no god nor man did create, but it ever was and is and will be: everliving fire, kindling in measures and being quenched in measures." Heraclitus

This is the first time that the word "cosmos" appears in Greek philosophy and he's clearly insisting that the cosmos always was the case and always will be the case. That's interesting since Plato deliberately labeled him inaccurately as naively saying that "everything changes" and that nothing is permanent.

You and I think of MOTION ON THE EARTH as moving from one place to another during some time, right? Remember, the Greeks were just beginning to do this analysis, and moving from place to place was not their primary concern. Change by itself was, and Heraclitus was the first to abstract *any* change as basically a form of motion, seeming to assert that universal "flux" was an important feature in the world.

> "It is not possible to step twice into the same river." Heraclitus

This is a famous paraphrase of a translation of his most famous of three "river aphorisms," The idea is that the river is always flowing, and if you step into "the river" once and then step into it a second time, it's a different river. So two rivers sort of functioning at the same time. It's a little different from this one:[13]

> "As the same thing in us are living and dead, waking and sleeping, young and old. For these things having changed around are those, and those in turn having changed around are these." Heraclitus

A young person is connected to their older self through the changes that they undergo. A is different from B, but linked because *A changes into B*. But, living and dead? This is a deep idea and seems to suggest that A and its opposite, B, are actually the same thing. In fact, Change here has a job: it's a sort of glue that links together different things or different aspects of a thing. So apparent opposites are connected, meaning that everything in the world is connected. One.

Plato used Heraclitus as a punching bag and said that connecting opposites, as Heraclitus suggests, gives us logical contradictions. Plato had an agenda. Aristotle was a little more forgiving, and we'll see how he codified and categorized change, which will explicitly include our notion of locomotion. But it seems that he had to go through Heraclitus to get there.

It's easy to be amused by Heraclitus' words, and for millennia, that's been a sport, and I have more for you in *Presocratic Greeks, Today* in Section 1.2.1 below.

1.1.3.2 Nothing Gets Done: The Parmenides Problem

Parmenides took the extreme, opposite position, probably writing after Heraclitus. His argumentation is tightly logical, so much so that it's possible to be swayed by the apparent inevitability of his arguments. If you can penetrate the denseness of it. I'll call his oddly persuasive but troubling conclusions the **Parmenides Problem**. It will seem to us like the Parmenides Problem will not go away.

He is the first in a long line of philosophers of both **metaphysics** (the philosophy of the nature of being) and **epistemology** (the philosophy of knowledge). He wrote a single book in verse (and according to Aristotle, not very well). It's a narrative story about his meeting with a goddess and how she teaches him about two kinds of knowledge.

There is the "first path" to knowledge: knowledge that is true by necessity. This "Way of Truth" is confined to your reasoning, not your senses. The second path to knowledge, that of perception, is "habit" and from "your heedless eye." This "Way of Seeming" is needed in order to get along in the world, but you can't trust it because you can be fooled. For that reason, the "Way of Seeming" can't tell you what is true. So:

▻ **The Parmenides Problem**: True means permanent. So, anything that changes cannot be true.

[13]While the most famous Heraclitus aphorism, there are at least three versions of it, and some dispute as to its overall authenticity.

Remember our own experiences: our senses can fool us and the objects of our perceptions can evolve between observations. What can you trust in the world if not your eyes? So he got rid of both issues. Truth can only refer to permanent things.

Accepting his premises, his logic seems oddly persuasive. In a nutshell, which could be on a T-shirt, I can sum up Parmenides in his two words (read it carefully…if nobody's around maybe even read this out loud): "**It is**." It's punchy. He also then reasons that "**It is** and cannot, **not be**." *It cannot…* ***not be***. If something **is**, it can't be **not-is** at the same time. Further, if something **exists**, then **it is**. Consequently, if it **doesn't exist**, then it is **not-is**. So knowing what **is**, is to know what **exists**. So far, so good. Something can't exist and not exist simultaneously. (Can you see how this is against Heraclitus, who seemed to welcome A and not-A simultaneously?)

He goes further. If something exists (it **is**), then also it could never have been different in the past, nor will it be different in the future. For if it came into existence as **is**, then before that event, it must have been: **not-is**. It changed. If it changes into something else in the future, then it goes from being **is** to then being **not**-is. How can something at one time be **not-is** and at another time be **is**? That can't happen! So if something **is**, it's always been **is**. In some sense, then the past and the present are one. Whew. Are you with me?

He's staked out clever ground in two new ways: His approach seems so logical that it launched philosophical analysis as an appropriate way to make arguments. And, he's defined what it is to be real: what's real must be true, and therefore, it must be unchanging. The only place where truth can be realized is in your head. Where you reason.

Parmenides' sidekicks ran with this. Zeno took his arguments to the extreme and that's our connection with MOTION. Maybe you remember the story of how Achilles couldn't beat a tortoise in a race?

> This is one of 10 of "Zeno's Paradoxes," **The Achilles**. Achilles, being the fastest human, is to race a tortoise, maybe the slowest animal, so he gives the tortoise a head start, halfway to the finish line. They both start, but poor Achilles is faced with an impossible task. In order to traverse half of the distance to the tortoise's starting point, he has to traverse half of that half. Then half again of *that* half. In fact, he needs to travel through an infinite number of paths, which is impossible, so he can't catch the tortoise! There are three other paradoxes on motion (The Dichotomy, The Arrow, and The Stadium), all designed to support Parmenidean conclusions about motion. In Technical Appendix A.2 I explain how we think of Zeno's paradoxes today as...well, not paradoxical.

Zeno gets this from Parmenides, and since the reasoning seemed to be impenetrable, with an apparent gloss of a mathematical sheen lending a seeming validity, all of those races that you've seen with your lyin' eyes were apparently fooling you. I touch on two others in *Zeno and His Paradoxes*, Section 1.2.3 below.

> We've now encountered examples of significant philosophical or scientific commitments. Sides were beginning to be drawn in natural philosophy that continues to this day: Can knowledge about the world be gained by thinking? Or must knowledge come from observation? The former is called **rationalism**, and the latter, **empiricism**, and physicists still argue about this. Clearly, Pythagoras is in the first camp, and so was Parmenides—distrust of the senses disqualified observation as a source of truth. And the geometrical argument seems like a good example of what must be true. The Ionians pioneered the second camp, gleaning knowledge and theories about the universe by looking and hypothesizing from their observations.

Finally, the void. The **vacuum**. A state of actual nothing! By now, you can imagine what Parmenides thinks of such an idea: it's impossible since it's the state of **non-being**. Another Eleatic, Melissus, took this to the ultimate conclusion without the need of Zeno-like paradoxes. Just logic: anything that **is** cannot move since it would need a place to move to— it would need an open space where **nothing is** in order to relocate. But a place where **nothing is**. . . is nothing. But nothing can't be the case, so there is no motion. Another MOTION problem.

Parmenides was the first to seriously question what can be known and by what means. Your senses deceive you all the time, and so you can't depend on your observations for truth. But at the same time, your rational, logical thought—an argument assembled before Aristotle invented the actual rules of logic—is dependable. He then laid out a dispassionate argument that leaves one wondering what in the world is wrong with it.

> How do we get around this? In order to do science, or frankly, to live, one has to be able to hold a tentative, hypothetical idea in your head, but less than "True." But Parmenides was worried about that Truth with a capital "T" and so he couldn't abide an idea that is not true or even tentative as a stand-in for what's true and so his philosophy was sterile. Scientists don't deal with that kind of truth.

Well, this is embarrassing. My project here is an account of MOTION, and now we've just encountered what seems to be a persuasive argument that MOTION is impossible. That's not progress, is it?

The Parmenides Problem is an important stepping-off point for Plato.

1.1.4 ACT IV: Antidotes to Parmenides?

Empedocles •Anaxagoras •Leucippus •Democritus
(Set the context with the timeline in Figure 1.2 on page 22.)

Parmenides' arguments were unsettling. The notion of a tightly logical argument was brand new, and yet even if its conclusions seemed nonsensical, you've got to struggle to find holes in his reasoning. But that didn't stop four intrepid souls. We still call them "Presocratics," but really they were "Co-socratics" (I made that up) since they all lived around the time of Plato's mentor. They're our last stop before Plato.

GREEK RESEARCH PROGRAM #3b : Attempts at solutions: Back to Monism for solutions to the Parmenides Problem?

1.1.4.1 Empedocles and Anaxagoras

One philosophical god was apparently not enough. **Empedocles of Sicily (–494 to maybe –434)** was another self-appointed deity. He was a contemporary to the Ionian, **Anaxagoras of Ionia (–500 to maybe –428)** who had a similar solution to the Parmenides Problem. Both took the position that the world is made of multiple entities and that *those* entities are what's permanent, but their *combinations* are multitude and accommodate change. In some ways, a modern approach.

Empedocles was a character. Legend has it that he dressed in a purple robe, with wreaths around his neck. He claimed to have performed miracles, raising folks from the dead, curing illness, and so on and he claimed to have been reincarnated from previous lives as a bird, a fish, a girl, a bush (really? shrubbery?) ...His brand was very Pythagorean he lived and worked in that same region of the Greek confederacy as the still functioning Pythagorean society, so there might have been some influence. He famously wore bronze-soled shoes everywhere. They figure into his legendary ascendence at the end. He was supposed to have leaped into the active volcano at Etna and disappeared, but one of those distinctive shoes was left behind, casting doubt on that last miracle. It seemed that the volcano spit the sandal out after consuming him. These stories came two centuries after his lifetime.

We only have fragments from him, who wrote in verse, as seemed to be the custom in the West. It is from him that we get the familiar **ancient elements** of Earth, Air, Fire, and Water as basic elements (he called them "roots"). We will see that Plato and Aristotle took hold of this idea and ran with it all the way to Galileo's time. These four roots accommodate change by mixing with one another driven by two opposing forces, "Love" and "Strife." Again, a simplistic but modern-sounding notion of fundamental forces acting on the basic constituents of matter.

This is inspired. The roots are indivisible and have always existed, as have the two "forces" of Love (an attractive force) and Strife (a repulsive force). He also agreed that no-thing can come from nothing. So, we can check off both the Parmenides permanence and not-nothing boxes. But he also accommodates our senses, while warning of their fragility. What we observe is that things in our world are different from one another and that there are many of them. Some rocks are hard, and some rocks are brittle. They're both rocks, so how do we build our observed rocks with only four roots?

> ▻ Empedocles contribution was that everything we observe is constructed of varying *proportions of the root elements*.

All rocks might be made of the same combinations of the roots, but a hard rock would have more of the Earth root than the brittle rock. With infinitely mixing proportions of the four roots, you can make the variety of the world. Sounds a little like a proto-chemistry.

Empedocles insisted that there was no purpose to the universe and that we're all subject to chance, postulating that we actually live in an undulating, repetitive cycle of a spherical universe in which Love and Strife compete for dominance.

His contemporary, Anaxagoras, was from the other side of the West-East divide. He was an Ionian who ended up in Athens, establishing the first of a long string of Athenian philosophers. His arrival came during the classical period when the architecture, sculpture, literature, and, yes, philosophy that we think of when we think "Greek" began.

Rather than only four substances, Anaxagoras presumes as many elements as there are things. Things... are themselves infinitely divisible. How do you acquire hair and bones? Well, you eat foods that contain elements of... hair and bones. Everything is in everything. He insisted that the senses give us a window or a picture into aspects of reality that are not directly observable but, nonetheless, exist. Again, another modern idea from one of our "Co-socratics."

Notice that neither of our two characters explicitly addresses the issue of locomotion. This is a confusion that Aristotle promulgates, as we'll see. "Change" *per se* is broader than a thing moving from one place at one time to another place at a later time. So, as you'll see in *Zeno and His Paradoxes*, Section 1.2.3 while Zeno works on that problem, he starts with the presumption that change is not possible, and so by extension, locomotion is impossible, and hence, the paradoxes try to persuade you of that. Our next two "Co-socratics" do find a way to explain locomotion, which, again, Aristotle rejects out of hand.

1.1.4.2 Atoms

I'll bet that you first learned the origin of the word "**atom**" in elementary school. "*Atomon*" is Greek for indivisible, and the origin of that idea was, again, the anxious need to find a way around the Parmenides Problem. You probably also learned that the inventor of atomism was **Democritus of Abdera (about –445 to –370)**, originally from a region that's closer to **Macedonia** (see the map in Figure **??** than it is to **Athens**, so a northerner. Here are three interesting things about Democritus. First, we classify him as a Presocratic, but that's really a misnomer. He's a "Post-socratic," younger than Socrates by more than 20 years. Secondly, he didn't invent the idea of atoms. He inherited it from **Leucippus of Miletus (about –480 to –420)**. Finally, Plato doesn't mention him! He apparently burned Democritus' books. Aristotle knew him very well, maybe because of their shared northern roots.

Obviously, the idea of atoms is one with legs, albeit with ups and downs over the next two millennia, usually, unwelcome and only accepted when Einstein found two ways to demonstrate that there are indeed invisible chunks of matter. (That's a story that's not our current Einstein focus, but a large part of his miraculous 1905 year.)

However, the atoms (typically a mixture of Leucippus and Democritus' contributions) of classical Greece and our idea of atoms are very different. First, there are an infinite number of Greek atoms of all possible shapes. Some have hooks and can attach to others (think Velcro), while some pairs have shapes that fit together. They move around and bounce off of one another, or they cling to one another, forming compounds that eventually become the substances that we're familiar with. We know of them because of the sensible qualities that they bring to objects we can deal with using... our senses. For example, things that taste sweet are composed of smooth atoms, while things that are acidic are composed of sharp-edged, angular atoms.

How is this an antidote for the Parmenides Problem? First, the atoms are permanent, but second, they are constantly in motion, and all change is due to their arrangements and re-arrangements.

But the real way in which this works is that both atomists insist that what's real are atoms... *and the void*. The void is the place where moving things can go *to*. So, locomotion is possible. There. That does it for Parmenides. So, the atomists are happy to make room (so to speak) for MOTION ON THE EARTH.

> The void is an unpopular idea, and to this day, we continually redefine what the vacuum is (or isn't). Our current understanding, again, my scientific playground, is that there is no place in the universe where there is nothing. The vacuum is full, but it's a quantum mechanical fullness that has no connection to any ideas before about 1950.

But, as I said, Plato ignored this singular, logical conclusion to the Parmenides Problem, which seems a cowardly way of dealing with an idea. As we'll see, Aristotle could not abide the void so he's no atomist either.

There's one more interesting fact about this pair's ideas, and that's an idea that Plato would embrace but with only partial credit to the right people. Everyday objects are not real things, and the attributes that we ascribe to visible, touchable, tasty, smelly, and loud objects of our sensible world are all based on convention. Democritus wrote:

> "By convention sweet and by convention bitter, by convention hot, by convention cold, by convention color; but in reality atoms and void." Democritus

Even though we can't see atoms, we know they're there because our minds tell us about what we can't see. A reality that's beyond our senses. Now this is a very modern idea and also a very Plato-idea and we'll see it emerge in a slightly different guise when we talk about Galileo and how he invented physics when he used this notion—now labeled "Platonic," but could be labeled Democritus-ian.

1.1.5 What's Important For Us

G2E is about MOTION and LIGHT. Does it make any sense to speak of either of them without numbers? MOTION implies speed (to us), immediately bringing to mind numbers: miles per hour, for example. LIGHT involves brightness, color, reflection, and refraction… qualities that we can describe using words, but they're a stand-in for actual numbers as well: you'd evaluate a lightbulb's brightness by "lumens" and its color by "Kelvin" which are numbers. "Red" is a name for a particular frequency of light.

This is so much a part of our thinking now, that it almost seems trivial to mention it. Wouldn't it seem odd to think in any other way for almost everything, from cooking to taking a pain reliever to deciding when to buy new tires? Attaching numbers to the physical world is a gift of the Presocratics and in particular, the Pythagoreans. Trivial or not, before the Pythagoreans, numbers as more than just counting would have been a foreign concept, after them, well, numbers are *in* everything.

But their gifts were generous beyond just this. Let me quickly summarize what the Pre-, Co-, and Post-socratics have brought to the scientific table.

The invention of the scientific commitments that we use today came from them:

1. They eliminated the supernatural as an acceptable argument for why things happen in the world. We can know about the physical world.
2. They conceived of the notion that the universe is made of naturalistic stuff: the water, aperion, and air first guesses, to more intricate and even modern-sounding permanent entities that go together in proportions to build the stuff we experience.
    (a) They toyed with the idea that these entities had to obey rules that allowed for their interactions and, in some cases, motions.
3. They invented the notion that mathematics is tied both to geometry and to things in the world, essentially birthing modern mathematics. We literally have no other way to describe and predict the properties and behavior of the physics world.
4. Some Greeks realized that learning about the universe involved seeing, touching, and hearing what the universe of things does. But others noted that our senses are unreliable, and so couldn't deliver truth if "truth" meant "permanent," setting up the problematic notion of Change. Taking a page from their high school geometry class, mathematics was a pretty good model of what is constant and true. But we can only deal with geometrical objects through

reason. So: don't look at the world, *think* about the world. That's what I've called the Parmenides Problem: is change in the world an illusion?

5. Reactions to the Parmenides Problem led to at least two directions: primary substances mixed in proportion, Earth, Water, Air, and Fire... or atoms. It also confused everyone that followed and heavily motivated Plato and in a different way, Aristotle.

And, proto-science, and now science as we know it, is a social activity.

6. They argued. One philosopher added to or reacted to what another said, creating the social structure and behavior necessary to support the scientific enterprise.

We're now ready for Plato.

1.2 Presocratic Greeks, Today

1.2.1 Tweeting With Heraclitus

Heraclitus is challenging because he's tough to analyze and because the available material is. . . pithy. The general view is that he really did write in these short aphorisms and that they aren't somehow surviving snippets of something larger.

The most famous of them, which tends to support his historical brand that "everything changes", is the River Analogy. The most famous version is due to Plato's rendition, which he wrote in *Cratylus*:

> "Heraclitus, I believe, says that all things pass and nothing stays, and comparing existing things to the flow of a river, he says you could not step twice into the same river." Plato

But there are actually three versions of the river tweet:

> "On those stepping into rivers staying the same other and other waters flow." *Cleanthes*, a Greek Stoic from two centuries after Heraclitus' life and almost a contemporary of Plato

> "Into the same rivers we step and do not step, we are and are not." *Heraclitus Homericus*, a commentator from 500 years after Heraclitus' life

> "It is not possible to step twice into the same river according to Heraclitus, or to come into contact twice with a mortal being in the same state." Plutarch, Roman philosopher and neo-Platonist

The first is probably the most likely and doesn't contradict the more popular version. However, this story illustrates the difficulty, once again, of the detective work involved in assigning credit (or blame) to the Presocratics.

I mentioned that he wasn't a people-person, probably unsuited for political leadership (notice the disdain for his Italian contemporary, Pythagoras):

> "One is worth ten thousand to me, if he is the best."

> "Eyes and ears are poor witnesses to people if they have uncultured souls."

> "War is the mother of everything."

> "The best of men choose one thing in preference to all else, immortal glory in preference to mortal good; whereas the masses simply glut themselves like cattle."

> "It is not good for men to get all that they wish to get."

> "What sense or thought do they have? They follow the popular singers, and they take the crowd as their teacher."

> "Learning many things does not teach understanding. Else it would have taught Hesiod and Pythagoras, as well as Xenophanes and Hecataeus."

> "Poor witnesses for men are the eyes and ears of those who have barbarian souls."

> "The adult citizens of Ephesus should hang themselves, every one, and leave the city to children, since they have banished Hermodorus, a man pre-eminent among them, saying, Let no one stand out among us; or let him stand out elsewhere among others."

His unity of opposites appears in multiple places:

> "Sea is the purest and most polluted water: for fish drinkable and healthy, for men undrinkable and harmful."

> "Collections: wholes and not wholes; brought together, pulled apart; sung in unison, sung in conflict; from all things one and from one all things."

> "Every pair of contraries is somewhere coinstantiated; and every object coinstantiates at least one pair of contraries."

> "Good and ill are one."

But, he's also inspirational:

> "Nature loves to hide."

> "Sound thinking is the greatest virtue and wisdom: to speak the truth and to act on the basis of an understanding of the nature of things."

> "Abundance of knowledge does not teach men to be wise."

> "This world-order [kosmos], the same of all, no god nor man did create, but it ever was and is and will be: everliving fire, kindling in measures and being quenched in measures."

> "The character of man is his guardian spirit."

> "The sun is new every day."

. . . and amusing:

> "And they pray to these images, as if one were to talk with a man's house, knowing not what gods or heroes are."

> "Souls smell in Hell."

"Every beast is driven to the pasture with blows."

"Asses would rather have straw than gold."

1.2.2 Modern Day Pythagoreans

Want to liven a party? Raise the following question:

1. Is mathematics invented? Or,
2. Is mathematics discovered?

That is, are the theories, proofs, and concepts of mathematics the creation of human thought, or are they "out there" waiting to be revealed by thinking about them? "**Platonism**" would rally around #2. and I'll tell you about that in the next chapter.[14]

Now if you want to rejuvenate your now yawning party-goers, narrow the question to:

3. Is mathematics invented in order to explain the physical universe? Or,
4. Is mathematics discovered to be already "in" the physical universe?

Number 3 suggests that mathematics is only a tool —a language—to describe the universe. Maybe it's a lucky break that we've invented it and that it seems to do pretty well. Perhaps another tool might have worked? For example, a musical score for guitar could be represented by standard musical notation. But it can also be represented by chord diagrams.

Number 4 suggests that the discovery of mathematical and especially numerical relationships and their match to what we observe in the universe represents an uncovering of its fundamental mathematical fabric. Here, Pythagoreans do find a place: their discovery was that #4 is how it goes. Numbers (and in modern language, patterns) are *in* physical objects.

Most rough-and-ready physicists would lean towards #3, but not everyone. I'm close to #4, but in a practical and not spooky way. (Some of my contemporaries are okay with spookiness when it comes to math and reality.)

We owe a debt to the Pythagoreans and while their application of "number" to the world is primitive, there are vestiges of their discovery that make science (and modern life) possible.

Mathematics describes the universe There is this nagging feeling that math and physical reality share a pretty special bond. Before the advent of Pythagoreanism, we saw that the Ionian approach to parting ways with deities was to ascribe a

[14]Want to start an argument? Try to defend any definition of what Pythagoreanism is. (You can also spice up the conversation by trying to defend what Platonism is, which is the next chapter.) I'm not a philosopher, but I do have a sense of how my interpretation of these two ideas fits my experience in modern physics research.

fundamental "stuff" as the basis of all physical things. Now, we don't depend only on that. We use math.

Take the weather. Before Pythagoreanism took hold, numbers meant "one apple," "two apples," and so on. Counting and nothing more. Before Pythagoras, I think that describing the weather using numbers might have seemed as strange as us saying that the weather is "happy." While the ancient Pythagoreans didn't use numbers in most of the ways that we do, they might not be surprised that we are now comfortable to describe the properties of our weather more completely with numbers than with words. I just looked at the weather in Pythagoras' modern Crotone in Italy, and it's not happy: it's 22° C (79° F), with a relative humidity of 76% and since the dew point is 71°, that's uncomfortable.[15] The barometric pressure is 1016 mb of Mercury and rising with a cloud cover of only 11%, so visibility is 10 miles. This short narrative puts a picture in your mind of the weather conditions that words would do much less efficiently or accurately. But there's more. I could take those numbers and recreate exactly those conditions in a lab. They are a natural measuring stick for us, and that's due to our Pythagorean inheritance.

MIT cosmologist Max Tegmark holds an extreme view that the numbers in our story aren't just *in* the weather; they *are the weather*. That is, if there's a one-to-one correspondence between a number and my interpretation of what the number means, then they're the same.

A taste from his controversial book, regarding the electric field:

> "If you can thus pair up every entity in our external physical reality with a corresponding one in a mathematical structure ('This electric-field strength here in physical space corresponds to this number in the mathematical structure,' for example), **then our external physical reality meets the definition of being a mathematical structure—**indeed, that same mathematical structure." (emphasis, mine) Max Tegmark, 2014, page 280

That he's under attack suggests that physicists do have strong opinions about #3 versus #4, as much as they'd probably outwardly profess disinterest.

> "So the bottom line is that if you believe in an external reality independent of humans, then you must also believe that our physical reality is a mathematical structure. Nothing else has a baggage-free description. In other words, we all live in a gigantic mathematical object—one that's more elaborate than a dodecahedron, and probably also more complex than objects with intimidating names such as Calabi-Yau manifolds, tensor bundles and Hilbert spaces, which appear in today's most advanced physics theories. *Everything in our world is purely mathematical—including you.* (my emphasis)" ibid., page 260

Or, in his technical publication Max Tegmark, 1998,

> "Physical existence is equivalent to mathematical existence."

[15]When the air temperature is equal to the dew point, the air is saturated with water vapor and the relative humidity is 100%.

I've heard him ask what is a tree. To most it's a barky, green, leafy structure with roots and a hardness and so on. To him it's a collection of electrons and quarks and reflecting and absorbing light. In turn, here's what each electron is: "-1, $\frac{1}{2}$, 1, and 0.511." That is, the properties of trees are the collection of the properties of electrons and electrons are uniquely described as a negative electrical charge of -1 unit,[16] a quantum mechanical "spin" of $\frac{1}{2}$, a "lepton number" of 1, and a mass of 0.511 MeV/c^2. Protons, neutrons, and quarks... and the light that's absorbed and emitted are also described completely and uniquely by a different set of numbers.

Now the labels that the numbers have are entirely human-defined. But no matter how an alien species might define the unit of electric charge, the electron (and proton) have ± 1 of it. So, to him what is a tree is defined by what are the properties of a tree, which are entirely defined by a small set of numbers.

Tegmark is not alone, but his is a very small club.

There are special numbers This is a book about the precursors to Einstein's *Special Theory of Relativity* which is based on the discovery of the importance of a single number: the speed of light, c. Arguably, no number is more special than $c = 3 \times 10^8$ meters per second!

While I'd not be prepared to say that marriage is "5" and when justice is done, that "9" is involved, there are many special numbers that our universe seems to have latched onto that both explain what we observe and were some of these numbers different, we would not be here. I just referred to one such special number, the electric charge of an electron or a proton.

Many numbers in nature play a role that designates unique properties of substances or processes that substances undergo. There are static properties of matter that have conventionally defined, critical numeric values. Here's one: 1836.153. This is the ratio of the mass of the proton divided by the mass of the electron. An alien species might not use the same units that we do, but whatever system they use would have to replicate this ratio. Otherwise, their Big Bang and chemistry would be completely different from ours. The formation of hydrogen atoms in the early universe would have occurred at a different temperature, and our early universe would not have formed galaxies.

Another one: Water freezes at a particular temperature. What the number is depends on a conventional scale ($^\circ$ C or $^\circ$ F), but that there is a definitive event and that it can be quantified by a unique number of degrees makes it special. If that freezing point of water were slightly different, then the geological history of the Earth would have been different.

[16]The "fundamental electrical charge" is traditionally 1.6×10^{-19} Coulombs, usually denoted by "e." An electron's is $-1e$, a proton's is $+1e$, and a neutron's is $0e$.

▻ Inherent in a Pythagorean view of the physical universe is that the "numbers are in the thing" and that we can poke at nature with experiments and extract the mathematical essence that's embedded inside. Just like Pythagoras did…before anyone else.

1.2.2.1 Unreasonable?

Generally, we physicists don't generally lack in confidence. So, in the interest of full disclosure, here's a complete capitulation, a sort of reluctant confession that we don't know why math and physics are so linked up:

Ask Mr Google to search just for the words "unreasonable effectiveness" and stand back. In less than a second, you'll be treated to a list of 12 million references to the Nobel Laureate, Eugene Wigner's 1960 article, *The unreasonable effectiveness of mathematics in the natural sciences.* It's actually a written version of a lecture he gave at NYU, and it's among the most famous documents in physics. It's so ubiquitous, that Wiley Publishing is pleased for you to download it for free.

In that same vein, here's a word that you won't find physicists using: "miracle." The last paragraph in Wigner's article states:

> "Let me end on a more cheerful note. **The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift** which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning." [emphasis mine]

"A more cheerful note"? "The *Miracle*"? for heavens' sake? If that's his conclusion, can you get a sense of how his previous nine pages went?

There's a straight line from Pythagoras (and Pythagoreans…remember) to Plato and Platonism and to physics! But we don't understand this "unreasonableness," and sometimes it is kind of uncomfortable. Gloves come off when physicists and astronomers argue about multiverses, string theories, and measurement theory in quantum mechanics.

So, by now maybe you're a little more aware of the possibility that we may *all be a little bit Pythagorean*. Over and over we learn this.

1.2.3 Zeno and His Paradoxes

Parmenides had a following, and his most devoted and enthusiastic partner was the younger **Zeno of Elea (ca –490 to ca –430)**. What he did was mess with everyone's mind about simple, common-sense experiences. He's remembered primarily for 10 paradoxes, two of which are about motion. I'll remind you of here as the most famous. He wants to show you that what you think you know, you don't and that common sense deceives. (Like in Quantum Mechanics and Relativity, where common sense left the building a long time ago.) I'll do them in reverse order. (By the way, how do we know of his arguments? Plato, again, in a dialog where Socrates deals with the young Zeno, playing himself. And Aristotle, who goes after Zeno.)

"The "Dichotomy." This is the famous race. In order to run the 100-meter dash, you've got to get to 50 meters. In order to get to 50 meters, you've got to get to 25 meters. See where I'm going (or maybe *not going*)? According to Zeno, there are an infinite number of distances that have to be traversed in order to move in space at all. So you can't get to 100 meters, in fact, you can't move at all. MOTION ON THE EARTH is impossible. Aristotle noticed that this is like the Achilles and the Tortoise paradox, except the conclusion of no motion is reserved to the Dichotomy.

Now, this has been dissected for centuries. Ask Mr. Google about "Zeno," and you'll see 36 million hits. The push-back begins with Aristotle, who argued persuasively, but in the end, inconclusively, that you can move through an infinite number of spaces if the time intervals become shorter and shorter while you do it. Aristotle hated infinity, so this must have been hard for him. But this presumes that Zeno was suggesting that the motion would take an infinite amount of time, but maybe it's because he was trying to cram an infinite number of steps into a finite period of time. So, Aristotle's argument is not general enough.

The modern solution requires an understanding of how speed relates to time and space, a very modern set of ideas that are the heart of Relativity. I'll show you a complete explanation in Technical Appendix A.2.

The Paradox of Infinite Divisibility. This paradox is the jumping-off point to an entirely different way of dealing with Heraclitus and Parmenides: If an object is made of parts, then one should be able to start cutting... into two parts, then four parts, and so on. At some point, you reach some end: 1) If after an infinite number of slices, you find nothing..., then the object was made of nothing—a **not-is**. 2) If after a finite number of slices, you find something... but it has zero size, then the object was made of something that has no size. Another kind of **not-is**. 3) If after a finite number of slicings, you find something that has finite size, like an element? Well, you're just not done slicing!

> This is a modern thing as we are perfectly content to imagine that quarks that make of the proton and neutron have no size,[a] likewise the electron. But we have a field description of elementary particles and the forces among them, so we have a quantum mechanical push-back against Zeno here. But prior to the 20th century, a physics solution was not possible.
>
> [a] Yes. Our word "particle" creates an image of a little billiard ball, doesn't it? In actuality, the size of quantum mechanical objects is so ill-defined as to have little meaning outside of agreed-upon criteria involving waves.

You can see how this works. Zeno was apparently clever enough to waste the pixels on your computer screen in 36 million hits... all in service to the Parmenides' two arguments: **Nothing changes** and **knowledge from perception cannot lead to**

truth.

Chapter 2

Can’t Live With ’Em Or Without ’Em : Plato and Aristotle

“The safest general characterization of the European philosophical tradition is that it consists of a series of footnotes to Plato.”

- A.N Whitehead (1861-1947), *Process and Reality*

Bert and Ernie, Kirk and Spock, Mantle and Maris, Venus and Serena, Abbott and Costello...Plato and Aristotle. One can’t have one without the other, and like the other pairs in that list, these last two are deep subjects. My need for Plato and Aristotle’s contributions to the study of MOTION are for two ideas: following Pythagorean inspiration, Plato and his collaborators built the first spherical working models of MOTION BY THE EARTH and MOTION IN THE HEAVENS. Aristotle expanded on it, and they were both wrong.

And, while Plato didn’t concern himself with MOTION ON THE EARTH (except in an almost impenetrable portion of his last book), Aristotle was all over MOTION ON THE EARTH and invented its systematic study, informing—and infecting—science for 2000 years. It took until the 17th century before we could be all over with Aristotle. His models of MOTION ON THE EARTH, MOTION BY THE EARTH, and MOTION IN THE HEAVENS became Medieval and Renaissance Church dogma but are wrong in almost every respect. By pushing back, scientists learned what was better and why.

So why does Plato’s shadow hang around while Aristotle’s importance for physics disappeared more than 400 years ago? We still talk about Platonic worldviews in some fundamental branches of physics, but nobody talks about Aristotelian–anything. Plato put important questions that remain troubling: What can we know? How do we know when we're right? And, most importantly, what is the role of mathematics in the fabric of the universe?

It was the worst-kept secret sneak attack in history. Everyone knew that the Persians were coming as under King Darius' son Xerxes the Great's command, the invading infantry slowly marched along in parallel to the Persian navy counter-clockwise around the inside of the Aegean basin, subjugating the Ionians along the way. Anaximenes lived under the locally sourced Persian rule that drove Pythagoras to Italy. About 100 years before Socrates' execution, following a 10-year advance of −480, the battle was joined with an amassed Persian force of 150,000 soldiers and 600 warships. Athens was evacuated, and the Persians destroyed the city.

The Greek confederation then organized itself: the wounded Athens mounted the naval campaign, and Sparta, the foot soldier command. What followed was a series of military maneuvers, which are still studied today. Spartan heroism of King Leonidas, with 300 Spartan troops and 9,000 allied soldiers, met and slaughtered the Persians at the pass at Thermopylae. The movie and the comic book series *300* might jog your memory (Snyder, 2006). While this was happening, the Athenian navy engaged and overwhelmingly defeated the larger Persian naval force. Finally, during the summer of −479, the Persians were defeated in a decisive land battle. Yet, the war continued in one form or another for thirty more years until the Persians fled the Aegean, leaving behind a Sparta with a greatly enhanced reputation. Proud Athens rebuilt after that disaster in −480 and under Pericles' leadership — throughout the decades of extended conflict, began its 75-year Golden Age when everything you think of as Greek in culture, art, architecture, and philosophy was intentionally created.

Ironically, even though Sparta could be credited as the major military force in the Greeks' victory, its isolated and belligerent nature simply did not equip it to lead during peacetime. In contrast, while Athens had been destroyed, its nature was to rebuild more robust, to organize politically, and to lead—all while doing what Greeks did best: fighting.

While the Golden Age was unrolling, Athens simultaneously managed to battle with Sparta −465; Corinth and Sparta −459; Samos −440; Corinth again −433; Potidaea −433; Mageria −433; Sparta again −431 (Socrates was active as a soldier during this period), (Score: **Sparta 1, Athens 0**) Syracuse and Sparta −415, (Score: **Sparta 2, Athens 0**); Sparta −414, (Score: **Sparta 3, Athens 0. Game, Set, Match**).

After that third war with Sparta,[1] Athens surrendered to Spartan general Lysander

[1] who allied with Persia!

in −404. Plato was 23 years old, and Socrates had five years to live.

Athens poorly handled its unfortunate overreach and eventual defeat. In the final stages of the war, it managed to expel its leading general, execute six other military leaders, and flip from autocracy to democracy and back to autocracy. Socrates was on the autocracy side, and it was the democrats who condemned him to drink the hemlock in −399.

Athens' subjugation by Sparta after the two Peloponnesian Wars was tumultuous — governance of the city jerked back and forth between oligarchs and democrats. In the same way that the Golden Age of Classical Greece emerged during the war with the Persians, amid the city's internal chaos, western philosophy began and was followed quickly by the first systematic attempts to understand MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS by our two lead actors. Yet the catalyst to all of this progress was not interested in either. Socrates' persistent question was how to live a virtuous life, not how things move. As his talented acolyte, Plato adopted the older man's voice and wrote truly engaging tales but expressed his ideas, and while his program was ostensibly one of ethics, the Socrates/Plato approach opened a new front in the battle with the Parmenides Problem which resonates in modern physics today. And, as so often happens in philosophy (and physics), the next productive steps were in opposition, launched by Aristotle, one of the most remarkable intellects in history whose words we have are probably from lecture notes and not intended as his legacy literature. Yet in physics, Plato endured, and Aristotle is gone.

2.1 Act V A Little Bit of Plato

Plato (–429 to –348) is a nickname, suggesting someone of broad shoulders or perhaps a wrestler. The name on his driver's license would have been Aristockes, and his aristocratic family had been influential for generations. Plato was no democrat and grew up during the Peloponnesian War (−431 to −405)[2] and the subsequent subjugation of Athens by the victorious Spartans. In many ways, Plato's idea of the correct form of government was informed by the collectivism and brutality of the Spartan way. But he was close to Athenian politics as a young man. His family connections allowed him to join the Thirty Tyrants—the oligarchy that overthrew Athenian democracy—but he was so put out by the violence that he stepped away. The democrats retook Athens (Set the context with the timeline in Figure 1.2 on page 22.)

[2] He fought in the war and then again served in the military, perhaps during the Corinthian War.

> One of the signature events of his life was his attempt to help form a government in Syracuse, where he somehow got the idea that he could turn the tyrant Dionysius into a philosopher-king since, in Plato's opinion, leaders should be philosophers. That got him imprisoned and even sold into slavery for a while (or so the story goes) until he was ransomed. He tried two more times, which brings to mind one's questionable mental state as per Einstein's observation much later about repeating the same mistake over and over and expecting a different outcome..

His life's direction was formed when he, like many young men in the newly democratic Athens, started to associate with **Socrates (–470 to –399)** who, after his (apparently distinguished) service as a foot soldier in the war, took philosophy on an entirely different course from investigating the nature of reality to how best to live a satisfactory life. Many of us learned in school about Socrates' self-administered execution at the hands of democratic Athenian politics—one of the reasons that Plato was distrustful of democracy. It was traditional to give the convicted criminals options on how they would like to do away with themselves, and Socrates suggested that he be given free food for life. That was turned down, and eventually, death by poisoning was prescribed.

Plato's 35 books are all in the dialogue form, conversations between Socrates and various fictional and real persons. Unlike Aristotle's largely academic writing (which might have been lecture notes), Plato's books are literature and are valued for their style and lyricism. Plato himself is only mentioned twice, and he never speaks directly. The assumption is that he's talking through his mentor and that the ideas are his and not meant to be those of the older man. (One book, *The Apologies*, might have been more personally Socrates as in that volume he defends himself against his accusers.) So, the ideas are Plato's, and the books comprise his philosophy as it evolves over his productive, long life. Almost all of his work follows a general theme, and what he seems to struggle with is what I've called the Parmenides Problem. Plato wants to contrast what we experience in our everyday world—objects (physical things) and ideas (like virtue, justice, beauty, what's good)—with abstract concepts that are the source of the variety of physical things and the imperfect values we associate with more aspirational ideas.

It might be reasonable to view the Socrates of Plato's dialogs as a literary invention. Still, he was known to broader Athens and even parodied in the *Clouds*, a vicious comedy by Aristophanes and figured in other writers' accounts, including in dialog form. But the world now knows of Socrates through Plato, and he figures into every one of Plato's dialogues as "that guy" who irritates everyone. However, in the later dialogues, his role diminishes. His job is to ask simple-seeming questions (the "Socratic Method") of an assembled group of friends (or foes), often about an ethical matter. What's temperance? What is virtue? What is justice? The course of these sorts of innocent-sounding conversations is repeated: the folks being questioned are maneuvered into impossible rhetorical cul-de-sacs, shown to be incapable of logical thinking, and more often than not, shown not to know things that they should have known.

Meanwhile, Socrates rarely says what he thinks; in fact, he usually hides behind the assertion that he doesn't know either, but at least he knows that he doesn't know. Superior to a fault. These questions also often segue into something more than they seem, and many move to more weighty topics like how *do* you know what you know. They form the beginning of serious Epistemology, one of the foundational philosophical disciplines.

Plato's output was enormous and I'll choose only a few topics that inform our scientific project. We have complete writings unlike almost all of the previously considered Greek philosophers. He famously started **The Academy**, a school that lasted more than 700 years and whose star pupil was Aristotle, whom I'll tell you about below. Bertrand Russell (in his Literature Nobel Prize-winning, *A History of Western Philosophy*) appropriately sums up what I'm about to dive into:

> "Aristotle's metaphysics, roughly speaking, may be described as Plato diluted by common sense. . . He is difficult because Plato and common sense do not mix easily." (Russell, Bertrand, 1946) *A History of Western Philosophy*

My focus is on two aspects of Plato's philosophy and then his physics and how they're related. I'll leave his modeling in astronomy to Chapter 3 when I review early Greek astronomy, but I'll consider his overall approach to astronomy here. Of concern then (and now) are Plato's Epistemology—what does it mean to know something (from the *Meno* and *Phaedo*), his Metaphysics—what is the nature of reality (from *Phaedo*, *Parmenides*, and *Republic*), and his physics (from *Republic*, *Timaeus* and Book X of the *Laws*).

2.1.1 What Is True Knowledge?

Our Parmenides Problem deeply influenced Plato and took this on with a study of the broader question of what constitutes true knowledge. He thought deeply about this, and his conclusions became grist for philosophical mills for the next 2500 years.[3] He decided that there are two hallmarks to knowing: that knowledge should be infallible and that it should be "of something that is." Typical was the exchange between Socrates and the 16-year-old Theaetetus in the dialogue by that name. Socrates teases out of the boy his ideas of four kinds of knowledge and demolishes every one of them. First up, what do we learn by *perception* as a source of knowledge? Socrates dispatches that since your *internal* perceptions are infallible (what you think is true to you), but *act* of perception cannot prove that the *objects* of perception exist. So, it fails on the second hallmark. Second up is *belief* as a source of knowledge. That results in a blistering dissertation on subjectivity. And, finally, third up is "true belief." Naive belief and even true belief are fallible, so failing on the first hallmark. Three outs. But what about *belief with a reason* to hold that belief, what in the context of *Theaetetus* is sometimes called "true belief plus

[3] I'm grateful to philosopher Professor Harold I. Brown for essential discussions on this complex topic in Platonic philosophy.

an account" or, "**Justified True Belief**"? This is sometimes incorrectly described as Plato's theory of knowledge, but Socrates makes a hash of JTB and leaves the question in an unsatisfying state. Let's look at a couple of examples.

J+T+B was considered among the best efforts into the present day and relies on the three aspects memorialized in its name. The B: one can't claim knowledge about something you don't believe. (I read that my calendar reports that today is Tuesday, but I think it's Monday, which certainly doesn't qualify as knowledge of Monday.) The T: the fact must be true (if the fact is not true, then you cannot be said to have knowledge of it.) The J: whatever you claim about the fact, you need to be able to justify it.

Consider this claim: It is 3 o'clock. I believe it's 3 o'clock because I looked at my watch and saw that time displayed. B, T, and J are all in play, and this seems a reasonable example of knowledge.

> But there are holes and weaknesses. Instead of that J, how about J2: It is 3 o'clock, it's 3 o'clock, because 3 is my favorite number. I'm right since it is 3 o'clock, but that justification is silly and certainly doesn't qualify as knowledge of the time. How about this, J3: It is 3 o'clock; I believe it's 3 o'clock because I looked at my watch and saw that time displayed. But...I didn't know my watch was broken and had stopped at precisely 3 o'clock. So it was just luck that my reading corresponded to the right time. So that's hard to accept as knowledge. In fact, it was only in 1963 that Edmund Gettier found counterexamples to JTB, which are now called "Gettier Cases."

Clearly, justification is the rub, and many efforts have been made to turn J+T+B into J+T+B+X, where X is something added to take care of the Gettier Cases. It's an ongoing problem. For scientific claims of knowledge, sometimes Justification weaknesses turn on issues of observation and even the senses (for direct or instrumental observation), so we're right back to the Parmenides Problem.

Plato had an answer that turns out to be more than a theory of knowledge, but also a theory of what's real: fixing epistemological problems resulting in metaphysical commitments.

True knowledge, for Plato, can only come from permanent, unchanging things. Thanks, Parmenides. If something is true, it must be so forever, which means that it was never untrue, nor will it ever become untrue. He falls squarely in the Being camp, as opposed to the Becoming camp.

> Notice how this demand of permanence as the qualifying feature of true knowledge is an **unquestioned commitment**. In this thinking, there's no room for degrees of knowing—yet we all know things with varying levels of trust, and this is especially true in science where not being able to question an assertion is now the very definition of "unscientific." I think their insistence on permanence is a function of their being impressed with geometry and the fact that it was very early days in the brand-new field of epistemology.

Plato differed from ardent Eleatics like Parmenides by insisting that knowledge is possible, but there's a catch.

He proposed after *Theaetetus* that there are two worlds:

- The world of the Forms.
- The world of the senses.

2.1.2 The Forms

Plato's theory of the **Forms** is one of the most challenging ideas in philosophy, but comprehending it is critical for an understanding of his projects, *but also for appreciating physics*. He gives abstract concepts an existence of their own and a job to do with a consequence that grates on you.

Take high school (please): If you ever took a geometry class, you were presented with a set of elementary pieces from which you could create new, bigger pieces with just a ruler and a compass. These pieces included things like points with no extent and lines with no thickness. You manipulated and proved theorems about angles, relations, rectangles, perfect triangles, and circles. Let's focus on that last one.

Think of all the "circular" things you come in contact with in your everyday life. Coins, dials on appliances, buttons on your shirt, a camera lens, a cookie, maybe a rendition of something circular in an image or on a screen. You know that none of these circles are the circles of your geometry class. But these circular things share the property of *circularity*. They may woefully miss the perfection of that high school circle in your mind, but by thinking about it, you know that your Oreo is almost circular.

Plato would say that that unique abstract circle *actually exists* as a "Form." That there's a kind of reality—a realm— that's different from the reality you think of when you drop that circular plate in the kitchen. That abstract realm is where the Forms exist.

That high school geometry's perfect circle is such an abstract notion. But you can grasp that reality, you can apply it, engineers can use it, and you recognize it when you see it...only in your mind. Try an experiment: construct the best circular thing you can and measure its diameter in a hundred points around the center at micron precision— while your rendition may be a good one, it's not that abstract high school one, is it? The Form of a circle is aspirational but can't be studied by measuring regular-life circular things rather it can only be brought to life through your intellect. The Form of a circle has always been there (circles were not born), and that realm is outside of space and time. Can you get on board with abstract things being real?

Maybe Plato's assignment of "real" to mathematical abstractions is slightly less odd than at first glance. But he went further than geometry; you might have experience with non-mathematical abstractions. Here's one: "We hold these truths to be self-evident, that all men are created equal..." What is a self-evident truth? If it's a "truth," then questioning it is a waste of effort; it's permanent in a Parmenides sort of way. If an idea is self-evident, then in some sense, it's always been there, imprinted in us, while accessible, but at the same time, distant.

You can't hold such truth in your hand, and you know it's not universal in our everyday life since "all men are created equal" is untestable since the ones we know are tall, some are smart, and, yes, some are disadvantaged. That they're "equal" is an abstraction— an aspirational idea of perfection— that we can hold in our minds but know won't be realized in "our world." But a nation of 300 million "Platonists" swears by that truth.

What about realities outside of our plate-dropping reality? If one is a Christian, then you've been brought up to believe in such a reality—heaven (and hell) are outside of our everyday lives.

But more to Plato's point, I see hundreds of sofas when I go to a furniture store. They're all different, but they all share. . . a "sofa-ness." They're all *participating (sharing) in the Form of the Sofa* which I can (only) know of in my mind. It's a perfect sofa.

With the forms, the Parmenides Problem is dealt with in a brand new way: there is a world of Being and a world of Becoming and they are connected, but in a hierarchical way. And, it's not just living room furniture that has Forms. There is a Form for everything: even Justice, Virtue, Beauty, and the Good. . . the latter of which is somehow a super Form.

2.1.3 The Republic

Plato's contribution to science is not any particular theory or practice, but as (G. E. R. Lloyd, 1970) suggests, it is more his philosophy of science that we value. This is laid out most explicitly in *Republic*, probably his most famous book, ostensibly, a treatise on politics and good governance. It's here where he describes how a city should be ruled, certainly not by popular election, but by the training of a special category of people bred and educated in order to be rulers, the philosopher-kings, the guardians. Their lives would be scripted from early ages, living communally, and essentially the pool of potential candidates for leadership. Their educations would be scripted as well, relying on an intensive study of mathematics to foster a habit of mind. The goal is for them to be completely comfortable with the most abstract concepts, including Justice and what's Good. Learning mathematics is a primary route to that appreciation. *Republic* includes a few analogies to try to get Plato's point across. Two are relevant to physics.

Analogy of the Divided Line.

Along with the Allegory of the Cave, the "Analogy of the Divided Line" is important for Plato and, I think, important for physics—as Galileo and modern physics will eventually enlighten us. A rendition of the Divided Line is in Figure 2.1. What we can know is a hierarchy, from muddled to perfectly clear and divides into two broad “realms,” one representing our *Becoming* world—The Visible Realm—which we occupy in everyday life, and the other representing the *Being* world—The Intelligible Realm—which is outside of space and time and only recognized through thought.

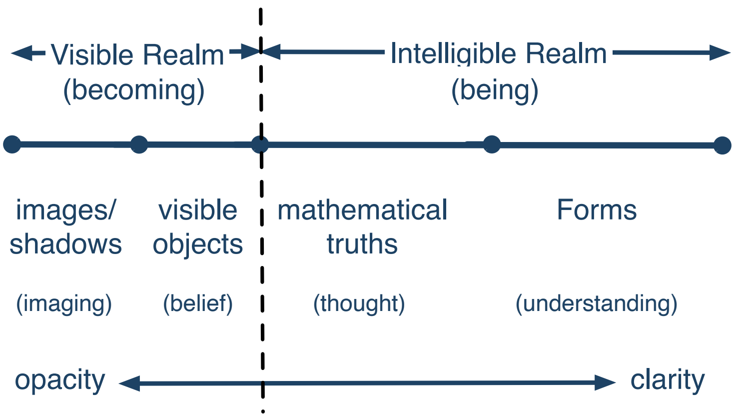


Figure 2.1: The line represents a kind of knowledge-hierarchy, from unclear to perfectly clear.

The Becoming realm is broken into two levels, of which the objects of the first and lowest segment are shadows and illusions of objects in our experience. The shaky knowledge we have about them are mere illusion and dreams. The objects of the second stage are actual, everyday objects themselves, and the knowledge we have about them are opinions and beliefs gleaned through our (untrustworthy) senses. Taken together, these two stages constitute our knowledge of our everyday world, where things change: the Visible Realm is where you and I use our senses and dreams to navigate our lives.

The Intelligible Realm is only accessible through thought and reason and is likewise divided into two more sophisticated segments. The first includes knowledge gained through mathematics and hypotheticals (think high school geometry) about which we have knowledge through reasoning. Finally, the highest segment of the Intelligible Realm is of the Forms, the pinnacle of clarity, “beyond hypothesis,” which is aspirational and not easily realizable.

Earlier, I opined that “degrees of knowing” is a more modern way of thinking, and the Divided Line actually sneaks up on just that. As I’ll emphasize, when we study Galileo, there is a realm of the universe that is very hard to observe (on Earth) but which is our goal when we theorize about nature. So I’m not quite willing to pass this off as silly, while at the same time, I don’t agree with the realm of the Forms as an ethereal parallel universe that we cannot access but through rationality. Stay tuned.

Allegory of the Cave.

Plato famously tries to work out more of these distinctions in the *Republic* with the famous "Allegory of the Cave" and in the *Meno* with the idea of" Reminiscence." In the former, prisoners in a dark cave are shackled to the ground facing a wall. They can only look straight ahead and what they see are shadows of objects and puppets that are held in front of a fire behind them so that they project on the wall. If they see a sofa on the wall, it's because the Form of the sofa, which is behind them and out of sight, is projected as a shadow of the real Sofa in front of the fire.

Now, if one of the prisoners escapes her bonds and looks around, she'll see the fire and the contrived circumstances. The light from the fire would hurt, and she'd want to go back to her former spot. And if she were dragged out of the cave and into the sun, she'd be blinded, but slowly, she'd look around her and realize that there are actual things in the world and not just shadows. Notice that in the Allegory, she's moving from left to right in the Divided Line in Figure 2.1. She ventures back into the cave and tries to describe that true reality to her still-captured colleagues. But in the dark, she'd not see well, and the prisoners would not allow her to persuade them to follow her into the sun since it apparently takes away one's sight. Plato even worries that the prisoners might kill the one who escaped.

Obviously, Plato is describing the daunting project that he's taken on as the enlightened former prisoner trying to explain what's Real and True to everyday people who don't want to accept it. The similarities to Neo's trip out of the realm of perceptions and into the realm of the real is not an accident as the *The Matrix* (L. Wachowski, 1999) is full of philosophical allegories, and the Cave is one of them.

What we can learn in the realm of the Forms is true knowledge and a goal of mastering philosophy. What we can know of the world of appearances is simply opinion. The Forms inspired many in the centuries to follow, from Neo-Platonic Christian images to modern science. We'll come back to them when we discuss Galileo where finally, properly characterizing MOTION begins. By the way, Plato despised art. A painting of a mountain is nothing but an imitation (the painting) *of an imitation* (a sensible mountain) of the *actual Form* of a True Mountain.

2.1.4 Mathematics For Plato from Republic

Plato's experience in Italy wasn't limited to a failed experiment in his theory of governance but began as a deliberate project to study with Pythagoreans. Pythagoras had been gone for a century by that point, but two schools grew up around his legacy. The *acusmatici* viewed themselves as the guarantors of Pythagoras' the man's legacy as a complete system. Not only his mathematics but the other aspects of the Brotherhood were preserved and defended without expansion or elaboration. On the other side were the *mathematici* who bought into reverence for the man, but intentionally expanded the mathematics to new areas of research, an unwelcome sin in the eyes of the *acusmatici* who eventually died out.

Recall that Plato and Aristotle probably learned most of Pythagoreanism from Philolaus, but Plato's mathematical inclinations came from a contemporary, one of the mathematicians that Plato befriended and learned from, **Archytas of Tarentum (ca –420 to –355)** who is one of our characters in Chapter 3. Our title character in the next chapter is **Eudoxus of Cnidus (–408 to –355)**, a student of Archytas and the most significant mathematician before Archimedes. Both influenced Plato and Aristotle's cosmology, and that subject kicked off two millennia of modeling and, eventually, dogma. The mathematics required in the guardians' education came from Architas, arithmetic, geometry, astronomy, and harmonics. Plato didn't fully agree and added a fifth subject, solid geometry.

Maybe you can begin to understand Plato's elevation of mathematics—in the Greek life of his day, geometry and proportions—to the point of his famous sign above the door, "Let no one who is not a geometer enter." (Well, that sign only crops up in the 4th century AD, so it's probably a myth.) Geometry is venerated by Plato and all who follow for centuries.

This is hit directly in *Republic* where Socrates extracts from Glaucon[4] the reasoning behind requiring astronomy for guardian training. As usual, Socrates/Plato starts out with a theme which in the course of explaining it, evolves into a matter of serious philosophical interest. Glaucon tries to guess at why astronomy is important. Maybe because it's useful for recognizing seasons, or timing agricultural events. Practical things. That doesn't go over well, and so he tries again: maybe astronomy is "good for the soul"... that looking at the sky takes us away from looking at everyday things. Again, not productive for Socrates. Here's where geometry comes in and where Plato earns an uncertain reputation for suggesting that "armchair astronomy" is the only way to go: doing astronomy without ever looking at the stars. Here's how I interpret this:

Back to the literal drawing board: Take out a ruler and the sharpest pencil you have and carefully draw the most precise triangle you can create. now get the best protractor you can buy and try to verify that the interior angles of that triangle all add up to 180°. No matter how careful you are, you'll fail to perfectly measure 180.000...°. In fact, Socrates/Plato would tell you to not bother since studying an everyday triangle won't help. The perfect 180° is in your head, and its truth is one of reasoning and geometrical proof.

Socrates/Plato suggests that the same is true for astronomy.

> "We shall therefore treat astronomy, like geometry, as setting us problems for solution," I said, "and ignore the visible heavens, if we want to make a genuine study of the subject and use it to convert the mind's natural intelligence to a useful purpose." Socrates/Plato, *Republic*.

He says that you can look at the stars, but discerning their actual motions cannot be done by measuring their apparent motions. You can only understand MOTION IN THE HEAVENS by reasoning; astronomy without looking up! Like the triangle, you might get hints from the world of Becoming, but only through reasoning can you learn what the stars and planets do in the perfect world of Being.

[4] Possibly, Plato's older half-brother.

> Here is **another unquestioned commitment** by Plato. That the stars and planets would necessarily execute perfect motion is an assumption. Again, this is the very earliest days of astronomy and philosophy and it's built on a variety of prejudices.

Plato's "Doctrine of Reminiscence" is another idea that comes from the Forms. In the *Meno* Socrates demonstrates that a slave boy actually knows geometrical proofs without knowing that he knows them! By asking questions in his Socrates way. In the *Meno*, the protagonist, Meno (a real, young aristocrat), asks Socrates if Virtue can be taught, and of course, Socrates begins by asking the young man to define what Virtue is and then dismembers his multiple attempts at an answer. The scene degenerates into Meno now becoming frazzled and paralyzed as the discussion evolves. As often happens, more than the problem at hand emerges, including what's called "Meno's Paradox": the realization that if you know something, you don't need to ask about it, but if you don't know it, then you don't know enough to ask. Of course, this all leaves everyone unsatisfied. It's surprising to me that anyone ever wanted to talk to Socrates!

The discussion turns to a religious view that the soul has always existed and will exist after we die and that the soul knows all that there is to know before and after and, therefore, we already know everything we've just forgotten. He then proceeds to demonstrate this idea by asking a slave boy the geometrical proof of how to double the area of a square. By asking him successive questions, he pulls the proof out of the boy. (You can see the proof in Technical Appendix B.1.

> In school, did you ever successfully work out a proof in geometry or mathematics? Don't you do a little victory dance inside, maybe with a knowing nod — Aha!— that solution seems like it was there all along, and all you did was *reveal* it. That you almost *remembered it*. This is the basic characteristic of **deductive reasoning**. It doesn't lead to anything new but reinforces—-(or recalls, suggests Plato)— something that was already in the premises. I know I've had that feeling, and I can understand why Plato chose a geometric proof to illustrate his idea, which is broader than just math for him, of remembrance. What Plato was really after was the fact that the Form of that geometric proof was there all along, in that Intelligible Realm, all the time.

2.1.4.1 The Soul

The "Soul" is a very Greek idea that functions at multiple levels for Plato; in one dialogue, he assigns three separate jobs to the Soul. For our purposes, he's impressed with the idea that some things are inanimate — like a rock — and that some things appear to be animate. The very word "animate" gives you a sense of what he thought might be the distinguishing feature between animate objects: they can self-generate their motions, move themselves without an outside cause. So in some ways, this is a question of MOTION ON THE EARTH (but he extends it to MOTION IN THE HEAVENS). He found the Soul a useful cause for all things that can move of their own accord — he would speak of “self-motion” — as imbued with the Soul. It’s not only humans, but birds, flowers, and even planets that appear to be able to execute locomotion on their own and enjoy their very own Soul. I’ll show you that this idea actually figures into some of his astronomy, so in a backdoor sort of way… this is an example of MOTION BY THE EARTH! It is this very talented Soul that causes self-motion among animate objects, but also persists before and after death. We get a glimpse of the all-knowing Soul when we do a mathematical deduction, as Socrates illustrated with the slave boy.

2.1.5 Timaeus

Boy, the European medievals must have been confused about Plato. Until the early 12th century, the only Latin translation of any of his works was just one: *Timeaus*. It’s notoriously difficult, convoluted, and ripe for repackaging by the “neo-Platonists” up to Augustine. In this difficult late dialogue, the title character is Timaeus of Tauromenium, a fictional Greek statesman and scientist from southern Italy (ah, as we’ll see, surely a Pythagorean), who is encouraged by Socrates at yet another get-together to tell the origins story of the universe. *Timaeus* is less a dialogue than a monologue, and it covers a lot of ground without Socrates being his usual, obnoxious self. Obviously, Plato had a lot on his mind in this book.

He was so enamored of mathematics that through Timaeus’ voice, he built what he calls a “likely story” of cosmology by mixing geometricized ideas of the atomists with a relentlessly Pythagorean numerology (that he learned directly from Archytas?), a major focus in Chapter 3.

Timaeus relates that the universe was assembled (not created) through the actions of a “Craftsman”[5] who builds everything—animals, planets, stars—from a blueprint of eternal ideas, which are surely the Forms, and does so using existing materials at hand. It’s not created from nothing (so Parmenides’ influence is apparent). He’s an artisan, more than just a laborer and less than a creative deity. Plato leaves the impression that the Craftsman does the best that he can — a best-effort universe! There is a difficult overall purposefulness and expectation that the Craftsman is “. . . . greatest and best and fairest and most perfect.” This is the best possible world.

The dialog begins with Socrates counting, “One, two, three,. . . ” a portending of the strange, mystical use of numbers as the Craftsman does his job. I’ll reserve the cosmology part of Timaeus for Chapter 3 and make reference here to only those parts of the dialogue that overlap with our project. That leaves most of *Timaeus* untouched.

Referring to Plato’s invention of the fable of Atlantis and Athens 9000 years before leads to the idea that Earth is periodically destroyed, erasing memories for everyone

[5] In Greek, the “*Demiurge*.”

but somehow, not the Egyptians. This prompts a discussion of how the universe began. Timaeus asks (with Parmenides looking over his shoulder?):

> "What is that which *always is and has no becoming*, and what is that which is *always becoming and never is*? That which is apprehended by intelligence and reason is always in the same state, but that which is conceived by opinion with the help of sensation and without reason is always in a process of becoming and perishing and never really is." (emphasis, mine) Plato, *Timaeus*

Suffice it to say that the Sun, Moon, and planets all take their familiar places according to a mathematical (even musical—Pythagoras, again) format and that Time itself is created along with the planets. In fact, the motions of those most nearly perfect celestial bodies are the cause of time. The ancients told the days, months, and years by the motions of the Sun, planets, and stars and so it's maybe not a surprise that Time and those objects have a causal relationship to one another.

The Craftsman isn't omnipotent and is restricted to using those Empedocles' four elements — the materials at hand.

> "The starting-point is, of course, universally accepted: that fire, earth, water, and air are material bodies. Now, this means that, like all bodies, they have depth, and anything with depth is necessarily surrounded by surfaces, and any rectilinear surface consists of triangles. There are two basic triangles from which all triangles are derived, and each of them has one right angle and two acute angles." Plato *Timaeus*

That seems deceptively straightforward and here's what he means. There are three kinds of plane triangles: equilateral (all sides are equal, so all angles are 60°), isosceles (two sides are equal and so two angles are equal), and scalene (no sides are the same length and no angles are equal). He concentrates on two, the isosceles and his favorite triangle:[6]

> "...we posit one as the most excellent...whose longer side squared is always triple its shorter side" [and] "...one whose hypotenuse is twice the length of its shorter side..." Plato *Timaeus*

Those two descriptions are identical and the hypothenuse being twice that of the shorter leg specifies a particular scalene triangle with interior angles of 30°/60°/90°. With an isosceles triangle with interior angles of 45°/45°/90°, he has the two "elementary particles" of his universe: everything is made of their various combinations.

Figure 2.2 shows the two primitive triangles at the top.

The *Timaeus* outlines the way in which Fire, Water, Air, and Earth are represented as solid shapes which are themselves built out of those two kinds of primitive triangles, and Figure 2.2 shows how he suggests this happened for his "most excellent" triangle: On the left, he uses 6 scalene triangles to make an equilateral triangle, and then multiple equilateral triangles can be fitted together to make three

[6]Everyone should have their own favorite triangle.

kinds of 3-dimensional volumes: the tetrahedron (a three-sided solid, made of 4 equilaterals, so 24 scalenes), the octahedron (an 8-sided solid, made of 48 scalenes), and the icosahedron (a 20-sided solid, so made of 120 scalenes). In the figure, I've shown just the tetrahedron.

For the isosceles triangle, the right of Figure 2.2 shows how it can construct a square: four of the primitive ones. Then, he makes a cube (a 6-sided solid with 24 primitive isosceles) out of six of his squares.

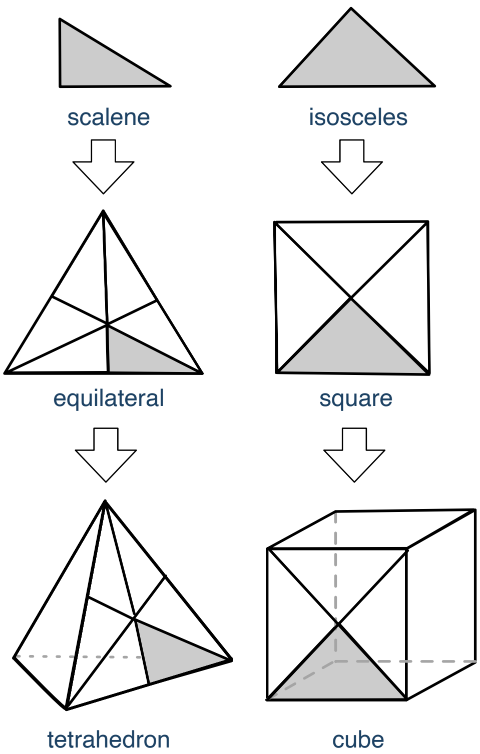


Figure 2.2: Plato's favorite triangles. The scalene triangle pieces together to form an equilateral triangle and then a tetrahedron. The isosceles triangle is used to make a square (in an odd way) and then a cube.

Whew. There was an easier way, and I believe it's not understood why he did things this way. For example, a square can be easily made of two isosceles triangles rather than four, and an equilateral triangle can be made from only two of his particular scalene triangles. As a card-carrying particle physicist, were I to make a model of matter out of more than the fewest necessary fundamental particles, I'd lost that membership card.

The four fundamental solids represent the four elements: Fire is made of tetrahedrons, Air is made of octahedrons, Water is an icosahedron, and Earth is made of cubes. Then he imagines a kind of chemistry with "reactions" among the elements. For example, Air = 2 Fires, Water = 2 Airs + 1 Fire. And so on. It must have been great fun. By the way, Earth can't be broken into or made of any of the other elements.

He's used up four of the five known three-dimensional solid forms, historically (but inaccurately) called the **Platonic Solids**. So, having bought into a theory, he did what many modern theoretical physicists might do. If the solids are important and only four of the five seem to immediately come to good use, then maybe there might be a job for the fifth shape, the dodecahedron (12-sided). He assigned that to be representative of the universe itself. Maybe its 12 faces are kin to the zodiac, its shape is rather close to being a sphere?

Plato refers to a fifth element as "...the most translucent kind which is called by the name of the **aether**...," but he sticks to the four elements of Empedocles for "stuff." Aristotle does something similar, but with a twist.|

> There is some ambiguity among the terms "aether," "**quintessence**," and "**ether**." In this book I'll use the term "ether" to refer the 19th century substance that all thought "carried" the propagation of light waves throughout the universe. "Aether" and "quintessence" are Greek references and are often used interchangeably. In Chapter 3 I'll use "aether" to refer to Aristotle's fifth element.

So, in the Timaeus, Plato again reveals his Pythagorean biases: The world is geometry—pure, abstract form.

But he's just getting started as his Pythagoreanism knows no bounds, as we'll see when I introduce his influential cosmology in Chapter 3.

> Platonism is not just confined to philosophy or mathematics. The Medici family in Renaissance Florence was instrumental in reacquiring Greek philosophical texts from the Byzantine Empire by importing Greek-speaking academics. They set up a school dedicated to Greek philosophy and a school for the children of the court. One of those children was a ward of Lorenzo the Magnificent, and he would have learned of this worldview that permeated so much of his sculpture. So when Michelangelo later noted, "I saw the angel in the marble and carved until I set him free," he was expressing a very Platonic idea that he absorbed as a young student in the Medici household.

2.1.6 Platonic Legacy

We've skimmed only a thin slice of Plato's influential work and modern physics—my life's work—didn't fully emerge until the focus shifted towards his ideas and away from Aristotle's. Our modern reliance on rarified mathematical descriptions of nature reflects our move toward abstraction, aligning more with Plato's philosophy, particularly his concept of the Forms.

There is one unfortunate legacy that's more complicated than is normally presented: the idea of "Saving the Phenomenon," or "the Appearances." This is his statement used to assign this idea to him:

> "This was the method I adopted: I first assumed some principle, which I judged to be the strongest, and then I affirmed as true whatever seemed to agree with this, whether relating to the cause or to anything else, and that which disagreed I regarded as untrue." Plato, *Pheado*

Certainly, Socrate's apparent, "don't look, imagine" armchair-astronomy leans in that direction. In essence, Socrates/Plato seems to argue that one should only assign truth to a model or even observation that agrees with a pre-determined principle or theory. People still argue about this, but he never wrote specifically about , "saving the appearances." That came from the 6th century Neoplatonist, Simplicius, who reported that Plato proposed the problem of finding "by the supposition of what uniform, circular, and ordered motions the appearances of planetary movements could be saved." Indeed, as we follow the twists and turns in future modeling of MOTION IN THE HEAVENS the commitment to circles was a guiding principle.

In any case, "saving the appearances" had legs. Can you see how *un*scientific this is?

As I've hinted, his positive legacy is critical and abstract. His ideas were reformulated a number of times and Neo-Platonism was a pre-medieval version that eventually found its way into Catholic Church doctrine, much through Augustine, only to be reassessed centuries later.

What can't be overstated is Plato's influence on our project of describing the universe using mathematics: Platonism is an enduring feature of fundamental physics. Johannes Kepler, in the 16th century, was among the first truly Platonic (or even Pythagorean) scientists, and as I joked earlier, my particle physics sub-discipline is very Platonic.

Notice that MOTION has not been a feature of my discussion of Plato. In part, we think of Plato's ideas about motion as focused on astronomical topics, which we'll cover later in this chapter. But also his ideas as expressed in *Timaeus* (and to some extent in the *Laws*) are so esoteric as to be mostly unintelligible. That the Soul is responsible in part for "self-motion" is all very unsatisfying.

In fact, "unsatisfying" is a good stepping-off point as I'll next consider the often-scientifically-unsatisfying Aristotle and his unfortunate impact on the development of physics and astronomy. For someone so wrong, it's ironic that we can't ignore him.

2.2 Act VI A Little Bit of Aristotle

> "Aristotle is a Foal. When a foal has had enough milk, it's known that it kicks its mother." ascribed to Plato

While Plato's practical impact on physics was limited to abstract and esoteric notions, not so with **Aristotle of Stagira (–384 to –322)** an even bigger subject. He was a systems builder with practicality and abstraction as joint projects. The extent of his intellectual reach was incredible, and not only did he further philosophical ideas, but he invented whole fields of science and philosophy.

Aristotle was born in Stagira, near Macedonia, north of Greece, and was connected to Macedonian royalty as the son of the king's physician. He was orphaned and presumably precocious and so was sent to Greece by his step-parents to study at Plato's Academy at the age of 17... and then stayed for almost 20 years. One can imagine his surprise when this teenager showed up to find no Plato but Eudoxus (who we'll meet in the next chapter) running the Academy since Plato was in Syracuse on one of three disastrous "consulting" jobs in that Sicilian kingdom.

While he was in residence in Athens, probably just beginning his writing, the

Macedonian King Philip II (perhaps Aristotle's childhood friend) began his conquest of northern Greek cities, including Athens. . . , which came under his control through concession and only limited conflict. (Set the context with the timeline in Figure 1.2 on page 22.)

When Plato died in −348, Aristotle went to Assus in the northwestern area of modern-day Turkey, married, and began (or continued) an impressive series of biological, marine biological, and zoological research which he wrote about in *The History of Animals* and *On the Parts of Animals*. He was a details-person who described animals and insects with minute detail through dissection and description, beginning the classification exercise that established the science of biology for centuries. He classified more than 500 different species into genus and species forming categories of likeness and habit of mammals, fish, reptiles, and insects. It was here that he established his insistence on observation as the source of knowledge, an evolution away from Plato that was obviously severe. Think of his approach as taking a deck of cards swirled together on a table, and ordering them all by identifying and sorting for like features—suit, color, and number. That kind of organization came naturally to Aristotle, it's very modern and seems to have first been apparent to him as a scientific practice.

His range was remarkable, covering: Law, physical science, psychology, natural science, philosophy, logic, ethics, and the arts. Words that we have from him include: energy, dynamic, induction, demonstration, substance, attribute, essence, property, accident, category, topic, proposition, universal. . . His metaphysics informed the development of his science and confused the awakening Western world from about 1100 to 1600. In particular, his astronomy, and especially his physics, didn't make sense, and I'll show you that the Medievals knew it didn't make sense. Everything was a part of his system, and so abandoning or selectively adjusting one nonsensical would bring something else down. It was a philosophical game of Jenga.

One positive thing, if only his followers had preserved it: we have Aristotle to thank for dampening enthusiasm for the unwelcome Platonic idea of "Saving the Phenomena":

> ". . . speaking of phenomena, they say things that do not agree with the phenomena. . . They are so fond of their first principles that they seem to behave like those who defend theses in dialectical arguments; for they accept any consequence, thinking they have true principles—as though principles should not be judged by their consequences. . . " Aristotle, *On the Heavens*

For our narrow project, we have three Aristotelian issues to consider, which together only sample a small sliver of his whole universe: what is real, how change happens, and his physical science.

2.2.1 Aristotle and What's Real and What's Knowledge?

Unlike Plato, Aristotle rejected the idea of a super-sensible realm housing the ethereal Forms. He had a different job for his Form that linked it with actual substance, here on Earth, closer to our idea of the form of a physical object. His focus—which was refreshing after the Parmenides Problem and now the Plato Problem—was on *individual things*, which we learn about through a personal experience with the world, not through some intellectual abstraction. What's real for him are *particular objects*.

> "If we did not perceive anything we would not learn or understand anything."
> Aristotle, *On the Soul*

Like I said, refreshing.

Substance—stuff—and Form work together to make the world. The oft-used metaphor of a house is instructive. In order to make a house you need stuff—wood, nails, and so on—and a plan, an organizing principle. Substance and Form. An individual thing is then matter which has been given a form, and you can't separate them. An individual thing must have both.

For Aristotle, perceived facts are the necessary ingredients for knowledge. We organize them in our memories, looking for commonalities and differences. We categorize our facts into bins of like and unlike with relationships among them. We have an individual perception of things, collect facts, ruminate on them by comparing in our memory with our internal database, and categorize. This is classical Empiricism, as opposed to Plato's classical Rationalism. So far, so good. (Think about that deck of cards, now abstracted as a philosophical goal.)

2.2.2 Change and Cause

But we still can't get away from the Parmenides Problem, and Aristotle also battled change and permanence. Let's race through how he thought about change and how it functioned in his physics.

For him, Change relieves a… tension. An actual thing, what **is**, has within it the potential to become something new. As long as it's not in that newer state—it's "deprived," and it is obligated to go there. Inevitably. So everything is also in a Hericlitean flux but in a very particular and interesting way. In sympathy, perhaps, with Parmenides, in order for something to change into something else, it had to **be** in the first place, and taking that all the way back takes him into an abstract place where there needed to have been an original Unmoved Mover. I'll not follow that line of thought.

What's important about change for Aristotle, which fits into his bigger system, is that in order to acquire knowledge of something that changes you can identify the Cause of change. Because: *all change must be caused* and what can be caused comes from within a set of Aristotelian "Categories" (of being). The ten Categories is a complicated idea so I'll skim. They are substance, quality, quantity, relation, time, place, position, state, activity, and passivity — his complete set of predicates that can be assigned in a statement. For example, what can you say about Galileo:

- Galileo was human (substance)

- Galileo was smart (quality)
- Galileo was 5 feet tall (quantity)
- Galileo was older than Kepler (relation)
- Galileo lived during the 16th and 17th centuries (time)
- Galileo lived in Florence (place)
- Galileo sometimes sat at his desk (position)
- Galileo sometimes wore shoes (state)
- Galileo sometimes wrote with a pen (activity)
- Galileo was sometimes ill (passivity)

A particular substance must be all of these things in order to be a thing. In order to exist. Like I said, you have to be impressed with Aristotle's ability to take a complex topic and break it into its constituents. Remember, he invented Logic.

> " Nature is a principle of motion and change, and it is the subject of our inquiry. We must therefore see that we understand what motion is; for if it were unknown, nature too would be unknown." Aristotle, *Physics*

Substances have "motions" but not the kind you're thinking of. They're very Greek motions and can be quite abstract. For Aristotle, *motion is anything that goes to* something. In this change a substance remains a substance, but Form adjusts, characterizing the natural evolution of a state in which a goal is not achieved into a state in which a goal is achieved. And that idea of a "goal" is very important and in part, where Aristotle's physics goes astray. So the form of the seed and its various guises changes. Stay with me.

Motions can be of any of the Categories of being, but usually are among just three of them:

- change of quality
- change of quantity
- change of place

For example:

- Galileo changed from a boy to a man. That's a change of quality.
- Galileo changed from a person who weighed 50 pounds to a person who weighed 150 pounds. That's a change of quantity.
- Galileo moved from Padua to Florence. That's a change of place.

That last one, a change of place, is our modern idea of "motion" which he called "locomotion." But for him, locomotion is no more fundamental than any other kind of motion and that's very Greek. But, again, he's thought deeply and by accident, all three kinds of motion have examples today:

- Modern Change of Place: We tend to think of locomotion as the only one of his categories to apply to change in physics: objects moving from this place to that place, during some time.

That's familiar. But two of his other "motions" have modern examples that he would not have known of:

- Modern Change of Quality: A phase transition, such as water boiling or freezing, could be considered a change of quality.
- Modern Change of Quantity: Aristotle could not have imagined a nuclear or particle decay from one thing into three different things, like the decay of a neutron into a proton, electron, and neutrino.

As for goals, it's easiest to think of the nature of something, and that involves potentiality and deprivation. An acorn becomes an oak tree. An acorn does not become a Galileo, so it has within it the potential only to be an oak from the beginning. That inevitability is also universal and directed, and that even becomes an argument against infinity since there is no such thing as unrealized or unconstrained potential.

> "It is not what has nothing outside that is infinite, but what always has something outside it." Aristotle, *Physics*

Now we know what properties a thing must have in order to exist and what kinds of change can happen. Again, to understand a change, one must understand the causes—in fact, there are four causes. They are the material cause, the efficient cause, the formal cause, and the final cause.

Take a house:

- The material cause of the house is the wood, nails, and so on.
- The efficient cause of the house is the action of the carpenter.
- The formal cause of the house is the blueprint in the mind of the carpenter.
- The final cause of the house is the purpose for which it was made.

There is sometimes a discussion about whether these function as causation or explanation. Are they the four "becauses"? In any case, the last one of them is problematic for physics as the notion that everything moves for a purpose (that "goal" again) doesn't work in modern terms. This is called "teleological." (One can imagine an argument for Aristotle that there is some teleological logic to how plants and animals "move" from one kind to another, seeds to plants, kittens to cats, and so on.) Of the four (and there's a lot more detail in Aristotle than just enumerating them), Efficient Cause comes the closest to a modern physics cause. That's splitting hairs! Which I guess would be a Change of Quantity?

2.2.3 Aristotle's Physics

Aristotle inherited his **ontology** (the philosophy of being) from his teacher, who inherited it from Empedocles. That is, the four elements of earth, air, fire, and water are supplemented by one more, "aether," which is outside of the earth-bound region of the universe. Like the reactions to Parmenides, Aristotle envisions "stuff" as mixtures of the four elements. But he goes further than just classification, as their makeup, Causes, and Categories all feed into his explanation for the sort of motion that we think of. So understanding locomotion is intimately tied to the entirety of the Aristotelean system.

With respect to our familiar MOTION, he was very much an empiricist, and locomotion, in particular, fit his overall philosophy. Watch a high kick of a soccer ball, a towering home run in baseball, or a shot in the shot-put. The projectile will race to the top of its trajectory and then appear to fall steeper and faster than its rise. Drop a feather, a crumpled-up piece of paper, and a metal key. Will they hit the ground at the same time?

In each of these everyday examples, it seems like the heavier object will hit the ground first. That fits his philosophy, or maybe his philosophy grew from watching things fall since the heavier an object is, the more deprived it is of its most natural place: the Earth. So any object seeks its place by virtue of the amount of earthiness it has in its composition. Heaviness is an attribute, and the natural motion associated with heaviness is down, toward the center of the Earth. *Lightness is also an attribute* for Aristotle (for us, that's just less heaviness). Natural motion for a light object is up, toward the sky. So, below the orbit of the Moon, objects have two kinds of natural motion:

- Natural locomotion for heavy objects is down, and natural motion for light objects is up. These Earth-bound motions — MOTION ON THE EARTH — both follow straight lines toward their preferred places. So firey things want to be at the edge of the Moon's orbit and earthy things want to be at the center of the universe (the Earth).

But MOTION IN THE HEAVENS of the Sun, Moon, planets, and stars don't move in straight lines and have no apparent pushing force, so they must be composed of different stuff from Earth, Water, Air, or Fire and have a different sort of natural motion:

- Cosmic objects are made of "aether" and have circular natural motion.

Like all motions, Earth-bound objects not at their natural places are deprived and realization of their potential is to … go there. Celestial objects move naturally in circles. To fulfill their essence.

There is another kind of locomotion which is un-natural, dubbed "violent," and for Aristotle, what causes violent motion must be a contact force. So throwing a ball is violent and unnatural since it's not directed down. When the ball is in contact with your hand, you're making it move. When it leaves your hand? Well, here Aristotle had trouble and everyone knew it. The contortions that he went through to explain projectiles are pretty contrived. But he was wedded to his system and in spite of his scorn for Plato's Saving the Phenomenon, he seems all about that here.

When the ball leaves your hand, it doesn't immediately head towards the center of the Earth. The medium of the air is critical in two ways:

1. The motion of the hand is (somehow) transferred to the air which (somehow) successively creates forces in steps… air moves the projectile, then another segment of air moves the projectile… and so on until the ability of the air to

perform that critical contact-force job is used up. Somehow the forces of air meet some dissipative force... of the air(!), and it stops.

2. Then the object falls directly to the ground because the air stops it.

The air both moves it and stops it! Also, the projectile doesn't share both unnatural, forced motion and a component of natural, downward motion. There's a lot not to like about this. Even probably including Aristotle given his complicated explanation. Figure 2.3 is a 16th-century depiction of Aristotle's projectile paths: straight line up, then straight line down.

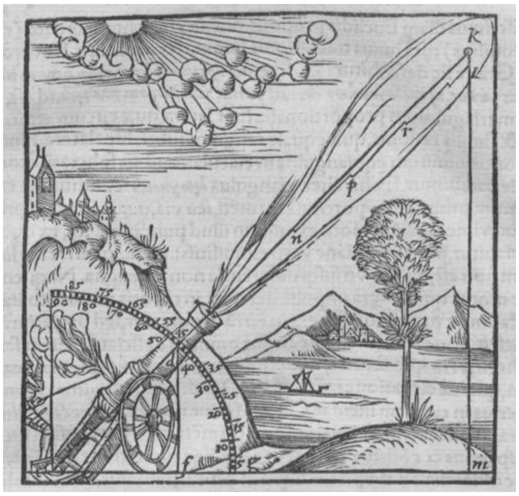


Figure 2.3: A drawing of Aristotlean projectile motion in a mathematics text by Daniel Santbech in 1561.

Now he's not entirely consistent in his descriptions. In his *Physics*, he says:

> "Again, as it is, things thrown continue to move, though that which impelled them is no longer in contact with them, either because of mutual replacement“ as some say, or because the air which has been thrust forward thrusts them with a movement quicker than the motion by which the object thrown is carried to its proper place." Aristotle, *Physics*, IV, 8

Later in Book VIII, he says:

> "Therefore, we must say that the original mover gives the power of being a mover... to air... naturally adapted for imparting and undergoing motion... The motion ceases when the motive force produced in one member of the consecutive series [of forces imparted by the air] is at each stage less, and it finally ceases when one member no longer causes the next member to be a mover but only causes it to be in motion. The motion of these last two— the one as mover and of the other as moved—must cease simultaneously, and with this, the whole motion ceases.

The first extract seems to make reference to an idea that's in *Thaeatus* called antiperistasis, in which Plato tries to explain respiration, suction, and falling bodies as displacing the air and back-filling it to avoid a vacuum. This either evolved too, or was also a suggestion by Aristotle that the air in front of a ball rushed around to the back and pushed the ball forward. I know. It makes no sense. The Medievals were very critical and modified the ideas.

Aristotle didn't know algebra, but I can most easily summarize his points with some simple proportions. The mathematical symbol for "proportional to" is $\sim$.

He would describe the locomotion of a projectile with these ideas:

- Heavier objects (made of more earth than other elements and so highly deprived of its natural place) would fall faster than light objects: $t \sim \frac{1}{W}$ where W is the weight, a stand-in for earthiness. Heavier objects would then fall faster than light objects —have a higher velocity.
- He had some sense of the resistance of air, and so the velocity relates to weight and resistance as $v \sim \frac{W}{R}$ where R is some measure of the resistance that air or water or some medium asserts on the falling object.
- This leads to a convenient conclusion. If there is no resistance, then $R = 0$, and the speed that falls would become infinite. But nothing can be infinite in Aristotle's philosophy, so there is no vacuum allowed, no medium with zero resistance.
- And finally, for violent motion, which requires an external force in contact with the object, $v \sim \frac{F}{R}$. No force, no speed. More force, more speed.

Each of the bullets describes exactly what you and I experience every day in a sport with a ball or just life. Aristotle is clearly a champion Empiricist.

There's more. If linear motion is the only natural motion, then his Earth *must be stationary* otherwise, we would feel the effects of some tangential wind force rotating the Earth. And we don't, so his Earth *does not rotate*, a philosophically inclined explanation for MOTION BY THE EARTH. For objects in the heavens, since they move naturally but in circles, a different material is required, a fifth element.

2.2.4 Summary of Aristotle and Locomotion

So to sum up the first real study of MOTION. . . ever.

1. MOTION ON THE EARTH is of two types:
    1. Natural motions are toward or away from the center of the Earth according to the degree of heaviness (among the four elements, Earth would dominate the others) or lightness (among the four elements, fire would dominate the others) that compose their substance. Natural motions are in straight lines. They represent the fulfillment of an object's potential.
    2. Unnatural, or violent motions are those which are not natural. They all require that an external force is applied throughout whatever trajectory a body experiences. Take away the force, and the motion would cease. These motions can be of any shape.

2. And MOTION BY THE EARTH?
    1. It's zero. The Earth is stationary because no forces can be detected that would be required to make it move. And, motion on the Earth doesn't suggest that the Earth is moving. Throw a ball up and it doesn't fall behind you, as he suggested would be the case if the Earth were moving. So he has an explanation as to why it must be stationary, but not a prediction. He's justifying his contention.
3. And MOTION IN THE HEAVENS?
    1. That motion is circular. Objects outside of the Moon's orbit are of an entirely different substance than what we experience: aether. Why? Since if they were of the same material that that of and on the Earth, its natural motion would be in straight lines.[7]

Aristotle's theories of MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS are relentlessly empirical: they are theories of what we all observe in our everyday lives. His theories of motion are wrong, relentlessly abstract and hidebound to the rules of his overarching philosophy.

2.2.5 Plato and Aristotle on LIGHT

2.2.5.1 ELECTRICITY and MAGNETISM

The Greeks' reasoning about ELECTRICITY and MAGNETISM wasn't their best effort. But when you're inventing the business of accounting for natural phenomena using non-spiritual explanations, you need to define the playing field. They start with the expectation that all objects in the universe must be either inert and not alive, or alive. How can one tell? Well, some objects can only be moved by external agents, and some objects can move themselves. The complicated notion of "Soul" weaved in and out of the conversation from Thales to Plato and Aristotle who roughly agreed that objects endowed with Soul could move themselves, going so far as to nearly characterize Soul as the cause of self-motion.

So there's the rub: they knew of materials that could cause other objects to move but were surely not themselves alive. Did they have soul? It seems an academic question, but remember, this is the early days of scientific thinking.

There were two problematic and naturally occurring substances in Greece. "**lodestone**" was found in the Greek region of Magnesia, so hence, our name for the magnet ("magnetes lithos" in Greek, "stone of Magnesia," or "Heraclean stones") are nicknames for a kind of brown/black iron ore called magnetite, one of only a few naturally occurring, permanently magnetic materials. Just how this mineral acquired its permanent magnetism is not entirely understood, but the best theory is that the high currents in primeval lightning strikes could have been the cause.[8] It would have been remarkable to stumble on a "regular" rock and find that iron

[7] some circular reasoning there, no pun intended

[8] This has been demonstrated in artificially produced lighting in New Mexico.

pieces would stick to it and be drawn from afar to it. There are Chinese references to Lodestones from −400, while the Greek references are indirect through Aristotle, who credited Thales as having studied them in around −600.

The other naturally occurring substance with a similarly eerie property of attracting objects is the resin, amber, which has a long history as jewelry. Amber is a pretty, clear substance that sometimes even has insects embedded, and probably polishing it to enhance its appearance led to the discovery that when it's rubbed, it attracts little bits...of almost anything, including flakes of iron.

So here we have ancient recognition that Lodestone naturally attracts iron (and nothing else) and amber attracts iron and other stuff after being rubbed. And they both seem to cause motion from across distances without touching. They are different and similar in strange ways.

Lodestone magnetism is stronger, and so it was more readily apparent. Aristotle referenced Thales, who seemed prepared to endow the magnet with Soul. In *On the Soul*, Aristotle struggled with magnetism, and his philosophical system was so rigid that there was just no place for it. His mention of magnetism in *On the Soul* was in reference to Thales, and he only mentions it in one other place in the *Physics*. Almost in passing. So Aristotle ignored it because he couldn't accommodate it!

The Presocratic who deeply thought about magnetism was Empedocles (the Earth, Water, Air, and Fire elements originator), and he came up with the first mechanical model that worked at a micro-level. He envisioned that both Lodestone and iron have surface pores that are normally covered by air, but that "effluences" (a fluid? a field?) are emitted by both substances and that from the magnet can actually displace the air-lid on the iron's pores and then the iron follows that effluence back to the magnet to which they attach since their pores are similar. (Apparently, Empedocles also had a theory of vision that worked similarly.) Democritus also had a magnetism theory that was basically like-attracts-like, and that notion was attractive to Timaeus, who expounded on it in Plato's book of that name.

Plato didn't do much better and worried about motion as the need for a moved object to have the place it vacated replenished with displaced air in front—an idea we tend to attribute to Aristotle. So when an object is moved by a magnet, there's a direct contact (a precursor to Aristotle again), as the iron would be moved by the circulating air. But this discussion came incidentally, as it was actually about the act of breathing, not magnetism or electricity!

> "Moreover, as to the flowing of water, the fall of the thunderbolt, and the marvels that are observed about the attraction of amber and the Heraclean stones,—in none of these cases is there any attraction; but he who investigates rightly, will find that such wonderful phenomena are attributable to the combination of certain conditions—the non-existence of a vacuum, the fact that objects push one another round, and that they change places, passing severally into their proper positions as they are divided or combined." Plato *Timaeus*

This single off-hand reference to amber is the first time electricity is hinted at in

Western writing and completes the sum of Plato's interest in either magnetism or electricity. Once.

The Greek philosopher and biographer Plutarch of Chaeronea (c.46 to c.125) wrote in his *Moralla* about Plato's ideas, and he expanded on them to his own theory. He borrows Empedocles' "effluvia" but their nature and the pores are different among Lodestone, iron, and amber. For example, the air "lid" on amber is removed when it's rubbed, and then the effluvia can connect. Plus, the effluvia for amber is weaker than that for Lodestone. So, for the first time, Plutarch distinguishes ELECTRICITY and MAGNETISM as having different strengths and consequences.

2.2.5.2 OPTICS

The history of OPTICS calles on strands from Greece, India, and China, but it's the Greek approach, with crucial contributions from Arab scientists, that informed medieval European ideas about optics, which evolved to modern interpretation. Here I'll focus on the Greek approach before Euclid and Ptolemy, reserving their critical work for Chapter 4. Before them, what we would call optics was more about vision than it was about the physics of light.

Remember that Parmenides (and eventually, Plato) disparaged the acquisition of knowledge from the senses, and that meant their approach to vision was different from those who were more interested in the objects of vision, and less so, the degree of trust that could be ascribed to seeing. The Milesians used the senses, as did Pythagoras, Empedocles, and Democritus — with warnings. What we know of objects comes from our perception of them. But what's the source?

One branch ascribed visual perception as a consequence of the eye emitting rays that interact with the perceived object. For Hipparchus (circa), the "fire" from the eye takes the role of a visual hand. The always thoughtful Empedocles distinguished two kinds of rays from things that themselves emit light (the Sun, fire) and rays from the eye. Perhaps it's not surprising that the atomists ascribed vision to the observed object's atoms themselves meeting the visual fire between the observer and the observed, while later, Lucretius dispensed with the visual rays and assigned the perception of an object to be the result of an object's emission of atoms.

Plato merges the visual fire with the Sun's light as a collaboration that caresses an object where it meets emanations from the object itself to reveal its Soul, which is conveyed to us. In his standard way, Aristotle reviewed and then criticized all previous theories in favor of his own. Remember that he's the ultimate empiricist relying on his senses to process and categorize almost all of the natural world's variety. In his logical manner he wondered about the ability to perceive the stars from so far away as a reasonable criticism of Plato's emanation theory.

Aristotle was impressed with the idea that the liquid in an eye is transparent like the air[9] and that together, they make a continuous medium that (somehow) conveys the

[9] Did he discect a human eye?

nature of the observed to the observer. In particular, color. Information doesn't flow in either direction, but that common eye-air medium is aware like a touch. Color sets that medium in motion and the eye's job is to form the image of the object's color from sensing that motion in the transparent medium. There's much to be confused about here and discussions of, for example, whether vision is a physical or mental process. Is light, say from a fire, a physical entity and different from "seeing" an object with the transparent medium at work? Most interpret Aristotle's view as not assigning the status of "substance" to light but that it's more like an event.

Much like their astronomy as we'll see, the Classical Greeks' theories of vision, and of electricity and magnetism are stories, not explanations. Neither ELECTRICITY, MAGNETISM, nor OPTICS fit their philosophies or worldviews and so, like I said, unsatisfying.

They were qualitative and not quantitative and we have to wait until Euclid and Ptolemy for geometrical explanations to emerge and become the Greek optics that the Arabs then worked on, setting up the medievals who took the subject further.

2.3 Plato and Aristotle, Today

2.3.1 Modern Day Platonists

> "I imagine that whenever the mind perceives a mathematical idea, it makes contact with Plato's world of mathematical concepts. . . When mathematicians communicate, this is made possible by each one having a direct route to truth, the consciousness of each being in a position to perceive mathematical truths directly, through this process of "seeing." ' Roger Penrose (1931-), theoretical physicist, Nobel Laureate

It's unlikely that anyone today would wonder about the application of Aristotelianism into the physics of MOTION ON THE EARTH, MOTION BY THE EARTH, nor MOTION IN THE HEAVENS but thousands of pages of writing (and links) have been devoted to the application of Platonism into modern physics, and especially in mathematics. Recall my party-question in the previous chapter: Is mathematics discovered or invented? Many mathematicians and physicists have concluded that it's discovered, and that's the bumper-sticker version of modern Platonism: suitable for the 21st century.

In this *Plato and Aristotle, Today* section, I'll describe a more modern version of Platonism that might function in physics in two different aspects, which I'll call "The Platonic Process in Physics" and "The Platonic Reality in Physics." It's about an evolved notion of the Forms.

2.3.2 The Platonic Process in Physics

The Forms were by far the Platonic idea with an impact on all branches of philosophy, mathematics, and physics. His premise is that reality consists not only of everyday stuff (that's the Ionian "monist" position that all of reality is made of matter) but that there is an additional reality realm that consists of non-material entities outside of space and time. This is the premise of *The Matrix* in which Morpheus gives Neo the choice of two pills: if he takes the blue pill, he's choosing to continue to live his life in an artificial but comfortable world in which we don't examine what's true and happily accept opinion as knowledge. If he takes the red pill, he's chosen the more difficult path: to live in the truth. The references to the *Allegory of the Cave* are obvious, but it's also the old biblical story of eating from the Tree of Knowledge.

Paying homage to Morpheus' red and blue pills, let's call our everyday, physical world, the **Blue World** (BW) and the ethereal, maybe more truthful world, the **Red World** (RW...in order to help us remember, think of it also as the "Real World."). And let me try to suggest that to be a modern physicist might be to be partly Platonist—engaging a BW while simultaneously leaning on a RW. Stay with me.

Plato's classical RW is where the Forms reside, in which they had two broad characteristics:

1. Forms exist in the RW, are permanent, outside of space and time, and represent the essences of all things and ideas. All objects in the BW—objects we would call physical objects—"participate" in the Forms. My example was the perfect sofa.
2. The RW contains the only true things and so acquiring Truth (with a capital "T") means somehow realizing the Forms in their natural, unusual habitat uniquely through our intellect.

So Plato's is both a story about ontology (the philosophy of what exists) and epistemology (the philosophy of what we can know).

The heated debates of the last 50 years about Platonism are largely about mathematics. In this literature, it's not hard to find questions like whether the reality of a tree is different from the reality of $\sqrt{2}$. In some way, the latter is more permanent. And, of course, there are also the perfect objects of geometry...and maybe the rules of geometry. I think it's fair to generalize that there are three schools of thought in the Philosophy of Mathematics that can be labeled as:

- **Intuitionalism**, where mathematics is just the product of mental activity and a mathematical entity is constructed by the mind and lives solely in the mind. This is also sometimes called "structuralism" or "constructivism."
- **Formalism** is probably the most popular camp in which there is no truth-value assigned to any mathematical property or entity. It's all just the study of logical consequences dubbed "if-thenism." There's no commitment to anything beyond manipulating marks on paper according to the rules of the game.
- **Mathematical Platonism**, suggests that mathematics is the study of abstract entities that have an existence that's as real as the external world targets of scientific experiment. So the question for Platonism is: do abstract mathematical

things exist? Do abstract rules exist?

2.3.2.1 Quine–Putnam Indispensability Argument

I've had the misfortune. . . or fortune. . . of doing physics research for half a century after a master's degree in the philosophy of science. That means that I've never been able to avoid standing back and looking at what I do and what my colleagues do and categorizing and analyzing the process, what counts as a valid argument, what counts as a valid scientific question, and what counts as an acceptable answer. And what about "reality"?

I'm intrigued with a particular strand of Platonism that's due to **Willard Quine (1908- 2000)** in the 1950s through 1990s, and **Hilary Putnam (1926-2016)**, who later found common cause with Quine. Together, their ideas are called the **Quine–Putnam Indispensability Argument**. To an aw-shucks, country-physicist like myself, I interpret it to say:

1. Science (read "physics") works and interacts with real objects in the BW through experiments.
2. Mathematics works and interacts with abstract quantities and rules in the RW.
3. Physics can not work without mathematics, and so the two are *indispensable*. This is a partial answer to Wigner. "Unreasonable effectiveness" becomes "indispensability."
4. Given the impossibility of physics without mathematics, abstract mathematical-physics entities in the RW should enjoy the same level of reality as the objects of experiment in the BW.
5. So there are at least two realities: a physical reality and a mathematical reality.

The Quine–Putnam Indispensability Argument both rhymes with Wigner and demands a new definition of physics.

> "[talk of" mathematical entities is indispensable for science. . . therefore we should accept such talk. . . [which] commits us to **accepting the existence of the mathematical entities in question** [emphasis mine]." Hilary Putnam, 1971, *Philosophy of Logic*.

Quine called himself a "reluctant Platonist," and I think that physics has joined that club. And as I'll show in Chapter **??**, Galileo was the charter member, and he showed us all how to make progress in unraveling MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS once the club's Platonism was embraced.

A few random comments about the Quine–Putnam Indispensability Argument.

- Do I have to be a believer in order to do physics? No. You might be surprised how little philosophical thinking goes into a professional physics education. Long ago, the pain inherent in thinking too hard about, first, quantum mechanics and then general relativity taught those of us who teach these subjects to undergraduate and graduate students to not go there. "Shut up and calculate" is not just a funny phrase, it's actually an instruction that you must follow if you're going to make scientific progress. We physicists don't tend to analyze physics any more than a bird analyzes the dynamics of flight.

- Where does this leave mathematics and its philosophical problems? Well, first, we pretty much don't care! Second, Mathematical Platonism adherents think it's perfectly fine for there to be a plethora of mathematical realities. A multi-verse of mathematical worlds, if you will. Some of them have that special connection with physics...and some of them don't.
- I've concluded that we are relentlessly *both* Platonic and Pythagorean. We can't make progress nor explain the incredible success we've enjoyed without the rules of physics (the "laws") nor without the commitment to the numbers required to make predictions and then contact with experiment. The Platonic is joined with the Pythagorean, in contrast to Plato's Divided Line, the division is blurred and crossable.
- Is it just too unreasonable (sorry) to deal with this multiple reality stuff? A reasonable person might say that if I can touch it or kick it, then it's real. A pretty good working definition of "reality." Stay with me.

2.3.3 The Platonic Reality in Physics

What I described above is about a *process*. But there's also an "ontology." What are the objects of fundamental physics and do they live in the BW or the RW? Let's look at two objects and then go kick a rock.

2.3.3.1 Their Own Forms

There is no sofa that's identical to its Form. Even two sofas designed and constructed in the same manufacturing facility will not be identical. Patterns on one will be slightly altered from the other. Tolerances on color fabric structure, or leg shape cannot be perfect. A BW sofa is not identical to its RW Form. They're separated into the two Realms.

The 20th century has upended this very Platonic separation, and Plato might have been intrigued with the result.

A molecule of hemoglobin in your blood contains 10,000 atoms of hydrogen, oxygen, nitrogen, and iron. Each of these atoms has protons, neutrons, and electrons. Isn't it remarkable that each of the many thousands of electrons in that single hemoglobin molecule is identical to one another?

Isn't it even more remarkable that each of those electrons in my blood is absolutely identical to an electron in an atom of hydrogen in the outer edges of the Andromeda Galaxy? Or to every electron that was flying around the early universe before Hydrogen atoms formed 370,000 years after the Big Bang. (I might note that every hydrogen atom in your hemoglobin was, in fact, formed in the Big Bang.)

A perfect form of an electron — the ideal electron in the RW— is identical to its BW counterpart electron. No imperfection. No difference.

So the distinction between Forms and the objects in the BW that participate in the Forms evaporates as soon as we begin to deal with elementary particles. That is, when we begin to confront the universe as it is composed in the BW.

▻ Elementary particles in our everyday world (the Blue World) are their own Platonic Forms.

2.3.3.2 Are Wavefunctions BW Or RW Or Not Real At All?

Want some serious Plato? I give you Quantum Mechanics, the theory of the very small: atoms, electrons, nuclei, elementary particles, and quantum fields. Atoms and all of chemistry is precisely determined by a single equation called the Schrödinger Equation which can be solved to determine the "state" of an atom and make predictions about properties of matter. For example, the model of the optical spectra that result from electrons falling from high orbits to low ones by emitting unique colors of light is the first prediction of quantum atomic theory and was bang-on correct. Quantum mechanics is exquisitely precise and its predictions match experimental results to mind-boggling precision. It works better than any theory ever invented.

But Quantum Mechanics comes with a very strange substance that we cannot see, hear, touch, or measure. I can arrive at predictions only by calculating the evolution of the spooky entity called the "wave function," ψ. The wave function seems to me to be the very definition of an RW-existent, mathematical entity. Essential to the physics, but with an existence on paper only—a very Quine-Putnam idea.

I can predict the results of an experiment involving atoms, molecules, or elections by mathematically evolving their wavefunctions using the Schrödinger Equation, which takes $\psi(t_1)$ at some time, t_1 and tells you precisely how $\psi(t_2)$ will behave at time t_2 in the future. This works perfectly. Every time.

But here's the rub: *ψ is intrinsically undetectable*. It doesn't exist in the BW, but it does have a communicable existence as mathematical marks on paper. We make a connection in the BW by predicting the *probability* that a particle will be here…or there….or over there…or on the Moon. That comes from the *square of the wavefunction*, ψ^2. Remember that party you un-livened up with the question about mathematics? Ask two physicists in attendance, "Is the wavefunction real?" Then stand back. That will liven it back up.

Let me repeat: We can calculate the value of ψ at any time or place in the future, but to connect with a measurement, we can only predict probabilities, no certainties are allowed. Ever. We cannot get from the equations of Quantum Mechanics to a measurement in the BW without passing through an RW Platonic manipulation of the mathematical entity, ψ.

If you ever needed a definition of a mathematical entity that behaves as if it has a reality only in the Intelligible Realm, the wavefunction, ψ, is exactly that. For Quantum Mechanics to function, we must work wholly inside of a very strange mathematical RW which indispensably (in that Quine-Putnam sense) is very real. Quantum Mechanics works better than any theory ever devised in any science.[10]

So every entity in physics is ultimately an elementary particle, which is its own Platonic Form and which is described by a mathematical entity which cannot be observed.

2.3.3.3 "I refute him thus!"

In a different context, it was the British writer of the *Dictionary* Dr. Samual Johnson claimed to be able to refute the Idealism of Bishop Berkeley that to be real was to be observed. He kicked a rock and declared, "I refute him thus!" Well, there's a lot inside of a rock and Sam's foot.

It's quite natural to insist, "I know there's a real world out here because I can see and touch stuff!" Okay, let's talk about touching. That rock that you kicked with your foot is not a solid hunk of stuff. It's made of minerals in crystalline structures of definite chemical elements: atoms with electrons in their atomic shells which have complicated bonding with their "home" nucleus and across the crystals with neighboring atoms. Your foot is made up mostly of water in cells and tissues, so, of course, different atoms in different arrangements.

These atoms of the "kick-er" and the "kick-ee" interact with one another as you bring your foot very, very close—molecularly close. There would be some deformation of the two materials (to your foot's disadvantage) since the rock's lattice is relatively rigid in comparison to the tissues of your foot. But what's going on? The electrons at the surface of your foot are electrically repelled by the electrons in the outer orbits of the atoms at the surface of the rock. To make it even more complicated, there's a region of quantum mechanical attraction and repulsion that is active between the whole molecules of the two materials called the "Van der Waals force." But the dominant reason that your foot doesn't go right through the rock is called the Pauli Exclusion Principle. That is the name for the quantum behavior of electrons that prohibits more than one of them from occupying the same energy level. (Why atoms have electrons in "shells.") So *your real-life-kick is inherently a quantum mechanical process and is as real as the wavefunction* of the previous section, and the electrons and photons of the section before that. You think you kicked a solid thing that's a rock in the BW, *but what you did was cause a quantum mechanical interaction only describable in our RW*.

Again. As a practicing physicist, do I stay up at night worrying about the different realities that our description of nature presents to us? Or do I just keep on calculating...because it works? For almost all of us, it's the latter. We're actually

[10]Einstein famously washed his hands of Quantum Mechanics, immensely uncomfortable with its lack of certainty, related to the reliance on the wavefunction. And he was one of its inventors!

all trained to be highly skilled “Quantum Mechanics” seemingly working in the BW of experiment, without concern for the philosophical niceties of the RW of the equations. This is the same as a skilled engine mechanic working under the hood of your car who doesn’t need to know the material science or engineering of the digital electronics of the engine and control systems to solve BW problems.

But Plato is there. He’s changed his mind about a few things, but when it comes to philosophical longevity— when it comes to physics—he outlasted Aristotle. By millennia. But Aristotle also had his moment. Take out your phone.

2.3.4 Aristotle’s Legacy in Physics and Engineering

Aristotle invented the iPhone. Well, not exactly all of it, but he created the basic language that all electronics use to process instructions and communicate internally. This language allows digital components in integrated circuits to do arithmetic, compare number strings, turn peripherals on and off like pixels on a screen, and many other functions. All of this comes from seemingly endless strings of logical operations performed by mind-boggling numbers of individual digital “gates” of silicon, each of which do very simple things and all of which do complicated things together.

You see, Aristotle invented that language, and I think that’s his modern legacy: he first conceived of the rules of **formal logic**, which were so powerful that they instantly became active research projects for ancient and medieval philosophers for a thousand years. “Logic” is now the primary subject in whole fields: Philosophy of Logic, Physics, Discrete Mathematics, and Computer Engineering! If winning an argument is important and if you can reliably create valid arguments and always identify invalid ones, then you possess a superpower.[11] That was his goal. Making that superpower. For a more detailed introduction to the field of Formal Logic, see Technical Appendix B.2. Here, I just want to hit some broad ideas.

Look at these two arguments:

Example 1.
- (All apples)(are fruit)
- (All red objects in that tree) (are apples)
- Therefore, (All red objects in that tree) (are fruit)

Example 1. hits you right, I’ll bet. This is the kind of argument called a **syllogism** consisting of two *premises* followed by a *conclusion*. Here’s another one:

Example 2.
- (All elephants)(are English speakers)
- (All squirrels) (are elephants)
- Therefore, (All squirrels) (are English speakers)

[11] We’ll see in Chapter **??** the re-discovery and use of Aristotle’s Logic was arguably the major threat to the dogmatic Augustinian Catholic Church in the 12th century. An uneasy truce was pieced together by Thomas Aquinas by the 13th century.

Now Example 2. kind of hurts. These seem like very different arguments and you'd want to say that that this second one is absurd or wrong—more about that in a bit. But can you see that they share an important feature: they are both structured in the same way—they have the same **form**. Try this:

Example 3.

- (All A)(are B)
- (All C) (are A)
- Therefore, (All C) (are B)

This shows the structure of both arguments. In both examples, we can identify: A = apples/elephants, B = fruit/English speakers, and C = red objects in that tree/squirrels. Many substitutions will work for A, B, or C if the premises and conclusion are arranged like the above.

There's more: in any argument arranged as in Example 3. the conclusion is "forced" on you. The easiest way to see that is to look carefully at the "Euler Diagram" in Figure 2.4.

2.3.4.1 Valid, Invalid, and Sound Arguments

In the courtroom, the board room, in science, and in everyday life, having the facts in hand is only part of a winning strategy to persuade others. Your argument has to be, we colloquially say, "logical." We all have a sense of what that means, but it can be nuanced. Let's look at two examples of arguments. Notice that in the three arguments above, I've inserted parentheses that demarcate important phrase chunks in each of their three lines.

Can you see that in Figure 2.4, there are three circular areas, the biggest of which is B? All of region A is inside of the bigger region B so the first premise that (All A)(are B) is evident and that all of C is inside of A, so the second premise that (All C) (are A) is evident. So from the picture, you forcefully conclude that (All C) (are B)—the conclusion of Example 1. You're worried about talking elephants. Stay tuned.

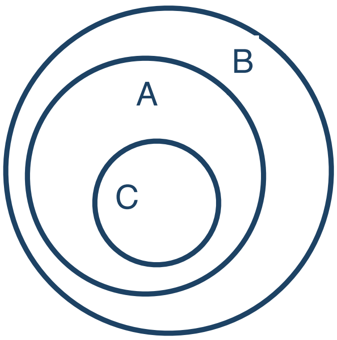


Figure 2.4: In a valid argument shows that one is forced to conclude that All C are B.

2.3.4.2 Greatest gift

Aristotle's unique invention that makes general rules possible for argumentation was to create what I think of as an *algebra of language*. Here is a seminal moment in history, from the first book of his *Prior Analytics* (focus on the last sentences):

> "...if every B is A then some A is B. For if no A were B, then no B could be A....e.g. **let B stand for animal and A for man. Not every animal is a man; but every man is an animal.**" (emphasis, mine) Aristotle, *Prior Analytics*.

Look at the sentences that I've highlighted: he's using variables A and B, to stand for things, here in his example, A = man and B = animal. Instead of men and animals, the variables could be squirrels or fruit. As long as the *form* is proper, we say that the argument is "valid."

Let's be clear—because Logic is all about clarity and bottom-up reasoning. We all use words that sometimes have specific meanings in specialized fields like Logic. Here are some that I'll make use of in this section. Some definitions for us:

- Here, I will use the term *statement* as a kind of a sentence that can be true or false. "Elephants are larger than squirrels." is a true statement. "All bachelors are talking squirrels" is a false statement.
- When a statement includes a "quantifier" (an example of which is "all"), a subject, a connective (often called a copula, a form of the verb "to be"), and a predicate I'll refer to these as *propositions*. (All apples are fruit.) is a true *proposition*.
- Not all sentences are *statements* or *propositions*. Our two here are aimed at logical argumentation.
- *Statements* and *propositions* can be true or false.
- I will use the term *Arguments* in two ways. In this subsection, a *Syllogistic argument* will stand as an ordered collection of *propositions* (here, the *premises* of the argument). As I showed you, Syllogistic arguments are constructed as specific forms. (In the next section, I'll refer to a different kind of argument, a *Propositional argument*.)
- Syllogisms were Aristotle's first venture into Logical arguments, and he identified 16 valid forms, but others after him found additional ones. Most likely, it was the 13th-century University of Paris scholar, William of Sherwood, who gave names and hints to identifying the 19 valid syllogisms (out of 256), and this particular one is called "BARBARA."[12]
- Syllogistic arguments consist of:
  - two propositions which are premises, which in the above examples are the first two sentences and
  - a single proposition which is a conclusion.
- A Syllogistic argument, which is properly constructed according to one of the defined forms, is simply *valid*, without regard to the terms (the A, B, or C).
- A Syllogistic argument constructed according to one of the defined forms which have true premises is called valid and *sound*. That is: If the premises

[12] BARBARA wasn't a person, but a mnemonic invented by Sherwood in order to remember the kinds of statements are in the premises and conclusion. Here the three are "All" statements, and hence his name, "A" statements. So they are "All x are y." E-statements are of the form "No x is y," and for such a syllogism, he invented the mnemonic, CELARENT, with two E's and one A statement. He did this for each of the 19. Medieval analysis of Logic was exhaustive and probably exhausting. This dedication has carried on to this day.

are true, and the argument is properly formed, then the conclusions must be true in a sound argument.

- A Syllogistic argument that is not ordered according to one of the defined forms is *invalid* and *unsound*.

▻ Introducing variables as a placeholder for the subjects and objects in a statement is a seminal moment in the history of mathematics.

Amazing. Out of this beginning, your mobile phone was born.

Now, about talking elephants and talking elephant-squirrels. Elephants can't speak English, and squirrels aren't elephants. So Example 2. is a *valid, but unsound argument* according to the rules of Logic that Aristotle invented. Why? Well, remind yourself of the "Euler Diagram" in Figure 2.4. Its conclusion is forced on you. Now consider this argument:

Example 4.

- (All elephants)(are English speakers)
- (All elephants)(are squirrels)
- Therefore, (All squirrels) (are English speakers)

This has the form:

Example 5.

- (All A)(are B)
- (All A)(are C)
- Therefore, (All C) (are B)

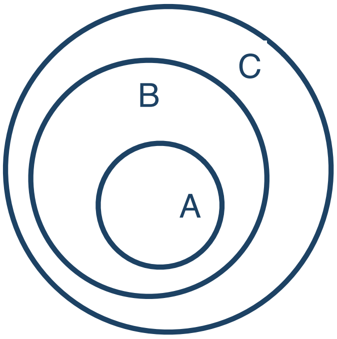


Figure 2.5: Here, the invalid argument is clear. All of the region A (elephants) are indeed included in region B (English speakers), but "all C (squirrels) are B (English speakers)" does not hold since there are regions in C (the squirrel's region) that are *outside* of region B. Only some of region C is inside of region B.

Notice that between Example 3. and Example 5, that the order of A and C in the second premise is switched, which is enough to make Example 4. invalid. So not only are the premises not true (so not sound), but it's also logically invalid, and to get a sense of that, look at Figure 2.5. The caption explains why one is valid and the other is not.

Aristotle covered this new-born subject in a number of his books, including: *Categories*, *On Interpretation*, *Prior Analytics*, *Posterior Analytics*, *Topics*, and *On Sophistical Refutations* which collectively, were much later dubbed "*Organon*" which means "instrument."

What I've chosen for my elephant-squirrel example is one of 256 possible syllogistic forms. Maybe you can see why studying Logic became a matter of intense research following Aristotle's death and into the first 1000 years of both Arab and Western philosophy. There was lots of work to do.

These arguments are examples of **deductive logic** which is often contrasted with **inductive logic**. In Deduction, if the form of the argument is according to the rules, then the argument is guaranteed to be valid. That's the sort of argumentation that was used in Socrates' discussion with the slave boy in the sense that the conclusion of a deductive argument is, in some sense, already in the premises. Inductive logic is not reliable because it is not rule-bound and delivers conclusions that can seem persuasive but aren't true.

Here's a personal, inductive argument about squirrels:

- (As a child) There's a brown squirrel
- (As an adult. . . many times) There goes another brown squirrel
- Wow. . . more brown squirrels and no other ones
- What is it with all of the brown squirrels?
- Gosh, I conclude that all squirrels are brown!

Induction can sound persuasive and scientific. It is an important form of reasoning in science, but it must be used with care. Aristotle knew of both kinds of logic.

Here's a problem with my induction about squirrels: Before I moved to Michigan, the only squirrels I'd ever seen were brown. Now my yard is full of black squirrels. They're everywhere. Many times in science, a deduction uses premises that came from inductive reasoning, so even if the deduction is proper, the argument might be unsound. Induction is always vulnerable to being questioned, but the soundness of properly formed deductive arguments can only be challenged by questioning whether their premises are true. That's where a lot of the scientific action is.

From this point, when I refer to "logic," I'll mean deductive logic. By the way, Sherlock Holmes is reputedly the Master of Deduction. Well, sorry. That's not true. If you look at his stories, you'll see very, very few examples of deductive reasoning. He's the Master of Induction![13]

2.3.4.3 Propositional Logic

Theophrastus (−371 to −287) was a favorite student of Aristotle's who led the **Lyceum** for 37 years after his teacher's death. Aristotle even willed him the guardianship of his children...and his library. While a devoted student, Theophrastus went beyond his teacher and expanded and modified some basic Aristotelian notions. He also moved the study of botany forward and worked extensively in Logic. Theodor Geisel (Dr. Seuss) used "Theophrastus" as a pen name.

He is probably the one who extended the idea of syllogistic argumentation into a new direction with the invention of "propositional logic" in which (for our examples here) there can be two variables, rather than the three of a syllogism.[14] In the same spirit as our definitions above, I'll call these *Propositional arguments*. This is where the modern engineering action is.

[13]Or more appropriately, the Master of Abduction, a third kind of logic. Look it up.

[14]Propositional arguments can have any number of premises and variables.

Propositional arguments are different in form and content from Syllogistic arguments and they involve a statement that is conditional: "If this,....then that." Let's contrast them. Here's a Syllogistic argument:

- (All apples)(are fruit)
- (All red objects in that tree) (are apples)
- Therefore, (All red objects in that tree) (are fruit)

Notice that the variables In Syllogisms are kinds of things (called classes in Logic).

Here's a Propositional argument which seems similar, but is very different:

- (If those red objects are apples) (then they are fruit.)
- (They are apples.)
- Therefore, (they are fruit.)

Here's how a Propositional argument is very different in an important way. The variables have a "truth-value," TRUE or FALSE.

Just as before it's useful to abstract the specific terms in the premises with general symbols and Table 2.1 does this on the left in words, and on the right using logical symbols. The → symbol means "implies" and is associated with an "If...then" kind of statement. The lone A is a standard way to say that "A is the case" or "A is true." Finally, the symbol ∴ means "therefore." It doesn't seem like much, but it's powerful. Establishing the truth-value of the conclusion of a Propositional argument

A Conditional in Words	A Conditional in Symbols
• If A is true, then B is true	• A→ B
• A is true	• A
• Therefore, B is true.	• ∴ B

Table 2.1: A Conditional argument and its concise symbolic equivalent.

can be straightforward, or complicated. The game is to analyze the argument, again, for formal validity and ask whether the truth value of the premises guarantees to the truth of the conclusion.

> An argument of this particular form (If A then B), (A), (therefore B) is called "Modus Ponens" (Latin for "method of affirming") and is one of six basic forms of propositional logical arguments. Another common propositional argument is "Modus Tollens," which also seems intuitive. For example: (If it is an apple) (then it is a fruit), (It is not an apple), (therefore it is not a fruit.)

The engineering action is associated with Propositional Logic. In Technical Appendix B.2, I'll show you how a few digital electronics elements can turn the Modus Ponens argument into a digital circuit. The clue is in the prominent appearance of "true" and "false" in Table 2.1. On and off. 1 and 0. Binary logic as the backbone of digital circuitry.

The first digital computers relied on thousands of vacuum tubes and filled whole rooms with hot, clunky racks of tubes and wires, but when the transistor became commercially viable in the 1960s, the digital world came alive. With binary arithmetic, gates can be combined to do arithmetic functions, logical functions, and importantly, storage of bits. A one bit digital memory consists of four so-called NAND gates—four transistors—and it's the basic cell of a computer's memory.

All of these—and more—transistor components can be imprinted in tiny silicon wafers in which a single transistor package might be only 20 nanometers in size or soldered to a circuit board as a package about half of the size of an AA battery. With the logical functions and the manufacturing techniques of today, my current Apple Watch has 32GB of random access memory (RAM) so it can manage 32,000,000,000 Bytes of information, which is 25,6000,000,000 bits, and so 102,400,000,000 individual transistors are inside my watch, just for the memory! The CPU and control circuitry would add millions of additional imprinted transistors and their gate-equivalents. All on my wrist. All speaking "Aristotle."

Obviously, the 2500-year path from Classical Athens to cat videos on YouTube is full of breakthroughs and smart ideas. But it all started with Aristotle.

Chapter 3

The Most Important Mathematician You've Never Heard Of : Eudoxus and Greek Astronomy

"If I were at the outside, say at the heaven of the fixed stars, could I stretch my hand or my stick outward or not? To suppose that I could not is absurd: and if I can stretch it out, that which is outside must be either body or space...We may then in the same way get to the outside of that again, and so on, asking on arrival at each new limit the same question; and if there is always a new place to which the stick may be held out, this clearly involves extension without limit. If now what so extends is body, the proposition is proved; but even if it is space, then, since space is that in which body is or can be, and in the case of eternal things we must treat that which potentially is as being, it follows equally that there must be body and space extending without limit."

- Argument for the infinity of space attributed to Archytas, circa. −400, *Quoted by Simplicius, Physics*

I'll bet that many of you have seen the solar system arrangement as imagined by Copernicus (surprises await in Chapter **??**) with the Sun in the center and all of the planets, including Earth, obediently orbiting it in perfect circles. What he challenged was the ancient and universally-held idea that it's the stationary Earth that's in the center of the universe, not the Sun. Fascination with that older picture is prevalent in many decorated medieval manuscripts through the centuries, and one of the earliest is shown in Figure 3.1.

This is from a 10th-century edition from the British Museum of a poem by the Greek poet, **Aratus (–315/310 to –240)** from about -275 called *Phaenomena*, which was named for a book of the stars and constellations by the Greek mathematician, Eudoxus, of probably a century before. It was he who created that 2000-year-old "geocentric" model of the universe—one in which the Sun, Moon, planets, and stars all orbit around the stationary Earth. I'll show you that the poem *Phaenomena* figures crucially in the history of astronomy two centuries after Aratus wrote it, so watch for it reappearing as we go along.

I took some pains in the last chapter to underscore that the

Figure 3.1: Aratus, the poet, lived about a century after Eudoxus (and hence, Aristotle) and turned his astronomy book into a poem. Later, Cicero translated it, and this 10th-century manuscript is an illustrated copy of that work.
https://sarahjbiggs.typepad.com/.a/6a013488b5399e970c01bb07c8696d970d-pi

model of motion on the Earth belongs in Aristotle's corner as he really invented the dynamics of motion. But we tend to ascribe that geocentric model of the universe largely to him as it became the authoritative, unquestioned dogma of the medieval and renaissance periods even though it made no numerical predictions and was known since Aristotle's time to be just wrong. In fact, it was pure larceny as

I'll show you in this and the next chapter. The lead-up to Aristotle's model—which became Dante's model—which had become the Church's model—started with Plato and his colleague, Eudoxus.

When we last saw Pythagoras, around −495, he was on the run from Croton to Locris to Metapontum in the instep of the Italian boot—an inglorious escape by land and water, trying and failing to be allowed to settle anywhere. People were afraid to protect him for fear of being the subject of attack by followers of the wealthy and thin-skinned ruler of Croton, Cylon who was apparently unused to the standard brusk treatment by our philosopher. (Or not. Remember, Presocratic stories are often just that...stories.) Just how Pythagoras came to his eventual end isn't clear, and of course, there are many versions. The bottom line is that his cult's welcome had soured, and Pythagoreans spread out from Croton, migrating further east within the instep of the Italian boot and also to Syracuse, Thebes, Corinth, and some to Athens. Philolaus was one of those emigrants and, probably near Athens, wrote the account of Pythagoreanism that Plato read.

After Socrates' forced suicide, Plato and other followers abandoned Athens for nearby Megara where a school of Socratic philosophy was established. He served in the military again and began the project that became his life's work, writing probably more than 10 of his first books during that time. One of the first of these might have been *Gorias*, which contains some Pythagorean references, so it's reasonable to suspect that he's become interested in that mathematical philosophy. About that time he started traveling: to Egypt (perhaps), **Syracuse** in Sicily, and Tarentum in southern Italy. Pythagoras' territory.

The foremost mathematician of the time was Philolaus' student, **Archytas of Tarentum (–428 to –347)** whom we met on page 62, and so he stopped in **Tarentum**, one of those "boot instep" Magna Greek[1] sanctuaries and one of the most powerful Greek city-states. (See the map in Figure 1.1 (a).) He seems a reasonable thinker:

> To become knowledgeable about things one does not know, one must either learn from others or find out for oneself. Now, learning derives from someone else and is foreign, whereas finding out is of and by oneself. Finding out without seeking is difficult and rare, but with seeking it is manageable and easy, though someone who does not know how to seek cannot find. Archytas, *fragment*.

Plato's relationship with Archytas has been much discussed over the centuries. Were they friends or competitors? We have a sense of it for in addition to Plato's famous writings, there are also a set of letters which are maybe or maybe not written by him. Letter VII is perhaps the most reliably from Plato's hand, in which he describes his multiple harrowing escapes in Syracuse. It's a self-serving description of what he did and why and suggests that Archytas sat at Plato's knee, rather than the other

[1] the Roman name for the Greek-speaking colonies in the coast of southern Italy

way around. The other school of thought is that Archytas taught Plato mathematics. I'm inclined towards this interpretation, given Archytas' undoubted skills.

Plato wouldn't have written *The Republic* by that time, but ideas about what constituted the best ruler must have begun to form as he became interested in Syracuse at the southern tip of the island of Sicily, which was ruled by a ruthless "tyrant"[2] Dionysius I and then his successor son. The trip went badly as Dionysius didn't take kindly to Plato's criticism of the debauchery and cruelty that marked his reign, and so he sold him to slavery, as I mentioned on page 56.

Figure 3.2: This is a 19th century woodcut from an unknown artist. We know of it because it appeared in a book on meteorology by the French astronomer Camille Flammarion in 1888. Some attribute its inspiration to Archytas' "stick experiment."

In that first trip, when he was about 40 years old, he must have split his time between Italy and Syracuse, and there he formed a bond with the tyrant's brother-in-law, Dion, who two decades later took it upon himself to arrange for his undisciplined nephew's education and brought Plato back—now almost 60 years old—on a special ship sent to Athens just to bring him to Syracuse as a tutor. (Aristotle was about to arrive in Athens and would have found Plato missing!) It *again* went badly when Dionysius II expelled his uncle and imprisoned Plato with (according to some legends) intentions of selling him— again— into slavery. Plato managed to send word to his friend, Archytas, who, during those two decades after their first encounter, had acquired the stature necessary to rescue Plato with yet another, Plato-exclusive ship.

As I noted in the last chapter, Archytas was a committed Pythagorean and a mathematician of great skill. But he also was a civic leader and an elected military general. In spite of Tarentum law, he was re-elected general seven times because he never lost a battle. (Did I mention that Greeks fought constantly?) When he did step down, the army started losing.

Figure 3.2 is a famous engraving (by an unknown artist...maybe late 18th century)[3] suggesting the quotation attributed to Archytas at the head of this chapter. Among the most famous arguments in cosmology is whether the universe is infinite or finite

[2] meaning someone in power who didn't inherit it, but took it

[3] It's associated with the popular science writer Camille Flammarion as he used in his 1888 book *L'atmosphère: météorologie populaire.*

in size, and Archytas had the first of many similar inspirations that the universe cannot be finite: He did a thought experiment, imagining traveling to its presumed edge and attempting to thrust his stick beyond that limit. If he could extend it, then, well, that's not the edge...and so he'd have to go further, repeating the experiment without end. This is a good example of the kind of intuitive cleverness that seemed to be built into this great Greek mathematician, politician, and military leader.

Archytas was reported to be an even-tempered, cultured man who led Tarentum through a period of democracy and about whom Aristotle apparently wrote more (lost) books than any other person. There is some evidence that he wrote a book on mechanics and that he enjoyed making toys for children—very un-Plato-like in spirit.

His mathematical skills were legendary, and he solved an old problem with mystical roots: Apollo sent a plague to the city of Delos, and a delegation was sent to Delphi to learn from the Oracle how to rid themselves of the pestilence. The instructions were to take their cubical altar to Apollo...and build a new one with double its volume. This is called the problem of "duplicating the cube" (also called the Delian Problem), and it required cleverness on Archytas' part and inventive tools beyond pure, plain geometry, which caused Plato to disparage his solution. Archytas contributed to many branches of mathematics and Euclid's *Elements* includes some of his proofs.

All in all, Archytas was the most accomplished Pythagorean, and in the spirit of the opening to this chapter, we're indebted to him for his products and also to one of his students: the most accomplished of all Greek mathematicians before Archimedes, namely, Eudoxus, from whom 2000 years of cosmology originated.

3.1 A Little Bit of Eudoxus

Recall that Philolaus was the source of Plato and Aristotle's knowledge of Pythagoreanism—for example, the "Pythagorean" cosmology came through him or probably originated from him. Was he a student of Pythagoras? Their overlaps are nearly right in order to imagine that relationship, but that's controversial. He's certainly the closest we get to the great man, so it's not far-fetched to imagine a teacher → student theme of Pythagoras → Philolaus → Archytas → Eudoxus. Lunar craters are named after each, which is not the normal teacher-student legacy. (Set the context with the timeline in Figure 1.2 on page 22.)

Eudoxus of Cnidus was the son of a physician and became one himself, but we know of him as a gifted mathematician and astronomer. As I'll show you, astronomy and medicine were connected through astrology, and mathematics and astronomy have always been kin, so these seemingly disparate skills go together. Cnidus was a city founded by Sparta on the southern Aegean coast of modern Turkey and was where he started... and finished, between which times he traveled all over the Aegean to study and teach. As a young man, Eudoxus went to Tarentum to study mathematics with Archytas. So two ways that Plato connects with Archytas. Sometimes students shine above their teachers and Eudoxus became arguably one of the most influential mathematicians in antiquity. He likely invented the theory of proportions, the basis of the fifth book of Euclid's *Elements* — and the primary tool for mathematics and physics through Galileo. He also snuck up on integral calculus by inventing the "method of exhaustion"—the logical notion that one geometrical figure can be made smaller than another by repeatedly halving it. Archimedes used this technique to prove that the area of a circle is proportional to the square of its radius.

He seemed to be unable to stay in one place. After his mathematics instruction, he went to Sicily to study medicine, then by the age of 23, he went to Athens and stayed briefly (and apparently, unhappily) with Plato's Academy (rooming 7 miles away, so a long commute to lectures). After less than a year, he was back on the road to home in order to raise funds...so that he could travel even further! He went to Egypt with what we'd call a scholarship and studied astronomy there for 16 months, shaving his head and learning from the priestly-cast astronomers, before leaving for the northern modern Turkish Black Sea coast and the Greek colony of Cyzicus. By this point, he's lecturing on his own and established a popular school and an observatory. With data from his observations in the north and from Egypt, he published his first book, *Phaenomena*, which was a compendium of star locations and *On Speeds*, of their motions. Recall that this is the subject of Aratus' important poem.

Around -368, during his 30s, he moved his school to Athens, by which time Plato was 60 years old and Aristotle had left for Macedonia. It was here, as the legend goes, that Eudoxus was challenged by Plato to form a geometrical model of the heavens. The legend is unlikely as, by this point, Eudoxus was the mathematical champion of the Greek-speaking world and more likely to issue challenges than accept them. Plato's mathematical skill was no match for Eudoxus' whose work was memorialized in a number of Euclid's *Elements*. As we'll see below, his model was born and, in various guises, persisted until Copernicus, Galileo, Kepler, and Newton.

He first proposed a solar cycle of four years, three of 365 and one of 366 days. It's Eudoxus' astronomy and cosmology that are our concern here and we;ll begin on the same footing as any Greek astronomer by reviewing the problems that everyone in antiquity faced when trying to describe what we observe from Earth. Then, we'll work through Plato's ideas, which formed an almost linear line of inspiration: from Pythagoreans to Plato and to Eudoxus.

3.2 A Little Bit of the Sky

We're about to begin one of the main problems that all ancient cultures studied, but which the Greeks took on as my last — but many centuries-long, research programs: cosmology. And here, we can sympathize.

GREEK RESEARCH PROGRAM #4 : How is the Universe structured, and what are the rules that govern its beginning and current state?

There are very few objective experiences that we can share with people who lived thousands of years ago. But if you watch the Sun's path across your sky and the night sky over many days, you'll see the same things as all of prior humanity—consistency, punctuated by usual events. We can disagree about a lot, but every human has experienced the same MOTION IN THE HEAVENS.

For millennia the skies seemed memorable and intimate. Cultures all over the world adopted the periodic motions in the sky as a to-do list for planting, religious observances, expectations of periodic floods, and other natural events. The heavens seem perfect, and so it was natural to associate deities with the cycles (and picture their images in the star patterns and planets) but also to look to the heavens when unfortunate terrestrial events happened for correlation with unusual events like eclipses and conjunctions of planets with one another.

Remember that for Aristotle, everything changes, and that any change is a "motion," and unnatural motions on the Earth are caused by something. In his *Meteorology*, he found it persuasive that large-scale but continually changing phenomena like the weather should be caused by the continually but predictably changing MOTION IN THE HEAVENS. Certainly, the Sun seems to influence the life of plants and animals, and the Moon's motion seemed to be connected with women's physiology (and later, Ptolemy associated the tides with the Moon).

The Babylonians were the first to create a systematic observation program, with extensive data recorded over centuries in cuneiform tablets. With a nascent astrological bent, in order to predict future Earth-bound events, they created huge positional data sets and invented an algorithmic approach to making predictions. The Greeks inherited their and Egyptian data but made the program geometric. The former approach seems sterile, while the latter approach creates pictures, which is a very modern physics approach.

Horoscopic **astrology** became important and popular during the Hellenistic period, and geometric tools were developed and deployed to better record astronomical events and match them to both personal lives and medical treatments. The distinction between astrologers and astronomers blurred and stayed entangled into the 17th century, each serving the other.

How to make sense of complicated MOTION IN THE HEAVENS? Many cultures tried, but the Greek geometrical approach was best suited to prediction and explanation. The problem was hard.

Let's imagine partnering with a Hellenistic Greek as we each observe the sky and note the puzzles that confront us both.

> Let's go out tonight at my home, which has **latitude** and **longitude** of 42.7° N and 84.5° W. In what follows, I'll use "EL" to mean "East Lansing, Michigan," and you and I will agree that this corresponds to that latitude. If you're an ancient Greek, then my latitude is identical to that of Greek colonies in the south Black Sea. So, around where Eudoxus worked for a while!

3.2.1 What Ancients Saw and What We Still See

Suppose you're indeed a smart Greek with time on your hands and able to spend years just recording what the sky presents to you during the days and nights. A few things would stand out...and if you were a patient and persistent observer, nuance would start to emerge. In *Greek Astronomy, Today*, in Section 3.7.1 I'll "set the record straight" with full, modern explanations for each of these scenes and motions but here we'll just observe.

The Sun The Ancients' —personified by that smart Greek with free time —and your and my relationships to our Sun are the same. From the northern hemisphere, we all see it come up in the East in the morning, rise to a peak in the southern sky at midday, and settle into the western sky in the evening. Where it rises, sets, and peaks almost unnoticeably changes from day to day, but from season to season, it dramatically changes—with the weather.

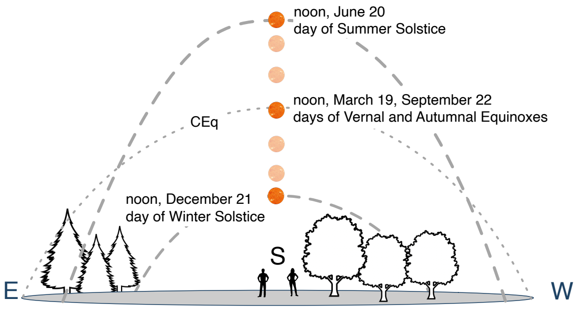


Figure 3.3: An observer looking south would see the Sun take very different paths through the year. Of course, the Sun moves from east to west, but at various altitudes. This figure shows the situation for EL. On December 21st, the Sun takes its lowest path; on June 20th, the Sun is nearly overhead, and between those extremes, the paths are slightly different each day. The equinoxes are right in the middle.

Look at Figure 3.3 in which our Greek and one of us are both watching our Sun's paths through a year for EL during 2024. On December 21st, the Sun takes its lowest path, and the days are the shortest because the Sun rises south of east and sets south of west (behind the trees), so it's visible in the sky for only about ten and a half hours on that day. The day of that lowest Sun-path is called the **Winter solstice**—the shortest day of our year. Every day after that, we notice that the Sun's eastern rise is a little bit north from the day before and that it would set a little bit further north as well so each day would be a little longer. Furthermore, at noon, the point each day when it's at its peak would be just a little higher in the sky than the previous day. Then on June 20th, the Sun has gone as far up as it will and is nearly overhead at noon, rising and setting quite a bit north of east and west, so that day is the longest of the year: it will be light for more than 13 hours. That's the **Summer Solstice**. Then the situation reverses, and the Sun is lower every day until the next December. Between those extremes, the paths are different slightly each day.

In that round trip, there's one day on the way up and one day on the way down when the Sun rises precisely in the east and sets precisely in the west, and at noon, its height above your horizon is exactly between those two solstice extremes during late December and June. Also, on those two days, the day and night durations are the same all over the world: 12 hours. Each of these two special days is called an **equinox**.[4] and they happen in late March (called the **vernal equinox**)[5]and late September (the **Autumnal Equinox**).[6] Each equinox is a precise astronomical event and marks the point when the Sun passes through an imaginary circle in the sky called the Celestial Equator on its way up or down (we'll talk about the Celestial Equator in the next section). In Figure 3.3, you can see that the trajectory of the Sun's path in the middle is dotted rather than dashed to highlight that the Sun's path that day is very close to that Celestial Equator. It crosses the Celestial Equator at the precise moments at 11:06 PM EDT on March 19th in 2024 and that moment officially defines the Vernal Equinox. On September 22nd 8:44 AM EDT in 2024, is the official moment of the Autumnal Equinox.

Equinoxes were striking events throughout ancient history, and across cultures. The Vernal Equinox was celebrated from the Mayans to the ancient Germanic tribes to the ancient Saxons, as a time of renewal and rebirth. Structures like Stonehenge, the Mayan pyramids, the Egyptian Pyramid of Khafre, and events in China, India, Cambodia, Ireland, and New Mexico celebrate the VE. Understanding them, though, only became a goal among a few Hellenistic Greeks when solar models were invented by mathematically clever and imaginative astronomers. As our story unfolds, notice how the Sun figures into every corner of ancient astronomy—and yet, it was considered to be just another orbiting object.

There is another imaginary circle in the sky and that's is constructed in your mind's eye, by completing the path of the Sun during an equinox. On that day, you can imagine tracing out the Sun's path overhead and then continuing it around the other side of the Earth centered on the Earth's center. If you looked at the sky 12 hours later, you'd find that all of the planets are following that same, Sun-path circle. In fact, if you imagine an imaginary band across the sky as wide as the Sun's excursion between the solstices and centered on its path during equinoxes, the planets' paths would *also* be contained within it. This path where the planets and Sun move is called the **ecliptic**, another very old, very universal observation across ancient cultures. What's more, cultures identified star patterns within that ecliptic

[4]This derives from the Latin *aequus*, for "equal" and *nox*, for "night."

[5]Latin for "spring" is *ver*.

[6]In 2023, the WS, VE, SS, and AE occur on December 22, 2023, 3:27 AM, March 20, 2023, 9:24 PM, June 21, 2023, 2:57 PM, and September 23, 2023, 6:49 AM, GMT

band and for the Greeks, and us later, the patterns of star constellations are called the **zodiac**. Now that we've moved our story to the night, let's talk about what we see when it's dark.

The celestial sphere. Let's look up after sunset and watch the stars' motions through a particular night. Figure 3.4 is what we'd see on March 19, 2024 from EL. Here, I've again positioned our ancient and modern partners looking south with the eastern horizon on their left and the western horizon on their right. Directly overhead is the **zenith**, which would be 90° from all points on the horizon. Let's follow one familiar constellation through a band of star groups that

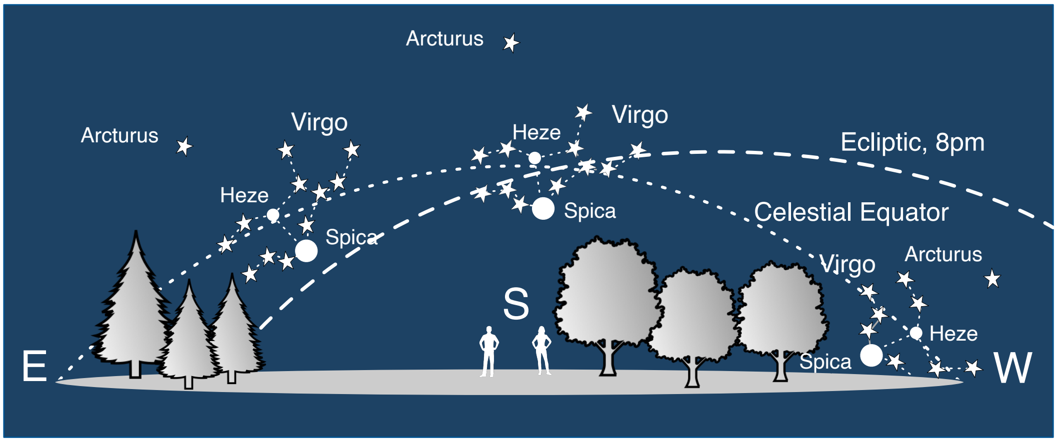


Figure 3.4: An image of the constellation Virgo at three times — 4 PM, 9 PM, and 2 AM— during the night of March 19, 2024 from EL. The apparent single star, Heze, follows very closely the outline of the Celestial Equator. The dashed line is the curve of the ecliptic at 8 PM that night (the "ecliptic" will be defined in a bit).

Virgo, the "maiden" is the largest constellation in the zodiac and is most evident in the spring. Its shape presents two "legs" and two "arms" seemingly attached to a "body." The downward "hip" is Spica, one of the brightest stars in the sky. The two outstretched arms reach toward the spectacular Virgo Cluster of thousands of elliptical and spiral galaxies. Our interest is more modest.

The naked-eye star, Heze, is joined at the other hip to Virgo, so to speak, and is actually two relatively modest stars appearing to be close together as one object. What's useful for us is Heze's location because it traces out an important circular path. Figure 3.4 shows it as a dotted circle with three replicas of Virgo showing its positions from late in the afternoon (invisible since the Sun is still up) to overhead about 9 PM, and then at about 2 AM when it sets. That dotted curve to which Heze appears to be attached is special; it starts directly in the east and ends directly in the west. Also pictured is Arcturus, the fourth brightest star in the sky, which likewise follows another circular path that is parallel to Heze's. In fact, as you watch, you can imagine all of the stars in the sky following concentric, circular paths every night. Figure 3.5 shows a time-lapse photograph of the northern sky where all of the circular star trails are evident with the axis of all of those circles centered at the **north celestial pole**, which for us now is very close to the North Star, Polaris.

Figure 3.5: A time-lapse photograph of the star positions during a single night in the northern hemisphere is shown, clearly demonstrating the circular "inside" of the Celestial Sphere. The pole is conveniently located (now) at the North Star, Polaris.

The most natural impression is that you're standing in the middle of an enormous 24-hour spinning sphere — the **celestial sphere**—with stars attached to its inside surface. If the Earth were to become transparent, you'd see the whole stellar panorama turning around you and its axis from Polaris to the other side poking out below you near the south pole. Heze's path is special since that dotted line traces out the equator of that spinning sphere, the **celestial equator**, Ceq.

One of those nuances is that the stars' appearances are not repeatable night after night. The times that stars begin to appear on the eastern horizon change each night by four minutes early out of 24 solar hours, which is called "**heliacal rising**." This rising time advances through the year, and the "ascendency" of stars in the east became milestones on a calendar that people could use to predict when astronomical events would occur. For example, when the bright star Sirius in the constellation Canis Major appears in the eastern sky just before dawn each year, Egyptians knew that the Nile's flooding was coming.

The Sun's motion. By Hellenistic times (after Alexander's conquests), everyone knew that the Earth was spherical and that some of the angular quantities in the sky matched angular quantities on the Earth's surface. Greeks were spread between northern Africa (about 30° north of the equator) and the northern shores of the Black Sea (about 45° north), so the apparent position of the stars was easily seen to be different when viewed from different locations. For example, Figure 3.6 is a perspective view from EL corresponding to Figure 3.3 where the angle that the Celestial Pole makes with the northern horizon is identical to the observer's latitude in that image; in this case, the 42.7° N of EL. That means that the angle that the celestial equator (and hence the Sun's path on the day of equinoxes) makes with the southern horizon is (90° − the observer's latitude). Finally, the angular separation of the Sun's extreme altitudes is 23.5° up and down from the Sun's equinox path.

Of particular importance to the Greeks and all concerned later with astrology were the constellations in which the "Sun resides" during the time of an equinox.[7] During the times of the Greeks, the special point in the sky when spring would begin was when the Sun passed through the leading edge of the zodiacal constellation of Aries—the "First Point of Aries" and it became the origin of a coordinate system in order to document the location of stars and planets and became particularly important to astronomers in the −200's.

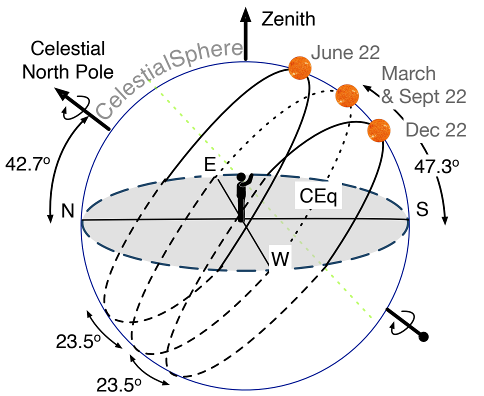


Figure 3.6: A perspective view of the Celestial Sphere from one's horizon, here for the latitude of 42.7° of East Lansing, Michigan, is shown. The three bands show the Sun's path in the sky at the Summer Solstice (top), Winter Solstice (bottom), and the Equinox (middle). Each of the bands around that central arc is 23.5° above and below it.

Clearly associated with the Sun are the seasons, and they aren't the same length—spring and summer are longer than fall and winter, but there are definite times of cold and warm weather in the two hemispheres. In 2023, in the northern hemisphere, after 89 days in 2022, winter ended; spring was 93 days long; Summer was 94; and Autumn was 89. The Athenian astronomers Meton and his student, Euctemon, found 92, 93, 90, and 90 days in about −432, so the seasons' durations were a known problem. (The student also has a lunar crater named for him.) Then, as today, we start spring at the Vernal Equinox, summer at the Summer Solstice, fall at the Autumnal Equinox, and winter at the Winter Solstice.

Planets' apparent motions. There are other objects that execute similar east-west motions through an individual night, are brighter than stars, don't twinkle like stars, and occupy strange, un-star-like positions from night to night. Of course, these are the "planets," probably named by the Greeks from their word for "wanderer," *planetai*. Figure 3.7 shows a striking event in the sky at 2:30 AM on June 23rd, 2022

[7]Of course, they could not see the stars when the Sun is out, but they knew to look at the sky exactly 12 hours later and then extrapolate 180° around the zodiac to determined where that point of "residence" was.

from EL in which four of the five naked-eye planets were all above the horizon at once. The bright circles are naked-eye planets, and the gray circles are the rest of the complement, which require a telescope to see, but notice they, too, are all lined up with the others and the Moon. Pluto is added for nostalgia. The Sun is about to rise, following Venus on that same dashed curve. Obviously, their paths are somehow related.

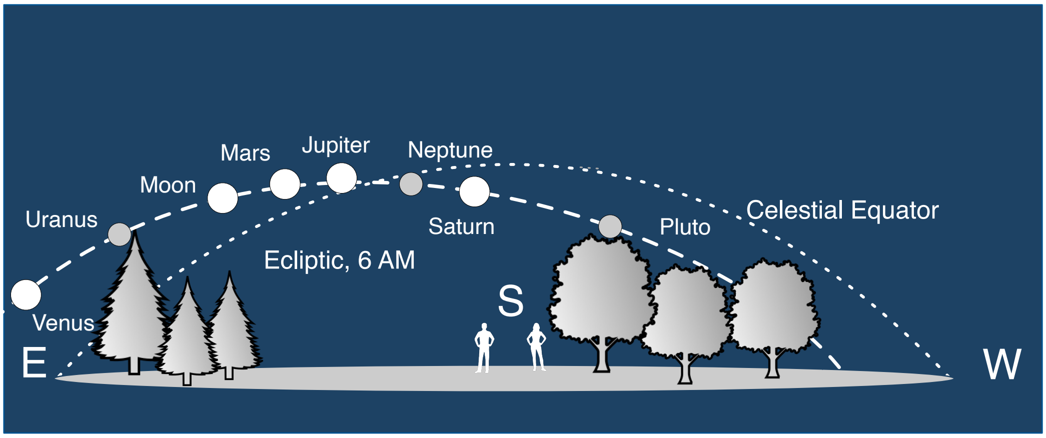


Figure 3.7: The position of the naked-eye planets (white circles) from EL at 2:30 AM on June 23rd, 2022. The dotted line is the Celestial Equator, and the dashed line is the ecliptic. The gray circles indicate where planets that the Greeks could not have seen with the naked eye.

All of the planets and the Sun are within $\pm 7^\circ$ of the dashed mean curve (except Pluto, which is 17°, one of the reasons it's no longer considered a planet of ours). This common "lane" in which all of the solar system (and the Moon) objects reside is called the **ecliptic**, and the central path is sometimes called the "**mean Sun**." At a different day and time, the Celestial Equator won't have moved, but note that the ecliptic traces out a *different* curve relative to the horizon, and you can see that in Figure 3.4, where it's represented again as a dashed curve, but for a different day, March 19, 2024. This must have been confusing!

The ecliptic plane is inclined to the Celestial Equator by 23.5°. The constellations of the zodiac are distributed around the sphere within that strip of the sky[8] and the center of it is the path of the Sun.

Finally, there are two kinds of "motions" spoken of for the planets, which is confusing.

- If you watch a planet during a single night, you'll see it move from east to west in line with the stars behind it. This is called "**prograde motion**."
- But there's another kind of "motion" which is not during a single night, but appears when one does a comparison from night to night. After all, the planets have their own motions relative to the speckled stellar background

[8]There are 13 zodiac signs, but that's inconvenient for astrologers, so they ignore one of them.

on the Celestial Sphere, so if you look at, say, Mars every night at 10 PM and take note of what stars are behind and around it, you'll notice that it usually appears east of where it had been the previous night. But then, periodically, something strange happens. Suppose Star A and Star B are on either side of Mars. On some successive nights, the arrangement of the three objects will go something like this table below facing the south:

Night #1	East	........*A*..........*M*.........*B*	West
Night #2	East	........*A*........*M*...........*B*	West
Night #3	East	........*A*......*M*.............*B*	West
Night #4	East	........*A*....*M*...............*B*	West
Night #5	East	........*A*..........*M*.........*B*	West
Night #6	East	........*A*............*M*........*B*	West
Night #7	East	........*A*................*M*....*B*	West
Night #8	East	........*A*..............*M*......*B*	West
Night #9	East	........*A*............*M*........*B*	West
Night #10	East	........*A*..........*M*..........*B*	West
Night #11	East	........*A*........*M*............*B*	West
Night #12	East	........*A*......*M*..............*B*	West
Night #13	East	........*A*....*M*................*B*	West

Each night Mars seems to be more east of the star pattern near it—that separate motion of Mars at work. But between nights 4 and 11 Mars appears more west and after a number of nights, it then reverses course and continues its nightly progression eastward. This is called "**retrograde motion**" and it confused everyone. Certainly, the common description of retrograde motion as a "motion" is confusing nomenclature since the "movement" is actually a displacement over many nights. This happens to Mars every 26 months and the retrograde loop takes about four months to complete.

The apparent motion of the Moon. Our Moon is prominent for its size and its regularly changing features. If looked at from overhead, it travels in a clockwise orbit, nearly circular, with a period of 27.322 days, changing its appearance through phases during that cycle.

Unlike the Sun and the stars, the Moon changes its appearance every single night. Sometimes it's "full" and a bright circle. Sometimes, it's not there at night, but maybe visible during the daytime. Most times the bright part of the Moon is a crescent shape, culminating in a half-circle, and then back to crescent. Occasionally, the Moon gets in the way of the Sun and we have a solar eclipse. Sometimes the Earth blocks the Moon from the Sun, and we have a lunar eclipse. Why these events didn't happen every month was a puzzle. One thing doesn't change about the Moon, and that's the face that we all see each night—another puzzle.

The accumulated puzzles from our simple observations include at least these:

1. Why are the seasons of different durations?

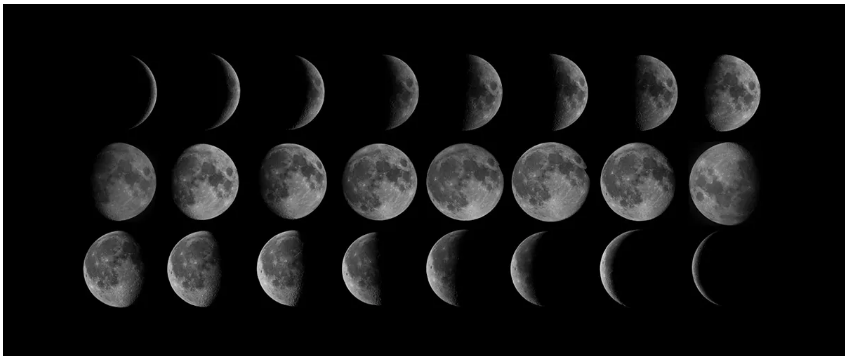


Figure 3.8: ;

Faces of the MoonViews of the familiar faces of the Moon through a month, not showing the new Moon phase. Getty

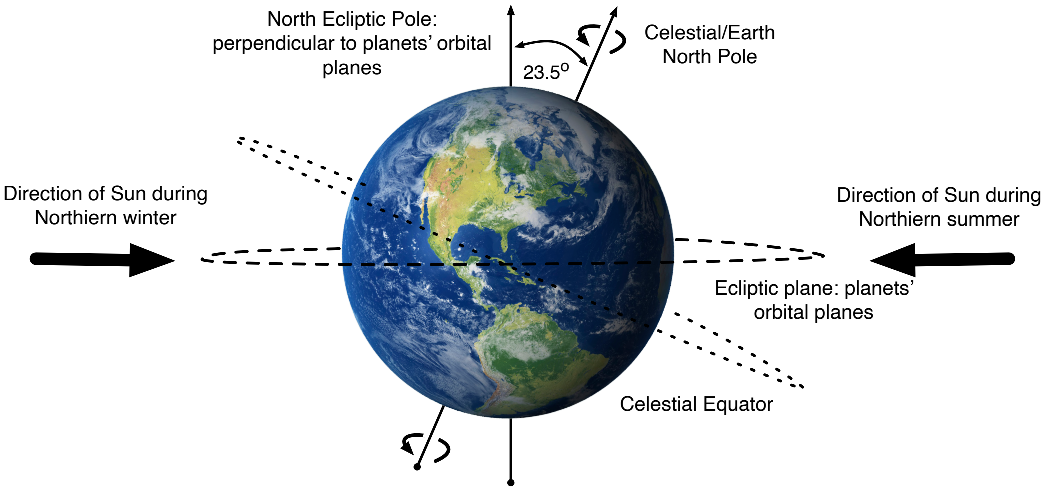


Figure 3.9: The facts of the matter are: The Earth and all of the planets orbit the Sun in a plane, the ecliptic plane; the Earth spins on an axis which is 23.5° inclined from the vertical to that plane. The Celestial Sphere then is also inclined and the stars appear to revolve at that inclination. The Sun's rays on the left are spread out over the Earth's surface in the northern hemisphere, and we have winter, when the Earth is on the other side, six months later, the Sun's rays (on the right) are more concentrated over the surface, and we have summer.

2. Each planet's speed varied as it went around the Earth. At **apogee** (furthest from Earth), they moved slower than at **perigee** (closest to Earth). This has historically been called the "**first anomaly**."
3. Why do the planets undergo retrograde motion? This has been historically called the "**second anomaly**."
4. What is the nature of the spherical shell that seems to carry the stars around in celestial circles?
5. What is the reason for the appearance of the 23.5° inclination of the CEq and the ecliptic?
6. Why are the planets sometimes bright and sometimes dim?
7. Why don't lunar and solar eclipses happen every month?

Puzzled — like our Greek friend —about these observations? If you can't wait for Copernicus, Tycho, Kepler, and Galileo...then skip ahead to *Greek Astronomy, Today* in Section 3.7.1 for the modern interpretation of how it goes. Figure 3.9 is a taste of the solutions to many of the puzzles.

3.3 A Little Bit of Presocratic Astronomy

Pythagoras •Philolaus •Parmenides •Archytas
(Set the context with the timeline in Figure 1.2 on page 22.)

In Chapter 1, I briefly discussed the Presocratics' cosmologies with two ideas among them that were shared: all but two appeared to believe in a flat, and stationary Earth. The two who thought differently were Pythagoras and Parmenides.

Parmenides had a number of original ideas about the heavens—in particular, he may have been the first to conceive of the whole universe as being spherical (Pythagoras/Philolaus might also have determined this) and finite.

> "...like the mass of a well-rounded sphere, from one middle, equal in every respect." Parmenides

He was also apparently the first Greek to note that the Moon reflected the light of the Sun and must be spherical, and he was even poetic about it:

> "[the moon is a body] shining by night, wandering around earth with borrowed light..." Parmenides

"Borrowed light" is a nice phrase. If the Moon "borrows" its light from the Sun and doesn't shine on its own, then the shape of the phases of the Moon leads to a spherical shape conclusion.[9] Ironic, isn't it that Parmenides can perhaps be credited with a scientific discovery—one that requires observation— when we tend to think of him as anti-scientific and untrusting of what he might observe?

[9] It was traditional to credit Parmenides with extrapolating from a spherical Moon to declaring that the Earth, too, is spherical. But that's not authenticated and Pythagoreans' claim to a spherical Earth is perhaps more likely.

The Pythagorean team (probably more Philolaus than Pythagoras, so I'll call it collectively "Pythagorean/Philolaus") extrapolated their fondness for regular motions, musical tones, and numbers and built a cosmology that tried to put all of these commitments into one model. They were responsible for many "firsts" in Greek astronomy: they too hypothesized that the Universe is spherical; most credit them with establishing that the Earth is spherical (for metaphysical and symmetry reasons); they proposed a popular ordering of the planets (Earth, Moon, Sun, Mercury, Venus, Mars, Jupiter, and Saturn...surrounded by the stars), they hypothesized that the planets' speeds are inversely proportional to the size of their orbits, and they concluded that the "morning star" and "evening star" (our Venus) were not two different planets but the same one which is close to the Sun. And, crucially, they were the first to propose that the planets follow circular orbits around a center.

There was a first version of Pythagorean/Philolaus cosmology in which the Earth is at the center of the universe containing a "central fire" or "'Hestia," in homage to the immobile goddess of the hearth. But that morphed into the cosmology of Chapter 1 with the "central fire" situated in the center of the universe, relegating Earth to be just another celestial object orbiting around it in circular orbits. Figure 3.10 (a) shows the whole system with the Earth, Moon, Sun, and the planets orbiting counterclockwise around the center and inside an outer shell of the stars. The Earth orbits the central fire once a day and the Sun, once a year. So the Earth catches up daily and passes the Sun, accounting for day and night.

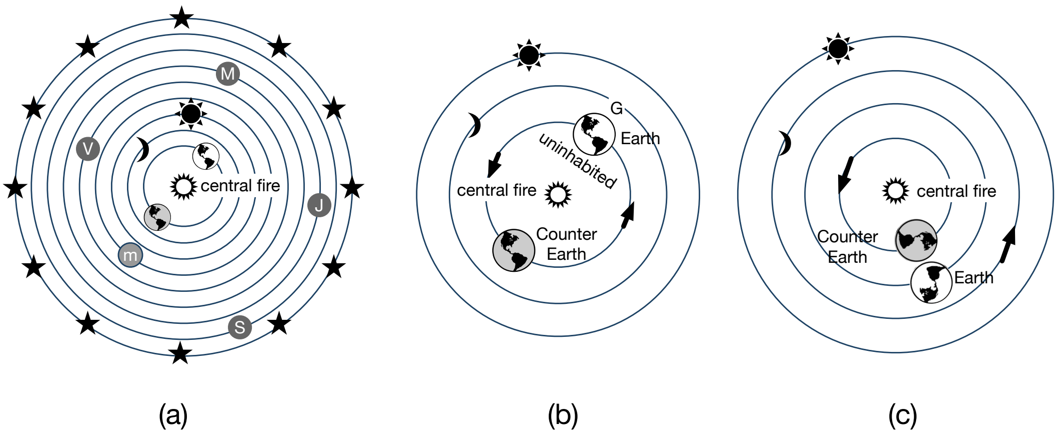


Figure 3.10: (a) shows the Pythagorean system with all of the heavenly bodies and the Earth orbiting the central fire in a counterclockwise sense. In (b), the Earth is shown in one of a number of interpretations of Philolaus' system. Greece (G) is on the far side, leaving the side facing the fire without people. In this orientation it's morning as the Earth is catching up with the slower-moving Sun. In (c), the counter earth is positioned so that it blocks the central fire.

We don't see a "central fire," and there were two proposals as to why, shown in Figure 3.10 (b) and (c). The standard interpretation is the second one in which inhabitants of the Earth are shielded from the fire by the presence of a "counter earth" which strategically blocks it (see J. L. E. Dreyer, 1953). Without the counter earth, there are only nine components to the universe, and so Aristotle was critical of them for perhaps arbitrarily adding the counter earth just to make the total 10 (as suggested in D. R. Dicks, 1970) and many others.

This is the first cosmology based on a *regular, circular* MOTION IN THE HEAVENS and a model in which MOTION BY THE EARTH is not zero. The idea, of course, stimulated 2000 years of astronomical research! Circles, everywhere.

3.3.1 Summary of the Astronomy of Parmenides, Pythagoras, and Philolaus

(Set the context with the timeline in Figure 1.2 on page 22.)

- Parmenides (−514 to −450):
  - He was first to assert that the whole universe was spherical.
  - He was perhaps the first to recognize that the Moon does not shine by its own light, but reflected ("borrowed") light from the Sun. The Pythagoreans might also have realized that.
- Pythagoreans [Pythagoras (−575 to −500) especially including Philolaus (−470 to −385)]:
  - "They" were first to realize that the Earth is spherical.
  - "They" were first to hypothesize a particular ordering of the planets, perhaps with the their orbit size inversely proportional to their speeds.
  - "They" realized that the "morning" star and "evening" star were the same planet, Venus.
  - "They" were to propose a model in which the planets (including Earth and Sun) all orbited a central point (for them, the mysterious "central fire.") in perfectly circular orbits.
  - Their insistence on heavenly motions being uniform and circular outlived their specific model.

3.4 Act VII Plato and Exodus' Models

Plato •Eudoxus •Aristotle
(Set the context with the timeline in Figure 1.2 on page 22.)

In Chapter 1 I touched on Plato's cosmology in *Timaeus* but noted that it was a late development for him as his ideas about the nature of the cosmos grew over almost his whole career. His mathematics from Archytas and Pythagoreans' tendency to rely on symmetry launched him in the direction of building everything around circles, and then spheres.

Recall that the *Republic* was nominally a treatise on the nature of justice and how to build a just state, which he proposed be totalitarian. When philosophy and political science students read it, they're probably surprised by its ending, which is a full-on Pythagorean cosmology, the "Myth of Er."

> "Once upon a time he died in war; and on the **tenth day**, when the corpses, already decayed, were picked up, he was picked up in a good state of preservation. Having been brought home, he was about to be buried on the twelfth day; as he was lying on the pyre, he came back to life, and, come back to life, he told what he saw in the other world." (emphasis, mine) Plato, *Republic*

Socrates is trying to motivate why someone should live a good life and relates a cosmic carrot-and-stick story is not unfamiliar with other religious admonitions. Er is a soldier who was killed and does what all deceased do... they go to a place where their lives are evaluated, not by St. Peter at the Pearly Gates, but by four judges who tell him that he's got a job to do: after 10 days[10] his body will be retrieved from the battlefield, and on day 12, he's to be resurrected from the dead, dramatically on his own pyre before it's lit. He's to tell others what he's seen, which includes a strange vision of a pillar of light that extends to the heavens, which Plato describes as a spindle and whorl used for spinning wool. Figure 3.11 (a) shows a Roman woman spinning wool with the weighted whorl at the bottom, which spins as she works. Figure 3.11 (b) is the umbrella-like structure (the whorl upside down) that Socrates describes:

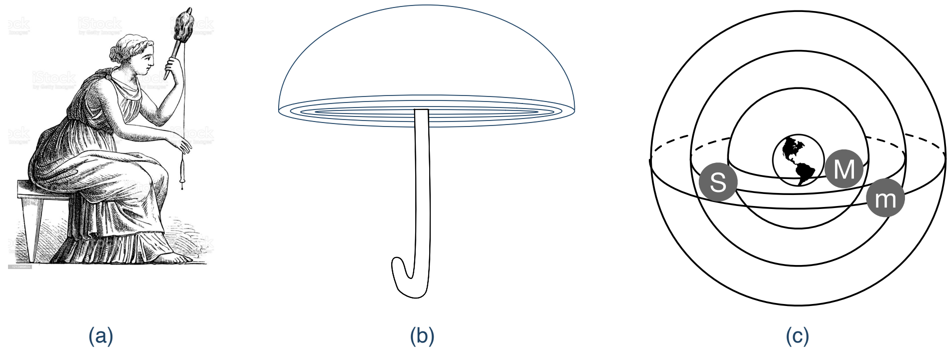


Figure 3.11: The figure in (a) is a Roman sketch of a woman spinning wool using a spindle and whorl, which is the weight at the bottom with a hook. The image in (b) is Plato's description of the whorl actually hollowed out with nested layers of whirl-shaped half-spheres. The image in (c) is the cosmos that the onion-layered whorl represents with the Moon, Sun, and the first planet, Mercury, attached to the first three of eight spheres. I've only included three in this cartoon.

> "Its shape was that of (whorls) in our world, but... it was as if in one great whorl, hollow and scooped out, there lay enclosed, right through, another like it but smaller, fitting into it **as containers** that fit into one another, and in like matter another... There were **eight of the whorls** in all, lying within one another..." (emphases, mine) Plato, *Republic*

The eight "containers" are hinted at in my sketch in Figure 3.11 (b) and the whole is abstracted as nested spheres in Figure 3.11 (c), where I've only shown three spheres

[10]Why 10 days? some Pythagoreanism is maybe showing?

(remember, "containers") for simplicity. Earth is no longer a "regular" planet but is in the center with concentric spheres of the Moon, Sun, the outer planets, and again, the stars on the furthest shell, which Socrates says is "speckled." So, Plato's first cosmology has MOTION BY THE EARTH as zero, and MOTION IN THE HEAVENS is described as Pythagorean, but using spheres, not just circles. He also tells you how they move and the sounds that they emit as a Siren sits on each sphere and sings a tone. This is the world's first three-dimensional cosmological model. But it didn't match what the planets do, and Plato actually tried to remedy it in the *Timaeus*. Given his penchant for not modeling appearances, this was an unusual move and suggests to me that getting it right was (briefly?) important to him.

The *Timaeus* is Plato's "origin story," and in the previous chapter, I described the Craftsman's efforts to create matter using geometric three-dimensional shapes. It's also his cosmology update from the *Republic* and quite different. Socrates teases the story out of the main character, Timaeus—a Pythagorean—and then uncharacteristically allows the speaker to have the floor without much interruption. It's where Plato becomes mathematical, in a spooky, Pythagorean way.

Does this string of numbers mean anything to you: 1,2,3,4,9,8,27? Me neither, but they function as a part of the instructions to the Craftsman in order to build the universe following a numerology algorithm described in a nearly unintelligible paragraph:

> "And he began the division in this way. First he took **one portion** from the whole, and next a **portion double of this**; the **third half as much again as the second**, and **three times the first**; the **fourth double of the second**; the **fifth three times the third**; the **sixth eight times the first**; and the **seventh twenty-seven times the first**." Plato, **Timaeus**

Timaeus is tough to read (impenetrable in some places), and so I've unpacked the algorithm—pure Pythagoras— from the paragraph in Technical Appendix C.1. The upshot is that the Craftsman has fashioned a universe with two rotating spheres. One of them he calls "the same," and it represents the (unavoidable) rotating Celestial Sphere. The other he calls "the different," which is inclined at an angle relative to the "same." That strange string of numbers represents the relative sizes of the layers inside of that inclined sphere where the planets are arranged. His Er story didn't account for the ecliptic, and this "different" sphere set is that correction.

> "This whole fabric, then, he split lengthwise into **two halves**; and making the **two cross one another** at their centers in the form of the letter X, he bent each round into a circle and joined it up, making each meet itself and the other at a point opposite to that where they had been brought into contact." (emphases, mine) Plato, *Republic*

Figure 3.12 is my silly attempt to illustrate this. Figure 3.12 (a) is a person playing with a hula hoop, perfectly aligned so that the axis of the toy's rotational plane points through our person's head. This represents the axis and equator of the Celestial Sphere around the Earth. Figure 3.12 (b) shows just how good this person is at hula hoops: two are rotating, the original, and another that somehow our friend manages to get to rotate at an angle relative to the first one, requiring some serious hip action. This represents the ecliptic, inclined by that spacing corresponding to the latitude of the observer. Those strange numbers? Well, there would actually be seven hoops with diameters proportional to those numbers: 1–2–3–4–8–9–27. Figure 3.13 shows what this is really about.

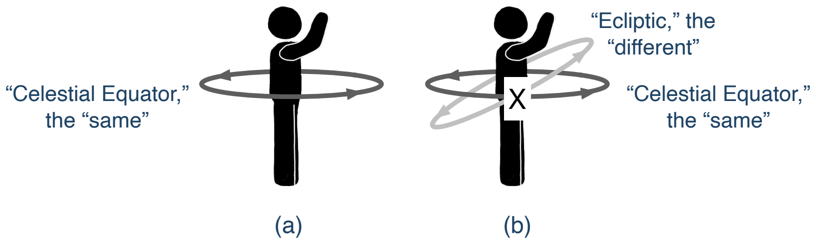


Figure 3.12: Pretty good hula hoops chops. Notice Plato's "X" at the points of intersection of the two hoops.

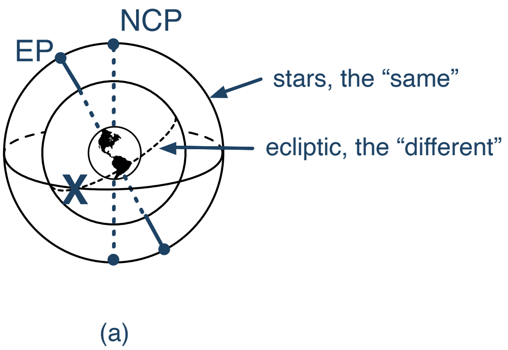


Figure 3.13: (a) shows the two spheres with their equators. One is the Celestial Sphere (carrying the stars around the Earth each night, so an axis centered on the North Pole of the Earth), and the other is the ecliptic (in which the planets reside as they appear to go around the Earth) with the pole of that sphere, the North Ecliptic Pole. Again, the X marks where the ecliptical and the celestial equators overlap. (b) takes away the three-dimensional view and will be a useful sketch for these kinds of constructions in what follows.

The celestial sphere and its axis I've called the **NCP** (north celestial pole) in the diagram. The other strip is the equator of the other, ecliptic, sphere (with axis labeled EP), which makes an "X" where it crosses in two places with the Same. (These are the points of the equinoxes, when the Sun on the ecliptic crosses the Celestial Equator.) Inside of this strip, the segments correspond to the locations of the Moon, Sun, Mercury, Venus, Mars, Jupiter, and Saturn. Of course, this is a little mad, but Eudoxus took on the task of turning this story into a geometrical model.

3.4.1 Eudoxus' Model

By the time Eudoxus had returned to the Academy, he would have been familiar with the *Republic* and probably *Timeaus*. Once Plato had inserted the ecliptic path, he still needed to explain retrograde motion. And he knew it:

> "...as for the dances of these and how they relate to each other, the **backward-cycles and forward-progressions** of the circles to each other...to speak without visual representations of these same would be a **vain effort**." Plato, *Timaeus*

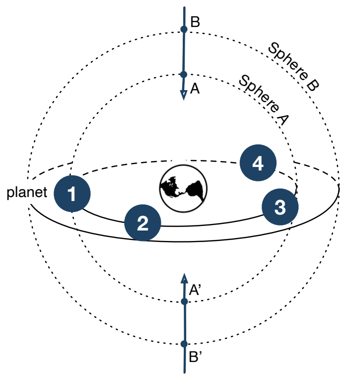
So, he realized the problem but had no solution and just gave up ("vain effort"). He was out of his depth, but Eudoxus was ready and came up with a brilliantly complex model, and while it's not known what Plato thought of it, it's clear how Aristotle reacted: he made it his. It's intricate, so let's go to the box and work out the inner workings of the idea and then skip to the end. Look at Figure Box 3.14 on page 115. After you've read the material in that Box, return to this point and continue reading.

The figure in Box 3.14 describes the tool-kit that Eudoxus used to construct a full model of each planet in which they ride on the equators of coupled, spinning spheres. The two spheres shown in the box form the minimal number of moving parts unique to every planet, and they are each embedded inside of two other spheres, one for the ecliptic, whose equator includes the rough paths of the planets, and the other is the Celestial Sphere which includes the motions of the stars around the Earth every nearly 24 hours. Let's take it slow in Figure 3.15.

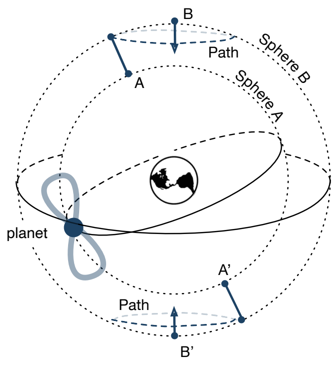
The basic Eudoxus planetary building block was a set of four spheres, centered on the Earth. Using the nomenclature from Figure 3.15 and Box 3.14, labeling them from the inside out:

- A: the sphere to which the planet is attached,
- B: the next sphere which precesses around that inner sphere (producing Eudoxian figure-eight)
- C: the sphere that rotates around the ecliptic—that stretches out that Eudoxian figure 8 in Figure 3.14 to produce retrograde motion, and
- D: the outer-most sphere that rotates daily, showing the pattern of the starry Celestial Sphere.

FIGURE BOX 3.14



(a)



(b)

The model that Eudoxus created is an impressive bit of geometry mixed with inspired imagination. It's the famous "nested spheres" model that made it all the way to the Baroque as an explanation for the odd motions of the planets. In a very modern way, it's full of parameters that could be tweaked to make it fit the observations...some of which he made himself at the observatory he created in his school before he returned to Athens.

Imagine taking two hoops, one of which is slightly smaller than the other and is attached inside the larger one across their mutual diameters. Figure 3.14 (a) shows this with a "planet" attached to the equator of the inside hoop. Now, if we spin that hoop around its axis AA', the planet will follow a circle from position 1 through 2, 3, 4, and so on. This spinning *observed from the outside* essentially defines a sphere, Sphere A, here centered on the Earth. If the two hoops are attached, and if the outer hoop spins around its axis, BB', creating the surface of Sphere B, then the motion of the planet will be the sum of the two speeds at the hoop pair equators since the AA' axis, and so Sphere A is attached to that spinning Sphere B. So if the outer hoop spins at the same rate as the inner hoop, but in the opposite direction, then the planet would appear to the Earth to remain stationary at position 1.

Now imagine that the axis of the inner hoop is attached at a point *off-axis* on the surface of the Sphere B as shown in Figure 3.14 (b). Now when Sphere B spins, it takes the AA' axis of Sphere A around with it tracing the path shown. In addition, if Sphere B spins while its following that path independently, the motion is a complicated figure eight pattern as shown. Eudoxus figured this out and named the shape a "hippopede" which is "horse fetter" in Greek. (A fetter is like a chain.) Now, there are many variables at work that would alter the shape of the hippopede: the speeds of the two spheres and the angle at which AA' axis of Sphere A is inclined to the BB" axis of Sphere B.

Now go back to page 114 and pick up where you left off.

All of these separate motions are coupled. . . , and that's just for one planet! By

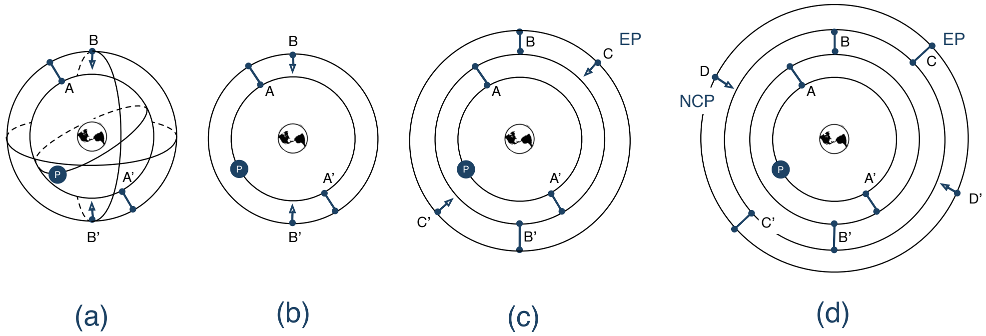


Figure 3.15: (a) is a slightly different rendering of Figure 3.14. (b) is (a) redrawn but as an abstraction for clarity, removing some of the circular lines that suggest a solid sphere. (c) includes the sphere of the ecliptic (EP for Ecliptic pole is shown) with the axis of rotation CC′. Notice that it's attached to the outer sphere of Eudoxus' tool-kit pair. And (d) includes the sphere of the outer stars, the celestial sphere (NCP for the North Celestial Pole is shown), and the ecliptic sphere is attached to it.

tuning the inner two spheres' rotation speeds and the inclination of their inner axes, the motions of the planet can be made to do the figure-eight dance at just the right time of year and with the right elongation in the sky—to make the planet appear to reverse direction and recover, and resume as viewed by the Earth. Each planet required four spheres and the Sun and Moon required three each, plus the Celestial Sphere: 26 spheres to do the job. This was a mammoth intellectual puzzle that Eudoxus created and then solved with those relatively simple pieces of interlocking spheres.

It still didn't quite do the job as well as it might and in the best tradition of what Thomas Kuhn would have called "Normal Science," **Callippus of Cyzicus (–370 to –300)** tried to make it better without starting over. He was a student of Plato and worked with Aristotle and worried about the seasons' length problem and some finer points of the planets' motions. He added two additional spheres for the Sun and Moon and one each for Mercury, Venus, and Mars, for a total of seven more. So now: 34 spheres. Was it all just an exercise in geometry? Perhaps. The Eudoxian program of research was pictures without numbers, and so it had no predictive capability—it was purely explanatory. In a sense, it was more of a story than a scientific model, like Plato and Aristotle's will be.

Around −370, Eudoxus also apparently created a star catalog in his book *Phenomena* of at least 47 stars, which a century later were memorialized in the famous poem of that same name by Aratus that I introduced in the preface to this chapter. In the same way as his spheres, these entries record the times of the rise, set, and position overhead of constellations or stars near parts of constellations—but as stories. For example,

> "As a guide the Ram and the knees of the Bull lie on it, the Ram as drawn lengthwise along the circle, but of the Bull only the widely visible bend of the

> legs. On it is the belt of the radiant Orion and the coil of the blazing Hydra; on it too are the faint Bowl, on it the Raven, on it the not very numerous stars of the Claws, and on it the knees of Ophiuchus ride. It is certainly not bereft of the Eagle: it has the great messenger of Zeus flying nearby, and along it, the Horse's head and neck move round." Eudoxus from *Dennis Duke, 2008.*

What we know of Eudoxus' catalog come to us from the body of Aratus' poem and then Hipparchus' later critique of the poem and by extension, of Eudoxus' work.

3.5 Act VIII Aristotle's Astronomy

When it came to astronomy, Aristotle was downright derivative. Ironically, his model that became Church dogma wasn't exactly his, and to make matters worse, it was flawed and largely ignored soon after he died. How it went from forgotten to dogma is the story of Chapter **??**, but let's see what he actually did and why. His astronomical writings were scattered throughout two large books, *On the Heavens* and *Meteorologies* and his solutions to known problems were a mixture of pure metaphysics, his physics—often relying on his own rules of motion as authoritative,—and the observations of others. Aristotle didn't observe the heavens.

3.5.1 Properties of the Earth, Aristotle-style

Aristotle vigorously disagreed with the Pythagorean/Philolaus cosmology in which the Earth orbits the center of the universe and devised the challenges that anyone defending a moving Earth would have to meet squarely.

The Earth Pythagorean/Philolaus adherents proposed that the Earth is spherical, arguing largely from aesthetic grounds, namely that circles and spheres are good, and so the Earth should be spherical as well. Oh, and that the universe is spherical, and so must the Earth.

Aristotle proposed multiple, more concrete reasons why. First, when one observes a lunar eclipse, one sees that the shape of the demarcation between light and dark is always convex. So if the Earth's shadow is the explanation for the eclipse, then the Earth must be at least circular, if not spherical. He knew from reports that people in the southern latitudes saw different stars on their horizon than those in the northern latitudes. He argued against those who insisted (still) that the Earth was flat by noting that the horizon looks flat, but that's simply because the Earth is large.[11]

He also had a physics reason. Since earthy material would naturally be aimed at the center of the universe then all earthy material would be drawn to a single point and highly compressed equally in all dimensions with the result: a sphere of earthiness. That sphere would be surrounded by a thick sphere of water. That

[11] Nowhere in Aristotle is the famous alleged argument attributed to him that when ships begin to appear on the horizon that first the mast and then the hull are observed.

would be surrounded by a sphere of air and then fire. So a spherical double-double-decker sandwich of the four terrestrial elements fills up the whole volume below the Moon, the "sub-lunar realm." This argument supported two other Aristotelian–imperatives: that the Earth finds itself in the center of the universe and that it's stationary.

The Stellar Parallax Argument Finally, he makes a good argument for the stationary Earth, which becomes the essential challenge to any future moving Earth cosmology.

Look at a point across your room with one eye closed and put your finger in front of you, and notice what's behind it on a wall or distant surface. Now switch eyes and notice that what's behind your finger now seems to have moved. If you open and close each alternate eye successively, the background will appear to jump from side to side relative to your finger. This is called "**parallax**", and it's because your eyes are attractively located inches apart from one another on your face and enough so that the lines of sight from each are slightly different.

If the Earth is orbiting a center, then at one point of the year, a particular star would appear as a line at a particular angle (like your right eye open). Then, at the halfway point around its orbit (six months later if the orbit is around the Sun), when the Earth is on the other side of that center (like your left eye open), look for that same star, and it will be at a completely different angle. "**stellar parallax**" or "annual parallax" is the name of this phenomenon, and I'll point this out more than once in our story.

Nobody observed stellar parallax, leaving only two explanations. Either the Earth doesn't move around a center of revolution, or the stars are so far away that parallax isn't visible. Nobody was prepared to imagine a universe that big, and so the conclusion was that MOTION BY THE EARTH is zero.[12]

He agreed with Parmenides and the Pythagoreans that the light from the Moon is reflected light and that the shape of the crescent of the Moon's phases suggests that it must be a sphere. From that and his spherical Earth hypothesis, he reasoned that all of the heavenly bodies are likely spherical, albeit made from different stuff.

For millennia, Aristotle has been held responsible for the theory of five elementary substances: in *On the Heavens*, he added what he called the "first body" to the familiar earth, water, air, and fire. Much later, this was renamed "the fifth element," and later, the "**aether**;" and later than that, the Latinate, "quintessence." In spite of almost all popular and even scholarly sources, Aristotle never identifies his first body as "aether" although he was surely aware that Plato used that term explicitly. History assigns Cicero, from the first century BCE, as the source of Aristotle's reference to "aether" with the assumption that the famous Roman orator had access to now-lost Aristotelean manuscripts. Or, given our repeated reminder that much

[12]It took until the 19th century to actually observe stellar parallax because the universe really is that big.

of what we know of the Greeks is muddled...it's possible that Aristotle never used the word.

I'll use "aether" as it will become a useful contrast with the 19th century "ether," the direct experimental lead-in to Relativity. And, by the way, Aristotle is often said to have insisted that the Eudoxian spheres were crystalline; the "**crystalline spheres**" were indeed an assumption in Medieval and Renaissance times, but nowhere does Aristotle refer to this. (See, David E. Hahm, 1982)

Aristotle's aether is eternal, not composite, neither heavy nor light, and is the most divine of all of the heavenly objects. So it's not anything like the four Aristotelian elements, but for some reason he holds heavenly objects to some of the same physics as terrestrial objects.

The Sky The heavens differ from terrestrial objects in an obvious way: the night sky repeated, every night, while everything on the Earth seems less ordered. Sure, falling objects executed their motions according to rules, but every object's behavior is different, so the eternal permanence of the heavenly motion contrasts with the impermanence and changeability of MOTION ON THE EARTH. Furthermore, for Aristotle, natural motions near the Earth were in straight lines—with a beginning and an end. But the motions of the heavenly bodies seem circular, and so, never-ending...eternal. Obviously, then, the deep sky is made of special, different stuff.

Aristotle's universe is a finite volume in space all the way to the outermost starry sphere, like that of the Pythagoreans. Furthermore, it's always been there, and he speculates on, and rejects an argument about the possible creation of the universe. So he disagrees with Plato. That, for him, would presume that before that event, there was already a notion of up and down, and that bothered him. So, the universe is a finite volume in space, but of infinite extent in time.

3.5.2 Aristotle's Cosmology

The basic features of Aristotle's cosmology were the same as Plato's, as were his ordering of the planets (and different from what Philolaus assumed for the Pythagorean model): Earth–Moon–Sun–Mercury–Venus–Mars–Jupiter–Saturn and the stars. Ever the mechanist, he worried about real material concerns: *how* do they *actually* move as a composite unit?

First, he knew that what was required was a model of the whole universe—Eudoxus' model was a template for each planet, not a whole cosmos— and so each of those sets of spheres needed to all be packaged together into one big onion of spheres, one set inside of another. And this became his problem: since he couldn't have Jupiter's motions affecting Saturn and Mars' motions, he needed to "mechanically" decouple each one.

Remember that I noted that if you had two connected Eudoxian spheres rotating at the same speeds but in opposite directions, their motions would cancel one another.

Aristotle took that idea and intentionally inserted "rewinding spheres" to do that in such a way to preserve the spheres' connections to the ecliptic and celestial spheres but to isolate them.

Table 3.1 shows that for all of the planets but the Moon and Sun, four spheres were sufficient for Eudoxus. (The Sun and Moon didn't need the daily, celestial sphere rotation.) Callippus added spheres for the inner planets, Sun, Moon, and Mars. It was these 33 spheres that Aristotle then tried to turn into an actual seven-object, whole system.

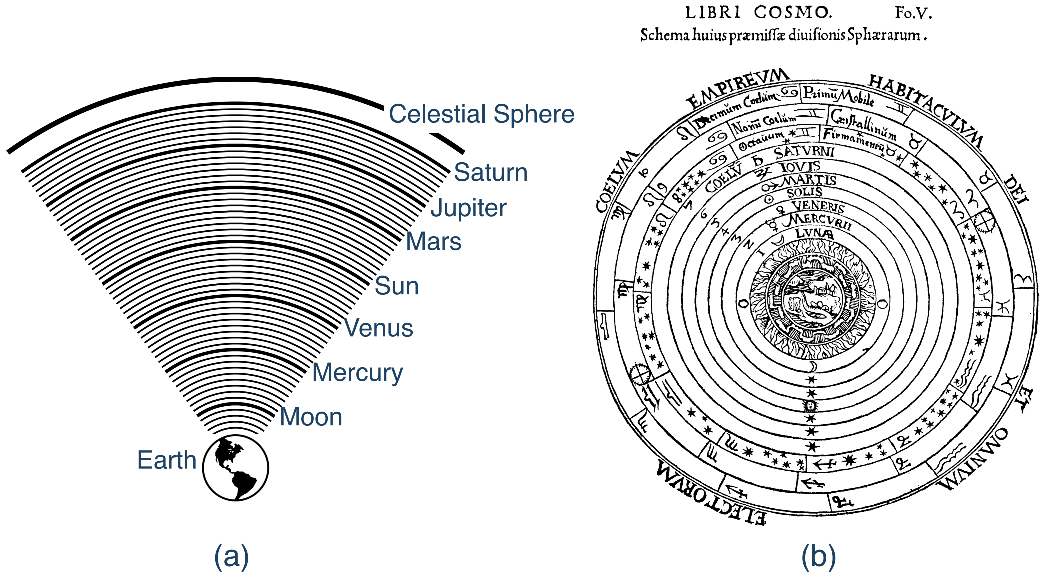


Figure 3.16: (a) Representation of the 55 spheres of Aristotle's model. (b) is a typical Medieval representation of the Aristotelean cosmology. At the top you can make out the sphere of the Prime Mover.

Table 3.1: The number of spheres for each of the Eudoxian systems for the Moon, Sun, and planets—not including the outer sphere of the fixed stars— with the Aristotelian unwinding spheres counted separately in the last column.

Planet	Eudoxus	Callipus	Aristotle	Unwinding
Saturn	4	4	4	3
Jupiter	4	4	4	3
Mars	4	5	5	4
Sun	3	5	5	4
Venus	4	5	5	4
Mercury	4	5	5	4
Moon	3	5	5	
Total:	26	33	33	+22 = 55

> It is necessary, if all the spheres put together are going to account for the observed phenomena, that for each of the planetary bodies there should be

> other counteracting ["unrolling"] spheres, one fewer in number [than Calluppus]...for only thus is it possible for the whole system to produce the revolution of the planets." Aristotle, *Meteorologies*.

Figure 3.16 (a) shows a rendering of the 55 Aristotelean spheres (b) shows a typical Medieval picture of Aristotle's cosmology, the Prime Mover is noted (see below), and in the center, the four Aristotelean elements are drawn. But there's an interesting difference: the planetary order is not Aristotle's but from later.[13] Maybe this will help: Figure 3.17 is a cartoon of his universe in a way that nobody from his time would have drawn it and I've left out the individual shells for simplicity.

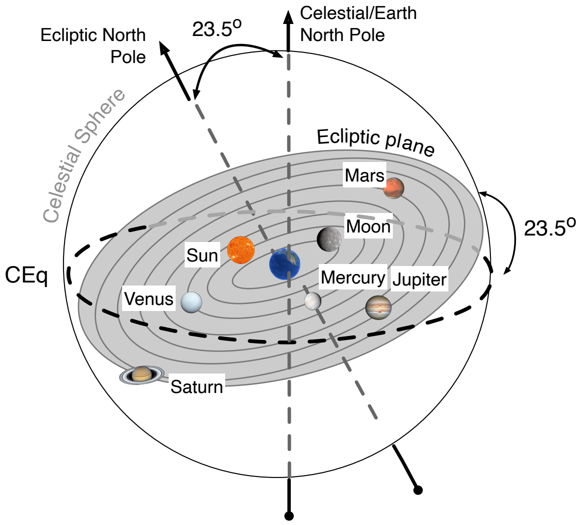


Figure 3.17: A cartoon of what Aristotle's model implied for the universe.

He always seemed fascinated by his own ideas about Earthly motion, and yet when modeling planetary motions, he carried over some Earthly rules to that very different realm. For example, he assumed that bodies made of that completely unique aether still needed to follow his physics and causal rules. Why didn't he just say that aether spheres just naturally isolate themselves, one set from another? In that same sticking-to-the-terrestrial-rules spirit, he seemed to believe that the spheres needed a cause in order to execute their natural, circular motion, and that drives his model into strange places. Just like *unnatural motion* for terrestrial objects required a contact pusher, inexplicably, he decided that the *natural, circular motion* of his spheres *also needed contact pushers*. That creates an embarrassing regress problem. Every sphere had its very own pusher, and so did the outer star sphere, but how does that last pusher itself remain stationary in order to be able to move that last sphere? Another pusher? He complicated this by insisting that the pushers had themselves no substance, were outside of space and time, and were essentially pure intellect. He called them "unmoved movers" or "Prime Movers," and the idea was a soft toss to Thomas Aquinas 1600 years later to equate the Primer Mover with the Catholic deity.

Aristotle's astronomy is underwhelming and unsatisfying, and it didn't solve the major issues endemic to an Earth-centered cosmology: Since the model required each planet to be always the same distance from Earth througout a year, why do they vary in brightness? And a relatively new problem in his time: Why are the

[13] Aristotle seems to have made at least one mistake and actually had two models, one of 47 and the other of 55 spheres. Nobody knows why.

seasons, autumn, winter, spring, and fall, all of different durations? These brought Aristotelean modeling to a halt. New ideas were required.

3.5.3 Summary of the Astronomy of Plato, Eudoxus, and Aristotle

(Set the context with the timeline in Figure 1.2 on page 22.)

By the time Aristotle was done, astronomy had converged on a qualitative ”picture model” built by two philosophers and a mathematician.

- Plato (-427 to -348):
  - He placed the Earth at the center of the universe.
  - He modeled the planets as attached to spinning spheres.
  - He proposed that the outer star-sphere spins around the Earth once a day.
  - He placed the sphere of the planets to be inclined to that of the stars so that they all orbit at an angle inclined to the Earth's equator—on the ecliptic.
- Eudoxus (-390 to -340)
  - He modeled each planet's motion as created by four spheres, with axes inclined to one another to replicate retrograde motion and motion relative to the stars. (The Sun and Moon only needed three spheres.)
  - He modeled each planet's model as separate from the others, and he did not propose a whole solar system, just pieces.
  - Callipus added spheres for some of the planets in order to slightly tune some of the motions to better match observation.
  - He apparently created one of the first published star catalogs, memorialized in the poem by Aratus, *Phaenomena*.
- Aristotle (-384 to -322):
  - He adopted Eudoxus and Callipus' approach in order to model all of the planets by piecing together the Eudoxian sets of spheres, one inside of the other from Saturn to the Moon.
  - Since each is tied to the one beneath, Aristotle felt that additional spheres were needed in order to isolate the motions of the planets from one another. These were the rewinding spheres.
  - He insisted that the volume outside of the orbit of the Moon was made of a different element from the four elements that operated within. That fifth element, the aether, filled the remaining volume to the outer stars, providing the material of the heavenly bodies. Natural motion in the aether is perfectly circular.
  - He originated the idea that the universe was "full" of the aether—-no gaps or emptiness. This demand became necessary in all future Greek cosmologies.
  - Aristotle's physics guided (or handcuffed) speculation about any motion that the Earth might have had. The Earth had to be in the center of the universe, not spinning, nor orbiting any point.

  - He was very critical of the Pythagorean idea of an orbiting Earth for (his) physics reasons, but also because there was no apparent parallax which meant that the stars were so far away as to hide parallax (too far for anyone's taste) or that the Earth was stationary.

Modeling of this sort stopped after Aristotle as there were problems with any model in which the planets orbit in perfect circles with their common center on the Earth:

- The seasons would all have the same durations, but everyone knew that was not the case.
- The brightness of the planets would not change, but everyone knew that was not the case.
- The ordering of the planets was arbitrary.

3.6 Aristotle's Cosmology Project

In the Prologue I identified the components of a Project, and Aristotle's Cosmology is where I choose to begin to lay those out. Recall that I proposed that every Project commits to the following categories:

1. Numbers (prior measurements or numerical facts),
2. Theories (concepts, accepted views),
3. Techniques (best practice mathematical or experimental practices),
4. Norms (community expectations), and
5. Curiosity (a puzzle to be solved...the goals of the Project).
6. Influences and Products

At the end of a Project, some of these might change, some might be abandoned, and new ones might be added. Table 3.2 lays out Aristotle's Cosmology Project.

Aristotle's Cosmology Project	
1. Numbers project inputs	**Numbers project outputs**
1. there are seven planets 2. there are 33 spheres in the universe	1. no change 2. there are 55 spheres in the universe 3. there are as many movers as planets plus one
2. Theoretical project inputs	**Theoretical project outputs**
1. his physics of circular motions beyond the Moon 2. his physics of a stationary Earth 3. motion in the heavens is circular. 4. Earth is spherical 5. heavenly objects are spherical 6. heavenly objects are unblemished 7. universe is eternal, no creation 8. universe is finite in volume 9. heavenly objects are made of aether 10. Eudoxus' sphere tool-kit for each planet 11. the planetary order is Plato's	1. no change 2. no change 3. no change 4. no change 5. no change 6. no change 7. no change 8. no change 9. no change 10. no change 11. no change 12. Mode;ling should be of a complete system of all heavenly objeccts 13. Modeling should be a real representation 14. Spheres will interact with one another and so that must be neutralized with additional "unwinding" spheres 15. The spheres' motions require "prime movers" with one who sits outside of the planets
3. Technique project inputs	**Technique project outputs**
1. geometry 2. self-consistency with his whole philosophy	1. no change 2. no change
4. Norms project inputs	**Norms project outputs**
1. quantitative observation is not expected	1. no change
5. Curiosity project puzzle	**Curiosity project conclusion**
1. How would a full system of seven planets and the outer celestial sphere be constructed?	1. A complete Universe could be modeled.
6. Project influences	**Project products**
1. Plato's teaching, Eudoxus and Callipus' geometry	1. His books: *On the Heavens*, *Physics*, and *Meteorologies*

Table 3.2: Aristotle's Cosmology Project.

3.7 Greek Astronomy, Today

3.7.1 Let's Set The Record Straight: How we now understand the sky

From our more advanced vantage point, every one of the puzzles mentioned on page 106 in Section 3.2.1 was slowly explained in the 16th, 17th, and 18th centuries which will correspond to our Chapters **??**, **??**, **??**, and **??**. We understand MOTION BY THE EARTH and MOTION IN THE HEAVENS and some of these details you learned in school: the planets all orbit around the Sun (which is not the center of the universe) and the eight planets (including Earth but not including Pluto) orbit the Sun in nearly circular paths. Earth has an orbiting moon, as do many of the other planets, as we see in Table 3.4; some have many dozens.

That broad picture is usually attributed to Copernicus, but I'll show you in Chapter **??** that it's not quite so simple. But nonetheless, it's close enough to serve as a worthy mental image, and Figure 3.18 (a) presents that picture known to all schoolchildren. In (b), an on-edge view of the planets shows that they all orbit in approximately the same plane where we take Earth's orbital plane to define the ecliptic (0°). Mercury's orbit is the most inclined at $\pm 7^\circ$ so that defines the breadth of the ecliptic containing all of the other planets: a 14° band.[14]

[14] For those of you mourning the elimination of Pluto from the planetary family, its inclination to the ecliptic is more like $\pm 17^\circ$, as are other dwarf planets in the outer edges of the solar system. The undisputed opinion now is that Pluto's existence is due to some event that is not of the same origin of the other planets. Hence, it's being voted off of the planetary island. When asteroids were first discovered, they were thought to have been planets. So early 19th century books listed 11 planets!

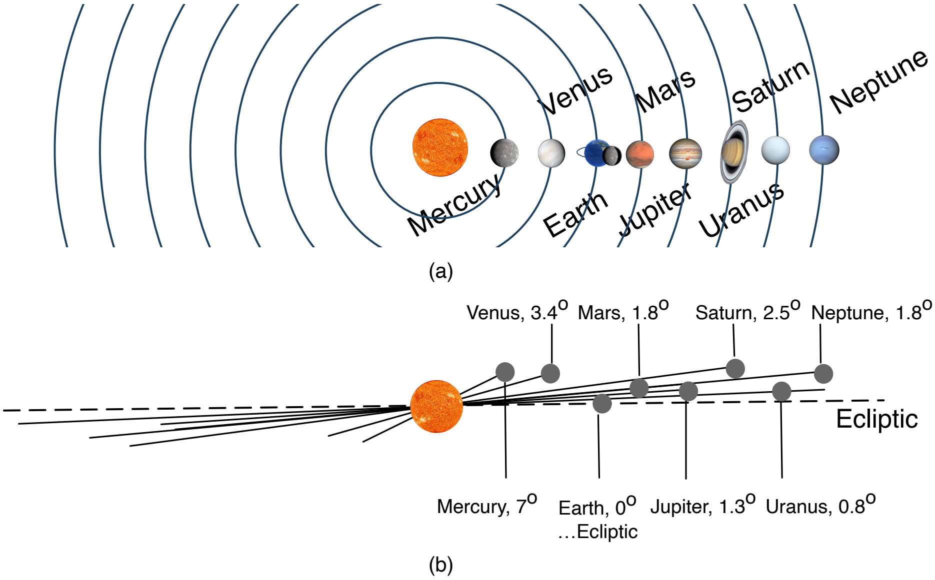


Figure 3.18: (a) is a sketch of the solar system as we picture it today and which we credit to Copernicus. For display purposes, the actual relative radii of the orbits are not anything like those shown, and the orbits are elliptical, not circular. (b) shows what the relative orbital planes are for each planet, inclined slightly to the overall ecliptic (the dashed horizontal line is the edge of the ecliptic plane).

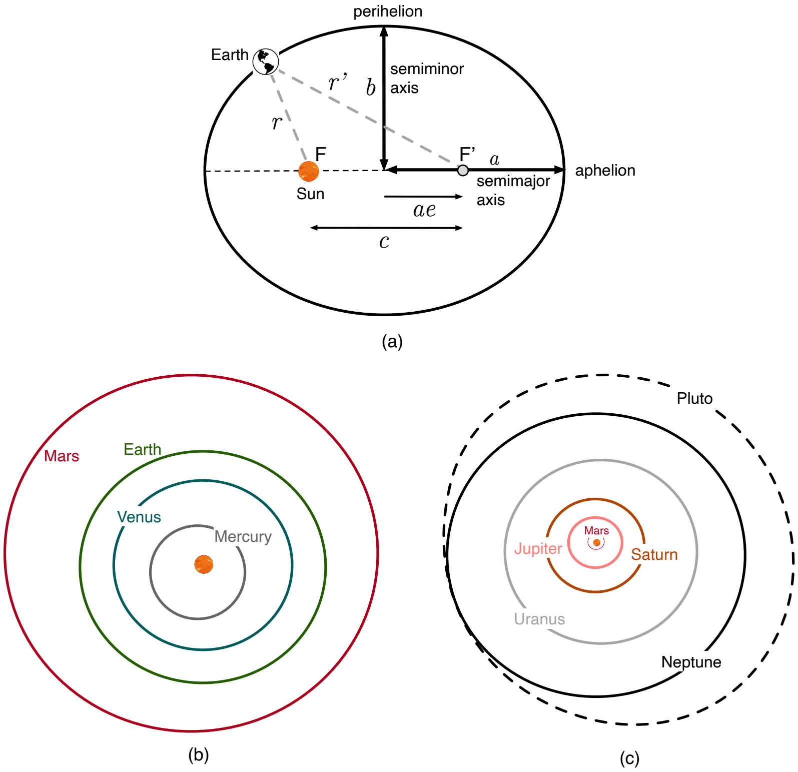


Figure 3.19: (a) shows the basic construction of an ellipse. (b) is a scale drawing of the first four planets where their elliptical shapes can be clearly seen, and (c) extends that view to the outer planets.

Elliptical orbits. The infatuation with heavenly circles persisted beyond Copernicus and Galileo, and I'll show you that it painfully goes away in the work of Johannes Kepler in 1609, every physicist's scientific hero (Chapter **??**). He figured out that planetary orbits aren't circular, but that they are in the shape of an **ellipse**.

> Ellipses are among a set of two-dimensional figures called "conic sections," so named because by cutting a three-dimensional cone with planes at various angles, the intersections create the shapes of circles, ellipses, parabolas, and hyperbolas. I'll introduce you to the Greek who made the most progress on this subject in the next chapter. Figure 3.19 (a) describes the basic configuration of an ellipse. There are two axes, major (the long one, length, a) and minor (length, b), and two special points called foci, which are offset from the geometrical center. The primary relationship of an ellipse relates the r and r' lengths as: $r + r' = 2a$. Notice that a circle is then just a special case of a general ellipse in which $r = r'$ and the two foci are collapsed together at the geometrical center. How non-circular an ellipse is can be characterized by its "**eccentricity**," e, which is the fraction of the major axis that the foci are displaced from the center.

The Sun is positioned at one of the foci of each orbit and nothing happens at the other. Isaac Newton explained how that worked in our Chapter **??**. The planet's orbits are not very elliptical but sufficiently so to have frustrated any attempt to describe orbits as circles from −200 through 1600 CE. Cue Kepler. In Tables 3.3 and 3.4 we can see that Venus has the most circular orbit, with an eccentricity of only 0.007, while Mercury has the largest eccentricity of 0.206, 20%.[15] Mars will figure into our story as it's easily visible and has a significant enough eccentricity of about 10%, to be measurable. Figure 3.19 (b) and (c) show the shapes of the orbits to scale where you can see the relative eccentricities. Beginning to characterize the orbits by means of points not at the center of orbits will begin to emerge as a technique in the next chapter where astronomers from the Hellenistic Greeks through Copernicus built models that desperately tried to preserve their circular bias by introducing many different offsets as centers of motion—cheating in effect, in order to retain circles. They tried very hard to make circles do the work of ellipses. And couldn't succeed.

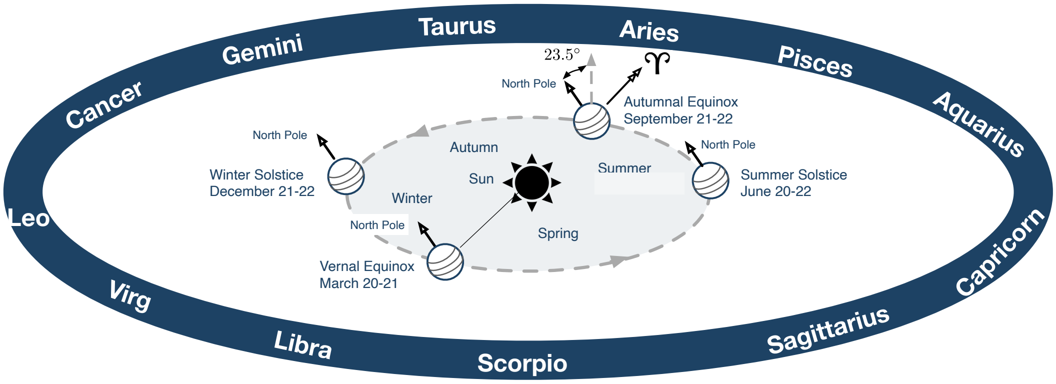


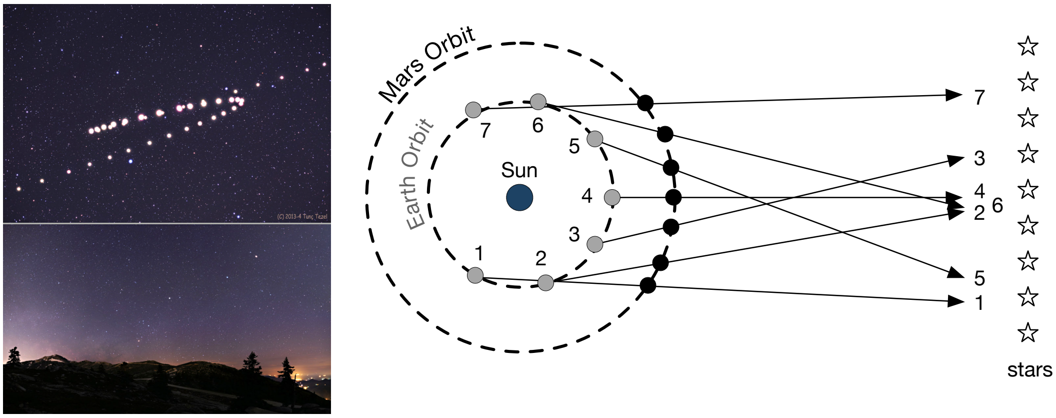
Figure 3.20: There's a lot in this image. The Sun (☉) is at the center, and the ecliptic is shown as the gray circle around which the Earth orbits. The 23.5° inclination is pictured showing how the solstices are inclined in our northern hemisphere's summer and winter. The Vernal Equinox (♈) is pointing at the zodiacal constellation of Aries, as it was in ancient times (today, it's in Pisces).

The "punchline" image above in Figure 3.9 shows that the Earth is tilted by that seemingly random 23.5° that figured so prominently in the stories above and in Figure 3.20 the Earth is shown at the four seasonal points of the two equinoxes and the two solstices. The dark band includes the ecliptic and is the plane with all of the planets, including Earth. The ancients ascribed special significance to the constellations that appear in that band, the zodiac, and they served as a rough coordinate system against which risings and settings, planetary motions, and the

[15]Pluto's is larger, but again, there is much about Pluto's orbital parameters that lead to the reasoning that it's not a regular planet in our solar system. Fun fact: From this writing in 2024, the last time Pluto had made a complete revolution was 1776, a revolutionary year. Another fun fact: Because of their eccentricities, sometimes Neptune's distance from the Sun is further than Pluto's, which was the case from 1979 to 1999.

Moon and Sun's positions could be located.

The Earth is titled by that 23.5° as measured from the plane of the ecliptic, and its direction does not move throughout the year and points to the Celestial Pole. The Vernal Equinox is shown when the Sun is within the Aries constellation (as in antiquity...now it's in Pisces). The "Age of Aquarius" is next!.



(a) (b)

Figure 3.21: Retrograde motion by Mars. In (a), the sky in Turkey shows a photograph of Mars from December 5, 2013, in the upper right-hand corner and then an overlayed photograph is taken every five or six nights until August 8, 2014. The looping behavior in the middle is the retrograde motion. (b) shows how this happens (see the text for an explanation) https://twanight.org/gallery/tracing-the-red-planet/?preview=true

Now we can understand both causes of the seasons and why they are of different durations, and Figure 3.20 tells the whole story. When the Earth's orbit is closest to the Sun, it's moving the fastest in its elliptical orbit, so it spends less time between the two equinoxes, here on the left side of its orbit. Notice that the tilt of the Earth's axis is away from the Sun, and so the full force of the Sun's rays are directed not to the northern hemisphere but the southern. In fact, at the Tropic of Capricorn at a latitude of 23.5° South (slicing Australia in almost northern and southern halves), the Sun would be overhead at the winter solstice. So less radiation intensity falling on the northern hemisphere means it's cooler. So yes, winter happens when the Earth is nearest to the Sun. On the other side, at the summer solstice, the Sun's rays are intense on the northern hemisphere as the Earth's tilt is now towards it, and the Sun is overhead at noon on the summer solstice at the latitude of the Tropic of Cancer—where the city of **Syene** in the Aswan in Egypt is located at 23.5° North and will play a role in the next chapter.

Earth and the Moon The Earth has at least two motions, as do all of the planets. It orbits the Sun in a nearly circular path in a counterclockwise sense when viewed from above the Sun's north pole. The Earth also spins on its own axis, also in a counterclockwise sense.[16] That the Earth spins on its axis explains the apparent motion of the Sun through our sky from E-W each day. The speed of the surface of the Earth at the equator is due to its spinning is about 460 m/s (about 1000 mph), while the speed of the Earth's track along its orbit is 220 km/s (about 490,000 mph). We don't feel these motions since they are constant, the Earth is large, the atmosphere moves with us, and we're held to the surface by the Earth's gravity.

Figure 3.22 shows that the Moon's orbit is inclined to the ecliptic by about 5°, which is why we don't see lunar and solar eclipses every month. (Hipparchus determined this angle.) Finally, Earth has a third motion that was very confusing to the Greeks, who began to compare contemporary data with that of astronomers of previous centuries. The location of the Vernal Equinox appeared to have moved: that Aries-to-Pisces movement that I mentioned above. This was very confusing and while it was possible to estimate how much the shift happens (about a degree per century), there was no understanding of what caused it. It took Isaac Newton to figure that out. The spinning of the Earth's motion around its pole actually precesses like a top relative to the ecliptic: sometimes that axis points there, and centuries later, it will point somewhere else. It takes 26,000 years for that precessional axis to make it all the way around. Currently, it points toward the North Star, Polaris. In about 12,000 years, it will point towards the star Vega.

Retrograde motion. The strange retrograde motion is easily explained in the heliocentric system. Earth and Mars, for example, have different "years" as they go around the Sun. Sometimes, the Earth will lap Mars and leave it behind. That's the story and Figure 3.21 explains it. In (a), we see a time-lapse photograph of Mars in successive nights from December to August. Clearly, Mars appears to "move" against the stars. (b) shows how.

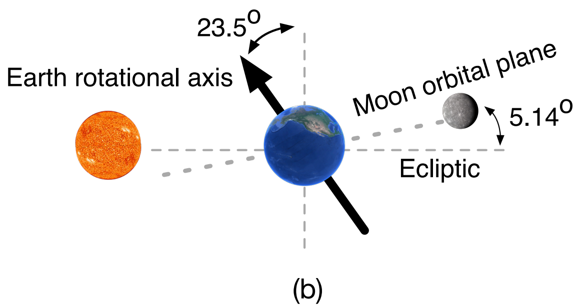


Figure 3.22: The inclination of the Earth's spinning is oriented away from being perpendicular to the ecliptic in which the Earth's orbit is fixed. Also, the orbital plane of the Moon's orbit around the Earth is slightly inclined relative to the ecliptic as well.

Tables 3.3 and 3.4 show the most important orbital parameters for the planets plus the Moon and Pluto. I've already pointed out the eccentricities, and I'll refer to other parameters in later chapters.

[16] only Venus among the planets spins in a clockwise sense while Uranus has a spin axis which is on its side, relative to the others. One explanation is that, like the Moon was created some billions of years ago in a collision with the Earth, something massive might have struck the adolescent Venus and Uranus. Multiple hypotheses exist.

Table 3.3: Add caption

	MERCURY	VENUS	EARTH	MOON
Mass (1024kg)	0.33	4.87	5.97	0.073
Diameter (km)	4879	12104	12756	3475
Gravity (m/s2)	3.7	8.9	9.8	1.6
Rotation Period (hours)	1407.6	-5832.5	23.9	655.7
Length of Day (hours)	4222.6	2802	24	708.7
Distance from Sun (106 km)	57.9	108.2	149.6	0.384*
Perihelion (106 km)	46	107.5	147.1	0.363*
Aphelion (106 km)	69.8	108.9	152.1	0.406*
Orbital Period (days)	88	224.7	365.2	27.3*
Orbital Velocity (km/s)	47.4	35	29.8	1.0*
Orbital Inclination (degrees)	7	3.4	0	5.1
Orbital Eccentricity	0.206	0.007	0.017	0.055
Mean Temperature (C)	167	464	15	-20
Number of Moons	0	0	1	0
Ring System?	No	No	No	No

Table 3.4: Add caption

	MARS	JUPITER	SATURN	URANUS	NEPTUNE	PLUTO
Mass (1024kg)	0.642	1898	568	86.8	102	0.013
Diameter (km)	6792	142984	120536	51118	49528	2376
Gravity (m/s2)	3.7	23.1	9	8.7	11	0.7
Rotation Period (hours)	24.6	9.9	10.7	-17.2	16.1	-153.3
Length of Day (hours)	24.7	9.9	10.7	17.2	16.1	153.3
Distance from Sun (106 km)	228	778.5	1432	2867	4515	5906.4
Perihelion (106 km)	206.7	740.6	1357.6	2732.7	4471.1	4436.8
Aphelion (106 km)	249.3	816.4	1506.5	3001.4	4558.9	7375.9
Orbital Period (days)	687	4331	10747	30589	59800	90560
Orbital Velocity (km/s)	24.1	13.1	9.7	6.8	5.4	4.7
Orbital Inclination (degrees)	1.8	1.3	2.5	0.8	1.8	17.2
Orbital Eccentricity	0.094	0.049	0.052	0.047	0.01	0.244
Mean Temperature (C)	-65	-110	-140	-195	-200	-225
Number of Moons	2	95	146	28	16	5
Ring System?	No	Yes	Yes	Yes	Yes	No

Chapter 4

Greek Astronomy Becomes Scientific : Ptolemy and Hellenistic Astronomy

"We shall try to note down everything which we think we have discovered up to the present time; we shall do this as concisely as possible and in a manner which can be followed by those who have already made some progress in the field. For the sake of completeness in our treatment we shall set out everything useful for the theory of the heavens in the proper order, but to avoid undue length we shall merely recount what has been adequately established by the ancients. However, those topics which have not been dealt with [by our predecessors] at all, or not as usefully as they might have been, will be discussed at length, to the best of our ability."

- Ptolemy, *Almagest*, *Book I, 1*

The passage above is the opening stanza of the last verse of Greek astronomy and is at the threshold of a strange 1500-year dance between the rigorously mathematical (Ptolemy) and achingly abstract (Aristotle) models of the universe. How we got there is the purpose of this chapter as it lays the groundwork for two millennia of mutually supportive and mutually conflicting views of MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS .

I took some pains in the last chapter to underscore that models of MOTION ON THE EARTH belong in Aristotle's corner as he really invented the dynamics of motion. But while we tend to ascribe that geocentric model of the universe to him as well, he borrowed it lock stock and barrel from Eudoxus and Plato.

This "geocentric" picture became the authoritative, unquestioned dogma of the medieval and renaissance periods even though it made no numerical predictions and was known since Aristotle's time to be just wrong. The other game in town was precise and predictive and was the model of the Greek astronomer Claudius Ptolemy from the first century CE.

Alexander the Great radically and violently altered the Greek world—indeed, the whole Mediterranean world—and between Aristotle and Cleopatra's reign, astronomy became an experimental and quantitive science. The culmination of astronomy came after Greek–everything became Roman–everything and just before the Roman Empire began its decline. One last Greek, in our long string of Greek philosophers, mathematicians, and scientists remained and we'll close our chapter with Ptolemy's "turn-the-crank" model for MOTION IN THE HEAVENS.

A game that many scientists play is to trace their scientific lineage back for centuries—their major professor's professor and so on (there's an app for that). I followed mine back through centuries and found that I descended from Copernicus![1] I'd like to think I've made him proud.

Sometimes it turns out that someone's student ends up in the history books. But not many students actually take over the known world by force!

When Plato died, the Macedonian King Philip II "encouraged" Aristotle to relocate to Macedonia in order to teach his 13-year-old son, Alexander. He set up a school, taught Alexander (and perhaps the future general/king/Pharaoh, Ptolemy I Soter[2]) for three years, and then stayed for seven more before returning to Athens, where he started his school, the Lyceum. By this time, the teenage Alexander was already on the battlefield and, with his father, had occupied the entirety of the Peloponnese and Attica. So Athens was once again ruled by outsiders—now connected to Aristotle!

After Philip II was assassinated,[3] and Alexander, soon to be "The Great," ascended to the throne and began his brutal, lightening-fast, nine-year conquest of the entire western world: modern Turkey, the middle east, Egypt, Arabia, and all the way across Afghanistan to India, leaving military oversight over Athens and the rest of Greece. While he stayed in touch with Aristotle, sending him botanical, zoological, and geological samples from all over Asia, his teacher became distant, put off by Alexander's adaptation of Persian customs, dress, and persona.

Alexander died in Babylon in −323 under suspicious circumstances, and within a

[1]Everyone I know seems to come from Copernicus. A mark that what he started had legs?
[2]Not to be confused with Ptolemy the astronomer!
[3]Assassination, murder, and betrayal were all family hobbies.

year, Aristotle himself died at the age of 63 at his mother's family estate outside of Athens. His Macedonian connections had become dangerous, and his adopted city turned on him: impiety was charged, and a death sentence issued. So he fled to his mother's home, uttering his famous remark (invoking Socrates' fate) about the city not sinning against philosophy for a second time. In his absence, the Lyceum stayed active under new management for another century.

Alexander's senior commanders divided up the sprawling kingdom among a dozen generals and aides and they did what came naturally: they fought among themselves for 40 years. In the end, three kingdoms and a dizzying array of city-states were established: the survivors were Macedonia and Greece, Seleucia (roughly modern-day Iraq), and Egypt.

Hundreds of thousands of Greeks migrated into the newly acquired territories, establishing an international Greek-ness of culture, arts, and philosophy which was the beginning of the **Hellenistic Age**.[4] The entire western world became "Greek." Of the two dozen cities that Alexander created or conquered named for himself, the "Alexandria" that mattered most to him, and to us, was the new Egyptian port city of **Alexandria**.

Egypt became unusually secure under Alexander's former bodyguard and general (and rumored Aristotle student), **Ptolemy I Soter (–367 to –282)** who eventually fashioned himself, "Pharaoh." He adopted Egyptian customs,[5] and was an intellectual of sorts, creating the first state-supported national laboratory and library. The "**Alexandrian Museum**" was a national facility devoted to research and for centuries and was home to scores of recruited Greek scholars, all supported by the dozen Ptolemys from the Ist to the final one, Cleopatra.

The Library of Alexandria probably contained all the manuscripts of classical and Hellenic philosophers, poets, playwrights, and physicians. There was a hunger for knowledge of all sorts, and agents of Ptolemy's library director searched every ship that docked, stealing or copying any books on board and renting or stealing manuscripts from all of the major cities.

Among the scores of Alexandrian scientists are the astronomers Eratosthenes of Cyrene, Aristarchus of Samos, and especially Claudius Ptolemaeus, who will figure into our story, while only Heraclides of Athens, Hipparchus of Nicaea, and Apollonius of Perga played major roles outside of Alexandria. The Greek Ptolemy dynasty lasted 300 years until the legendary feud involving "the" Cleopatra (a common name for female Ptolemy-family successors), Marc Antony, and Julius Caesar. The Library and Museum lasted into the first five centuries CE until the Muslim conquests of the Near East, north Africa, and Spain, when it was eclipsed by great Muslim libraries in Baghdad, Cairo, and Cordoba in Spain.

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[4] Often the pre-Alexandrian Greek era is called "**Hellenic**."

[5] including that of rulers marrying their siblings

4.1 A Little Bit of Hellenistic Astronomy

Euclid •Aristarchus •Eratosthenes •Archimedes •Apollonius •Hipparchus •Ptolemy

(Set the context with the timeline in Figure 1.2 on page 22.)

After Plato, Eudoxus, Callippus, and Aristotle's fanciful modeling, there were two basic thrusts. Hellenistic astronomy became both observationally and mathematically sophisticated, culminating with Claudius Ptolemy's enduring model in the second century CE. Let's unwrap this extraordinary period of Alexandrian astronomy and set the stage for 1500 years of surprisingly authoritarian science.

4.1.1 A Moving Earth

Heraclides of Pontus (–387 to –312), from the southern coast of the Black Sea, was a contemporary of Plato and Aristotle and as the son of a wealthy family and an apparently smart young man, he was able to emigrate to Athens, where he became a favorite student of Plato and was put in charge of the Academy when Plato went on his last, ill-fated trip to Syracuse. He also studied with Aristotle (who was 10 years his senior) and the Pythagoreans in Athens, so he was fully rounded in the three major pillars of classical Greek philosophy. Plato died in −348, and his successor, Speusippus, died in −339, and when Heraclides lost the election for the next leader, he returned north to Pontus. That's where he probably did his astronomy and where he had two good ideas, neither of which went anywhere for 2000 years.

It should have bothered Aristotle that his model required the outside starry sphere to rotate at an astonishing rate to make it all the way around each day. The obvious alternative was a spinning Earth and stationary stars, and Heraclides proposed just that.

His other imaginative idea addressed a second interesting fact: Mercury and Venus have a different relationship to the Sun from all of the other heavenly bodies. They seem to cling to it, appearing and disappearing as the Sun rises and sets. It was Heraclides who first suggested that this special relationship could be explained by making those two inner plants satellites of the Sun. His cosmology was that the Earth is at the center of the universe, spinning on its axis, orbited by the Sun as "normal," but the Sun, in turn, was itself a second center of rotation with Mercury and Venus orbiting it. So Aristotle's grip wasn't overwhelming, even in his own time and we'll see this idea repeat in the early Middle Ages and later in Copernicus' writings.

4.1.1.1 The Greek Copernicus

While Heraclides could be thought of as ushering in the post-Athens era, it was **Aristarchus of Samos (–210 to –230)**, a toddler when Heraclides died, who conceived a completely new way to deal with the cosmos: by measuring it. He studied with Strato of Lampsacus, who was the third director of Aristotle's Lyceum, and when Strato went to Alexandria to tutor and counsel Ptolemy II, he brought Aristarchus along as his pupil. Strato returned to Athens, but Aristarchus stayed in Alexandria and did his mathematics and astronomy in that growing Greek-Egyptian intellectual center. He probably overlapped with the senior Euclid and surely learned all of Greek mathematics known to that time, conceivably from its most famous chronicler. He fashioned his single surviving text *On the Sizes and Distances of the Sun and the Moon* like Euclid's *Elements*: propositions followed by orderly proofs.

As the Moon orbits the Earth half of it is always illuminated, but as shown in Figure 4.1 (a) we see different fractions of its illumination—the phases—as it makes its way around us. When it's on the other side of the Earth from the Sun, and we're in the nighttime, we see it fully illuminated ("full Moon") by the Sun. When the Moon is between us and the Sun ("new Moon") the side that's illuminated is toward the Sun, so it's invisible during the day. And of course during that new Moon phase, our nighttime sky is Moon-less (a good night for telescopes). But just before sunrise or just after sunset, a bright sliver reflecting from the Sun can be seen, along with a dimmer image of the rest of the Moon's shape. That's due to reflection of sunlight from the Earth ("earthshine"). But Aristarchus realized that the two-quarter Moon phases are special because at exactly that point, we see the Moon illluminated into two equal halves, one dark: a unique geometrical arrangement.

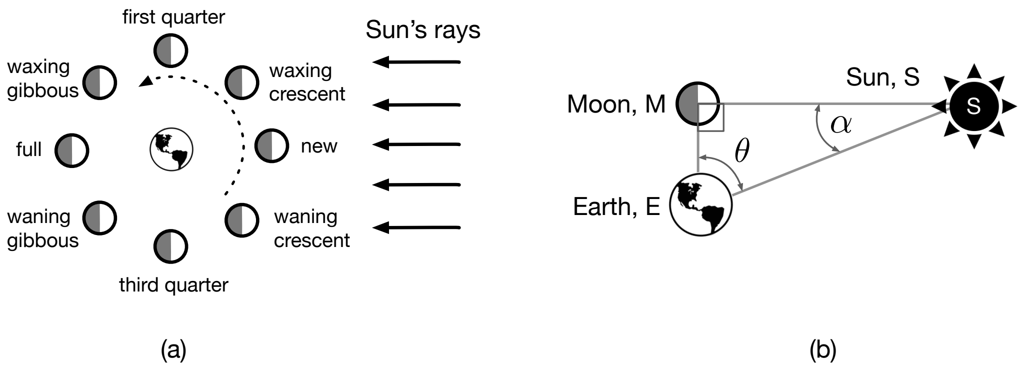


Figure 4.1: The Moon's phases and positions are shown in (a) relative to the Earth and Sun. From this vantage point, the Moon orbits counterclockwise. In (b) the particular position and phase that makes the Aristarchus calculation possible with the right angle shown occurring at just the first or third quarter when the Moon is half lit.

While Aristarchus didn't anticipate the Moon orbiting the Earth, he did realize that this quarter phase had a particular geometric arrangement with respect to the Sun and Figure 4.1 (b) shows his idea. At that moment, the angle between the Sun and the Earth is a right angle, $\angle EMS = 90^\circ$.

> "...when the Moon appears to us halved, the great circle which divides the dark and the bright portions of the Moon is in the direction of our eye...when the Moon appears to us halved, its distance from the Sun is less than a quadrant

> by one-thirtieth of a quadrant." Aristarchus, *On the Sizes and Distances of the Sun and the Moon.*

By "distance from the Sun," he means angle α in Figure 4.1 (b), $\angle MSE$. With that angle, in one line of modern trigonometry you can calculate $\frac{ES}{EM}$, the ratio of the distance of the Earth to the Sun over the distance of the Earth to the Moon. Without modern trigonometry, it's still a straightforward exercise in geometry. Aristarchus did just that and found:

$$\frac{\text{Distance, Earth to Sun}}{\text{Distance, Earth to Moon}} = 19 - 20$$

where the range is his own estimate of how well he could determine the angle. Technical Appendix D.1 completes this calculation and some other interesting measurements that he and others made. Their originality is stunning and beautifully simple. He also subsequently calculated three additional things about the universe, for a total of four groundbreaking conclusions (the symbol $\approx$ stands for "approximately equal to"):

1. the distance of the Earth to the Sun) $\approx 20\times$ distance of the Earth to the Moon
2. the diameter of the Sun $\approx 19\times$ the diameter of the Moon
3. the diameter of the Moon is $\approx 20/57\times$ the diameter of the Earth
4. the distance of the Earth to the Moon $\approx 10\times$ the diameter of the Earth

His mathematics and methods are correct but he had some mistakes, crucially because α is very hard to measure and so his determination of $\theta = 87^\circ$ was wrong...it's actually closer to 89.853° which makes the distance of the Earth to the Sun) $\approx 390\times$ distance of the Earth to the Moon.[6]

But that's not all. Let's let Aristarchus' Italian/Greek contemporary **Archimedes of Syracuse (–287 to –312)** take over from here:

> "Aristarchus has brought out a book consisting of certain hypotheses, wherein it appears, as a consequence of the assumptions made, that the universe is many times greater than the "universe" [expected]...**His hypotheses are that the fixed stars and the sun remain unmoved, that the earth revolves about the sun on the circumference of a circle, the sun lying in the middle of the orbit,** and that the sphere of fixed stars, situated about the same centre as the sun, is so great that the circle in which he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the centre of the sphere bears to its surface." (emphasis, mine) Archimedes, *The Sand-Reckoner*.

Aristarchus was apparently the first to envision a Sun-centered ("heliocentric") universe, and, oh, by the way, he also apparently adopted Heraclides' notion of a

[6]The point of First Quarter would be in the same part of the sky as the Sun, just before Sunset. Without modern tools, measuring that angle would essentially impossible, if not dangerous! James Evans, 1998 suggests that Aristarchus concocted the "one-thirtieth" as an extrapolation of the time that it takes for the Moon to reach the First Quarter as the largest angle that could come from a month of 30 days to orbit and one quarter of that for the phase. That's almost even more impressive reasoning.

spinning Earth. Copernicus-in-training. Nobody knows how he came to this conclusion...even though it solves many of the problems (planets' changing brightness, for example). His model was largely ignored, and the fact that Archimedes tossed that reference off so casually is indicative of what must have been an overwhelming concern for the parallax problem (which is a prejudice about the possible enormity of the universe) and Aristotle's authority when it came to terrestrial physics.

> In Book II we'll see that using no more mathematics than what Aristarchus knew, Copernicus probably determined the periods of the planets using the broad Aristarchus' plan of a Sun-centered solar system. It's astonishing to me that Aristarchus—and those who immediately followed him— could have anticipated our modern view by almost 2000 years!

But there it is: the first modern-sounding MOTION BY THE EARTH and MOTION IN THE HEAVENS .[7] Copernicus later took comfort in Aristarchus' idea.

▻ This is an auspicious moment! Aristarchus' body of work ushers in the beginning of quantitative astronomy. Understanding the cosmos now requires more than story-telling. It will now require making measurements.

Aristarchus' work was quickly taken up by his contemporary, **Eratosthenes** (−276 to −194), who became the Chief Librarian of the Alexandria Library just following Aristarchus' death. (He was also a geographer, mathematician, astronomer, and poet. The nickname given to him was Pentathlos, implying a Greek pentathlon athlete of many talents.) Remember the ancient Egyptian city of Syene near modern Aswan from page 129 in Chapter 3? It's located at the Tropic of Cancer at latitude and so directly overhead at the summer solstice. With his access to Library data, Eratosthenes learned that in Syene on that day at noon, the Sun's rays were known to go right into a vertical well without hitting the sides so a vertical stick would not cast a shadow.

Meanwhile, Alexandria is directly north of Syene at the same longitude and so Eratosthenes reasoned that the Sun is so far away that it's okay to presume that its rays were parallel at both cities. Therefore, for a spherical Earth, the shadow of the Sun on a vertical stick in Alexandria would cast a shadow—which he measured! It was 7.2° at Alexandria, which is 1/50th of the 360° of a circle so that the circumference of the Earth must be 50 times the distance between the two cities, which is 875 km (in modern units, and with uncertainties of at least 30% in calculating the conversion from the Greek measure of distance of "stades" to kilometers). Fifty times 875 km is about 43,000 km for Earth's circumference— only a few percent higher than a more modern value! Honestly, that's clever reasoning. Technical Appendix D.1 shows his calculation in modern terms.

Eratosthenes wasn't done. He also devised a way to measure the obliquity of the ecliptic—that angle 23.5° of inclination of the ecliptic from the Celestial Equator. He

[7] Of course, remember that Pythagoras' model was actually the first to require a moving Earth.

made a star catalog of 650 stars...and he wrote a poem about himself. He reportedly went blind in his old age and chose to commit suicide rather than live in darkness.

So, for the first time, astronomers learned the size of the Earth, and more could be learned: for example, using Aristarchus and Eratosthenes' results, from Aristarchus' #3 above, they could conclude that the diameter of the Moon is about 4700 km, where the actual value is about 3500 km.

▻ I hope you can appreciate that Greek astronomers are no longer merely telling stories. They're measuring our universe.

4.1.2 Casting Aside Aristotle and Eudoxus

The next important step is by another storyteller, but an important mathematician who had a clever idea. **Apollonius of Perga** (−240 to −190) migrated from Turkey to Alexandria as a young man to study in the successor school of Euclid. "The Great Geometer" became his historical label and he's remembered for discovering the mathematics of "conic sections" (circles, parabolas, ellipses, and hyperbolas)—a subject beyond Euclid's geometry.

For our story, we know of him as the geometer who puzzled over the seasons problem and found a way to modify the Eudoxian model to loosen the requirement of all spheres centered on the Earth. One of his discoveries is shown in Figure 4.2 (a) in which E shows the location of the Earth, S is the location of the orbiting Sun, and D is a point in space—attached to no object— which is displaced from E. The distance $\overline{EC} = e$ is called the **eccentricity**.[8] The Sun uniformly follows the dashed **eccentric circle**, centered on D and not the Earth! Notice that the result is a Sun's path, sometimes further from, and sometimes closer to, the Earth. When it's further, it would take longer to go halfway around and so the seasons during that path segment would be longer. This is poking at Aristotle: a model of solar motion which is uniform and circular but centered ...not on the Earth.

Epicycles But there's more to this as Apollonius discovered a geometric equivalence illustrated in Figure 4.2 (b). Here, a circle, called the **deferent**, is centered on the Earth but doesn't act as an orbital path for the Sun. Rather, the Sun rides on another circle, the clockwise rotating **epicycle** with its center (A) attached to the rim of the counterclockwise, rotating deferent. Notice that the rotational sense (here, clockwise) of the epicycle is opposite to that of the orbit of its center, A, on the deferent. If the parallelogram EDAS is maintained, then this second model would trace out the same path for the Sun as the first. So this too provides a solution to the problem of unequal seasonal durations. But again, it's a story, not a numerical model.

[8]Remember that the quantity "eccentricity" is a defining feature of ellipses as I introduced on page 127 in Chapter 3.

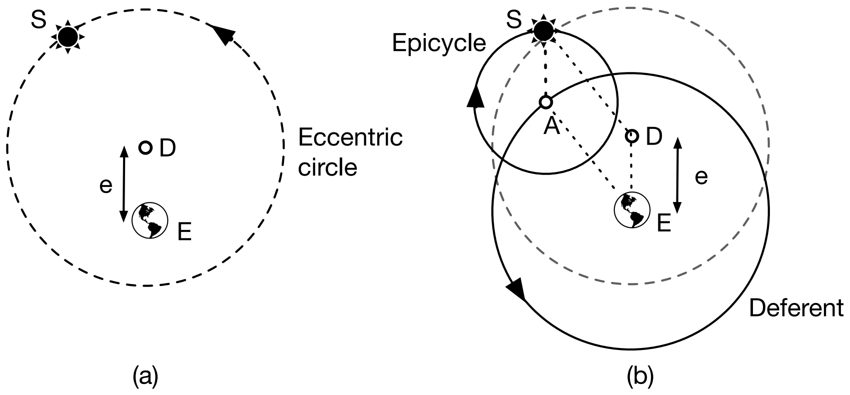


Figure 4.2: In both figures, E is the location of the Earth, and S is the location of the Sun. In (a) an eccentric circle is shown for a proposed Sun orbit around the Earth. By putting the center at a spot in space displaced from the Earth by the eccentric, *e*, the seasons would appear on Earth to be of different durations. In (b) the equivalent (under the conditions described in the text) epicycle solution is shown with an overlay of the eccentric circle shown in a light dashed line for comparison. The deferent is centered on the Earth, and the epicycle is centered on the rim of the deferent. The magnitude of *e* is grossly exaggerated.

The idea of an epicycle is not easy to grasp since we don't use them anymore in planetary astronomy. But if you look up some night, you'll see an example of an epicycle. Think modern (for a moment): we know that the Earth goes around the Sun and that the Moon goes around the Earth...and when looked at in a particular way, ours is an epicycular system. The Earth's (nearly) circular orbit around the Sun is the deferent and the Moon's orbit around the Earth is an epicycle. What's important is an observer's perspective.

Epicycles will become the most important motions for planets from Ptolemy—300 years after Apollonius—through Copernicus. In fact, we briefly noted on page 137 that Heraclides had a story model with Mercury and Venus orbiting the Sun, while the Sun orbits the Earth. Either of those planet orbits would appear to be epicycles from the Earth, with the Sun's orbit playing the role of the deferent. So epicycle shapes were "in the air" but not as a focus in and of themselves.

He found one more thing about an epicyclical model. If the rotational sense of the epicycle is the same as its center's rotation on the deferent, then the path of the object (now, not the Sun, but an arbitrary planet) would have a loop-the-loop path. So it would sometimes be close to the Earth, sometimes far away, and when it's close, it would appear to move backward against the stars. So, a possible solution to the problem of retrograde motion. Figure 4.3 shows an example. Look at each numerical position which successively take the planet (the closed, gray dot) around the Earth. The thin, black circle is the deferent, centered on the Earth. The tiny gray open dots centered on the deferent denote the center of the epicycle at successive times around its route with the light gray dot-planet following its course around the open-dot-epicycle center. The identical clockwise sense of both the epicycle and its motion around the deferent results in the looped trajectory shown as the dash-dot curve. You can follow the planet around its loop-the-loop path with the sequentially numbered positions, which are sequential times. Points 6-7-8 denote the retrograde period.[9]

Numerical predictions were not the goal for Apollonius, but suggestive framework was—and probably the geometry was also an attraction for him. So his ideas were one more step away from Aristotle toward a new way of doing science.

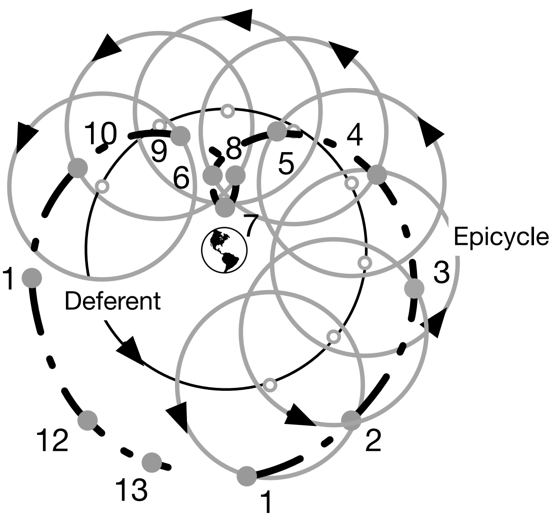


Figure 4.3: Apollonius' model for retrograde motion using epicycles. See the text for description of the path and the sequence.

4.1.3 The Greatest Astronomer: Hipparchus

The most celebrated astronomer of antiquity was, yet another Greek about whom we don't have many biographical details. However, **Hipparchus of Nicea** (about −190 to about −120) was so accomplished that his feats were detailed in later Hellenistic astronomy texts and most completely two centuries later by Ptolemy. His mature astronomy work appears to have been done on the island of Rhodes, a large island to the west of Cyprus and far from his home near Constantinople. There, he built an observatory and created or improved instruments for measuring the positions of stars and planets. He was a serious observer of astronomical objects and events and a mathematician of significance. Finally, the world was ready for a complete astronomer...The Greatest Astronomer, he was later called.

Let's be clear: **astronomy was different after Hipparchus**. He dedicated himself to an entirely different purpose from the "picture stories" of Plato and Aristotle. Hipparchus measured the numerical features of the cosmos.

Hipparchus' Solar Model. Hipparchus figured out that if he used the eccentric model, only a few measurable parameters were required in order to determine e, and so the problem of the seasons' unequal durations could be solved geometrically, almost like being a cosmic surveyor. His model is shown in Figure 4.4 with the

[9]Another proof that Apollonius created was to show what conditions between the angular speeds of epicycle and deferent and the different radii would identify the "stationary point," number 7 in the diagram.

anchor for astronomical positioning chosen to be the Vernal Equinox (VE, ♈). The Sun orbits the center of the eccentric orbit at C, and the Earth is displaced by the eccentricity, e (which is usually quoted as the fraction of the distance $\overline{CE}$ to the radius, $\overline{CA}$). The dash-dot lines denote the axis from the Vernal Equinox (mid-March), the Autumnal Equinox (AE, mid-September), the Summer Solstice (SS, mid-June), the Winter Solstice (WS, mid-December), and the four unequal quadrants delineate the four seasons. Here, it's drawn for antiquity in which spring was the longest season and autumn was the shortest (while in our time, summer is the longest and winter is the shortest). In astronomy, the furthest point of a celestial object's orbit from a reference is called the "**apogee**" and the closest approach, the "**perigee**." The figure shows the arrangement for antiquity when the angle of the dotted line through E and C was about $\alpha = 65^\circ$. Today, it's greater than 90° which is why our summers are longer than antiquity's summers.

His result was that the eccentric is displaced from the Earth by about 1/24th (about 0.04) of its orbital radius so it is almost a circle centered on Earth, which could explain why the seasons' durations are within a few days of one another.[10] (Of course, it doesn't explain this, but it was clearly suggestive as a model.) Notice that our summer and spring is when the Sun is at apogee and fall and winter are at perigee.[11]

Hipparchus could use his solar model to predict the location of the Sun at any time in the future. It was accurate and used for hundreds of years.

Hipparchus' Lunar Model. The Moon's motion is more complicated than the Sun's with at least three parameters required to determine its motion. He managed that as well, this time using an epicycle model. Finally, that legend ascribed to Thales from 400 years before is made whole: Hipparchus could predict both solar and lunar eclipses!

In addition to his modeling of the Moon's motion, he found a way to determine the distance from the Earth to the Moon. With his version of trigonometry (see below), he found that the distance from the Earth to the Moon is 65.5 times the radius of the Earth, and that's about right (it's about 60.336). (Newton used his result in his invention of his Law of Gravitation.) Hipparchus attempted the same thing for the distance to the Sun but underestimated it by a factor of 50.

Hipparchus' Fixed Star catalog. Hipparchus began the first quantitative survey of the fixed stars—the ones thought to be on the inside of the Celestial Sphere. Prior to him, locations of bright stars were noted by identifying their rough, relative positions in words: that a star in the "shoulder" of one in one constellation is rising when the star in the "sword" of another constellation is setting and that the star on the "right leg" of a third constellation appears right overhead when this happens.

[10] Had $e = 0$, then all four season would have been the same length and the Sun's orbit would have been Aristotle-like, centered on the Earth.

[11] Why the Sun is *furthest* away during the summer is a reasonable question and understanding that waited for Kepler and Newton.

More stories. Hipparchus took a different approach.

His data were extensive and would have required impressive patience (night after night) and commitment to a multi-year research project. Ptolemy tells us that Hipparchus cataloged around 850 stars, their positions, and their brightnesses, and they were in use for centuries afterward. Others had made catalogs (Eudoxus and Eratosthenes), but his was different: he invented a coordinate system and assigned positional numbers to each star. Think about how your GPS specifies a location on the Earth: my phone tells me that the location of the Library of Alexandria is 31.20870° N, 29.90911° E. What that tells me is that the library is a little more than 31° north of the equator (the **latitude**) and about 30° east of some point that's worldwide agreed to be the observatory at Greenwich, England (the **longitude**). Hipparchus adopted the same thing, but applied to the stars—the underside, if you will, of that Celestial Sphere above us. (More about this and how his system is essentially identical to modern astronomy is discussed in *Greek Astronomy, Today* in Section 4.4.1.

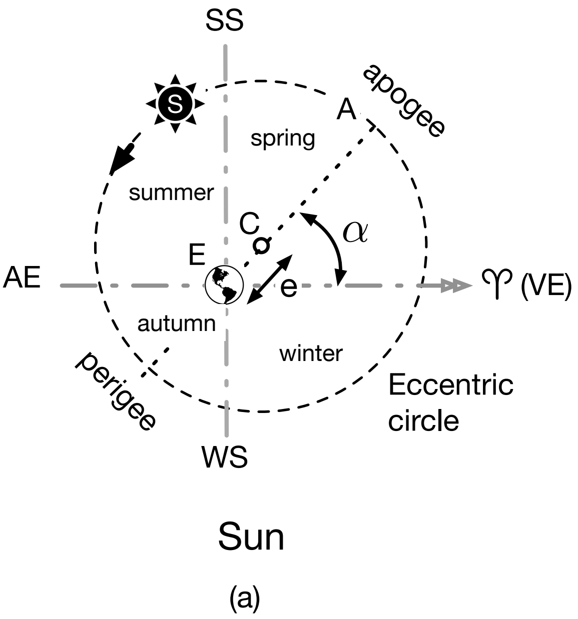


Figure 4.4: Hipparchus and Ptolemy's solar model showing the seasons in antiquity (today, winter is shorter and summer is longer). SS and WS are the Summer and Winter Solstices, VE (♈), AE are the Vernal and Autumnal Equinoxes, and the seasons are then defined as the four quadrants among them. The Earth (⊕) is displaced from the Sun (⊙) by the eccentricity, *e*, the distance in space from Earth to the center of the eccentric circle about which the Sun orbits. The dotted line is described in the text.

A many-decade detective story unfolded in trying to figure out which (if any) of Hipparchus' data were included in Ptolemy's more extensive star catalog. And there's a clue. Remember Aratus' poem, *Phaenomena* from Figure 3.1, which was written as an ode to Eudoxus? The one book we have of Hipparchus' is his *Commentary on the Phaenomena of Eudoxus and Aratus* in which he severely criticized mistakes of fact in the poem regarding the relative positions of stars in the constellations. He included a set of positions for 22 stars of his own observation, and these have been extensively compared with Ptolemy's catalog, and the agreement is pretty good. Without Hipparchus' grumpiness about a 200-year-old poem,[12] we wouldn't

[12]He wrote other ill-tempered reviews of other people's writings.

have any corroborating information that Hipparchus really did create the first-ever quantitative star catalog. Well, maybe until 2022! For that breaking story, look at *Greek Astronomy, Today* in Section 4.4.2.

Hipparchus' Trigonometry. The mathematical problems he had to solve for his solar and lunar models were surely the inspiration for a tool that marked the invention of trigonometry. Figure 4.5 shows his idea. A chord inside of a circle with radius R and center O is shown as the length $\overline{AB}$ where the chord subtends the angle θ. By hand, Hipparchus divided carefully drafted circles into degrees based on 360° (which came from the Babylonians), but much finer: 21,600 segments, which is the number of arc minutes in 360°. Then he painstakingly created "tables of chords" of varying lengths for each segment, giving him a fairly precise lookup table of angles, radii, and chords. Given a radius, and the length of a cord, an angle could be looked up in the table. Or visa versa. It's equivalent to a table of trigonometric sines since as in the figure if one divides the chord in two so that there are two right angles at point C, then the $\sin(\frac{\theta}{2}) = \frac{1}{2}\left(\frac{\overline{AB}}{R}\right)$.

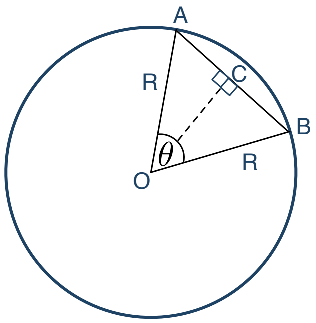


Figure 4.5: Showing how ancient "chords" related to a modern sin for a given angle θ.

Hipparchus' Discovery of the Precession of the Equinoxes

The discovery for which he's most known was that the Earth's seasons might shift over time. He found this in two, complimentary ways. Remember that we see arcs of two equators in the sky: the ecliptic, which is the lane in which the planets' orbits around the Sun all lie, and the celestial equator, which revolves around the axis through the north pole of the Earth and about which the stars revolve at night. What Hipparchus did was note that over centuries, the points of intersection of those two equators were not at the same place relative to the background of the stars. Here's how to think about this. Imagine drawing a big chalk circle on the ground, labeled like a clock, 1–12. Now imagine turning a beach umbrella the size of your clock upside down and spinning it like a top. The pole of the umbrella precesses as a top would, which means that sometimes it points to the sky, say towards that cloud over there and later the top of that tall tree over here. At the first of those two points the rim of the umbrella might point at 2 o'clock and at the second at 7 o'clock.

The point of intersection that he worked on was at the location of the Vernal Equinox, and in two very clever and different ways, he found that the VE pointed one direction comparing some star positional data from an Alexandrian astronomer, Timocharis, in −294 and −283, with those from his own time almost two centuries later. That intersection point moved at about 1° across the zodiac in 75 years, and so a repeat rate (he didn't calculate this) of every 27,000 years.[13] Ptolemy did a

[13] $75 \times 360 = 27,000$

similar experiment 265 years later and compared it with Hipparchus' and got about 1° per 100 years. Hipparchus' measurement is closer to the modern repeat value of 25,920 years! This phenomenon is called the **Precession of the Equinoxes** and had to be taken into account every time models were compared from the time of Hipparchus to that of Copernicus. The VE that pointed to the constellation Aries in ancient times now points to Pisces, and it's on its way to the "Age of Aquarius" as the next constellation over in the zodiac.

As I alluded to in Chapter 3 we know now that the precession of equinoxes has a physical cause: the Earth's axis of rotation (the umbrella pole) points at an angle that's not perpendicular to the plane of its orbit around the Sun (the chalk clock). So just like our chalk drawing is stationary and the umbrella rotates, for these purposes, the ecliptic is stationary and the Earth's axis rotates since It's tilted by close to that 23.5° from Figure 3.20. So it's like a top, the mass of the Earth causes it to precess around the Celestial Pole and Newton explained this.

4.1.4 Summary of the Astronomy of Aristarchus, Eratosthenes, Apollonius, and Hipparchus

(Set the context with the timeline in Figure 1.2 on page 22.)

- Aristarchus (-310 to -230):
  - He made the first attempts to use geometry to measure distances among and sizes of the Earth, Moon, and Sun.
  - He proposed the first model of a Sun-centered cosmology, apparently without geometrical modeling.
- Eratosthenes (-276 to -194):
  - He measured the diameter of the Earth to impressive accuracy.
  - He measured the obliquity of the ecliptic—that 23.5° tilt of the ecliptic from the celestial equator.
  - He apparently created a star catalog of more than 600 stars. This would have been, in other words, itemizing the apparent locations of stars relative to constellation points.
- Apollonius (-240 to -190):
  - He was a mathematician of the first rank and found a picture-way to model the Sun's motion around the Earth to create seasons of different lengths through the introduction of the deferent and eccentricity.
  - He also found a mathematically identical, but geometrically different form for planetary motion called epicycles. His proof of their equivalence was lauded as an important step by Ptolemy.
- Hipparchus (-190 to -120):
  - He built on Apollonius' deferent model and found a way to measure the actual eccentricity of the Sun's orbit and the longitude of the apogee. This was the first attempt to not only geometrically model the cosmos (or any physical mechanism) but also to quantitatively measure the shape parameters of the model.

- He found a way to determine the distance to the Moon in terms of Earth radii, a value used by Newton much later.
- His star catalog of more than 800 entries went beyond the stories that had been told previously: he invented a coordinate system that could be used by anyone to find the actual numerical positions of objects relative to an "origin" of essentially a celestial longitude and latitude.
- He discovered that the Earth's seasons shift relative to the star's positions over time—the precession of the equinoxes. Understanding the physical cause of this phenomenon waited for Newton's explanation of the precession of the Earth's axis of rotation...slowly: about 1° per 75 years.

4.2 The End of Greek Astronomy: Ptolemy

While Aristotle's concentric spheres model lay dormant for centuries, it was to rise again in the Middle Ages and take on a strange parallel existence next to a model that made precise predictions. This is the framework of the astronomer, geographer, and mathematician Claudius Ptolemaeus, known for nearly two millennia as **Ptolemy of Alexandria** (100 to 170 CE). He created the most complete model of the cosmos before Copernicus, and refreshingly, the content of his books survived almost intact thanks to Arab intellectuals' commitment to preserving and commenting on the works that they encountered from the Islamic conquest of the Near East, all of Northern Africa, and Spain.

Ptolemy wrote six books on astronomy (and additional books on astrology, music, optics, and cartography) for which we have mostly Arabic translations. *Mathematical Syntaxis* or *Synthaxis Mathematica* (*Μαθηματικγ Συνταξιϱ*) is his great work written in Hellenistic Greek but through translation, for 2000 years has been known by its Arabic title of *Almagest*, a corruption of the Arabic *Al* with the Greek word *megistē*, for "*the greatest.*" *Almagest* lays out the entirety of the **Ptolemaic System**, the longest-running scientific model in history. His second important astronomy test is the *Handy Tables*, which has two parts: the second part lists the tables of his planets and stars, and they've been preserved for us from medieval versions 200 years after Ptolemy. The first part is the instruction manual on how to use the tables, surviving only in Greek. *Almagest* is too complicated to have been absorbed by most, and so the *Handy Tables* assured widespread use of Ptolemy's work. *Planetary Hypotheses*, his third astronomy text came later (last?) and is an upgrade of the earlier *Almagest* and an attempt to build a plausible physical model of the purely mathematical *Almagest*. It was only appreciated and fully translated as two books in the 1960s!

Even though we finally have a nearly complete set of one of our astronomer's works, ironically, we know little about his life, except for a few self-references that bracket when he must have lived. Ptolemy certainly worked in Alexandria, as his extensive observations come from that latitude. He's the first of our Greeks to have two names! "Claudius" indicates that he was a Roman citizen, probably during the time of Emperors Hadrian to Marcus Aurelius. "Ptolemaeus" indicates that he was of Greek ancestry (although our "Ptolemy" is not from that original Alexandrian ruling family). For a scientific working life during about 130 CE, Alexandria would have been ideal. The intellectual culture was diverse, and the Museum would have been fully active, a magnet for intellectuals from throughout the Mediterranean, and it would have included a thousand years of astronomical results. Not tables as we think of, but the story-telling references to positions and events. Smith, 1996 points out that the earliest observation referenced by Ptolemy was Babylonian from −720.

His influence was wide and deep and his work in astronomy, geography, and of course astronomy was both a source of knowledge and a target if criticism. Even in that role, his work was influential since it stimulated new ideas and his optics was such an example.

Figure 4.6: Annonymous Portrait of Ptolemy from 1584. (https://www.britishmuseum.org/collection/image/1613222995)

4.2.1 Hellenistic Theories of LIGHT

4.2.1.1 Euclid

Please don't confuse the first-of-the-line General Ptolemy, Aristotle's student and the first ruler of Alexander's Egypt, with our astronomer Claudius Ptolemy, who lived 400 years later. The mathematician **Euclid of Alexandria (perhaps –325 to –265)** was among General Ptolemy's Museum's first recruits—a good move, since Euclid wrote ***Elements*** at the Museum, the most-read book in history after the Bible. For 2500 years, from the Romans to the Arabs, Copernicus, Thomas Jefferson, and to modern times, mastering *Elements* was the route to mathematical literacy."[14] *Elements* is a compendium of all of Greek mathematics with many

[14]General Ptolemy found it rough-going and asked for an easier way to learn it, but was told by the author that "...there is no Royal Road to geometry," a sentiment still applicable today.

proofs from him and most from others, but it was more than just an edited collection.

> Elements is comprised of 13 books, the first six of which deal with plane geometry— the backbone of his work. From 23 definitions (introducing the building blocks of points, lines, and planes), five general axioms (the necessary basics, such as “Things which are equal to the same thing are also equal to one another”), and five geometrical postulates (unproven assumptions, such as “Given two points there is one straight line that joins them.”), he derives 48 basic propositions that constitute the properties of triangles, rectangles, squares, parallelograms, and circles. Book five is on proportion (relying on Eudoxus’ work). The remaining books deal with number theory, geometrical progression, irrational numbers, and three-dimensional geometry. His famous “fifth postulate”[a] figures into Einstein’s work as “non-Euclidean geometries” were studied for the first time at the end of the 19th century and were necessary for his General Theory of Relativity. Book 2 shows how to use geometry to solve algebraic problems, many centuries before algebra was to be conceived. Technical Appendix D.3 shows one such solution.
>
> [a]The Fifth Postulate states that if you have a line and a point not on that line, that only one parallel line can be drawn through that point.

Geometry organized this tightly isn’t just a way to frustrate secondary school students; *Elements* introduced a powerful set of tools and a new approach to discovery. While much of this geometrical content was true then (he made some mistakes), and is true now, its most important consequence was *a new way of thinking*.[15] The world after *Elements* is one of shapes, real or ideal and we can learn about their features by deductively manipulating patterns and following rules. Euclid codified a process of analysis: define your terms and objects of analysis, state what’s true in axioms, lay out postulates using them, and deductively reach conclusions. Many mathematical and technical books were written in his style, including Copernicus and Newton. His was not just a Platonic tool for mental mathematics, Euclid applied his method in one practical direction: Optics seemed like a natural subject.

4.2.1.2 Euclid’s Optics

As we saw in Section 2, Greek theories of vision followed one of three mechanisms all presuming that seeing an object requires direct contact. (Maybe underscoring the problems that they all had with magnetic and electrostatic phenomena.) Such direct contact might go from (1) the eye to the object—some flux or “fire” emitted by the eye (Plato’s, “emission model”); (2) from object to the eye, (Democritus, the “intromissionist model”); or (3) some combination of a flux originating from the eye

[15]Did I say “comprehensive“? Not many scientists or mathematicians have sub-disciplines named after them (”Euclidean Geometry“), but nobody’s ”not-name“ (”non-Euclidean Geometry“) is a whole additional field!

meeting up with and combining with an object's emission (Aristotle). In each case, there is a physical connection between the observer and the observed.

Euclid envisioned cones and straight lines and the rules governing their shapes. He started with seven postulates and deductively derived his conclusions. He embraced the emission model that vision occurs because straight "rays" are emitted from the eye to the object which are confined to a cone whose apex originates in the eyes, and whose base encompasses the objects. The rays from the eye uniformly spread out with the cone's radius increasing the further away they travel. So he had in hand a model for optical focusing since objects further away would have fewer rays over their surface and be less well defined than an object close to the viewer since the cone would be smaller and the rays more plentiful and dense. He worked out the first ideas of perspective by imagining angles and likewise worked out the angles to portray the relative size of objects in relation to their distance. His geometry and analysis seemed to uniquely fit the problem of vision.

4.2.1.3 Ptolemy's Optics

Almost 500 years after Euclid, the prolific Ptolemy also wrote his *Optics*, one of his last books which we know of from a truly bad Arabic-Latin translation (the Arabic version is lost). He carried Euclid's geometrical approach further to include models of how rays reflect from mirrors and refract in liquids, relying on experiments that he performed himself.

If you hold a pencil perpendicular to a mirror with its eraser pointing at it, you'll see it appear *behind* the glass with that end appearing closest to you. The pencil is in your hand, but there's an image—we'd say a virtual image—of it as if it's behind the mirror. This is **reflection** a common experience for all of us.

Prior to Ptolemy, Hero of Alexandria (about 60 CE), an imaginative inventor and mathematician, concluded that the angle of incidence of light reflecting from a mirror would emerge at an equal "angle of reflection." He attributed that result to light having the property of following the shortest path between two points.[16] Three centuries before Hero, Archimedes may have had a practical appreciation for the optics of reflection and refraction. Ptolemy accepted that and used a mechanics of a ball bouncing from a wall as a metaphor. This idea must have stimulated his study of refraction.

Figure 4.7 shows a stick in a glass of water appearing to be disjointed and shortened at the boundary between the air in my kitchen and the water surface., You've maybe seen this with an oar in the water. This apparent bending is called **refraction** and it happens for all wave phenomena that go from one medium to another—including sound.[17]

[16] This is an idea of great importance and beauty and was formally inserted into mathematical physics by Pierre de Fermat in 1662.

[17] At night sound will appear to be more clear across a lake than during the day. There the "boundary" is not distinct, but a factor of the thermal gradient in the air above the water.

For a modern understanding of reflection and refraction, look at Figure Box 4.8 on page 152. After you've read the material in that Box, return to this point ☟ and continue reading.

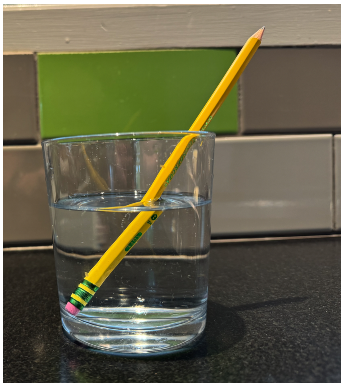
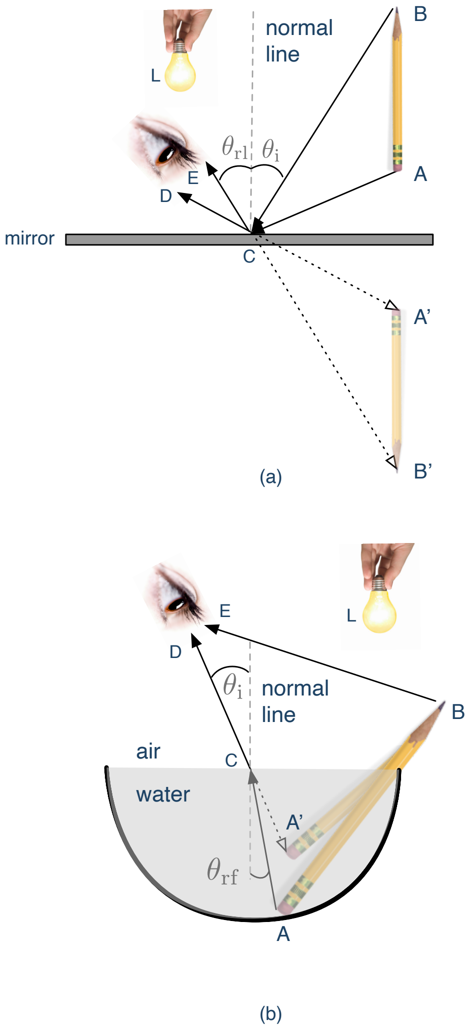


Figure 4.7: A photograph of a pencil bent in a glass of water.

In Figure 4.8 (b) an image is drawn of that pencil half-submerged in a bowl of water viewed side-on. Something happens to light rays at the air-water boundary with an explanation that we'll visit later in this series. But in practice, refraction is the change of direction of light rays at a boundary between two substances. It's standard to characterize this phenomenon by comparing the angles of the ray, before (θ_{i}) and after (θ_{rf}), relative to a perpendicular to the boundary (the **normal** line).

Many optical illusions are due to refraction, and as we'll see later, it is the principle behind a "**refracting telescope**" of the sort that Galileo adopted. Ptolemy must have been impressed with the regularity of the angles of deformation having a direct correlation with the angle of an object in water as his *Optics* described a bronze circular measuring device which he vertically half-submerged in water with a sighting tube. It was finely etched with angular ticks and he recorded tables of results of angles of incidence and their subsequent angles of refraction.

FIGURE BOX 4.8



Reflection of light from a mirror is illustrated in the top left figure. Here a pencil (endpoints, A and B) is illuminated by the bulb (L) and the light rays from the tip and the bottom hit the whole pencil and spread out in all directions and some hit the mirror at point C and again reflect with some of them going into that eerie detatched observer's eye, from A to D through B to E. The rules of reflection require that relative to a perpendicular from the mirror (the "normal line"), the **angle of incidence** (θ_i) is equal to the **angle of reflection** (θ_{rl}) as measured relative to the normal line. What do we see? Well, the ray from C to E, *appears to be on the other side of the mirror* at point A'. So we'd see the eraser closest to the mirror's plane. Likewise, the point of the pencil would appear to us to be further away behind the mirror at point B'. So the image of the pencil is not the actual pencil, but a "virtual image" on the "other side" of the mirror.

Refraction of light passing through a transparent medium is shown in the bottom figure. There are many examples of refraction (bending) of light between air and water, air and glass, and even in the atmosphere and with sound. When a wave passes from one medium to another, the speed of the wave changes and that affects the path of the wave. So thought of as rays of light, when one puts a stick, or here a pencil, in water and looks at it from the outside, it appears to be bent. In the figure a pencil (endpoints A and B) is half submerged in a bowl of water and is illuminated from a bulb, L. The light from the part of the pencil would reflect from A and some of it would reflect and be captured by the observer's eye at E. The other end of the pencil would also reflect light back out of the water and into the air passing from A to C, but then bending at the interface, towards D. The observer sees that ray of light as pointing back, not to A, but to A' so it appears to be both shortened and bent at C.

Now go back to page 151 and pick up where you left off.

Ptolemy also linked color and illumination —one can change into the other. Color might be activated in an object by an external source of light like the Sun, but color was a property of the object. Both illuminated light and color were then somehow sensed by the emitted rays of our visual apparatus, where an idea similar to Euclid's cones contained the rays that separately flow around the center of each eye's gaze. He forced those rays to converge in order to avoid dual vision. And, by virtue of how far they traveled, objects are deemed to be near or far (how that sense works is not explained). He delegated to the mind the job of then discerning position, size, and color. This mix of visual rays and illuminated flow from objects led him to a sophisticated theory that addressed optical illusions—which he attributed to the medium between the eye and the object. The medium was important to him, and he considered the effects of the atmosphere when measuring astronomical objects calculating corrections to the apparent positions of stars.

As we'll see, Ptolemy's astronomy denotes the peak and end of Greek astronomy. Although his writings on Optics are perhaps unreliable through many stages of translations, his optical and vision theories were highly influential. Like his astronomy, his optics was carried forward by Arab scholars who preserved and commented on it.

4.2.2 Ptolemy's Astronomy

Almagest is a huge subject. It is 700 pages long in a modern edition, and more than a thousand pages are required to fully lay out the considerable mathematics of the book (N. M. Swerdlow and O. Neugebauer, 1984). It's not for the faint of heart. It's also pure mathematics and little philosophy and *not a physical model*.

> Here's what that's like. I could imagine building a mechanical model of the economic principle of supply and demand. Suppose I build a playground teeter-totter with an arrow on the right end that points to a dial indicating high or low prices of goods. Right side up, prices high, right side down, prices are low. If we start with the teeter-totter level and add weights to the right to represent *supply* of that product and weights to the left to represent *demand* for that product...we've got a mechanical model of the economy. When the supply, right weight is larger than the left demand weight, the arrow points down—prices fall. Likewise, when demand outweighs (sorry) supply, then the left side goes down, and the arrow points up for higher prices.
>
> This is a perfectly predictable model of the economy, and through careful analysis of past economic history, one could tune the amounts of weight that would correspond to a prediction of prices and mark the dial with $ indicators. But, while it's a good model, *it's not a realistic representation of the economy.* *Almagest* is like that. It's a very complicated model of moving and spinning circles, lots of numbers to characterize the circles, scores of huge tables of numbers,[a] and could accurately predict positions of the heavenly bodies. But Ptolemy made no claim that the Sun, Moon, and planets actually performed the motions in his model.
>
> [a] Perhaps the first use of tables in any manuscript in history.

4.2.2.1 Instruments for Naked Eye Astronomy

Ptolemy was both a theoretician and a practical and skilled observer. Because of his knowledge of spherical geometry, he, like other Greek astronomers, was also a geographer and could use many of the same tools for both projects. His astronomical and astrological motives were the determination of the positions and timing of events relative to rise, set, and other objects in the sky, like the Sun and Moon and edges of the zodiac constellations. Measuring angles was key.

The sundial is the most important and oldest measuring device. While we often think of it as a garden decoration that tells time by the vertical structure (gnomon) casting a shadow over a graduated plate, it served a more precise purpose for the Greeks. They discovered that by measuring the length of the shadow at noon on the day of an equinox, they could determine their latitude on the Earth. This knowledge was crucial for passing around astronomical catalogs for use from different locations, making the sundial a critical tool for public astronomy.

How to measure the angle between two points far away? Imagine taking two chopsticks and spreading them in an open jaw to encompass the left and right angular spread of a doorway in your kitchen. From that open position, you'd need a reliable way to translate that into an angle, and there are many ways to imagine doing that and many ways for such a determination to be imprecise! Your hand might tremble, you might not hold that angle constant during your translation of the sticks to a compass, and your eyeball sighting of the two ends of the chopstick pair might be off.

From Hipparchus' work to the 17th century, continuous improvements were made to our hypothetical chopstick measuring device to increase precision and reliability. These devices were constructed at increasingly large scales using durable and precision-milled brass or bronze devices equipped with leveling attachments, precision screw-controls, sighting tubes, and engineered with etched, graduated angular scales.

The first likely enhancement of the chop-stick tool was the cross-staff, your chopsticks with a perpendicular sliding member that keeps the opening steady and can convert the geometry to read the angular separation. The well-known, but imaginary image of Ptolemy in Figure 4.6 shows him holding a cross-staff. A quadrant was also an ancient tool for measuring altitudes[18] A diaptra is another angular-measuring device, as is a plinth, and a triquetrum.

The astrolabe and the armillary sphere were innovative Hellenistic Greek inventions, although the Chinese also developed an astrolabe. The astrolabe is a circular plate with the zodiac around the outside, usually about the size of a frisbee. Your position is meant to be at the center of the outer plate. Additional plates can be inserted, each etched with the position of the horizon, the ecliptic, and important stars, which are projections of the celestial sphere onto its flat surface. Apollonius is often credited with the idea, and Hipparchus with improvements. On the back is a sight that crosses the diameter of the outer plate, which can be used to line up an object and then, by adjusting the plates, determine where stars would be, the time, the direction to Mecca, and many other uses. In medieval times, before clocks, it was produced in pocket-sized wood versions and elaborate brass works of art. Chaucer wrote a tract for his son on how to use one. It was essentially a portable analog computer.

The armillary sphere is an ingenious three-dimensional device consisting of circular bands mounted concentrically around the center where the Earth would be depicted as a small ball. Each band represents one of the great circles, such as the ecliptic (zodiac), the celestial equator, the meridian, the tropics, and the equator. It sits in a frame where a fixed, horizontal circular band surrounds the inner circles, representing the horizon. The position of the circles within the frame is adjusted for the user's latitude. By turning the celestial sphere circle, all the others turn appropriately. With graduated scales, diagrams, and pointers to stars, one can see exactly where everything is at any time and make predictions for any time.

Ptolemy's Philosophical Roots and Prerequisites for the Book: Books I and II of *Almagest* describe his working philosophy, defending it with standard arguments. But apart from the actual heavenly body motions, it's Aristotle, top to bottom. The mathematics required was Euclidean plane geometry and the use of Hipparchus'

[18] Another famous portrait shows him using a quadrant...while wearing a crown...which was an incorrect mixing up of his name with the Alexandrian General Ptolemy.

chord tables, except Ptolemy made them even more precise. He used the new "spherical geometry," and he developed it from scratch for the reader. With this introduction, he's ready to solve the world.

Ptolemy's Solar Model: Book III This was relatively easy and critically important. All of positional astronomy—to this day— depends on understanding where objects in the sky are relative to the Vernal Equinox, which in turn depends on the Sun's motion and position at any time. He didn't invent a solar model—he replicated Hipparchus and was generous with his praise for the original author.[19] So, Ptolemy's model of the Sun's is exactly the same: Figure 4.4. He repeated Hipparchus' determination of the eccentricity and agreed, but with higher precision: $e = 0.0415$ as compared with Hipparchus' $e = 0.04$.

Ptolemy's Lunar Model: Book IV and V. The motion of the Moon is difficult to grasp even today. Ptolemy's solution was ugly and also his biggest mistake: he could solve for eclipses (lunar and solar), but his model predicts that the Moon's apparent size would vary by a factor of two in a month, which obviously isn't the case. His solution is tortured and from our modern perspective, clearly an indication that there must have been something wrong. One has the impression of him just giving up and declaring successful eclipse predictions as a victory. He made careful tables of predictions of the eclipses—which were accurate— for any date, and washed his hands of the Moon problem.

Ptolemy's Fixed Star Catalog: Books VII and VIII. It was Ptolemy who told us of Hipparchus' catalog of the positions of 850 stars. He takes on the same task but also includes the positions and apparent star brightness of 1022 objects from 48 constellations in his catalog, and with this began almost two centuries of fights among historians. Did Ptolemy copy Hipparchus' 850 stars (shifting their longitudes by $2^\circ 40'$ to correct for the precession of the equinox over 265 years) or did he measure their positions as he claimed? Or had Hipparchus' catalog been wrong? The comparison of Hipparchus' 22 stars' from his *Commentary* to Aratus' poem with their counterparts in Ptolemy's catalog is the key. There are translation problems since Greek numbers were written using Greek *letters* (*A* was a letter and the number 1, and so on) and obvioiusly mistakes happened in the transcription of centuries-old media. Stars were not always named, but a little story was told about each one to locate it within a constellation. So mistakes happened. The argument has largely subsided: within the uncertainties that can reasonably be attributed to each, most of Hipparchus' 22 stars do match their Ptolemaic counterparts, and each astronomer is likely vindicated. I'm sure you're glad that I've cleared that up.

The bottom line about Ptolemy's catalog is this: it represented an enormous effort over probably decades and with updates, was the best star chart all the way to Tycho de Brahe in the late 16th century (Copernicus used much of it). A remarkable achievement and legacy.

Ptolemy's Planetary Theories: Books IX through XIV. His planetary models (yes,

[19] He has been accused of plagiarizing Hipparchus, but that's not fair as he gave ample credit.

there were three) were the target of the Muslim astronomers Copernicus, Galileo, Tycho, Kepler, and Newton, and it took all of them to bring Ptolemy down. Its accuracy is still impressive so something besides getting the right numbers was behind its downfall, an important part of our story later.

The end product of his planetary research is a chapter for each of the five planets, including its geometrical model, the particular parameters built into each model, a description of how he determined each parameter from his observations, and deliverables: tables of positional coordinates for each planet, for any day in the future. It was these tables that were reprised in his User's Manual, the *Handy Tables* and maybe the first time that numerical tabular organization was used in ancient writing.

He must have struggled mightily to make Aristotelean circular orbits work, but he held accuracy to a higher standard than the Classical Greeks, for whom a nice picture story was sufficient. In order to "get it right"—which meant making predictions that worked— he had to deviate from some of Aristotelian rules. For example, the eccentric model for the Sun and a strange epicyclic model of the Moon had heavenly bodies orbiting seemingly arbitrary points in space apart from the Earth! But as painful as the Moon solution was, getting the motions of the planets right was another story altogether.

4.2.3 Mars, Jupiter, and Saturn

> "...in a tour de force of possibly the most complex and extended calculation in all of ancient mathematics, he developed a method of successive approximation that allows the numerical values of the eccentricity and the direction of the apsidal [direction of the apogee of Mars' orbit] line to be found to any degree of accuracy. Both the problem and the solution are remarkable...his solution shows a very high order of mathematical intuition...The number of astronomers after Ptolemy who understood and could apply the method must have been very small." N. M. Swerdlow and O. Neugebauer, 1984, Vol 1, p307.

The prominent retrograde motion of especially Mars, as well as Jupiter and Saturn, added an entirely different set of complications from the naive Apollonius and Hipparchus' epicycle model. The simple epicycle picture of Figure 4.2 wouldn't do. Ptolemy had to insult Aristotle one more time, and that particular solution offended Copernicus and his Arab predecessors. Let's look at his solution for the outer planets, as they're a little simpler. Figure 4.10 shows his model that functions for Mars, Jupiter, and Saturn, and it's slightly and importantly different from Apollonius' model in Figure 4.3. Look at Figure Box 4.10 on page 159. After you've read the material in that Box, return to this point and continue reading.

As Box 4.10 shows, his new wrinkle is the introduction of a third point in space, the **equant** (Q), displaced from the deferent point by the same amount as D is from E, also called the eccentricity. A superior planet's epicycle's center P doesn't undergo uniform circular motion about the deferent center, D, *but about the equant*, Q. That is, the angle θ uniformly increases in time around the epicycle's path, so it appears to perform *non-uniform* rotation around D (its center) *and non-uniform around Earth*. The Sun is shown with its orbit centered on the Earth (since its eccentric center is too small to explicitly show). So there are two centers of motion here—one for the Sun and another for Mars' deferent.

Not always appreciated was the fact that in *Almagest*, the planet's deferents were all taken to be the same radius and that the distances were all set by the epicycle's individual radii. He chose 60 "units" (always influenced by the Babylonian base-60 sexagesimal system we use today for time and angles) for that common deferent radius. I've explicitly noted that "60" in Figures 4.9, 4.10 and 4.11. While the deferent is of fixed radius, the epicycle radii vary according to his parameter determinations: Mars:Jupiter:Saturn epicycle radii are in proportions of approximately 7:2:1. This was because the planetary models in *Almagest* were not a system. Much like Eudoxus before him, he treated each planet separately and made no attempt to merge them, until much later in his life. Figure 4.9 shows Ptolemy's independent planetary pieces.

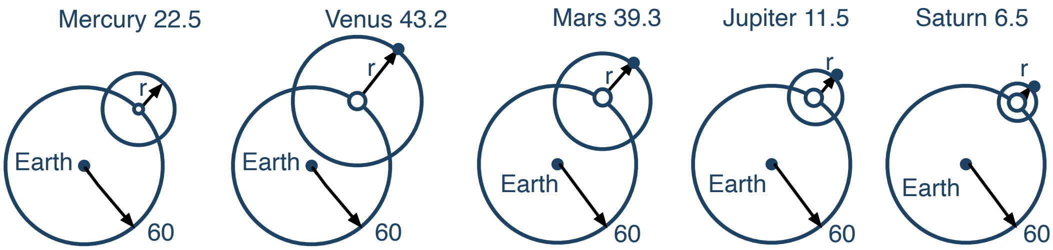
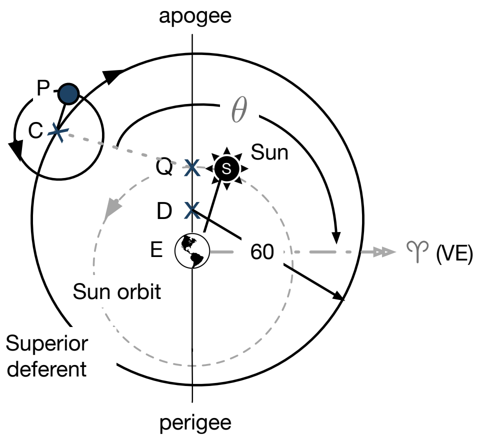


Figure 4.9: Each of the planets' epicycles is shown with their differing r values listed above as they ride on their deferents, which each of the same radius. The units are arbitrary, so the relative epicycle radius to deferent is a measure of their relationship to the Earth. So the larger is r, the closer that planet is to Earth.

An important point that will figure prominently in Ptolemy's models is that the relationship between the pieces and the Sun is very particular. In this case, Figure 4.10 shows a constraint that his model must satisfy: the radius of the epicycle $\overline{CP}$ must always be parallel to the line from the Earth to the Sun, $\overline{ES}$. This will receive inspired attention in the 15th century by the astronomer and mathematician Regiomontanus, whom we will meet in Chapter **??**, and his observation will be a direct influence on Copernicus.

FIGURE BOX 4.10



The figure to the left shows Ptolemy's model (not to scale) for a superior planet like Mars, Jupiter, or Saturn and its relationship to the Sun. Here, one of them (P) is on an epicycle with its center at C. C rotates clockwise around the circular deferent path with its center at D. The Earth is close to the center of the (slightly eccentric Sun's orbit). What Ptolemy had to do was introduce a wrinkle: the angular speed of P around D — the amount that the angle θ increases with time is constant, but about the "equant" point Q...not D.

Each planetary "kit" looks like this for superior planets and slightly different for the inferior planets. Every circular deferent radius was chosen *for all planets* to be 60 in an arbitrary set of units. The necessary parameters were determined by Ptolemy separately for each planet, including the epicycle radius, the separation of Earth from the deferent point, D, (the eccentricity), which is also the separation of D from the equant, Q, the orientation of the apogee to the Vernal Equinox direction, and the angular speed at which θ increases in time.

Now go back to page 157 and pick up where you left off.

4.2.3.1 Example: Mars

Let's pick on Mars since it figures prominently in our story now, and will reappear a number of times through Kepler's understanding of the solar system. It's easy to observe that its "year" is sufficiently short to facilitate many measurements in an astronomer's lifetime. In short, it's a fine laboratory to tune a mathematical model.

Mars orbits Earth about every 687 Earth days, or 1.88 Earth years, and undergoes retrograde motion about every 2.1 years, or a little more than one revolution around the Sun. The backward appearance lasts a little more than two Earth months, or about 72 days. Ptolemy's model with the equant rather precisely describes Mars' retrograde motion as it forces a kind of loop-the-loop as viewed from Earth.

In Figure 4.11 I've calculated the Mars model to show its epicycle and eccentricity (separation among Earth, D, and Q) using parameters taken from *Almagest*. Mars' path is, well, unusual. There are 4 points identified on the actual path that Mars takes while riding on its epicycle. Let's start at position 1, and as the epicycle turns

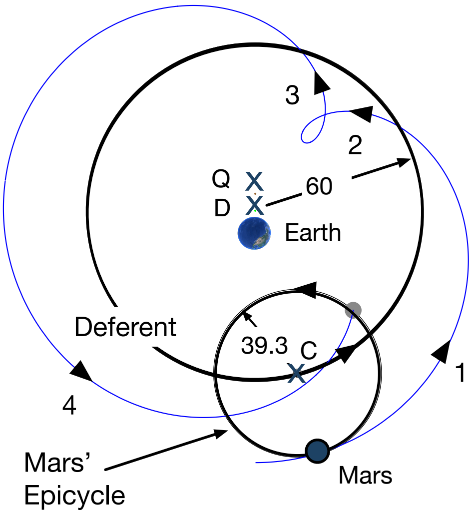


Figure 4.11: Mars (♂) is shown on its epicycle with its center, C, rotating around the deferent with its center at D. I've used Ptolemy's actual relative sizes for Mars. All deferents were in units of 60. Mars' epicycle's radius is 39.3/60, and the distance from Q to Earth is 12/60. One can see the strange loop motion described in the text.

and as the deferent turns, Mars moves to position 2, where it starts to appear to slow, making that loop which makes it appear to go backward during 72 nights. Then it comes out of retrograde and continues its forward-appearing path at 3 and nearly completes its 1.8-year-long path at 4. In each Mars year, the location of the loop shifts a bit relative to the Vernal Equinox.

This is what's seen from Earth with a bonus: it also addresses the fact that in retrograde, the planets are brighter here because they would literally be closer to Earth. Just how often and how fast would be determined by the parameters—Jupiter and Saturn's parameters are quite different.

It works very well as seen in Figure 4.12 from James Evans, 1984 (inspired by James Evans, 1998). This shows seven bands that should encompass the retrogrades of Mars as viewed from Earth for seven years of Ptolemy's observations, from 109–122 CE. The loops are the Mars retrograde events relative to the Vernal Equinox (the trajectory between points 2 and 3 in Figure 4.11), and the wedges show predictions of where that should happen. Shown in (a) are predictions for a straight epicycle model (like Apollonius and Hipparchus) *without an equant* while (b) shows the same thing, but *including the equant*. This, and other successful measurements, surely convinced Ptolemy that he was right. He needed the equant.

4.2.3.2 Venus and Mercury

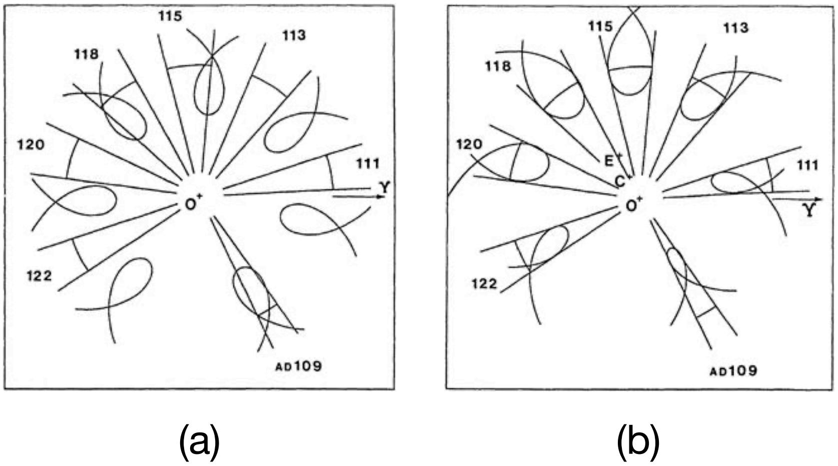


Figure 4.12: Seven retrograde loops of Mars for times of Ptolemy's observations (a) without the equant and (b) with the equant.

The relationship that Mercury and Venus have with the Sun was very problematic. Today, we know that they orbit very close to the Sun, but even now, measuring their positions is challenging. The Sun's in the way! Observations had to be done just after sunrise and just before sunset...and carefully so as not to blind one's self. So, they presented a set of problems that couldn't be solved without separate models for each. And those solutions are strange, especially for Mercury with more moving centers of deferents.

Think about all of the major ways in which Ptolemy has bent Aristotelian imperatives. Is Earth at the center now? Of what? The outer planets and the Sun no longer orbit around it symmetrically. They also don't orbit at constant speeds except now around an uninhabited point in space, not around the Earth. It's torturously pieced together in ways that Aristotle could never have imagined—and that a modern physicist would not have tolerated. "Simplicity" is nice in physical models, not guaranteed, but when your model is so bizarre you'd tend to think that it's trying to tell you that the world is probably not that way. But this is the first time.

▻ Going from pictures and stories to numerical prediction is a revolutionary step, changing the norms of scientific behavior, a feature of Ptolemy's Astronomy Project from Table 4.1

The late 16th century Johannes Kepler models the real solar system and we'll have to wait 1400 years to Chapter **??** for him to appear and save the day.

4.2.4 Ptolemy's Cosmology.

Just as it was important for Aristotle to build a multi-planet system out of Eudoxus' separate planets, it eventually seemed incomplete to Ptolemy also. So he later wrote *Planetary Hypotheses*, which upgraded some of his measurements but also presented a whole cosmology of all of the heavenly objects. There are two views of his whole universe. First, there is the geometry of the orbits, and second, there's the physical model of the whole in three dimensions (with motions that are physically impossible).

Figure 4.13 (a) shows the geometry in a simplified format where I've abstracted the epicycles for each planet: the line in each epicycle shows the relationship of the planet to the center of its epicycle. Notice that for the outer planets, the epicycles are constructed so that for each planet, those lines are parallel to one another—and parallel to a line connecting Earth to the Sun. So, you have to imagine all of them rotating about their individual centers while maintaining that parallel relationship. For the inner planets, it's the *centers* of their epicycles that all lie on that parallel line connecting the Earth to the Sun. These constraints would have been brutal to calculate. As I warned above, the Sun figures prominently.

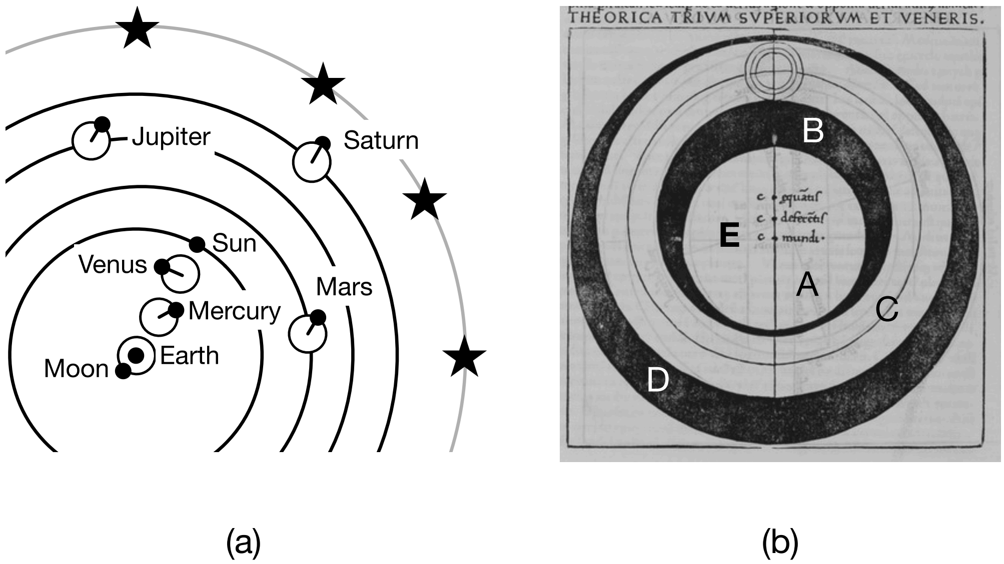


Figure 4.13: The whole cosmology of Ptolemy. In (a), the planets and the Sun are arranged in a very particular way relative to the Sun. The lines in the circles for each planet represent the center of the epicycle to the planet. In (b), an image from *Theoricae novae planetarum* by Georg Peurbach is shown, which represents a slice through the Medieval idea of Ptolemy's 3-dimensional model for one planet. Notice the epicycle inside of the region labeled C. The other labels are described in the text. (Aiton, 1987)

> Recall in Section 3.5.2, I noted that that the classical planet ordering was Plato's and Aristotle's: Earth–Moon–Sun–Mercury–Venus–Mars–Jupiter–Saturn and the stars. Ptolemy made the executive decision to change that to Earth–Moon–Mercury–Venus–Sun–Mars–Jupiter–Saturn, and because of his authority, it stuck. (Again, notice that the Sun sits between (our) inner and outer planets. Interestingly, many times when a Medieval or Renaissance rendering of Aristotle's cosmos was presented in books, it was Ptolemy's, not Aristotle's, ordering that was used. Sometimes Ptolemy's name is included on an image, even though the picture might be Aristotle's equal-orbit, totally geocentric geometry. Ptolemy's and Aristotle's pictures get mixed up during Medieval and Renaissance depictions.

Planetary Hypotheses also presented a physical model for his cosmology. In it, there are solid aether spheres that carry the epicycles through...pathways in the solid aether around the Earth. This wasn't interpreted as an image until the early part of the 15th century when Georg Peurbach's 1454 *New Theories of the Planets* included the image shown in Figure 4.13 (b).[20] Think of this as a slice through a spherical aether unit required to support and guide a planet. The light volume labeled A would contain another such unit, and so on...so that together they would nest together like Russian dolls. It's what's in a unit that's hard to swallow. The light region, C, is a kind of hollowed-out shell within which an epicycle rolls around a diameter. It's off-center since the planet follows the epicycle, sometimes close to the Earth, E, and sometimes away from it. In the figure, you can maybe just make out the three points that he marked and labeled in Latin as the equant, the deferent, and "mundi"...the "center of the world," which would be the Earth. The cavity labeled C is centered on the deferent, while the whole volume is centered on the Earth.

He imagined that the largest excursion of, say, Mercury's orbit in its epicycle, constrained inside of Mercury's C cavity, would just match the smallest excursion of Venus' orbit in its epicycle, within its C cavity. Then the largest excursion of Venus' orbit would just match the inner excursion of the Sun's and so on. He packed them together with minimal spacers of aether (D and B in Figure 4.13 (b)).

He demanded uniform motion of the spheres, but the shifting of their centers is a problem. Imagine a soccer ball spinning around an axis at a uniform rate. Can it spin around another axis parallel to the first one at a uniform rate? No! It's physically impossible and this truly offended many Muslim astronomers and mathematicians who attacked his physical model in no uncertain terms.

While his planetary orbits were independent of one another, their relative orbital sizes could be calculated as each is determined by the tight fit. So if you knew the size of one of them, you could then establish the size of others, working your way from edge to edge of each "spherical space-shell."

He knew the distance from the Earth to the Moon (from studies like that of Aristarchus) and the Earth to the Sun, and in this way, he actually calculated

[20]We'll meet Peurbach in the next chapter.

the distance from Earth *to each planet and to the stars themselves*! For example, he calculated that the maximum distance from the Earth to Venus was 1079 Earth radii. (Today, we know that the maximum Earth-Venus distance across the Sun, pretending that they are as far away from one another as possible, is more like 25,000 Earth radii.) For fun, he predicted that the distance from the Earth to the Stars—*the size of the entire universe*—would be $20,000 \times E_R$, or 126,000 km. Both are an astonishing feat—-calculating the size of the entire universe—and wildly wrong. His universe's size is smaller than the actual furthest separation of Earth and Venus in our world.

4.2.5 The End of Greek Astronomy

Think about the conceptual leap that we've taken: we've gone from Aristotle, who told picture stories about the planets, to Ptolemy, who quantitatively modeled his entire universe! It's an astonishing feat, and nobody successfully challenged it for 1400 years (although there were many attempts by the Muslim astronomy and mathematics community), which is a pretty good record. Here's perhaps a surprise:

▻ The Ptolemaic model is mathematically identical to the Copernican model.

In fact with modern parameters from modern instruments, Ptolemy's model predicts the planetary positions and astronomical events with high precision, within a few percent (Rushkin, 2015). And yet, you're wondering how that could be the case since we now know that this was not an actual model of how the planets go?

In Volume II, I'll explain how, and we'll watch the slow evolution of scientists' goals from just getting the numerical predictions right to the mandate to build a model of how the planets really move. That commitment is Copernicus' and then those who followed through the 18th century.

Ptolemy was a kind of intellectual Greek island: little progress for 400 years before him and none after him. He was the last Greek astronomer. Science would explore new frontiers, but the Greeks would no longer be on board. Rather, Western research[21] in MOTION BY THE EARTH and MOTION IN THE HEAVENS shifted to India and among the Muslim scholars who did original astronomical and mathematics work and translated, preserved, and commented on Greek writings—especially Ptolemy.

[21] There was a parallel research path in China, but it didn't influence the eventual progress Europe

4.3 Ptolemy's Astronomy Project

Ptolemy's Astronomy Project	
1. Numbers project inputs	**Numbers project outputs**
1. number of planets is seven 2. Hipparchus' star catalog of 850 3. Hipparchus equinox precession 4. 23.5° tilt between equinox and CE 5. solar eccentricity $e = 0.04$	1. no change 2. 1022 stars' positions and brightnesses 3. his own measurement 4. no change 5. solar eccentricity improved $e = 0.0415$ 6. dozens of measured inputs were measured 7. deferent radii set to "60," epicycles and eccentricities uniquely determined
2. Theoretical project inputs	**Theoretical project conclusion**
1. Adherence to all of Aristotle's physics 2. modeling using eccentrics and epicycles 3. commitment to cataloging heavenly objects' positions	1. no change 2. modeling framework requiring measured input parameters 3. enhanced, precise numerical precision 4. models must match observation
3. Technique project inputs	**Technique project outputs**
1. spherical trigonometry 2. altitude-azimuth coordinate system 3. use of common instruments: sundial, cross-staff, dioptra, astrolabe, armillary sphere, etc.	1. spherical trigonometry improved 2. coordinate system improved 3. same instruments but often re-designed for higher precision including armillary sphere 4. complicated, predictive model eccentricities and equant
4. Norms project inputs	**Norms project outputs**
1. circular motion for heavenly motions 2. beginnings of quantitative positional determination	1. uniform circular motion, but a loosening of the definition of a strict Earth-centered system 2. a demand for very high precision 3. Tables become deliverables, facilitating predection
5. Curiosity: project puzzle	**Curiosity: project outputs**
1. Could a consistent, predictive, and precise model be constructed for heavenly objects' positions and astronomical events?	1. Yes. A predictive and precise model based on epicycles and equants for the planets and Moon with an eccentric model for the Sun.
6. Project influences	**Project products**
1. Aristotle's physics 2. Hipparchus' writings and techniques	1. books: *Almagest*, *Handy Tables* , *Planetary Hypotheses* and *Tetrabiblos* (astrology),

Table 4.1: Ptolemy's Project for Astronomy

Table 4.1 is my representation of Ptolemy's Astronomy Project. By contrast, his Cosmology Project was not well-developed and served more as a target of criticism than an actual stimulus for other Projects. (Set the context with the timeline in Figure 1.2 on page 22.)

- Ptolemy (85 to 165):
  - He focused on creating precise and predictive modeling of all of the planets and major astronomical events. His commitment to numerical modeling requiring precision and accuracy became the standard for astronomy and physics to this day.
  - He wrote the mammoth book, *Mathematical Composition*, nicknamed by Islamic astronomers as *Almagest*, which became its label to this day (it's in the dictionary of your word processor). It was the definitive tool for predicting the positions of all of the heavenly bodies. The naive Copernican heliocentric model is mathematically identical to the epicyclic model of Ptolemy. No better, no worse than Ptolemy's.
  - He created a star catalog of more than a 1000 stars, including a subjective measure of each's brightness.
  - He continued Hipparchus' solar model with a separate, and corroborating measurement of the eccentric.
  - He adopted the epicycle model of Apollonius and found ways to assign measured parameters to the epicycle variables: the deferent radii he took as constant and found epicycle speeds of rotation, radius, and orbital speeds on the deferents, separately for each planet.
  - He wrote a "handbook" (*Handy Tables*) that would teach an astronomer, physician, or astrologer how to predict the positions of planets using his model, without having to absorb the considerable mathematics of *Amalgest*. Tables became a feature of all astronomical modeling for almost 2000 years.
  - He later wrote a complete cosmology that attempted to put all of the planets, epicycles and all, into one nested cosmological model. This allowed him to make predictions about the sizes of orbits.

4.3.1 What's Next?

The scene is now set for the full story of MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS. Here's a fascinating coda to our Ptolemy story. He was so close! His reliance on Aristotle's physics would prove to be less well-founded, but that took 1000 years. And, as for his astronomy, it took Arab astronomers, Medieval mathematicians, and Renaissance scholars that same period to prepare the World's intellectual stage for Nicolaus Copernicus. That's the subject of Volume II of G2E: *Rennaissance Astronomy and Medieval Investigations of Motion*.

4.4 Greek Astronomy, Today

4.4.1 Hipparchus and Modern Celestial Coordinate Systems

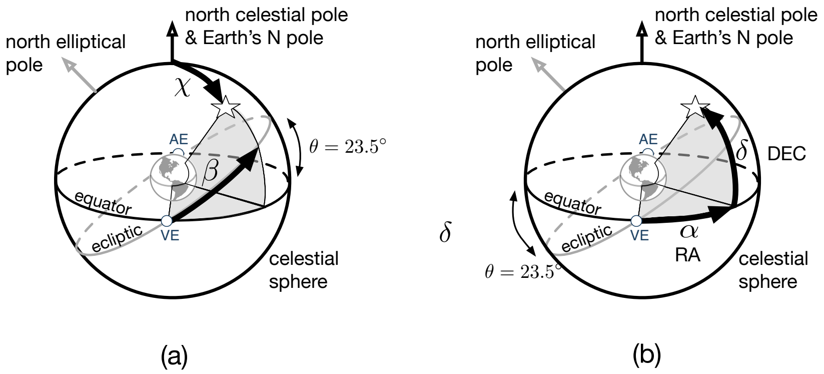


Figure 4.14: The Celestial Sphere is shown in both diagrams for two different coordinate systems that can be used to locate a star on the Sphere. In (a) the "longitudinal" coordinate (β) is along the ecliptic starting from the position of the Vernal Equinox along the ecliptic and the "latitude" coordinate (χ) is measured from the Celestial Pole to the star along a great circle. In (b) the longitude (α) is along the Celestial Equator from the Vernal Equinox (and so identical in angle to β) and the latitude is measured up from the Celestial Equator (δ). The coordinate system in (a) is called the Ecliptic Coordinate System and (b), the Equatorial Coordinate System. (b) is the standard modern system for star charts in which δ is called "declination" and α is called "Right Ascension" (and is recorded in modern tables in units of time, rather than angle where 24 hours equals 360°). A modern version of the Ecliptic Coordinate System uses $\lambda = 66.5^\circ - \chi$, but I represented it here from the pole because Ptolemy measured χ for "latitude." Hipparchus seems to have used both of these systems while Ptolemy used (a).

(Dennis Duke, 2002) correctly argues that the coordinate system that Hipparchus seems to have originated and Ptolemy perpetuated is essentially identical to what is used today in astronomy, called the "equatorial system." Figure 4.14 (a) shows the situation. What Hipparchus did was measure the angle of a star relative to the North Celestial Pole and an angle along the ecliptic. If you look at Figure 3.20 you'll see that the Earth is surrounded by the 12 constellations of the zodiac. The Greeks (and Babylonians) divided the whole circular pattern into 12 signs, each of 30° each and his coordinate system referred to the constellation and then the number of degrees within that constellation. This is like the longitude on the Earth's surface—degrees around. The "zero" of this coordinate system is located at the position of the Vernal Equinox, which, recall, is where the Sun on the ecliptic crosses the Celestial Equator during the spring. The Sun was in the constellation Aries during these times (which is why the symbol for the Vernal Equinox is ♈, which is the symbol for that constellation. Today, the VE has moved to the constellation Pisces precisely because of the precision phenomenon that Hipparchus discovered.[22] (More about the Vernal Equinox below.) So in the *Commentary*, he wrote about the constellation

[22]The "Age of Aquarius" is next, as precession continues.

Bootes (not among the 12 zodiac members):

> "Bootes rises together with the zodiac from the beginning of the Maiden to the 27th degree of the Maiden... Hipparchus, "

The "Maiden" is Virgo which is the 6th constellation ("sign") around from Aries (Figure 3.20). So the angle, α in the figure where the constellation Bootes rises is $(6-1)\times 30^\circ + 27^\circ = 177^\circ$.[23] A modern version of Bootes extends 202° to 237°, so it doesn't appear to match? Ah, but the precession of the equinoxes is worth $1^\circ/72$ years, so we need to add that factor times the number of years since Hipparchus recorded his measurement 2153 years ago—that's an additional 30° which makes that edge be 207°: Hipparchus is just right.

For the other coordinate, he measured from the North Celestial Pole *down to the object* of interest, χ in the figure. That's the "polar angle" and is the opposite of our Earth-faced latitude, which measures up from the equator.

The modern equatorial system uses the same idea. For the polar angle, a star or object's "latitude" coordinate is measured *up from the Celestial Equator*. This is called the "Declination, δ." So it's identical through a difference of 90°:

$$\chi = 90 - \delta.$$

This north-south polar angle measure is called "co-declination."

The modern longitude, called the Right Ascension, α, is measured also from the location of the Vernal Equinox, but typically recorded as a time, rather than an angle. This is natural, since the whole Celestial Sphere rotates 360° in 24 hours. So while the edge of Bootes is 202° for Hipparchus' units, it's $13^h 36.1^m$.

About the Vernal Equinox. I don't believe that there's any record of just how Hipparchus could have determined the location of the VE in the zodiac. After all, the Vernal Equinox for the Greeks was determined at noon on that day when the Sun is precisely between its altitude at the two solstices, and equivalently, when it rises and sets precisely in the east and the west. His accuracy was about 1/4 of a day for observations and I can think of two ways he might have done this.

He would surely already know roughly when the equinox was to happen and would start measuring the Sun's location, rise, and set for days before and days after the expected event. Then, later he could figure out precisely which day. But along with his altitude measurements, he might look at the east just before the Sun rises each of those days and precisely located which constellations were still visible before it becomes bright. Likewise, he would look just after sundown to see what constellations would be "coming out" as it gets dark.

He could also have noted when the equinox occurred, waited exactly 12 hours and then looked to see which constellation would be at the altitude of the Sun at noon.

In both of these, he would presumably conclude that it was Aries and the "First Point of Aries" became the nickname for where the Vernal Equinox is in the sky.

[23] Because Aries the first sign starts at 0°, so the 6th sign starts with 150°

4.4.2 New Evidence for Hipparchus' Lost Star Catalog

When we're talking about millennia, "breaking news" needn't be "yesterday." So there is remarkable Breaking News when it comes to Hipparchus' star catalog. Parts of it might have been found.

In 2012 Jamie Klair, an undergraduate at the University of Cambridge was studying a multi-spectrum image of folio pages of an ancient Greek palimpsest[24] known as the *Codex Climaci Rescriptus* at St Catherine's Monastery on the Sinai Peninsula (now in Museum of the Bible's collection in Washington, D.C.). It was a summer project assigned by a biblical historian at the University of Cambridge, Peter Williams, who continued the work, and in 2017, he and French collaborators confirmed the observation and found more of it. They recently published it in (V. J. Gysembergh, 2022). In that image, an under-text is slightly visible, which he realized appeared to contain astronomical notations—actually a quotation from Eratosthenes. It appears that the original writings were erased in the 9th or 10th century and overwritten. However, the multispectral imaging brings out the original impressions on 9 of the 146 pages.

By digitally bringing out the faint background writing, it's apparently astronomical data, coordinates, actually. Almost certainly from Hipparchus' observations. For example, one of the decoded and translated phrases in the hidden text is:

> Corona Borealis, lying in the northern hemisphere, in length spans $9^\circ 1/4$ from the first degree of Scorpius to $10^\circ 1/4$ in the same zodiacal sign (i.e. in Scorpius). In breadth it spans $6^\circ 3/4$ from 49° from the North Pole to $55^\circ 3/4$.

They noted that "length" is the east-west measure and "breadth" is the north-south measure. The north-south measure is as above, the co-declination and the east-west measure is again the Right Ascension, in angular units. Scorpio is the 8th constellation, so from the previous section, that's $7 \times 30^\circ + 1 = 211^\circ$. Adding the 30° for precession since then would give a RA today of 240°. The edge of Corona Borealis is almost exactly that.

The stars in the 9 pages refer mostly to Ursa Major, Ursa Minor, and Draco, and the values are essentially those in Hipparchus' *Commentary*. The general consensus is that this is the first concrete evidence for the long-lost Star Catalog of Hipparchus!

[24] a document that has been reused by scrubbing out the original content

Technical Appendices

A Technical Appendices: Presocratic Greeks

A.1 Proof of Pythagoras' Theorem

A.2 Zeno's Paradox

blah

B Technical Appendices: Plato and Aristotle

B.1 Socrates' Geometrical Problem

B.2 Aristotle's Legacy in Physics and Engineering

B.2.1 Logical Fallacies

Propositional logic lays bare some logical fallacies, which can be mistakes. Logical fallacies can also be used to convince people of the truth of a conclusion using an argument that appears to be valid but is not. Look at the argument on the left in

A Valid Modem Monens Argument	A Logical Fallacy
• If a reactor leaks radiation, people nearby will get cancer. • A reactor leaded radiation • Therefore, people nearby got cancer.	• If a reactor leaks radiation, people nearby will get cancer • People nearby got cancer • Therefore, the reactor leaked radiation.

Table 2: On the left, is a valid Modus Ponens argument. But on the right is a logical fallacy called Affirming the Consequent.

Table 2. Its validity is forced on you in the way that deductive arguments must do. A subtle change can take a valid argument and turn it into an invalid logical fallacy called "Affirming the Consequent," by switching the consequence for the hypothesis in the second premise. Can you see that the argument on the right in the table is sneaky and invalid? People get cancer from all sorts of causes, and that someone got cancer does not mean that the reactor leaked radiation. Health care is often a target for this form of fallacy.

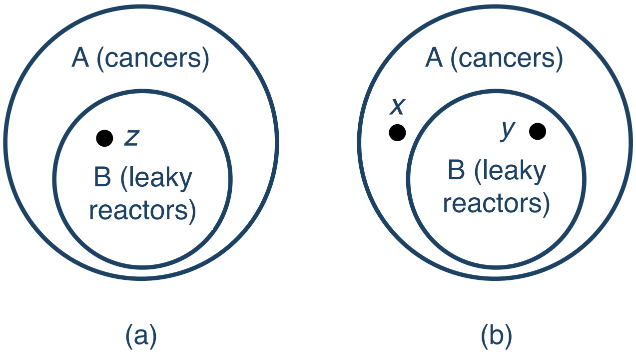


Figure 15: On the left is the valid argument that says that the placement of *z* with both cancer and near a leaky reactor is the only result of the valid argument. But the right says that there is a cancer, but it could be either coincident with a leaky reactor (*y*) or have nothing to do with a reactor (*x*), and so the argument is invalid.

The objects in Figure 15 —which are not strictly Euler Diagrams— but similar to them— help to capture the argument. The conclusion of the valid and invalid arguments is apparent by the way the circles are arranged. The left diagram and the right diagram are the same since they represent the "If...Then" part of the argument. So within that arrangement, we can ask about validity by looking at entities that might fit the discussion. Look at entity "*z*" in the left diagram. It has the property B and since B is inside of A, it also has the property A. So, given the argument that the reactor leaked and entity z is inside that leaked region, it also is inside of the cancer region, completing the Modus Ponens true conclusion.

The diagram on the right has the same two regions, but now, in the spirit of the invalid argument, assert that entity *y* has the "attribute" of having cancer, so begin inside of region A. But this doesn't exhaust all of the possibilities for an entity having cancer. Entity *x* is also asserted to have the property of cancer, but it doesn't support the conclusion that it overlaps with the leaky reactor region. So that second argument is not valid.

B.2.2 The Connection with Our Modern World

Aristotle's logical writing came from a deep level of analysis of language and thought. From the ground up. One might think that some ideas are just too trivial to write them down, but he wrote them down and defended his definitions, even the most trivial bits. Here's one:

> "...there cannot be an intermediate between contradictories, but of one subject we must either affirm or deny any one predicate" Aristotle, *Metaphysics*

This is called the Law of the Excluded Middle. *A proposition is either true or its negation is true.* There's no in-between. It's binary. This is a "two-valued" logic, and Aristotle's structure was always built around that requirement: he didn't admit the (modern) idea of "degrees of truth" or "fuzzy logic." Trivial? Centuries of ink have been spilled over precisely understanding the implications of Law of the Excluded Middle and how to unequivocally state it symbolically. It's a simple idea that's deep, and he had a number of such crisply defined notions so his Logic was really built from first principles.

What else can you think of that's strictly two-valued? How about binary arithmetic, where the only numbers are 0 and 1. How might you trivially represent 0 and 1? How about a pair of fixed voltages, say $V = 0$ and $V = 5$ volts.[25] There are a handful of seminal discoveries about Logic that extend to our modern usage. **Gottfried Wilhelm Leibniz (1646–1716)** refined binary arithmetic. In 1854, **George Boole** (1815–1864) invented the algebra of two-valued logic...how to combine multiple conjunctives into meaningful outcomes which can only be T or F, 1 or 0. In 1921 in his dense and terse *Tractatus Logico-Philosophicus*, **Ludwig Wittgenstein** (1889–1951) presented the Truth Table, which can be used in logical proofs (and circuit design). Finally, in 1938 **Claude Shannon** (1916–2001) realized that Boole's algebra could be realized in electronic, "on-off" circuits. This was put into practice in the 1940's with vacuum tubes and then in the 1960's with transistors.

B.2.3 Truth Tables

My goal here is to give you a hint about how important logical analysis has become, by following two of Aristotle's ideas: First, that statements and propositions can be written as abstract sentences with *variables* rather than with named things. And, that The Law of the Excluded Middle leads us to a *two-valued logic*.

Here's a statement: (It is raining.) This could be true (T) or false (F), depending on the circumstances. But it's verifiable since we could determine T or F by looking out the window. I'll call that statement p. Here's another: (the grass is wet.), another verifiable statement which could be T or F and I'll call it q.

I can put these together into a compound statement using a "logical connective": (It is raining.) AND (The grass is wet). "AND" joins the two statements. I can write this using the logical symbol, $\wedge$, which stands for AND, so our sentence—in general— can be abstracted in the Aristotle-variable-way as $p \wedge q$.

Our question of interest is: when will the compound statement, (It is raining.) AND (the grass is wet) be true? That is, what is the truth-value of "$p \wedge q$"...for the four possible T and F values that p and q might take on? Thought of in a different way, if I asserted that compound statement, when am I telling the truth?

- If it is raining and the grass is wet, then $p = T$ and $q = T$, I would be telling the truth if I said, "It is raining and the grass is wet."
- If it is raining and the grass is not wet. $p = T$ and $q = F$ then I would be lying if I said, "'It is raining and the grass is wet." (since $q = F$ means that the grass is dry).

[25] the voltage range for transistor-transistor logic (TTL) logic used in many applications.

- If It is not raining and the grass is wet. $p = F$ and $q = T$ then I would be lying if I said, "It is raining and the grass is wet."
- If it is not raining and the grass is not wet. $p = F$ and $q = F$ then I would be lying if I said, "It is raining and the grass is wet."

So of the four possible combinations of p and q, there is only one instance where the combination $p \wedge q$ is TRUE. This begs for an ordered way to present these possibilities and for each p and q, we can generate rows in a **Truth Table**. For AND, this is shown in Table 3. Notice that the entries in the last column correspond to the bullets just above and complete the possible p's and q's states.

Raining?	Wet?	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Table 3: The Truth Table for the AND connective.

Primitive logical connectives come by different names depending on one's discipline. They include: NOT, AND, OR, XOR ("exclusive OR"), NAND ("not-AND"), NOR (negate), XNOR ("exclusive NOR"), Implication, and Biconditional. They all have their own truth tables. And they're useful. What this means is that we can take many arguments and turn them into symbols using the connectives as "puzzle pieces."

Let's think about analyzing an everyday situation, like planning a picnic. Weather can be a problem for picnicking since wet grass can make it unpleasant. So the morning of the planned outing, a picnic planner might muse something like:

- If it is raining, then the grass is wet
- It is raining
- And so the grass is wet.

Notice that this has the form of Modus Ponens, and I'm going to make a 21st century realization of it 2000 years after it was discovered. Here, $p =$ (It is raining.) and $q =$ (The grass is wet.). Let's set the stage and flesh out the single possibility for a valid Modus Ponens argument.

- (If it is TRUE that it is raining, then it will be TRUE that the grass is wet)
- AND (it is TRUE that it is raining)
- THEN (it is TRUE that the grass is wet)

But a Propositional argument contains phrases that have truth values, and in general, they are not necessarily all true. Recall the "am I lying" test from above: I could have $p =$ T or F and $p =$ T or F and only one combination of the four possible arrangements completes our valid raining-wet argument.

The entire set of possibilities can be compactly and completely captured in one big truth table, and here I just present this result in Table 4. It's a picnic table (sorry). (In Technical Appendix B.2 I build that whole table.) Notice that the AND operation between the third and first columns creates the third column's results, by comparing them using the rows of Table 3 as an instruction. The only combination that's true is the first one, the Modus Ponens argument itself. Validity of the argument is

Variables		Conditional	Conclusion
p	q	$(p \rightarrow q)$	$(p \rightarrow q)$ AND p
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

Table 4: The truth table for the Propositional argument above. The last column comes from comparing the third column with the first column according to the the T and F values in Table 3.

assured only if $p = T$ and $q = T$. Our connective, AND, figures prominently in this Propositional argument.

B.2.4 Modern Digital Arguments

Inspired by Aristotle, this "regular" conversation about the consequence of raining and the state of the grass can actually be embedded into a digital circuit using very basic digital packages[26] called "gates" (NOT, AND, OR, XOR, NAND, NOR, XNOR, and buffers). You'll recognize them as some of the logical connectives from above, plus one more that has a single input and just holds its value, called a buffer. The magic of the second half of the twentieth century is that particular combinations of transistors can produce digital packages corresponding to the gates which in turn can be soldered to a circuit board to make a decision-making circuit. With all of the individual gates, an electrical engineer can piece them together to do a job. In the background, if not in the engineer's notebook, is the equivalent of a complicated truth table.

> Think about the decision-making that's required in order for an ATM machine to process your card, the keypad, your PIN, your request, and that you took out your bills. That each step was accomplished—and checked to have been done correctly— is actually a set of questions with T or F answers that a digital circuit is happy to perform for you.

Figure 16 is a cartoon of what this might mean. In the top figure, I show the engineering symbol for an AND gate. Below it, the black box could consist of a single digital gate element or hundreds of digital gates, each receiving inputs from

[26] You can go on Amazon and purchase integrated circuit packages of usually multiple gates in a single element that can be soldered onto a circuit board.

the outputs of others. Here the box receives two binary inputs, each of which could be T or F.[27] and it outputs a result, *r*, either T or F. So there could be four possible inputs but one result. What's inside of the box are circuits of connected gates built on the logical structure of the problem.

Our complete Modus Ponens picnic argument presented here as set of English statements could be recreated in a digital circuit (what might be inside the black box in Figure 16 (b)). For our particular example the circuit would consist of three gates (made from five transistors which would be so small that you cannot see them): an electronic circuit of the English sentences covering all of the possibilities of the argument.

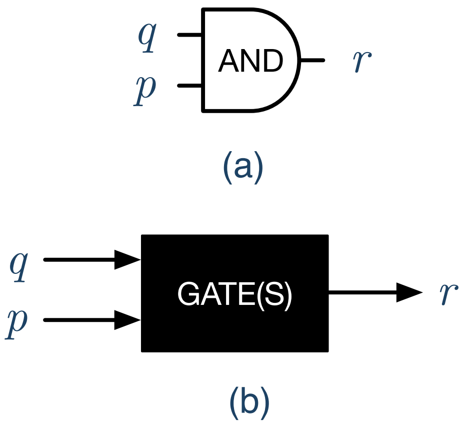


Figure 16: In (a) the engineering symbol for an AND gate is shown. The output of the AND gate, *r*, corresponds to the result of the truth table in Table 3. In (b) a black box of digital logic gates is suggested. The two inputs, *p* and *q*, are each either T or F, and the output, *r*, is either T or F. This could be one gate or a thousand gates.

I hope you can get a sense of how digital circuits are designed. There's a job to do, it's described in logical terms (*p*'s and *q*'s), a truth table (or equivalent) abstraction is done, and from (millions of) combinations of the seven digital gates that exist, a circuit design is created. Humans used to do this indeed at the beginning of my career, we laid out digital circuits by hand, but now computer-aided design workstations do the work of creating schematics, simulating what electrical signals would do in the design, and preparing the instructions for printed circuit board (PCB) fabrication by specialized companies.

C Technical Appendices: Eudoxus and Greek Astronomy

C.1 Plato's Timaeaus Cosmology—The Numerology

> "And he began the division in this way. First he took **one portion** from the whole, and next a **portion double of this**; the **third half as much again as the second**, and **three times the first**; the **fourth double of the second**; the **fifth three times the third**; the **sixth eight times the first**; and the **seventh twenty-seven times the first**. Next, he went on to fill up both the double and the triple intervals, cutting off yet more parts from the original mixture and placing them between the terms, so that within each interval there were two means, the one (harmonic) exceeding the one extreme and being exceeded by

[27] which in practice, of course, is a 1 or 0 ("low" or "high") bit, and a the transistor level, a low and high voltage in a circuit

> the other by the same fraction of the extremes, the other (arithmetic) exceeding the one extreme by the same number whereby it was exceeded by the other."
> Plato, **Republic**

Okay the numbers seem arbitrary. But there's an algorithm:

- one portion of the whole: ○, 1
- double of this: ○○, 2
- half as much again: ○ ○ ○, 3
- double of the second: ○ ○ ○○, 4
- three times the third: ○ ○ ○ ○ ○ ○ ○ ○ ○, 9
- eight times the first: ○ ○ ○ ○ ○ ○ ○, 8
- twenty-seven times the first: ○, 27

Now manipulate:

- The first four are the famous 1,2,3,4 and since they're the special numbers, they have a job to do:
  - Square each of the first numbers—remember, 1 is not a number— (Greeks knew how to multiply): and you get 4 and 9.
  - Cube those same first two important numbers: and you get 8 and 27.

So all of the numbers in that excerpt are some manipulation of the numbers 2 and 3—he stopped at 3 because there are only three dimensions. Collecting all of the numbers, but now into even and odd strings (remember, 1 is neither even nor odd for Pythagoreans and apparently also, for Plato):

Then, Timaeus says that if you take the number strings you actually construct the intervals of the diatonic musical scale. More Music of the Spheres. Whew. Wait until we get to Kepler.

D Technical Appendices: Hellenistic Greeks

D.1 Some Aristarchus Measurements

blah

Glossary & Acronyms

aether In ancient and medieval science, the fifth element, thought to fill the universe beyond the terrestrial sphere..

Alexandrian Museum An institution in ancient Alexandria, Egypt, part of the Library of Alexandria, serving as a major center of scholarship and research..

ancient elements From the Greek Presocratic, Empedocles, that all of reality is made of the set of "classical elements," earth, water, air, and fire..

apogee The point in the orbit of a celestial body where it is farthest from the Earth..

astrology The belief and practice of interpreting the positions and movements of celestial bodies to predict events and influence human affairs..

atom The smallest unit of a chemical element, consisting of a nucleus surrounded by electrons..

c The speed of light, $c = 3 \times 10^8$ meters per second, or 6.7×10^8 miles per hour..

celestial equator The projection of Earth's equator onto the celestial sphere, dividing it into the northern and southern celestial hemispheres..

celestial sphere An imaginary sphere of arbitrary large radius centered on the observer, on which all celestial objects are projected..

cosmology The study of the origin, structure, and evolution of the universe..

crystalline spheres Ancient Greek concept of transparent, concentric spheres to which the stars and planets were thought to be attached..

deductive logic A type of reasoning that moves from general premises to a specific conclusion, where the conclusion logically follows from the premises..

deductive reasoning Logical process where a conclusion follows necessarily from given premises, moving from general to specific..

ecliptic The apparent path of the Sun and planets across the sky over the course of a night or the year, intersecting the celestial sphere..

Eleatics Pre-Socratic philosophical school that argued for the unity and unchangeability of reality, founded by Parmenides..

Elements A mathematical treatise by Euclid, composed of 13 books covering many aspects of mathematics, including geometry and number theory..

ellipse An oval-shaped curve, the shape of the orbits of planets around the Sun, defined by two focal points..

empiricism Philosophical belief that knowledge comes primarily from sensory experience and experimentation..

empiricism Philosophy which emphacizes knowledge as derived from sensory experience and observation..

epistemology The philosophical study of knowledge, its nature, sources, and limits..

equinox Either of the two times in the year when the Sun crosses the celestial equator, resulting in nearly equal day and night lengths..

ether THEDEFINITIONremove2.

first anomaly In the Geocentric model, referring to the apparent variation of the speeds of the planets as they orbit around the Earth..

formal logic Branch of logic dealing with the structure of arguments and the formal properties of logical systems..

Formalism In philosophy of mathematics, the view that mathematics is not about numbers or other abstract entities but about the manipulation of symbols according to rules..

Forms Abstract, perfect, non-material templates for all things, according to Plato's philosophy..

G2E The name of this series: *Greeks to Einstein*..

Hellenic Pertaining to ancient Greek history, culture, or art, especially before the Hellenistic period..

Hellenistic Age The period from the death of Alexander the Great (323 BCE) to the Roman conquest of Egypt (30 BCE), characterized by the spread of Greek culture..

Hz Hertz, the unit to characterize the rate for a periodically changing entity in cycles per second..

inductive logic A type of reasoning that moves from specific observations to broader generalizations and theories..

Intuitionalism The view that mathematical knowledge is primarily based on intuitive insights and mental constructions, rather than on external logical systems or empirical observations..

Justified True Belief A traditional definition of knowledge, stating that a belief must be true and justified to count as knowledge. Often associated with Plato..

latitude The geographic coordinate that specifies the north-south position of a point on the Earth's surface, measured in degrees from the equator..

lodestone Naturally magnetized piece of the mineral magnetite, historically used as a magnet and in navigation..

longitude The geographic coordinate that specifies the east-west position of a point on the Earth's surface, measured in degrees from the Prime Meridian..

Lyceum The school founded by Aristotle in Athens, focused on research and teaching across various disciplines..

materialistic A philosophical viewpoint that posits that physical matter is the only or fundamental reality, and that all phenomena, including mental phenomena and consciousness, are the result of material interactions..

Mathematical Platonism Philosophical view that mathematical entities exist independently of the human mind, similar to Plato's Forms..

mean Sun A hypothetical sun that moves uniformly along the celestial equator at a constant speed, used in timekeeping..

metaphysics Branch of philosophy exploring the fundamental nature of reality, existence, and the universe..

monist A philosophical perspective that asserts that reality is composed of a single substance or principle. Monism contrasts with dualism and pluralism, which posit two or more fundamental substances or principles..

Mycenaeans Ancient Greek civilization (c. 1600-1100 BCE) known for its fortified palace complexes and contributions to Greek culture. These were the antagonists during the legendary Battle of Troy..

NCP North Celestial Pole.

normal A line perpendicular to a surface at the point of incidence where a wave strikes the surface..

north celestial pole The point in the sky directly above Earth's North Pole, around which the stars appear to rotate..

ontology Branch of metaphysics concerned with the nature and relations of being and existence..

parallax The apparent displacement or difference in the apparent position of an object viewed along two different lines of sight..

Parmenides Problem THEDEFINITIONremove2.

perigee The point in the orbit of a celestial body where it is closest to the Earth..

Platonic Solids Five regular, convex polyhedra (tetrahedron, cube, octahedron, dodecahedron, icosahedron) that Plato associated with the elements..

Platonism The branch of philosophy built on the ideas of Plato, emphasizing the existence of abstract, non-material realities called Forms..

Presocratics THEDEFINITION.

prograde motion The normal, eastward movement of a celestial body across the sky relative to the stars, seen in planets..

Ptolemaic System The Ptolemaic System is an ancient geocentric model developed by Claudius Ptolemy, where the Earth is at the center and all celestial bodies revolve around it in complex paths. It was widely accepted until the heliocentric model by Copernicus emerged in the 16th century..

Pythagoreanism An ancient philosophical and religious movement founded by Pythagoras, emphasizing the importance of numbers and mathematical relationships in understanding the universe. It also includes beliefs in the immortality and transmigration of the soul..

Quine–Putnam Indispensability Argument Argument that asserts the indispensability of mathematical entities to scientific theories as evidence for their existence, leading to the assignment of reality to those mathematical entites..

quintessence The fifth and highest element in ancient and medieval philosophy, believed to compose the heavenly bodies and fill the universe. Another name for aether..

rationalism Philosophical doctrine that reason and logic are the primary sources of knowledge and truth..

reflection The change in direction of a wavefront at an interface between two different media, so that the wavefront returns into the original medium..

refracting telescope An optical telescope that uses lenses to gather and focus light from distant objects for magnified viewing..

refraction The bending of a wave as it passes from one medium to another, caused by a change in its speed..

retrograde motion The apparent westward, backward movement of a celestial body across the sky, relative to the stars, as observed from Earth..

second anomaly In the Geocentric model, referring to the apparent backward motion of planets as viewed night to night..

solstice Either of the two times in the year when the Sun is at its greatest distance from the celestial equator, resulting in the longest and shortest days..

stellar parallax The apparent shift in position of a nearby star against the background of distant objects, caused by the Earth's movement around the Sun..

syllogism Form of deductive reasoning consisting of two premises and a conclusion, each sharing a common term with the conclusion logically following from the premises..

The Academy The philosophical school founded by Plato in Athens, considered the first institution of higher learning in the Western world..

vacuum A space devoid of matter..

vernal equinox The equinox occurring around March 21, marking the beginning of spring in the Northern Hemisphere..

zenith The point in the sky directly above an observer..

Names

Anaxagoras of Ionia (–500 to maybe –428) Anaxagoras introduced Nous (Mind) as the organizing cosmic force and believed that everything is composed of infinitely divisible particles..

Anaximander (ca –610 to –545) Anaximander introduced the concept of the "apeiron" as the origin of all things and created one of the earliest maps of the known world..

Anaximenes (ca –570 to –525) Anaximenes proposed that air is the fundamental element from which all matter is derived, with changes in air density forming different substances..

Aratus (–315/310 to –240) Aratus was a poet and didactic writer known for his work *Phaenomena*, which describes constellations and weather signs in a poetic manner..

Archimedes of Syracuse (–287 to –312) Archimedes was a mathematician, physicist, and engineer known for his work on the principles of levers, buoyancy, and the value of pi..

Archytas of Tarentum (ca –420 to –355) Archytas was a mathematician and philosopher who made significant contributions to geometry and mechanics, and is often associated with the Pythagorean school. Friend, or rival, of Plato..

Aristarchus of Samos (–210 to –230) Aristarchus was an ancient Greek astronomer who made many measurements of astronomical bodies and their relationships to one another and the Earth and apparently proposed the heliocentric model, placing the Sun at the center of the known universe..

Aristotle of Stagira (–384 to –322) Aristotle, a student of Plato and tutor to Alexander the Great, made extensive contributions to numerous fields, including logic, metaphysics, physics, biology, and ethics. Not mathematics..

Callippus of Cyzicus (–370 to –300) Callippus was an ancient Greek astronomer who refined the Metonic cycle, improving the accuracy of the lunar and solar calendar systems..

Democritus of Abdera (about –445 to –370) Known as the father of atomism, Democritus theorized that the universe is composed of small, indivisible particles called atoms, forming all matter..

Empedocles of Sicily (–494 to maybe –434) Empedocles proposed that all matter is composed of four elements—earth, air, fire, and water—and introduced Love and Strife as cosmic forces..

Euclid of Alexandria (perhaps –325 to –265) Euclid was a mathematician known as the "father of geometry" for his work *Elements*, which systematically laid out the principles of geometry..

Eudoxus of Cnidus (–408 to –355) Eudoxus was a significant mathematician and astronomer and developed the theory of homocentric spheres to explain planetary motion and made important contributions to mathematics and astronomy. A student of Archytas..

Gottfried Wilhelm Leibniz (1646–1716) Leibniz was a philosopher and mathematician who co-invented calculus, and proposed the idea of a pre-established harmony in the universe..

Heraclides of Pontus (–387 to –312) Heraclides was an ancient Greek philosopher who suggested that the Earth rotates on its axis and proposed that Venus and Mercury orbit the Sun..

Heraclitus of Ephesus (ca –540 to –480) Heraclitus believed that change is the essence of the universe and introduced the concept of the unity of opposites, with fire as the fundamental substance..

Hilary Putnam (1926-2016) Hilary Putnam was a 20th-century philosopher known for his contributions to philosophy of mind, philosophy of language, and philosophy of science, particularly for his argument against the brain-in-a-vat hypothesis..

Leucippus of Miletus (about –480 to –420) Leucippus is credited with developing the earliest theory of atomism, proposing that everything is composed of small, indivisible particles..

Parmenides of Elea (ca –514 to –450) Parmenides argued that reality is unchanging and that change is an illusion, presenting a vision of a singular, eternal, and indivisible being..

Philolaus of Croton (ca –470 to –385 Philolaus posited that reality is fundamentally mathematical and proposed that the Earth revolves around a central fire, not the center of the universe..

Plato (–429 to –348) Plato, a student of Socrates, founded the Academy and introduced the theory of Forms, positing that non-material abstract forms represent the most accurate reality..

Ptolemy I Soter (–367 to –282) Ptolemy I Soter, a general under Alexander the Great, founded the Ptolemaic dynasty in Egypt and established the Library of Alexandria..

Pythagoras of Samos (ca –582 to –497) Known (incorrectly) as the originator of the Pythagorean theorem, Pythagoras founded a school combining religious rites with studies of mathematics, music, and astronomy..

Socrates (–470 to –399) Socrates is famed for his method of questioning to stimulate critical thinking and illuminate ideas, focusing on ethics and human behavior..

Thales of Miletus (ca –624 to –547) Considered the father of Western philosophy, Thales proposed that there is a single substance, water, as the fundamental substance of all matter. Rational understanding rather than deity as explanation for phenomena..

Willard Quine (1908- 2000) Quine was a 20th-century philosopher who challenged the analytic-synthetic distinction and contributed significantly to logic and the philosophy of language..

Zeno of Elea (ca –490 to ca –430) Zeno is best known for his paradoxes, which challenge the notions of motion and plurality, supporting Parmenides' views on the unchanging nature of reality..

Places

Athens The capital of Greece, renowned as the birthplace of democracy and Western philosophy..

Attica A historical region of Greece, home to Athens, known for its cultural and intellectual achievements..

Croton An ancient Greek colony in southern Italy, famous for its athletes and as the home of philosopher Pythagoras..

Ionia A region along the Anatolian coast, notable for its cities' contributions to Greek philosophy, science, and art..

Macedonia An ancient kingdom in northern Greece, famous for Alexander the Great and its role in spreading Greek culture..

Miletus An ancient Greek city on the Anatolian coast, known for its wealth and producing notable pre-Socratic philosophers..

Minoa An ancient civilization on Crete known for its advanced Bronze Age culture and palatial centers like Knossos..

Peloponnese A peninsula in southern Greece with historical cities like Sparta and Mycenae, significant in Greek history and mythology..

Syene An ancient city on the Nile, now Aswan, significant for Eratosthenes' calculations of the Earth's circumference. Its latitude is zero degrees..

Syracuse A powerful Greek city in Sicily, known for its cultural heritage and as the home of Archimedes. Also, Plato's laboratory for trying to train the despot, Dionysius, as a philosopher-king as envisioned in *Republic*. It failed..

Tarentum A major Greek colony in southern Italy, known for its naval power and cultural influence in Magna Graecia..

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