From the Greeks to Einstein
How the stories of motion and light became the Special Theory of Relativity

Draft

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Contents

Chapter 1

It’s All Greek To Me :
The Greeks

“We are all Greeks. Our laws, our literature, our religion, our arts have their root in Greece.”

- Percy Bysshe Shelley (1792-1822), *poet*

“There is a land called Crete, in the midst of the wine-dark sea, a fair, rich land begirt with water; and therein are many men past counting, and ninety cities.”

- Homer, *The Odyssey*

It may have once been the case that all roads lead to Rome, but for most of western philosophy, physical science, and mathematics, all roads lead *from* Greece. This chapter is the first stop in our path towards Einstein’s Special Relativity: our MOTION themes start with the Greeks, eventually centered on Plato and Aristotle.

Since this is a book on physics, and since you can only invent something once, I want to tell you how physics started. This is the first of three chapters on the Greeks and they will be different from the ones that follow as I’ll talk about many people, rather than one. In this chapter we’ll learn about the first steps to new habits of mind that evolved two centuries before Plato and drive us still. This is the beginning of asking three of our four questions: how (or whether) the Earth itself moves, how (or whether—looking at you, Zeno) things on the Earth move, and how the objects in the heavens move. When you see the tags MOTION BY THE EARTH, MOTION ON THE EARTH, or MOTION IN THE HEAVENS it signifies an important step on our path to developing our MOTION theme.

The quotes above are a small sampling of how we modern scientists should look back at the Greeks. In many ways my field of particle physics is relentlessly Platonic (but don't tell anyone that I said that!). Plato (and to a lesser extent, Aristotle) continues to challenge us: *What* can we know? And, *how* do we know we know that something is true? And, of course, *how* do things move?

The next chapter will deal with them—but Plato was reacting to the thinkers who came before him, traditionally called "Presocratics," obviously meant to cover those who came before Socrates. Now, "Presocratics" is an all-purpose label that applies to people before Socrates, but also those who were contemporary to Socrates, and even some who where younger than Socrates. For all practical purposes, it essentially means: pre-Plato. This chapter is about the Presocratics.

Here I will lay out four Greek Research Programs which still motivate us today, but which were really first identified by the Presocratics. Each theme was seeded before Plato and Aristotle and then watered and then harvested. I'll highlight them as we move along. They are:

1. **Is the universe constructed of fundamental building blocks and might those fundamental entities behave together according to rules?** This is the nature of physics today: my field of particle physics is dedicated to finding and characterizing the fundamental entities that make up everything else. Quarks and Leptons are those entities. But just stockpiling particles is merely stamp-collecting. They have to interact with one another and so the rules are deeply important. We call them the four fundamental forces today.

2. **Is the universe inherently mathematical?** It's long been appreciated that the universe seems to operate according to rules that are mathematical or can be described as mathematical. Discoveries in physics and mathematics have each influenced the other. Why that relationship exists isn't understood and is yet so persuasive to some theoretical physicists, that they postulate—still— that the universe is not just mathematical, but *is mathematics*. I'll have a lot to say about this as it underpins not only MOTION and LIGHT but all of modern science.

3. **How can we reconcile permanence with change?** This is a

tricky issue and one that bedeviled not only the Greeks, but much of philosophy to the present day. Unraveling this tension is intimately connected to theories of knowledge: what can we know and what can we trust? The permanent part of physics today refers to the various "conservation laws"...the Conservation of Energy, for example. But our elementary particles move around, they mix together, they annihilate and are born out of the vacuum. All the time. Change and permanence, agonized over by the Presocratics, are firmly a part of our modern story.

4. **How is the Universe structured and what are the rules that govern its beginning and current state?** “Cosmology” is the Greek word for this study that mashes together their word *cosmos* for "the world" or "universe," and *logos*, the word for "study of." It’s now a modern term and Cosmology is an entire discipline in physics and astronomy. It started with the Greeks and their ideas became, just like motion, mangled by Aristotle’s authority. It took 2000 years to get it right.

Greeks reveled in drama and it’s within the turmoil and bloodshed between the Persian Wars and Alexander the Great that western philosophy and nascent science had its beginnings. So we’ll picture this as a play in eight acts. The curtain rises...on a catastrophe.

I imagine that it started out like any bright day on the northern coast of Crete. A lazy afternoon in this peaceful paradise.

Then, total darkness.

Without warning, the **loudest sound** ever experienced by humans was followed on the northern horizon by a hint of fire and smoke erupting tens of miles into the previously clear sky. Slowly the sun dimmed, and then the sky became black as six inches of ash fell all over the island like a dirty rain. In fact, debris fell as far as the whole of modern Turkey, northern Egypt, and the middle east. Following that sooty deluge, tidal waves fifty feet high engulfed the sea-side areas of Crete and destroyed everything for kilometers inland.

That terrifying -1650 day...

> Wait...Negative years? I’m sorry, but in my head the timeline of history is a number line with positive and negative numbers—years. Sure, it’s a number line without a zero, but BC or BCE isn’t separated from AD, or CE by a year 0 either. The names are too clumsy to compare one year with another and so I prefer almost-straight-up arithmetic for years since it makes it a breeze.

...in the capital city of Knossos was the consequence of a massive volcanic eruption on the island of Santorini, about 100 miles to the north. Look at your map application and navigate to 36°23’41.46" N 25°23’57.55" E. There you’ll see a little Packman-like, backwards “C” feature in the Aegean Sea. That’s the scar—the caldera from the “Minoan Eruption”—left behind by the opening act in what might have been the story of us in the West.

Our tragic Minoa—modern-day Crete—was a refined culture of master architects, mariners, and traders, an apparently relaxed and leisure-loving people. Their cities didn’t seem to need much fortification—they seem to be secure among themselves and were rulers of the sea. They were literate and created the first *symbolic*, written language—two of them, actually. Their ancestors were pre-Bronze-Age migrants from the north, seasoned with Egyptian influence from about –3000.

Over the next thousand years, Minoans and Phoenicians became Mediterranean, international sea-going powerhouses trading across its entire breadth. Think about that: 1000 years of prosperity! Trading partners inclusive of hundreds of different cultures. After the volcano, they rebuilt but were never the same and were likely absorbed by a rougher crowd from the Greek mainland (which is called the “Peloponnese”). The Minoans are our literate ancient scientific ancestors, influencing the Greek culture even though they ceased to exist.

I like to think of those long-gone cultured Minoans as the polite part of our western scientific ancestors—the smart side of the family. But the famously disagreeable, Homeric Greeks came from that side of the family that you’d like to hide from your friends.

That “rougher crowd” were the Mycenaeans who evolved into the heroic Greeks of Homer’s *Iliad*, made perhaps slightly more civilized by their Minoan acquisition. The centuries following were eventful and then blank: Iron-weapon-wielding northerners created chaos with the Mycenaeans and eventually initiated a multi-century dark age. What emerged around −800 included the still-standing Athens, Sparta, and Corinth; the singing and eventual writing of the Homeric sagas; and an explosive emigrant population prominently on the Aegean islands, western Ionian shores, and the southern boot of Italy. Established by −650, these colonies were active traders, especially in Melitus in Ionia. Figure 1.1 shows the Greek colonial expanse and details of the immediate Aegean and Italian city-states.

1.1 A Little Bit of The Presocratic Greeks

Around 2800 years ago a proto-science began by people asking modern-sounding questions. We’ll concern ourselves with our scientific parents: the Milesians (in Ionia, on the modern day west coast of Turkey) who invented the idea of substructure and natural rules, the Pythagoreans (in Italy) who emphasized the fundamental nature of mathematics, the Eleatics (in Italy) who fleshed out the tension between change and permanence, and the Pluralists (in Italy and Ionia), who found a rational

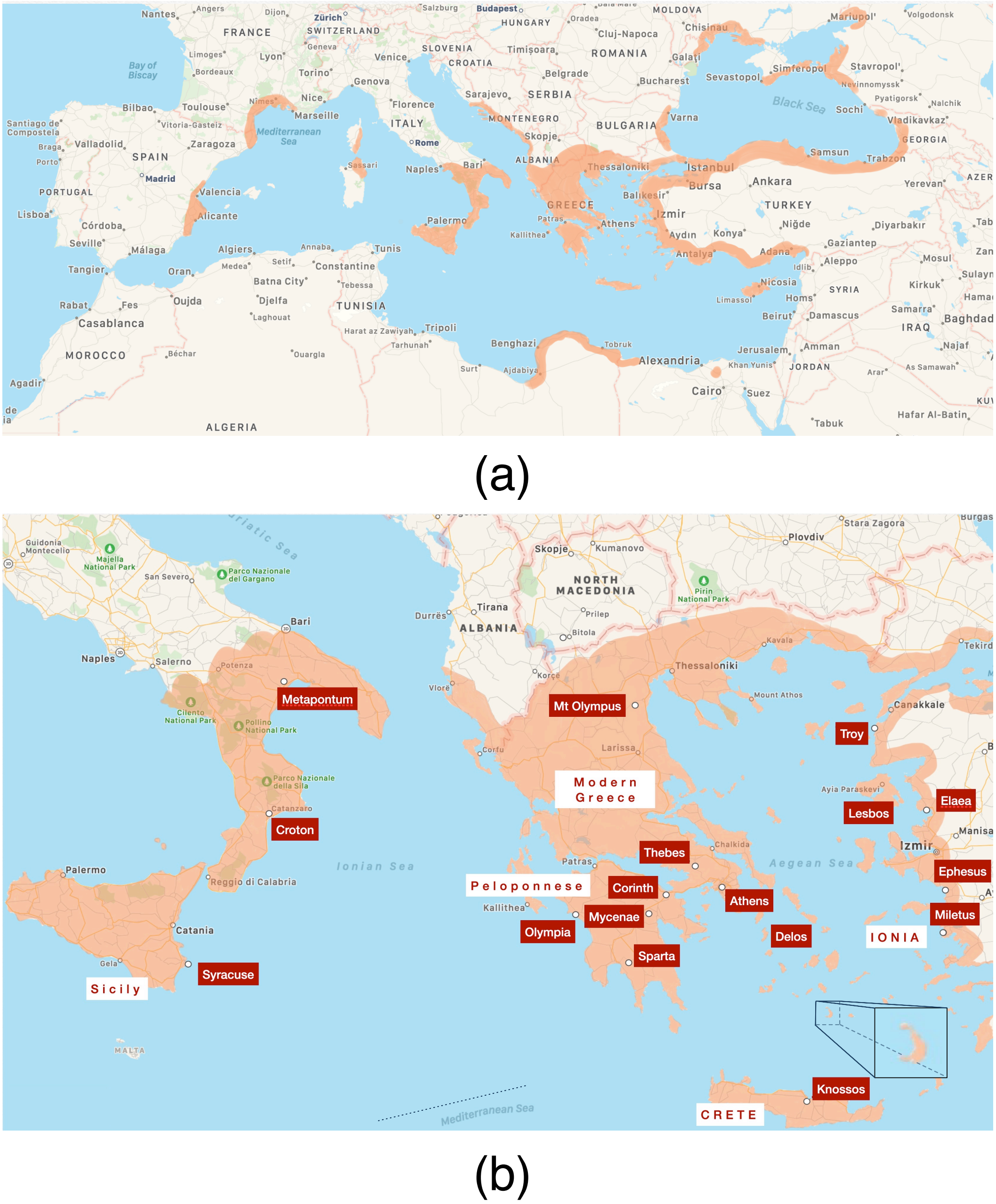

Figure 1.1: (a) The Presocratic and classical Greek colonial empire was vast, although I hesitate to use the word “empire” which implies cohesiveness since Greek colonies were only loosely connected to the mother ship. Eventually, the Egyptian port (to be called), Alexandria became the final storehouse of Greek learning, outside of Baghdad. All of this came at a price. Greeks were almost constantly at war. (b) The regions around Home Base show the eastern Ionian and western Italian Greek cities where the Presocratics lived. The inset in the lower right highlights the island of Santorini, the caldera left from the massive “Minoan Eruption” of approximately –1600.

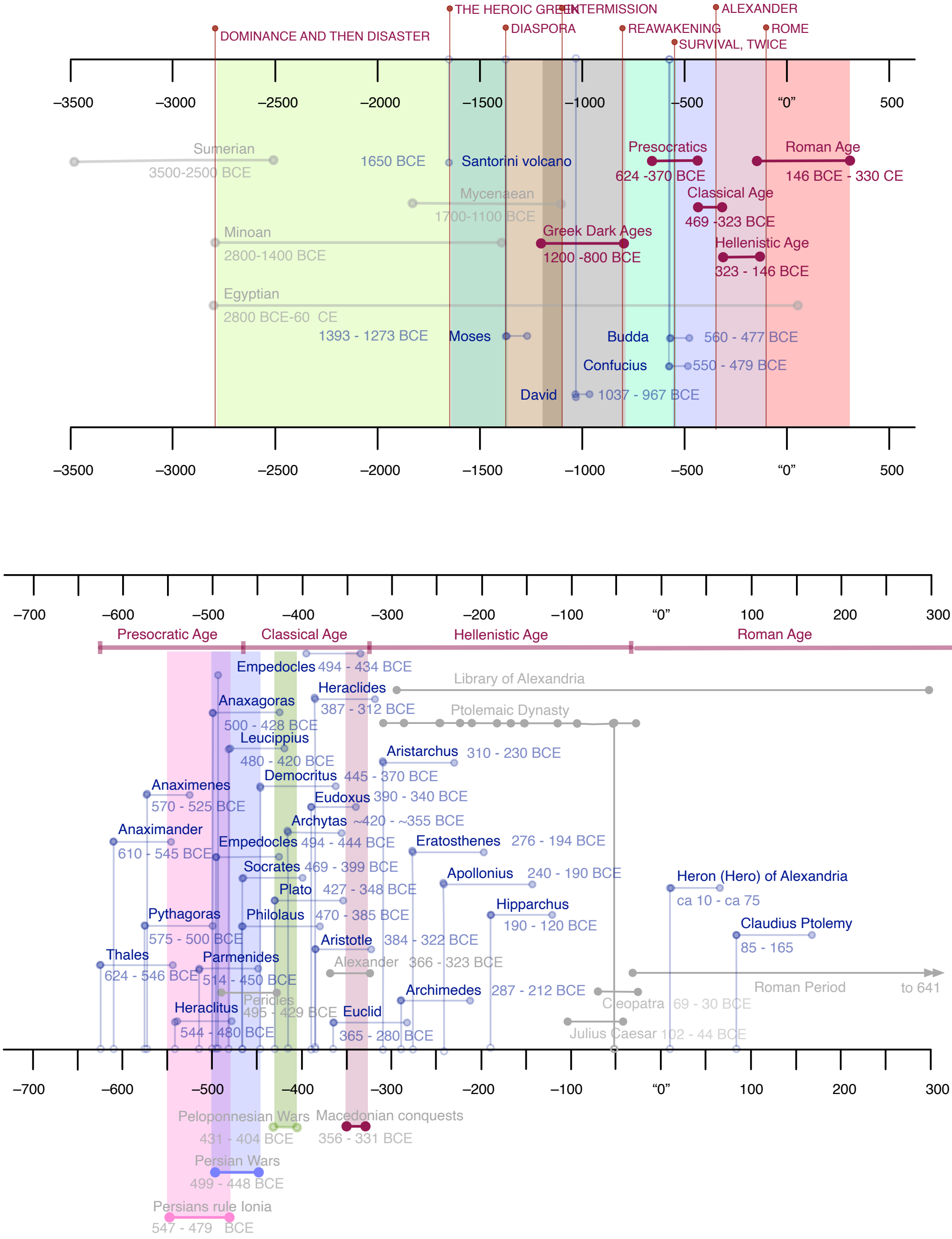

Figure 1.2: On the top, a Mediterranean timeline stretches from pre-biblical times to the end of the Roman empire. The bottom lays out the life spans of all of the Greeks you probably ever heard of...and the overlapping disasters that surrounded their lives.

alternative to the most persuasive and extreme of the Eleatics.

Brief relative (and rare) peace in the Ionian colonies, their positioning in the Mediterranean as a shipping crossroad, and the growth of large city-states led to a period suitable for growth of a new culture. And this was what emerged: The beginning of western philosophy. The time of the "Presocratics," literally those early philosophers who came before (or overlapped with) Socrates. These folks and their "Post-socratics (?)" asked modern-sounding questions of their surroundings.[1]

The timeline in Figure 1.2 shows roughly three distinct periods with names you might recognize. There are the Presocratics (from about –600 to about –430), the classic philosophers (from about –430 to about –250), and then the Hellenistic philosophers and scientists (from about -250 to +165). Notice that each of these periods overlap with war: Greeks fighting Persians, Greeks fighting Greeks (after the Persian wars, an over-confident Athens precipitated a dozen conflicts with Corinth and Sparta until the major Peloponnesian war), Macedonians fighting Greeks, and Greeks fighting the rest of the Mediterranean and Middle East. Notice that the whole of western history since the Magna Carta in 1215 would fit within a tick mark and a half in that top timeline.

1.1.1 ACT I: Is Nature Made From Stuff Governed By Rules?

Thales •Anaximander •Anaximenes •Pythagoras •Philolaus
(Set the context with the timeline in Figure 1.2 on page 8.)

Over my career I've published hundreds of scientific articles. Every publication has a common element: a bibliography with references to dozens or even more than a hundred other scientific works. Science doesn't happen in isolation as we're constantly building on, disputing, or confirming work of other scientists.

Take out a piece of blank paper. In many ways what your looking at is the bibliography of the first western philosopher, or even proto-scientist: **Thales of Miletus** (ca –624 to –547). Plato and Aristotle (and neo-Platonic philosophers who came centuries later) tell stories of him which form a lot of what we know. The fellow who invented history, Herodotus, also is a source.[2] Thales left no first-hand writings but stories about him abound.

Here's one: my favorite *New Yorker* cartoon is a Robert Weber's 1981 image of professorial-looking, tweedy fellow with pipe on a NYC street corner asking a cop, "Excuse me, Officer. I'm an academic. Where am I?" That image of us academics didn't originate in a fancy magazine. Plato told the story that Thales was walking along looking at the stars and deep in thought and dropped straight into a well that he didn't see in his path. That embarrassment wasn't enough, as Plato also notes

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[1] But the next century would see Ionia ruled by Persian-installed kings and tyrants.

[2] Herodotus was the first to tell about the past by trying to justify his assertions and find reasons for events. He's best known for his detailed history of the Greco-Persian Wars.

that a passing servant girl was on-hand to make fun of him in his reduced state.[3] But we also know that he was savvy enough to predict some weather changes and a possible bumper olive crop so he bought up all of the olive-presses in Miletus, and made a fortune selling them back.[4]

Maybe that happened. Here's another. It was suggested by Herodotus that Thales studied in Egypt, learned geometry and astronomy sufficiently to be able to predict an eclipse of the Sun on (our dating) May 28, -585 that pretty much stunned everyone, including causing a battle to pause. How did he do that?

Well, he couldn't have. That didn't happen. Available data wouldn't have allowed anyone to make such a prediction. It's trivial *now* to point back to the line of totality (the swath on Earth that would be dark) which would maybe have indeed been over the historical battle site, at that time. But a prediction? No.

Determining the veracity of stories like these is an example of a detective-story-approach to unraveling Thales and the other Presocratics: The eclipse fable suggests that Thales might have been an adult in -585 and thought by Herodotus to be a well-enough respected personage that his "predictions" might have mattered. So this story, while fiction, did contribute to the picture of the man called Thales and his reputation and his timeline. Thales was a real person.

What's not in dispute is that he initiated, or was a part of, a new way of asking questions *and* a new standard of what constitutes acceptable answers. Nobody thought like him and his immediate successors, and now we all do.

1.1.1.1 The World Before Thales & Co.

Why does it rain? Why are there earthquakes? Why are some people honest and others not? Why did my crop succeed and yours fail? Why is the Earth suspended under the sky? If you're Greek before about -500, there's a god for that. Why are there clouds? Yup, a god for that too. Why does the Sun shine? Another god. I tried to count all of the Greek gods, titans, minor deities, spirits, sea gods, agricultural gods, "rustic" gods, plus health and sleep gods. Oh, plus almost 30 mortals who earned promotion to god-like eternal life. It's hundreds. There is the varsity team—the 12 gods of Olympus and the 12 Titans. But the god-team bench is really deep.

Take earthquakes. Currently, Greece ranks fifth or sixth in propensity for seismic activity. So Greeks were used to their ground moving. What everyone knew was that earthquakes happen because Poseidon (the god of the sea) is irritated. Without enough attention, he bangs his trident on the ground from Olympus and they get an earthquake. Or rain. If water falls from the sky it's also the case that another

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[3] Plato's references to the Presocratics are often to make fun of them.

[4] He was also an astronomer of note and a mathematician with theorems to his credit. An all-around academic.

petulant god is unhappy, this time Zeus (the god of a lot, including the weather) using his lightening bolt symbol to make trouble.

There's a madness to this, but also some sort of understandable urge to assign every human experience to an outside influence. While Homer's tales include the gods as major actors, it was Homer's contemporary, Hesiod who thought that the story of the gods needed a rational and believable origins story. His *Theogony* is basically the story of the origins of the world including the genealogy of the gods. There's also a cosmology in these myths: the gods are themselves born... they've not always been around. And they have lives—outrageous ones.

That's interesting. They could have just "been there" outside of time, like in other religions, but Greek myth seemed to require a logical structure: Earth (Gaia) and Sky (Ouranos) were the first and their union is followed by scenes from *Animal House*... no, much worse. Infanticide, incest, fratricide, cannibalism, mutilation, and betrayal follow among the gods and the Titans, and between them and regular humans. Murders are the most light-hearted events in Hesiod's story.

The bottom line of Greek mythology is that everything happens for a reason. Why? Because a god is benevolent or unhappy or just doing their job.

1.1.1.2 Thales' Science and His Successors

GREEK RESEARCH PROGRAM #1 : Thales ushers in the first Greek Research Program, that the world is made of some fundamental substance that behaves according to natural laws.

Thales was the first that we know of to take a different approach. He's best known for asking what is the underlying, common structure of the universe, what Aristotle called on his behalf, the First Cause.[5] Thales reasoned that all of our universe depended on a single substance, and for him that substance is: water. After all, without water or moisture, things perish. Water is in the air and condenses and wets surfaces. It evaporates and reappears, sometimes revealing (creating?) soil underneath. Nothing lives without water and when things die, they become dry. So as a single substance acting as the basis of all things, it's not too bad. This description of the world is **materialistic** and **monist** (the view that there is one underlying substance).

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[5] Aristotle uses that word. But Aristotle was fond of Aristotle's philosophy and his reliance on "Cause" and "Substance" in his own work, motivated his description of the Presocratics' work. Those words were not available to the early Presocratics.

This concept is the first of three novel features of Presocratic proto-scientific thinking.

1. Thales suggested that humans could understand how the world works, including what causes the events and things that we experience. His suggestion is that the world is made of fundamental stuff guided by rules—laws of nature, so to speak—that govern how that stuff operates. The world needn't be a mystery.
2. Their "how" commitment searches for naturalistic reasons for events and existence. The previous "why" commitment was satisfied that "a god did it." For the "how" answers, the gods aren't involved. For example, the early Greeks inherited an ancient idea that the Earth is a flat disk with a dome of sky overhead, surrounded by a river (the Ocean or *Okeanos*) and the whole thing is held up by Atlas as a punishment handed out by Zeus. Thales agreed with the geographical part of this cosmology that the disk floats on water but earthquakes happen when the water sloshes. A wildly wrong explanation, but completely naturalistic. Poseidon is not involved.
3. Finally, the Presocratics jousted with one another: an idea or a research program from one, might be incorporated in another's account. Or, an idea or research program of one might be a focus of criticism resulting in an alternative account.

> This is not yet science, but science can't happen without at least these three commitments: **we can know about a rules-based universe**, **"how" cannot depend on the supernatural**, and **competition and collaboration are essential** to carry a project forward. All of this was new and now familiar.

Others who came after Thales adopted the same "research program" hypothesizing and defending an underlying substance for the world. Thales' "A" students, **Anaximander** (ca -610 to -545) and **Anaximenes** (ca -570 to -525) asked that question and answered it in different ways, but with the same basic motivation. Each of them had their own underlying substance idea.

Anaximander gave us one of the first maps, perhaps the sundial, and a full cosmology including a hockey-puck-like cylindrical Earth floating at the center of the universe. He watched the stars go around us and concluded that the Earth can't be falling…so it must be balanced at the center of the cosmos.

> Here, is our first reasoned theory of MOTION, in particular MOTION BY THE EARTH. He concluded that there is no motion for the Earth, but for a reason. Because of symmetry and balance.

Anaximenes went a step further and realized that what's important is *process*—things turn into other things. Cycles happen. Lawlike behavior is evident. Neither Anaximander nor Anaximenes went along with Thales' contention that water could be the sole source of stuff—how can water be the source of its opposite, fire? That's not the point, though! They rejected his specifics, but bought into the project: While Anaximander chose something etherial and not itself one of the substances (the spooky "Apeiron"), Anaximenes chose air as the fundamental substance, but he had a scheme whereby air's various guises could account for the actual things we experience.

By this point, proto-scientific practice is pretty much up and running. They were naturalists, materialists, and the first Empiricists—using their powers of observation to study their world and attempt to explain it without recourse to a deity or a dogma.

1.1.2 ACT II: Pythagoreans in the West

It must be exhausting being a philosopher in your day job while also moonlighting as a deity and yet **Pythagoras of Samos** (ca −582 to −497) seemed to function as both, or so his followers asserted. Yes, that Pythagoras: of the triangle, although it's probably not what you think. What Pythagoras taught and what evolved out of the long Pythagorean school is difficult to parse today so it's not fair to attribute all of "Pythagoreanism" to that one person. The ideas that are attributed to him originated in Italy but evolved considerably becoming a dispersed movement that spread throughout the Hellenic world and beyond to the Renaissance hundreds of years later. Indeed by Plato's time, Pythagoras was already an enigma. As we'll see, Plato probably learned about him through Philolaus of Croton and Archytas of Tarentum, two acknowledged second generation Pythagoreans and mathematicians in their own right. So we have a nearly mythical figure: In the near-term there was Pythagoras, "so-called Pythagoreans" (as Aristotle called them), and Pythagoreanism…the seed-philosophy of mathematics that has lasted in some form to the present day. I'll mostly use the plural "them" rather that the singular, "him." "Pythagoras" is essentially the name of a movement and a culture and no more reliably an individual.

His biographical details are from Roman-era writers and enthusiasts and it's difficult to know what's believable. He grew up in the Aegean island of Samos and reportedly met the elderly Thales, and maybe studied with both Anaximander and Anaximenes. So suggested Heraclitus, from whom we do have actual written (critical) fragments. He may have traveled around the Aegean with his merchant-marine father and probably on his own lived in Egypt and maybe Babylon for at least two decades, absorbing language, philosophy, and mathematics. So, a well-traveled, probably comfortable young intellectual. The politics of Samos became tenuous and in spite of the fact that he'd established a following of students, at the age of 40, he relocated to the large Greek city of Croton in the "instep" of the boot of Italy. Some accounts suggest that he was accompanied by a number of loyal followers—the Pied Piper of Samos?—but most suggest that he moved by himself. In Italy he again established a following of reputedly as many as 600 (some say thousands) men and women in Italy and actually wielded some civic influence in Croton, serving as both an advisor and unwelcome busybody. He eventually founded a school that was to last 300 years, twice as long as my own Michigan State University has been around.[6] The ideas generated from that time evolved and the border between the man and the movement is impossible to demarcate today.

This unusual school also functioned as a mystical, essentially religious cult. Its members were regimented as to how to dress, what they could eat, what they may believe. . . and what secrets they must keep. They loved secrets. Pythagoras was its head and was by legend, supreme. He taught reincarnation and himself thought he remembered past lives.

The basic discovery moment came from thinking deeply about musical tones which they extrapolated to the proposition that numbers and mathematics are a fundamental fabric of the universe. Although they were not in competition with the Ionians, reliance only on a substance-based first principle wasn't sufficient for them. Not given the discoveries that they believed they'd made about mathematics and the world:

> "All things have form, all things are form; and all forms can be defined by numbers." Pythagoras
>
> "The Pythagorean . . . having been brought up in the study of mathematics, thought that things are numbers . . . and that the whole cosmos is a scale and a number." Aristotle *Metaphysics*

1.1.2.1 The Most Durable Discovery in History

GREEK RESEARCH PROGRAM #2 : Pythagoras ushers in the second Greek Research Program, that the world is mathematical. Or even that the world is mathematics.

Pythagoras left no writings, but stories/fables/tales reported by dozens of others abound. He claimed (or it was claimed for him) to have discovered integer relationships among the strings of a lyre[7] and the pleasant chords it could make. The lyre was probably a 7-string variety although he reportedly built a one-stringed tool ("kanon" or "monochord") to study its behavior (although that story is disputed). A quick taste of what the Pythagoreans left for us (and for Plato!):

When you pluck a string, clamped at the ends, you cause the string to vibrate with a fundamental frequency related to the length (and tension—think, a guitar). Call

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[6] But both his and mine are mere babes, as compared with Oxford University, the University of Paris, or the Academy of Plato.

[7] and the tones from cups filled with different amounts of water which were noted for their pleasing sounds

that the “ground note.” (A Pythagorean scale is different from how a piano is tuned, but I’ll use piano as my analogy.) A piano’s middle C is a natural ground note and has a frequency of 261 Hertz (Hz, cycles per second). Pressing the lyre string at a half-way point and then plucking one of the two halves will cause the ground note to be repeated, but an octave higher. (On the piano, C above middle C is a frequency of 522 Hz, twice 261 Hz.) Pressing a lyre string at 2/3 of the length and plucking the long remaining string, causes the fifth above the ground to sound (for the ground of middle C, that would be G, or 392 Hz, 3/2 of middle C’s frequency) and pressing 3/4 of the length, a fourth above that (A above middle C at 348 Hz, 4/3 times that of middle C’s frequency).

Play those intervals on a lyre or chords on a modern piano and your ears will be happy. These are pleasant-sounding combinations while other combinations are not so sweet—we say dissonant. To the Pythagoreans, the difference between pleasant and dissonant was in the integer ratios of the string lengths—the *numbers themselves* were important.[8]

> To them, this revealed an **intimate link between numbers and the world**: integer ratios 2/1, 3/2, and 4/3 → to specific lyre string lengths → to pleasing your ear (your soul). This relationship made the numbers 1, 2, 3, and 4 very special to them.

Lyres had been around for millennia, so surely this particular discovery was not news. But what Pythagoreans did was new. They elevated numbers to a significance that’s *beyond just counting*. They **invented the concept of number itself**: from 2 oranges to the abstract concept of “2.” This direct connection between a few integer numbers, their ratios, and special numbers with important meanings[9] influenced all that’s “scientific” up to the present day. This discovery is one of a few discontinuous events in the history of science. To them and many to the present day, mathematics is embedded in the physical world’s tissue: A brand new commitment.

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[8] It’s a matter of current physiological research to understand why some combinations of tones are pleasing and others are dissonant.

[9] Notwithstanding "42" as the numerical explanation of everything in *Hitchhiker’s Guide to the Galaxy*

When it comes to Pythagoreans, who did what, when is murky. In the lower timeline of Figure 1.2 between Pythagoras and Plato you'll see **Philolaus of Croton** (ca -470 to -385) who was the first Pythagorean to write about their program, although only fragments and references from others remain. A lot of what Plato and Aristotle knew probably originated from his writings. (Plato only mentions "Pythagoras" and "Pythagorean" once each, but Aristotle was more expansive.) Plilolaus was a scholar in his own right and it's hard to discern what ideas were his and what came from Pythagoras himself, or even in Pythagoras' lifetime. What Plato and Aristotle knew of Pythagoreanism probably came from Philolaus or Archytas, another Pythagorean known well to Plato[a] Highly readable accounts are Ferguson (2008) and Lloyd (1970).

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[a] And, what we know of Philolaus might have come from the Pythagorean, Hippasus. The most unlucky Pythagorean. He is remembered as having constructed bronze disks who's thicknesses matched the lyre string ratios. When struck they would then create the same pleasing sounds as strings. He's also historically the poor guy who found the non-integer problem with the Pythagorean Theorem. Stay tuned.

This connection between integers and one's soul seemed to have been just the beginning. They also connected numbers with shapes and so geometry and by extension, to space itself. Keep them in mind: 1, 2, 3, and 4.

What can you do with them? Well, you can add them: $1 + 2 + 3 + 4 = 10$ which is not such a complicated thing, but these are special numbers after all and so their combinations must be special as well: "10" was important.

You can also make patterns with numbers—and a highly useful definition of modern mathematics (especially in physics)—is that it's the process of finding patterns. Figure 1.3 shows examples of Pythagorean patterns with integer numbers.

Figure 1.3 (a) starts with one stone, and adds the first odd number, 3,[10] arranged around it turning $1 + 3$ into 4, *but it also laying them out as a pattern in space.* Numbers = geometry for the first time. This is a "square number" which follows the rule (in modern notation) of $1 + 3 = 4 = 2^2$. We can expand this into more square numbers and the next one is in Figure 1.3 (b) which shows that $1 + 3 + 5 = 9 = 3^2$. One can also take two stones and add the next even number around them in (c), say three above and one to the right, to get a "rectangular number."

Especially important is the arrangement shown in Figure 1.3 (d). Remember, $1, 2, 3, 4$ are special. Lay out four stones, then layer three on top, then two, and finally one. You've now made a special triangle—-the tetraktys ("fourness")—with 4 stones on each of three sides. So it's an equilateral triangle and all four of the important numbers are contained in it... adding to 10. Maybe they liked bowling.[11]

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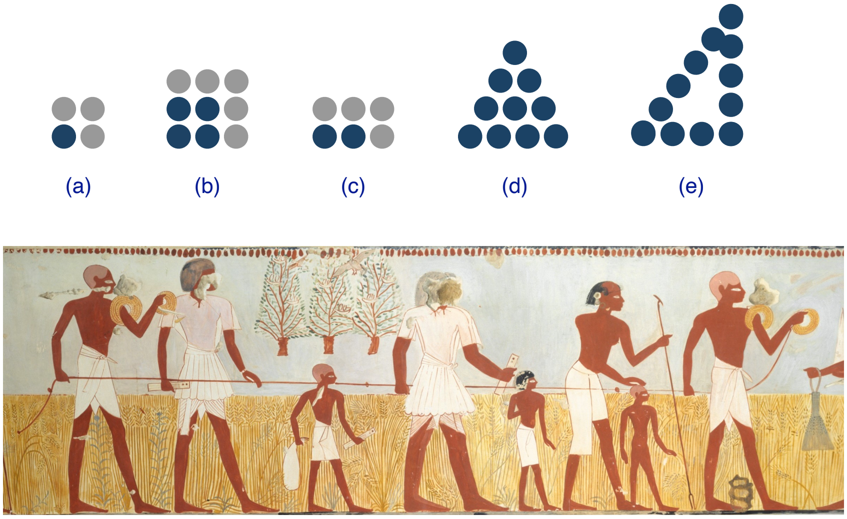
[10] The number 1 was not a number for them: numbers meant a plurality. One is not "odd" nor is it "even." It's unique.

[11] There is a fable that a Pythagorean became ill at an inn while traveling but had no money to compensate the owner for his care while convalescing. The traveler told the owner to hang an image


(a) (b) (c) (d) (e)

(f)

Figure 1.3: Dots represent stones that they would have used to signify numbers—precisely like the dots on dice. The image (f) is from the Tomb of Menna showing Egyptian workers getting ready to do surveying with a knotted rope. See the text for a description.

There's another connection between numbers and geometry—again, connected with the physical world. "1" was a special number, neither odd nor even (for them) and plays the role of a beginning. The source. A single isolated point is the starting point (no pun intended) for everything. "2" represents a line, which starts with a point and is constructed of points. "3" represents a triangle which delineates a flat plane and is constructed of lines, and "4" represents a tetrahedron, a three-dimensional solid constructed of triangles. That's it. Three dimensions to our physical space is all there is and so "4" represents completion and its encoding in the tetraktys (count the stones in any direction in the tetraktys and you'll count 1, 2, 3, and the base, 4) and that relationship with "10" tied it all together for them. (Of course today multidimensional spaces are a mathematical walk in the park. We know that our physical world consists of at least four dimensions. So stopping at "4" was premature!) There's more. "5" is special as it's the sum of the first even and first odd number. "6" is special since it's both the sum of the first three numbers and simultaneously, the product of the first three numbers. And so it goes.

Notice that there's another triangular pattern in (e). If you count the spaces between stones, you'll find that they delineate $3-4-5$ which is a familiar triangle to some of you, but a familiar triangle to thousands of years of Egyptian builders. This triad of numbers has practical value as it's a sure-fire way to make a right angle. Take a length of rope and tie 12 knots equally spaced from end to end (like the stones in the figure). Then have a worker hold one end, another hold the third knot, and a

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of the tetraktys and other Pythagorean travelers would compensate him far beyond his original costs. And they did. So it goes.

third 4 more knots along. If the other end is then given to the first worker. The only way that making each of the three segments taunt is for there to be a right angle between the 3 and 4 knot segments. There are other such triads that make a right angle in this way, for example $6-8-10$. The ancient Babylonians and Egyptians knew of many of them and used them in surveying and building without realizing that this was an important thing. Figure 1.3 (f) is from the Tomb of Menna showing a knotted rope for surveying. As you know from high school, Pythagoreans figured out what this means in an abstract way.

There was a mystical quality to numbers and numerology was a thing and so the numbers also had special meanings for things beyond just "quantity." For example, 5 is the sum of the first even and odd numbers $2+3$ and since 2 symbolized female and 3 male, then 5 symbolized marriage. The first even number is 2 and squared is 4 and so that first square number, 4 symbolized *justice*. Likewise, the first odd number is 3 and its square is 9 and so it also symbolized *justice*. (Even today, we refer to a "square deal" as a proper deal.)

In fact, 10 was such an important number that in one version of Pythagoras' cosmology we have another early moment of MOTION BY THE EARTH. The Earth and all of the other celestial objects moved around something called the "central fire." This actually comes from Philolaus:

> "The first thing fitted together, the one in the center of the sphere, is called the hearth." Philolaus *Fragment 7*

The bodies are, from the inside-out, Earth, Moon, Sun, Venus, Mercury, Mars, Jupiter, Saturn, and the celestial sphere, but... wait. That adds up to 9. It must be 10 in order to be right, so they added the "counter-Earth" who's orbital mechanics are such to be perfectly positioned to block our view of the central fire since we don't see it.

> "... inasmuch as ten seemed to be the perfect number and to embrace the whole nature of numbers, they asserted that the number of bodies moving through the heavens were ten, and when only nine were visible, for the reason just stated they postulated the counter-earth as the tenth."
> Aristotle *Metaphysics*

That's a very modern interpretation of the use of mathematics in physics. You postulate the importance of a principle ("10 is magic"), you create a model of the universe (or some small part of it) built within the model, and then using the basic rules of the model (like arithmetic or something fancier) tweak it while still committing to the model. Here the counter-earth was such a tweak. That's actually how physicists work within models until they become unwieldy or are ruled out by experiment. I'll have more to say about a modern day view of Pythagoreanism *Presocratic Greeks, Today* in Section 1.2 and their cosmology in Chapter 3. It's a matter of much discussion (a polite way of saying, "argument") today. It gets worse when we add Plato to the mix.

> The connection of music and integers led Pythagoras to assert that the regular harmonies of the cosmos were everywhere. The planets and stars all move and emit tones that ordinary humans can't hear since it forms a background to everyday noise. But Pythagoras? Ah, he's different. He's the only human who can hear the Harmonies of the Spheres. Being a deity has its privileges.

You're wondering about that theorem, I know you are. Look at Figure 1.4 and relive high school for a moment. Notice that Figure 1.4 (b) is the knot/stones-version of the Egyptian right-angle trick.

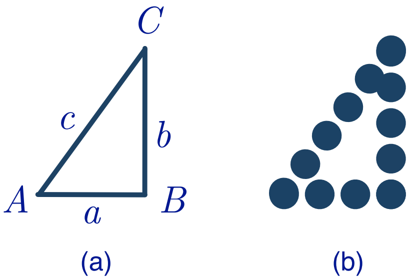

Figure 1.4: In (a) is a modern-day version of drawing a triangle, while in (b) is the same thing but with stones or knots delineating distances. The length c is the hypothenuse.

Maybe you remember the little song for a right-angled triangle: "…the square of the hypotenuse is equal to the sum of squares of the other two sides."

Or less lyrically,

$$c^2 = a^2 + b^2.$$

> There's no evidence that Pythagoras first proved this, and in fact, plenty of evidence that it was long known before him. (There are now about a hundred different proofs of the "Pythagorean Theorem.") The Egyptians had a real estate problem to solve: the Nile overflowed its banks every year and the fertile crop land alongside of it would be covered with water. That meant a problem: once the water receded, whose land was whose? Out of a need, geometry for Egyptians was a necessity. This was another job for the practical $3^2 + 4^2 = 5^2$. But the Babylonians were the champs. Not only did they keep accounting records, they did so in a base-60 number system...which must be 6 times better than our base-10 system, right? We've fragments that showed that they had worked out things like $119^2 + 120^2 = 169^2$ which admittedly doesn't come up every day.

There is a Pythagorean-Theorem story that tells you much of what you need to know about his cult. Remember, integers were the thing and so we feel sorry for the poor guy (historically, maybe Hippasus) who noted that a triangle with legs of 1 would have an hypotenuse that's Pythagorean-impossible since $1^2 + 1^2 = (\sqrt{2})^2$. This

$\sqrt{2} = 1.4142135624\ldots$[12] never ends—the definition of an "irrational number"—it goes on forever and so decidedly not one of the mandated integers. Since he'd found a non-integer, for his trouble, as the story goes, he was thrown overboard from a ship in order that his little discovery not be revealed to the other cult members. Maybe this happened.

In the end, as sometimes happens with cults, Pythagoras' welcome in Croton wore out. His house was burned and he escaped, only to die in his escape. . . or not. We don't know. But what he and his colleagues created lived far beyond them.

1.1.3 ACT III: The Eleatics in the West

Heraclitus •Parmenides •Zeno
(Set the context with the timeline in Figure 1.2 on page 8.)

What happened next unsettled the young enterprise of philosophy and, after Plato and Aristotle, initiated millennia of philosophical controversy. We saw that the Ionians relied on their senses and took it for granted that events in the world changed in time. But you and I have both learned that our senses can be tricky and not always accurate. And, even if we see/hear/feel accurately, the targets of our perception themselves change. So if that's the case, then what about our "scientific" observations? Can we trust our senses to gather accurate impressions of our surroundings and base our theories on those impressions? This investigation traditionally pits two Presocratics against one another, the "Riddler" of Philosophy, **Heraclitus of Ephesus** (ca –540 to –480) and the first "Lawyer" of Philosophy, **Parmenides of Elea** (ca –514 to –450). The former was an Ionian from the big city of Ephesus, not far from Miletus. The latter, was from the colony of Elea in southern Italy. Pythagoras' territory.

Heraclitus was a loner, while Parmenides evolved a school of philosophy called the "Eleatics." You might not have heard of that, but you may recognize one of Parmenides' significant followers: Zeno. . . of Achilles and the Tortoise fame. Heraclitus (by himself) and Parmenides and his followers took up the subject of change. Heraclitus was decidedly on the side of, sure, things change. But he took it in an abstract direction. On the other side, Parmenides concluded that change is an illusion. He even *proved* that change is an illusion. At first glance, that seems strange, but his novel method of philosophizing was persuasive and as a consequence he created two branches of philosophy. And in the course of digging into the problematic nature of Change, set off a huge argument over centuries. Obviously, this is prior to any kind of physics-like analysis of MOTION!

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[12] "dot dot dot," ... is mathematics-speak for "never ends."

GREEK RESEARCH PROGRAM #3a : The Problem: Tension between Change versus Permanence begins with Heraclitus and Paremenides.

1.1.3.1 The Riddler

Although we know few details of Heraclitus' life, he was apparently prominent in Ephesus. His father was said to have been an aristocrat, but Ionia was under Persian control during his life and suggestions that Heraclitus might consider a political life might be hard to picture. He wasn't a people-person. He would have been a child when Anaximenes died but he was critical of the Milesians and scathing in his criticism of his contemporary, Pythagoras. About 100 fragments of Heraclitus' work remain showing that his style was. . . unusual. He wrote very short tweets which have puzzled and delighted readers for thousands of years.

He was a monist as well: fire was his fundamental substance. And as interpreted by one of his aphorisms, he had a cosmology,

> "This world-order [*kosmos*], the same of all, no god nor man did create, but it ever was and is and will be: everliving fire, kindling in measures and being quenched in measures." Heraclitus

This is the first time that the word "cosmos" appears in Greek philosophy and he's clearly insisting that the cosmos always was the case and always will be the case. That's interesting since Plato deliberately labeled him inaccurately as naively saying that "everything changes" and that nothing is permanent.

You and I think of MOTION ON THE EARTH as moving from one place to another during some time, right? Remember, the Greeks were just beginning to do this analysis and moving from place to place was not their primary concern. Change by itself was and Heraclitus was the first to abstract *any* change as basically a form of motion, seeming to assert that universal "flux" was an important feature in the world.

> "It is not possible to step twice into the same river." Heraclitus

This is a famous paraphrase of a translation of his most famous of three "river aphorisms," The idea is that the river is always flowing and if you step into "the river" once, and then step into it a second time, it's a different river. So two rivers sort of functioning at the same time. It's a little different from this one:[13]

> "As the same thing in us are living and dead, waking and sleeping, young and old. For these things having changed around are those, and those in

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[13] While the most famous Heraclitus aphorism, there are at least three versions of it and some dispute as to its overall authenticity.

> turn having changed around are these.” Heraclitus

A young person is connected to their older self through the changes that they undergo. A is different from B, but linked because *A changes into B*. But, living and dead? This is a deep idea and seems to suggest that A and its opposite, B, are actually the same thing. In fact Change here has a job: it’s a sort of glue that links together different things or different aspects of a thing. So apparent opposites are connected meaning that everything in the world is connected. One.

Plato used Heraclitus as a punching bag. He said look what Heraclitus gives us: logical contradictions! Plato had an agenda. Aristotle was a little more forgiving and we’ll see how Aristotle codified and categorized change, which will explicitly include our notion of loco-motion. But it seems that he had to go through Heraclitus to get there.

It’s easy to be amused by Heraclitus’ words, and for millennia, that’s been a sport and I have more for you in *Presocratic Greeks, Today* in Section 1.2.1 below.

1.1.3.2 Nothing Gets Done: The Parmenides Problem

Parmenides took the extreme, opposite position, probably writing after Heraclitus. His argumentation is tightly logical so much so that it’s possible to be swayed by the apparent inevitability of his arguments. If you can penetrate the denseness of it. I’ll call his oddly persuasive but troubling conclusions the **Parmenides Problem**. It will seem to us like the Parmenides Problem will not go away.

He is the first in a long line of philosophers of both Metaphysics (the philosophy of the nature of being) and Epistemology (the philosophy of knowledge). He wrote a single book in verse (and according to Aristotle, not very well). It’s a narrative story about his meeting with a goddess and how she teaches him about two kinds of knowledge.

There is the “first path” to knowledge: knowledge that is true by necessity. This “Way of Truth” is confined to your reasoning, not your senses. The second path to knowledge, that of perception, is “habit” and from “your heedless eye.” This “Way of Seeming” is needed in order to get along in the world, but you can’t trust it because you can be fooled. For that reason the “Way of Seeming” can’t tell you what is true. So:

> The Parmenides Problem: True means permanent. Change means not true.

Remember our own experiences: our senses can fool us and the objects of our perceptions can evolve between observations. What can you trust in the world if not your eyes? So he got rid of both issues. Truth can only refer to permanent things.

Accepting his premises, his logic seems oddly persuasive. In a nutshell, which could be on a T-shirt, I can sum up Parmenides in his two words (read it carefully…if nobody's around maybe even read this out loud): "**It is**." It's punchy. He also then reasons that "**It is** and it cannot, **not be**." *It cannot…**not be***. If something **is**, it can't be **not-is** at the same time. Further, if something **exists**, then **it is**. Consequently, if it **doesn't exist**, then it is **not-is**. So knowing what **is**, is to know what **exists**. So far, so good. Something can't exist and not exist simultaneously. (Can you see how this is against Heraclitus, who seemed to welcome *A* and not-*A* simultaneously?)

He goes further. If something exists (it **is**) then also it could never have been different in the past, nor will it be different in the future. If it came into existence as **is**, then before that event it must have been: **not-is**. If it changes into something else in the future, then it goes from being **is** to then being **not**-is. How can something at one time be **not-is** and at another time be **is**? That can't happen! So if something **is**, it's always been **is**. In some sense, then the past and the present are one. Whew. Are you with me?

He's staked out clever ground in two new ways: His approach seems so logical that it launched philosophical analysis as an appropriate way to make arguments. And, he's defined what it is to be real: what's real must be true and therefore, it must be unchanging. And the landscape of truth is in your head. Where you reason.

Parmenides' sidekicks ran with this. Zeno took his arguments to the extreme and that's our connection with MOTION. Maybe you remember the story of how Achilles couldn't beat a tortoise in a race?

> This is one of 10 of "Zeno's Paradoxes," **The Achilles**. Achilles, being the fastest human, is to race a tortoise, maybe the slowest animal, so he gives the tortoise a head start, halfway to the finish line. They both start but poor Achilles is faced with an impossible task. In order to traverse half of the distance to the tortoise's staring point, he has to traverse half of that half. Then half again of *that* half. In fact he needs to travel through an infinite number of paths, which is impossible so he can't catch the tortoise! There are three other paradoxes on motion (The Dichotomy, The Arrow and The Stadium), all designed to support Parmenidean conclusions about motion.

Zeno gets this from Parmenides and since the reasoning seemed to be impenetrable, all of those races that you've seen with your lyin'eyes were apparently fooling you. I touch on two others of Zeno's Paradoxes in*Presocratic Greeks, Today* in Section 1.2.3 below.

> We've now encountered examples of significant philosophical or scientific commitments. Sides were beginning to be drawn in natural philosophy that continue to this day: Can knowledge about the world be gained by thinking? Or must knowledge come from observation. The former is called Rationalism and the latter, Empiricism and to this day physicists argue about this. Clearly Pythagoras is in the first camp and so was Parmenides—distrust of the senses disqualified observation as a source of truth. And, geometrical argument seems like a good example of what must be true. The Ionians pioneered the second camp gleaning knowledge and theories about the universe by looking and hypothesizing from their observations.

Finally, the void. The vacuum. A state of actual nothing! By now you can imagine what Parmenides thinks of such an idea: it's impossible since it's the state of **non-being**. Another Eleatic, Melissus took this to the ultimate conclusion without the need of Zeno-like paradoxes. Just logic: anything that **is** cannot move since it would need a place to move to— it would need an open space where **nothing is** in order to relocate. But a place where **nothing is**. . . is nothing. But nothing can't be the case so there is no motion. Another MOTION problem.

Parmenides was the first to seriously question what can be known and by what means. Your senses deceive you all the time. You can't depend on your observations for truth. But at the same time, your rational, logical thought—an argument assembled before Aristotle invented the actual rules of logic—is dependable. He then laid out a dispassionate argument that leaves one wondering what in the world is wrong with it.

> How do we get around this? In order to do science, or frankly, to live, one has to be able to hold a tentative, hypothetical idea in your head. Something less than demonstrably the case. But Parmenides was worried about Truth with a capital "T" and so he can't abide an idea that is not true or even tentative as a stand-in for what's true and so his philosophy was sterile. Scientists don't deal in that kind of truth.

Well, this is embarrassing. My project here is an account of MOTION and now we've just encountered what seems to be a persuasive argument that MOTION is impossible. That's not progress, is it.

The Parmenides Problem is an important stepping-off point for Plato.

1.1.4 ACT IV: Antidotes to Parmenides?

Empedocles •Anaxagoras •Leucippus •Democritus
(Set the context with the timeline in Figure 1.2 on page 8.)

Parmenides' arguments were unsettling. The notion of a tightly logical argument was brand new, and yet even if its conclusions seemed nonsensical, you've got to struggle to find holes in his reasoning. But that didn't stop four intrepid souls. We still call them “Presocratics” but really they were “Co-socratics” (I made that up) since they all lived around the time of Plato’s mentor. They’re our last stop before Plato.

GREEK RESEARCH PROGRAM #3b : Attempts at solutions: Back to Monism for solutions to the Parmenides Problem?

1.1.4.1 Empedocles and Anaxagoras

One philosophical god was apparently not enough. **Empedocles of Sicily** (–494 to maybe –434) was another self-appointed deity. He was a contemporary to the Ionian, **Anaxagoras of Ionia** (–500 to maybe –428) who had a similar solution to the Parmenides Problem. Both took the position that the world is made of multiple entities and that *those* entities are what’s permanent, but their *combinations* are multitude and accommodate change. In some ways, a modern approach.

Empedocles was a character. He dressed in a purple robe, with wreaths around his neck. He claimed to have performed miracles, raising folks from the dead, curing illness, and so on. He claimed to have been reincarnated from previous lives as a bird, a fish, a girl, a bush (really? shrubbery?) . . . His brand was very Pythagorean he worked in that same part of the Greek confederacy as the still functioning Pythagorean society, so there must have been some infuence. He famously wore bronze-soled shoes everywhere. They figure into his legendary ascendence at the end. He was supposed to have leaped into the active volcano at Etna and disappeared but one of those distinctive shoes was left behind casting doubt on that last miracle.

We only have fragments from Empedocles who wrote in verse, as seemed to be the custom in the west. It is from him that we get the familiar **Earth, Air, Fire, and Water** as basic elements (he called them “roots”). We will see that Plato and Aristotle took hold of this idea and ran with it all the way to Galileo’s time. These four roots accommodate change by mixing with one another driven by two opposing forces, “Love” and “Strife.” Again, a simplistic but modern-sounding notion of fundamental forces acting on the basic constituents of matter.

This is inspired. The roots are indivisible and have always existed, as have the two “forces” of Love (an attractive force) and Strife (a repulsive force). He also agreed that no-thing can come from nothing. So, check the Parmenides permanence and not-nothing boxes. But he accommodates our senses, while warning of their fragility. What we observe is that things in our world are different from one another and that there are many of them. Some rocks are hard and some rocks are brittle. They’re both rocks, so how do we build our observed rocks with only four roots? His contribution was that everything we observe is constructed of varying *proportions of the roots*. All rocks might be made of the same combinations of the roots, but a hard rock would have more of the Earth root than the brittle rock. With infinitely mixing proportions of the four roots, you can make the variety of the world. Sounds a little like a proto-chemistry.

Empedocles insisted that there was no purpose to the universe and that we're all subject to chance, postulating that we actually live in an undulating, repetitive cycle of a spherical universe in which Love and Strife compete for dominance.

His contemporary, Anaxagoras was from the other side of the West-East divide. He was an Ionian who ended up in Athens, establishing the first of a long string of Athenian philosophers. His arrival came during the classical period when the architecture; sculpture; literature; and yes, philosophy that we think of when we think "Greek" began.

Rather than only four substances, Anaxagoras presumes as many elements as there are things. Things…are themselves infinitely divisible. How do you acquire hair and bones? Well you eat foods that contain elements of…hair and bones. Everything is in everything. He insisted that the senses give us a window or a picture into aspects of reality that are not directly observable, but nonetheless, existing. Again, another modern idea from one of our "Co-socratics."

Notice that neither of our two characters explicitly address the issue of locomotion. This is a confusion that Aristotle promulgates, as we'll see. "Change" *per se* is broader than a thing moving from one place at one time to another place at a later time. So as you'll see in *Presocratic Greeks, Today* Section 1.2.3 while Zeno works on that problem, he starts with the presumption that change is not possible and so locomotion is logically not possible and hence the paradoxes that try to persuade you that motion is not possible. Our next two "Co-socratics" do find a way to explain locomotion which again, Aristotle rejects out of hand.

1.1.4.2 Atoms

I'll bet that you first learned the origin of the word "atom" in elementary school. "*Atomon*" is Greek for indivisible and the origin of that idea was again, the anxious need to find a way around the Parmenides Problem. You probably also learned that the inventor of atomism was **Democritus of Abdera** (about –445 to –370), originally from a region that's closer to Macedonia than it is to Athens, so a northerner. Here are three interesting things about Democritus. First, we classify him as a Presocratic, but that's really a misnomer. He's a "Post-socratic," younger than Socrates by more than 20 years. Secondly, he didn't invent the idea of atoms. He inherited it from **Leucippus of Miletus** (about –480 to –420). Finally, Plato doesn't mention him! He apparently burned Democritus' books. Aristotle knew him very well, maybe because of their shared northern roots.

Obviously, the idea of atoms is an idea with legs, albeit with ups and downs over the next two millennia, usually, unwelcome and only accepted when Einstein found two ways to demonstrate that there are indeed invisible chunks of matter. (That's a story that's not our current Einstein focus, but a large part of his miraculous 1905 year.)

However, the atoms (typically a mixture of Leucippus and Democritus' contributions) of classical Greece and our idea of atoms are very different. First, there are an infinite number of Greek atoms of all possible shapes. Some have hooks and can attach to others (think velcro), while some pairs have shapes that fit together. They move around and bounce off of one another, or they cling to one another forming compounds that eventually become the substances that we're familiar with. We know of them because of the sensible qualities that they bring to objects we can deal with using…our senses. For example, things that taste sweet are composed of smooth atoms while things that are acidic are composed of sharp-edged, angular atoms.

How is this an antidote for the Parmenides Problem? First, the atoms are permanent but second they are constantly in motion and all change is due to their arrangements, and re-arrangements.

But the real way in which this works is that both atomists insist that what's real are atoms…*and the void*. The void is the place where moving things can go *to*. So locomotion is possible. There. That does it for Parmenides. So, the atomists are happy to make room (so to speak) for MOTION ON THE EARTH.

> The void is an unpopular idea, and to this day we continually redefine what the vacuum is (or isn't). Our current understanding, again, my scientific playground, is that there is no place in the universe where there is nothing. The vacuum is full, but it's a quantum mechanical fullness that has no connection to any ideas before about 1950.

But, as I said, Plato ignored this singular, logical conclusion to the Parmenides Problem, which seems a cowardly way of dealing with an idea. As we'll see, Aristotle could not abide the void so he's no atomist either.

There's one more interesting fact about this pair's ideas and that's an idea that Plato would embrace, but with only partial credit to the right people. Everyday objects are not real things, and the attributes that we ascribe to visible, touchable, tasty, smelly, and loud objects of our sensible world are all based on convention. Democritus wrote:

> "By convention sweet and by convention bitter, by convention hot, by convention cold, by convention color; but in reality atoms and void."
> Democritus

Even though we can't see atoms, we know they're there because our minds tell us about what we can't see. A reality that's beyond our senses. Now this is a very modern idea and also a very Plato-idea and we'll see it emerge in a slightly different guise when we talk about Galileo and how he invented physics when he used this notion—now labeled "Platonic," but could be labeled Democritus-ian.

1.1.5 What's Important For Our Project

Our project is about MOTION and LIGHT. Does it make any sense to speak of either of them without numbers? MOTION implies speed, immediately bringing to mind numbers: miles per hour, for example. LIGHT involves brightness and color... qualities that we can describe using words, but they're a stand-in for actual numbers as well: you'd evaluate a lightbulb's brightness by "lumens" which is a number and "red" is a name for the frequency of light.

This is so much a part of our thinking now, that it almost seems trivial to mention it. Wouldn't it seem odd to think in any other way for almost everything, from cooking to taking a pain reliever to deciding when to buy new tires? Attaching numbers to the physical world is a gift of the Presocratics and in particular, the Pythagoreans. Trivial or not, before the Pythagoreans, numbers as more than just counting would have been a foreign concept, after them, well, numbers are *in* everything.

But their gifts were generous beyond just this. Lets quickly summarize what the Pre-, Co-, and Post-socratics have brought to the scientific table.

The invention of the scientific commitments that we use today came from them:

1. They eliminated the supernatural as an acceptable argument for why things in the world happen. We can know about the physical world.
2. They conceived of the notion that the universe is made of naturalistic stuff: the water, aperion, air first-guesses, to more intricate and even modern-sounding permanent entities that go together in proportions to build the stuff we experience.
    (a) They toyed with the idea that these entities had to obey rules that allowed for their interactions, and in some cases, motions.
3. They invented the notion that mathematics is tied both to geometry and to things in the world, essentially birthing modern mathematics. We literally have no other way to describe and predict the properties and behavior of the physics world.

They committed themselves to the idea that truth allowed for no compromise: truth is eternal, unchanging, and in conflict with our senses. That created all manner of consternation from what I've called the Parmenides Problem. It's also wholly incompatible with modern science, so this was a dead-end... except the struggle to come to grips with the Parmenides Problem led to Plato and the atomists. That's not unusual: a bad idea can often lead to good ideas in the quest to kill the bad idea! We'll see that over and over in this book.

4. Some Greeks realized that learning about the universe involved seeing, touch-

ing, and hearing what the universe of things does. But our senses are unreliable and so couldn't reliably deliver truth, if "truth" meant "permanent" and "never changing." Since they questioned the reliability of sense data as a source of knowledge by demanding that things that are True must be unchanging, they squarely set up the problem that Change itself is problematic. They should have questioned that all-or-nothing definition of truth! Taking a page from their high school geometry class, mathematics become a pretty good model of what is constant and true. But we only can deal with geometry at the level necessary through reason. So: don't look at the world, *think* about the world. That's what I've called the Parmenides Problem: is change really an illusion?

5. Reactions to the Parmenides Problem led to at least two directions: primary substances mixed in proportion, Earth, Water, Air, and Fire... or atoms.

And, proto-science, and now science as we know it, is a social activity.

6. They argued. One philosopher added to or reacted to what another said. This created the necessary social structure and behavior necessary to support the scientific enterprise.

We're now ready for Plato.

1.2 Presocratic Greeks, Today

1.2.1 Tweeting With Heraclitus

Heraclitus is challenging because he's tough to analyze and because the available material is... pithy. The general view is that he really did write in these short aphorisms and that they aren't somehow surviving snippets of something larger.

The most famous of them, that tends to support his historical brand that "everything changes" is the River Analogy. The most famous version is due to Plato's rendition which he wrote in *Cratylus*:

> "Heraclitus, I believe, says that all things pass and nothing stays, and comparing existing things to the flow of a river, he says you could not step twice into the same river." Plato

But there are actually three versions of the river tweet:

> "On those stepping into rivers staying the same other and other waters flow." *Cleanthes*, a Greek Stoic from two centuries after Heraclitus' life and almost a contemporary of Plato
>
> "Into the same rivers we step and do not step, we are and are not." *Heraclitus Homericus*, a commentator from 500 years after Heraclitus' life
>
> "It is not possible to step twice into the same river according to Heraclitus, or to come into contact twice with a mortal being in the same state." Plutarch, from the Renaissance

The first is probably the most likely and doesn't contradict the more popular version. However, this story illustrates the difficulty, once again, of the detective work involved in assigning credit (or blame) to the Presocratics.

I mentioned that he wasn't a people-person, probably unsuited for political leadership (notice the disdain for his Italian contemporary, Pythagoras):

> "One is worth ten thousand to me, if he is the best."
>
> "Eyes and ears are poor witnesses to people if they have uncultured souls."
>
> "War is the mother of everything."
>
> "The best of men choose one thing in preference to all else, immortal glory in preference to mortal good; whereas the masses simply glut themselves like cattle."
>
> "It is not good for men to get all that they wish to get."
>
> "What sense or thought do they have? They follow the popular singers, and they take the crowd as their teacher."
>
> "Learning many things does not teach understanding. Else it would have taught Hesiod and Pythagoras, as well as Xenophanes and Hecataeus."
>
> "Poor witnesses for men are the eyes and ears of those who have barbarian souls."
>
> "The adult citizens of Ephesus should hang themselves, every one, and leave the city to children, since they have banished Hermodorus, a man pre-eminent among them, saying, Let no one stand out among us; or let him stand out elsewhere among others."

His unity of opposites appears in multiple places:

> "Sea is the purest and most polluted water: for fish drinkable and healthy, for men undrinkable and harmful."
>
> "Collections: wholes and not wholes; brought together, pulled apart; sung in unison, sung in conflict; from all things one and from one all things."
>
> "Every pair of contraries is somewhere coinstantiated; and every object coinstantiates at least one pair of contraries."
>
> "Good and ill are one."

But, he's also inspirational:

> "Nature loves to hide."
>
> "Sound thinking is the greatest virtue and wisdom: to speak the truth and to act on the basis of an understanding of the nature of things."
>
> "Abundance of knowledge does not teach men to be wise."

> "This world-order [kosmos], the same of all, no god nor man did create, but it ever was and is and will be: everliving fire, kindling in measures and being quenched in measures."
>
> "The character of man is his guardian spirit."
>
> "The sun is new every day."

. . . and amusing:

> "And they pray to these images, as if one were to talk with a man's house, knowing not what gods or heroes are."
>
> "Souls smell in Hell."
>
> "Every beast is driven to the pasture with blows."
>
> "Asses would rather have straw than gold."

1.2.2 Modern Day Pythagoreans

Want to liven a party? Raise the following question:

1. Is mathematics invented? Or,
2. Is mathematics discovered?

That is, are the theories, proofs, and concepts of mathematics the creation of human thought, or are they "out there" waiting to be revealed by thinking about them? "Platonists" would rally around #2. and we'll talk about that in the next chapter.[14]

Now if you want to rejuvenate your now yawning party-goers, narrow the question to:

3. Is mathematics invented in order to explain the physical universe? Or,
4. Is mathematics discovered to be already "in" the physical universe?

Number 3 suggests that mathematics is only a tool —a language—to describe the universe. Maybe it's a lucky break that we've invented it and that it seems to do pretty well. Perhaps another tool might have worked? For example, a musical score for guitar could be represented by standard musical notation. But it can also be represented by chord diagrams.

Number 4 suggests that the discovery of mathematical and especially numerical relationships and their match to what we observe in the universe represents an uncovering of its fundamental mathematical fabric. Here, Pythagoreans do find a place: their discovery was that #4 is how it goes. Numbers (and in modern language, patterns) are *in* physical objects.

---

[14] Want to start an argument? Try to defend any definition of what Pythagoreanism is. (You can also spice up the conversation by trying to defend what Platonism is, which is the next chapter.) I'm not a philosopher, but I do have a sense of how my interpretation of these two ideas fits my experience in modern physics research.

Most rough-and-ready physicists would lean towards #3, but not everyone. I'm close to #4, but in a practical and not spooky way. (Some of my contemporaries are okay with spookiness when it comes to math and reality.)

We owe a debt to the Pythagoreans and while their application of "number" to the world is primitive, there are vestiges of their discovery that make science (and modern life) possible.

Mathematics describes the universe There is this nagging feeling that math and physical reality share a pretty special bond. Before the advent of Pythagoreanism, we saw that the Ionian approach to parting ways with deities was to ascribe a fundamental "stuff" as the basis of all physical things. Now, we don't depend only on that. We use math.

Take the weather. Before Pythagoreanism took hold, numbers meant "one apple," "two apples," and so on. Counting and nothing more. Before Pythagoras, I think that describing the weather using numbers might have seemed as strange as for us saying that the weather is "happy." While the ancient Pythagoreans didn't use numbers in most of the ways that we do, they might not be surprised that we are now comfortable to describe the properties of our weather more completely with numbers than with words. I just looked at the weather in Pythagoras' modern Crotone in Italy and it's not happy: it's 22° C (79° F), with a relative humidity of 76% and since the dew point is 71°, that's borderline uncomfortable. The barometric pressure is 1016 mb and rising and with a cloud cover of only 11%, and so visibility is 10 miles. This short narrative puts a picture in your mind of the weather conditions that words would do much less efficiently or accurately. I could take those numbers and recreate exactly those conditions in a lab. They are a natural measuring stick for us and that's due to our Pythagorean inheritance.

MIT cosmologist, Max Tegmark holds an extreme view that the numbers in our story aren't just *in* the weather, they *are the weather*. That is, if there's a one-to-one correspondence between a number and my interpretation of what the number means, then they're the same.

A taste from his controversial book, regarding the electric field:

> "If you can thus pair up every entity in our external physical reality with a corresponding one in a mathematical structure ('This electric-field strength here in physical space corresponds to this number in the mathematical structure,' for example), *then our external physical reality meets the definition of being a mathematical structure* (emphasis, mine)—indeed, that same mathematical structure. Tegmark (2014), page 280

That he's under attack suggests that physicists do have strong opinions about #3 versus #4, as much as they'd probably outwardly profess disinterest.

> "So the bottom line is that if you believe in an external reality independent of humans, then you must also believe that our physical reality is

> a mathematical structure. Nothing else has a baggage-free description. In other words, we all live in a gigantic mathematical object—one that's more elaborate than a dodecahedron, and probably also more complex than objects with intimidating names such as Calabi-Yau manifolds, tensor bundles and Hilbert spaces, which appear in today's most advanced physics theories. *Everything in our world is purely mathematical—including you.* (my emphasis)" Tegmark (2014), page 260

Or, in his technical publication Tegmark (1998),

> "Physical existence is equivalent to mathematical existence."

I've heard him ask what is a tree. To most it's a barky, green, leafy structure with roots and a hardness and so on. To him it's a collection of electrons and quarks and reflecting and absorbing light. In turn, the electrons are "-1, 1/2, 1, and 0.511." That is, the properties of trees are the collection of the properties of electrons and electrons are uniquely described as a negative charge of -1 unit, a spin of 1/2, a lepton number of 1, and a mass of 0.511 MeV/c^2. Protons, neutrons, and quarks... and the light that's absorbed and emitted are also described completely and uniquely by a different set of numbers.

Now the labels that the numbers have are entirely human-defined. But no matter how an alien species might define the unit of electric charge, the electron (and proton) have ± 1 of it. So, to him what is a tree is defined by what are the properties of a tree, which are entirely defined by a small set of numbers.

Finally, this is a book about Einstein's Special Theory of Relativity and it can almost be completely thought of as discovering the importance of a single number: the speed of light, c. No number is more special than c.

Tegmark is not alone, but his is a very small club.

There are special numbers While I'd not be prepared to say that marriage is "5" and when justice is done, that "9" is involved, there are special numbers that our universe seems to have latched onto that both explain what we observe, and were some of these numbers different, we would not be here. I just referred to one such special number, the charge of an electron or a proton.

Many numbers in nature play a role that designates unique properties of substances or processes that substances undergo. There are static properties of matter which have conventionally-defined, critical numeric values. Here's one: 1836.153. This is the ratio of the mass of the proton divided by the mass of the electron. An alien species might not use the same units that we do, but whatever system they used would have to replicate this ratio. Otherwise, their big bang and chemistry would be completely different from ours. The formation of hydrogen atoms in the early universe would have occurred at a different temperature and our early universe would not have formed galaxies.

Another one: Water freezes at a particular temperature. What the number is depends on a conventional scale (° C or ° F), but that there is a definitive event and that it can be quantified by a unique number of degrees makes it special. If that freezing point of water were slightly different, then the geological history of the Earth would have been different.

Inherent in a Pythagorean view of the physical universe is that the "numbers are in the thing" and that we can poke at nature with experiments and extract the mathematical essence that's embedded inside. Just like Pythagoras did...before anyone else.

1.2.2.1 Unreasonable?

Generally, we physicists don't generally lack in confidence. So in the interest of full-disclosure, here's a complete capitulation, a sort of a reluctant confession that we don't know why math and physics are so linked up:

Ask Mr Google to search just for the words "unreasonable effectiveness" and stand back. In less than a second, you'll be treated to a list of 12 million references to the Nobel Laureate, Eugene Wigner's 1960 article, *The unreasonable effectiveness of mathematics in the natural sciences.* It's actually a written version of a lecture he gave at NYU and it's among the most famous documents in physics. It's so ubiquitous, that Wiley Publishing is pleased for you to download it for free.

In that same vein, here's a word that you won't find physicists using: "miracle." The last paragraph in Wigner's article states:

> "Let me end on a more cheerful note. **The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift** which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning." [emphasis mine]

"A more cheerful note"? "The *Miracle*"? for heavens' sake? If that's his conclusion, can you get a sense of how his previous nine pages went?

There's a straight line from Pythagoras (and Pythagoreans...remember) to Plato and Platonism and to physics! But we don't understand this "unreasonableness," and sometimes it is kind of uncomfortable. Gloves come off when physicists and astronomers argue about multiverses, string theories, and measurement theory in quantum mechanics.

So, by now maybe you're a little more aware of the possibility that we may *all be a little bit Pythagorean*. Over and over we learn this.

1.2.3 Zeno and His Paradoxes

Parmenides had a following and his most devoted, and enthusiastic partner was the younger **Zeno of Elea** (ca –490 to ca –430). What he did was mess with everyone's mind about simple, common-sense experiences. He's remembered primarily for 10 paradoxes, two of which about motion I'll remind you of here as the most famous. He wants to show you that what you think you know, you don't, that common sense deceives. (Like in Quantum Mechanics and Relativity, where common sense left the building a long time ago.) I'll do them in reverse order. (By the way, how do we know of his arguments? Plato, again, in a dialog where Socrates deals with the young Zeno, playing himself. And Aristotle, who goes after Zeno.)

"The "Dichotomy." This is the famous race. In order to run the 100 meter dash, you've got to get to 50 meters. In order to get to 50 meters, you've got to get to 25 meters. See where I'm going (or maybe *not going*)? According to Zeno, there are an infinite number of distances that have to be traversed in order to move in space at all. So you can't get to 100 meters, in fact, you can't move at all. MOTION ON THE EARTH is impossible. Aristotle noticed that this is like the Achilles and the Tortoise paradox, except the conclusion of no motion is reserved to the Dichotomy.

Now this has been dissected for centuries. Ask Mr Google about "Zeno" and you'll see 36 million hits. The push-back begins with Aristotle, who argued persuasively, but in the end, inconclusively, that you can move through an infinite number of spaces if the time intervals become shorter and shorter while you do it. Aristotle hated infinity, so this must have been hard for him. But this presumes that Zeno was suggesting that the motion would take an infinite amount of time, but maybe it's because he was trying to cram an infinite number of steps into a finite period of time. So Aristotle's argument is not general enough.

The modern solution requires an understanding of how speed relates to time and space, a very modern set of ideas that are the heart of Relativity. I'll show you a complete explanation in Appendix A.1.1.

The Paradox of Infinite Divisibility. This paradox is the jumping-off point to an entirely different way of dealing with Heraclitus and Parmenides: If an object is made of parts, then one should be able to start cutting. . . into two parts, then four parts, and so on. At some point you reach some end: 1) If after an infinite number of slices, you find nothing. . . then the object was made of nothing—a **not-is**. 2) If after a finite number of slices, you find something. . . but it has zero size, then the object was made of something that has no size. Another kind of **not-is**. 3) If after a finite number of slicings, you find something that has finite size, like an element? Well, you're just not done slicing!

This is a modern thing as we are perfectly content to imagine that quarks that make of the proton and neutron have no size,[a] likewise the electron. But we have a field description of elementary particles and the forces among them, so we have a quantum mechanical push-back against Zeno here. But prior to the 20th century, a physics solution was not possible.

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[a] Yes. Our word “particle” creates an image of a little billiard ball, doesn’t it? In actuality, the size of quantum mechanical objects is so ill-defined as to have little meaning outside of an agreed-upon criteria involving waves.

You can see how this works. Zeno was apparently clever enough to waste the pixels on your computer screen in 36 million hits. . . all in service to the Parmenides two arguments: **Nothing changes** and **knowledge from perception cannot lead to truth**.

Chapter 2

Can’t Live With ’Em Or Without ’Em :
Plato and Aristotle

“The safest general characterization of the European philosophical tradition is that it consists of a series of footnotes to Plato.”

- A.N Whitehead (1861-1947), *Process and Reality*

Bert and Ernie, Kirk and Spock, Mantle and Maris, Venus and Serena, Abbott and Costello...Plato and Aristotle. One can’t have one without the other and, just like the other pairs in that list, these last two are deep subjects. My need for Plato and Aristotle’s contributions to the study of MOTION are for two ideas: following Pythagorean inspiration, Plato and his collaborators built the first spherical working model of MOTION BY THE EARTH and Aristotle expanded on it. They were both wrong.

And, while Plato didn’t concern himself with MOTION ON THE EARTH (except in an almost impenetrable portion of his last book), Aristotle was all over MOTION ON THE EARTH and invented its systematic study, informing—and infecting—science for 2000 years. It took until the 17th century before we could be all over with Aristotle. His models of MOTION ON THE EARTH, MOTION BY THE EARTH, and MOTION IN THE HEAVENS became Medieval and Renaissance Church dogma, but are wrong in almost every respect. By pushing back scientists learned what was better and why.

So why is it that Plato’s shadow hangs around while Aristotle’s

> importance for physics disappeared more than 400 years ago? We still talk about Platonic worldviews in some fundamental branches of physics, but nobody talks about Aristotelian–anything. Plato put important questions in play that remain troubling: What can we know? How do we know when we're right? And, most importantly, what is the role of 'mathematics in the fabric of the universe?

It was the worst-kept secret sneak attack in history. Everyone knew that the Persians were coming as under King Darius' son Xerxes the Great's command, the invading infantry slowly marched along in parallel to the Persian navy counter-clockwise around the inside of the Aegean basin, subjugating the Ionians along the way. Anaximenes lived under that locally-sourced, Persian rule that drove Pythagoras to Italy. About 100 years before Socrates' execution following a 10 year advance in −480 the battle was joined with an amassed Persian force of at 150,000 soldiers and 600 warships. Athens was evacuated and the Persians destroyed the city.

The Greek confederation then organized itself: the wounded Athens mounted the naval campaign and Sparta, the foot soldier command. What followed was a series of military maneuvers, still studied today. Spartan heroism of King Leonidas with 300 Spartan troops and a total of 9,000 allied soldiers met and slaughtered the Persians at the pass at Thermopylae. The movie and the comic book series *300* might jog your memory **?**. While this was going on, the Athenian navy engaged and overwhelmingly defeated the much larger Persian naval force. Finally during the summer of −479, the Persians were defeated in a decisive land battle. Yet, war continued in one form or another for thirty more years until the Persians fled the Aegean leaving behind a Sparta with a greatly enhanced reputation. Proud Athens rebuilt after that disaster in −480 and under Pericles' leadership — throughout the decades of extended conflict, began its 75 year Golden Age when everything you think of as Greek in culture, art, architecture, and philosophy was intentionally created.

Ironically, even though Sparta could be credited as having been the major military force in the Greeks' victory, its isolated and belligerent nature simply did not equip it to lead during peacetime. In contrast, while Athens had been destroyed, its nature was to rebuild stronger, to politically organize, and to lead. All while doing what Greeks did best: fighting.

While the Golden Age was unrolling, Athens simultaneously managed to battle with: Sparta −465; Corinth and Sparta −459; Samos −440; Corinth again −433; Potidaea −433; Mageria −433; Sparta again −431 (Socrates was active as a soldier during this period), (Score: **Sparta 1, Athens 0**) Syracuse and Sparta −415, (Score: **Sparta 2, Athens 0**) ; Sparta now allied with Persia −414, (Score: **Sparta 3, Athens 0. Game, Set, Match**).

After that third war with Sparta,[1] Athens surrendered to Spartan general Lysander in −404. Plato was 23 years old and Socrates had five years to live.

Athens badly handled their unfortunate overreach and eventual defeat and in the final stages of the war they managed to: expel their leading general. execute six other military leaders and flip from autocracy to democracy and back to autocracy. Socrates was on the autocracy side and it was the democrats who condemned him to drink the hemlock in −399.

Athens' subjugation by Sparta after the two Peloponnesian Wars was tumultuous — governance of the city jerked back and forth between oligarchs and democrats. In the same way that the Golden Age of Classical Greece emerged during war with the Persians, in the midst of the city's internal chaos, western philosophy began and was followed quickly by the first systematic attempts to understand MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS by our two lead actors. Yet the catalyst to all of this progress was interested in neither. Socrates' persistent question was: how to live a virtuous life, not how do things move. As his talented acolyte, Plato adopted the older man's voice and wrote truly engaging tales, but expressed his own ideas and, while his program was ostensibly one of ethics, the Socrates/Plato approach opened a new front in the battle with the Parmenides Problem which resonates in modern physics today. And, as so often happens in philosophy (and physics), the next productive steps were in opposition, launched by Aristotle, one of the most remarkable intellects in all of Western history and whose voice is probably from lecture notes and not intended as his legacy literature. Yet in physics: Plato endured and Aristotle is gone.

2.1 Act V A Little Bit of Plato

Plato (−429 to −348) is actually a nickname, suggesting someone of broad shoulders or perhaps a wrestler. The name on his driver's license would have been Aristockes and his aristocratic family had been influential for generations. Plato was no democrat and grew up during the Peloponnesian War (−431 to −405) and the subsequent subjugation of Athens by the victorious Spartans. In many ways Plato's idea of the correct form of government was clearly informed by the collectivism and brutality of the Spartan way. (Set the context with the timeline in Figure 1.2 on page 8.)

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[1] who actually allied with Persia!

> One of the signature events of his life was the story of his attempt to help form a government in Syracuse where he somehow got the idea that he could turn the tyrant Dionysius into a philosopher-king, since in Plato's opinion leaders should be philosophers. That got him imprisoned and even sold into slavery for a while (or so the story goes), until he was ransomed. He actually tried two more times, which brings to mind one's questionable mental state as per Einstein's observation much later about repeating the same mistake over and over and expecting a different outcome..

His life's direction was formed when he, like many young men in the newly democratic Athens, started to associate with **Socrates** (-470 to -399) who, after his (apparently distinguished) service as a foot soldier in the war, took philosophy on an entirely different course from investigating the nature of reality to how best to live a satisfactory life. We learn in school about Socrates' self-administered execution at the hands of the democratic Athenian politics—one of the reasons that Plato was distrustful of democracy. It was traditional to give the convicted criminal options on how they would like to do away with themselves and Socrates suggested that he be given free food for life. That was turned down and eventually death by poisoning was prescribed.

Plato's 35 books are all in the dialogue form, conversations between Socrates and a variety of fictional and real persons. Unlike Aristotle's largely academic writing (which might have been lecture notes), Plato's books are literature and valued for their style and lyricism. Plato himself is only mentioned twice and he never speaks directly. The assumption is that he's speaking through his mentor and that the ideas are his, and not meant to be those of the older man. (One book, *The Apologies*, might have been more personally Socrates as in that volume he defends himself against his accusers.) So the ideas are Plato's and the books comprise his philosophy as it evolves over his productive, long life. Almost all of his work follows a general theme, and what he seems to struggle with is what I've called the Parmenides Problem. Plato wants to contrast what we experience in our everyday world—objects (physical things) and ideas (like virtue, justice, beauty, what's good)—with abstract ideas that are the source of the variety of physical things and the imperfect values we associate with more aspirational ideas.

It might be reasonable to view the Socrates of Plato's dialogs as a literary invention, but he was known to broader Athens and even parodied in the *Clouds*, a vicious comedy by Aristophanes and figured in other writers' accounts, including in dialog form. But the world now knows of Socrates through Plato and he figures into every one of Plato's dialogues as "that guy" who irritates everyone, although in the later dialogues his role diminishes. His job is to ask simple-seeming questions (the "Socratic Method") of an assembled group of friends (or foes), often about an ethical matter. What's temperance? What is virtue? What is justice? The course of these sorts of innocent sounding conversations is repeated: the folks being questioned are maneuvered into impossible rhetorical cul-de-sacs, shown to be incapable of any kind of logical thinking, and more often than not, shown to not know things that they should have known. Meanwhile, Socrates rarely says what he thinks, in fact, he usually hides behind the assertion that he doesn't know either, but at least he knows that he doesn't know. Superior to a fault. These questions also often segue into something more than they seem, and many of them move to more weighty topics like how *do* you know what you know. That is, they form the beginning of serious Epistemology, one of the foundational philosophical disciplines.

Plato's output was large and I'll choose only a few topics that inform our scientific project. Unlike almost all of the previously considered Greek philosophers, we have complete writings. He famously started *The Academy*, a school that lasted more than 700 years whose star pupil was Aristotle, whom we will consider below. Bertrand Russell (in his Literature Nobel Prize winning, *A History of Western Philosophy*) sums up what we're about to dive into appropriately:

> "Aristotle's metaphysics, roughly speaking, may be described as Plato diluted by common sense... He is difficult because Plato and common sense do not mix easily." **?**, *A History of Western Philosophy*

My focused concern is with two aspects of his philosophy and then his physics and they're related. I'll leave his modeling in astronomy to the next chapter when I will consider all of the Greek astronomy at once, but I'll consider his overall approach to astronomy here. Of concern then (and now) are Plato's Epistemology—what does it mean to know something (from the *Meno* and *Phaedo*), his Metaphysics—what is the nature of reality (from *Phaedo*, *Parmenides*, and *Republic*), and his physics (from *Republic*, *Timaeus* and Book X of the *Laws*).

2.1.1 What Is True Knowledge?

Plato was deeply influenced by our Parmenides Problem and took this on with a study of the broader question of what actually constitutes true knowledge. He thought deeply about this and his conclusions became grist for philosophical mills for the next 2500 years.[2] He decided that there are two hallmarks to knowing: that knowledge should be infallible and that it should be "of something that is." Typical was the exchange between Socrates and the 16 year old Theaetetus in the dialogue by that name. Socrates teases out of the boy his ideas of four kinds of knowledge, and demolishes every one of them. First up, what do we learn by *perception* as a source of knowledge? That's dispatched by Socrates, it's infallible (since your internal evaluation of a perception is true to you), but perception is incapable of demonstrating that the objects of perception actually exist. So it fails on the second hallmark. Next, is *belief* as a source of knowledge? That results in a blistering dissertation on subjectivity. And the same thing happens with "true belief." They are both fallible, so failing on the first hallmark. Strike three. And finally, what about *belief with a reason to hold that belief,* what in the context of *Theaetetus* is sometimes

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[2] I'm grateful to philosopher, Professor Harold I. Brown for important discussions on this complex topic in Platonic philosophy.

called “true belief plus an account” or, “Justified True Belief”? This is sometimes incorrectly described as Plato’s own theory of knowledge, but Socrates makes hash of JTB and leaves the question in an unsatisfying state. Let’s look at a couple of examples.

J+T+B was considered to be among the best efforts into nearly the present day and relies on the three aspects memorialized in its name. The B: one can’t claim knowledge about something you don’t believe. (I read that my calendar reports that today is Tuesday, but I believe it’s Monday certainly doesn’t qualify as knowledge of Monday.) The T: the fact must be true (if the fact is not true, then you cannot be said to have knowledge of it.) The J: whatever you claim about the fact, you need to be able to justify it.

Consider this claim: It is 3 o’clock, I believe it’s 3 o’clock, because I looked at my watch and see that time displayed. B, T, and J are all in play and this seems a reasonable example of knowledge.

> But there are holes and weaknesses. What about instead of that J, how about J2: It is 3 o’clock, I believe it’s 3 o’clock, because 3 is my favorite number. I’m right, since it really is 3 o’clock but that justification is silly and certainly doesn’t qualify as knowledge of the time. How about this: It is 3 o’clock, I believe it’s 3 o’clock, because I looked at my watch and see that time displayed. But...I didn’t know that my watch was broken and had stopped at precisely 3 o’clock. So it was just luck that my reading corresponded to the right time. So that’s hard to accept as knowledge. In fact, it was only in 1963 that Edmund Gettier apparently found counterexamples to JTB which are now called "Gettier Cases."

Clearly Justification is the rub and many efforts have tried to turn J+T+B in to J+T+B+X. . . where X is some thing added to take care of the Gettier Cases. It’s an ongoing problem. For scientific claims of knowledge, sometimes Justification weaknesses turn on problems with observation and even the senses so we’re right back to the Parmenides Problem.

Plato had an answer and it turns out to be more than a theory of knowledge, but also a theory of what’s real: fixing epistemological problems resulting in metaphysical commitments.

True knowledge for Plato can only come from permanent, unchanging things. Thanks, Parmenides. If something is true, it must be so forever, which means that it was never not true, nor will it ever become not true. He falls squarely in the Being camp, as opposed to the Becoming camp.

> Notice how this demand of permanence as the qualifying feature of true knowledge is an **unquestioned commitment**. There's no room for degrees of knowing—we all know things with varying levels of trust and this is especially true in science where not being able to question an assertion is actually now the very definition of "unscientific." I think that their insistence on permanence is a function of their being impressed with geometry and the fact that it was very early days in the brand new field of epistemology.

Plato differed from ardent Eleatics like Parmenides by insisting that knowledge is indeed possible, but there's a catch.

He proposed after *Theaetetus* that there are two worlds:

- The world of the Forms.
- The world of the senses.

2.1.2 The Forms

Plato's theory of the Forms is one of the most difficult ideas in philosophy but comprehending it is critical for an understanding of his projects, *but also for appreciating physics.* He gives abstract concepts an existence of their own and a job to do with a consequence that sort of grates on you.

Take high school (please): if you ever took a geometry class you were given a set of pieces out of which you could create new pieces with just a ruler and a compass. These pieces include things like points with no extent and lines with no thickness. You manipulated and proved theorems about perfect triangles and perfect circles. Let's focus on that last one.

Think of all of the "circular" things that you come in contact with in your everyday life. Coins, dials on appliances, buttons on your shirt, a camera lens, a cookie, maybe a rendition of something circular in an image or on a screen. You know that none of these circles are the circles of your geometry class. But all of these circular things share their property of *circularity*. They may woefully miss in the perfection of that high school circle in your mind—but through thinking about it, you know that your Oreo is circular, almost.

Plato would say that that unique abstract circle *actually exists* as a "Form." That there's a kind of reality—a realm— that's different from the reality that you think of when you drop that circular plate in the kitchen. That realm is where the Forms exist.

That high school geometry-circle is such an abstract notion. But you can grasp that reality, you can apply it, engineers can use it, and you recognize it when you see it...only in your mind. But try an experiment: construct the best circular thing that you can and measure its diameter in a hundred points around the center at micron precision— while your rendition may be a good one, it's not that abstract high school one, is it. The Form of a circle is aspirational but can't be studied by measuring regular-life circular things, rather it can only be brought to life through your intellect. The Form of a circle has always been there (circles were not born) and that realm is outside of space and time. Can you get on board with abstract things being real?

Maybe Plato's assignment of "real" to mathematical abstractions is a little less odd than at first glance. But he went further than geometry and you might have experience with non-mathematical abstractions. Here's one: "We hold these truths to be self-evident, that all men are created equal..." What is a self-evident truth? If it's a "truth" then questioning it is a waste of effort, it's permanent in a Parmenides sort of way. If an idea is self-evident, then in some sense it's always been there, imprinted in us, while apparently accessible, but at the same time, distant.

You can't hold such a truth in your hand and you know it's not universal in our everyday life since, "all men are created equal" is untestable since the ones we know are tall, some are smart, and yes, some are disadvantaged. That they're "equal" is an abstraction—again, an aspirational idea of perfection—that we can hold in our minds but we know won't be realized in "our world." But a nation of 300 million “Platonists” swears by that truth.

What about realities outside of our plate-dropping reality? If one is a Christian, then you've been brought up to believe in such a reality—heaven (and hell) are outside of our everyday lives.

When I go to a furniture store I see hundreds of sofas. They're all different, but they all share…a “sofa-ness.” They're all *participating (sharing) in the Form of the Sofa* which I can (only) know of in my mind. It's a perfect sofa.

With the forms, the Parmenides Problem is dealt with in a brand new way: there is a world of Being and a world of Becoming and they are connected, but in a hierarchical way. And, it's not just living room furniture that has Forms. There is a Form for everything: even Justice, Virtue, Beauty, and the Good…the latter of which is somehow a super Form.

2.1.3 The Republic

Plato's contribution to science is not any particular theory or practice, but as Lloyd (1970) suggests it is more his philosophy of science that we value. This is laid out most explicitly in *Republic*, probably his most famous book, ostensibly a treatise on politics and good governance. It's here where he describes how a city should be ruled, certainly not by popular election, but by the training of a special category of people bred and educated in order to be rulers, the philosopher-kings, the guardians. Their lives would be scripted from early ages, living communally, and essentially the pool of potential candidates for leadership. Their educations would be scripted as well, relying on an intensive study of mathematics to create a habit of mind.

The goal is for them to be completely comfortable with the most abstract concepts, including Justice and what's Good and mathematics is a primary route to that appreciation. *Republic* includes a few analogies to try to get Plato's point across. Two are relevant for physics.

Analogy of the Divided Line.

Along with the Allegory of the Cave, the "Analogy of the Divided Line" is important for Plato and I think important for physics—as Galileo and modern physics will eventually enlighten for us. A rendition of the Divided Line is in Figure 2.1. What we can know is a hierarchy, from muddled to perfectly clear and divides into two broad "realms," one representing our *Becoming* world—The Visible Realm—which we occupy in everyday life, and the other representing the *Being* world—The Intelligible Realm—which is outside of space and time and only recognized through thought.

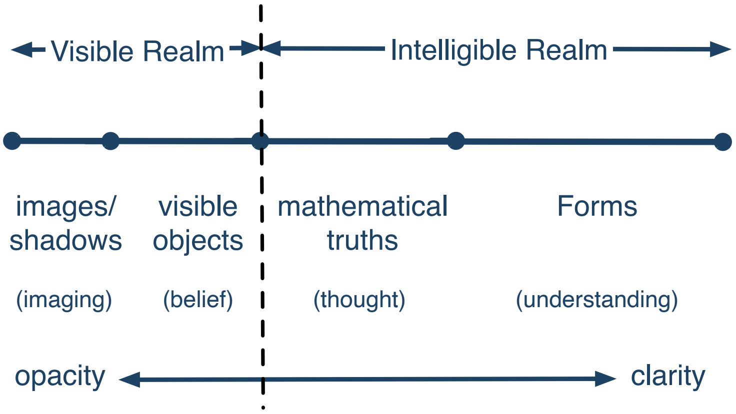

Figure 2.1: The line represents a kind of knowledge-hierarchy, from unclear to perfectly clear.

The Becoming realm is broken into two levels of which the objects of the first, and lowest segment are shadows and illusions of objects in our experience. The shaky knowledge we have about them are mere illusion and dreams. The objects of the second stage are actual ordinary, everyday objects themselves, and the knowledge we have about them are opinion and belief gleaned through our (untrustworthy) senses. Taken together these two stages constitute our knowledge of our everyday world, where things change: the Visible Realm is where we we use our senses and dreams to navigate our lives.

The Intelligible Realm is only accessible through thought and reason and is likewise divided into two more sophisticated segments. The first of these includes knowledge gained through mathematics and hypotheticals (think high school geometry) about which we have knowledge through reasoning. And finally, the highest segment of the Intelligible Realm is of the Forms, the pinnacle of clarity, "beyond hypothesis" which is aspirational, not easily realizable.

Earlier I opined that degrees of knowing is a more modern way of thinking and the Divided Line actually sneaks up on just that. As we'll see when we study Galileo, there is a realm of the universe which is very hard to observe (on Earth) but which is our goal when we theorize about nature. So I'm not quite willing to pass this off as silly, while at the same time I don't agree with the realm of the Forms as an ethereal parallel universe that we cannot access but through rationality. Stay tuned.

Allegory of the Cave.

He famously tries to work out more of this progression in the *Republic* with the "Allegory of the Cave" and in the *Meno* with the idea of" Reminiscence." Prisoners in a dark cave are shackled to the ground facing a wall. They can only look straight ahead and what they see are shadows of objects and puppets that are held in front of a fire behind them so that they project on the wall. If they see a sofa on the wall, it's because the Form of the sofa, which is behind them and out of sight, is projected as a shadow of the real Sofa in front of the fire.

Now, if one of the prisoners escapes her bonds and looks around she'll see the fire and the contrived circumstances. The light from the fire would hurt and she'd want to go back to her former spot. But if she were dragged out of the cave and into the sun, she's blinded but slowly she'd look around her and realize that there are actual things in the world and not just shadows. Notice that in the Allegory, she's moving from left to right in the Divided Line in Figure 2.1. She ventures back into the cave and tries to describe that true reality to her still captured colleagues. But in the dark she'd not see well and the prisoners would not allow her to persuade them to follow her into the sun since it apparently takes away one's sight. Plato even worries that the prisoners might kill the one who escaped.

Obviously, Plato is describing the daunting project that he's taken on as the enlightened prisoner trying to explain what's Real and True to everyday people who don't want to accept it. The similarities to Neo's trip out of the realm of perceptions and into the realm of the real is not an accident as the movie *The Matrix* is full of philosophical allegories, and the Cave is one of them.

What we can know of the world of the Forms is true knowledge. That's an aspiration of philosophy. What we can know of the world of appearances is simply opinion. The Forms inspired many in the centuries to follow, from Neo-Platonic Christian images to modern science. We'll come back to them in Galileo where finally, properly characterizing MOTION begins. By the way, Plato despised art. A painting of a mountain as nothing but an imitation (the painting) of an imitation (a sensible, actual example mountain) of the form of Mountain, which is the only real thing.

2.1.4 Mathematics For Plato from Republic

Plato's experience in Italy wasn't limited to a failed experiment in his theory of governance, but began as a deliberate project to study with Pythagoreans. Pythagoras had been gone for a century by that point, but two schools grew up around his legacy. The *acusmatici* viewed themselves as the guarantors of Pythagoras' the man's legacy as a complete system. Not only his mathematics, but the other aspects of the Brotherhood were preserved and defended without expansion or elaboration. On the other side were the *mathematici* who bought into reverence for the man, but intentionally expanded the mathematics to new areas of research, an unwelcome sin in the eyes of the *acusmatici* who eventually died out.

Recall that Plato and Aristotle probably learned most of Pythagoreanism from Philolaus, but Plato's mathematical inclinations came from a contemporary, one of the mathematici that Plato befriended and learned from, **Archytas of Tarentum** (ca −420 to −355) who is one of our characters in Chapter 3.2.2. Our title character in the next chapter is **Eudoxus of Cnidus** (−408 to −355), a student of Archytas and the most significant mathematician before Archimedes. Both influenced Plato and Aristotle's cosmology, and that subject kicked off two millennia of modeling and eventually, dogma. The mathematics required in the guardians' education came from Architas, arithmetic, geometry, astronomy, and harmonics. Plato didn't fully agree and added a fifth subject, solid geometry.

This is very much physics as we'll see. Maybe you can begin to understand Plato's elevation of mathematics—in the Greek life of his day, geometry and proportions—to the point of his famous sign above the door, "Let no one who is not a geometer enter." (Well, that sign only crops up in the 4th century AD, so it's probably a myth.) Geometry is venerated by Plato and all who follow for centuries.

This is hit directly in *Republic* where Socrates extracts from Glaucon[3] the reasoning behind requiring astronomy for guardian training. As usual, Socrates/Plato starts out with a theme which in the course of explaining it, evolves into a matter of serious philosophical interest. Glaucon tries to guess at why astronomy is important. Maybe because it's useful for recognizing seasons, or timing agricultural events. Practical things. That doesn't go over well and so he tries again: maybe astronomy is "good for the soul"... that looking at they sky takes us away from looking at everyday things. Again, not productive for Socrates. Here's where geometry comes in and where Plato earns an uncertain reputation for suggesting that "armchair astronomy" is the only way to go: doing astronomy without ever looking at the stars. Here's how I interpret this:

Back to the literal drawing board: Take out a ruler and the sharpest pencil you have and carefully draw the most precise triangle you can create and now get the best protractor you can buy and try to verify that the interior angles of that triangle all add up to 180°. No matter how careful you are, you'll fail to perfectly measure 180.000...°. In fact, Socrates/Plato would tell you to not bother since studying an everyday triangle won't help. The perfect 180° is in your head and its truth is one of reasoning and geometrical proof.

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[3] Possibly, Plato's older half-brother's name.

Socrates/Plato suggest that the same is true for astronomy.

> "We shall therefore treat astronomy, like geometry, as setting us problems for solution", I said, "and ignore the visible heavens, if we want to make a genuine study of the subject and use it to convert the mind's natural intelligence to a useful purpose." Socrates/Plato, *Republic*.

He says that you can look at the stars but discerning their actual motions cannot be done by measuring the apparent, but flawed motions of the imperfect stars and planets. You can only understand their motions by reasoning; astronomy without looking up! Like the triangle, you might get hints from the world of Becoming, but only through reasoning can you learn what the stars and planets do in the perfect world of Being.

> Here is **another unquestioned commitment** by Plato. That the stars and planets would necessarily execute perfect motion is an assumption. Again, this is the very earliest days of astronomy and philosophy and it's built on a variety of prejudices.

Plato's "Doctrine of Reminiscence" is another idea that comes from the Forms. In the *Meno* Socrates demonstrates that a slave boy actually knows geometrical proofs without knowing that he knows them! By asking questions, in his Socrates-way. In the *Meno* the protagonist, Meno (a real, young aristocrat) asks Socrates if Virtue can be taught and of course Socrates begins by asking the young man to define what Virtue is and then dismembers his multiple attempts at an answer. The scene degenerates into Meno now becoming frazzled and paralyzed as the discussion evolves. As often happens more than the problem at hand emerges, including what's called "Meno's Paradox": the realization that if you know something, you don't need to ask about it but if you don't know it, then you don't know enough to ask. Of course this all leaves everyone unsatisfied. (It's surprising to me that anyone ever wanted to talk to Socrates.)

The discussion turns to a religious view that the soul has always existed and will exist after we die and that the soul knows all that there is to know before and after and therefore, we already know everything. . . we've just forgotten it. He then proceeds to demonstrate this idea by asking a slave boy the geometrical proof of how to double the area of a square. By asking him questions, he actually induces the boy to give the right answer. (You can see the answer by following Socrates' questions in Appendix A.1.3.

> In school, did you ever work out a derivation in geometry or math that you got right? Don't you do a little victory dance inside, maybe with a knowing nod — Aha!— that solution seems like it was there all along and all you did was *reveal* it. That you almost *remembered it*. This is the basic characteristic of Deductive Reasoning. It doesn't lead to anything new, but reinforces—-(or recalls, suggests Plato)—that something that was already in the premises. I know I've had that feeling and I can understand why Plato chose a geometric proof to illustrate his idea, which is broader than just math for him, of remembrance. What Plato was really after was the fact that the Form of that geometric proof was there all along, in that Intelligible Realm, all the time.

2.1.4.1 The Soul

The "Soul" is a very Greek idea which functions at multiple levels for Plato, in one dialogue, he assigns three separate jobs to the Soul. For our purposes, he's impressed with the idea that some things are inanimate — like a rock — and that somethings appear to be animate. The very word "animate" gives you a sense of what he thought might be the distinguishing feature between animate objects: they can they move on their own. So in some ways, this is a question of MOTION ON THE EARTH (but he extends it to MOTION IN THE HEAVENS). He found it useful to ascribe to all things that can move of their own accord — he would speak of "self-motion" — as imbued with Soul. It's not only humans, but birds, flowers, even planets which appear to be able to execute locomotion on their own that enjoy their very own Soul. We'll see that this idea actually figures into some of his astronomy, so in a backdoor sort of way… this is an example of MOTION BY THE EARTH! It is this very talented Soul that causes self-motion among animate objects, but also persists before and after death. We get a glimpse of the all-knowing Soul when we do a mathematical deduction, as Socrates illustrates with the slave boy.

2.1.5 Timaeus

Boy, the European medievals must have been confused about Plato. Until the early12th century, the only Latin translation of any of his works was just one: *Timeaus*. It's notoriously difficult, convoluted, and ripe for repackaging by the "neo-Platonists" up to Augustine. In this difficult late dialogue, *Timaeus* the title character, a fictional Greek statesman and scientist from southern Italy (ah, as we'll see, surely a Pythagorean), who when asked by Socrates at yet another get-together, tells the origins story of the universe. A sort of Greek Carl Sagan. Timaeus is less a dialogue than a monologue and it covers a lot of ground without Socrates being as much his usual, obnoxious self. Obviously, Plato had a lot on his mind in this book.

Plato was so enamored of mathematics that through Timaeus' voice, he builds what he calls a "likely story" of cosmology by mixing geometricized ideas of the atomists with a relentlessly Pythagorean numerology (that he learned directly from Archytas?), a major focus in Chapter 3.2.2.

The universe was assembled (not created) through the actions of a "Craftsman"[4] who builds everything—animals, planets, stars—from a blueprint of eternal ideas—surely, the Forms and does so using existing materials at hand. He's an artisan, more than just a laborer and less than a creative deity.

The dialog begins with Socrates counting, "One, two, three,. . . " a portending of the strange, mystical use of numbers as the Craftsman does his job. I'll reserve the astronomy part of Timaeus for Chapter 3 and make reference here to only those parts of the dialogue that overlap with our project. That leaves most of *Timaeus* untouched.

The story begins with fables about Athens of 9000 years previously, among which are its war with Atlantis and the idea that Earth is periodically destroyed, erasing memories for everyone. . . but somehow, not the Egyptians. This leads to a discussion of how the universe began. Timaeus asks (with Parmenides looking over his shoulder?):

> What is that which *always is and has no becoming*, and what is that which is *always becoming and never is*? That which is apprehended by intelligence and reason is always in the same state, but that which is conceived by opinion with the help of sensation and without reason is always in a process of becoming and perishing and never really is." (emphasis, mine)
> Plato, *Timaeus*

Of course, this is a reference to the Forms ("always is and has no becoming") in contrast to particulars and everyday things ("process of becoming and perishing and never really is"). In sympathy to Parmenides' poem, Timaeus also tells about both kinds of knowledge. This is his stepping off point to the fact that the universe has "become" and so was not always around which implies a creation act or a cause, or in any case, a creator. That's the Craftsman's job who follows a plan which is an aspirational blueprint.

The universe isn't created out of nothingness (more Parmenides?) but rather the Craftsman works with the material at hand using the Forms as a blueprint and fashions it into an Earth-centric ("geocentric") model, which we'll talk about in the next chapter. Plato leaves the impression that the Craftsman does the best that he can — a best-effort universe! There is a difficult overall purposefulness and expectation that the Craftsman is ". . . .greatest and best and fairest and most perfect." This is the best possible world.

Suffice it to say that the Sun, Moon, and planets all take their familiar places according to a mathematical (even musical—Pythagoras, again) format and that Time itself is created along with the planets. In fact the motions of those most-nearly-perfect celestial bodies is the cause of time. The ancients told the days, months, and years by the motions of the Sun, planets, and stars and so it's maybe not a surprise that Time and those objects have a causal relationship to one another.

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[4] In Greek, the "Demiurge"

The Craftsman isn't omnipotent and is restricted to Empedocles' four elements — the materials at hand. The *Timaeus* outlines the way in which Fire, Water, Air, and Earth go together (again, in proportion) by assigning them solid shapes: Fire is made of tetrahedrons, air is made of octahedrons, water is made of icosahedrons, and finally Earth is made of cubes. The solids themselves are made of two kinds of constituent triangles; the isosceles and scalene triangles. The former is what results from cutting a square into two parts along diagonals and the latter is a triangle in which the hypothenuse is twice the length of the shortest side. Two scalene triangles, side by side, attaching the long sides. . . makes an isosceles triangle. So the atoms (in a modern sense) of the four elements are made of two elementary, triangular constituents (like modern atoms are made of electrons and nuclei): tetrahedrons (4 faces of equilateral triangles), octahedrons (8 faces of equilateral triangles), icosahedrons (20 faces of equilateral triangles), and cubes (12 equilateral faces).

Water then could be broken down into fire and air as an icosahedron can be decomposed into two octahedrons of air and one tetrahedron of fire. In fact, that water evaporates can be modeled in his scheme by noting that two water solids can geometrically be reduced to five air solids. He's used up 4 of the 5 known three dimensional solid forms, historically (but inaccurately) called the Platonic Solids. So, having bought into a theory, he did what any theoretical physicist would do. If the solids are fundamental and only 4 of them seem to immediately come to good use, then there must above a job for the fifth shape, the dodecahedron, and he assigned that to some measure of the universe itself as it has so many faces, it's close to being a sphere?

Plato refers to a form of air as "...the most translucent kind which is called by the name of aether...," but he sticks to the four elements of Empedocles. Aristotle does something similar, but with a twist. There is some ambiguity among the terms "aether," "quintessence," and "ether." In this book I'll use the term "ether" to refer the 19th century substance that all thought "carried" the propagation of light waves throughout the universe. "Aether" and "quintessence" are Greek references and are often used interchangeably. In Chapter 3 I'll use "aether" to refer to Aristotle's fifth element.

So, Plato is revealing his Pythagorean biases: The world is geometry—pure, abstract form. But he's just getting started as his Pythagoreanism knows no bounds as we'll see when we consider his cosmology in Chapter 3!

Platonism is not just confined to philosophy or mathematics. The Medici family in Renaissance Florence was instrumental in reacquiring Greek philosophical texts from the Byzantine empire by importing Greek-speaking academics. They set up a school dedicated to Greek philosophy and a school for the children of the court. One of those children was a ward of Lorenzo and would have learned of this approach to the world. So when Michelangelo later noted, "I saw the angel in the marble and carved until I set him free" he was expressing a very Platonic idea that he absorbed as a young ward of Lorenzo di Piero de' Medici, modestly, Lorenzo the Magnificent.

2.1.6 Platonic Legacy

We've covered a lot, but only a little of the large subject that is Plato. I view the history of physics as ebbing and flowing between Plato's and Aristotle's influence and out of that I have concluded that our recognizable scientific discipline—my life's work—didn't happen until the history of physics swerved in the direction toward Plato and away from Aristotle. So our discussion of the Forms and how the mathematical picture is illuminated by his conclusion that there are two sorts of reality is necessary in order to tell the whole story of MOTION. There is one negative legacy that's more complicated than it's normally presented: the idea of "Saving the Phenomenon," or "Appearances." This is the statement that is used to assign this idea to him:

> "This was the method I adopted: I first assumed some principle, which I judged to be the strongest, and then I affirmed as true whatever seemed to agree with this, whether relating to the cause or to anything else; and that which disagreed I regarded as untrue." Plato, *Pheado*

It's more complicated and people still argue about it. I suspect that there are four reasons that this seems to lead to that direction:

1. Aristotle seems to be critical of that way of thinking (see his statement from *On the Heavens* below on page 2.2)
2. There's the "armchair astronomy" admonition by Socrates in *Republic*, described above.
3. There's the fact that his student/colleague Eudoxus takes on the task of describing the motion of celestial bodies using only circles. This will be discussed in the next chapter.
4. And there's this quotation from *Phaedo*.

The person that was most responsible for making this direct connection to Plato was the neoPlatonist, Simplicius, who flourished in the 6th century (CE) (He reported that Plato proposed the problem of finding "by the supposition of what uniform, circular, and ordered motions the appearances of planetary movements could be saved.")

In any case, this methodology had legs. Can you see how *un*scientific this is? First create the theory, and then interpret the facts only to support the theory. This is especially the case in his astronomy.

His positive legacy is, as I've hinted, critical and abstract. His ideas were reformulated a number of times and Neo-Platonism was a pre-medieval version that eventually found its way into Catholic Church doctrine, much through Augustine, only to be reassessed centuries later.

What can't be overstated is the influence that Plato had on our project of describing the universe using mathematics: "Platonism" is an enduring feature of fundamental physics. Johannes Kepler in the 16th century was among the first truly Platonic (or even Pythagorean) scientists and as I joked earlier, my particle physics subdiscipline is very Platonic.

Notice that MOTION has not been a feature of my discussion of Plato. In part, we think of Plato's ideas about motion as focused on astronomical topics, which we'll cover later in this chapter. But also his ideas as expressed in *Timaeus* (and to some extent in the *Laws*) are so esoteric as to be mostly unintelligible. There we learn that the Soul is responsible in part for "self-motion." It's all very unsatisfying.

"Unsatisfying" is a good stepping-off point as we consider Aristotle and his huge negative impact on physics. For someone so wrong, it's ironic that we can't ignore him.

2.2 Act VI A Little Bit of Aristotle

> "Aristotle is a Foal. When a foal has had enough milk, it's known that it kicks its mother." ascribed to Plato

While Plato's practical impact on physics was limited to abstract and esoteric notions, not so with **Aristotle of Stagira** (−384 to −322) an even bigger subject. He was a systems builder with practicality and abstraction as joint projects. The extent of his intellectual reach was incredible and not only did he further philosophical ideas, he invented whole fields of science and philosophy.

He was born in Stagira, near Macedonia north of Greece and was connected to Macedonian royalty as the son of the king's physician. He emigrated to Greece to study at Plato's Academy at the age of 17. . . and then stayed for almost 20 years. While he was in residence, probably beginning his writing, the Macedonian King Philip II began his conquest of northern Greek cities, including Athens. . . which came under his control through concession, and only limited conflict. (Set the context with the timeline in Figure 1.2 on page 8.)

When Plato died in −348, Aristotle went to Assus in the northwestern area of modern-day Turkey, married, and began (or continued) an impressive series of biological, marine biological, and zoological researches which he wrote about in *The History of Animals* and *On the Parts of Animals*. He was a details-person and described animals and insects with minute detail through dissection and description, beginning the classification exercise that established the whole science of biology for centuries. He classified more than 500 different species into genus and species forming categories of likeness and habit of mammals, fish, reptiles, and insects. It was here that he established his insistence on observation as the source of knowledge, an evolution away from Plato that was obviously severe. Think of his approach as like taking a deck of cards that's all swirled together on a table, and ordering the them all by identifying and sorting for like features—suit, color, and number. That kind of organization came naturally to Aristotle, it's very modern, and it seems to have first been apparent to him as a scientific practice.

His range was remarkable, covering: Law, physical science, psychology, natural science, philosophy, logic, ethics, and the arts. Words that we have from him include: energy, dynamic, induction, demonstration, substance, attribute, essence, property, accident, category, topic, proposition, universal... His metaphysics informed the development of his science and confused the awakening Western world from about 1100 to 1600. And, everything was a part of his system, and so abandoning one piece that might not make sense would bring the whole system down. It was a philosophical game of Jenga. In particular, his astronomy, and especially his physics, didn't make sense and we'll see that the Medievals knew that it didn't make sense. But selectively adjusting it seemed impossible.

One positive thing, if only his followers had preserved it: we have Aristotle to thank for dampening enthusiasm for the unwelcome Platonic idea of "Saving the Phenomena":

> "...speaking of phenomena, they say things that do not agree with the phenomena...They are so fond of their first principles that they seem to behave like those who defend theses in dialectical arguments; for they accept any consequence, thinking they have true principles—as though principles should not be judged by their consequences..." Aristotle, *On the Heavens*

We have three Aristotelian issues to consider for our narrow project which together only sample a small sliver of his whole universe: what is real, how does change happen, and his physical science.

2.2.1 Aristotle and What's Real and What's Knowledge?

Unlike Plato, Aristotle rejected the idea of a super-sensible realm housing the ethereal Forms. He had a different job for his Form that linked it with actual substance, here on Earth, closer to our idea of the form of a physical object. His focus—which was refreshing after the Parmenides Problem and now the Plato Problem—was on *individual things* which we learn about through a personal experience with the world, not through some intellectual abstraction. What's real for him are *particular objects*.

> “If we did not perceive anything we would not learn or understand anything.” Aristotle, *On the Soul*

Like I said, refreshing.

Substance—stuff—and Form work together to make the world. The oft-used metaphor of a house is instructive. In order to make a house you need stuff—wood, nails, and so on—and a plan, an organizing principle. Substance and Form. An individual thing is then matter which has been given a form and you can’t separate them. An individual thing must have both.

For Aristotle, perceived facts are the necessary ingredients for knowledge. We organize them in our memories, looking for commonalities and differences. We categorize our facts into bins of like and unlike with relationships among them. We have an individual perception of things, collect facts, ruminate on them by comparing in our memory with our internal database, and categorize. This is classical Empiricism, as opposed to Plato’s classical Rationalism. So far, so good. (Think about that deck of cards, now abstracted as a philosophical goal.)

2.2.2 Change and Cause

But we still can’t get away from the Parmenides Problem and Aristotle also did battle with change and permanence. Let’s race through how he thought about change and how it functioned in his physics.

For him, Change relieves a. . . tension. An actual thing, what **is**, has within it a potentiality to become something new. As long as it’s not in that newer state—it’s “deprived” of it—and it is obligated to go there. Inevitably. So everything is also in a Hericlitean flux, but in a very particular and interesting way. In sympathy, perhaps, with Parmenides, in order for something to change into something else, it had to **be** in the first place and taking that all the way back, takes him into an abstract place where there needed to have been an original Unmoved Mover. We’ll not follow that line of thought.

What’s important about change for Aristotle, which fits into his bigger system, is that in order to acquire knowledge of something that changes means you can identify the Cause of change. Because: *all change must be caused* and what can be caused comes from within a set of Aristotelian “Categories” (of being). The ten Categories is a complicated idea and we’ll skim. They are: substance, quality, quantity, relation, time, place, position, state, activity, and passivity — his complete set of predicates that can be assigned in a statement. For example, what can you say about Galileo:

- Galileo was human (substance)
- Galileo was smart (quality)
- Galileo was 5 feet tall (quantity)
- Galileo was older than Kepler (relation)

- Galileo lived during the 16th and 17th centuries (time)
- Galileo lived in Florence (place)
- Galileo sometimes sat at his desk (position)
- Galileo sometimes wore shoes (state)
- Galileo sometimes wrote with a pen (activity)
- Galileo was sometimes ill (passivity)

A particular substance must be all of these things in order to be a thing. In order to exist. Like I said, you have to be impressed with Aristotle's ability to take a complex topic and break it into its constituents. Remember, he invented Logic.

> " Nature is a principle of motion and change, and it is the subject of our inquiry. We must therefore see that we understand what motion is; for if it were unknown, nature too would be unknown." Aristotle, *Physics*

Substances have "motions" but not the kind you're thinking of. They're very Greek motions and can be quite abstract. For Aristotle, *motion is anything that goes* <u>to</u> something. In this change a substance remains a substance, but Form adjusts, characterizing the natural evolution of a state in which a goal is not achieved into a state in which a goal is achieved. And that idea of a "goal" is very important and in part, where Aristotle's physics goes astray. So the substance of a seed changes as it evolves into a flower. But the form of the seed and its various guises changes. Stay with me.

Motions can be of any of the Categories of being, but usually are among just three of them:

- change of quality
- change of quantity
- change of place

For example:

- Galileo changed from a boy to a man. That's a change of quality.
- Galileo changed from a person who weighed 50 pounds to a person who weighed 150 pounds. That's a change of quantity.
- Galileo moved from Padua to Florence. That's a change of place.

That last one, a change of place, is our modern idea of "motion" which he called "locomotion." But for him, locomotion is no more fundamental than any other kind of motion and that's very Greek. But, again, he's thought deeply and by accident, all three kinds of motion have examples today:

- Modern Change of Place: We tend to think of locomotion as the only one of his categories to apply to change in physics: objects moving from this place to that place, during some time.

That's familiar. But two of his other "motions" have modern examples which he would not have known of:

- Modern Change of Quality: A phase transition like water boiling or freezing could be considered a change of quality.
- Modern Change of Quantity: Aristotle could not have imagined a nuclear or particle decay from one thing into three different things, like the decay of a neutron into a proton, electron, and neutrino.

As for goals, it's easiest to think of the nature of something and that involves potentiality and deprivation. An acorn becomes a oak tree. An acorn does not become a Galileo, so it has within it the potential only to be an oak from the beginning. That inevitability also is universal and directed and that even becomes an argument against infinity since there is no such thing as unrealized or unconstrained potential.

> "It is not what has nothing outside that is infinite, but what always has something outside it." Aristotle, *Physics*

Now we know what properties a thing must have in order to exist and we know what kinds of change can happen. Again, to have knowledge of a change one must understand the causes: in fact, four causes. They are the material cause, the efficient cause, the formal cause, and the final cause.

Take a that house:

- The material cause of the house is the wood, nails, and so on.
- The efficient cause of the house is the action of the carpenter.
- The formal cause of the house is the blueprint in the mind of the carpenter.
- The final cause of the house is the purpose for which it was made.

There is sometimes a discussion about whether these function as causation or explanation. Are they the four "becauses"? In any case, the last one of them is problematic for physics as the notion that everything moves for a purpose (that "goal" again) doesn't work in modern terms. This is called "teleological." (One can imagine an argument for Aristotle that there is some teleological logic to how plants and animals "move" from one kind to another...seeds to plants, kittens to cats, and so on.) Of the four (and there's a lot more detail in Aristotle than just enumerating them), Efficient Cause comes the closest to a modern physics cause. That's splitting hairs!

2.2.3 Aristotle's Physics

Aristotle inherited his ontology (the philosophy of being) from his teacher, who inherited it from Empedocles. That is the four elements of earth, air, fire, and water are supplemented by one more, "aether" which is outside of the earth-bound region of the universe. Like the reactions to Parmenides, Aristotle envisions "stuff" as mixtures of the four elements. But he goes further than just classification, as their makeup, Causes, and Categories all feed into his explanation for the sort of motion that we think of. So understanding locomotion is intimately tied to the entirety of the Aristotelean system.

With respect to our familiar MOTION, he was very much an empiricist and locomotion in particular fits his overall philosophy. Watch a high kick of a soccer ball or a towering home run in baseball or a shot in the shot-put. The projectile will race to the top of its trajectory and then appear to fall steeper and faster than its rise. Drop a feather and a crumpled up piece of paper and a metal key. Will they hit the ground at the same time?

In each of these everyday examples it seems like the heavier object will hit the ground first. That fits his philosophy, or maybe his philosophy grew from watching things fall since the heavier an object is, the more deprived it is of its most natural place: the Earth. So any object seeks its place by virtue of the amount of earthiness it has in its composition. Heaviness is an attribute and the natural motion associated with heaviness is down, toward the center of the Earth. *Lightness is also an attribute* for Aristotle (for us, that's just less heaviness). Natural motion for a Light object is up, toward the sky. So, below the orbit of the Moon, objects have two kinds of natural motion:

- Natural locomotion for heavy objects is down and natural motion for light objects is up. These Earth-bound motions — MOTION ON THE EARTH — both follow straight lines toward their preferred places. So firey things want to be at the edge of the Moon's orbit and earthy things want to be at the center of the universe (the Earth).

But MOTION IN THE HEAVENS of the Sun, Moon, planets, and stars don't move in straight lines and have no apparent pushing force, so they must be composed of different stuff from Earth, Water, Air, or Fire and have a different sort of natural motion:

- Cosmic objects are made of "aether" and have circular natural motion.

Like all motions, Earth-bound objects away from their natural places are deprived and realization of their potential is to . . . go there. Celestial objects move naturally in circles. To fulfill their essence.

There is another kind of locomotion which is un-natural, dubbed "violent," and for Aristotle what causes violent motion must be a contact force. So throwing a ball is violent and unnatural, since it's not directed down. When the ball is in contact with your hand, you're making it move. When it leaves your hand? Well, here Aristotle had trouble and everyone knew it. The contortions that he went through to explain projectiles are pretty contrived. But he was wedded to his system and in spite of his scorn for Plato's Saving the Phenomenon, he seems all about that here.

When the ball leaves your hand, it doesn't immediately head towards the center of the Earth. The medium of the air is critical in two ways:

1. The motion of the hand is (somehow) transferred to the air which (somehow)

successively creates forces in steps... air moves the projectile, then another segment of air moves the projectile... and so on until the ability of the air to perform that critical contact-force job is used up. Somehow the forces of air meet some dissipative force... of the air(!), and it stops.

2. Then the object falls directly to the ground because the air stops it.

The air both moves it and stops it! Also, the projectile doesn’t share both unnatural, forced motion and a component of natural, downward motion. There’s a lot not to like about this. Even probably including Aristotle given his complicated explanation. Figure 2.2 is a 16th century depiction of Aristotle’s projectile paths: straight line up, then straight line down.

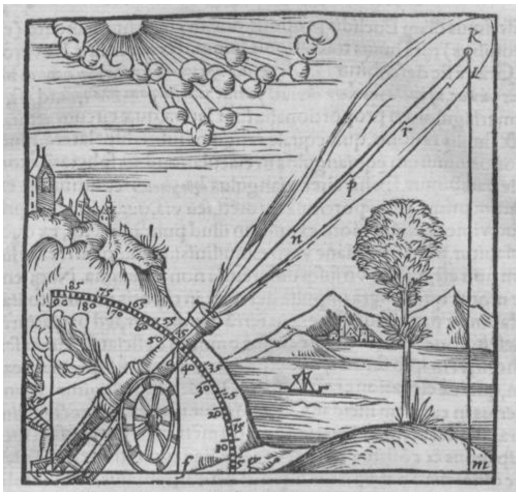

Figure 2.2: A drawing of Aristotlean projectile motion in a mathamatics text by Daniel Santbech in 1561.

Now he’s not entirely consistent in his descriptions. In his *Physics*, he says:

> “Again, as it is, things thrown continue to move, though that which impelled them is no longer in contact with them, either because of”mutual replacement" as some say, or because the air which has been thrust forward thrusts them with a movement quicker than the motion by which the object thrown is carried to its proper place." Aristotle, *Physics*, IV, 8

Later in Book VIII he says:

> "Therefore, we must say that the original mover gives the power of being a mover... to air... naturally adapted for imparting and undergoing motion... The motion ceases when the motive force produced in one member of the consecutive series [of forces imparted by the air] is at each stage less, and it finally ceases when one member no longer causes the next member to be a mover but only causes it to be in motion. The motion of these last two—of the one as mover and of the other as moved—must cease simultaneously, and with this the whole motion ceases.

The first extract seems to make reference to an idea that’s in *Thaeatus* called antiperistasis, in which Plato tries to explain respiration, suction, and falling bodies as displacing the air and back-filling it to avoid a vacuum. This either evolved too, or was also a suggestion by Aristotle that the air in front of a ball rushed around to the back and pushed the ball forward. I know. It makes no sense. The Medievals were very critical and modified the ideas.

Aristotle didn't know algebra, but we can most easily summarize his points with some simple proportions. The mathematical symbol for "proportional to" is $\sim$.

He would describe the locomotion of a projectile with these ideas:

- Heavier objects (made of more earth than other elements and so highly deprived of its natural place) would fall faster than light objects: $t \sim \dfrac{1}{W}$ where W is the weight, a stand-in for earthiness. Heavier objects would then fall faster than light objects —have a higher velocity.
- He had some sense of the resistance of air and so the velocity relates to weight and resistance as $v \sim \dfrac{W}{R}$ where R is some measure of the resistance that air or water or some medium asserts on the falling object.
- This leads to a convenient conclusion. If there is no resistance, then $R = 0$ and the speed that if falls would become infinite. But nothing can be infinite in Aristotle's philosophy, so there is no vacuum allowed…no medium with zero resistance.
- And finally, for violent motion, which requires an external force in contact with the object, $v \sim \dfrac{F}{R}$. No force, no speed. More force, more speed.

Each of the bullets describe exactly what you and I experience every day in a sport with a ball or just life. Aristotle is clearly a champion Empiricist.

There's more. If linear motion is the only natural motion then his Earth *must be stationary* otherwise, we'd would feel the effects of some tangential wind-force rotating the Earth. And we don't, so his Earth *does not rotate*. For objects in the heavens, since they move naturally but in circles, a different material is required, a fifth-element.

2.2.4 Summary of Aristotle and Locomotion

So to sum up the first real study of MOTION…ever.

1. MOTION ON THE EARTH is of two types:
    1. Natural motions are toward or away from the center of the Earth according to the degree of heaviness (among the four elements, Earth would dominate the others) or lightness (among the four elements, fire would dominate the others) that compose their substance. Natural motions are in straight lines. They represent the fulfillment of an object's potential.
    2. Unnatural, or violent motions are those which are not natural. They all

require that an external force is applied throughout whatever trajectory a body experiences. Take away the force, and the motion would cease. These motions can be of any shape.

2. And motion by the Earth?
    1. It’s zero. The Earth is stationary because no forces can be detected that would be required to make it move. And, motion on the Earth doesn’t suggest that the Earth is moving. Throw a ball up and it doesn’t fall behind you, as he suggested would be the case if the Earth were moving. So he has an explanation as to why it must be stationary, but not a prediction. He’s justifying his contention.
3. And motion in the heavens?
    1. That motion is circular. Objects outside of the Moon’s orbit are of an entirely different substance that what we experience: aether. Why? Since if they were of the same material that that of and on the Earth, its natural motion would be in straight lines.[5]

Aristotle’s theories of motion by the Earth, motion on the Earth, and motion in the heavens are relentlessly empirical: they are theories of what we all observe in our everyday lives. His theories of motion are wrong, relentlessly abstract, and hidebound to the rules by his overarching philosophy.

2.3 Plato and Aristotle, Today

2.3.1 Modern Day Platonists

> “I imagine that whenever the mind perceives a mathematical idea, it makes contact with Plato’s world of mathematical concepts...When mathematicians communicate, this is made possible by each one having a direct route to truth, the consciousness of each being in a position to perceive mathematical truths directly, through this process of “seeing.” ’
> Roger Penrose (1931-), theoretical physicist, Nobel Laureate

It’s unlikely that anyone today would wonder about the application of Aristotelianism into the physics of motion on the Earth, motion by the Earth, nor motion in the heavens but thousands of pages of writing (and links) have been devoted to the application of Platonism into modern physics, and especially in mathematics. Recall my party-question in the previous chapter: Is mathematics discovered or invented? Many mathematicians and physicists have concluded that it’s discovered and that’s the bumper-sticker version of modern Platonism: suitable for the 21st century.

In this *Plato and Aristotle, Today* section I’ll describe a more modern version of Platonism that might function in physics in two different aspects which I’ll call “The Platonic Process in Physics” and “The Platonic Reality in Physics.” It’s about an

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[5] some circular reasoning there, no pun intended

evolved notion of the Forms.

2.3.2 The Platonic Process in Physics

The Forms were by far the Platonic idea with impact for all branches of philosophy, mathematics, and physics. His premise is that reality consists, not of only everyday stuff (that's the Ionian "monist" position that all of reality is made of matter) but that there is an additional reality-realm which consists of non-material entities outside of space and time. This is the premise of the movie *The Matrix* in which Morpheus gives Neo the choice of two pills: if he takes the blue pill, he's choosing to continue to live his life in an artificial but comfortable world in which we don't examine what's true and happily accept opinion as knowledge. If he takes the red pill, he's chosen the more difficult path: to live in the truth. The references to the *Allegory of the Cave* are obvious, but it's also the old biblical story of eating from the Tree of Knowledge.

Paying homage to Morpheus' red and blue pills, let's call our everyday, physical world, the **Blue World** (BW) and the ethereal, maybe more truthful world, the **Red World** (RW...in order to help us remember, think of it also as the "Real World."). And let me try to suggest that to be a modern physicist is to be partly a Platonist, working as if the BW and the RW both exist simultaneously. Stay with me.

Plato's classical RW is where the Forms reside in which they had two broad characteristics:

1. For Plato, forms exist in the RW which are permanent, outside of space and time, and represent the essences of all things and ideas. All objects in the BW — objects we would call physical objects — "participate" in the Forms. My example was the perfect sofa.
2. The RW contains the only true things and so acquiring Truth (with a capital "T") means somehow realizing the Forms in their natural, unusual habitat uniquely through our intellect.

So Plato's is both a story about ontology (the philosophy of what exists) and epistemology (the philosophy of what we can know).

The heated debates of the last 50 years about Platonism are largely about mathematics. In this literature it's not hard to find questions like whether the reality of a tree is different from the reality of $\sqrt{2}$. In some way, the latter is more permanent. And, of course, there are also the perfect objects of geometry...and maybe the rules of geometry. I think it's fair to generalize that there are three schools of thought in the Philosophy of Mathematics that can be labeled as:

- Intuitionalism, where mathematics is just the product of mental activity and a mathematical entity is constructed by the mind and lives solely in the mind. This is also sometimes called "structuralism" or "constructivism."

- Formalism, is probably the most popular camp in which there is no truth-value assigned to any mathematical property or entity. It's all just the study of logical consequences... dubbed "if-thenism." There's no commitment to anything beyond manipulating marks on paper according to the rules of the game.
- Platonism suggests that mathematics is the study of abstract entities that have an existence that's as real as the external world targets of scientific experiment. So the question for Platonism is: do abstract mathematical things exist? Do abstract rules exist?

2.3.2.1 Quine–Putnam Indispensability Argument

I've had the misfortune... or fortune... of doing physics research for half a century after a masters degree in the philosophy of science. That means that I've never been able to avoid standing back and looking at what I do and what my colleagues do and categorizing and analyzing process, what counts as a valid argument, what counts as a valid scientific question, and what counts as an acceptable answer. And what about "reality"?

I'm intrigued with a particular strand of Platonism that's due to **Willard Quine** (1908- 2000) in the 1950s through 1990's, and **Hilary Putnam** (1926-2016), who later found common cause with Quine. Together, their ideas are called the **Quine–Putnam Indispensability Argument.** To an aw-shucks, country-physicist like myself, I interpret it to say:

1. Science (read "physics") works and interacts with real objects in the BW through experiments.
2. Mathematics works and interacts with abstract quantities and rules in the RW.
3. Physics cannot not work without mathematics, and so the two are *indispensable*. This is a partial answer to Wigner. "Unreasonable effectiveness" becomes "indispensability."
4. Given the impossibility of physics without mathematics, abstract mathematical-physics entities in the RW should enjoy the same level of reality as the objects of experiment in the BW.
5. So there are at least two realities: a physical reality and a mathematical reality.

The Quine–Putnam Indispensability Argument both rhymes with Wigner and demands a new definition of physics.

> "[talk of" mathematical entities is indispensable for science... therefore we should accept such talk... [which] commits us to **accepting the existence of the mathematical entities in question** [emphasis mine]." **?**, *Philosophy of Logic*.

Quine called himself a "reluctant Platonist" and I think that physics has joined that club. And as I'll show in Chapter 8, Galileo was the charter member and he showed us all how to make progress in unraveling MOTION BY THE EARTH, MOTION ON THE

EARTH, and MOTION IN THE HEAVENS once the club's Platonism was embraced.

A few random comments about the Quine–Putnam Indispensability Argument.

- Do I have to be a believer in order to do physics? No. You might be surprised how little philosophical thinking goes into a professional physics education. Long ago, the pain inherent in thinking too hard about, first quantum mechanics and then general relativity taught those of us who teach these subject to undergraduate and graduate students to not go there. "Shut up and calculate" is not just a funny phrase, it's actually an instruction that you must follow if you're going to make scientific progress. We physicists don't tend to analyze physics any more than a bird analyzes the dynamics of flight.
- Where does this leave mathematics and their philosophical problems? Well, first, we pretty much don't care! Second, Mathematical Platonism adherents think it's perfectly fine for there to be a plethora of mathematical realities. A multi-verse of mathematical worlds, if you will. Some of them have that special connection with physics...and some of them don't.
- I've concluded that we are relentlessly *both* Platonic and Pythagorean. We can't make progress nor explain the incredible success we've enjoyed without the rules of physics (the "laws") nor without the commitment to the numbers required to make predictions and then contact with experiment. The Platonic is joined with the Pythagorean, in contrast to Plato's Divided Line, the division is blurred and crossable.
- Is it just too unreasonable (sorry) to deal with this multiple reality stuff? A reasonable person might say that if I can touch it or kick it, then it's real. A pretty good working definition of "reality." Stay with me.

2.3.3 The Platonic Reality in Physics

What I described above is about a *process*. But there's also an "ontology." What are the objects of fundamental physics and do they live in the BW or the RW? Let's look at two objects and then go kick a rock.

2.3.3.1 Their Own Forms

There is no sofa that's identical to its form. Even two sofas designed and constructed in the same manufacturing facility will not be identical. Patterns on one will be slightly altered from the other. Tolerances on color or fabric structure or leg shape cannot be perfect. A BW sofa is not identical to it's RW Form. They're separated into the two Realms.

The 20th century has upended this very Platonic separation and Plato might have been intrigued with the result.

A molecule of hemoglobin in your blood contains 10,000 atoms of hydrogen, oxygen, nitrogen, and iron. Each of these atoms have protons, neutrons, and electrons. Isn't it remarkable that each of the many thousands of electrons in that single hemoglobin molecule are identical to one another?

Isn't it even more remarkable that each of those electrons in my blood is absolutely identical to an electron in an atom of hydrogen in the outer edges of the Andromeda Galaxy? Or to every electron that was flying around the early universe before Hydrogen atoms formed at 370,000 years after the big bang. (I might note that every hydrogen atom in your hemoglobin was in fact formed in the big bang.)

A prefect form of an electron — the ideal electron in the RW— is identical to its BW counterpart electron. No imperfection. No difference.

So the distinction between Forms and the objects in the BW that participate in the Forms evaporates as soon as we begin to deal with elementary particles. That is, when we begin to confront the universe as it is composed in the BW.

Elementary particles in our everyday BW are their own Platonic Forms.

2.3.3.2 Are Wavefunctions BW Or RW Or Not Real At All?

Want some serious Plato? I give you Quantum Mechanics, the theory of the very small: atoms, electrons, nuclei, elementary particles, and quantum fields. Atoms and all of chemistry is precisely determined by a single equation called the Schrödinger Equation which can be solved to determine the "state" of an atom and make predictions about properties of matter. For example, the model of the optical spectra that result from electrons falling from high orbits to low ones by emitting unique colors of light is the first prediction of quantum atomic theory and was bang-on correct. Quantum mechanics is exquisitely precise and its predictions match experimental results to mind-boggling precision. It works better than any theory ever invented.

But Quantum Mechanics comes with a very strange substance that we cannot see, hear, touch, or measure. We arrive at predictions by calculating the evolution of the spooky entity called the "wave function," ψ. The wave function seems to me to be the very definition of a RW-existent, mathematical entity. Essential to the physics, but with an existence on paper only—a very Quine-Putnam idea.

We predict the results of an experiment involving atoms, molecules, or elections by mathematically evolving their wavefunctions using the Schrödinger Equation which takes $\psi(t_1)$ at some time, t_1 and tells you precisely how $\psi(t_2)$ will behave at time t_2 in the future. This works perfectly. Every time.

But here's the rub: *ψ is intrinsically undetectable*. It doesn't exist in the BW, but it does have a communicable existence as mathematical marks on paper. We make a connection in the BW by predicting the *probability* that a particle will be here. . . or there. . . .or over there. . . or on the Moon. That comes from the *square of the wavefunction*, ψ^2. Remember that party you un-livened up with the question about mathematics? Ask two physicists in attendance, “Is the wavefunction real?” Then stand back. That will liven it back up.

Let me repeat: we can calculate the value of ψ at any time or place in the future, but to connect with a measurement, we can only predict probabilities, no certainties are allowed. Ever. We cannot get from the equations of Quantum Mechanics to a measurement in the BW without passing through a RW Platonic manipulation of the mathematical entity, ψ.

If you ever needed a definition of a mathematical entity that behaves as if it has a reality only in the Intelligible Realm, the wavefunction, ψ, is the poster child for exactly that. For Quantum Mechanics to function, we must work wholly inside of a very strange mathematical RW which indispensably (in that Quine-Putnam sense) is very real. And Quantum Mechanics works better than any theory ever devised in any science.[6]

So every entity in physics is ultimately an elementary particle, which is its own Platonic Form and which is described by a mathematical entity which cannot be observed.

2.3.3.3 “I refute him thus!”

In a different context, it was the British writer of the *Dictionary* Dr. Samual Johnson claimed to be able to refute the Idealism of Bishop Berkeley that to be real was to be observed. He kicked a rock and declared, “I refute him thus!” Well, there’s a lot inside of a rock.

It’s quite natural to insist, “I know there’s a real world out here because I can see and touch stuff!” Okay, let’s talk about touching. That rock that you kicked with your foot is not a solid hunk of stuff. It’s made of minerals in crystalline structures of definite chemical elements: atoms with electrons in their atomic shells which have complicated bonding with their “home” nucleus and across the crystals with neighboring atoms. Your foot is made up mostly water in cells and tissues, so, of course, different atoms in different arrangements.

These atoms of the “kick-er” and the “kick-ee” interact with one another as you bring your foot very, very close—molecularly close. There would be some deformation of the two materials (to your foot’s disadvantage) since the rock’s lattice is relatively rigid in comparison to the tissues of your foot. But what’s going on? The electrons at the surface of your foot are repelled by the electrons in the outer orbits of the atoms at the surface of the rock. And to make it even more complicated, there’s a region of quantum mechanical attraction and repulsion that is active between the whole molecules of the two materials called the “Van der Waals force.” So *your kick is inherently a quantum mechanical process and is as real as the wavefunction* of the

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[6] Einstein famously washed his hands of Quantum Mechanics, immensely uncomfortable with its lack of certainty, related to the reliance on the wavefunction. And he was one of its inventors!

previous section, and the electrons and photons of the section before that. You think you kicked a solid thing that's a rock in the BW, *but what you did was cause a quantum mechanical interaction only describable in our RW*.

Again. As practicing physicists do we stay up at night worrying about the different realities that our description of nature presents to us? Or do we just keep on calculating...because it works. For almost all of us, it's the latter. We're actually all trained to be highly skilled "Quantum Mechanics" seemingly working in the BW of experiment, without concern for the philosophical niceties of the RW of the equations. This is the same as a skilled engine mechanic working under the hood of your car who doesn't need to know the material science or engineering of the digital electronics of the engine and control systems to solve BW problems.

But Plato is there. He's changed his mind about a few things, but when it comes to philosophical longevity— when it comes to physics—he outlasted Aristotle. By millennia. But Aristotle also had his moment. Take out your phone.

2.3.4 Aristotle's Legacy in Physics and Engineering

Aristotle invented the iPhone. Well, not exactly all of it, but he created the basic language that all electronics use to process instructions and communicate internally. This language allows digital components in integrated circuits to do arithmetic, compare number strings, turn peripherals on and off like pixels on a screen, and many other functions. All of this comes from seemingly endless strings of logical operations performed by mind-boggling numbers of individual digital "gates" of silicon which do very simple things.

You see, Aristotle invented that language and I think that's his modern legacy: Aristotle first conceived of the rules of **Formal Logic** which were so powerful, they instantly became active research projects for ancient and medieval philosophers for a thousand years. "Logic" is now the primary subject in whole fields: Philosophy of Logic, Discrete Mathematics, and Computer Engineering! If winning an argument is important and if you can reliably create valid arguments and always identify invalid ones, then you possess a superpower.[7] That was his goal. Making that superpower. For a more detailed introduction to the field of Formal Logic, see Appendix A.2.2. Here I just want to hit some broad ideas.

2.3.4.1 Valid, Invalid, and Sound Arguments

In the courtroom, the board room, in science, and everyday life having the facts in hand is only part of a winning strategy to persuade others. Your argument has to be, we colloquially say, "logical." We all have a sense of what that means, but it

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[7] We'll see in Chapter 4 the re-discovery and use of Aristotle's Logic was arguably the major threat to the dogmatic Augustinian Catholic Church in the 12th century. An uneasy truce was pieced together by Thomas Aquinas by the 13th century.

can be nuanced. Let's look at two examples of arguments. Notice that I've added parentheses that demarcate important phrase chunks in each of the three lines.

Example 1.

- (All apples)(are fruit)
- (All red objects in that tree) (are apples)
- Therefore, (All red objects in that tree) (are fruit)

Example 1. hits you right, I'll bet. This is the kind of argument called a Syllogism consisting of two *premises* followed by a *conclusion*. Here's another one:

Example 2.

- (All elephants)(are English speakers)
- (All squirrels) (are elephants)
- Therefore, (All squirrels) (are English speakers)

Now Example 2. kind of hurts. These seem like very different arguments and you'd want to say that that this second one is absurd or wrong—more about that in a bit. But can you see that they share an important feature: they are both structured in the same way—they have the same **form**. Try this:

Example 3.

- (All A)(are B)
- (All C) (are A)
- Therefore, (All C) (are B)

This shows the structure of both arguments. In both examples we can identify: A = apples/elephants, B = fruit/English speakers, and C = red objects in that tree/squirrels. Many substitutions will work for A, B, or C if the premises and conclusion are arranged like the above.

There's more: in any argument arranged as in Example 3. the conclusion is "forced" on you. The easiest way to see that is to look carefully at the "Euler Diagram" in Figure 2.3.

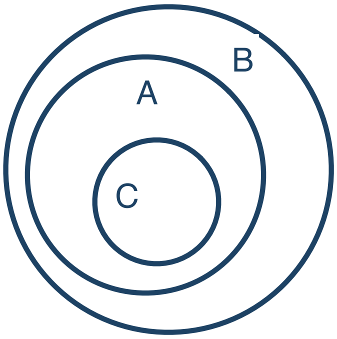

Figure 2.3: In a valid argument shows that one is forced to conclude that All C are B.

Can you see that in Figure 2.3 there are three circular areas, the biggest of which is B. All of region A is inside of the bigger region B so the first premise that (All A)(are B) is evident and that all of C is inside of A, so the second premise that (All C) (are A) is evident. So from the picture you forcefully conclude that (All C) (are B)—the conclusion of Example 1. You're worried about talking elephants. Stay tuned.

2.3.4.2 Greatest gift

Aristotle's unique invention that makes general rules possible for argumentation was to create what I think of as an *algebra of language*. Here is a seminal moment in history, from the first book of his *Prior Analytics* (focus on the last sentences):

> "...if every B is A then some A is B. For if no A were B, then no B could be A....e.g. **let B stand for animal and A for man. Not every animal is a man; but every man is an animal.**" (emphasis, mine) Aristotle, *Prior Analytics*.

Look at the sentences that I've highlighted: he's using variables A and B, to stand for things, here in his example, A = man and B = animal. Instead of men and animals, the variables could be squirrels or fruit. As long as the *form* is proper, we say that the argument is "valid."

Let's be clear—because Logic is all about clarity and bottom-up reasoning. We all use words that sometimes have specific meanings in specialize fields like Logic. Here are some that I'll make use of in this section. Some definitions for us:

- Here, we will use the term *statement* as a kind of a sentence which can be true or false. "Elephants are larger than squirrels." is a true statement. "All bachelors are talking squirrels" is a false statement.
- When a statement includes a "quantifier" (an example of which is "all"), a subject, a connective (often called a copula, a form of the verb "to be"), and a predicate we'll refer to these as *propositions*. (All apples are fruit.) is a true *proposition*.
- Not all sentences are *statements* or *propositions*. Our two here are aimed at logical argumentation.
- *Statements* and *propositions* can be true or false.
- We will use the term *Arguments* in two ways. In this subsection, a *Syllogistic argument* will stand as an ordered collection of *propositions* (here, the *premises* of the argument). As we saw, Syllogistic arguments are constructed as specific forms. (In the next section, we'll refer to a different kind of argument, a *Propositional argument*.)
- Syllogisms were Aristotle's first venture into Logical arguments and he identified 16 valid forms, but others after him found additional ones. Most likely it was the 13th century University of Paris scholar, William of Sherwood, who gave names and hints to identifying the 19 valid syllogisms (out of 256) and this particular one is called "BARBARA."[8]
- Syllogistic arguments consist of:

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[8] BARBARA wasn't a person, but a nemonic invented by Sherwood in order to remember the kinds of statements are in the premises and conclusion. Here the three are "All" statements, and hence his name, "A" statements. So they are "All x are y." E statements are of the form "No x is y" and for such a syllogism he invented the nemonic, CELARENT, with two E's and one A statement. He did this for each of the 19. Medieval analysis of Logic was exhaustive and probably exhausting. This dedication has carried on to this day.

  - two propositions which are premises, which in the above examples are the first two sentences and
  - a single proposition which is a conclusion.
- A Syllogistic argument which is properly constructed according to one of the defined forms is simply *valid*, without regard to the terms (the A, B, or C).
- A Syllogistic argument constructed according to one of the defined forms which has true premises is called valid and *sound*. That is: If the premises are true, and the argument is properly formed, then the conclusions must be true in a sound argument.
- A Syllogistic argument which is not ordered according to one of the defined forms is *invalid* and *unsound*.

Introducing variables as a placeholder for the subjects and objects in a statement is a seminal moment in the history of mathematics.

Amazing. Out of this beginning, your mobile phone was born.

Now, about talking elephants and talking elephant-squirrels. Elephants can't speak English and squirrels aren't elephants. So Example 2. is a *valid, but unsound argument* according to the rules of Logic that Aristotle invented. Why? Well, remind yourself of the "Euler Diagram" in Figure 2.3. Its conclusion is forced on you. Now consider this argument:

Example 4.

- (All elephants)(are English speakers)
- (All elephants)(are squirrels)
- Therefore, (All squirrels) (are English speakers)

This has the form:

Example 5.

- (All A)(are B)
- (All A)(are C)
- Therefore, (All C) (are B)

Notice that between Example 3. and Example 5, that the order of A and C in the second premise are switched which is enough to make Example 4. invalid. So not only are the premises not true (so not sound), but it's also logically invalid and to get a sense of that, look at Figure 2.4. The caption explains why one is valid and the other not.

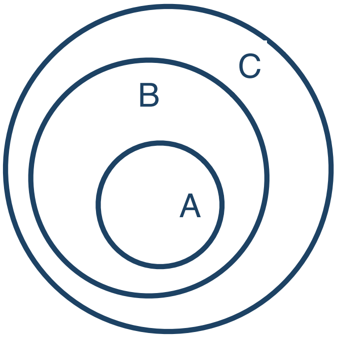

Figure 2.4: Here the invalid argument is clear. All of region A (elephants) are indeed included in region B (English speakers) but "all C (squirrels) are B (English speakers)" does not hold since there are regions in C (the squirrels region) that are *outside* of region B. Only some of region C are inside of region B.

Aristotle covered this new-born subject in a number of his books, including: *Categories*, *On Interpretation*, *Prior Analytics*, *Posterior*

Analytics, *Topics*, and *On Sophistical Refutations* which collectively, were much later dubbed "*Organon*" which means "instrument."

What I've chosen for my elephant-squirrel example is one of 256 possible syllogistic forms. Maybe you can see why studying Logic became a matter of intense research following Aristotle's death and into the first 1000 years of both Arab and Western philosophy. There was lots of work to do.

These arguments are examples of **deductive logic** which is often contrasted with **inductive logic**. In Deduction, if the form of the argument is according to the rules, then the argument is guaranteed to be valid. That's the sort of argumentation that was used in Socrates' discussion with the slave boy in the sense that the conclusion of a deductive argument is in some sense, already in the premises. Inductive logic is not reliable since it's not rule-bound and it delivers conclusions which can seem persuasive but aren't true.

Here's a personal, inductive argument about squirrels:

- (As a child) There's a brown squirrel
- (As an adult… many times) There goes another brown squirrel
- Wow… more brown squirrels and no other ones
- What is it with all of the brown squirrels?
- Gosh, I conclude that all squirrels are brown!

Induction not only can sound persuasive, it sounds scientific. And it is an important form of reasoning in science but it must be used with care. Aristotle knew of both kinds of logic.

Here's a problem with my induction about squirrels: Before I moved to Michigan, the only squirrels I'd ever seen were brown. Now my yard is full of black squirrels. They're everywhere. Many times in science a deduction uses premises which came from inductive reasoning, so even if the deduction form its proper, the argument might be unsound. Induction is always vulnerable to being questioned but the soundness of properly formed deductive arguments can only be challenged by questioning whether their premises are true. That's where a lot of the scientific action is.

From this point, when I refer to "logic" I'll mean deductive logic. By the way, Sherlock Holmes is reputedly the Master of Deduction. Well, sorry. That's not true. If you look at his stories you'll see very, very few examples of deductive reasoning. He's the Master of Induction![9]

2.3.4.3 Propositional Logic

Theophrastus (−371 to −287) was a favorite student of Aristotle's who led the Lyceum for 37 years after his teacher's death. Aristotle even willed him the

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[9] Or more appropriately, the Master of Abduction, a, third kind of logic. Look it up.

guardianship of his children...and his library. While a devoted student, Theophrastus went beyond his teacher and expanded and modified some basic Aristotelian notions. He also moved the study of botany forward and worked extensively in Logic. Theodor Geisel (Dr. Seuss) used "Theophrastus" as a pen name.

He is probably the one who extended the idea of syllogistic argumentation into a new direction with the invention of "propositional logic" in which (for our examples here) there are two variables, rather than the three of a syllogism.[10] In the same spirit as our definitions above, we'll call these *Propositional arguments*. This is where the modern engineering action is.

Propositional arguments are different in form, and content from Syllogistic arguments. They involve a statement that is conditional: an "If this ....then that" statement. Let's contrast them. Here's a Syllogistic argument:

- (All apples)(are fruit)
- (All red objects in that tree) (are apples)
- Therefore, (All red objects in that tree) (are fruit)

Notice that the variables In Syllogisms are kinds of things (called classes in Logic).

Here's a Propositional argument which seems similar, but is very different:

- (If those red objects are apples) (then they are fruit.)
- (They are apples.)
- Therefore, (they are fruit.)

Here's how a Propositional argument is very different in an important way. The variables have a "truth-value," TRUE or FALSE.

Just as before it's useful to abstract the specific terms in the premises with general symbols and Table 2.1 does this on the left in words, and on the right using logical symbols. The → symbol means "implies" and is associated with an "If...then" kind of statement. The lone A is a standard way to say that "A is the case" or "A is true." Finally, the symbol ∴ means "therefore." It doesn't seem like much, but it's powerful. Establishing the truth-value of the conclusion of a Propositional argument can be straightforward, or complicated. The game is to analyze the argument, again, for formal validity and ask whether the truth value of the premises guarantees to the truth of the conclusion.

A Conditional in Words	A Conditional in Symbols
• If A is true, then B is true • A is true • Therefore, B is true.	• $A \rightarrow B$ • A • $\therefore B$

Table 2.1: A Conditional argument and its concise symbolic equivalent.

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[10] Propositional arguments can have any number of premises and variables.

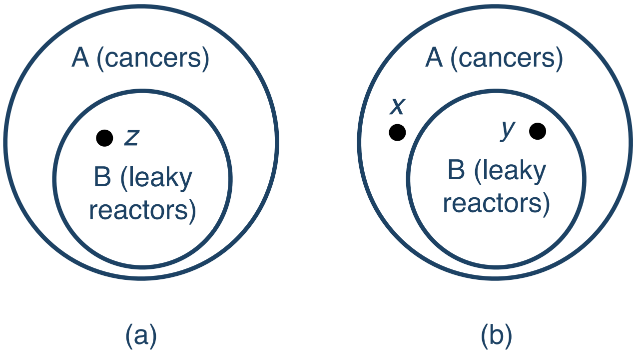
> An argument of this particular form (If A then B), (A), (therefore B) is called "Modus Ponens" (Latin for "method of affirming") and is one of six basic forms of propositional logical arguments. Another common propositional argument is "Modus Tollens," which also seems intuitive. For example: (If it is an apple) (then it is a fruit), (It is not an apple), (therefore it is not a fruit.)

2.3.4.4 Logical Fallacies

Propositional logic lays bare some logical fallacies which can be mistakes. Or logical fallacies can be used to convince people of the truth of a conclusion using an argument that appears to be valid, but is not. Look at the argument on the left in Table 2.2. Its validity is forced on you in the way that deductive arguments must do. A subtle change can take a valid argument and turn it into an invalid logical fallacy called "Affirming the Consequent," by switching the consequence for the hypothesis in the second premise. Can you see that the argument on the right in the table is sneaky, and invalid? People get cancer from all sorts of causes and that someone got cancer does not mean that the reactor leaked radiation. Health care is often a target for this form of fallacy.

A Valid Modem Monens Argument	A Logical Fallacy
• If a reactor leaks radiation, people nearby will get cancer. • A reactor leaded radiation • Therefore, people nearby got cancer.	• If a reactor leaks radiation, people nearby will get cancer • People nearby got cancer • Therefore, the reactor leaked radiation.

Table 2.2: On the left, is a valid Modus Ponens argument. But on the right is a logical fallacy called Affirming the Consequent.



(a) (b)

Figure 2.5: On the left is the valid argument that says that the placement of z with both a cancer and near a leaky reactor is the only result of the valid argument. But the right says that there is a cancer, but it could be either coincident with a leaky reactor (y) or have nothing to do with a reactor (x), and so the argument is invalid.

The objects in Figure 2.5 —which are not strictly Euler Diagrams— but similar to them— help to capture the argument. The conclusion of the valid and invalid arguments is apparent by the way the circles are arranged. The left diagram and the right diagram are the same since they represent the "If...Then" part of the argument. So within that arrangement, we can ask about validity by looking at entities that might fit the discussion. Look at entity "z" in the left diagram. It has the property B and since B is inside of A, it also has the property A. So given the argument, that the reactor leaked and entity z is inside that leaked region, it also is inside of the cancer region, completing the Modus Ponens true conclusion.

The diagram on the right has the same two regions, but now in the spirit of the invalid argument assert that entity y has the "attribute" of having cancer, so begin inside of region A. But this doesn't exhaust all of the possibilities for an entity having cancer. Entity x is also asserted to have the property of having cancer, but it doesn't support the conclusion that it overlaps with the leaky reactor region. So that second argument is not valid.

2.3.4.5 The Connection with Our Modern World

Aristotle's logical writing came from a deep level of analysis of language and thought. From the ground up. One might think that some ideas are just too trivial to write them down, but he wrote them down and defended his definitions even the most trivial bits. Here's one:

> "...there cannot be an intermediate between contradictories, but of one subject we must either affirm or deny any one predicate" Aristotle, *Metaphysics*

This is called the Law of the Excluded Middle. *A proposition is either true or it's negation is true.* There's no in-between. It's binary. This is a "two-valued" logic and Aristotle's structure was always built around that requirement: he didn't admit the (modern) idea of "degrees of truth" or "fuzzy logic." Trivial? Centuries of ink have been spilled over precisely understanding the implications of Law of the Excluded Middle and how to unequivocally state it symbolically. It's a simple idea that's deep and he had a number of such crisply defined notions so his Logic was really built from first principles.

What else can you think of that's strictly two-valued? How about binary arithmetic, where the only numbers are 0 and 1. How might you trivially represent 0 and 1? How about a pair of fixed voltages, say $V = 0$ and $V = 5$ volts.[11] There are a handful of seminal discoveries about Logic that extend to our modern usage. **Gottfried Wilhelm Leibniz** (1646–1716) refined binary arithmetic. In 1854, **George Boole** (1815–1864) invented the algebra of two-valued logic...how to combine multiple conjunctives into meaningful outcomes which can only be T or F, 1 or 0. In 1921 in his dense and terse *Tractatus Logico-Philosophicus*, **Ludwig Wittgenstein** (1889–1951) presented the Truth Table, which can be used in logical proofs (and circuit design).

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[11] the voltage range for transistor–transistor logic (TTL) logic used in many applications.

Finally, in 1938 **Claude Shannon** (1916–2001) realized that Boole's algebra could be realized in electronic, "on-off" circuits. This was put into practice in the 1940's with vacuum tubes and then in the 1960's with transistors.

2.3.4.6 Truth Tables

My goal here is to give you a hint about how important logical analysis has become, from following two of Aristotle's ideas: First, that statements and propositions can be written as abstract sentences with *variables* rather than with named things. And, that The Law of the Excluded Middle leads us to a *two-valued logic*.

Here's a statement: (It is raining.) This could be true (T) or false (F) depending on circumstances. But it's verifiable since we could determine T or F by looking out the window. We'll call that statement p. Here's another: (the grass is wet.), another verifiable statement which could be T or F and we'll call it q.

We can put these together into a compound statement using a "logical connective": (It is raining.) AND (The grass is wet). "AND" joins the two statements. We can write this using the logical symbol, $\wedge$, which stands for AND, so our sentence—in general— can be abstracted in the Aristotle-variable-way as $p \wedge q$.

Our question of interest is: when will the compound statement, (It is raining.) AND (the grass is wet) be true? That is, what is the truth-value of "$p \wedge q$"...for the four possible T and F values that p and q might take on? Thought of a different way, if I asserted that compound statement, when am I telling the truth?

- If it is raining and the grass is wet, then $p = T$ and $q = T$ and I would be telling the truth if I said, "It is raining and the grass is wet."
- If it is raining and the grass is not wet. $p = T$ and $q = F$ then I would be lying if I said, "'It is raining and the grass is wet." (since $q = F$ means that the grass is dry).
- If It is not raining and the grass is wet. $p = F$ and $q = T$ then I would be lying if I said, "It is raining and the grass is wet."
- If it is not raining and the grass is not wet. $p = F$ and $q = F$ then I would be lying if I said, "It is raining and the grass is wet."

So of the four possible combinations of p and q, there is only one instance where the combination $p \wedge q$ is TRUE. This begs for an ordered way to present these possibilities and for each p and q, we can generate rows in a **Truth Table**. For AND, this is shown in Table 2.3. Notice that the entries in the last column correspond to the bullets just above and complete the possible p's and q's states.

Primitive logical connectives come by different names depending on one's discipline. They include: NOT, AND, OR, XOR ("exclusive OR"), NAND ("not-AND"), NOR (negate), XNOR ("exclusive NOR"), Implication, and Biconditional. They all have their own truth tables. And they're useful. What this means is that we can take

Raining?	Wet?	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Table 2.3: The Truth Table for the AND connective.

many arguments and turn them into symbols using the connectives as “puzzle pieces.“

Let’s think about analyzing an everyday situation, like planning a picnic. Weather can be a problem for picnicking since wet grass can make the it unpleasant. So the morning of the planned outing, a picnic planner might muse something like:

- If it is raining, then the grass is wet
- It is raining
- And so the grass is wet.

Notice that this has the form of Modus Ponens and we’re going to make a 21st century realization of it 2000 years after it was discovered. Here, $p =$ (It is raining.) and $q =$ (The grass is wet.). Let’s set the stage and flesh out the single possibility for a valid Modus Ponens argument.

- (If it is TRUE that it is raining, then it will be TRUE that the grass is wet)
- AND (it is TRUE that it is raining)
- THEN (it is TRUE that the grass is wet)

But a Propositional argument contains phrases that have truth values, and in general, they are not necessarily all true. Recall the “am I lying” test from above: we could have $p =$ T or F and $p =$ T or F and only one combination of the four possible arrangements completes our valid raining-wet argument.

The entire set of possibilities can be compactly and completely captured in one big truth table and here I just present this result in Table 2.4. It’s a picnic table (sorry). (In Appendix A.2.2 I build that whole table.) Notice that the AND operation between the third and first columns creates the third column’s results, by comparing them using the rows of Table 2.3 as an instruction. The only combination that’s true is the first one, the Modus Ponens argument itself. Validity of the argument is assured only if $p = T$ and $q = T$. Our connective, AND, figures prominently in this Propositional argument.

2.3.4.7 Modern Digital Arguments

Inspired by Aristotle, this “regular” conversation about the consequence of raining and the state of the grass can actually be embedded into a digital circuit using

Variables		Conditional	Conclusion
p	q	$(p \to q)$	$(p \to q)$ AND p
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

Table 2.4: The truth table for the Propositional argument above. The last column comes from comparing the third column with first column according the the T and F values in Table 2.3.

very basic digital packages[12] called "gates" (NOT, AND, OR, XOR, NAND, NOR, XNOR, and buffers). You'll recognize them as some of the logical connectives from above, plus one more that has a single input and just holds its value, called a buffer. The magic of the second half of the twentieth century is that particular combinations of transistors can produce digital packages corresponding to the gates which in turn can be soldered to a circuit board to make a decision-making circuit. With all of the individual gates, an electrical engineer can piece them together to do a job. In the background, if not in the engineer's notebook, is the equivalent of a complicated truth table.

> Think about the decision-making that's required in order for an ATM machine to process your card, the keypad, your PIN, your request, and that you took out your bills. That each step was accomplished—and checked to have been done correctly— is actually a set of questions with T or F answers that a digital circuit is happy to perform for you.

Figure 2.6 is a cartoon of what this might mean. In the top figure, I show the engineering symbol for an AND gate. Below it, the black box could consist of a single digital gate element, or hundreds of digital gates, each receiving inputs from the outputs of other others. Here the box receives two binary inputs, each of which could be T or F.[13] and it outputs a result, r, either T or F. So there could be four possible inputs but one result. What's inside of the box are circuits of connected gates built on the logical structure of the problem.

Our complete Modus Ponens picnic argument presented here as set of English statements could be recreated in a digital circuit (what might be inside the black box in Figure 2.6 (b)). For our particular example the circuit would consist of three gates (made from five transistors which would be so small that you cannot see them): an electronic circuit of the English sentences covering all of the possibilities of the argument.

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[12] You can go on Amazon and purchase integrated circuit packages of usually multiple gates in a single element that can be soldered onto a circuit board.

[13] which in practice, of course, is a 1 or 0 ("low" or "high") bit, and a the transistor level, a low and high voltage in a circuit

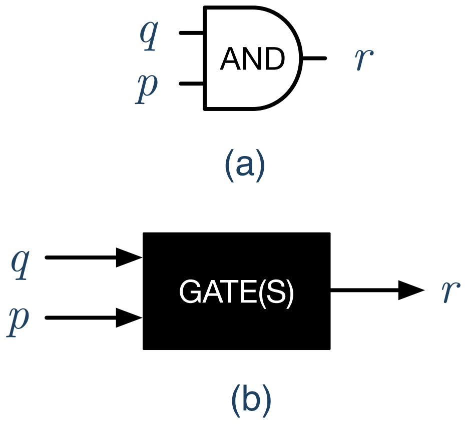

Figure 2.6: In (a) the engineering symbol for an AND gate is shown. The output of the AND gate, *r*, corresponds to the result of the truth table in Table 2.3. In (b) a black box of digital logic gates is suggested. The two inputs, *p* and *q*, are each either T or F and the output, *r*, is either T or F. This could be one gate or a thousand gates.

I hope you can get a sense of how digital circuits are designed. There's a job to do, it's described in logical terms (*p*'s and *q*'s), a truth table (or equivalent) abstraction is done, and from (millions of) combinations of the seven digital gates that exist, a circuit design is created. Humans used to do this, indeed at the beginning of my career we laid out digital circuits by hand, but now computer aided design workstations do the work of creating schematics, simulating what electrical signals would do in the design, and preparing the instructions for printed circuit board (PCB) fabrication by specialized companies.

The first digital computers relied on thousands of vacuum tubes and filled whole rooms with hot, clunky racks of tubes and wires but when the transistor became commercially viable in the 1960s the digital world came alive. With binary arithmetic, gates can be combined to do arithmetic functions, logical functions, and importantly, storage of bits. A 1 bit digital memory consists of four so-called NAND gates—four transistors—and it's the basic cell of a computer's memory.

All of these—and more—transistor components can be imprinted in tiny silicon wafers in which a single transistor package might be only 20 nanometers in size or soldered to a circuit board as a package about half of size of a AA battery. With the logical functions and the manufacturing techniques of today, my current Apple Watch has 32GB of random access memory (RAM) and so it can manage 32,000,000,000 Bytes of information, which is 25,6000,000,000 bits and so 102,400,000,000 individual transistors are inside my watch, just for the memory! The CPU and control circuitry would add millions of additional imprinted transistors and their gate-equivalents. All on my wrist. All speaking "Aristotle."

Obviously, the 2500 year path from Classical Athens to cat videos on YouTube is full of breakthroughs and smart ideas. But it all started with Aristotle.

Chapter 3

The Most Important Mathematician You've Never Heard Of : Eudoxus and Greek Astronomy

"We shall try to note down everything which we think we have discovered up to the present time; we shall do this as concisely as possible and in a manner which can be followed by those who have already made some progress in the field. For the sake of completeness in our treatment we shall set out everything useful for the theory of the heavens in the proper order, but to avoid undue length we shall merely recount what has been adequately established by the ancients. However, those topics which have not been dealt with [by our predecessors] at all, or not as usefully as they might have been, will be discussed at length, to the best of our ability."

- Ptolemy, *Almagest, Book I, 1*

The passage above is the opening stanza of the last verse of Greek astronomy and is at the threshold of a strange 1500 year dance between the rigorously mathematical (Ptolemy) and achingly abstract (Aristotle) models of the universe. How we got there is the purpose of this chapter as it lays the ground work for two millennia of mutually supportive and mutually conflicting views of MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS .

I'll bet that many of you have seen the solar system arrangement as imagined by Copernicus (surprises await in Chapter **??**) with the Sun in the center and all of the planets, including Earth, obediently orbiting it in perfect circles. What he challenged was the ancient, and universally-held idea, that it's the stationary Earth that's in the center of the universe, not the Sun. Fascination with that picture is prevalent in many decorated medieval manuscripts through the centuries and one of the earliest is shown in Figure 3.1. This is from a 10th century edition from the British Museum of a poem by the Greek poet, **Aratus** from about −275 called *Phaenomena* which was named for a book of the stars and constellations by the Greek mathematician, Eudoxus, of probably a century before. It was he who created that 2000 year old "geocentric" model of the universe—one in which the Sun, Moon, planets, and stars all orbit around the stationary Earth. We will see that the poem *Phaenomena* figures crucially in the history of astronomy two centuries after Aratus wrote it, so watch for it reappearing as we proceed.

I took some pains in the last chapter to underscore that models of MOTION ON THE EARTH belong in Aristotle's corner as he really invented the dynamics of motion. But while we tend to ascribe that geocentric model of the universe to him as well, he borrowed it lock stock and barrel from Eudoxus and Plato.

Figure 3.1: Aratus the poet lived about a century after Eudoxus (and hence, Aristotle) and turned his astronomy book into a poem. Later, Cicero translated it and this 10th century manuscript is an illustrated copy of that work.
https://sarahjbiggs.typepad.com/.a/6a013488b5399e970c01bb07c8696d970d-pi

This "geocentric" picture became the authoritative, unquestioned dogma of the medieval and renaissance periods even though it made no numerical predictions and was known since Aristotle's time to be just wrong. The other game in town was precise and predictive and was the model of the Greek astronomer, Ptolemy, from the first century, CE.

The Greek world—indeed, the whole world—was radically and violently altered by Alexander the Great and between Aristotle and Cleopatra, astronomy become an experimental and quantitive science. The culmination of Greek astronomy came after Greek–everything became Roman–everything and just before the Roman Empire began its decline. One last Greek, in our long string of Greek philosophers, mathematicians, and scientists remained and we'll close our chapter with Ptolemy's calculating-model of the heavens.

A game that many scientists play is to trace their scientific lineage back for centuries—their major professor's professor and so on (there's an app for that). I followed mine back through centuries and found that I descended from Copernicus! I'd like to think I've made him proud.

Sometimes it turns out that someone's student ends up in the history books. But not many students actually take over the known world by force!

When Plato died, the Macedonian King Philip II "encouraged" Aristotle to relocate to Macedonia in order to teach his 13 year old son, Alexander. He set up a school, taught Alexander (and perhaps the future general/king, Ptolemy) for three years, and then stayed for seven more before returning to Athens where he started the Lyceum. By this time the teen-aged Alexander was already on the battlefield and with his father, had occupied the entirety of the Peloponnese. So Athens was once again ruled by outsiders—connected to Aristotle.

After Philip II was assassinated,[1] and Alexander, soon to be "The Great," ascended to the throne and began his brutal lightening-fast, nine year conquest of the entire western world: modern Turkey, the middle east, Egypt, Arabia, and all the way across Afghanistan to India, leaving military oversight over Athens and the rest of Greece. While he stayed in touch with Aristotle, sending him samples from all over Asia, his teacher became distant, put off by Alexander's adaptation of Persian customs, dress, and persona.

Alexander died in Babylon in -323 under suspicious circumstances and, within a year, Aristotle himself died at the age of 63 at his mother's family estate outside of Athens. His Macedonian connections had become dangerous and his adopted

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[1] Assassination, murder, and betrayal were a family hobby.

city turned on him: impiety was charged, a death sentence issued, and so he fled to his mother's home uttering his famous remark about the city not sinning against philosophy for a second time. In his absence, the Lyceum stayed active under new management for another century.

Alexander's senior commanders divided up the sprawling kingdom among a dozen generals and aides and they did what came naturally: they fought among themselves for 40 years. In the end, three kingdoms and a dizzying array of city-states were established: the survivors were Macedonia and Greece, Seleucia (roughly modern-day Iraq), and Egypt.

Hundreds of thousands of Greeks migrated into the newly acquired territories establishing an international Greek-ness of culture, arts, and philosophy which was the beginning of the **Hellenistic Age**.[2] The entire western world became "Greek." Of the two dozen cities that Alexander created or conquered named for himself, the "Alexandria" that mattered most to him, and to us, was the new Egyptian port city of Alexandria.

Egypt became unusually secure under Alexander's former body guard and general (and rumored Aristotle student), **Ptolemy I Soter** (-367 to -282) who eventually fashioned himself, "Pharaoh." He adopted Egyptian customs,[3] and was an intellectual of sorts, creating the first state-supported national laboratory and library. The "Alexandrian Museum" was a national facility devoted to research and among its first recruits was the mathematician, Euclid, who while in residence, wrote *Elements*, the most-read book in history, besides the Bible. For 2500 years, from Copernicus to Thomas Jefferson, mastering *Elements* was the route to mathematical literacy.[4] For centuries the Museum was home to scores of Greek scholars, all supported by the dozen Ptolemy's from the I^{st} to the final one, Cleopatra.

The Library of Alexandria probably contained all of the manuscripts of the classical and Hellenic philosophers, poets, playwrights, and physicians. There was a hunger for knowledge of all sorts and agents of Ptolemy's library director searched every ship that docked, stealing or copying any books on board and renting or stealing manuscripts from all of the major cities.

Among the scores of Alexandrian scientists are the astronomers Eratosthenes of Cyrene, Aristarchus of Samos, Eratosthenes, and especially Claudius Ptolemaeus who will figure into our story, while only Heraclides of Athens, Hipparchus of Nicaea, and Apollonius of Perga played major roles outside of Alexandria. The Greek Ptolemy dynasty lasted 300 years until the legendary feud involving "the" Cleopatra (a common name for female Ptolemy-family successors), Marc Antony, and Julius Caesar. The Library and Museum lasted into the first five centuries CE

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[2] Often the pre-Alexandrian Greek era is called "Hellenic."

[3] including that of rulers marrying their siblings

[4] Ptolemy found it rough-going and asked for an easier way to learn it, but was told by the author that "...there is no Royal Road to geometry," a sentiment still applicable today.

until the Muslim conquests of the near east, north Africa, and Spain when it was eclipsed by great Muslim libraries in Babylon, Cairo, and Cordoba in Spain.

3.1 A Little Bit of Eudoxus

Recall that Philolaus was the source of Plato and Aristotle's knowledge of Pythagoreanism—for example, the "Pythagorean" cosmology came through him or probably originated from him. Was he a student of Pythagoras? Their overlaps are nearly right in order to imagine that relationship, but that's controversial. He's certainly the closest we get to the great man so it's not far-fetched to continue the teacher → student theme that began this chapter: Pythagoras → Philolaus → Archytas → Eudoxus. Lunar craters are named after each which is not the normal teacher-student legacy. (Set the context with the timeline in Figure 1.2 on page 8.)

Eudoxus of Cnidus (circa -408 to around -355) was the son of a physician and became one himself, but we know of him as a gifted mathematician and astronomer. As we'll see, astronomy and medicine were connected through astrology and mathematics and astronomy have always been kin, so these seemingly disparate skills go together. Cnidus was a city founded by Sparta on the southern Aegean coast of modern Turkey and was where he started... and finished, between which times he traveled all over the Aegean to study and teach. As a young man he went to Tarentum to study mathematics with the pre-eminent Pythagorean mathematician (and much more) **Archytas of Tarentum** (-428 to -347) who seemed like a sensible guy:

> To become knowledgeable about things one does not know, one must either learn from others or find out for oneself. Now learning derives from someone else and is foreign, whereas finding out is of and by oneself. Finding out without seeking is difficult and rare, but with seeking it is manageable and easy, though someone who does not know how to seek cannot find. Archytas, *fragment*.

Let's learn a little bit about him in Figure Box 3.2 on page 88. After you've read about Archytas, return to this point ☟ and continue reading about his student, Eudoxus.

He seemed to not be able to stay in one place. After his mathematics instruction, he went to Sicily to study medicine, then by the age of 23 he went to Athens and stayed briefly (and apparently, unhappily) with Plato's Academy (rooming 7 miles away, so a long commute to lectures). After less than a year, he was back on the road to home in order to raise funds...so that he could travel even further! He went to Egypt with what we'd call a scholarship and studied astronomy there for 16 months, shaving his head and learning from the priestly-cast astronomers, before leaving for the northern modern Turkish Black Sea coast and the Greek colony of Cyzicus. By this point he's lecturing on his own and established a popular school and an observatory. With data from his observing in the north and from Egypt, he published his first book, *Phaenomena*, which was a compendium of star locations and *On Speeds*, of their motions. Recall that this is the subject of Aratus' important poem.

Around −368, during his 30s, he moved his school to Athens, by which time Plato was 60 years old and Aristotle had left for Macedonia. It was here, as the legend goes, that Eudoxus was challenged by Plato to form a geometrical model of the heavens. The legend is unlikely as by this point, Eudoxus was the mathematical champion of the Greek-speaking world and more likely to issue challenges, than accept them. Plato's mathematical skill was no match for Eudoxus' whose work was memorialized in a number of Euclid's *Elements*. As we'll see below his model was born and in various guises, persisted until Galileo, Kepler, and Newton.

He first calculated/measured the length of a year of 365 days and 6 hours. and it's Eudoxus' astronomy and cosmology that are our concern here and so let's work up to that with a review of the problems that everyone in antiquity faced when trying to describe what we observe from Earth and then work through Plato's ideas that formed an almost linear line of inspiration: from Pythagoreans, to Plato, and to Eudoxus.

3.2 A Little Bit of Greek Astronomy

The biggest export of Greek astronomy before the Romans was Aristotle's model of the cosmos with its Earth-centered ("geocentric") description of MOTION BY THE EARTH and MOTION IN THE HEAVENS . It became popularized, petrified, and deified when it was officially incorporated into Church dogma after the work of Thomas Aquinas in the late 13th century. So from that point until the Baroque era, Aristotle reigned supreme. He was revolutionary and inventive in so many areas, so it's amusing that his cosmological model had the longest run and that it was almost entirely due to Eudoxus. We'll dig a little deeper into both of their ideas as both were influential. And was it really Aristotle's model that ruled?

3.2.1 What Ancients Saw

Suppose you're a smart Greek with time on your hands and able to spend years just recording what the sky presents to you during the days and nights. A few things would stand out...and if you were a patient and persistent observer nuance would start to emerge. In Section 3.5.1 of *Greek Astronomy, Today* I'll "set the record straight" with modern explanations for each of these scenes and motions but here we'll just observe. The vantage point on Earth suggests that we're on the inside of a vast sphere above us. This "Celestial Sphere" is our starting point.

Sun's apparent motion. Your days would be dominated by the Sun. If you're in the northern hemisphere, in general you'd see it appear to rise over your eastern horizon, pass not quite overhead, and then disappear over your western horizon.

The event seems to repeat itself, over and over, but if you kept track over a year you'd find that every day's solar performance is a little different. Suppose it's mid-December in the northern hemisphere. The Sun's path would be very low in the southern sky and you would be experiencing your winter. Every day after December 22nd or so, you would discover that the Sun's eastern rise is a little bit north from the day before and that it would set a little bit further north as well and so each day would be a little longer. Furthermore, at noon the point each day when it's at its peak would be just a little higher than the previous day.

This would continue until June when the Sun is very high in the sky at noon and the days are the longest of the year. Then the whole pattern would reverse until December. In that round trip, there's one day on the way up and one day on the way down when the Sun rises precisely in the east and sets precisely in the west and at noon, it's height above your horizon is exactly between those two extremes during late December and late June 20th. Also on those two days, the day and night durations are the same all over the world: 12 hours and so each is called an **equinox**.[5] These points happen in late March (called the Vernal Equinox)[6] and late September (the Autumnal Equinox).[7] Each **equinox** is a precise astronomical event, but as an ancient Greek you don't know this yet! If you imagine a great circle going from south to north over your head (called the "meridian"), the Sun's path at the equinoxes (when it "rises" and "sets" at exactly the east and west points on the compass) is in the middle of a band of possible paths up and down with the extremes (at the solstices) at exactly 23.5° from that midpoint Sun path. Figure 3.3 (a) shows the situation for a north latitude of Michigan State University in East Lansing, Michigan of 42.74°.

There are distinct patterns here known throughout history, especially the equinoxes. The Vernal (or Spring) Equinox was celebrated around the world: from the Mayans to the ancient Germanic tribes to the ancient Saxons the VE was celebrated as a time of renewal and rebirth. Structures like Stonehenge, the Mayan pyramids, the Egyptian Pyramid of Khafre, and others in Cambodia, Ireland, and New Mexico point out the VE. Understanding them, though, only became a goal among a few Hellenistic Greeks when "solar models" were invented by mathematically clever and imaginative astronomers. As this story unfolds, notice how the Sun figures into every corner of ancient astronomy—and yet, it was considered to be just another orbiting object.

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[5] This derives from the Latin *aequus*, for "equal" and *nox*, for "night."

[6] Latin for "spring" is *ver*.

[7] In 2023, the WS, VE, SS, and AE occur on December 22, 2023, 3:27 AM, March 20, 2023, 9:24 PM, June 21, 2023, 2:57 PM, and September 23, 2023, 6:49 AM, GMT

FIGURE BOX 3.2

Un missionnaire du moyen âge raconte qu'il avait trouvé le point
où le ciel et la Terre se touchent...

The image on the left is a famous engraving (by an unknown artist...maybe late 18th century) suggesting an ancient sentiment due to Archytas, a friend and competitor of Plato. Among the most famous arguments in cosmology is whether the universe is infinite or finite in size and Archytas had the first of many similar inspirations that the universe cannot be finite: He did a thought experiment, imagining traveling to its presumed edge and attempting to thrust his stick beyond that limit. If he could extend it, then, well, that's not the edge...and so he'd have to go further, repeating the experiment without end. This is a good example of the kind of intuitive cleverness that seemed to be built into this great Greek mathematician, politician, and military leader. The very model of a modern major—Pythagorean— general. Archytas was a committed Pythagorean and a mathematician of great skill. But he also he was a civic leader and an elected military general, in spite of Tarentum law, re-elected seven times because he never lost a battle. (Did I mention that Greeks fought constantly?) When he did step down, the army started losing.

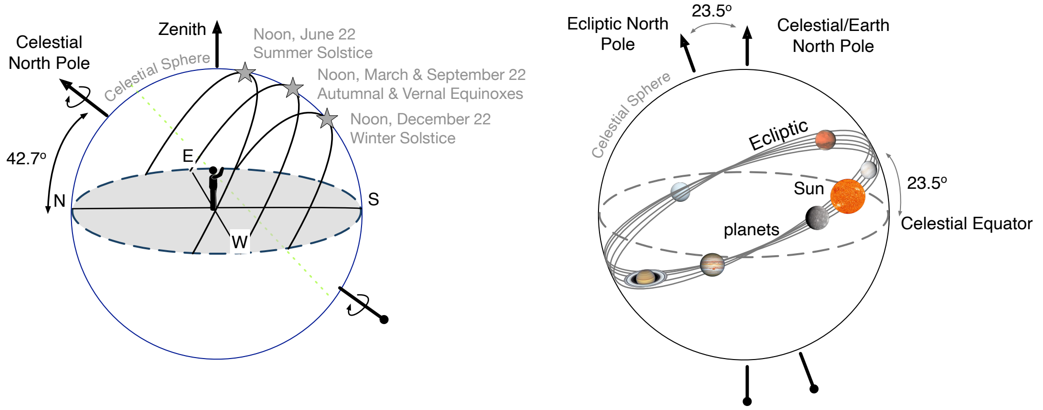
Archytas was reported to be an even-tempered, cultured man who led Tarentum through a period of democracy and that Aristotle apparently wrote more (lost) books about Archytas than he wrote about any other person. There is some evidence that he wrote a book on mechanics and that he enjoyed making mechanical toys for children—very un-Plato-like in spirit.

His mathematical skills were legendary and he solved an old problem with mystical roots: Apollo sent a plague to the city of Delos and a delegation was sent to Delphi to learn from the Oracle how to rid themselves of the pestilence. The instructions were to take their cubical altar to Apollo...and build a new one with double its volume. This is called the problem of "duplicating the cube" (also called the Delian Problem) and it required cleverness on Archytas' part, beyond just geometry, which caused Plato to disparage his effort. Archytas contributed to many branches of mathematics and Euclid's *Elements* includes some of his proofs.

All in all, Archytas was the most accomplished Pythagorean of all and in the spirit of the opening to this chapter, we're indebted to him for his products, but also one of his students. The most accomplished of all Greek mathematicians before Archemides, Eudoxus, from whom 2000 years of cosmology originated.

Now go back to page 85 and pick up where you left off.

Clearly associated with the Sun are the seasons and they aren't the same length—spring and summer are longer than fall and winter, but there are definite times of



(a) (b)

Figure 3.3: In (a) a view of the Celestial Sphere from one’s horizon, here for the latitude of 42.74° of East Lansing, Michigan. The three bands show the Sun’s path in the sky at the Summer Solstice (top), Winter Solstice (bottom), and the Equinox (middle). Each of the bands around that central arc are 23.5° above and below it. In (b) the orientation switches so that the axis of the Celestial Sphere is up (coinciding with the axis of the Earth’s rotation, and the ecliptic axis is inclined to it by 23.5°. The planets and the Sun all appear to move from east to west within the band that is the ecliptic, the equator of the ecliptic axis. Where the ecliptic passes the Celestial Equator are the two locations of the equinoxes.

cold and warm weather in the two hemispheres. In 2023 in the northern hemisphere: after 89 days in 2022, winter ended; spring was 93 days long; Summer was 94; and Autumn was 89. The Athenian astronomers Meton and his student, Euctemon found 92, 93, 90, and 90 days in about −432, so this was a known problem. (The student also has a lunar crater named for him.) Finally, as the Sun appears to us to go around us, it "visits” each of the constellations of the zodiac once per year, while staying on the ecliptic, of course (since the Sun’s path defines the ecliptic). When it intersects with the Celestial Equator (which, recall, is a projection of the Earth’s equator), that’s the point of passage from below to above (spring is coming) and then, above to below (autumn is coming) the midpoint. That intersection defines the equinoxes as shown in Figure 3.3 (b).

The apparent motion of the Moon. Prominent for its size and its regularly changing features is our Moon. If looked at from overhead, it travels in a clockwise orbit, nearly circular, with a period of 27.322 days., changing its appearance through phases during that cycle. Unlike the Sun and the stars, the Moon changes its appearance every single night. Sometimes it’s “full” and a bright circle. Sometimes it’s not there at night, but maybe visible during the daytime. Most times the bright part of the Moon is a crescent shape, culminating in a half-circle, and then back to crescent. Occasionally, the Moon gets in the way of the Sun and we have a solar eclipse. Sometimes the Earth blocks the Moon from the Sun and we have a lunar eclipse. Why these events don’t happen every month was a puzzle. One thing doesn't change about the Moon and that's the face that we see—another puzzle.

The apparent celestial sphere. Another reliable pattern in the sky also occurs at night. If you watch the stars for a few hours you'll find that they all appear to rotate around a single point in the north sky (for us, now, around the star Polaris). This motion seems to be always the same and its natural to imagine that you're at the center of a huge sphere with stars attached to the inside. Figure 3.4 shows a time-lapse photograph of the northern sky where the circular trails are evident. This "Celestial Sphere's" axis goes right through the axis of the Earth's poles and the equator of the CS is a projection into the sky of the Earth's terrestrial equator. Figure 3.3 (b) shows the relation between the Sun's path and the axis of the Celestial Sphere, inclined by 23.5° from the Celestial Equator.

Figure 3.4: A time-lapse photograph of the star positions during a single night in the northern hemisphere are shown clearly demonstrating the circular "inside" of the Celestial Sphere. The pole is conveniently located (now) at the North Star, Polaris.

The stars seem innumerable and for millennia people have found recognizable images of animals and deities in the stellar patterns, the constellations; particular bright stars were given names; and that region in the sky at night that corresponds to the equinox solar path had special constellations called the zodiac. Babylonians and Egyptians in particular took notes on when stars or parts of constellations rose, and when that event occurred, what stars were directly overhead, and what stars were disappearing in the west. Patiently, each night for hundreds of years these observations were recorded, to become useful during the Hellenistic period.

Of particular importance were the constellations in which the "Sun resides" during the time of an equinox. During the times of the Greeks, that point in the sky was in the leading edge of the zodiacal constellation of Aries—the "First Point of Aries" became the origin of a coordinate system in order to document the location of stars and planets and became particularly important in the −200's by important astronomers.

Planets' apparent motions. There are a few brighter objects (the planets) which execute similar east-west motions through an individual night, but strange positioning from night to night. The planets (and the Sun and Moon) appear to make their way across the sky in a "lane" or a strip about 14° wide, the **ecliptic**. Figure 3.3 (b) shows this band inclined to the Celestial Equator by 23.5°. The constellations of the zodiac are distributed around the sphere within that strip of the sky[8] and the center of it is the path of the Sun. Some planets' brightness changes during a year,

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[8] There are 13 zodiac signs, but that's inconvenient for astrologers so they ignore one of them.

dramatically for a few. Finally, there are two kinds of "motions" spoken of for the planets, which is confusing.

- If you watch a planet during a single night, you'll see it move from east to west in line with the stars behind it. This is called "prograde motion."
- But there's another kind of "motion" which is not during a single night, but appears as a comparison from night to night. Suppose you look at Mars every night at 10 pm and take note of what is the arrangement of stars are around the planet. About every two and a half years you'll see something strange happen. Suppose Star A and Star B are on either side of Mars. In some successive nights the arrangement of the three objects will go something like this cartoon facing the south (Mars' back and forth would actually take about three months):

Night #1 East ........*A*..........*M*.........*B* West
Night #2 East ........*A*........*M*...........*B* West
Night #3 East ........*A*......*M*............*B* West
Night #4 East ........*A*....*M*..............*B* West
Night #5 East ........*A*..........*M*.........*B* West
Night #6 East ........*A*............*M*........*B* West
Night #7 East ........*A*................*M*....*B* West
Night #8 East ........*A*..............*M*......*B* West
Night #9 East ........*A*............*M*........*B* West
Night #10 East ........*A*..........*M*..........*B* West
Night #11 East ........*A*........*M*............*B* West
Night #12 East ........*A*......*M*.............*B* West
Night #13 East ........*A*....*M*...............*B* West

Between nights 4 and 11 Mars appears to move backwards and then reverse course and continue its nightly progression westward. This is called "retrograde motion" and it surely must have confused everyone.

3.2.2 Greek Astronomy, Presocratics

Parmenides •Pythagoras •Philolaus
(Set the context with the timeline in Figure 1.2 on page 8.)

In Chapter 1, I briefly discussed the Presocratics' cosmologies with two ideas among them that were shared: all but two appeared to believe in a flat, and stationary Earth. The two who thought differently were Pythagoras and Parmenides.

Parmenides had a number of original ideas about the heavens—in particular, he may have been the first to conceive of the whole universe as being spherical (Pythagoras/Philolaus might also have determined this) and finite.

> "...like the mass of a well-rounded sphere, from one middle, equal in every respect." Parmenides

He was also apparently the first Greek to note that the Moon reflected the light of the Sun and must be spherical and he was even poetic about it:

> "[the moon is a body] shining by night, wandering around earth with borrowed light. . . " Parmenides

"Borrowed light" is a nice phrase. If the Moon "borrows" its light from the Sun and doesn't shine on its own, then the shape of the phases of the Moon lead to a spherical shape conclusion.[9] Ironic, isn't it that Parmenides can perhaps be credited with a scientific discovery—one that requires observation— when we tend to think of him as anti-scientific.

The Pythagorean team (probably more Philolaus than Pythagoras, so I'll call it collectively "Pythagorean/Philolaus") extrapolated their fondness for regular motions, musical tones, and numbers and built a cosmology that tried to put all of these commitments into one model. They were responsible for many "firsts" in Greek astronomy: they too hypothesized that the Universe is spherical, most credit them with establishing that the Earth is spherical (for metaphysical and symmetry reasons), they proposed a popular ordering of the planets (Earth, Moon, Sun, Mercury, Venus, Mars, Jupiter, and Saturn...surrounded by the stars), they hypothesized that the planets' speeds are inversely proportional to the size of their orbits, and they concluded that the "morning star" and "evening star" (our Venus) were not two different planets but the same one which is close to the Sun. And, crucially: they were the first to propose that the planets follow circular orbits around a center.

There was a first version of Pythagorean/Philolaus cosmology in which the Earth is at the center of the universe containing a "central fire" or "'Hestia," in homage to the immobile goddess of the hearth. But that morphed into the cosmology of Chapter 1 with the "central fire" situated in the center of the universe, relegating Earth to be just another celestial object orbiting around it in circular orbits. Figure 3.5 (a) shows the whole system with the Earth, Moon, Sun, and the planets orbiting counterclockwise around the center and inside an outer shell of the stars. The Earth orbits the central fire once a day and the Sun, once a year. So the Earth daily catches up and passes the Sun accounting for day and night.

We don't see a "central fire" and there were two proposals as to why, shown in Figure 3.5 (b) and (c). The standard interpretation is the second one in which inhabitants of the Earth are shielded from the fire by the presence of a "counter earth" which strategically blocks it, see **?**. Without the counter earth there are only nine components to the universe and so Aristotle was critical of them for perhaps arbitrarily adding the counter earth just to make the total 10, as suggested in **?**.

This is the first cosmology based on *regular, circular* MOTION IN THE HEAVENS and a model in which MOTION BY THE EARTH is not zero. The idea of course has spawned

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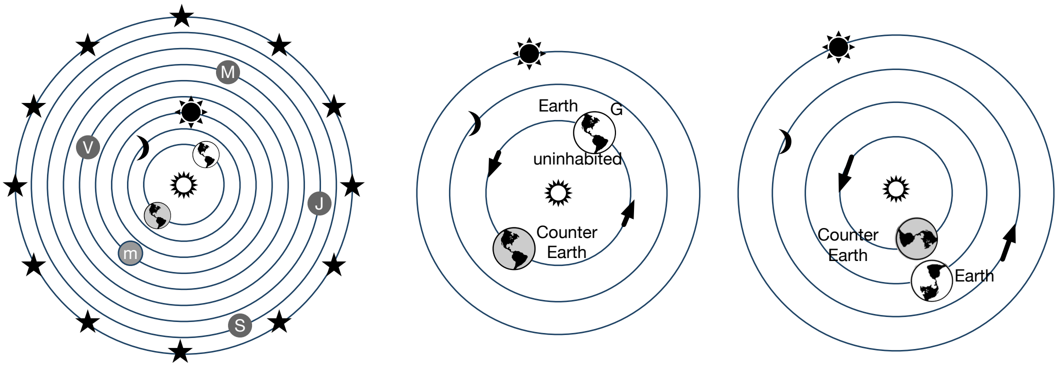
[9] It was traditional to credit Parmenides with extrapolating from a spherical Moon to declaring that the Earth, too, is spherical. But that's not authenticated and Pythagoreans' claim to a spherical Earth is perhaps more likely.



(a) (b) (c)

Figure 3.5: (a) shows the Pythagorean system with all of the heavenly bodies and the Earth orbiting the central fire in a counterclockwise sense. In (b) the Earth is shown in one of a number of interpretations of Philolaus' system. Greece (G) is on the far side, leaving the side facing the fire without people. In this orientation it's morning as the Earth is catching up with the slower-moving Sun. In (c) the counter earth is positioned so that it blocks the central fire.

2000 years of astronomical research! Circles, everywhere.

3.2.3 Summary of the Astronomy of Parmenides, Pythagoras, and Philolaus

(Set the context with the timeline in Figure 1.2 on page 8.)

- Parmenides (−514 to −450):
  - He was first to assert that the whole universe was spherical.
  - He was perhaps the first to recognize that the Moon does not shine by its own light, but reflected ("borrowed") light from the Sun. The Pythagoreans might also have realized that.
- Pythagoreans [Pythagoras (−575 to −500) especially including Philolaus (−470 to −385)]:
  - "They" were first to realize that the Earth is spherical.
  - "They" were first to hypothesize a particular ordering of the planets, perhaps with the their orbit size inversely proportional to their speeds.
  - "They" realized that the "morning" star and "evening" star were the same planet, Venus.
  - "They" were to propose a model in which the planets (including Earth and Sun) all orbited a central point (for them, the mysterious "central fire.") in perfectly circular orbits.
  - Their insistence on heavenly motions being uniform and circular outlived their specific model.

3.2.4 Classical Greek Astronomy

Plato •Eudoxus •Aristotle
(Set the context with the timeline in Figure 1.2 on page 8.)

In Chapter 1 we touched on Plato's cosmology in *Timaeus* but that was a late development for him as his ideas about the nature of the cosmos grew over almost his whole career. His learning from Archytas in mathematics and the symmetry tendencies of the Pythagoreans launched him in the direction of building everything around circles, and then spheres.

3.2.4.1 Plato's Two Models

Recall that the *Republic* was nominally a treatise on the nature of justice and how to build a just state which he proposes be totalitarian. When philosophy and political science students read it, they're probably surprised by its ending, which is a full-on Pythagorean cosmology, the "Myth of Er."

> "Once upon a time he died in war; and on the **tenth day**, when the corpses, already decayed, were picked up, he was picked up in a good state of preservation. Having been brought home, he was about to be buried on the twelfth day; as he was lying on the pyre, he came back to life, and, come back to life, he told what he saw in the other world."
> Plato, *Republic*

Socrates is trying to motivate why someone should live a good life and relates a cosmic carrot-and-stick story, not unfamiliar to other religious admonitions. Er is a soldier who was killed and does what all deceased do… they go to a place where their lives are evaluated, not by St. Peter at the Pearly Gates, but by four judges who tell him that he's got a job to do: after 10 days[10] his body will be retrieved from the battlefield and on day 12 he's to be resurrected from the dead, dramatically on his own pyre before it's lit. He's to tell others what he's seen which includes a strange vision of a pillar of light that extends to the heavens which Plato describes as a spindle and whorl used for spinning wool. Figure 3.6 (a) shows a Roman woman spinning wool with the weighted whorl at the bottom which spins as she works. Figure 3.6 (b) is the umbrella-like structure (the whorl upside down) that Socrates describes:

> "Its shape was that of (whorls) in our world, but… it was as if in one great whorl, hollow and scooped out, there lay enclosed, right through, another like it but smaller, fitting into it **as containers** that fit into one another, and in like matter another… There were **eight of the whorls** in all, lying within one another…" Plato, *Republic*

The eight "containers" are hinted at in my sketch in Figure 3.6 (b) and the whole is abstracted as nested spheres in Figure 3.6 (c), where I've only shown three spheres (remember, "containers") for simplicity. Earth is no longer a "regular" planet but

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[10] Why 10 days? some Pythagoreanism is maybe showing?

(a) (b) (c)

Figure 3.6: The figure in (a) is a Roman sketch of a woman spinning wool using a spindle and whorl, which is the weight at the bottom with a hook. The image in (b) is Plato's description of the whorl actually hollowed out with nested layers of whirl-shaped half-spheres. The image in (c) is the cosmos that the onion-layered whorl represents with the Moon; Sun; and the first planet, Mercury attached to the first three of eight spheres. I've only included three in this cartoon.

is in the center with concentric spheres of the Moon, Sun, the outer planets, and again, the stars on the furthest shell, which Socrates says is "speckled." So, Plato's first cosmology has MOTION BY THE EARTH as zero and MOTION IN THE HEAVENS is described as Pythagorean, but using spheres, not just circles. He also tells you how they move and the sounds that they emit as a Siren sits on each sphere and sings a tone. This is the world's first three dimensional cosmological model. But the it didn't match what the planets do and Plato actually tried to remedy it in the *Timaeus*. Given his penchant for not modeling appearances, this was an unusual move and suggests to me that getting it right was (briefly?) important to him.

The *Timaeus* is Plato's "origin story" and in the previous chapter I described the Craftsman's efforts to create matter using geometric three dimensional shapes. It's also his cosmology update from the *Republic* and quite different. Socrates teases the story out of the main character, Timaeus—a Pythagorean—and then uncharacteristically allows the speaker have the floor without much interruption. It's where Plato becomes mathematical, in a spooky, Pythagorean way.

Does this string of numbers mean anything to you: 1,2,3,4,9,8,27? Me neither, but they function as a part of the instructions to the Craftsman in order to build the universe following a numerology algorithm described in a nearly unintelligible paragraph:

> "And he began the division in this way. First he took **one portion** from the whole, and next a **portion double of this**; the **third half as much again as the second**, and **three times the first**; the **fourth double of the second**; the **fifth three times the third**; the **sixth eight times the first**; and the **seventh twenty-seven times the first**." Plato, **Timaeus**

Timaeus is tough to read (impenetrable in some places) and so I've unpacked the algorithm from the paragraph in Appendix A.3.1. The upshot is that the Craftsman has fashioned a universe with two rotating spheres. One of them he calls "the same" and represents the (unavoidable) rotating Celestial Sphere. The other he calls "the different" which is inclined to the first. Those numbers represent the relative sizes of the layers inside of that inclined sphere where the planets are arranged. His Er story didn't account for the ecliptic, and this "different" sphere set is that correction.

> "This whole fabric, then, he split lengthwise into **two halves**; and making the **two cross one another** at their centers in the form of the letter X, he bent each round into a circle and joined it up, making each meet itself and the other at a point opposite to that where they had been brought into contact." Plato, *Republic*

Figure 3.7 is a silly attempt to illustrate this. Figure 3.7 (a) is a person playing with a hula hoop, perfectly aligned so that the axis of the toy's rotational plane points through our person's head. This represents the axis and equator of the Celestial Sphere around the Earth. Figure 3.7 (b) shows just how good this person is at hula hoops: two are rotating, the original, and another that somehow our friend manages to get to rotate at an angle relative to the first one. Some serious hip-action would be required. This represents the ecliptic, inclined by that spacing corresponding to the latitude of the observer. Those strange numbers? Well, there would actually be seven hoops with diameters proportional to those numbers: 1–2–3–4–8–9–27. Figure 3.8 shows what this is really about.

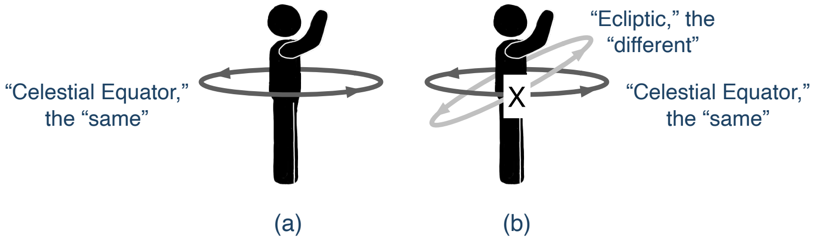
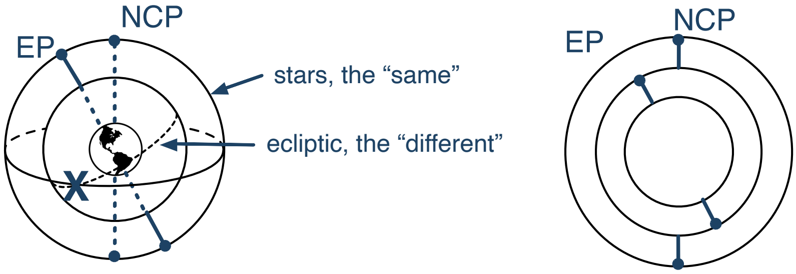


Figure 3.7: Pretty good hula hoops chops.

The celestial sphere and its axis I've called the NCP (north celestial pole) in the diagram. The other strip is the equator of the other, ecliptic, sphere (with axis labelled EP) which makes an "X" where it crosses in two places with the Same. (These are the points of the equinoxes, when the ecliptic crosses the Celestial Equator.) Inside of this strip, the segments correspond to the locations of the Moon, Sun, Mercury, Venus, Mars, Jupiter, and Saturn. Of course, this is a little mad but Eudoxus took on the task of turning this story into a geometrical model.



(a)

(b)

Figure 3.8: (a) shows the two spheres with their equators. One the Celestial Sphere (carrying the stars around the Earth each night, so an axis centered on the North Pole of the Earth) and the other is the ecliptic (in which the planets reside as they appear to go around the Earth) with the pole of that sphere, the North Ecliptic Pole. (b) takes away the three-dimensional view and will be a useful sketch for these kinds of constructions in what follows.

3.2.5 Eudoxus’ Model

By the time Eudoxus had returned to the Academy, he would have been familiar with the *Republic* and probably *Timeaus*. Once Plato had inserted the ecliptic path, he still needed to explain retrograde motion. And he knew it:

> “. . . as for the dances of these and how they relate to each other, the **backward-cycles and forward-progressions** of the circles to each other. . . to speak without visual representations of these same would be a **vain effort**.” Plato, *Timaeus*

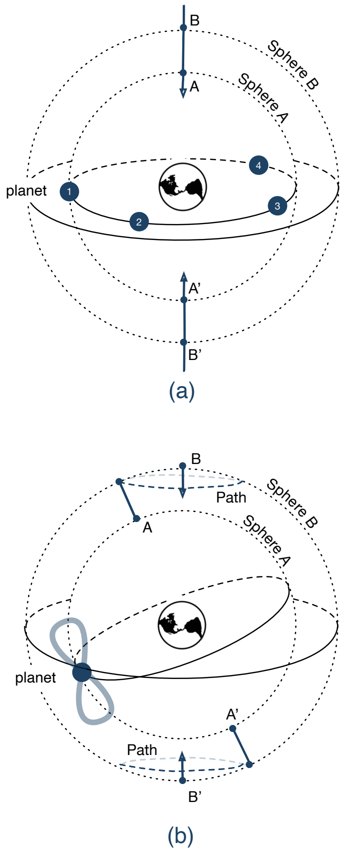
So, he realized the problem. . . but had no solution and just gives up (“vain effort”). He was out of his depth but Eudoxus was ready and came up with a brilliantly complex model and while it’s not known what Plato thought of it, it’s clear how Aristotle reacted: he made it his. It’s intricate, so let’s go to the box and work out the inner workings of the idea and then skip to the end. Look at Figure Box 3.9 on page 99. After you’ve read the material in that Box, return to this point ☟ and continue reading.

The figure in Box 3.9 describes the tool-kit that Eudoxus used to construct a full model of each planet in which they ride on the equators of coupled, spinning spheres. The two spheres shown in the box form the minimal number of moving parts unique to every planet and they are each embedded inside of two other spheres, one for the ecliptic whose equator includes the rough paths of the planets and the other is the Celestial Sphere which includes the motions of the stars around the Earth every nearly 24 hours. Let’s take it slow in Figure 3.10.

The fundamental Eudoxus set was four spheres, centered on the Earth. Using the nomenclature from Figure 3.10 and Box 3.9, labeling them from the inside out:

A: the sphere to which the planet is attached,
B: the next sphere which precesses around that inner sphere (producing Edoxian figure-eight)
C: the sphere that rotates around the ecliptic—that stretches out that Eudoxian figure 8 in Figure 3.9 to produce retrograde motion, and
D: the outer-most sphere that rotates daily showing the pattern of the starry Celestial Sphere.

FIGURE BOX 3.9



(a)

(b)

The model that Eudoxus created is an impressive bit of geometry mixed with inspired imagination. It's the famous "nested spheres" model that made it all the way to the Baroque as an explanation for the odd motions of the planets. In a very modern way, it's full of parameters that could be tweaked to make it fit the observations...some of which he made himself at the observatory he created in his school before he returned to Athens.

Imagine taking two hoops, one of which is slightly smaller than the other and is attached inside the larger one across their mutual diameters. Figure 3.9 (a) shows this with a "planet" attached to the equator of the inside hoop. Now if we spin that hoop around its axis AA' the planet will follow a circle from position 1 through 2, 3, 4 and so on. This spinning *observed from the outside* essentially defines a sphere, Sphere A, here centered on the Earth. If the two hoops are attached, and if the outer hoop spins around its axis, BB', creating the surface of Sphere B, then the motion of the planet will be the sum of the two speeds at the hoop pair equators. So if the outer hoop spins at the same rate as the inner hoop, but in the opposite direction, then the planet would appear to the Earth to remain stationary at position 1.

Now imagine that the axis of the inner hoop is attached at a point offset on the surface of the Sphere B as shown in Figure 3.9 (b). Now when Sphere B spins, it takes the AA' axis of Sphere A around with it tracing the path shown. In addition, if Sphere B spins while its following that path independently, the motion is a complicated figure eight pattern as shown. Eudoxus figured this out and named the shape a "hippopede" which is "horse fetter" in Greek. (A fetter is like a chain.) Now there are many variables at work which would alter the shape of the hippopede: the speeds of the two spheres and the angle at which AA' axis of Sphere A is inclined to the BB" axis of Sphere B.

Now go back to page 97 and pick up where you left off.

All of these separate motions are coupled... and that's just for one planet! By tuning the inner two spheres' rotation speeds and the inclination of their inner axes, the

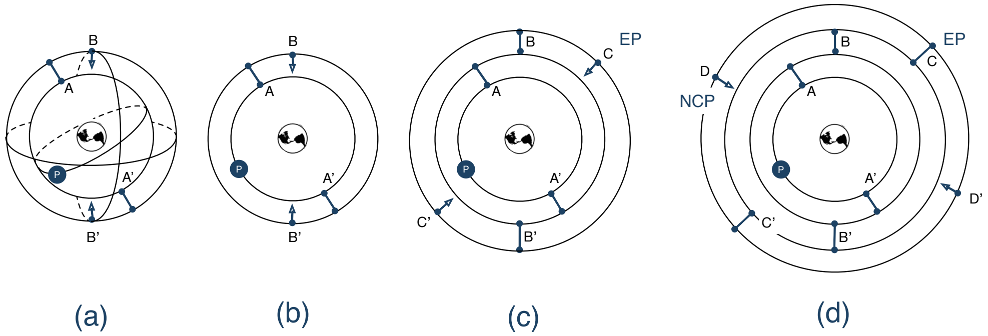


Figure 3.10: (a) is a slightly different rendering of Figure 3.9. (b) is an abstraction of (a) taking out some of the lines that suggest a solid sphere, for clarity. (c) includes the sphere of the ecliptic (EP for Ecliptic pole is shown) with axis of rotation CC′. Notice that it's attached to the outer sphere of Eudoxus' tool-kit pair. And (d) includes the sphere of the outer stars, the celestial sphere (NCP for the North Celestial Pole is shown) and the ecliptic shere is attached to it.

motions of the planet can be made to do the figure-eight dance at just the right time of year and with the right elongation in the sky—to make the planet appear to reverse direction and recover, and resume as viewed by the Earth. Each planet required four spheres and the Sun and Moon required three each, plus the Celestial Sphere: 27 spheres to do the job. This was a mammoth intellectual puzzle that Eudoxus created and then solved with those relatively simple pieces of interlocking spheres.

It still didn't quite do the job as well as it might and in the best tradition of what Thomas Kuhn would have called "Normal Science," **Callippus of Cyzicus** (−370 to −300) tried to make it better without starting over. He was a student of Plato's and worked with Aristotle and worried about the seasons' length problem and some finer points of the planets' motions. He added two additional spheres for the Sun and Moon and one each for Mercury, Venus, and Mars for a total of seven more. So now: 34 spheres. Was it all just an exercise in geometry? Perhaps. The Eudoxian program of research was abstract without numbers and so no predictive capability. It might indeed have been more of a story than a scientific model, like Plato, and like Aristotle's will be.

Around −370, Eudoxus also apparently created a star catalog in his book *Phenomena* of at least 47 stars which a century later were memorialized in the famous poem of that same name by Aratus that I introduced in the preface to this chapter. These entries were not numerical or with coordinates, but were story-like recording the times of the rise, set, and position overhead of constellations or stars near parts of constellations. For example, "As a guide the Ram and the knees of the Bull lie on it, the Ram as drawn lengthwise along the circle, but of the Bull only the widely visible bend of the legs. On it is the belt of the radiant Orion and the coil of the blazing

Hydra, on it too are the faint Bowl, on it the Raven, on it the not very numerous stars of the Claws, and on it the knees of Ophiuchus ride. It is certainly not bereft of the Eagle: it has the great messenger of Zeus flying near by; and along it the Horse's head and neck move round." **?** What we know of Eudoxus' catalog come to us from Aratus and the later Hipparchus' critique of the poem and by extension, Eudoxus' work.

3.2.6 Aristotle's Model

When it came to astronomy, Aristotle was downright derivative. Ironically, his model that became Church dogma wasn't his, and to make matters worse, it was flawed and largely ignored soon after he died. How it went from forgotten to dogma is the story of Chapter 3.5.3, but let's see what he actually did and why. His astronomical writings were scattered throughout two large books, *On the Heavens* and *Meteorologies* and his solutions to known problems were a mixture of pure metaphysics, physics—often relying on his own rules of motion as authoritative,—and the observations of others. Aristotle didn't observe the heavens.

3.2.6.1 Properties of the Earth, Aristotle-style

Aristotle vigorously disagreed with the Pythagorean/Philolaus cosmology in which the Earth orbits the center of the universe and devised challenges defending a stationary Earth that any future moving-Earth proponent would have to meet squarely.

The Earth Pythagorean/Philolaus adherents proposed that the Earth is spherical, arguing largely from aesthetic grounds, namely that circles are good and spheres are good and so the Earth should be spherical as well. Oh, and that the universe is spherical and so must be the Earth.

Aristotle proposed multiple, more concrete reasons why. First, when one observes a lunar eclipse, one sees that the shape of the demarcation between light and dark is always convex. So if the Earth's shadow is the explanation for the eclipse, then the Earth must be at least circular, if not spherical. He knew from reports that people in the southern latitudes saw different stars on their horizon than those in the northern latitudes. He argued against those who insisted (still) that the Earth was flat by noting that the horizon looks flat, but that's simply because the Earth is large.[11]

He also had a physics reason. Since earthy material would naturally be aimed at the center of the universe then all earthy material would be drawn to a single point and highly compressed equally in all dimensions with the result: a sphere of earthiness. That sphere would be surrounded by a thick sphere of water. That

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[11] Nowhere in Aristotle is the famous alleged argument attributed to him that when ships begin to appear on the horizon that first the mast and then the hull are observed.

would be surrounded by a sphere of air and then fire. So a spherical double-double-decker sandwich of the four terrestrial elements filling up the whole volume below the Moon, the "sub-lunar realm." Furthermore, this argument supported two other Aristotelian–imperatives: that the Earth finds itself in the center of the universe and that it's stationary.

The Stellar Parallax Argument Finally, he makes a good argument for the stationary Earth which becomes the essential challenge to any future moving Earth cosmology.

Look at a point across your room with one eye closed and put your finger in front of you and notice what's behind it on a wall or distant surface. Now switch eyes and notice that the what's behind your finger now seems to have moved. If you open and close each alternate eye successively, the background will appear to jump from side to side relative to your finger. This is called "parallax" and it's because your eyes are attractively located inches apart from one another on your face and enough so that the lines of sight from each are slightly different.

If the Earth is orbiting a center, then at one point of the year a particular star would appear as a line at a particular angle (like your right eye open). Then at the half-way point around its orbit (six months later if the orbit is around the Sun), when the Earth is on the other side of that center (like your left eye open), look for that same star and it will be at a completely different angle. "Stellar parallax" or "annual parallax" is the name of this phenomenon and we'll see it more than once in our story.

Nobody observed stellar parallax leaving only two explanations. Either the Earth doesn't move around a center of revolution, or the stars are so far away that parallax isn't visible. Nobody was prepared to imagine a universe that big, and so the conclusion was that MOTION BY THE EARTH is zero.[12]

He agreed with Parmenides and the Pythagoreans that the light from the Moon is reflected light, that the shape of the crescent of the Moon's phases suggests that the it must be a sphere. From that and his spherical Earth hypothesis, he reasoned that all of the heavenly bodies are likely spherical, albeit made from different stuff.

For millennia, Aristotle has been held responsible for the theory of five elementary substances: in *On the Heavens* he added what he called the "first body" to the familiar earth, water, air, and fire. Much later this was renamed "the fifth element;" and later, the "**aether**;" and later than that, the Latinate, "quintessence." In spite of almost all popular and even scholarly sources, Aristotle never identifies his first body as "aether" although he was surely aware that Plato used that term explicitly. History assigns Cicero from the first century BCE, as the source of Aristotle's reference to "aether" with the assumption that famous Roman orator had access to now lost Aristotelean manuscripts. Or, given our repeated reminder that much of what we

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[12] It took until the 19th century to actually observe stellar parallax because the universe really is that big.

know of the Greeks is muddled...it's possible that Aristotle never used the word. I'll use "aether" as it will become a useful contrast with the 19th century "ether," the direct experimental lead-in to Relativity. See, **?**. And, by the way: Aristotle is often said to have insisted that the Eudoxian spheres were crystalline, the "Crystalline Spheres" were indeed an assumption in Medieval and Renaissance times, but nowhere does Aristotle refer to this.

Aristotle's aether is eternal, not composite, neither heavy nor light, and is the most divine of all of the heavenly objects. So it's not anything like the four Aristotelian elements, but for some reason he holds heavenly objects to some of the same physics as terrestrial objects.

The Sky The heavens differ from terrestrial objects in an obvious way: the night sky repeated, every night, while everything on the Earth seems less ordered. Sure falling objects executed their motions according to rules, but every object's behavior is different so the eternal permanence of the heavenly motion contrasts with the impermanence and changeability of MOTION ON THE EARTH. Furthermore, for Aristotle natural motions near the Earth were in straight lines—with a beginning and an end. But the motions of the heavenly bodies seem circular, and so, never-ending...eternal. Obviously, then, the deep sky is made of special, different stuff.

Aristotle's universe is a finite volume in space all the way to the outermost starry sphere, like that of the Pythagoreans. Furthermore, it's always been there (and he speculates and rejects an argument about the possible creation of the universe and so the assembly of, say the Earth. That for him would presume that before that event, there was already a notion of up and down and that bothered him. So, the universe is a finite event in space, but of infinite extent in time.

3.2.6.2 Aristotle's Cosmology

The basic features of Aristotle's cosmology were the same as Plato's as were his ordering of the planets (and different from what Philolaus assumed for the Pythagorean model): Earth–Moon–Sun–Mercury–Venus–Mars–Jupiter–Saturn and the stars. Ever the mechanist, he worried about real material concerns: *how* do they *actually* move as a composite unit?

First, he knew that what was required was a model of the whole universe—Eudoxus' model was a template for each planet, not a whole cosmos— and so each of those sets of spheres needed to all be packaged together into one big onion of spheres, one set inside of another. And this became his problem: since he couldn't have Jupiter's motions affecting Saturns and Mars' motions, he needed to "mechanically" decouple each one.

Remember that I noted that if you had two connected Eudoxian spheres rotating at the same speeds, but in opposite directions, that their motions would cancel one another. Aristotle took that idea and intentionally inserted "rewinding spheres"

Table 3.1: The number of spheres for each of the Eudoxian systems for the Moon, Sun, and planets—not including the outer sphere of the fixed stars— with the Aristotelian unwinding spheres counted separately in the last column.

Planet	Eudoxus	Callipus	Aristotle	Unwinding
Saturn	4	4	4	3
Jupiter	4	4	4	3
Mars	4	5	5	4
Sun	3	5	5	4
Venus	4	5	5	4
Mercury	4	5	5	4
Moon	3	5	5	
Total:	26	33	33	+22 = 55

to do that in such a way to preserve the spheres' connections to the ecliptic and celestial spheres but to isolate them.

Table 3.1 shows that for all of the planets but the Moon and Sun, four spheres were sufficient for Eudoxus. (The Sun and Moon didn't need the daily, celestial sphere rotation.) Callippus added spheres for the inner planets, Sun, Moon, and Mars. It was these 33 spheres that Aristotle then tried to turn into an actual seven-object, whole system.

> It is necessary, if all the spheres put together are going to account for the observed phenomena, that for each of the planetary bodies there should be other counteracting ["unrolling"] spheres, one fewer in number [than Calluppus]...for only thus is it possible for the whole system to produce the revolution of the planets." Aristotle, *Meteorologies*.

Figure 3.11 (a) shows a rendering of the 55 Aristotelean spheres (from https://brunelleschi.imss.fi.it/vitrum/evtr.asp?c=8252. (b) shows a typical Medieval picture of Aristotle's cosmology, the Prime Mover is noted (see below), and in the center, the four Aristotelean elements are drawn. But there's an interesting difference: the planetary order is not Aristotle's but from later.[13] Again, he was always fascinated with his own ideas about motion and for some reason, he assumed that bodies made of the completely unique aether still needed to follow his physics and causal rules. Why didn't he just say that aether spheres just naturally isolate themselves, one set from another?

In that same sticking-to-the-terrestrial-rules spirit, he seemed believe that the spheres needed a cause in order to execute their natural, circular motion and that

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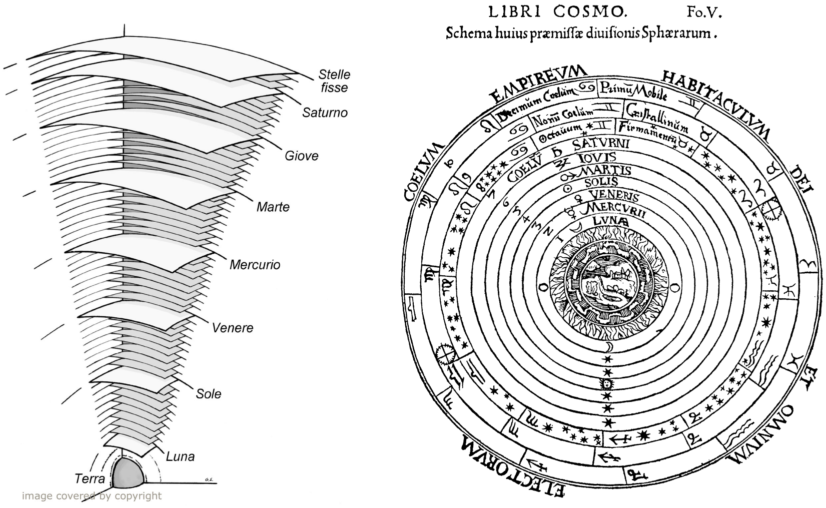
[13] Aristotle seems to have made at least one mistake and actually had two models, one of 47 and the other of 55 spheres. Nobody knows why.



(a) (b)

Figure 3.11: (a) Representation of the 55 spheres of Aristotle's model. Notice that Jupiter (Italian, Giove) has one too many layers and that the Moon (Luna) is depicted as having none. (Museo Galileo. (b) is a typical Medieval representation of the Aristotelean cosmology.

drives his model into strange places. Just like *unnatural motion* for terrestrial objects required a contact pusher, inexplicably he decided that the *natural, circular motion* of his spheres *also needed contact pushers*. That creates an embarrassing regress problem. Every sphere had its very own pusher and so did the outer, star sphere, but how does that last pusher itself remain stationary in order to be able to move that last sphere? Another pusher? He complicated this by insisting that the pushers had themselves no substance, were outside of space and time, and were essentially pure intellect. He called them "unmoved movers" or "Prime Movers" and the idea was a soft toss to Thomas Aquinas 1600 years later to equate the Primer Mover with the Catholic deity.

Aristotle's astronomy is underwhelming and unsatisfying and it didn't solve the major issues endemic to an Earth-centered cosmology: since the model required the planets to be always the same distance from Earth, why do they vary in brightness? And a relatively new problem in his time: why are the seasons, autumn, winter, spring, and fall, all of different durations? These brought Aristotelean modeling to a halt. New ideas were required.

3.2.7 Summary of the Astronomy of Plato, Eudoxus, and Aristotle

(Set the context with the timeline in Figure 1.2 on page 8.)

By the time that Aristotle was done, astronomy had converged on a qualitative, "picture-model" built by two philosophers and a mathematician.

- Plato (−427 to −348):
  - He placed the Earth is at the center of the universe.
  - He modeled the planets as attached to spinning spheres.
  - He proposed that the outer star-sphere spins around the Earth once a day.
  - He placed the sphere of the planets to be inclined to that of the stars so that they all orbit at an angle inclined to the Earth's equator—on the ecliptic.
- Eudoxus (−390 to −340)
  - He modeled each planet's motion as created by four spheres, with axes inclined to one another to replicate retrograde motion and motion relative to the stars. (The Sun and Moon only needed three spheres.)
  - He modeled each planet's model as separate from the others and he did not propose a whole solar system, just pieces.
  - Callipus added spheres for some of the planets in order to slightly tune some of the motions to better match observation.
  - He apparently created one of the first published star catalogues, memorialized in the poem by Aratus, *Phaenomena*.
- Aristotle (−384 to −322):
  - He adopted Eudoxus and Callipus' approach in order to model all of the planets by piecing together the Eudoxian sets of spheres, one inside of the other from Saturn to the Moon.
  - Since each is tied to the one beneath, Aristotle felt that additional spheres were needed in order to isolate the motions of the planets from one another. These were the rewinding spheres.
  - He insisted that the volume outside of the orbit of the Moon was made of a different element from the four elements that operated within. That fifth element, the aether, filled the remaining volume to the outer stars, providing the material of the heavenly bodies. Natural motion in the aether is perfectly circular.
  - He originated the idea that the universe was "full" of the aether—-no gaps or emptiness. This demand became necessary in all future Greek cosmologies.
  - Aristotle's physics guided (or handcuffed) speculation about any motion that the Earth might have had. The Earth had to be in the center of the universe, not spinning, nor orbiting any point.
  - He was very critical of the Pythagorean idea of an orbiting Earth for (his) physics reasons, but also because there was no apparent parallax which meant that the stars were so far away as to hide parallax (too far for anyone's taste) or that the Earth was stationary.

Modeling of this sort stopped after Aristotle as there were problems with any model in which the planets orbit in perfect circles with their common center on the Earth:

- The seasons would all have the same durations, but everyone knew that was

not the case.
- The brightness of the planets would not change, but everyone knew that was not the case.
- The ordering of the planets was arbitrary.

3.3 A Little Bit of Hellenistic Astronomy

Euclid •Aristarchus •Eratosthenes •Archimedes •Apollonius
•Hipparchus •Ptolemy
(Set the context with the timeline in Figure 1.2 on page 8.)

There were two basic thrusts after the fanciful modeling of Plato, Eudoxus, Callippus, and Aristotle. Hellenistic astronomy became both observationally intense—data collection became sophisticated— and mathematically sophisticated, culminating with Claudius Ptolemy's enduring model in the second century, CE. Let's unwrap this extraordinary period of Alexandrian astronomy and set the stage for 1500 years of surprisingly authoritarian science.

3.3.1 A Moving Earth

Heraclides of Pontus (-387 to -312), from the southern coast of the Black Sea, was a contemporary of Plato and Aristotle. As the son in a wealthy family and an apparently smart young man, was able to emigrate to Athens where he became a favorite student of Plato's and was put in charge of the Academy when Plato went on his last, ill-fated trip to Syracuse. He also studied with Aristotle (who was 10 years his senior) and the Pythagoreans in Athens, so he was fully rounded in the three major pillars of classical Greek philosophy. Plato died in -348 and his successor, Speusippus, died in -339 and when Heraclides lost the election for the next leader, he returned north to Pontus. That's where he probably did his astronomy where he had two good ideas, neither of which went anywhere for 2000 years.

It should have bothered Aristotle that his model required the outside starry sphere to be rotating at an astonishing rate in order to make it all the way around each day. The obvious alternative was a spinning Earth and stationary stars and Heraclides proposed just that.

His other imaginative idea addressed a second interesting fact: Mercury and Venus have a different relationship to the Sun from all of the other heavenly bodies. They seem to cling to it, appearing and disappearing as the Sun rises and sets. It was Heraclides who first suggested that this special relationship could be explained by making those two inner plants satellites of the Sun. His cosmology was that the Earth is at the center of the universe, spinning on its axis, orbited by Sun as "normal," but the Sun in turn was itself a second center of rotation with Mercury and Venus orbiting it. Aristotle's grip was not universal, even in his own time.

3.3.1.1 The Greek Copernicus

While Heraclides could be thought of as ushering in the post-Athens, Hellenic era, it was **Aristarchus of Samos** (−210 to −230), a toddler when Heraclides died, who conceived the best model of the universe and a completely new way to deal with the cosmos: by measuring it. He studied with Strato of Lampsacus, who was the third director of Aristotle's Lyceum, and when Strato went to Alexandria to tutor and counsel Ptolemy II he brought Aristarchus along as his pupil. Strato returned to Athens, but Aristarchus stayed in Alexandria and did his mathematics and astronomy in that growing Greek-Egyptian intellectual center. He probably overlapped with the senior Euclid and surely learned all of Greek mathematics known to that time, conceivably from its most famous chronicler. He fashioned his single surviving text *On the Sizes and Distances of the Sun and the Moon* like Euclid's *Elements*: propositions followed by orderly proofs.

As the Moon orbits the Earth half of it is always illuminated, but we see phases as it makes its way around us. From our modern understanding, Figure 3.12 (a) shows the named phase states as we see them. When it's on the other side of the Earth from the Sun and we're in nighttime, we see it fully illuminated ("full Moon"). When it's between us and the Sun ("new Moon") we don't see it at night (after all, we're looking away from the Sun at night), but can sometimes see it during the day. In between, it shows us partially illuminated crescents. But look at the two quarter Moons. From Earth, at exactly that point we see the Moon split into two equal halves, one dark and one bright.

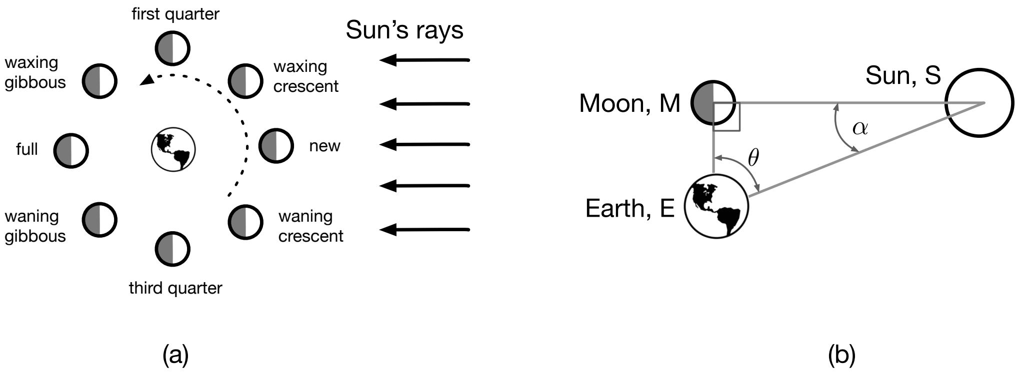


Figure 3.12: The Moons phases and positions are shown in (a) relative to the Earth and Sun. From this vantage point, the Moon orbits counterclockwise. In (b) the particular position and phase that makes the Aristarchus calculation possible with the right angle shown occurring at just the first or third quarter when the Moon is half lit.

While Aristarchus didn't anticipate the Moon orbiting the Earth, he did realize that this quarter phase had a particular geometric arrangement with respect to the Sun and Figure 3.12 (b) shows his idea. At that moment, the angle between the Sun and the Earth is a right angle, $\angle EMS = 90^\circ$.

> “...when the Moon appears to us halved, the great circle which divides the dark and the bright portions of the Moon is in the direction of our eye...when the Moon appears to us halved, its distance from the Sun is less than a quadrant by one-thirtieth of a quadrant.” Aristarchus, *On the Sizes and Distances of the Sun and the Moon.*

By “distance from the Sun” he means angle α in the diagram, $\angle MSE$. With a modest amount of modern trigonometry, it’s possible from the angles to calculate the ratio of the distance of the Earth to the Sun to the distance of the Earth to the Moon in one line. Without modern trigonometry it’s a straightforward exercise in geometry. Aristarchus did just that and found:

$$\frac{\text{Distance, Earth to Sun}}{\text{Distance, Earth to Moon}} = 19 - 20$$

where the range is his own estimate of how well he could determine the angle. Appendix A.3.2 completes this calculation and some other interesting measurements that he and others made. This are stunning in their originality and also in their simplicity. He also subsequently calculated three additional things about the universe, for a total of four groundbreaking conclusions:

1. the distance of the Earth to the Sun) ≈ 20× distance of the Earth to the Moon
2. the diameter of the Sun ≈ 19× the diameter of the Moon
3. the diameter of the Earth ≈ 2.85× the diameter of the Moon
4. the distance of the Earth to the Moon ≈ 10× the diameter of the Earth

His mathematics and methods are correct but he had some mistakes, crucially because α is very hard to measure and so his determination of $\theta = 87^\circ$ was wrong...it’s actually closer to 89.853° which makes the distance of the Earth to the Sun) ≈ 390× distance of the Earth to the Moon.[14]

But that’s not all. Let’s let Aristarchus’ Italian/Greek contemporary **Archimedes of Syracuse** (-287 to -312) take over from here:

> "Aristarchus has brought out a book consisting of certain hypotheses, wherein it appears, as a consequence of the assumptions made, that the universe is many times greater than the “universe” [expected]...**His hypotheses are that the fixed stars and the sun remain unmoved, that the earth revolves about the sun on the circumference of a circle, the sun lying in the middle of the orbit,** and that the sphere of fixed stars, situated about the same centre as the sun, is so great that the circle in which he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the centre of the sphere bears to its surface.” (emphasis, mine) Archimedes, *The Sand-Reckoner*.

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[14] The point of First Quarter would be in the same part of the sky as the Sun, just before Sunset. Without modern tools, measuring that angle would essentially impossible, if not dangerous! **?** suggests that Aristarchus concocted the “one-thirtieth” as an extrapolation of the time that it takes for the Moon to reach the First Quarter as the largest angle that could come from a month of 30 days to orbit and one quarter of that for the phase. That’s almost even more impressive reasoning.

Aristarchus was apparently the first to envision a Sun-centered ("heliocentric") universe and, oh by the way he also apparently adopted Heraclides' notion of a spinning Earth. Copernicus-in-training. Nobody knows how he came to this conclusion...even though it solves many of the problems (planets' brightness, for example). His model was largely ignored and the fact that Archimedes tossed that reference off so casually is indicative of what must have been an overwhelming concern for the parallax problem (which is a prejudice about the possible enormity of the universe) and Aristotle's authority when it came to terrestrial physics.

But there it is: the first modern-sounding MOTION BY THE EARTH and MOTION IN THE HEAVENS . Copernicus later took comfort in Aristarchus' idea.

This is an auspicious moment! Aristarchus'work ushers in the beginning of quantitative astronomy which was quickly taken up by his contemporary, **Eratosthenes** (−276 to −194), who became the Chief Librarian of the Alexandria Library just following Aristarchus' death. (He was also a geographer, mathematician, astronomer, and a poet. The nickname given to him was Pentathlos, implying a Greek pentathlon athlete of many talents.) With his access to Library data, Eratosthenes learned that at noon on the summer solstice (the first day of summer) in Syene, Egypt, the Sun's rays were known go right into a vertical well without hitting the sides. Syene (modern day Aswan) has a latitude of just about 24° which is at the northern tropic, the Tropic of Cancer which means at the Summer Solstice, the sun is directly overhead (the definition of the Tropic of Cancer) and so would not cast a shadow from a vertical stick in the ground. Meanwhile, Alexandria is directly north of Syene at the same longitude and so Eratosthenes reasoned that the Sun is so far away that it's okay to presume that its rays were parallel at both cities. Therefore, for a spherical Earth, the shadow of the Sun on a vertical stick in Alexandria would cast a shadow. He measured it rather than the 0° at Syene, it was 7.2° at Alexandria. That angle is 1/50th of the 360° of a circle so that the circumference of the Earth must be 50 times the distance between the two cities, which is 833 km (in modern units). Fifty times 833 km is 42,000 km for Earth's circumference— only a few percent higher than a more modern value! Appendix A.3.2 shows this calculation.

Eratosthenes wasn't done. He also devised a way to measure the obliquity of the ecliptic—that angle 23.5° of inclination of the ecliptic from the Celestial Equator. And he made a star catalog of 650 stars. And he wrote a poem about himself. He reportedly went blind in his old age and chose to commit suicide as a result.

So for the first time, astronomers learned the size of the Earth and more could be learned: for example, using Aristarchus and Eratosthene's results, from Aristarchus' #3 above they could conclude that the diameter of the Moon is 4700 km, where the actual value is about 3500 km. I hope you can appreciate that Greek astronomers are no longer merely telling stories. They're measuring our universe.

3.3.2 Casting Aside Aristotle and Eudoxus

The next important step is another storyteller, but an important mathematician who had a good idea. **Apollonius of Perga** (−240 to −190) migrated from Turkey to Alexandria as a young man to study in the successor school of Euclid. "The Great Geometer" became his historical label and he's remembered for discovering the mathematics of "conic sections" (circles, parabolas, ellipses, and hyperbolas)—a subject beyond Euclid's geometry.

For our story we know of him as the geometer who puzzled over the seasons problem and found a way to modify the Eudoxian model to loosen the requirement of all spheres centered on the Earth. His discovery is shown in Figure 3.13 (a) in which E shows the location of the Earth, S is the location of the orbiting Sun, and D is a point in space—attached to no object— which is displaced from E. The distance $\overline{\mathrm{EC}} = e$ is called the **eccentricity**. The Sun uniformly follows the dashed **eccentric circle**, centered on D and not the Earth! Notice that the result is a Sun's path sometimes further from, and sometimes closer to the Earth. When it's further, it would take longer to go halfway around and so the seasons during that path segment would be longer.

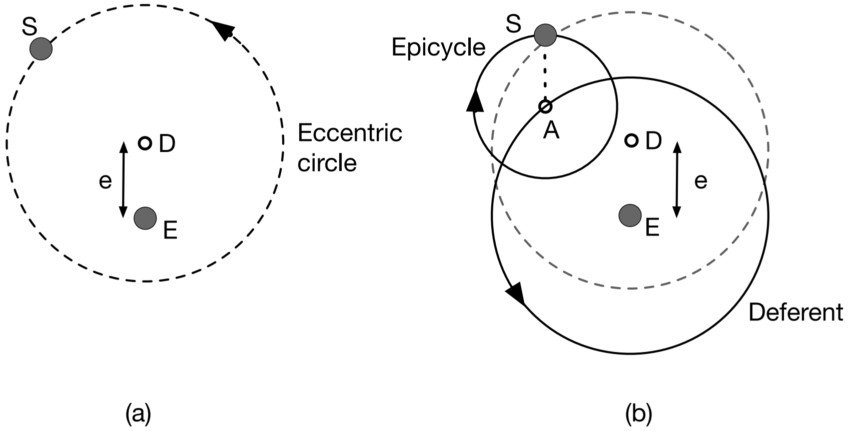

Figure 3.13: In both figures, E is the location of the Earth and S is the location of the Sun. In (a) an eccentric circle is shown for a proposed Sun orbit around the Earth. By putting the center at a spot in space displaced from the Earth by the eccentric, e, the seasons would appear on Earth to be of different durations. In (b) the equivalent (under the conditions described in the text) epicycle solution is shown with an overlay of the eccentric circle shown in a light dashed line for comparison. The deferent is centered on the Earth and the epicycle is centered on the rim of the deferent.

But there's more to this as Apollonius discovered a geometric equivalence also illustrated in Figure 3.13 (b). Here a circle, called the **deferent** is centered on the Earth but doesn't act as an orbital path for the Sun. Rather, the Sun rides on another circle, the clockwise rotating **epicycle** with its center (A) attached to the rim of the counterclockwise, rotating deferent. Notice that the rotational sense (here, clockwise) of the epicycle is opposite to that of the orbit of its center, A, on the deferent. Each of these models would cause Earth to experience more Sun during part of the year and less Sun the other parts, which would change the length of the seasons.

> The idea of an epicycle is not easy to grasp since we don't use them any more in planetary astronomy. But if you look up some night, you'll see an example of an epicycle. Think modern (for a moment): we know that the Earth goes around the Sun and that the Moon goes around the Earth. The Moon' s orbit can be thought of as an epicycle: the Earth's (nearly) circular orbit around the Sun would be the deferent and the Moon's orbit around the Earth is the epicycle. So looked at from the Sun, the Moon's orbit would be a slightly off-center orbit around the (orbiting) Earth. This particular epicycle is one in which in Figure 3.13 (b), E coincides with D. We're going to meet epicycles in a major way when we get to Ptolemy and Copernicus.

Numerical predictions were not the goal for Apollonius, but a more realistic framework was—and probably the geometry was also an attraction for him. So his ideas were one more step away from Aristotle toward a new way of doing science.

3.3.3 The Greatest Astronomer: Hipparchus

The most celebrated astronomer of antiquity was, yet another Greek about whom we don't have many biographical details. However, **Hipparchus of Nicea** (about −190 to about −120) was so accomplished that his feats were detailed in later Hellenistic astronomy texts and most completely two centuries later by Ptolemy. His mature astronomy work appears to have been done on the island of Rhodes a large island to the west of Cyprus and far from his home near Constantinople. There he built an observatory and created or improved on instruments for measuring positions of stars and planets. He was a serious observer of astronomical objects and events and a mathematician of significance. Finally, the world was ready for a complete astronomer...The Greatest Astronomer, he was later called.

Let's be clear: **astronomy was different after Hipparchus**. He dedicated himself to an entirely different purpose from the "picture-stories" of Plato and Aristotle. Hipparchus measured numerical features of the cosmos.

Hipparchus' Solar Model. Hipparchus figured out that if he used the eccentric model only a few parameters were required in order to determine, e and so the problem of the seasons' unequal durations could be solved geometrically, almost like being a cosmic surveyor. His model is shown in Figure 3.14 with the anchor for astronomical positioning, the Vernal Equinox (VE, ♈) (a convention used to this day). The Sun (☉) orbits the center of the eccentric orbit at C and the Earth is displaced by the eccentricity, e (which is usually quoted as the fraction of the distance $\overline{CE}$ to the radius, $\overline{CA}$). The dash-dot lines denote the axis from the Vernal Equinox (mid-March) and the Autumnal Equinox (AE, mid-September) and the

Summer Solstice (SS, mid-June) and the Winter Solstice (WS, mid-December) and the four unequal quadrants delineate the four seasons. Here it's drawn for antiquity in which spring was the longest season and autumn was the shortest (while in our time summer is longest and winter is shortest). In astronomy, the furthest point of a celestial object's orbit from a reference is called the "apogee" and the closest approach, the "perigee." The figure shows the arrangement for antiquity, when the angle of the dotted line through E and C was about $\alpha = 65^\circ$. Today, it's greater than 90° which is why our summers are longer than antiquity's summers.

His result was that the distance of the eccentric from the Earth's position was about 1/24th (about 0.04) of the radius of the orbit's radius so it's almost a circle centered on Earth, which is why the season durations are within a few days of one another.[15] Notice that the position when the Sun is furthest (called the "apogee," *A* here) from the Earth is when summer and spring occur and the closest (called "perigee") is when fall and winter happen.[16]

Hipparchus could use his solar model to predict the location of the Sun at any time in the future and it was accurate and used for many hundreds of years.

Hipparchus' Lunar Model. The Moon's motion is different and more complicated than the Sun's with at least three parameters required to determine its motion. He managed that as well, this time using an epicycle model. Finally that legend ascribed to Thales from 400 years before is made whole: Hipparchus could predict both solar and lunar eclipses!

In addition to his modeling of the Moon's motion, he found a way to determine the distance from the Earth to the Moon. With his version of trigonometry (see below), he found that the distance from the Earth to the Moon is 65.5 times the radius of the Earth and that's about right (it's about 60.336). (Newton used his result in his invention of his Law of Gravitation.) Hipparchus attempted the same thing for the distance to the Sun, but underestimated it by a factor of 50.

Hipparchus' Fixed Star catalog. Hipparchus began the first quantitative survey of the fixed stars—the ones thought to be on the inside of the Celestial Sphere. Prior to him, locations of bright stars were noted by identifying a rough relative position in words: that a the star in the "shoulder" of one in one constellation is rising when the star in the "sword" of another constellation is setting and that the star on the "right leg" of a third constellation appears right overhead when this happens. More stories. Hipparchus took a different approach.

His data were extensive and would have required impressive patience (night after night) and commitment to a multi-year research project. Ptolemy tells us that Hipparchus cataloged around 850 stars, their positions, and their brightnesses and

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[15] Had $e = 0$, then all four season would have been the same length and the Sun's orbit would have been Aristotle-like, centered on the Earth.

[16] Why the Sun is *furthest* away during the summer is a reasonable question and understanding that waited for Kepler and Newton.

they were in use for centuries afterwards. Others had made catalogs (Eudoxus and Eratosthenes), but his was different: he invented a coordinate system and assigned positional numbers to each star. Think about how your GPS specifies a location on the Earth: my phone tells me that the location of the Library of Alexandria is 31.20870° N, 29.90911° E. What that tells me is that the library is a little more than 31° north of the equator (the **latitude**) and about 30° east of some point that's world-wide agreed to be the observatory at Greenwich, England (the **longitude**). Hipparchus adopted the same thing, but applied to the stars—the underside, if you will, of that Celestial Sphere above us. (More about this and how his system is essentially identical to modern astronomy is discussed in Section 3.5.2 of *Greek Astronomy, Today*.)

A many-decade detective story unfolded in trying to figure out which (if any) of Hipparchus' data were included in Ptolemy's more extensive star catalog. And there's a clue. Remember Aratus' poem, *Phaenomena* from Figure 3.1 which was written as an ode to Eudoxus? The one book we have of Hipparchus' is his *Commentary on the Phaenomena of Eudoxus and Aratus* in which he severely criticized mistakes of fact in the poem regarding the relative positions of stars in the constellations. He included a set of positions for 22 stars of his own observation and these have been extensively compared with Ptolemy's catalog and the agreement is pretty good. Without that poem, and Hipparchus' grumpiness about a 200 year old poem,[17] we wouldn't have any corroborating information that Hipparchus really did create the first ever quantitative star catalog. Well, maybe until 2022! For that breaking story, look at Section 3.5.3 of *Greek Astronomy, Today*.

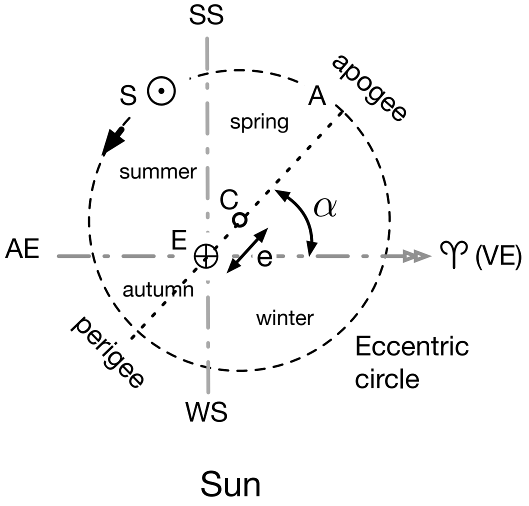

Figure 3.14: Hipparchus and Ptolemy's solar model showing the seasons in antiquity (today, winter is shorter and summer is longer). SS and WS are the Summer and Winter Solstices, VE (♈) and AE are the Vernal and Autumnal Equinoxes and the seasons are then defined as the four quadrants among them. The Earth (⊕) is displaced from the Sun (⊙) by the eccentricity, e, the distance in space from Earth to the center of the eccentric circle about which the Sun orbits. The dotted line is described in the text.

Hipparchus' Trigonometry. The mathematical problems he had to solve for his solar and lunar models were surely the inspiration for a tool that marks the invention of trigonometry. Figure 3.15 shows his idea. A chord inside of a circle

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[17] He wrote other ill-tempered reviews of other people's writings.

with radius R and center O is shown as the length $\overline{\mathrm{AB}}$ where the chord subtends the angle θ. By hand Hipparchus divided carefully drafted circles into degrees based on 360° (which came from the Babylonians), but much finer: 21,600 segments which is the number of arc minutes in 360°. Then he painstakingly created "tables of chords" of varying lengths for each segment giving him a fairly precise lookup table of angles, radii, and chords. Given a radius, and the length of a cord, an angle could be looked up in the table. Or visa versa. It's equivalent to a table of trigonometric sines since as in the figure, if one divides the chord in two so that there are two right angles at point C, then the $\sin(\frac{\theta}{2}) = \frac{1}{2}\left(\frac{\overline{\mathrm{AB}}}{R}\right)$.

Hipparchus' Discovery of the Precession of the Equinoxes.

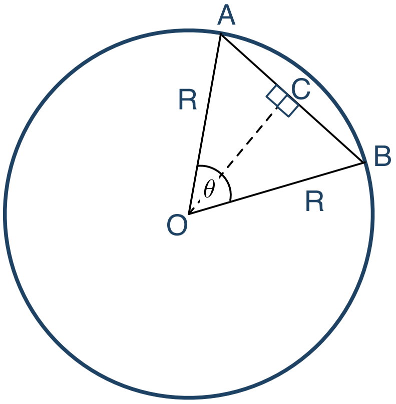

Figure 3.15: Showing how ancient "chords" related to a modern sin for a given angle θ.

The discovery for which he's most known was that the Earth's seasons might shift over time. He found this in two, complimentary ways. His first approach suggested the location against the zodiac of the summer solstice was 12 hours different from that recorded by Aristarchus, 145 years before. That inspired him to make a second, clever measurement to confirm that odd result.

He figured out how to determine the longitude of a star (the angular distance of the star relative to the Vernal Equinox) near the ecliptic and compare that to an earlier measurement from other astronomers. He focused on the bright star, Spica (the brightest in the constellation Virgo, or α Virginis) for which he had data from an Alexandrian astronomer, Timocharis in −294 and −283 almost two centuries before him. This could be done easily in principle. Just measure the angle between the Sun and the star, right? That is:

$$\text{Longitude, Spica} = (\text{longitude, Sun}) + (\text{arc-angle between Spica–Sun}).$$

He knew the longitude of the Sun from his Solar model which gave him the angle α from Figure 3.14. The arc-angle in longitude of Spica and the Sun is a different story since if the Sun is out, that's daytime (!) and so you can't see the star. But he was very clever. He made use of the fact that during a lunar eclipse, the Earth is directly between the Moon and the Sun...so they are 180° apart and at night, he would be looking away from the Sun, toward Spica. So measuring the arc of longitude of Spica relative to the eclipsed Moon gives him his answer:

$$\begin{aligned}\text{Longitude, Spica} &= (\text{longitude, Sun}) + (\text{arc-angle between Spica–Moon}) \\ &\quad + (\text{arc-angle between Sun–Moon}).\end{aligned}$$

At an eclipse, the (arc-angle between Sun–Moon) is 180°! Using Timocharis' Spica-Moon measurement, the longitudinal difference of Spica was 8° west of the Autumnal Equinox while he determined 6°: the longitude of Spica had increased by 2° in 150 years. (He actually did this as an average of two different eclipses 11 years apart.) That's about 1° per 75 years (consistent with his other measurement). Ptolemy did a similar experiment 265 years later and compared it with Hipparchus' and got about 1° per 100 years.

So what's going on here? Hipparchus concluded that the zero-point of longitude (the Vernal Equinox, which is where the ecliptic crosses the Celestial Equator) must be moving somehow over very long times.

This we know now has a physical cause: the Earth's axis of rotation points at an angle that's not perpendicular to the plane of its orbit around the Sun. It's tilted by close to that 23.5° from Figure 3.3 and like a top, the mass of the Earth causes it to precess around the Celestial Pole. This wobble of the Earth *looks* like a wobble of the ecliptic and so the equinoxes will be in a different location as time marches on. How fast? We know now that he precession rate is pretty close to Hipparchus' and Ptolemy's measurements: about 1° per 72 years. So to go all the way around, requires $72 \times 360^\circ = 25,920$ years.

3.3.4 Summary of the Astronomy of Aristarchus, Eratosthenes, Apollonius, and Hipparchus

(Set the context with the timeline in Figure 1.2 on page 8.)

- Aristarchus (−310 to −230):
  - He made the first attempts to use geometry to measure distances among and sizes of the Earth, Moon, and Sun.
  - He proposed the first model of a Sun-centered cosmology, apparently without geometrical modeling.
- Eratosthenes (−276 to −194):
  - He measured the diameter of the Earth to impressive accuracy.
  - He measured the obliquity of the ecliptic—that 23.5° tilt of the ecliptic from the celestial equator.
  - He apparently created a star catalog of more than 600 stars. This would have been in words itemizing apparent locations of stars relative to constellation points.
- Apollonius (−240 to −190):
  - He was mathematician of the first rank and found a picture-way to model the Sun's motion around the Earth to create seasons of different lengths through the introduction of the deferent and eccentricity.

  - He also found a mathematically identical, but geometrically different form for planetary motion called epicycles. His proof of their equivalence was lauded as an important step by Ptolemy.
- Hipparchus (-190 to -120):
  - He built on Apollonius' deferent model and found a way to measure the actual eccentricity of the Sun's orbit and the longitude of the apogee. This was the first attempt to not only geometrically model the cosmos (or any physical mechanism) but to also quantitatively measure the shape parameters of the model.
  - He found a way to determine the distance to the Moon in terms of Earth radii, a value used by Newton much later.
  - His star catalog of more than 800 entries went beyond the stories that had been told previously: he invented a coordinate system that could be used by anyone to find the actual numerical positions of objects relative to an "origin" of essentially a celestial longitude and latitude.
  - He discovered that the Earth's seasons shift relative to the star's positions over time—the precession of the equinoxes. Understanding the physical cause of this phenomenon waited for Newton's explanation of the precession of the Earth's axis of rotation...slowly: about 1° per 75 years.

3.4 The End of Greek Astronomy: Ptolemy

While Aristotle's concentric spheres model lay dormant, it was to rise again in the middle ages and assume a strange parallel existence next to the model that made precise predictions. This is the model of Claudius Ptolemaeus, known for nearly two millennia as **Ptolemy of Alexandria** (100 to 170 CE). He created the most complete model of the cosmos before Copernicus and, refreshingly, his books survived intact thanks to Arab intellectuals' commitment to preserving and commenting on the works that they encountered from the Islamic conquest of the Near East, all of Northern Africa, and Spain.

Ptolemy wrote three books on astronomy for which we have original Greek and some Arabic translations. *Mathematical Composition* is the main work, now known by its Arabic title of *Almagest*, a corruption of the Arabic *Al* with the Greek word *megistē*, for "*the greatest*." The second is the *Handy Tables* which consists of two parts: the second part includes tables of his planets and stars of which we know from medieval versions 200 years after Ptolemy's life. The first part is the instruction manual on how to use the tables, surviving only in its Greek origin. *Almagest* is too complicated to have been absorbed by most and so the *Handy Table*s assured widespread use of Ptolemy's work. The third, *Planetary Hypotheses*, is an upgrade of the earlier *Almagest* and an attempt to build a plausible physical model of the purely mathematical *Almagest*. It was only appreciated and fully translated as two books in the 1960s!

Even though we finally have a complete set of one of our astronomer's works, ironically we know little about his life, except for a few references of his and a few later narratives by Roman and medieval scholars. Ptolemy almost certainly worked in Alexandria as his extensive observations come from that latitude. He's the first of our Greeks to have two names! "Claudius" indicates that he was a Roman citizen, probably during the time of Emperors Hadrian to Marcus Aurelius. "Ptolemaeus" indicates that his was of Greek ancestry.

Almagest is a huge subject. It is 700 pages long in a modern edition and more than a thousand pages are required to fully lay out the considerable mathematics of the book (**?**). It's not for the faint of heart. It's also pure mathematics and little philosophy and *not a physical model*.

Here's what it's like. I could imagine building a mechanical model of the economics principle of supply and demand. Suppose we have a playground teeter-totter with an arrow on the right end that points to a dial indicating high or low for prices of goods. Right side up, prices high, right side down, prices are low. If we start with the teeter-totter level and add weights to the right to represent *supply* of that product and weights to the left to represent *demand* for that product...we've got a mechanical model of the economy. When the supply, right-weight is larger than the left demand-weight, the arrow points down—prices fall. Likewise, when demand outweighs (sorry) supply, then the left side goes down and the arrow points up for higher prices.

This is a perfectly predictable model of the economy and through careful analysis of past economic history, one could tune the amounts of weight that would correspond to a prediction of prices and mark the dial with $ indicators. But, while it's a good model, *it's not a realistic representation of the economy. Almagest* is like that. It's a very complicated model of moving and spinning circles, lots of numbers to characterize the circles, scores of huge tables of numbers,[18] and could accurately predict positions of the heavenly bodies. But Ptolemy made no claim that the Sun, Moon, and planets actually performed the motions in his model.

Ptolemy's Philosophical Roots and Prerequisites for the Book: Books I and II of *Almagest* describe his working philosophy, defending it with standard arguments. But apart from the actual heavenly body motions, it's Aristotle, top to bottom. The mathematics required was Euclidean plane geometry and the use of Hipparchus' chord tables, except Ptolemy made them even more precise. He used the new "spherical geometry," and he developed it from scratch for the reader. With this introduction, he's ready to solve the world.

Ptolemy's Solar Model: Book III This was relatively easy and critically important. All of positional astronomy—to this day— depends on understanding where objects in the sky are relative to the Vernal Equinox, which in turn depends on the Sun's motion and position at any time. He didn't invent a solar model—he replicated

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[18] Perhaps the first use of tables in any manuscript in history.

Hipparchus' and was generous with his praise the original author.[19] So, Ptolemy's model of the Sun's is exactly the same: Figure 3.14. He repeated Hipparchus' determination of the eccentricity and agreed, but with higher precision: $e = 0.0415$ as compared with Hipparchus' $e = 0.04$.

Ptolemy's Lunar Model: Book IV and V. The motion of the Moon is difficult to grasp even today. Ptolemy's solution was ugly and also his biggest mistake: he could solve for eclipses (lunar and solar), but his model predicts that the Moon's apparent size would vary by a factor of two in a month, which obviously isn't the case. His solution is tortured and from our modern perspective, clearly an indication that there must have been something wrong. One has the impression of him just giving up and declaring successful eclipse predictions as a victory. Figure 3.16 shows the model which is a prelude to the sorts of tools he had to use to get some of the planets right. Notice that the deferent of the epicycle of the Moon is not fixed, but while centered on the Earth, is driven around a little circle close to the Earth at point R (the "crank mechanism"). So the Moon is pushed away and yanked back, hence the apparent size change. He made careful tables of predictions of the eclipses—which were accurate— for any date, and washed his hands of the Moon problem.

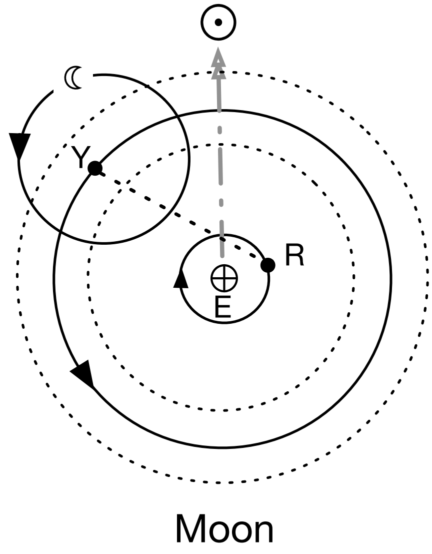

Figure 3.16: Ptolemy's model for the Moon (☾) is sketched (see below for explanation). Notice that the arrangement is in a particular relationship to the Sun.

Ptolemy's Model Fixed Star Catalog: Books VII and VIII. It was Ptolemy who told us of Hipparchus' catalog of the positions of 850 stars. He takes on the same task, but also includes the positions and apparent star brightness of 1022 objects from 48 constellations in his catalog and with this began almost two centuries of fights among historians. Did Ptolemy copy Hipparchus' 850 stars (shifting their longitudes by $2^\circ 40'$ to correct for the precession of the equinox over 265 years) or did he measure their positions as he claimed? Or had Hipparchus' catalog been wrong? The comparison of the Hipparchus' 22 stars' from his *Commentary* to Aratus' poem with their counterparts in Ptolemy's catalog is the key. There are translations problems since Greek numbers were written using Greek letters and sometimes mistakes happened in translation and transcription of centuries-old media. Stars were not always named, but a little story was told about each one to locate it within

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[19] He has been accused of plagiarizing Hipparchus, but that's not fair as he gave ample credit.

a constellation. So mistakes happened. This argument has largely subsided: within the uncertainties that can reasonably be attributed to each, most of Hipparchus' 22 stars do match their Ptolemaic counterparts and that each astronomer is likely vindicated. I'm sure you're glad that we've cleared that up.

The bottom line about Ptolemy's catalog is this: it represented an enormous effort over probably decades and was the best star chart all the way to Tycho de Brahe in the late 16th century (Copernicus used much of it). A remarkable achievement and legacy.

Ptolemy's Planetary Theories: Books IX through XIV. His planetary models (yes, there were three) were the target of the Muslim astronomers, Copernicus, Galileo, Tycho, Kepler, and Newton and it took all of them to bring Ptolemy down. Its accuracy is still impressive so something besides getting the right numbers was behind its downfall, an important part of our story later.

The end product of his planetary research is a chapter for each of the five planets including its geometrical model, the particular parameters built into each model, a description of how he determined each parameter from his observations, and then five deliverables: a set of tables of positional coordinates for each planet, for any day in the future. It was these tables that were reprised in his User's Manual, the *Handy Tables*.

He must have struggled mightily to make Aristotelean circular orbits work but he held accuracy to a higher standard than the Classical Greeks, for whom a nice picture-story was sufficient. In order to "get it right"—which meant, make predictions that worked— required him to make excursions from some of Aristotelian rules. For example, the eccentric model for the Sun and a strange epicyclic model of the Moon had heavenly bodies orbiting seemingly arbitrary points in space apart from the Earth! But as painful as the Moon solution was, getting the motions of the planets right was another story altogether.

3.4.1 Mars, Jupiter, and Saturn

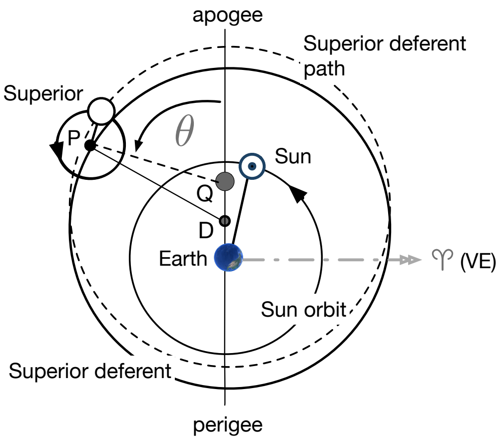
The prominent retrograde motion of especially Mars as well as Jupiter and Saturn added an entirely different set of complications from the naive epicycle model of Apollonius and Hipparchus. The simple epicycle picture of Figure 3.13 wouldn't do. Ptolemy had to insult Aristotle one more time and that particular solution offended Copernicus and his Arab predecessors. Let's look at his solution for the outer planets as they're a little simpler. Figure 3.17 shows his model that functions for Mars, Jupiter, and Saturn. Look at Figure Box 3.17 on page 121. After you've read the material in that Box, return to this point ☟ and continue reading.

The new wrinkle is the introduction of a third point in space, the **equant** (Q), displaced from the deferent point by the same amount as D is from *E*. A superior planet's epicycle's center P doesn't undergo uniform circular motion about the deferent center, D, *but about the equant*, Q. That is, the angle θ uniformly increases in time around the epicycle's path, so it appears to perform *non-uniform* rotation around D (its center) *and non-uniform around Earth*. The model constrains this movement such that the line from a superior planet to P, $\overline{\text{Superior-P}}$, is always parallel to the line connecting the Earth and the Sun, $\overline{\text{Sun-Earth}}$. Notice that this creates a special relationship among the Vernal Equinox, the Sun, and the planet.

So a superior planet orbits in its epicycle with center (P) following its deferent as originally imagined by Apollonius—except that as compared to Figure 3.13 the epicycle rotation is reversed from counterclockwise to clockwise. *That creates retrograde motion.* The Sun is shown with its orbit centered on the Earth (since its eccentric center is too small to explicitly show). So there are two centers of motion here—one for the Sun and another for Mars' deferent.

The dashed curve in the figure is the trajectory of Mars' deferent. So what Ptolemy knew was the various positions that Mars, Jupiter, or Saturn would have on the *dashed line*, but what he needed in order to build each model was its position on the deferent, the solid line. That's a formidable mathematical transformation.

FIGURE BOX 3.17



Ptolemy's model (not to scale) for a superior planet, Mars, Jupiter, or Saturn and its relationship to the Sun (⊙). Here, "Superior" (◯) is on an epicycle with its center at P. P rotates clockwise around the circular deferent path with its center at the center, D. The Earth is not at the center of anything, except close to that for the Sun! The angular speed of P around D — the amount that the angle θ increases with time is constant, but about the point Q...not D. The dashed circle is the path that P actually takes which its offset from the deferent's center.

Each planetary "kit" looks like this for superior planets and slightly different for the inferior planets. Every deferent radius for all planets was chosen to be 60 in an arbitrary set of units. The necessary parameters were determined by Ptolemy separately for each planet, including: the epicycle radius, the separation of Earth from the deferent point (D), which is also the separation of D from the equant, Q, the orientation of the apogee to the Vernal Equinox direction, and the angular speed at which θ increases in time.

Now go back to page 120 and pick up where you left off.

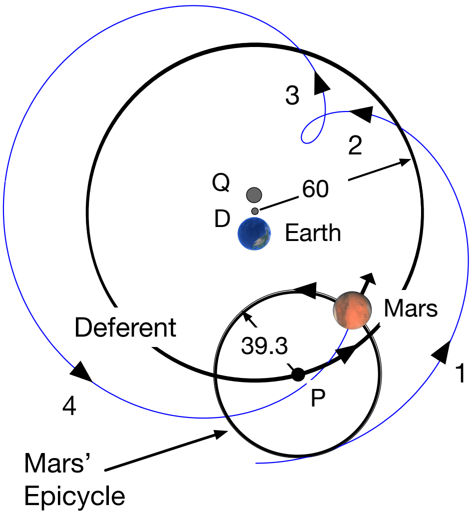


Figure 3.18: Mars (♂) is shown on its epicycle with its center, P, rotating around the deferent with its center at D. I've used Ptolemy's actual relative sizes for Mars. All deferents were in units of 60. Mars' epicycle's radius is 39.3/60 and the distance from Q to Earth is 12/60. One can see the strange loop motion described in the text.

> "...in a tour de force of possibly the most complex and extended calculation in all of ancient mathematics, he developed a method of successive approximation that allows the numerical values of the eccentricity and the direction of the apsidal [direction of the apogee of Mars' orbit] line to be found to any degree of accuracy. Both the problem and the solution are remarkable...his solution shows a very high order of mathematical intuition...The number of astronomers after Ptolemy who understood and could apply the method must have been very small." [**?**, Vol 1, p307.]

Let's pick on Mars. Mars orbits Earth (in our 20th century way of viewing things) about every 687 days, or 1.88 Earth years and undergoes retrograde motion about every 2.1 years, or a little more than one revolution around the Sun. The backwards appearance lasts a little more than two Earth months, or about 72 days. Ptolemy's model with the equant rather precisely describes Mars' retrograde motion as it forces a kind of loop-the-loop as viewed from Earth.

In Figure 3.18 I've calculated the Mars model to show its epicycle and eccentricity (separation among Earth, D, and Q) using parameters taken from *Almagest*. Mars' path is, well, unusual. There are 4 points identified on the actual path that Mars takes while riding on its epicycle. We start at position 1, and as the epicycle turns and as the deferent turns, Mars moves to position 2 where it starts to appear to slow making that loop which makes it appear to go backwards during 72 nights. Then it comes out of retrograde and continues its forward-appearing path at 3 and nearly completing it's 1.8 year long path at 4. In each Mars year, the location of the loop shifts a bit relative to the Vernal Equinox.

This is what's seen from Earth with a bonus: it also addresses the fact that in retrograde, the planets are brighter, here, because it would literally be closer to Earth. Just how often and how fast would be determined by the parameters—Jupiter and Saturn's parameters are quite different.

It works very well as seen in Figure 3.19 from **?** (inspired by **?**). This shows seven bands that should encompass the retrogrades of Mars as viewed from Earth for some of the years of Ptolemy's observations, from 109–122 CE. The loops are the Mars retrograde events relative to the Vernal Equinox (the trajectory between points 2 and 3 in Figure 3.18) and the wedges show predictions of where that should happen. In (a) predictions are for a straight epicycle model *without an equant* while (b) shows the same thing, but *including the equant*. This, and other successful measurements surely convinced Ptolemy that he was right. He needed the equant.

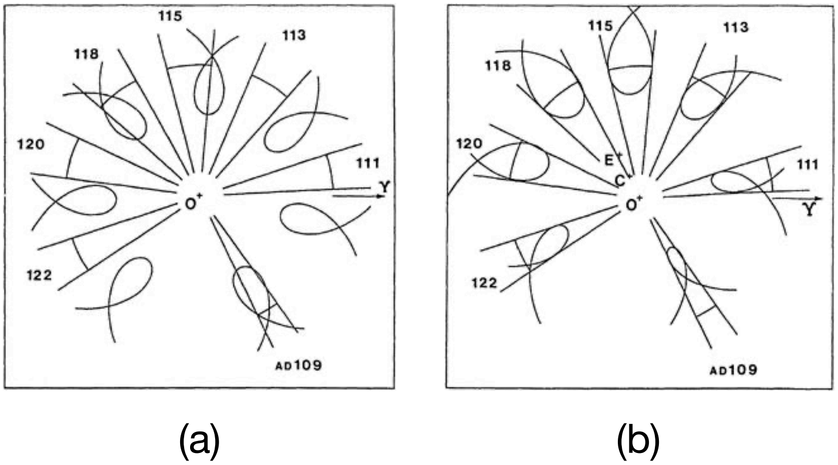


Figure 3.19: Seven retrograde loops of Mars for times of Ptolemy's observations (a) without the equant and (b) with the equant.

> The relationship that Mercury and Venus have with the Sun was very problematic. Today we know that they orbit very close to the Sun but even now measuring their positions is challenging. The Sun's in the way! Observations had to be done just after sunrise and just before sunset...and carefully as to not blind one's self. So they presented a set of problems which couldn't be solved without separate models for each. And those solutions are strange, especially for Mercury with more moving centers of deferents.

Think about all of the major ways in which Ptolemy has violated Aristotelian imperatives. Is Earth at the center now? Of what? The outer planets and the Sun no longer orbit around it symmetrically. They also don't orbit at constant speeds except now around a point in space, not around the Earth. It's torturously pieced together in ways that Aristotle could never have imagined—and that a modern physicist would not have tolerated. "Simplicity" is nice in physical models, not guaranteed, but when your model is so bizarre you'd tend to think that it's trying to tell you that the world is probably not that way. But this is the first time. Going from pictures and stories to numerical prediction surely meant that when predictions worked, then it must be some part of the truth. The late 16th century's Johannes Kepler is from whom we learn the real solar system model and we'll have to wait 1400 years to Chapter 6 for him to appear and save the day.

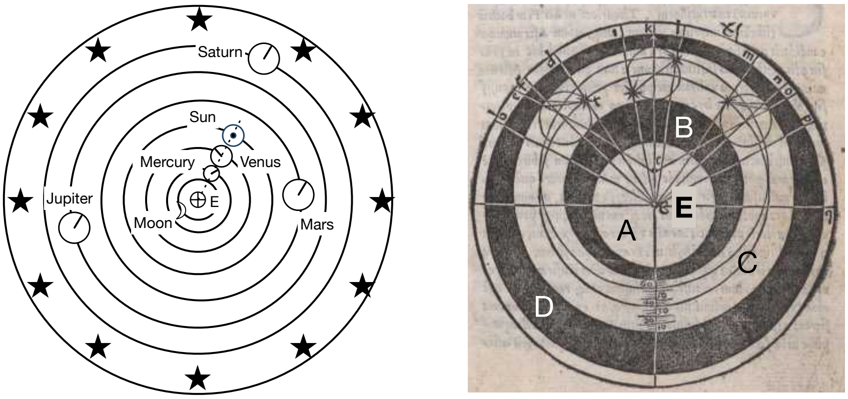
Not always appreciated, was the fact that in *Almagest*, the outer planet's deferents were all taken to be the same radius and that the distances were all set by the epicycle's individual radii. He chose 60 "units" (always working within the Babylonian base-60 sexagesimal system we use today for time and angles) for that common deferent radius with the Mars:Jupiter:Saturn epicycle radii in proportions of approximately 7:2:1. This was because the planetary models in *Almagest* were not a system. Much like Eudoxus before him, he treated each planet separately and made no attempt to merge them, until much later in his life.

3.4.2 Ptolemy's Cosmology.

Just as it was important for Aristotle to build a multi-planet system out of Eudoxus' separate planets, it obviously seemed incomplete to Ptolemy also. So he later wrote *Planetary Hypotheses* which upgraded some of his measurements but also presented a whole cosmology of all of the heavenly objects. Figure 3.20 (a) shows it in a simplified format with an abstraction of the epicycles for each planet: the line in each epicycle shows the relationship of the planet to the center of its epicycle. Notice that for the outer planets, the epicycles are constructed for that line-direction in each is parallel to one another and parallel to a line connecting Earth to the Sun. For the inner planets, it's the *centers* of their epicycles that all lie on that parallel line connecting the Earth to the Sun. **The Sun drives the whole machinery and the inner planets and outer planets have different models and constraints. But those clues weren't enough to resurrect the Aristarchus model with the Sun at the center. Such was still the strong pull of Aristotle's prejudices.**

Recall in Section 3.2.6.2, I noted that that the classical planet ordering was Plato's and Aristotle's: Earth–Moon–Sun–Mercury–Venus–Mars–Jupiter–Saturn and the stars. Ptolemy made the executive decision to change that to Earth–Moon–Mercury–Venus–Sun–Mars–Jupiter–Saturn and because of his authority, it stuck. (Again, notice that the Sun sits between (our) inner and outer planets. Interestingly, whenever a Medieval or Renaissance rendering of Aristotle's cosmos was presented in books it was Ptolemy's not Aristotle's ordering that was used. Sometimes Ptolemy's name is included on an image, even though the picture might be Aristotle's equal-orbit, totally geocentric geometry. Ptolemy's and Aristotle's pictures get mixed up during Medieval and Renaissance depictions.

Planetary Hypotheses also presented a physical model for his cosmology. In it, there are solid aether spheres which carry the epicycles through...pathways in the solid



(a) (b)

Figure 3.20: The whole cosmology of Ptolemy. In (a) the planets, and Sun are arranged in a very particular way relative to the Sun. The lines in the circles for each planet represent the center of epicycle to the planet. For the outer planets, they are *all parallel to one another and parallel to the line that connects the Earth to the centers of the inner planets, to the Sun.* The Sun is always key. In (b) an image from *Theoricae novae planetarum* by Georg Peurbach is shown which represents a slice through the Medieval idea of Ptolemy's 3-dimensional model for one planet. Notice the epicycle in various positions inside of the region labeled C. The other labels are described in the text. (Wikipedia, Georg Peurbach)

aether around the Earth. Here's a poor image of his model. Some hands-on-learning. Take your left hand and form a fist, and then back off a bit so that inside of your hand is a hollow sphere of nothing. This "nothing" is like region A in Figure 3.20 (b), while your hand—with its thickness—is like region B. Now take your right hand an wrap it around your left and then back away a bit so that there's space between the hands. That space is like region C in the image and your (thick) right hand is like region D.

Think of these four regions as if they are perfect spheres around that inner empty volume. This is close to Ptolemy's 3-dimensional model, but for one more step, which is hard to do with your hands. Region A is centered on the inner sphere of "nothing," at the center of which is the Earth, E. But the volumes of regions B, C, and D are then offset from no longer being centered on E. They're centered on the Deferent which in Figure 3.20 (b) is just a bit up from E.

With this introduction hopefully you can see what it looks like in Ptolemy's head in Figure 3.21 (a). (A very readable account is in **?**). Just like in Figure 3.20 (b): what we have is a slice through a three-dimensional, collections of spheres. Like before D is the right hand, B is the left hand, and C is that opening between them, but shifted to be centered on the deferent, D. A is that cavity in the center of both hands and the Earth is in the center. The Celestial Equator (CE) is shown, and the ecliptic plane is also, which is where the planets orbit. A planetary epicycle is labeled P shown twice: the dark one of the pair is the epicycle coming out of the page within that

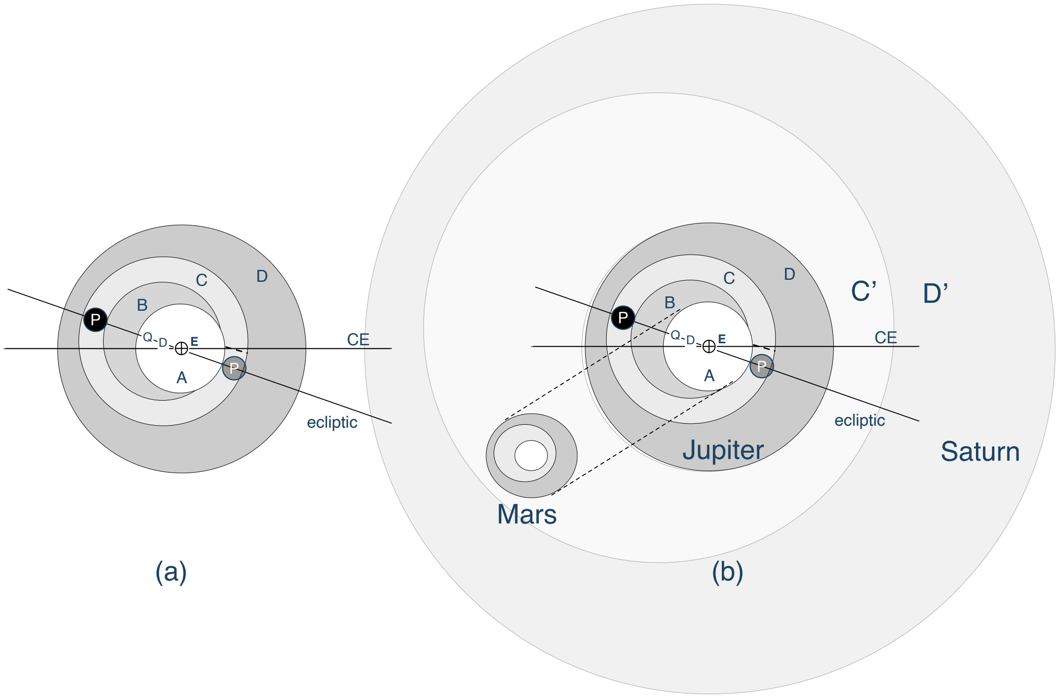


Figure 3.21: In (a), the 3-dimensional construct for Jupiter is shown and described in the text. In (b), an insert for Mars and Saturn are overlaid.

spherical cavity C. The light version shows P returning to the plane of the page and going back into the page...again, within the cavity C.

That inner cavity, A is important and Figure 3.21 (b) puts it together. Let's say that the first planet in (a) is Jupiter, then what goes inside of that originally, empty sphere A, is a similar structure for Mars with its own A, B, C, D...but it would neatly fill Jupiter's A void, filling it up. Around the outside, in (b), one can see the Saturn contraption in which our original Jupiter balls fit with its own A′,B′ (not shown, C′, and D′). Of course inside of the Mars inner cavity would be the Sun's structure...and so on, successively for Venus and Mercury, all the way to the Moon.

These spherical constructs for each planet are intentionally fitted together like a precision-made Russian doll. He imagined that the largest excursion of, say, Mercury's orbit in its epicycle, constrained inside of Mercury's C cavity, would just match the smallest excursion of Venus' orbit in its epicycle, within its C cavity. Then the largest excursion of Venus' orbit would just match the inner excursion of the Sun's and so on. He packed them together with minimal spacers of aether (D and B in Figure 3.21 (a).

He demanded uniform motion of the spheres, but the shifting of their centers is a problem. Imagine a soccer ball spinning around an axis at a uniform rate. Can it spin around another axis parallel to the first one at a uniform rate? No! It's physically impossible and this truly offended many Muslim astronomers and mathematicians who attacked his physical model in no uncertain terms.

While his planetary orbits were independent of one another, their relative orbital sizes could be calculated as each is determined by the tight-fit. So if you knew the size of one of them, you could then establish the size of others, working your way from edge to edge of each "spherical space-shell."

He knew the distance from the Earth to the Moon (from studies like that of Aristarchus) and the Earth to the Sun and in this way he actually calculated the distance from Earth *to each planet and to the stars themselves*! For example he calculated that the maximum distance from the Earth to Venus was 1079 Earth radii. (Today, we know that the maximum Earth-Venus distance, across the Sun pretending that they are as far away from one another as possible is more like 25,000 Earth radii.) For fun, he predicted that the distance from the Earth to the Stars—*the size of the entire universe*—would be $20,000 \times E_R$, or 126,000 km. Both an astonishing feat—-calculating the size of the entire universe—and wildly wrong. His universe's size is smaller than the actual furthest separation of Earth and Venus in our world.

3.4.3 Summary of the Astronomy of Ptolemy

(Set the context with the timeline in Figure 1.2 on page 8.)

- Ptolemy (85 to 165):
  - He wrote the mamoth book, *Mathematical Composition*, nicknamed by Islamic astronomers as *Almagest*, which became its label to this day (it's in the dictionary of your word processor). It was the definitive tool for predicting the positions of all of the heavenly bodies. The naive Copernican heliocentric model is mathematically identical to the epicyclic model of Ptolemy. No better, no worse than Ptolemy's.
  - He created a star catalog of more than a 1000 stars, including a subjective measure of each's brightness.
  - He continued Hipparchus' solar model with a separate, and corroborating measurement of the eccentric.
  - He adopted the epicycle model of Apollonius and found ways to assign measured parameters to the epicycle variables: the deferent radii he took as constant and found epicycle speeds of rotation, radius, and orbital speeds on the deferents, separately for each planet.
  - He wrote a "handbook" (*Handy Tables*) that would teach an astronomer, physician, or astrologer how to predict the positions of planets using his model, without having to absorb the considerable mathematics of *Amalgest*.
  - He later wrote a complete cosmology that attempted to put all of the planets, epicycles and all, into one nested cosmological model. This allowed him to make predictions about the sizes of orbits.

3.4.4 The End of Greek Astronomy

Think about the conceptual leap that we've taken: we've gone from Aristotle who told picture-stories about the universe to Ptolemy who quantitatively modeled the entire universe! He used measurable parameters that located all of the heavenly bodies, predicted their motions, and proposed numerical distances to every object including the size of the entire universe. It's an astonishing feat and nobody successfully challenged it for 1400 years (although there were many attempts by the Muslim astronomy and mathematics community) which is a pretty good record.

He was the last Greek astronomer. Science would explore new frontiers, but the Greeks would no longer be the explorers. Rather western research[20] in MOTION BY THE EARTH and MOTION IN THE HEAVENS shifted to India and among the Muslim scholars who did some original work, and translated, preserved, and commented on Greek writings—especially Ptolemy.

3.4.5 One More Thing?

This was an unusual set of chapters and what follows will be considerably less sweeping and more focused. But the scene is now set for the full story of MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS. Here's a fascinating coda to our Ptolemy story. He was so close!

Imagine a very simple auto race with two cars. The track consists of two lanes, both circular around a common center. One lane, in which car M stays has a larger radius than the other lane in which car E is constrained, So it's not a fair race, since M has further to go in a revolution than E. But, this is an analogy.

From the stands you can watch the two cars go in their counterclockwise circuit and here not only does E have an advantage as the inside lane, but E is also faster than M. So naturally, *it will periodically lap and pass* M. When that happens, to the driver in E it looks like M is in front...and then seems to E to go backwards as it's lapped!

By now you realize that in this race analogy I can substitute E for Earth, M for Mars, and S for Sun and we've just described a simple solar system of two planets viewed from two different perspectives (the people watching the race, and E). It should be, and is, possible to construct an algorithm (involving vectors) to translate the motions from one frame to the other. The spectator's view corresponds to a solar system of the sort that you have learned that Copernicus described: all of the planets orbiting the Sun in perfect circles and the other, is the solar system that Ptolemy discovered in which the Earth is stationary and the Sun and planets orbit it...but on epicycles.

The Ptolemaic model is mathematically identical to the Copernican model in which the orbit of an outer planet (like Mars) has the same dimension as the deferent circle of the Ptolemaic model.

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[20] There was a parallel research path in China, but it didn't influence the eventual progress Europe

What Ptolemy accomplished was an extraordinary mathematical feat. In fact, it's much more complicated than our modern view! He took a long, intellectual journey to his model whereas if he'd taken Aristarchus' model with the Sun in the center and circular orbits of the planets...he would have had a much simpler task. But what was in his way?

It was Ptolemy's commitment to the Aristotelian edict that the MOTION BY THE EARTH is zero, wrongly supported by a misunderstanding of the physics of MOTION ON THE EARTH *that was in the way of creating the better model*. Unraveling this is the task of this book: getting, first, the MOTION ON THE EARTH right and then applying it to MOTION BY THE EARTH and MOTION IN THE HEAVENS. It didn't come easy.

3.5 Greek Astronomy, Today

3.5.1 Let's Set The Record Straight: How we now understand the sky

From our more advanced vantage point: every one of the above points in Section 3.2.1 is explained overall by a Sun-centered solar system (with some nuance) around which the Earth and other planets orbit.

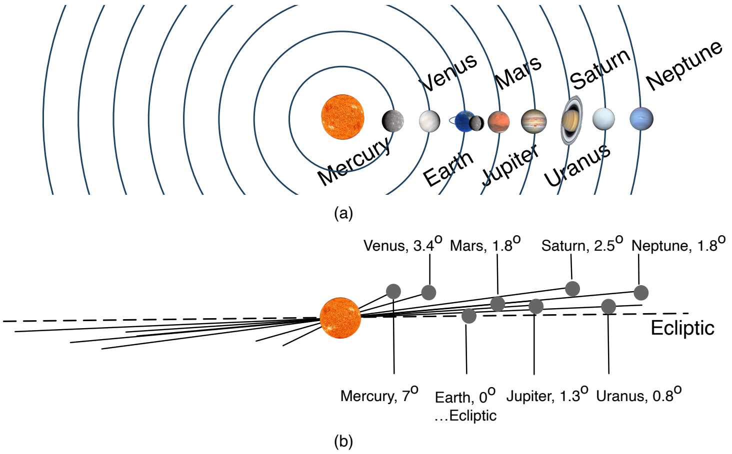

Figure 3.22: (a) is an abstract sketch of the solar system as we picture it today and and which we credit to Copernicus. "Abstract" because the alignment of the planets is for display purposes, actual relative radii of the orbits are not anything like shown, and the orbits are elliptical, not circular. (b) shows what the relative orbital planes are for each planet. The planets all have orbital planes inclined slightly to the overall ecliptic (the dashed horizontal line is the edge of the ecliptic plane). Notice that Mercury's is the one with the highest inclination of 7°. Pluto's is almost 17° up and down, indicative of its not belonging in the club of solar system planets.

Elliptical orbits. We know that our solar system is built of eight planets (Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune). Figure 3.22 (a) is familiar to all schoolchildren today. We know that their orbits are not circular, but slightly elliptical, with the Sun at a focal point and as such, when they are close to the Sun, they whip around it fast and when the are far from the Sun their motion is slower. They are nearly all in the same plane, which is shown in Figure 3.22 (b) where we take Earth's orbital plane to define the ecliptic (0°) so relative to that, Mercury's orbit is the most inclined at ±7° from the ecliptic. All of the other planets' orbits are within that 14° band. For those of you mourning the elimination of Pluto from the planetary family, it's inclination to the ecliptic is more like ±17°, as are other dwarf planets in the outer edges of the solar system. The undisputed opinion now is that Pluto's existence is due to some event that is not of the same origin of the other planets. Hence, it's being voted off of the planetary island.

(a) (b)

Figure 3.23: The inclination of the Earth's spinning is oriented away from being perpendicular to the ecliptic in which the Earth's orbit is fixed. Also, the orbital plane of the Moon's orbit around the Earth is slightly inclined relative to the ecliptic as well.

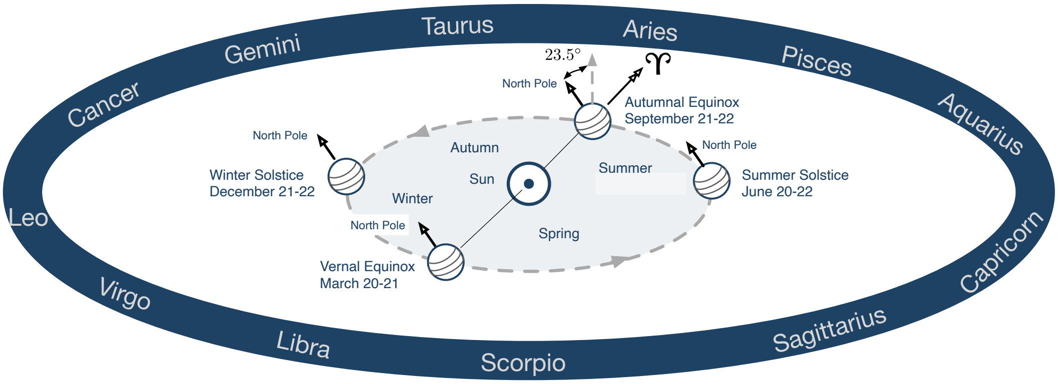

Figure 3.24: There's a lot in this image. The Sun (☉) is at the center and ecliptic is shown as the gray circle around which the Earth orbits. The 23.5° inclination is pictured showing how the solstices are inclined in our northern hemisphere's summer and winter. The Vernal Equinox (♈) is pointing at the zodiacal constellation of Aries, as it was in ancient times (today, it's in Pisces).

Figure 3.23 (a) shows a line-up of planets (in simulation) as they appeared in the eastern sky on June 24, 2022 just before dawn from East Lansing, Michigan. Notice that the Sun is just peeking over the horizon and Mercury, Venus, the Moon, Mars, Jupiter, and Saturn are all nearly in a line along the ecliptic. Figure 3.23 (b) shows that the Moon's orbit is inclined to the ecliptic by about 5° which is why we don't see lunar and solar eclipses every month. (Hipparchus determined this angle.)

The Earth is tilted by that seemingly random 23.5° that figured so prominently in the stories above and in Figure 3.24 the Earth is shown at the four seasonal points of the two equinoxes and the two solstices. The shaded circle is inscribed by the ecliptic and is the plane with all of the planets, including Earth. Notice that the Earth is titled by that 23.5° as measured from the plane of the ecliptic and that its direction does not move throughout the year and points to the Celestial Pole. The Vernal Equinox is shown when the Sun is within the Aries constellation (as in anquity).

Now we can understand both cause of the seasons and why they are of different durations and Figure 3.24 tells the whole story. When the Earth's orbit is closest to the Sun, it's moving the fastest in its elliptical orbit, so it spends less time between the two equinoxes, here on the left side of its orbit. Notice that the tilt of the Earth's axis is away from the Sun, and so the full-force of the Sun's rays are directed, not to the northern hemisphere, but the southern. In fact, at the Tropic of Capricorn at a latitude of 23.5° South, the Sun would be overhead at the winter solstice. So less radiation intensity falling on the northern hemisphere, means it's cooler. So yes, the winter happens when the Earth is nearest to the Sun. On the other side, at the summer solstice, the Sun's rays are intense on the northern hemisphere as the Earth's tilt is now towards it and the Sun is overhead at noon on the summer solstice at the latitude of the Tropic of Cancer—where Syene is located at 23.5$\circ$ North.

Spinning Earth. The Earth has two motions, as do all of the planets. It orbits the Sun in a nearly circular path in a counterclockwise sense when viewed from above the Sun's north pole. The Earth also spins on its own axis, also in a counterclockwise sense.[21] That the Earth spins on its axis explains the apparent motion of the Sun through our sky from E-W each day. The speed of the surface of the Earth due to its spinning is about 460 m/s (about 1000 mph) while the speed of the Earth's track along its orbit is 220 km/s (about 490,000 mph). We don't feel this motion since it is constant and we're held to the surface by the Earth's gravity. The same thing is true for the air and so we don't feel a wind as if the Earth were moving out from under the atmosphere.

Planets' orbits. The strange retrograde motion is easily explained in the heliocentric system. Earth and Mars, for example, have different "years" as they go around the Sun. Sometimes the Earth will lap Mars and leave it behind. That's the story and Figure 3.25 explains it. In (a), we see a time-lapse photograph of Mars in successive nights from December to August. Clearly Mars appears to "move" against the stars.

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[21] only Venus among the planets spins in a clockwise sense while Uranus has a spin axis which is on its side, relative to the others. One explanation is that, like the Moon was created through some billions of years ago collision with the Earth, so to something massive might have struck the adolescent Venus and Uranus. Multiple hypotheses exist.

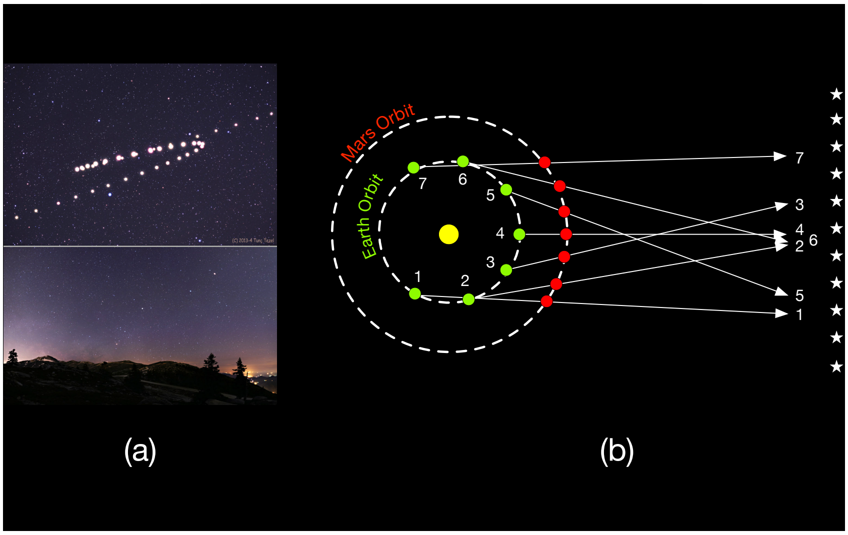

Figure 3.25: Retrograde motion by Mars. In (a) the sky in Turkey shows a photograph of Mars from December 5, 2013 in the upper right hand corner and then an overlayed photograph taken every five or six nights until August 8, 2014. The looping behavior in the middle is the retrograde motion. (b) shows how this happens (see the text for an explanation) https://twanight.org/gallery/tracing-the-red-planet/?preview=true

(b) shows how. Each

3.5.2 Hipparchus and Modern Celestial Coordinate Systems

(**?**) correctly argues that the coordinate system that Hipparchus seems to have originated and Ptolemy perpetuated is essentially identical to what is used today in astronomy, called the "equatorial system." Figure 3.26 (a) shows the situation. What Hipparchus did was measure the angle of a star relative to the North Celestial Pole and an angle along the ecliptic. If you look at Figure 3.24 you'll see that the Earth is surrounded by the 12 constellations of the zodiac. The Greeks (and Babylonians) divided the whole circular pattern into 12 signs, each of 30° each and his coordinate system referred to the constellation and then the number of degrees within that constellation. This is like the longitude on the Earth's surface—degrees around. The "zero" of this coordinate system is located at the position of the Vernal Equinox, which recall is where the Sun on the ecliptic crosses the Celestial Equator during the spring. The Sun was in the constellation Aries during these times (which is why the symbol for the Vernal Equinox is ♈, which is the symbol for that constellation. Today, the VE has moved to the constellation Pisces precisely because of the precision phenomenon that Hipparchus discovered.[22] (More about the Vernal Equinox below.) So in the *Commentary*, he wrote about the constellation Bootes (not among the 12 zodiac members):

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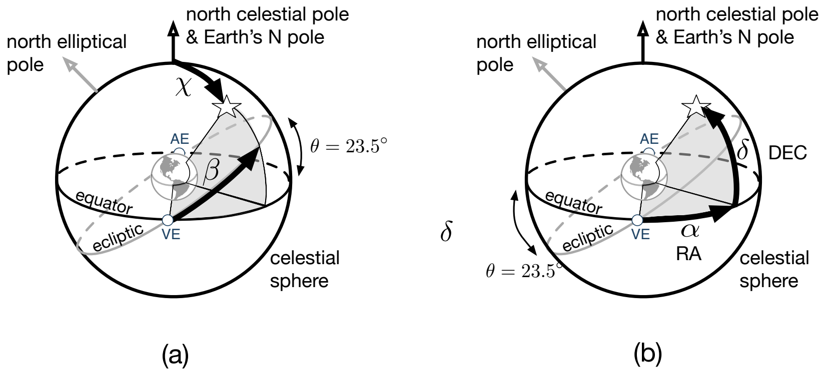
[22] The "Age of Aquarius" is next, as precession continues.

> "Bootes rises together with the zodiac from the beginning of the Maiden to the 27th degree of the Maiden... Hipparchus, "



(a) (b)

Figure 3.26: The Celestial Sphere is shown in both diagrams for two different coordinate systems that can be used to locate a star on the Sphere. In (a) the "longitudinal" coordinate (β) is along the ecliptic starting from the position of the Vernal Equinox along the ecliptic and the "latitude" coordinate (χ) is measured from the Celestial Pole to the star along a great circle. In (b) the longitude (α) is along the Celestial Equator from the Vernal Equinox (and so identical in angle to β) and the latitude is measured up from the Celestial Equator (δ). The coordinate system in (a) is called the Ecliptic Coordinate System and (b), the Equatorial Coordinate System. (b) is the standard modern system for star charts in which δ is called "declination" and α is called "Right Ascension" (and is recorded in modern tables in units of time, rather than angle where 24 hours equals 360°). A modern version of the Ecliptic Coordinate System uses $\lambda = 66.5^\circ - \chi$, but I represented it here from the pole because Ptolemy measured χ for "latitude." Hipparchus seems to have used both of these systems while Ptolemy used (a).

The "Maiden" is Virgo which is the 6th constellation ("sign") around from Aries (Figure 3.24). So the angle, α in the figure where the constellation Bootes rises is $(6-1) \times 30^\circ + 27^\circ = 177^\circ$.[23] A modern version of Bootes extends 202° to 237°, so it doesn't appear to match? Ah, but the precession of the equinoxes is worth $1^\circ/72$ years, so we need to add that factor times the number of years since Hipparchus recorded his measurement 2153 years ago—that's an additional 30° which makes that edge be 207°: Hipparchus is just right.

For the other coordinate, he measured from the North Celestial Pole *down to the object* of interest, χ in the figure. That's the "polar angle" and is the opposite of our Earth-faced latitude, which measures up from the equator.

The modern equatorial system uses the same idea. For the polar angle, a star or object's "latitude" coordinate is measured *up from the Celestial Equator*. This is called the "Declination, δ." So it's identical through a difference of 90°:

$$\chi = 90 - \delta.$$

This north-south polar angle measure is called "co-declination."

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[23] Because Aries the first sign starts at 0°, so the 6th sign starts with 150°

The modern longitude, called the Right Ascension, α, is measured also from the location of the Vernal Equinox, but typically recorded as a time, rather than an angle. This is natural, since the whole Celestial Sphere rotates 360° in 24 hours. So while the edge of Bootes is 202° for Hipparchus' units, it's $13^{\mathrm{h}}36.1^{\mathrm{m}}$.

About the Vernal Equinox. I don't believe that there's any record of just how Hipparchus could have determined the location of the VE in the zodiac. After all, the Vernal Equinox for the Greeks was determined at noon on that day when the Sun is precisely between its altitude at the two solstices, and equivalently, when it rises and sets precisely in the east and the west. His accuracy was about 1/4 of a day for observations and I can think of two ways he might have done this.

He would surely already know roughly when the equinox was to happen and would start measuring the Sun's location, rise, and set for days before and days after the expected event. Then, later he could figure out precisely which day. But along with his altitude measurements, he might look at the east just before the Sun rises each of those days and precisely located which constellations were still visible before it becomes bright. Likewise, he would look just after sundown to see what constellations would be "coming out" as it gets dark.

He could also have noted when the equinox occurred, waited exactly 12 hours and then looked to see which constellation would be at the altitude of the Sun at noon.

In both of these, he would presumably conclude that it was Aries and the "First Point of Aries" became the nickname for where the Vernal Equinox is in the sky.

3.5.3 New Evidence for Hipparchus' Lost Star Catalog

When we're talking about millennia, "breaking news" needn't be "yesterday." So there is remarkable Breaking News when it comes to Hipparchus' star catalog. Parts of it might have been found.

In 2012 Jamie Klair, an undergraduate at the University of Cambridge was studying a multi-spectrum image of folio pages of an ancient Greek palimpsest[24] known as the *Codex Climaci Rescriptus* at St Catherine's Monastery on the Sinai Peninsula (now in Museum of the Bible's collection in Washington, D.C.). It was a summer project assigned by biblical historian at the University of Cambridge, Peter Williams, who continued the work and in 2017 he and French collaborators confirmed the observation and found more of it. They recently published it in (**?**). In that image an under-text is slightly visible which he realized appeared to contain astronomical notations—actually a quotation from Eratosthenes. It appears that the original writings were erased in the 9th or 10th century and overwritten. But the multispectral imaging brings out the original impressions on 9 of the 146 pages.

By digitally bringing out the faint background writing, it's apparently astronomical

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[24] a document that has been reused by scrubbing out the original content

data, coordinates, actually. Almost certainly from Hipparchus' observations. For example, one of the decoded and translated phrases in the hidden text is:

> Corona Borealis, lying in the northern hemisphere, in length spans 9°1/4 from the first degree of Scorpius to 10°1/4 in the same zodiacal sign (i.e. in Scorpius). In breadth it spans 6°3/4 from 49° from the North Pole to 55°3/4.

They noted that "length" is the east-west measure and "breadth" is the north-south measure. The north-south measure is as above, the co-declination and the east-west measure is again the Right Ascension, in angular units. Scorpio is the 8th constellation, so from the previous section, that's $7 \times 30^\circ + 1 = 211^\circ$. Adding the 30° for precession since then would give a RA today of 240°. The edge of Corona Borealis is almost exactly that.

The stars in the 9 pages refer mostly to Ursa Major, Ursa Minor and Draco and the values are essentially those in Hipparchus' *Commentary*. The general consensus is that this is the first concrete evidence for the long-lost Star Catalog of Hipparchus!

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