Counting the Unmoved Movers: Astronomy and Explanation in Aristotle's *Metaphysics* XII.8

by Jonathan B. Beere (Princeton)

Abstract: I discuss Aristotle's use of astronomy in *Metaphysics* XII.8 to determine the number of divine intellects. Commentators have been perplexed by the astronomical system that Aristotle gives, because it involves mathematically superfluous spheres. I argue that this astronomical system is not merely a mathematical description of phenomena, but a causal account of the motions of the heavens. The idle spheres thus play an essential role in the system, because they are the proper cause of the diurnal revolution of the planets around the earth. I argue that this demand for explanation is neither immoderate nor unreasonable.

In Aristotle's *Metaphysics*, we are repeatedly promised a discussion of non-sensible immaterial substance. Yet only in Book XII does Aristotle fulfill this promise. His account begins in chapter 6 with an argument for the existence of at least one non-sensible immaterial substance, and continues in chapter 7 with a series of conclusions about the nature of such substances: They are purely active, immortal intellects, and substances of this kind are the ultimate principles of the world. Chapter 8 then describes how to determine the number of ultimate principles. There are as many ultimate principles as there are pure intellects, and as many pure intellects as there are heavenly motions. And there is already a science to tell us how many heavenly motions there are: astronomy.

Why does Aristotle care about the number of unmoved movers? Some suggest that the chapter is "gratuitous polemic" against the Platonists.[1] In fact, it is an integral part of the promised theory of non-sensible substance. Any account of the ultimate principles of being should include some reasoned method of determining how many principles there are. Similarly, in *Physics* I, Aristotle canvasses various answers to

[1] Cf. Lloyd 2000, 253. Lloyd provides quite a thorough discussion of chapter 8. See also Michael Frede's introduction to the same volume for a thorough discussion of the twelfth book as a whole and of chapter 8's role in it.

Archiv f. Gesch. d. Philosophie 85. Bd., S. 1–20

ISSN 0003-9101

the question of how many principles there are (although those are not the principles of all being, but only of changing things). This question does not spring from an arbitrary zeal for counting unmoved eternal substances[2], but from the general project of giving an account of the principles of being. This constraint would apply equally to an account according to which the ultimate principles of being are material substances, such as air, earth, fire, and water; that account has to fend off such questions as, 'Why not wine?' and 'Why include air, earth, and fire?' Any metaphysical theory is defective if it cannot answer these questions and others like them. And conversely, to show how to determine the number of principles with precision is a great virtue of a metaphysical theory, even if the number of principles remains unknown because of the limitations of our natural science.

In chapter 8 of *Metaphysics* XII, Aristotle wants to show that, given the best contemporary astronomical theories, the number of pure intellects can reasonably be accepted to be 55. But his primary goal is not that we accept that there are 55 pure intellects, but rather that we accept that, as far as his account of the ultimate principles of being is concerned, nothing is lacking for such a demonstration. As soon as the astronomy is in place, an answer to the question, 'How many principles are there?' is determined and readily available. Aristotle claims, very reasonably, that this is a strength of his theory, and he further claims that it is a weakness of 'the supposition of ideas' that it cannot provide a non-arbitrary criterion for the number of principles.[3] Aristotle's metaphysical theory is in a state comparable to a theory which estab-

---

[2] Cf. Lloyd 2000, 253.

[3] Whether this critical remark about the ideas is fair is not a question I will address. There is a great deal to say about whether or not it is arbitrary for the decade to be the principle or principles. My view of the criticism of the supposition of ideas differs from either of the views considered by Lloyd, who writes, following David Charles, "Aristotle's complaint might be [1] that the Platonists did not have demonstration as their goal: or [2] that they did not take themselves to be subject to proper scientific constraints in the first place" (2001, 253). If the former, the implicit contrast with Aristotle himself would have Aristotle taking demonstration as a goal. But Aristotle not only has demonstration as a goal; he can show us in detail how the demonstration would go, if we knew enough astronomy. There is nothing lacking from his account of the principles of being; what is lacking, rather, is some astronomy. He advances this as a strength of his theory. I am uncertain whether this is the same as the second option considered by Lloyd, but I am inclined to think not. On my view, Aristotle is not merely bound by the constraints of science in general; he has worked out in detail an account of the principles of being that makes immediately clear how to determine, in a feasible way, how many there are.

lishes that the elements to be listed on the periodic table of elements are the principles of being, but does so in the absence of a complete and definitive periodic table. In Aristotle's case, it is not chemistry but astronomy that provides the non-metaphysical information, and astronomy is said here to be "most proper to philosophy of the mathematical sciences" (1073b4f.). The main part of chapter 8, devoted to enumerating the postulated spheres, is thereby characterized as belonging to a branch of mathematics.

For this reason, it is puzzling to find certain spheres in the system which are mathematically superfluous. A system that differed from Aristotle's only by lacking these spheres would make precisely the same predictions about all heavenly phenomena. Two questions arise about those 'idle' spheres. (1) Since each sphere moves the one below it, why do the 'idle' spheres need their own movers? (2) Since they are mathematically superfluous, why do they have any role in this ostensibly mathematical system? My argument aims to answer these questions, which continue to puzzle commentators such as Heath, who writes, "Aristotle could […] have dispensed with the [redundant spheres] […] without detriment to the working of his system […] [and thereby] have saved six spheres out of his total number."[4]

We will respond to the first question by describing the *way* each sphere carries the one below it, and we will answer the second by recognizing that Aristotle is guided not only by mathematical considerations, but also by considerations about what constitutes a *per se* cause. Since

[4] See Heath 1913, 219. Mendell agrees: "Aristotle seems to over count the first sphere for every planet" (2001, 82; see generally 81–83). Yavetz also expresses perplexity about the issue (1998, 237 n. 16). Ross (1924, *ad loc.*) says, "Aristotle might have reduced the total number of spheres by six"; similarly, Neugebauer (1975, 685). Frede raises this question, too (2000, 38). Dicks attempts to rebut Ross, saying "But the poles of B [corresponding, in our discussion, to the last of Saturn's unwinding spheres] are not the poles of the sphere of the fixed stars […], whereas it is an essential part of the system that the first sphere of each set must represent the latter exactly; hence the 2 spheres *cannot* be replaced by 1. That is why Aristotle himself emphasizes that the purpose of his counteracting spheres is to 'restore to the same function *as regards position*' the first sphere of the following planetary set" (1970, 202). But why is this an "essential part of the system"? What sort of a system is this, such that the positions of the spheres' axes should be so important? Even granting Dicks' questionable gloss of 1074a3 – the Greek mentions neither the function nor the position of an axis – the deeper question is in what sense Aristotle thought the orientation of the axis matters, since it does not matter mathematically. According to my account below, it is not the orientation of axes that concerns Aristotle, but the importance of there being a *per se* cause for the diurnal revolution of the planets.

Aristotle does draw on non-mathematical assumptions, we will have to revise, or at least qualify, our statement that chapter 8 embodies a mathematical, as opposed to natural, science.

Let us sketch Aristotle's spherical system. Some aspects of the system are clear from Aristotle's text, while other aspects have been reconstructed based on later reports, especially Simplicius' commentary on *de Caelo*. The traditional reconstruction of Schiaparelli (1875) has been questioned recently, but my arguments are independent of this controversy. All accept that Aristotle begins with the theory of Eudoxus, along with its modification by Callippus, in which each complex heavenly motion is analyzed in terms of the motion of spheres. Eudoxus and Callippus analyzed the heavenly motions one at a time. One system of spheres describes the motion of the sun; a similar but separate system describes the motion of the moon; and so on with each of the five planets. In each system, the complex motion of a single heavenly body is represented as the composite motion of concentric spheres, each rotating equably around an axis and each (except, of course, the outermost) with the poles of its axis at rest relative to the surface of the preceding sphere.[5] The third and fourth spheres create a figure called by Simplicius a "horse-fetter" (ἱπποπέδη) which is a figure-eight.[6] The hippopede moves along the line of the ecliptic, while the planet moves along the hippopede.[7] The hippopede is a result of placing the fourth sphere

[5] It is not necessary that the spheres be conceived by Eudoxus and Callippus as progressively smaller; all may have the same radius. There is little evidence about whether Eudoxus and Callippus understood their scheme as a mere mathematical model or as (also) a physical model. I have tried to phrase this initial description so as not to beg the question. See below for further discussion. Wright (1973) discusses this question in connection with Eudoxus, Musgrave (1991) discusses it more generally from the pre-Socratics through Ptolemy.

[6] Yavetz questions even whether the sources require us to reconstruct a theory in terms of a hippopede. His argument hinges on an attack on the credibility of Simplicius. His view his rebutted by Mendell (2001). But Bowen (2001) makes a renewed attack on the reliability and informativeness of Simplicius. His paper begins with a discussion of general historiographical issues relevant to ancient astronomy.

[7] It had been thought until very recently that the hippopede was supposed to account for retrogression: When the planet's motion along the hippopede is in the same direction as the hippopede's motion along the ecliptic, the planet surges ahead; when the planet's motion along the hippopede is in the direction opposite to the hippopede's motion along the ecliptic, the planet stands still or retrogresses. The width of the hippopede determines how far above and below the ecliptic the planet wanders. This assumption has been challenged by a series of recent articles: Yavetz 1998, Mendell 1998 and 2001. See the previous note for some remarks on Yavetz. Mendell (1998) treats various cases in great detail, treating the slow and fast planets separately, and arguing that retrogradation might have been relevant to the slow planets (but that other phenomena too might be the relevant ones), but that retrogradation could not be relevant for the fast planets. These papers also contain citations of much other relevant second-

within the third, rotating (1) in the opposite direction to the third (2) at the same speed as the third (3) around a different axis from the third.[8] Henry Mendell has proved that "Any motion of two spheres may be decomposed into a motion of S_1 and a motion on a hippopede"[9]. The beauty of this theorem is that it shows us that the hippopede is not merely the figure that happens to be created in this spherical system. Rather, the hippopede is the key to understanding the composition of motions in any system of equably rotating, homocentric spheres.[10]

After sketching the Eudoxan systems and Callippus' modifications of them[11], Aristotle states his requirement that all the spheres for all the planets work together in one system. This requirement bears generally on the mathematical status of Eudoxus' and Callippus' work and Aris-

ary literature. A broader question, crucially important for the reconstruction of Eudoxus' and Callippus' systems, is whether retrogradation was known to early ancient astronomers at all. This is denied by Goldstein (1997), rebutted by Yavetz (1998, 225 n5) and Mendell (1998), and maintained again by Bowen (2001).

8 Demonstrations that this arrangement does produce a hippopede can be found in many of the relevant secondary texts. Of the demonstrations I have seen, by far the best, in my opinion, is that in Mendell 2001, 65ff. Mendell's exposition is entirely geometric, carefully avoids anachronism, and is supplemented by diagrams. He cites ancient texts that contain the relevant theorems. More detailed discussion of the hippopede can be found in Mendell 1998. Yavetz gives a reconstruction in terms of modern spherical coordinates in his Appendix B. Explanations can also be found in Heath (1913, 203 footnote) and Neugebauer (1975, 678).

9 Mendell 1998, 186.

10 Note, however, that this theorem does not in itself rebut Yavetz's view (for which see note 6), since the proof of the theorem relies on the very assumption that Yavetz questions, namely that the curve is traced by a point *on the equator* of the inmost sphere.

11 Callippus, Aristotle tells us, kept Eudoxus' spheres, but added several that he claimed were necessary "if one is going to account for [ἀποδώσειν] the phenomena" (1073b37). Aristotle delivers no explicit judgment or argument about the relative merit of Callippus' and Eudoxus' accounts. He seems ambivalent: He accepts Callippus' view for the planets, and countenances (but does not advocate) rejecting it for the sun and moon. This is taken by Lloyd as a sign of Aristotle's confusion: Aristotle "expresses his hesitancy in a context and in a manner that – if the reconstruction [of Eudoxus' and Callippus' theory] is sound – may suggest he is seriously out of his depth", writes Lloyd (2000, 261). Lloyd is in this paper reworking, in a more cautious vein, the argument he had presented in his 1996 paper. In the later paper, unlike the earlier, Lloyd accepts that it is far from clear whether the astronomy of Aristotle's day decisively favored Callippus' system over Eudoxus'. And it is, moreover, far from clear whether the traditional reconstruction of the Eudoxan-Callippan theory is sound (see note 7). Indeed, Neugebauer wrote that we should "admit our total ignorance of the character of Callippus' modification of the Eudoxan model" (1975, 684). Mendell tempers Neugebauer's claim, saying "although our ignorance may no longer be total, it is still quite profound" (1998, 256).

totle's use of it, but its immediate consequence is the introduction of 'unwinding' spheres to allow for a unified system:

> It is necessary, if the spheres *when put together* [*into one systematic whole*] [συντεθεῖσαι] are going to account for the phenomena, that for each of the planets there be additional [ἑτέρας] spheres, fewer by one [than the spheres in the isolated system], which reverse [the motions of those spheres] and always restore to the same placement [εἰς τὸ αὐτὸ τῇ θέσει] the first sphere of the star positioned below. (1073b38–74a4)

Before addressing the philosophical ramifications of the one-system requirement, we should understand its astronomical ramifications. What are these unwinding spheres and how do they make possible the integration of the various Eudoxan systems into a single Aristotelian one? The problem Aristotle faces is that Eudoxus' systems cannot simply be put together as they stand.[12] To see why, consider, for instance, the four spheres associated with Saturn; these are the four spheres most remote from the earth. The first corresponds to the sphere of the fixed stars, the second corresponds to the ecliptic, and the third and fourth, as a pair, create a hippopede with appropriate width. Suppose we add the next planet, Jupiter, simply by placing Jupiter's first sphere within Saturn's last, and then Jupiter's other spheres within that one. Jupiter itself would have a motion far more eccentric than any actual planet, because the positions and speeds of its second, third, and fourth spheres are calibrated on the assumption that its first sphere has the motion of the fixed stars. Jupiter's motion *relative to its own first sphere* would be unchanged, but its absolute motion (i.e., its motion relative to the earth) would no longer resemble its motion in the heavens, since Jupiter's first sphere would not move with an equable rotation, but rather with the motion imparted by the last of Saturn's spheres.

Aristotle solves this problem by interposing unwinding spheres between the two sets of Eudoxan spheres to cancel the motions of Saturn's 4 spheres. How many unwinding spheres are needed? If Saturn's four spheres are S_1 (fixed stars), S_2 (ecliptic), and S_3 and S_4 (hippopede), then the first unwinding sphere should undo the motion of S_4. How should this unwinding sphere move? Its *per se* motion should be a rotation around the same poles as S_4 with the same speed as the rotation of S_4, but in the opposite direction. Think of the motion of each sphere as the sum of its own rotation and the motion of the sphere above, and, for the case of the first unwinding sphere below Saturn, represent this as follows: *motion of the unwinding sphere* = *motion of S_4* + *rotation of the unwinding sphere*. In this formula, replace *motion of S_4* with its expansion according to the same principle. And replace *rotation of the unwinding sphere* with an alternative description, *cancellation of the rotation of S_4*. This yields, *motion of the unwinding sphere* = (*motion of S_3* + *rotation of S_4*) + *cancellation*

[12] This was pointed out already by Sosigenes *apud* Simplicius, *in libros de Caelo* II.12, The whole passage from 498.1 through 504.15 is relevant to the Aristotelian system, but on this problem in particular see 498.1 to 499.15, especially 499.1 ff.

of rotation of S_4 = motion of S_3. In short, since the first unwinding sphere cancels the motion of S_4, its resulting motion is that of sphere S_3.

Likewise, a second unwinding sphere, undoing the motion of S_3, has as resultant motion the motion of S_2, and a third, undoing the motion of S_2, has the resultant motion of S_1, i.e., the motion of the sphere of fixed stars. At this point, we can stop adding unwinding spheres and place Jupiter's Eudoxan spheres within the last of Saturn's unwinding spheres.[13] A corresponding system of unwinding spheres for Jupiter will make way for Mars's Eudoxan system, and so on. A complete table of the spheres can be found in the appendix.

A series of redundancies has entered the system with the unwinding spheres. The first of Jupiter's Eudoxan spheres appears redundant, since both it and the adjacent sphere, Saturn's last unwinding sphere, both move in the same way as the fixed stars. There are several points in the system at which this occurs: between Saturn and Jupiter, Jupiter and Mars, Mars and Venus, Venus and Mercury, Mercury and the Sun, and the Sun and the Moon. The number of spheres could be reduced by six without disrupting the mathematics of the system. It is all the more noteworthy that Aristotle neglects to mention the dispensability of these spheres, since he *does* mention that one might omit some of the Callippan spheres (1074a10–14).[14] Of course, the Callippan spheres and the 'idle' spheres are not on a par, since the Callippan spheres are putatively necessary to account for the phenomena, whereas the 'idle' spheres are not. But this makes it all the more noteworthy that he does not consider eliminating the 'idle' spheres. Aristotle is following the outstanding astronomers of his day, but not slavishly. He does not

---

[13] Mendell suggests that one might add yet another sphere (2001, 82). Aristotle, he says, "forgets to unwind the first sphere for every planet". He means that, although the third unwinding sphere (for Saturn) has the motion of the fixed stars, that sphere has itself unwound S_2, not S_1, which still needs unwinding. But why should S_1 be unwound? The mathematics of the system remains the same whether S_1 is unwound or not. Aristotle understands the mathematics well enough to know that no further unwinder is needed. The point of the unwinders is to prevent the various planetary systems from interfering with one another, and that has already been achieved without specifically unwinding S_1. This does not, of course, answer the question why the 'idle' spheres should be present, but the answer I will give below does not entail, or even suggest, that the S_1 needs an unwinder of its own.

[14] The alternative number given in the manuscripts, 47, appears not to be the correct number. If 55 is the correct number of spheres for Aristotle's version of the Callippan system, then, given the modifications he mentions, the alternative number should be 49. The number 47 is the *lectio difficilior* and it is in all the manuscripts cited in the standard apparatus. But I am inclined to conjecture, with Sosigenes *apud* Simplicius, that the text should read 49. There are, however, alternative explanations. See Ps.-Alexander and Ross 1924, *ad loc*.

simply take over the Eudoxan-Callippan systems by rote and therefore keep the 'idle' spheres. On the contrary, he must be strongly committed to them. Only if we cannot find any good reasons for including the 'idle' spheres should we concede that they are truly superfluous.

Even in antiquity, there was confusion about whether or not to include the 'idle' spheres. Simplicius praises Sosigenes for having understood that Aristotle intended these apparently superfluous spheres to be parts of the astronomical system:

> Next, one must realize that the eighth sphere [of the whole system] is the first sphere of Jupiter. Sosigenes understood rightly that the first sphere of Jupiter is *not* the last of the three unwinding spheres [= seventh sphere of the whole system] – which some people actually think, viz., that the last of the spheres that unwind the upper motions will be the first of those moving the star below, [e.g.,] that the seventh sphere and what we have called the eighth sphere, i.e., Jupiter's first sphere, are the same. [This must be wrong,] since they, in trying to save the number of unwinding spheres stated by Aristotle, turn out to count the same sphere twice. (502.20–27)[15]

Why then are the 'idle' spheres present?[16]

Aristotle's view looks even more perplexing when we consider that the spheres in question are not merely idle, but are downright problematic for his own project of counting the divine movers. Each 'idle' sphere has its own mover because each sphere requires a rotation about its axis, as well as the motion imparted by the sphere above. Yet each 'idle' sphere moves with exactly the same motion as the sphere above it, so that it would seem to have no need for an additional divine mover to rotate it. How then is Aristotle entitled to count movers for the 'idle' spheres?[17]

A more precisely imagined picture of the spheres will answer this question. The last of Saturn's unwinding spheres has a special feature, because its resultant motion, unlike that of most other spheres in the system, is an equable rotation. The special feature is that it has two sets of poles, the poles around which its unmoved mover rotates it and the poles around which its *resultant* rotation occurs. The latter set of poles

---

15 Μετὰ δὲ ταύτην ὀγδόην λοιπὸν νοητέον τὴν πρώτην τοῦ Διός, ὀρθῶς Σωσιγένους ἐπιστήσαντος, ὡς οὐκ ἔστιν ἡ τελευταία τῶν τριῶν ἀνελιττουσῶν πρώτη τῶν τοῦ Διός, ὅπερ τινές ᾠήθησαν, ὅτι ἡ τελευταία τῶν τὰς ἐπάνω φορὰς ἀνελιττουσῶν πρώτη ἔσται τῶν τὸν ὑποκάτω ἀστέρα φερουσῶν, ὡς εἶναι τὴν αὐτὴν ἑβδόμην τε καὶ ἥν ἡμεῖς φαμεν ὀγδόην πρώτην οὖσαν τῶν τοῦ Διός· τοῦτο γὰρ συμβαίνει αὐτοῖς δὶς τὴν αὐτὴν ἀριθμεῖν σῴζειν πειρωμένοις τὸν ἀριθμὸν τῶν ἀνελιττουσῶν τὸν ὑπὸ τοῦ Ἀριστοτέλους λεγόμενον.

16 See below, note 26, for Simplicius' answer.

17 Edward Hussey (private communication) drew my attention to this problem.

corresponds to the poles of the fixed stars.[18] The first set of poles is in motion; the second set of poles is at absolute rest (like the poles of the sphere of the fixed stars). The latter poles, at absolute rest, are the very points in which the poles of the *next* sphere, Jupiter's first, are fixed. Hence the upper sphere imparts no motion to the lower, which in turn needs its own unmoved mover in order to move at all. The same obtains for every 'idle' sphere. Hence each 'idle' sphere requires its own mover, without which it would be at absolute rest.

Indeed, quite generally, no sphere is *rotated* by any other. The spheres rotate not because of other spheres, but because of unmoved movers. In most cases, the upper sphere does impart some motion to the lower sphere, namely by causing its poles to revolve, but in the special cases of the 'idle' spheres, this does not occur.

It may be objected that my interpretation helps itself too easily to the counterfactual, *if the 'idle' spheres were not rotated by unmoved movers, they would be motionless.* This objection would emphasize that the heavenly spheres are mechanically interrelated physical bodies, and urge that any interpretation should accept the mechanical fact of friction. If friction plays a role, the counterfactual is falsified: the 'idle' spheres would *not* be motionless, even if they were not rotated by unmoved movers.

I offer three replies. First, it is far from clear that there is friction in the celestial realm, filled as it is with aether and topical matter. Because Aristotle believes that celestial substances are of a radically different nature from sublunary substances, the assumption that the celestial spheres are bodies does not entail that a complete description of their motion and its causes will mention friction. The evidence of *de Caelo* is equivocal (see book II, chapters 1, 4, and 7). Second, even granting that there is friction in the heavens, it is far from clear that the friction would produce an equable rotation. Thus the objection must make not the relatively modest claim that there is some friction in the heavens, but that this friction would produce precisely the correct rotation; otherwise, a special mover will be required for each sphere. Aristotle evidently does assume that the 'idle' spheres require movers, and so evidently assumes either that there is no friction or, at least, that such friction would not have the appropriate effect. Third, even if there were in the celestial realm friction with the appropriate effect, the 'idle' spheres need unmoved movers for the same reason that, I argue below, the 'idle' spheres are needed, namely that, without them, a phenomenon (in this case, the rotation of an 'idle' sphere) would lack a *per se* cause. The unmoved movers make it the case that the 'idle' spheres are moved *per se*, even if it is false that the 'idle' spheres would be at rest, if they were not moved by unmoved movers.

Another objection to my interpretation might lead in the opposite direction, alleging that I have overemphasized the mechanical aspects

---

[18] They correspond to the poles of the sphere of the fixed stars in the sense that the line joining the poles of the sphere of the fixed stars will pass through them.

of the spherical system, which should be understood as a purely mathematical model. It might be a purely mathematical model in (at least) two ways: by being an empirically adequate but literally false representation of the motions and their causes, or by making no causal claims of any kind.

If it is purely mathematical in the former sense, then the spheres do not exist, but are mathematical fictions employed to save the phenomena. But there is no reason to suppose that this is what Aristotle believed, it requires an instrumentalist view of geometry that Aristotle did not hold, and it is unclear what motivation an ancient astronomer would have for developing such a model.[19] While it is clear that the system does remarkably well at saving the phenomena, the system is not well-suited to making predictions, as is well-argued by Wright.[20] He points out that the Babylonian predictive method made for easier computations, since it relied simply on numerical astronomical coordinates and algorithms for extrapolating from them, while the Eudoxan scheme, under the constraint that the earth be located at the center of the concentric spheres all rotating *equably*, not only makes computation tremendously complex (as can be seen by anyone who tries to work out merely how the hippopede is created), but also seems to make it impossible to save some important phenomena, as was recognized quite early in antiquity.[21]

Even if the Eudoxan system is not a predictive tool, it may be claimed to be a mathematical model in the sense that it includes no causal claims or information; the mathematics requires only that each sphere have its poles at rest relative to the sphere above it, not that those poles be attached so that one sphere is literally carried by another. The theory is silent as to why the poles are at rest relative to the sphere in question.

---

[19] Ptolemy's system does not provide a counter example. His alleged motivation for thinking in terms of a mathematical model is that his system of spheres seems mechanically impossible. But even so, he may well have thought of the spheres postulated by the system as existing physical bodies. See Lloyd 1991 for a very helpful criticism of Duhem's view of ancient mathematical astronomy, from Plato to Proclus. He concludes, "Where it is perfectly fair to say that the Greeks distinguished, even contrasted, mathematics and physics, it is an exaggeration to claim they advocated a mathematical astronomy divorced from physics or sought to liberate astronomy from all the physical conditions imposed on it" (275).

[20] See Wright 1978. Mendell (1998, § 4) sketches an ingenious method of plotting points using the Eudoxan system, not by calculation, but by the use of fixed-length strings and model globes. Yavetz too discusses this issue (1998, 241ff.). It is quite possible that such a method was used by ancient astronomers.

[21] Why precisely the system of homocentric spheres was rejected is a very difficult question, discussed in detail in Mendell 2001.

But this interpretation too attributes to Aristotle a notion of mathematical model without a motivation. The surrounding context clearly indicates that Aristotle is concerned about causes, e.g., unmoved movers. Why is he silent about the causes of the fact that various spheres have motions other than the motion caused by their unmoved movers? Because he took for granted the obvious and modest conclusion that the poles of every sphere (except the first) are in fact fixed in the surface of the previous sphere.

Even more important, Aristotle's insistence on the unwinding spheres shows that the Eudoxan system (as Aristotle took it) does not stop short at the claim that the poles of inner spheres are at rest with respect to outer spheres, but specifies the cause of this relation, namely that the poles of every sphere are fixed in the surface of the preceding sphere. If Aristotle's Eudoxus had said only that the poles of inner spheres are at rest relative to the outer spheres, it is very hard to see why Aristotle would have invented the unwinding spheres, rather than taking the simpler route of rejecting the constraint that *every* sphere's poles be at rest relative to the preceding sphere. If the only motivation for this constraint were mathematical, i.e., to make the predictions correspond to the phenomena, then one would expect Aristotle to nest the systems without connecting them to one another. Jupiter's first sphere would then not be disturbed by Saturn's last sphere because there would be no mechanical link between them. The fact that Aristotle refuses to take this route shows that even the original Eudoxan system, or Aristotle's version of it, is not a mathematical as opposed to mechanical model. It contains claims about causal connections. We need not accept Heath's assertion that "Aristotle […] transformed the purely abstract and geometrical theory [of Eudoxus] into a mechanical system of spheres"[22]. Rather, I

[22] Heath 1913, 217. See also 225. This view has been accepted by later commentators. As Dicks writes, "Obvious difficulties arise if we enquire too closely into the actual *physical* connection of the spheres [in Eudoxus' theory]. For example, if the heavens really operated in this manner […] how did astronomers ever manage to make the observations that lay behind the original Eudoxan scheme […]?" (1970, 203; my italics) I think that this specific question is rather shallow (obviously the spheres are not visible), but the general concern is important. Behind it lies Dicks' understanding of "[Aristotle's] mechanistic view of the structure of the universe" (*ibid*.). Similarly, Ross writes, "Eudoxus and Callippus had offered a purely *geometrical* account of the planetary system; Aristotle aims at a *mechanical* account, and cannot isolate the system of one planet from that of the next" (1924, 391; my italics). There is no good evidence that the contrast between Eudoxus' astronomy and Aristotle's should be drawn in these terms. (The astronomy described in *Republic* VII is not good evidence about Eudoxan astronomy.)

suggest that Eudoxus' theory is the skeleton of a causal account, in the sense that it explains the motion of each sphere insofar as it is carried by the sphere above, but not the rotation of each sphere. Aristotle, preserving the basic structure of the skeleton, puts flesh on it and brings it to life, by providing causes for the rotations of the several spheres and by putting them all together into one system. The one system requirement does not run counter to Eudoxus' project, but rather extends it.

The concern with causes suggests a way to answer the second question raised at the beginning of this paper, the question why the 'idle' spheres are present at all. Since Saturn's last unwinding sphere has the same motion as the fixed stars, it has two axes, with different explanations for its motion around each. It has a 'proper' axis, around which its unmoved mover rotates it, thereby creating the motion which is essential to its role in the whole system and which cancels the motion of the sphere above. It also has an 'improper' axis, being the *last* of Saturn's unwinding spheres, so that its resultant motion, like its motion in its own right, is an equable rotation. If, among Jupiter's spheres, there is no sphere that *properly* (rather than incidentally) has the motion of the fixed stars, then Jupiter's motion will be said, on Aristotle's standards, to lack a proper cause, because nothing in the world will be responsible for Jupiter's daily motion around the earth. Since Jupiter is a planet, a wanderer detached from the fixed stars, an astronomical theory must account for the fact that, despite its detachment, Jupiter makes the same daily orbit as the fixed stars. And it must account for this fact not only in the sense of saving it as a phenomenon, but in the sense of giving a *per se* cause for it. It is in this sense that Aristotle's astronomy is not merely mathematical; it is required not only to save the phenomena, but to explain them in a richer sense, namely by way of *per se* causes.

Let us compare the explanations of Jupiter's motion with an 'idle' sphere and without it. If Jupiter's four Eudoxan spheres are labelled J_1, J_2, J_3, and J_4, so that J_1 is the 'idle' sphere, the explanation of Jupiter's motion would begin something like this:

> Jupiter moves as it does because the sphere on which it sits, i.e., J_4, moves as it does. But why does J_4 move as it does? (1) Because a divine mover rotates J_4 about its axis and (2) because sphere J_3, in which J_4 is situated, has precisely the motion it has.[23]

[23] I take the divine mover to be self-explanatory, not in need of further explanation; whereas (2) does need further explanation. "Self-explanatory" is to be understood in a very strong sense: they account for themselves, are responsible for themselves, as no other beings in the universe are. One might or might not be satisfied with this, but that is irrelevant to the character of Aristotle's theory.

Why does J_3 move as it does? (1) Because a divine mover rotates J_3 about its axis and (2) because sphere J_2, in which J_3 is situated, has precisely the motion it does have. But why does J_2 move as it does?

The explanation of J_2's motion will, like the other explanations, refer to (1) J_2's divine mover and (2) the motion of another sphere in which J_2 is situated. This other sphere must have the motion of the fixed stars, since Jupiter orbits the earth daily, but this constraint is compatible with this sphere's either being sphere J_1, the 'idle' sphere, or being the last of Saturn's unwinding spheres; both of these have the motion of the sphere of the fixed stars. To continue the story on the hypothesis that the 'idle' sphere, J_1, is absent:

J_2 moves as it does (1) because a divine mover rotates J_2 around its axis and (2) because *the last of Saturn's unwinding spheres* has the motion it has. The explanation of (2) will, of course, take the same form as the explanations already given, the motion for each sphere being partly explained by its own divine mover, partly by the motion of a higher sphere. The motion of the higher sphere will stand in need of further explanation until we reach the sphere of the outer heaven. In this way the motion of sphere J_2 is explained, and so is the motion of Jupiter.

Why then is Jupiter swung round the heaven with the same daily motion as the stars? There is no being, divine or otherwise, which explains this motion as such, for the motion merely supervenes on some brute facts about the arrangement of spheres and the divinely caused motions of the other spheres. We need not only to explain why the sphere in which J_2 is situated has such and such a speed around such and such an axis, but also to give a *per se* cause for its motion being *the same as* the motion of the fixed stars. One could not say that the sphere of the fixed stars itself is responsible for this, since its motion has been filtered out by unwinding spheres. But lacking a *per se* cause for this crucial feature of the motion of the sphere prior to J_2, we also lack a *per se* cause for the most obvious of Jupiter's motions, its daily orbit around the earth.

Now let us consider how the explanation would run with the 'idle' sphere, J_1, restored:

J_2 moves as it does (1) because a divine mover rotates it around its axis and (2) because sphere J_1 has precisely the motion it has. J_1 moves as it does (1) because a divine mover rotates it around its axis and (2) because the last of Saturn's unwinding spheres *has no effect* on the motion we are trying to explain.

On this account, Jupiter orbits the earth daily because sphere J_1 is rotated by its own divine mover with the same motion as the outer heaven. The last of Saturn's spheres should be mentioned in any candidate explanation of Jupiter's motion, since, if the motion of Saturn's last sphere were different, Jupiter's motion would be different. But in the explanation as I have sketched it, the force of the reference to Saturn's last sphere in (2) is not to indicate in what direction the explanation must continue, but to specify why the explanation stops here. The spheres in question, while mathematically idle, are not explanatorily idle, for without them Jupiter's daily orbit would be an incidental feature of the cosmic harmony, not properly explained by a cause of its own, just as the motion of a barnacle on a ship's hull lacks a proper cause.[24] The barnacle is *per se* stationary; it is only the ship (or the ship-barnacle composite) that is moved *per se*. Indeed, Aristotle himself concludes that not only the daily rotation of the fixed stars but "each of these motions too must be caused *per se* [καθ' αὑτήν] by an unmoved and eternal mover" (XII.8, 1073a32–34).[25] We need not assume that all events whatsoever have *per se* causes, only that, faced with a choice between two theories about eternal features of the world, one of which leaves certain eternal motions without full-fledged, *per se* explanations, we surely should prefer the theory that offers the more complete explanations, other things being equal.[26]

---

[24] This solution can be directly extended to solve a problem raised by Yavetz (1998, 237 n16), who observes that not only the first, but also the second sphere for each planet might have been eliminated. This requires a modification of the system of unwinders. In Aristotle's system, the unwinder of the ecliptic sphere of (say) Saturn has precisely the speed that cancels the motion of Saturn's ecliptic sphere; but that sphere might have a speed such that its resultant motion is the motion of Jupiter's ecliptic sphere. If the unwinder's speed is set in this way, then Jupiter's ecliptic sphere is redundant. And so throughout the system, the ecliptic spheres are eliminable. But this would result in a theory according to which there is no *per se* cause for the motion of the planets along the ecliptic, and this, if I am correct, is a worse theory, not a better one.

[25] He is speaking here of the criterion by which we count unmoved movers, and hence presumably he has in mind primarily the relationship between a given sphere and its unmoved mover, not the question how many spheres there should be in the system. What is important for our purposes is that he here clearly accepts the having of a *per se* cause as a desideratum for the theory. I am arguing that we can see that this criterion as relevant not only to the unmoved movers, but also to the 'idle' spheres.

[26] Simplicius reports that Theophrastus called the unwinding spheres "compensating" (ἀνταναφεροῦσαι), by which he meant something different from what Aristotle meant by "unwinding" (504.5–6). The poles of the spheres must (δεῖ) line up (κάθετον πίπτειν) "for only in this way, says Theophrastus, is it possible for the motion of the fixed stars to produce all things (as we have already said [it does]), and he is correct" (οὕτως γὰρ μόνως, φησίν, ἐνδέχεται τὴν τῶν ἀπλανῶν φορὰν ἅπαντα ποιεῖσθαι, καθάπερ ἤδη ἔθαμεν, εὖ λέγων; 504.14–15). This solution is similar in spirit to the one I offer, but I cannot see how the 'idle' spheres could

But are other things equal? An objector might develop a counter-argument in two stages, saying first that the 'idle' spheres do not complete the explanations of planetary motions, since in either case Jupiter's motion is partly caused by features of a network of spheres, and second that the two theories are not equal in respects other than explanatory completeness, since the theory with 'idle' spheres violates the principle of parsimony.

Our objector begins by observing that our formulations mentioned only two components for the explanation of, say, the motion of Jupiter's sphere, J_4: 'J_4 moves as it does (1) because a divine mover rotates the sphere about its axis and (2) because sphere J_3 has precisely the motion it does'. We omitted a crucial part of the explanation: 'and the poles of the axis of J_4 are fixed in the surface of sphere J_3 and the angle between the axes of J_4 and J_3 is *x*'. Indeed, determining the angles between the third and fourth spheres of the various planets was a crucial step in reconstructing the Eudoxan theory. When comparing two versions of the theory, we saw that the 'idle' spheres allow for fuller elaboration of the second part of the explanation, but our objector points out that they cannot eliminate or elaborate the (omitted) third component, the relations between spheres. According to the objector, the conceit of the 'idle' spheres theorist is that the arrangement of the network, being a brute fact, itself stands in need of explanation, whereas the postulated divine movers are self-explanatory, and therefore do not stand in need of any further explanation. But why are the angles of inclination between the axes of the spheres not also in need of explanation?

The objector confronts Aristotle with a dilemma. On the one hand, if Aristotle would extend the demand for explanation to cover all brute facts, then he must postulate divine beings as causes for every last feature of it – to explain why the angle of the ecliptic is 1/15 of a circle, why the number of spheres is 55. Are we to countenance a host of divine beings that cause the angles between various spheres to be just so many degrees?[27] Such a strategy, because it ignores all considerations of parsimony, would undermine our explanations, not enrich them. It would not explain, but merely stipulate that certain features of the world count as explained. On the other hand, if Aristotle balks at this proliferation of causes and agrees that parsimony is a consideration, then he should eliminate not just a few of these gratuitous divine movers, but all of them, and their 'idle' spheres too. Whether we account for the motion of Jupiter with the 'idle' sphere or without it, *both* explanatory factors must come into play – the arrangement of spheres and the *per se* motion of the preceding sphere. The alleged superiority of the explanations with 'idle' spheres is exposed as spurious, since its guiding principle leads to unbridled postulating of causes.

---

make such a difference. It is true that the motion of the 'idle' spheres mimics that of the fixed stars, but how could that entail that their presence allows the motion of the fixed stars to produce everything?

27 Lloyd does think of the unmoved movers as causing the spheres not only to rotate with a certain speed, but to have their axes at certain angles (2000, 254).

I grant that features of the network remain unexplained, but this does not undermine Aristotle's justification for postulating divine movers in the cases where he does. Our objector agrees that the divine movers make some contribution to explanation.[28] He should, therefore, also agree that the explanation of the heavenly phenomena, in particular of the diurnal rotations of the planets, is more complete to the extent that it can refer to some entity whose causal efficacy is directed toward these effects, rather than merely to brute facts about the network. The principle of parsimony should here be applied at the level of *per se* causes, not of spheres or divine movers. Aristotle should advocate the theory that uses the fewest causes while giving all the heavenly motions *per se* causes, even if this theory uses more spheres and divine movers than another theory which robs some heavenly motions of *per se* causes. But Aristotle should prefer this same theory to another in which all the heavenly motions have *per se* causes *and* unneeded *per se* causes are postulated for a variety of irrelevant facts. There is no more reason to postulate an unmoved mover to account for the angle between the ecliptic and the equator than there is reason to postulate an unmoved mover accounting for the existence of worms or of two basic pairs of opposites in the simple bodies.

The acceptability of the 'idle' spheres becomes clearer if we achieve greater precision about what these divine movers are, or rather, about what it means for there to be *many* such movers. Given their divine perfection, why is a single unmoved mover insufficient to cause all the heavenly motion, as long as that motion is conceived as a single extremely complicated motion? In the argument about how to count the unmoved movers (1073a26ff.), Aristotle adduced the premise, "one eternal motion is caused by one eternal mover", rather than (in the spirit of the parsimonious objector) postulating a single unmoved mover for the whole. What notion of explanation could have brought the philosopher to invoke a whole array of unmoved movers when it is not even clear whether they can be distinct from one another? All that is said in the text emphasizes their likeness to one another: ever-living, self-thinking thoughts. At the end of the chapter, Aristotle says that whatever is one in form can be many in number only by having matter (1074a32–34). Since these divine beings lack matter but are many in number, they must differ in form. I conjecture that the form of a divine mover is "what causes such and such a motion", from which it follows

[28] This might be challenged, of course, but it is a challenge beyond the scope of this paper.

that two movers cannot cause one motion because their forms would be identical and so they would be just one mover.[29]

This does not, however, eliminate the possibility that one mover causes all the heavenly motions: Why cannot the first unmoved mover have the form 'what causes *all* these motions' or 'what causes this *single*, very complicated motion'? If one were to insist that the motion of the heaven is one motion, could one in fact describe it as such, i.e., without reference to the multitude of spherical motions? Even if so, the heavenly motion would thereby lose its circularity, to which Aristotle is committed on other grounds (*de Caelo*, I.2).[30] The loss of the circularity of the motion would, furthermore, make the Aristotelian theory an empirically adequate but false mathematical model, which is not the kind of theory it is.

If, on the other hand, the first unmoved mover were thought to cause a plurality of heavenly motions, then the first unmoved mover could be only one part among several of the explanation of the motion of any particular sphere. The governing principle here was first thematically discussed by Socrates in the *Phaedo* where he avoids either giving a single cause for both being smaller and being larger or giving many causes for being two (100c9 ff.); it is that one cause has one effect and one effect has one cause. If we do not preserve this principle, then our explanations lose their force altogether, because the crucial question remains unanswered even after the cause is cited, namely, 'Why did it have

[29] I do not find in the literature a satisfying discussion of the premise, 'one eternal motion is moved by one mover'. Ross provides no more than a reiteration of the conclusion: "Since every eternal motion requires an eternal cause, and there are other eternal motions […], each of these requires an eternal substance as mover. […] There must be as many such substances as there are motions" (Ross 1924, 382). Lloyd (2000, 254) presents an alternative interpretation grounded on the perfection of the motions rather than the perfection of their movers: Because the motions are perfect, we cannot explain the differences between motions on the grounds that some achieve their goals more effectively than others; we must therefore have recourse to a multitude of movers (which are, in a way, goals). This alternative has two weaknesses. First, it omits the possibility of a single motion being caused by multiple movers. Second, on the assumption that a single mover can cause all the heavenly motions, why must the differences between the motions betoken an imperfection in the spheres (i.e., lesser ability to fulfil a goal)? This begs the question, which is whether a single unmoved mover can be a τέλος for the system of motions *as a whole*, a reasonable notion if one is, like Aristotle, impressed by the perfection of the whole system. In that case, the differences in the motions of the spheres would reflect not differing degrees of perfection, but differing roles in the fulfillment of a complicated τέλος.

[30] I am indebted to Sarah Broadie for this point.

this effect (out of the range of possible effects)?' Certainly one cause can in a sense have many effects, for the same man can build many houses; but Aristotle shows his awareness of this problem by saying that, properly, what produces a house is the art of house-building, which produces nothing but houses and by which all houses are produced.[31] Whatever the complexities and difficulties of this view about artistic production, we can see that the features of a particular house are explained either by the art itself (which, for instance, made the best of a bad building site) or by external interferences (for instance, the roof is missing because of a tornado). But this yields no helpful analogy with the heavenly realm, for no interferences occur there and so no causes account for the differences in the ways that the various spheres follow the unmoved mover's lead. As Lloyd observes[32], given the perfection of the spherical motions, their differences cannot reflect varying degrees of success in imitating the first unmoved mover. To count one mover per motion is to compromise between parsimony and extravagance, on rather complicated grounds.

Aristotle might thus offer a convincing rebuttal to the objection that his astronomical system is overburdened with unmoved movers. There are precisely as many unmoved movers as there are heavenly spheres. And however many heavenly spheres there are, certainly each planet has a sphere that an unmoved mover causes to revolve daily. Without these mathematically superfluous spheres, a cardinal feature of the heavenly motion would have no more explanation than a chance encounter in the agora. These apparently idle spheres are not idle in a system that, while apparently mathematical, is actually governed by a notion of explanation that insists on the principle of one cause-one effect and that has a strong but not overriding preference for including *per se* causes for eternal features of the world.[33]

---

31 This is not a strictly accurate characterization of Aristotle's view, since he thinks that arts can produce both members of a pair of opposites (*Metaphysics* IX.2). But this is not relevant for our purposes.

32 See note 29 above.

33 I would like to thank Sarah Broadie, Ursula Cooper, John Cooper, Michael Frede, Kinch Hoekstra, Edward Hussey, Andrew Sage and Christian Wildberg, without whose criticisms and suggestions this paper would have remained a private experiment, and Verity Harte, without whose encouragement this paper would never have been written in the first place. Henry Mendell provided enormously helpful comments, which saved me from serious errors.

Appendix: Table of Heavenly Spheres According to Aristotle and Callippus

Aristotle's unwinding spheres are listed in the form "$\text{sphere}_1/\text{sphere}_2$", where the unwinding sphere has the *same poles* as sphere_1 but opposite motion and hence cancels sphere_1's motion, and the unwinding sphere has the same *resultant motion* as sphere_2. "Sphere_1" indicates the poles, "sphere_2" indicates the motion. Thus S_4/S_3 cancels the motion of S_4, and has a resultant motion just like S_3. Reading a row of the chart from left to right, one sees how the unwinding spheres progressively reverse the motions of the spheres above.

S_1 and S_2 represent the sphere of the fixed stars and the 'ecliptic' sphere respectively. Each planet after Saturn has a pair of spheres that corresponds to but is distinct from S_1 and S_2. Beside those lower, corresponding spheres, I have marked (S_1) and (S_2) to bring out the correspondence. Each sphere marked (S_1) revolves once per day, but the speeds of the spheres marked (S_2) vary from planet to planet.

Saturn	S_1	S_2	S_3	S_4		S_4/S_3	S_3/S_2	S_2/S_1	
Jupiter	J_1 (S_1)	J_2 (S_2)	J_3	J_4		J_4/J_3	J_3/J_2	J_2/J_1	
Mars	A_1 (S_1)	A_2 (S_2)	A_3	A_4	A_5	A_5/A_4	A_4/A_3	A_3/A_2	A_2/A_1
Venus	V_1 (S_1)	V_2 (S_2)	V_3	V_4	V_5	V_5/V_4	V_4/V_3	V_3/V_2	V_2/V_1
Mercury	H_1 (S_2)	H_2 (S_2)	H_3	H_4	H_5	H_5/H_4	H_4/H_3	H_3/H_2	H_2/H_1
Sun	X_1 (S_1)	X_2 (S_2)	X_3	X_4	X_5	X_5/X_4	X_4/X_3	X_3/X_2	X_2/X_1
Moon	L_1 (S_1)	L_2 (S_2)	L_3	L_4	L_5				

Aristotle. 1957. *Metaphysica*. Ed. W. Jaeger. Oxford.
–. 1950. *Physica*. Ed. W. D. Ross. Oxford.
Bowen, A. C. 2001. "La scienza del cielo nel periodo pretolemaico". In *Storia della Scienza: I. La scienza antica.* Rome.
Dicks, D. R. 1970. *Early Greek Astronomy to Aristotle*. Bristol, England.
Frede, M. 2000. Introduction. In *Aristotle's* Metaphysics *Lambda*. Eds. Michael Frede and David Charles. Oxford.
Goldstein, B. R. 1997. "Saving the Phenomena: The Background to Ptolemy's Planetary Theory". *Journal for the History of Astronomy* 28: 1–12.
Heath, T. 1913. *Aristarchus of Samos: The Ancient Copernicus.* Oxford.
Lloyd, G. E. R. 1991. "Saving the appearances". In *Methods and Problems in Greek Science*. Cambridge.
–. 1996. "Heavenly Aberrations: Aristotle the Amateur Astronomer". In *Aristotelian Explorations.* Cambridge.
–. 2000. Chapter 8. In *Aristotle's* Metaphysics *Lambda*. Eds. Michael Frede and David Charles. Oxford.

Mendell, H. 2001. “The Trouble with Eudoxus”. In *Ancient and Medieval Traditions in the Exact Sciences: Essays in Memory of Wilbur Knorr*. Eds. Patrick Suppes, Julius Moravcsik, and Henry Mendell. Stanford.

–. 1998. “Reflections on Eudoxus, Callippus, and their Curves: Hippopedes 23 and Callippopedes”. *Centaurus* 40: 177–275.

Musgrave, A. 1991. “The Myth of Astronomical Instrumentalism”. In *Beyond Reason*. Ed. G. Munevar. Dordrecht.

Neugebauer, O. 1975. *A History of Ancient Mathematical Astronomy*. 3 vols. New York.

Platon. 1995. *Phaidon. Platonis Opera*. Oxford.

Ross, W. D. 1924. *Aristotle’s Metaphysics: A Revised Text with Introduction and Commentary*. Oxford.

Schiaparelli. 1875. “Le sfere omocentriche di Eudosso, di Callippo, e di Aristotele”. In *Pubblicazioni del R. Osservatorio di Brera in Milano*, No. ix, Milan. Translated into German by W. Horn, *Abhandlungen zur Geschichte der Mathematik*, I. Heft, Leipzig, 1877, 101–98.

Simplicius. 1894. *Simplicii in Aristotelis de Caelo Commentaria*. Commentaria in Aristotelem Graeca, vol. 7. Ed. I. L. Heiberg. Berlin.

Wright, L. 1973. “The Astronomy of Eudoxus: Geometry or Physics?” *Studies in History and Philosophy of Science* 4: 165–72.

Yavetz, I. 1998. “On the Homocentric Spheres of Eudoxus”. *Archive for History of Exact Sciences* 51: 221–78.