"Superfluous": The Stories of Einstein's Special Relativity

Draft

Raymond Brock

December 4, 2022

Contents

3 Archytas, Eudoxus, and Greek Astronomy **5**
3.1 Greek Astronomy 6
3.2 A Little Bit of Archytas 9
3.3 A Little Bit of Eudoxus 11
3.4 A Little Bit of Greek Astronomy 14
3.4.1 What We Know Now 15
3.4.1.1 Observation #1: stars' motion 15
3.4.1.2 Observation #2: planets' motions 16
3.4.1.3 Observation #3: that fuzzy stripe 19
3.4.1.4 Observation 4: The Moon 20
3.4.1.5 Observation 5: The Sun 21
3.4.2 Greek Astronomy, Presocratics 21
3.4.3 Plato's Two Models 23
3.4.4 Eudoxus' Model 27
3.4.5 Aristotle's Model 32
3.4.5.1 Critique of the Pythagorean Cosmology 32
3.4.5.2 Details of Aristotle's Model 33
3.4.5.3 The Physics of Aristotle's Model 33
3.5 More of Plato's, Aristotle's, and Ancient Astronomy's Stories 33
3.5.0.1 Modern Day Platonists 33
3.5.0.2 Indebtedness to Eudoxus 34

Appendices **37**

A Greeks Technical Appendix **37**
A.1 Zeno's Paradox 37
A.2 Proof of Pythagoras' Theorem 37
A.3 Socrates' Geometrical Problem 37

B Plato–Aristotle Technical Appendix **39**
B.1 Digital Gates 39

Sources **41**

Chapter 3

Two Most Important Mathematicians You've Never Heard Of : Archytas, Eudoxus, and Greek Astronomy

> "I think that modern physics has definitely decided in favor of Plato. In fact the smallest units of matter are not physical objects in the ordinary sense; they are forms, ideas which can be expressed unambiguously only in mathematical language." Werner Heisenberg (1901-1976), theoretical physicist, Nobel Laureate

> "I imagine that whenever the mind perceives a mathematical idea, it makes contact with Plato's world of mathematical concepts... When mathematicians communicate, this is made possible by each one having a direct route to truth, the consciousness of each being in a position to perceive mathematical truths directly, through this process of 'seeing."' Roger Penrose (1931-), theoretical physicist, Nobel Laureate

3.1 Greek Astronomy

I'll bet that many of you have seen the solar system arrangement as imagined by Copernicus with the Sun in the center and all of the planets, including Earth, obediently orbiting it in perfect circles. While we tend to think of our solar system as Copernicus' (surprises await in Chapter ??), the roughly 500 years since is a short era relative to the competing view of his time. He was challenging the ancient, and universally-held idea, that it's the stationary Earth that's in the center of the universe. Fascination with that picture as is prevalent in many decorated medieval manuscripts through the centuries. One of the earliest is from a poem by the Greek poet, Aratus called *Phaenomena*, named after a book of the stars and constellations by the Greek mathematician, Eudoxus, who created that 2000 year old "geocentric" model of the universe—one in which the Sun, Moon, planets, and stars all orbit around the stationary Earth. Figure 3.1 is from a 10th century version of *Phaenomena* from the British Museum.

Figure 3.1: Aratus the poet lived about a century after Eudoxus (and hence, Aristotle) and turned his astronomy book into a poem. Later, Cicero translated it and this 10th century manuscript is an illustrated copy of that work.
https://sarahjbiggs.typepad.com/.a/6a013488b5399e970c01bb07c8696d970d-pi

I took some pains in the last chapter to underscore that models of MOTION ON THE EARTH belong in Aristotle's corner. He was beholden to few for his physics and really invented the dynamics of motion. But while we tend to ascribe that geocentric model of the universe to him as well, he borrowed it lock stock and barrel from Eudoxus and Plato. We'll develop that in this chapter and go beyond them.

The Greek world—indeed, the whole world—was radically and violently altered by Alexander the Great and during the period between Aristotle and Cleopatra, astronomy become an experimental and quantitive science. The culmination of Greek astronomy came after Greek-everything became Roman-everything and just before the Roman Empire began its decline. One last Greek, in our long string of Greek philosophers, mathematicians, and scientists remained and we'll close our chapter with Ptolemy's calculating-model of the heavens.

A game that many scientists play is to trace their scientific lineage back for centuries—their major professor's professor and so on. I followed mine back through centuries and found that I descended from Copernicus! I'd like to think I've made him proud.

Sometimes it turns out that someone's student ends up in the history books. But not many students actually take over the known world by force!

After nearly 40 years of the Peloponnesian War , Sparta had to govern. Not just Athens, but all of the vast Athenian empire which had been loosely held together by agreement, but in which Sparta had to govern through force. They felt relentless pressure from everywhere—Thebes, internal strife, and the emboldened northern Macedonian threat. Remember that "tumultuous" is the signature skill of political Greeks: during the time of Plato and Aristotle, if you randomly threw four colors of paint at the wall, you'd pretty much replicate a map of the four different allied and opposing regional Greek and Macedonian states. When Plato died, encouraged by the Macedonian King Philip II, Aristotle relocated, first to the northern Aegean and then to Macedonia, acceding to a royal summons to teach his 13 year old son, Alexander. There he set up a school, taught Alexander (and perhaps the future general, Ptolemy) for three years, and then stayed for seven more before returning to Athens where he started the Lyceum. By this time the teen-aged Alexander was already on the battlefield and with his father, had occupied the entirety of the Peloponnese. So Athens was once again ruled by outsiders.

After Philip II was assassinated,[1] Alexander, soon to be "The Great," ascended to the throne and began his brutal lightening-fast, nine year conquest of three continents of the entire known western world: modern Turkey, the middle east, Egypt,

[1] Assassination, murder, and betrayal were a family hobby.

and Arabia all the way across Afghanistan to India, leaving military oversight over Athens and the rest of Greece. While he stayed in touch with Aristotle, sending him samples from all over Asia, his teacher became distant, put off by Alexander's adaptation of Persian customs and persona.

Alexander died in Babylon in −323 under suspicious circumstances and Ironically, within a year Aristotle himself died at the age of 63 of natural causes at his mother's family estate outside of Athens. Athens had turned on him and his Macedonian connections were dangerous. Impiety was charged, a death sentence issued, and he fled uttering his famous remark about the city not sinning against philosophy for a second time. In his absence, the Lyceum stayed active under new management for another century.

Meanwhile, in the "Partition of Babylon" Alexander's senior commanders divided up the sprawling kingdom among a dozen generals and aides and then did what came naturally: they fought among themselves for 40 years. In the end, three kingdoms and a dizzying array of city-states were established: Macedonia and Greece, Seleucia (roughly modern-day Iraq), and Egypt.

Hundreds of thousands of Greeks migrated into the newly organized territories establishing an international Greek-ness of culture, arts, and philosophy which was the beginning of the **Hellenistic Age.** During the final two centuries, BCE, "Greek" ceased to be the province of only a loosely-constructed, turbulent nation but rather characterized the entire, known world. Of the two dozen cities that Alexander created or conquered named for himself, the "Alexandria" that mattered most to him, and to us for physics and history, was the Egyptian Alexandria, a new port city on the Mediterranean.

Egypt became unusually secure under Alexander's former body guard and general, **Ptolemy I Soter** (−367 to 282) who eventually fashioned himself, "Pharaoh." He adopted Egyptian customs, including that of rulers marrying their siblings, and was an intellectual of sorts, creating the first state-supported national laboratory and library. The "Alexandrian Museum" was a national facility devoted to research and among its first recruits was the mathematician, Euclid who then wrote *Elements*, the most-read book in history, besides the Bible. For 2500 years, from Copernicus to Thomas Jefferson, mastering *Elements* was the route to mathematical literacy. Ptolemy found it rough-going and asked for an easier way to learn it, but was told by the author that "...there is no Royal Road to geometry," a sentiment still relevant today. For centuries the Museum was home for scores of Greek scholars, all supported by the dozen Ptolemy's from the I^{st} to the final, Cleopatra.

The Library of Alexandria probably contained all of the known—and now mostly lost—manuscripts of all of the philosophers, poets, playwrights, and doctors we've discussed in the previous chapters. There was a hunger for knowledge of all sorts and Ptolemy and agents of his library director searched every ship that docked, stealing or copying any books on board. They rented or stole manuscripts from all of the major cities for copying. The Library and Museum suffered through a couple of wars, including during Marc Antony's escape, and probably survived until the conquest of the Muslims in the 7th century CE.

Among the scores of Alexandrian scientists and astronomers are those whom we will mention below: Eratosthenes of Cyrene, Aristarchus of Samos, Hipparchus, and especially Claudius Ptolemaeus. The Ptolemy dynasty lasted 300 years until the legendary feud involving "the" Cleopatra (a common name for female Ptolemy-family successors) and Marc Antony and Julius Caesar. The Library and Museum lasted into the first five centuries CE until it was eclipsed by great Muslim libraries in Babylon, Cairo, and Cordoba in Spain. Astronomy is our concern in this chapter.

3.2 A Little Bit of Archytas

Recall that Philolaus was the source of Plato and Aristotle's knowledge of Pythagoreanism—for example, the "Pythagorean" cosmology came through him or probably originated from him. Was he a student of Pythagoras? The dates of their overlaps almost work out to imagine that relationship, but it's controversial. He's certainly the closest we get to the great man so it's not far-fetched to continue the teacher → student theme that began this chapter: Pythagoras → Philolaus → Archytas → Eudoxus. Lunar craters are named after each which is not the normal teacher-student legacy.

Figure 3.2 is a famous engraving often mistaken as a medieval work, when in fact it's from 1888. Its sentiment, however, is ancient and due to a Greek Pythagorean, a friend of or competitor to Plato. Among the most famous arguments in cosmology is whether the universe is infinite or finite in size and Archytas had the first of many similar inspirations (followed by Newton and then Einstein) that the universe cannot be finite. He did a thought experiment, imagining traveling to its presumed edge and attempting to thrust his stick beyond that limit. If he could extend it, then, well, that's not the edge...and so he'd have to go further, repeating the experiment without end. This is a good example of the kind of intuitive cleverness that seemed to be built into this great Greek mathematician, politician, and military leader, **Archytas of Tarentum** (−428 to −347). The very model of a modern major—Pythagorean— general.

When we last saw Pythagoras, he was on the run from Croton in the instep of the Italian boot and in an inglorious escape on land and by water, trying and failing to be allowed to settle anywhere. People were afraid to protect him for fear of being the subject of attack by followers of the wealthy and thin-skinned Cylon, unused to apparently standard brusk treatment by our philosopher. Just how Pythagoras came to his eventual end isn't clear and of course there are many stories. The bottom line is that the cult's welcome had soured and Pythagoreans spread out from Croton, migrating further east within the instep of the Italian boot, and also to Syracuse, Thebes, Corinth, and some to Athens. Philolaus was one of those

Figure 3.2: CAPTION

emigrants and probably near Athens wrote his account of Pythagoreanism that Plato read. Plato wanted to learn mathematics and Philolaus' student, Archytas, was the best in the land, and so off he went in his 38th year in −398 to Tarentum, one of those "instep" sanctuaries and one of the most powerful Greek city-states.

Plato wouldn't have written *The Republic* by that time, but ideas about what constituted the best ruler must have begun to form as he became interested in Syracuse at the southern tip of the island of Sicily which was ruled by a ruthless "tyrant"[2] Dionysius the Elder and his successor son.

In that first trip, Plato formed a bond with the tyrant's brother-in-law, Dion who took it upon himself to arrange for his undisciplined nephew's education and twenty years after his first visit, Dion brought Plato back—now almost 60 years old—on a special ship sent to Athens just for him to Syracuse as a tutor. It went badly when Dionysius expelled his uncle, and imprisoned Plato with (according to some legends) intentions of selling him into slavery, Plato managed to send word to his friend, Archytas, who by this time had acquired the stature necessary to rescue Plato with yet another, Plato-only ship.

As I noted in the last chapter, Archytas was a committed Pythagorean and a mathematician of great skill. But he also he was a civic leader and an elected military general. In spite of Tarentum law, he was re-elected seven times because he never lost a battle. (Did I mention that Greeks fought constantly?) When he did

[2]meaning someone in power who didn't inherit it, but took it

step down, the army started losing.

Archytas was reported to be an even-tempered, cultured man who led Tarentum through a period of democracy and his reputation was widely known beyond antiquity. Aristotle wrote more (lost) books about Archytas than any other person and one of his students, Aristoxenus wrote Archytas' life story so his impressions were probably closest to being accurate. There is some evidence that he wrote a book on mechanics and that he enjoyed making mechanical toys for children—very un-Plato-like in spirit.

His mathematical skills were legendary and he solved an old problem with mystical roots: Apollo sent a plague to the city of Delos and a delegation was sent to Delphi to learn from the Oracle how to rid themselves of the pestilence. The instructions were to take their cubical altar to Apollo...and build a new one with double its volume. This is called the problem of "duplicating the cube" (also called the Delian Problem) and it required cleverness on Archytas' part, but his resolution wasn't welcomed by Plato since it was not strictly geometrical since Archytas actually deployed his favored arithmetic tools. Little did he nor Plato know that the Delian problem was proved later to not be solvable through geometrical means alone. Archytas contributed to many branches of mathematics and Euclid's *Elements* includes some of his' proofs.

All in all, Archytas was perhaps the most accomplished Pythagorean of all and in the spirit of the opening to this chapter, we're indebted to him for his products, but also one of his students. The most accomplished of all Greek mathematicians before Archemides, Eudoxus, from whom 2000 years of cosmology originated.

3.3 A Little Bit of Eudoxus

Eudoxus of Cnidus (-408 to -355) was the son of a physician and became one himself, but we know of him as a gifted mathematician and astronomer. As we'll see, astronomy and medicine were connected through astrology and mathematics and astronomy have always been kin, so these seemingly disparate skills go together. Cnidus was a city founded by Sparta on the southern coast of modern Turkey and was the location of his start and finish, between which he traveled all over the Aegean to study and teach. As a young man he went to Tarentum to study mathematics with the pre-eminent Pythagorean mathematician (and much more) **Archytas of Tarentum** (-428 to -347). Let's learn a little bit about Archytas in Figure Box **??** on page **??**. After you've read about him, return to this point and continue reading about his student, Eudoxus.

After his mathematics instrucxtion, he went to Sicily to study medicine, then by the age of 23 he went to Athens and stayed briefly with Plato's Academy (rooming 14 miles away, so a long commute to lectures). After less than a year, he was back on the road to home in order to raise funds...so that he could travel even further to

Egypt where he studied astronomy for 16 months, before leaving for the northern modern Turkey Black Sea coast and the Greek colony of Cyzicus. By this point he's lecturing on his own and established a popular school and an observatory. With data from his observing in the north and from Egypt, he published his first book, *Phaenomena*, which was a compendium of star locations and *On Speeds*, of their motions.

Around −368, during his 30s, he moved his school to Athens, by which time Plato is 60 years old and Aristotle has left for Macedonia. His arrival in Athens may have had a diplomatic purpose since, of course, a war broke out between Athens and Persian forces which oversaw much of the Ionian colonies. An over-extended Athenian army enjoyed a short-lived victory and then faced the threat of a full-on conflict with Persia and there's reason to believe that Eudoxus' relocation to Athens helped to calm the situation. It was here, as the legend goes, that Eudoxus was challenged by Plato to form a geometrical model of the heavens. The legend is challenged as by this point, Eudoxus was the mathematical champion of the Greek-speaking world and more likely to issue challenges, than accept them. As we'll see below his model was born and in various guises, persisted until Galileo, Kepler, and Newton.

FIGURE BOX **??**

Figure 3.3: CAPTION

Figure 3.2 is a famous engraving often mistaken as a medieval work, when in fact it's from 1888. Its sentiment, however, is ancient and due to a Greek Pythagorean, a friend of or competitor to Plato. Among the most famous arguments in cosmology is whether the universe is infinite or finite in size and Archytas had the first of many similar inspirations (followed by Newton and then Einstein) that the universe cannot be finite. He did a thought experiment, imagining traveling to its presumed edge and attempting to thrust his stick beyond that limit. If he could extend it, then, well, that's not the edge...and so he'd have to go further, repeating the experiment without end. This is a good example of the kind of intuitive cleverness that seemed to be built into this great Greek mathematician, politician, and military leader. The very model of a modern major—Pythagorean— general. As I noted in the last chapter, Archytas was a committed Pythagorean and a mathematician of great skill. But he also he was a civic leader and an elected military general. In spite of Tarentum law, he was re-elected seven times because he never lost a battle. (Did I mention that Greeks fought constantly?) When he did step down, the army started losing.

Archytas was reported to be an even-tempered, cultured man who led Tarentum through a period of democracy and his reputation was widely known beyond antiquity. Aristotle wrote more (lost) books about Archytas than any other person and one of his students, Aristoxenus wrote Archytas' life story so his impressions were probably closest to being accurate. There is some evidence that he wrote a book on mechanics and that he enjoyed making mechanical toys for children—very un-Plato-like in spirit.

His mathematical skills were legendary and he solved an old problem with mystical roots: Apollo sent a plague to the city of Delos and a delegation was sent to Delphi to learn from the Oracle how to rid themselves of the pestilence. The instructions were to take their cubical altar to Apollo...and build a new one with double its volume. This is called the problem of "duplicating the cube" (also called the Delian Problem) and it required cleverness on Archytas' part, but his resolution wasn't welcomed by Plato since it was not strictly geometrical since Archytas actually deployed his favored arithmetic tools. Little did he nor Plato know that the Delian problem was proved later to not be solvable through geometrical means alone. Archytas contributed to many branches of mathematics and Euclid's *Elements* includes some of his' proofs.

All in all, Archytas was perhaps the most accomplished Pythagorean of all and in the spirit of the opening to this chapter, we're indebted to him for his products, but also one of his students. The most accomplished of all Greek mathematicians before Archemides, Eudoxus, from whom 2000 years of cosmology originated.

His mathematical skill was by that point unprecedented. As a student of Archytas, he was well aware of the Pythagorean horror of irrational numbers (recall Hippasus's reputed unfortunate end by discovering that the Pythagorean Theorem as applied to triangles of length 1 for the legs leads to the hypthenuse of length $\sqrt{1}$). He nearly perfected the mathematics of proportion, demonstrating that lengths corresponding to irrational values could be compared to those of rational values. He also (perhaps invented) the "method of exhaustion" in which the area of a circle, for example, could be approximated to any degree of precision by adding up the areas of triangles positioned side by side with their apexes at the center. The more triangles, the better the area derivation. This is of course a precursor to taking a limit in calculus and was expanded on by Archemedes. He also showed that in comparing the area of one circle (a curved shape) to another, was related to the square of their radii (linear quantities) and extended that to spheres. Eudoxus' work was memorialized in a number of Euclid's *Elements*.

But it's Eudoxus' astronomy and cosmology that are our concern here and so let's work up to that with some review of the problems that everyone in antiquity faced when trying to describe what we observer from Earth.

3.4 A Little Bit of Greek Astronomy

Greek cosmology came in fits and starts and I've alluded to some of it in this and Chapter 1. What we think of today as Greek astronomy is centered on work commissioned by and then described by Plato and Aristotle. Since it's mostly all of one piece, I've reserved this section to cover it. Their picture is highly derivative from... you guessed it: the Pythagoreans, but Plato is the real instigator for the traditional Earth-centered ("geocentric") description of MOTION BY THE EARTH.

If you go outside after dark and over many hours watch the sky throughout a few hours, you'd reach same conclusions that Greek observers did:

1. We seem to be stationary (you don't feel like you're moving) and through the night the whole panorama of stars seems to revolve east to west around an axis that points to the north pole. It's as if we're at the center of a vast sphere on which the stars are painted on the inside. However, the times that stars begin to appear on the eastern horizon changes each night by four minutes early out of 24 solar hours, which is called heliacal rising. This time advances through the year and the "ascendency" of stars in the east became a calendar that people could use to identify where they were during the course of a year. When Sirius ascends just before dawn, then in Egypt they knew that the Nile's flooding was near.
2. There are a few brighter objects (the planets) which each independently execute similar east-west motions through an individual night but which don't follow the overall star motions. Over the course of many nights, where the planets are at the same time each night is different when compared with

the stars and constellations and some of that overall movement relative to the background stars actually reverses course through a year.

3. From the northern hemisphere, there is a fuzzy broad stripe of stars across the southern sky.
4. The Moon has its own independent motion against the background stars which shows dramatic phases on its surface of light and dark through a month.
5. During the day, the Sun also has an east-west motion similar to the nighttime objects. All of the planets, the Moon, and the Sun move in a band across the sky through the constellations of the zodiac.

(a) (b)

Figure 3.4: The figure on the left is a time-lapse photograph of the north celestial sphere (essentially pointing the camera at Polaris, the North Star). One can clearly see that the stars in the sky all follow circular tracks around that pole position. The figure on the right shows a night photograph showing two planets, Jupiter, Venus, and the Moon just as the Sun is setting. The blowup is of that part of the sky more clearly showing Jupiter (J) and Venus (V).

3.4.1 What We Know Now

In order to get a sense of what the Greeks were contending with, let's remind ourselves of what we now know about our sky's appearance and how each of our above five ancient observations can be explained. Then we'll go back and see what they came up with.

3.4.1.1 Observation #1: stars' motion

Figure 3.4 (a) shows a time-lapse photo of the northern sky from the northern hemisphere showing that every single star seems to be rotating around a single point in the sky around the Earth's north pole—now the star Polaris— every 24 hours. What we know of course is that this apparent motion is not of the stars—which look to be stationary on human timescales— but is due to the Earth's daily rotation exposing different parts of the star fields all around us. That the Earth also moves around the Sun means that the stars will emerge each night in a slightly different place than the previous night. So, the Earth's rotation and its orbit around the Sun explains observation #1.

> That we see Polaris at nearly the axis of rotation (it's less than a degree away), is an accident! The Greeks observed this circular motion but Polaris wouldn't have been at that point because like a spinning top, the Earth's axis of rotation precesses in a circle, pointing at different parts of the sky over millennia. It will continue to do so, and the pole star will appear to slowly move away from our familiar Polaris-center-point, returning to it in 25,900 years.

3.4.1.2 Observation #2: planets' motions

Our local neighborhood is the solar system in which we're the third planet from the Sun. FigureFigure 3.5 is a cartoon of that system and while the scales are arbitrary, there is one thing that's accurate about this figure: the planets are all in a plane. This has to do with their original formation out of the dust and elements that were swirling around the Sun almost five billion years ago. They gravitationally clumped together, slowly forming the eventual eight planets. This common plane of the planets' orbits (and the Sun) is called the "ecliptic." If the Earth's rotational axis were perpendicular to the plane of its orbit, then we'd see all of the planets, the Sun, and the Moon staying within a lane that would be above the Earth's equator. But the Earth's axis is slight tilted by 23.5° so that plane appears at an angle relative to the Earth's rotational axis.

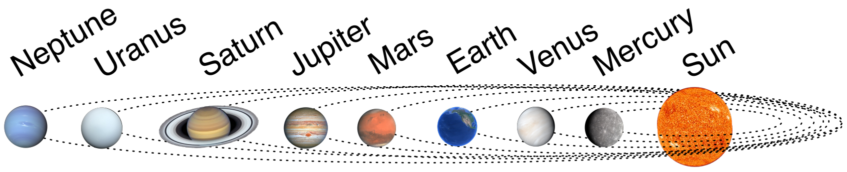


Figure 3.5: Our solar system, dramatically not to scale: the Sun is enormous compared with the individual planets, they are much further from it than the picture suggests, and the size differences among them are significant. The eliptical orbits of the planets is greatly exagerated.

There are three spheres that orient the Earth to the solar system and stars. A sphere centered on the Earth's rotational axis is the "Celestial Sphere." When the Earth rotates on that axis it looks to us like the stars are all rotating. Because the Earth is inclined by that small angle, the Sun, Moon, and planets are all in that lane and form the equator of another sphere, the "Ecliptic Sphere," the axis of which is inclined by 23.5° from the Celestial Sphere (Earth's rotational) axis. Finally, unless you live on the equator, the plane that you see as your horizon is related to your longitude and your "zenith" is directly overhead and your horizon is the equator of that personal sphere.

Figure Box ?? summarizes much of what I just described about the three spheres on page 18. After you've read the material in that Box, return to this point and continue reading.

The paths that the planets take around the Sun are not circles, but they're ellipses in which the Sun is not at the center, but at one of the "focii" meaning that the planets' speeds around the Sun are not constant. When they're closest to the Sun, they are going faster than when they're the furthest from the Sun. (We'll understand that in Chapter **??**).

Figure 3.4 (b) is a photograph of the southern sky in which Jupiter and Venus are visible. Notice that the horizon is still a little bit bright and so the Sun is just setting. Venus and Mercury are close to the Sun and the Greeks noted that uniquely they seem to cling to it as it rises and sets. The Greeks could observe the Sun, the Moon, and the closest five planets: seven celestial objects plus the background stars.

The planets also have varying brightnesses during different periods of the nights which is especially the case for Venus which shows phases like the Moon, as it passes in front of, and then around the back of the Sun as we view it.

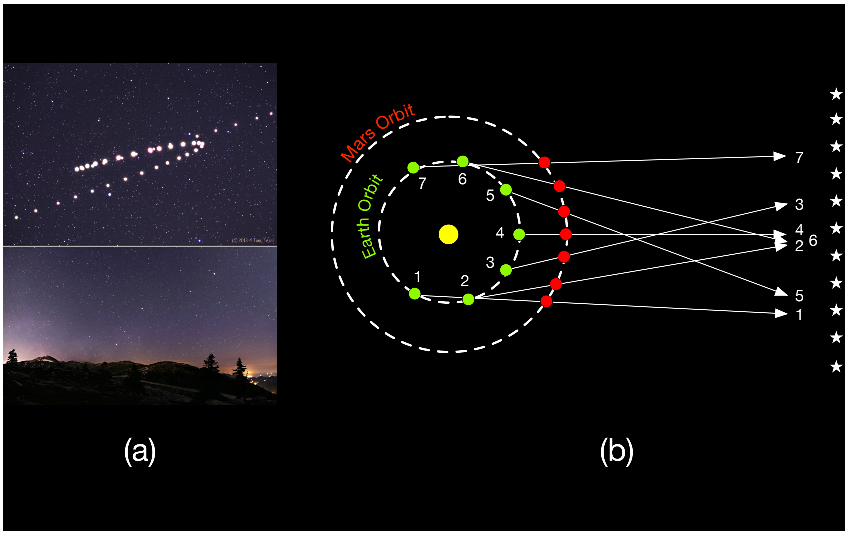
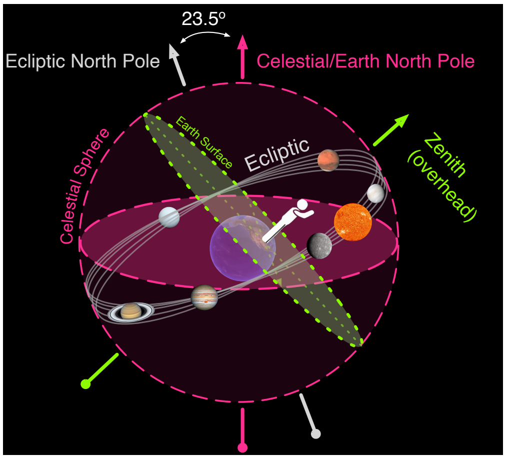


Figure 3.6: Retrograde motion by Mars. In (a) the sky in Turkey shows a photograph of Mars from December 5, 2013 in the upper right hand corner and then an overlayed photograph taken every five or six nights until August 8, 2014. The looping behavior in the middle is the retrograde motion. (b) shows how this happens (see the text for an explanation) https://twanight.org/gallery/tracing-the-red-planet/?preview=true

But there's a very odd dance that the planets execute called "retrograde motion" depicted in Figure 3.6. Suppose you took a wide-view snapshot of, say Mars each night at, say midnight, noting where it is in relation to the (nearly fixed) background stars. You'd see that each night at midnight it's moved a little bit east against the background stars.[3] At some point in your nightly observation the planet would slow its nightly rate and turn around and start moving west against the stars... and then reverse its course again and continue. Figure 3.6 (a) shows exactly this situation, overlaying multiple nightly pictures of Mars over Turkey.

FIGURE BOX ??



This image shows the Earth and a person standing in the Northern Hemisphere. The outside dashed sphere is the "Celestial Sphere" with its "Celestial North Pole" axis going through the Earth's geographic North Pole, which is of course the axis of Earth's daily rotation. Their observed horizon in all directions appears to him as a plane, with the "Zenith" directly overhead. Notice that if the observer were at the Earth's equator, then the Zenith would be perpendicular to the axis of rotation by the Earth. If they were at the North Pole, then the Zenith would point along the Earth's rotational axis. In East Lansing, Michigan my latitude is $42.7325^\circ\ N$ and so the angle between my northern horizon and the celestial/Earth pole is 42.732° and the zenith is 47.268° from the celestial/Earth pole.

The lineup of planets appears around the Earth in the ecliptic plane which is inclined to the Earth's equator by 23.5°, that angle of tilt that the Earth has relative to the plane of its orbit around the Sun. The ecliptic's North Ecliptic axis is inclined to the Celestial Sphere's axis as shown by that same 23.5° angle. The paths of the planets are not exactly arranged, but the ecliptic's equator forms a lane in which the planets are all contained as they pass overhead.

Now go back to page 17 and pick up where you left off.

In Figure 3.6 (b) provides the explanation. Here, a cartoon of the Sun (yellow circle in the center) and orbits of the Earth and Mars are drawn not to scale (this could be any planet outside of our orbit). The numbers refer to hypothetical nights' of observing Mars (the outer red circles) from Earth (the inner green circles). The arrows refer to an Earth observer each night sighting Mars in reference to the distant star background. So night #1 points low in the field, night #2 points towards the middle, #3 a bit higher, and then #4 Mars *appears* to go backwards against the stars. That backwards motion continues until night #6 when it's back to its original trajectory. So a heliocentric ("Sun-centered") model of the solar system explains

[3]Note, this does not mean that Mars moves east during a single night, but that if you looked at the same time in successive nights.

observation #2.

(a) (b)

Figure 3.7: The figure on the left is a photograph of the Andromeda galaxy, which is very similar to our own Milky Way galaxy. Pretending that this is the Milky Way, the approximate location of our solar system is shown in one of the spiral arms. The arrow is our line of sight to the center of our galaxy. The right figure shows what the Milky Way looks like from Earth as we look along that arrow in (a) toward the galatic center in the constellation Sagitarius. We're looking from the edge to the center through all of the stars between us and the galactic center.

3.4.1.3 Observation #3: that fuzzy stripe

Our Sun is about four and a half billion years old and is one of 100 thousand million stars in the Milky Way galaxy, which is itself more than 13 billion years old. It's not much but it's home—in fact, it's a relatively average galaxy. Figure 3.7 (a) is a photograph of the Andromeda galaxy (not ours!), which is the largest galaxy near us and similar in structure to and a little bigger than the Milky Way. It's about 2.5×10^6 light-years away from us so the image shows what Andromeda looked like almost three million years ago.[4] The Milky Way shape is broad, with arms of stars in a spirals outward, hence its classification as a "barred spiral galaxy." It has a diameter of about 87,000 light-years across and a thickness of roughly 1,000 light-years, so its flat like a pancake. Our Sun is about 25,000 light-years from the center in one of the spiral arms and we're rotating around the center of the galaxy at a speed of about 800,000 kilometers per hour (~500,000 mph) and the arrow in Figure 3.7 (a) suggest the relative position of our Sun, using Andromeda as a stand-in for the Milky Way.

[4] Each galaxy is moving through the local cluster of galaxies...toward one another. They will collide in about four and a half billion years from now and likely form a merged galaxy, but their trillion stars are so widely dispersed that the likelihood that any two stars will collide is negligible. Their two central black holes will possibly collide and emit substantial gravitational radiation. Simulations suggest that the solar system might be ejected, but probably in-tact. Eventually, all of the galaxies in the local group will merge. Not to worry.

Figure 3.7 (b) is a photograph of the southern sky showing what it looks like to look through the edge of the Milky Way—through the pancake—towards its center in the constellation Sagittarius (where there is an enormous black hole with a mass of about 4 million suns.) The arrow in Figure 3.7 (a) shows our line-of-sight towards that center. So our location in our Milky Way galaxy explains Observation 3.

3.4.1.4 Observation 4: The Moon

If we were above the Earth's orbital plane (the ecliptic) and positioned ourselves above the Earth's north pole we would see it rotating counter clockwise, once around every 24 hours. We'd also see the Moon 384, 399 km (238, 855 miles) from Earth's center orbiting in the same direction, always presenting the same face to Earth, and taking 27.322 days to complete an orbit. [The Moon is also rotating on its axis, but in sych (it's "tidally locked") so even though it rotates once around per orbit, we still see the same side.] If we looked the Earth-Moon system from the side (in the ecliptic) with the Sun to our left and Earth to our right, we'd see that the Moon's orbital plane is about 5° inclined relative to the ecliptic and the Sun-Earth line, so that sometimes the shadow that it casts from the Sun is above or below the Earth so we don't have a solar eclipse every month.

What does this dance look like from an observer on the Earth? The Sun always illuminates half of the Moon's surface, but because its rotational period is different from Earth's the Moon's appearance to us changes in a unique pattern which we call the "phases." If the Moon is between Earth and the Sun, during our night, we're looking away from the Moon and not see it: that's the "new moon." At that position it's at its closest position to the Sun relative to us but when we revolve back around and see the Sun during our day we'd also see the Moon for a few days even in the sunlight.

Each night, then the Moon's surface would be more and more visible (Waxing Crescent, First Quarter, and Waxing Gibbous) until we see the whole Moon's surface when it's on the other side of Earth from the Sun and the Full Moon brightly reflecting the Sun's light back to us when we're looking during our night-time. Then each night the Moon would slightly start showing the next phases (Waning Gibbous, Last Quarter, and Waning Crescent) until it's back between Earth and the Sun. Notice that when the Moon is in one of its slimmest slivers of Sun reflection, sometimes we can see the rest of the Moon's surface very dimly showing. That's reflection of light from the Earth bouncing off the Moon, back to us... "Earthshine." This complicated and somewhat random arrangement of Moon-Earth explains Observation 4.

3.4.1.5 Observation 5: The Sun

It sure looks like the Sun "rises" in the east and "sets" in the west and of course we speak that way. But we know of course that this apparent motion is due to the fact that the Earth is rotating toward the sun and illuminating our surface and then away leaving us dark. But the Sun's appearance is a little more nuanced than that. That 23.5° inclination of the Earth's rotational axis relative to the ecliptic plane means that as we move around the Sun in our "year" at one point the Earth's axis is pointed *away* from the Sun (our winter) and six months later, it's pointed towards the Sun (our summer). Ironically, that winter configuration is when the Earth is physically closest to the Sun and the summer configuration, it's further away. The warmth that the Earth's surface experiences is a matter of how dense the Sun's rays are when they illuminate the ground—when the axis is tilted toward the Sun, the light-intensity per unit area is highest. The tilt of our axis also explains why the Sunrise seems to happen at a different point on the eastern horizon every morning and why the Sun's position at noon—when it's the highest in the sky—changes every day. Again, when it's summer, the Sun is up higher and stays up longer, and in the winter it's the opposite.

Since the summer is when the Earth's axis is pointed towards the Sun, but is further away, our summer is longer than our winter. Because of the elliptical path, when the Sun is closest (winter), it's moving faster than when it's further away, it's slower. That the seasons are not the same length was confusing to anyone trying to explain the sky.

How high the Sun appears at noon depends on where you're watching from the Earth's surface. When it's at its highest point during the year, it's the summer solstice and directly above the Tropic of Cancer, which happens on June 20th, and the day that it is the lowest is December 21st, called the winter solstice when the Sun is directly above the Tropic of Capricorn. The sunset on the summer solstice is 23.5° above due west and at the winter solstice, it's 23.5° below due west. Every day between those extremes, the Sun takes a slightly different path in the sky during the day within the lane of the ecliptic. When the Sun appears to rise exactly at due east, or set due west we call that the spring (or vernal) equinox on March 21st and fall (or autumnal) equinox on September 23rd. On those days, the Sun is directly above our equator.

Without modern astronomical techniques, the Greeks (and everyone for the next 2000 years) had their hands full trying to explain what they saw.

3.4.2 Greek Astronomy, Presocratics

Let me remind you of the Presocratic cosmologies that I touched on in Chapter **??**. As I emphasized, what we know of the Presocratics is 2nd, 3rd, 4th. . . hand, but Aristotle was the first to systematically chronicle their approach to astronomy and cosmology.

Attributed to Thales was a picture of a flat Earth floating on water, surrounded by a river. He inherited this story from even more ancient Egyptian ideas but used it to his advantage in order to explain how earthquakes happen. So for him, MOTION BY THE EARTH was nil. In fact in Homer's *Iliad*, the description of the shield Achilles used in his famous battle with Hector at Troy contains a number of astronomical allusions, including the Oceanus river around its rim and reference to a "starry," solid dome held up by pillars.

His student Anaximander made a brilliant leap to an entirely different picture in which the Earth (a squat cylinder for him) is not held up by anything, but at the same time it's not moving because, he said, there's no particular reason to move since all around it is the same:

> "The earth is aloft, not dominated by anything; it remains in place because of the similar distance from all points [of celestial circumference]." attributed to Anaximander

This is either reasoning by a symmetry argument or reasoning by reference to balanced forces, or both. Either way, it's very much a mathematical inference.

> "Even if we knew nothing else concerining its author, this alone woudl guarantee him a place among the creators of a rational science of the natural world." (\cite kahn1994).

So for Anaximander, MOTION BY THE EARTH is zero as well.

In fact, most of the Presocratics believed that the Earth is flat, including many of those who came after Parmenides.

Parmenides had a number of original ideas about the heavens. He was the first to conceive of the whole universe as being spherical and finite.

> "...like the mass of a well-rounded sphere, from one middle, equal in every respect." Parmenides

He was apparently the first Greek to note that the Moon must be spherical and was even poetic about it:

> "[the moon is a body] shining by night, wandering around earth with borrowed light..." Parmenides

"Borrowed light" is a pretty good phrase. If the Moon "borrows" its light from the Sun and doesn't emit it on its own, then the shape of the phases of the Moon lead to the spherical shape conclusion. Thales is sometimes credited with this observation, but it's not mentioned in any of the commentaries on him from before Parmenides. It was traditional to credit Parmenides with extrapolating from a spherical Moon to declaring that the Earth, too, is spherical. But that's not authenticated and Pythagoreans' claim to a spherical Earth is perhaps more likely. After the Eleatics, Anaxagoras and Empedocles both assume that the Moon reflects the Sun's light and even formed a model for eclipses.

Ironic, isn't it that Parmenides can perhaps be credited with a scientific discovery— one that requires observation— when we tend to think of him as anti-scientific (which is quite a feat given that science really hadn't been born yet).

The Pythagorean team extrapolated their fondness for the number 10 into their cosmology out of southern Italy to Greece through Philolaus (recall, the one to first write down Pythagorean thoughts) and and we learn of their cosmology largely through Aristotle, and his students. In the haze that followed over centuries, the spherical shape of the Earth was attributed to Pythagoras, but that too is sometimes disputed. What's not in dispute is that he (they!) did take the Earth to be spherical and his (their!) model was the first in which the Earth moves.

There was a first version of his cosmology in which the Earth is at the center of the universe and inside of the Earth is a "central fire" or "'Hestia," in homage to the immobile goddess of the hearth. But that morphed into the cosmology that I described in Chapter **??** with the central fire now a central object of its own with the Earth —and the counter-earth— plus all of the celestial objects revolving around it. Figure 3.9 (a) shows a rendering of the Pythagorean idea.

The idea of a cosmology based on *regular, circular motions* is due first to the Pythagoreans who in effect, kicked off two millennia of astronomical and physics research. Motions are for all of the celestial objects!

So, to sum up: our standard Pythagorean cosmology likely comes from Philolaus who allowed for the MOTION BY THE EARTH to be in circles, orbiting around a central point in the cosmos.

3.4.3 Plato's Two Models

Recall that in the *Republic* proper education of the Guardians included astronomy which led to the controversial idea that Socrates/Plato denounced observational astronomy for a contemplative exercise determined to visualize and understand the true motions of the celestial objects. His first idea borrowed was from Pythagoras in broad ways, and actually serves as maybe a beta-version of the final Platonic model described in more detail, and with more realism in the *Timaeus*. Which is full-on Pythagorean.

Students from philosophy to political science study the *Republic* and are often surprised by its ending: the *Myth of Er*. Socrates is trying to motivate why someone should live a good life and relates a cosmic carrot-and-stick story, not unfamiliar to other religious admonitions. Er is a soldier who was killed on the battlefield and does what all deceased do... they go to a place where their lives are evaluated, not by St. Peter at the Pearly Gates, but by four judges in an ante-room with four doors, two up and two down. Er is told to wait and watch what happens as he's got a job to do: after 12 days he's to be resurrected from the dead, dramatically it seems on his own pyre before it's lit.

> "Once upon a time he died in war; and on the **tenth day**, when the corpses, already decayed, were picked up, he was picked up in a good state of preservation. Having been brought home, he was about to be buried on the twelfth day; as he was lying on the pyre, he came back to life, and, come back to life, he told what he saw in the other world." Plato, *Republic*

(Why 10 days? Well, some Pythagoreanism is maybe showing?) Er watches good deceased people sent upstairs, and good people coming downstairs to go back into the world as Pythagorean reincarnation is a part of the whole cycle of life. Their spirits remain intact, but are housed in different bodies (remember the remembrance from the *Meno*). The other two doors are for those who's past lives are unsavory. Some are fined and punished and sent back to the world to try again, but some remain in the underworld which is a less than pleasant place.

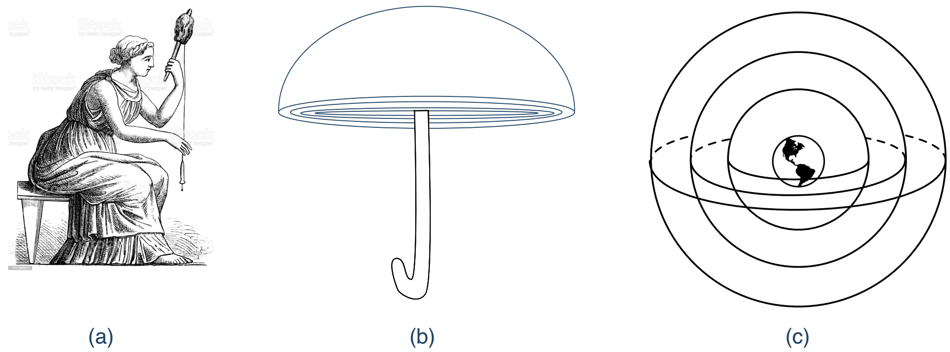


Figure 3.8: The figure in (a) is a Roman sketch of a woman spinning wool using a spindle and whorl, which is the weight at the bottom with a hook. The image in (b) is Plato's description of the whorl actually hollowed out with nested layers of whirl-shaped half-spheres. The image in (c) is the cosmos that the onion-layered whorl represents. I've not included all eight of the spheres in my cartoon.

Er mingles with some of the lucky ones and they observe a spectacle— a pillar of light that extends to the heavens which when they look closer, appears to be what we'd call today an umbrella structure. Plato describes a spindle and whorl used for spinning wool. Figure 3.8 (a) shows a Roman woman spinning wool with the weighted whorl at the bottom which spins as she works. Figure 3.8 (b) is the umbrella-like structure (the whorl upside down) that Socrates describes:

> "Its shape was that of (whorls) in our world, but... it was as if in one great whorl, hollow and scooped out, there lay enclosed, right through, another like it but smaller, fitting into it as containers that fit into one another, and in like matter another... There were eight of the whorls in all, lying within one another..." Plato

The eight "containers" are hinted at in my sketches in Figure 3.8 (b) and the whole is abstracted as nested spheres in Figure 3.8 (c). He also tells you how they move and the sounds that they emit as a Siren sits on each sphere and sings a tone. The nested whorls actually have a structure that's tied to the musical scale—again, homage to the Harmony of the Spheres that only Pythagoras could hear.[5]

This is Plato's (and the worlds) first three dimensional cosmological model and it's shown in Figure 3.9 in the style that we normally see ancient cosmic players.

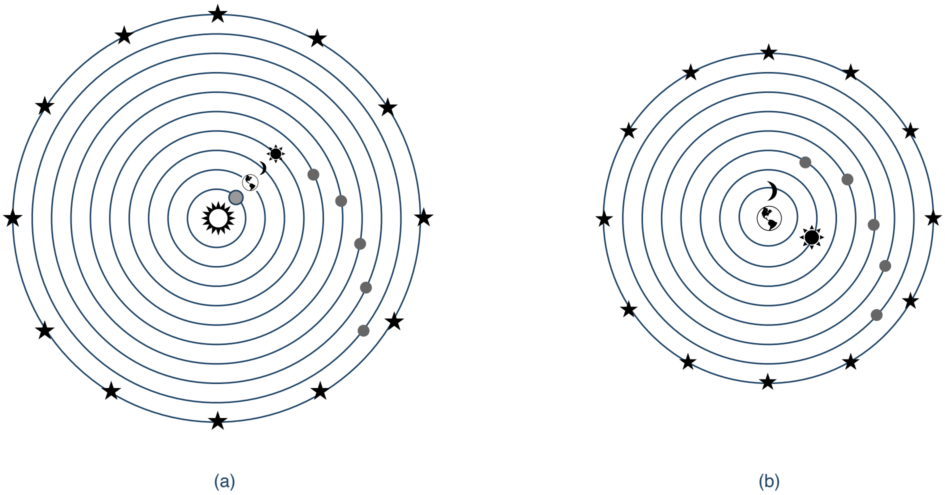


Figure 3.9: (a) shows the Pythagorean system and (b), the Platonic system as described in the *Republic*.

Figure 3.9 (a) shows the now familiar Pythagorean cosmology with the central fire in the center, with the counter-earth, then Earth, Moon, Sun, and the outer five planets, and finally the outer region with all of the stars. Figure 3.9 (b) is Plato's rendition along these same lines. Now Earth is in the center (a "geocentric" system has the Earth as the central object), with concentric spheres of the Moon, Sun, the outer planets, and again, the stars on the furthest shell, which Socrates says is "speckled." So, Plato's first cosmology has MOTION BY THE EARTH as zero.

Plato must have realized that the Er model wasn't sufficient. For example, everything is on one plane, so the motions of the planets, Sun, and Moon through the night sky in the ecliptic would have the same rates as the stars themselves and the ecliptic and celestial equators would have been the same. His attempted to fix this led to the very complex solution from the *Timaeus*.

The *Timaeus* is another story, literally and figuratively. We discussed the material and almost chemical aspects of the use that the Craftsman made of the four solid geometrical pieces that make up our "stuff" but Socrates teases out of Timaeus—the Pythagorean—a geocentric picture that's more sophisticated than that planar version from the Republic. Now Plato becomes mathematical, in a very Pythagorean way.

[5] I've shown the spacings as equal, but the plan was different from that.

The spheres now are a living being, moving at speeds dependent again on their distance. But numbers are the cause:

> "And he began the division in this way. First he took **one portion** from the whole, and next a **portion double of this**; the **third half as much again as the second**, and **three times the first**; the **fourth double of the second**; the **fifth three times the third**; the **sixth eight times the first**; and the **seventh twenty-seven times the first**. Next, he went on to fill up both the double and the triple intervals, cutting off yet more parts from the original mixture and placing them between the terms, so that within each interval there were two means, the one (harmonic) exceeding the one extreme and being exceeded by the other by the same fraction of the extremes, the other (arithmetic) exceeding the one extreme by the same number whereby it was exceeded by the other." Plato

Okay the numbers seem arbitrary. But there's an algorithm:

- one portion of the whole: ○, 1
- double of this: ○○, 2
- half as much again: ○ ○ ○, 3
- double of the second: ○ ○ ○○, 4
- three times the third: ○ ○ ○ ○ ○ ○ ○ ○ ○, 9
- eight times the first: ○ ○ ○ ○ ○ ○ ○, 8
- twenty-seven times the first: ○, 27

Now manipulate:

- The first four are the famous 1,2,3,4. So they have a job to do:
- Square each of the first numbers—remember, 1 is not a number— (Greeks knew how to multiply): and you get 4 and 9.
- Cube those same first two important numbers: and you get 8 and 27.

So all of the numbers in that excerpt are some manipulation of 2 and 3. Remember, that the Pythagoreans only had use for three dimensions, so Plato stops his calculation with cubes. So collecting all of the numbers, but now into even and odd strings (remember, 1 is special and neither even nor odd for Pythagoreans and apparently also, for Plato):

- 1, 2, 4, 8
- 1, 3, 9, 27

Then, Timaeus says that if you take the number strings and fill in the arithmetic and harmonic means between each of the numbers, you actually construct the intervals of the diatonic musical scale. More Music of the Spheres. Whew. Wait until we get to Kepler.

Then the Craftsman did some rearranging. He

> "This whole fabric, then, he split lengthwise into two halves; and making the

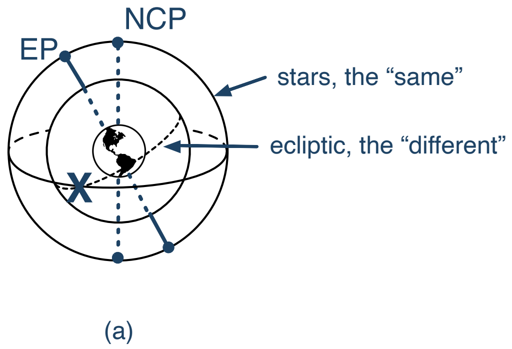


Figure 3.10: CAPTION

> two cross one another at their centers in the form of the letter X, he bent each round into a circle and joined it up, making each meet itself and the other at a point opposite to that where they had been brought into contact.
>
> "He then comprehended them in the motion that is carried round uniformly in the same place, and made the one the outer, the other the inner circle. The outer movement he named the movement of the Same; the inner, the movement of the Different. The movement of the Same he caused to revolve to the right by way of the side; the movement of the Different to the left by way of the diagonal." Plato

Figure 3.10 shows what this is about. The strip that Plato called the "Same" is the equator of a sphere that doesn't change, the furthest most out. That's the celestial sphere and its axis I've called the NCP (north celestial pole). The other strip is the equator of another sphere that Plato called the "Different" and makes an "X" where it crosses in two places with the Same. Inside of this strip, he's broken it into segments which differ in radii from the center according to the six spaces between the pairs of: 1, 2, 3, 4, 8, 9, and 27. Of course, the Different is the ecliptic and the individual bands in which the planets reside are at those radial spacings are the locations of the Moon, Sun, Mercury, Venus, Mars, Jupiter, and Saturn.

Of course, this is a little mad. But it's Plato being an empiricist when he doesn't really want to admit it! He had to fix the Er model in order to fit the apparent nature of the passage of the planets, Moon, and Sun in a plane that's different from the rotation observed by the stars.

This still leaves a lot unlearned about our list of what we know. But don't worry, Plato had a student. Maybe the most significant mathematician before Archimedes.

3.4.4 Eudoxus' Model

Eudoxus of Cnidus (−408 to −355) was born and educated in Cnidus on the Black Sea. He studied first as a physician and spent two months in Athens studying with Plato supported by what we'd call a scholarship from a former teacher. However, he left (prematurely?) and studied medicine and astronomy in Egypt for a year and a half and then went to Tarentum, Pythagorean territory, and studied with the Pythagorean, Archytas who also tutored Plato, as you'll recall. Plato's and Eudoxus' mathematics have common origins. But Plato's skill was no match for Eudoxus'.

Ever moving around, Eudoxus created his own popular school in the eastern Greek colonies, but actually went back to Athens in −368, again, to work with Plato bringing students with him. Th nature of their relationship is not understood and accounts differ about a lack of respect between them and a sense of competition.

There are stories that Plato assigned the problem of working out the astronomy of his nested spheres more completely, but this too is disputed with the argument that Eudoxus took this on his own, influenced entirely by the Pythagorean emphasis on spheres are perfect shapes along with the co-centerd circular orbits of celestial objects. The real situation is surely a combination of Plato's idea of the nested spheres, an Earth-centered model, and Pythagorean commitment to a mathematical solution.

Eudoxus was a ground-breaker in mathematics with ideas that, through their inclusion in Euclid's best-selling mathematics book of a few years later, set the stage for 1500 years of development. Some of what he did is described in More of Eudoxus in Section 3.5.0.2. Our concern here is his model of the universe, picked up by Aristotle and promulgated into the Baraque era. By the time Eudoxus had returned to the Academy, he was surely familiar with the *Republic* and probably *Timeaus*. The timing of his work with those of Plato is not clear, but the consensus is that it probably came after those late Platonic tests.

Plato clearly struggled with his vision of what the True cosmology should be and how to account for even the most rudimentary celestial appearances. Once he'd fixed the ecliptic path for the Sun and planets in their individual circular bands, he still needed to explain retrograde motion. And he knew it:

> "...as for the dances of these and how they relate to each other, the backward-cycles and forward-progressions of the circles to each other...to speak without visual representations of these same would be a vain effort." Plato

So, he gets it...but without a solution, let's give up and move on ("vain effort")?

Well, Eudoxus came up with a brilliantly complex solution and it's not known what Plato thought of it, but it's clear how Aristotle reacted: he adopted it and made it his. The model is intricate, so let's go to the box and work out the inner workings of the idea and then skip to the end. Look at Figure Box 3.12 on page 31. After you've read the material in that Box, return to this point ☟ and continue reading.

Figure Box 3.12 forms the tool-kit that Eudoxus used to construct a full model of all of the planets. Instead of the planets carried in bands around the Earth, they ride on the equators of spheres. That's a clever idea because the spherical shape makes it possible to connect the motion of one planet to the other celestial shapes. Bands would not allow that. The two spheres show in the box aren't enough to replicate the observations. The inner two form the minimal number of moving parts that will be unique to every planet and they need to be each embedded inside of two other spheres, one for the ecliptic whose equator include the rough paths of the planets and the other is the celestial sphere which includes the motions of the stars around the Earth every nearly 24 hours. The punch-line is in Figure 3.11. Let's take it slow:

- (a) is a slightly different rendering of Figure 3.12 (b)
- (b) is an abstraction of (a) taking out some of the lines that suggest a solid sphere, for clarity
- (c) includes the sphere of the ecliptic (EP for Ecliptic pole is shown) with axis of rotation CC′. Notice that it's attached to the outer sphere of Eudoxus' toolokit pair.
- (d) includes the sphere of the outer stars, the celestial sphere (NCP for the North Celestial Pole is shown) and the ecliptic shere is attached to it.

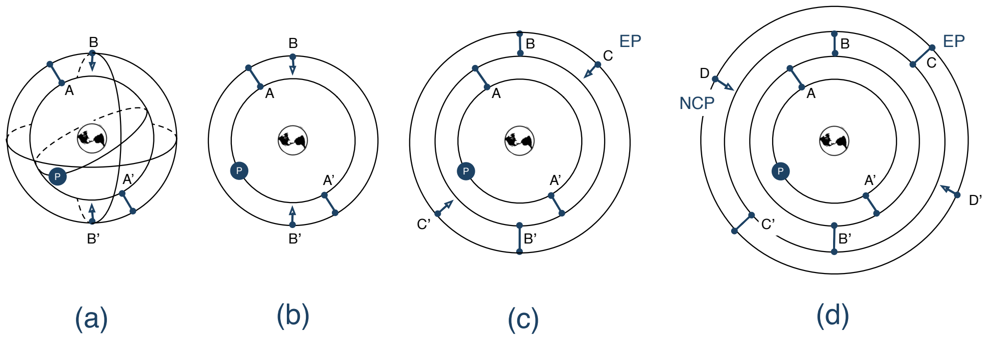


Figure 3.11: CAPTION

Now all of these separate motions are coupled... and that's just for one planet! By tuning the inner two sphere rotation speeds and the inclination of the inner axes, the motions of the planet can be made to do the figure-eight dance at just the right time of year and with the right elongation in the sky.

Each planet requires four spheres and the Sun and Moon require three each, plus the Celestial sphere, or 27 spheres to do the job. This was a mammoth intellectual puzzle that Eudoxus solved with those relatively simple pieces of interlocking spheres.

It still didn't quite do the job as well as it might and in the best tradition of what Kuhn would have called Normal Science or Lakatos, a Research Programme, someone tried to make it better without starting over. **Callippus of Cyzicus** (-370 to -300) was a student of Plato's and worked with Aristotle and he worried about details like the fact that the seasons are not the same lengths (which we know is because of the Earth's elliptical orbit). So this is getting into the details, but trying to make it more accurate. Calliopus added two additional spheres for the Sun and Moon and one each for Mercury, Venus, and Mars for a total of seven more. So now the model has 34 spheres.

Was it all just an exercise in geometry?

FIGURE BOX 3.12

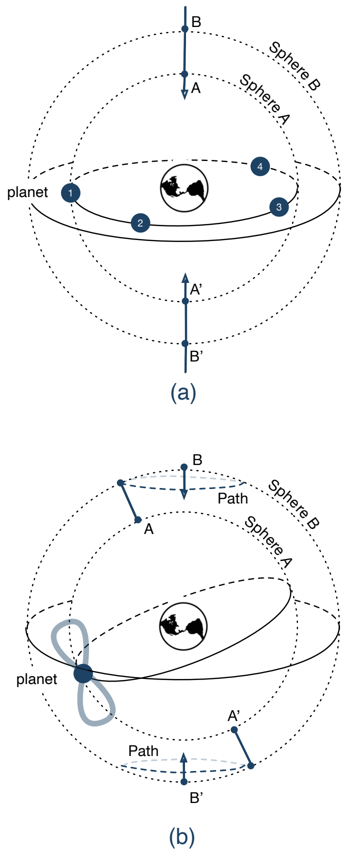


Figure 3.12: CAPTION.

The model that Eudoxus created is an impressive bit of geometry mixed with inspired imagination. It's the famous "nested spheres" model that made it all the way to the Baroque as an explanation for the odd motions of the planets. In a very modern way, it's full of parameters that could be tweaked to make it fit the observations...some of which he made himself at the observatory he created in his school before he returned to Athens.

Imagine taking two hoops, one of which is slightly smaller than the other and is attached inside the larger one across their mutual diameters. Figure 3.12 (a) shows this with a "planet" attached to the equator of the inside hoop. Now if we spin that hoop around its axis AA' the planet will follow a circle from position 1 through 2, 3, 4 and so on. This spinning essentially defines a sphere, Sphere A, here centered on the Earth. If the two hoops are attached, and if the outer hoop spins around its axis, BB', creating the surface of Sphere B, then the motion of the planet will be the sum of the two speeds at the hoop pair equators. So if the outer hoop spins at the same rate as the inner hoop, but in the opposite direction, then the planet would appear to the Earth to remain stationary at position 1.

Now imagine that the inner hoop is attached at a point offset on the surface of the Sphere B as shown in Figure 3.12 (b). Now when Sphere B spins, it takes the AA' axis of Sphere A around with it tracing the path shown. In addition, if Sphere B spins while its following that path independently, the motion is a complicated figure eight pattern as shown. Eudoxus figured this out and named the shape a "hippopede" which is "horse fetter" in Greek. (A fetter is like a chain.) Now there are many variables at work which would alter the shape of the hippopede: the speeds of the two spheres and the angle at which AA' axis of Sphere A is inclined to the BB" axis of Sphere B.

Now go back to page 28 and pick up where you left off.

3.4.5 Aristotle's Model

By now we know that Aristotle insisted that natural motion was straight and since he wanted to account for the celestial objects which obviously don't move in straight lines, then those objects must be special and made of different stuff: quintessence. The hierarchy is such that if all of the motions of all substances were separated into their individual natures, then all of the Earth would condense at the center of the universe. That sphere would be surrounded by a thick sphere of water. That would be surrounded by a sphere of air and then fire. So a spherical sandwich of the four terrestrial elements filling up the whole volume below the Moon, the "sub-lunar realm." Only outside of that larger sphere was the realm of the celestial objects, individually made of quintessence.

So Aristotle's universe is of finite volume in space all the way to the outermost starry sphere, like that of the Pythagoreans. Furthermore, it's always been there. So, a spatially finite spacial event, but an infinite temporary extent. But Aristotle did not like the Pythagorean picture in which the Earth revolves around a central point and his argument — shared by others — was a scientific argument.

3.4.5.1 Critique of the Pythagorean Cosmology

Look at a point across your room with one eye closed and put your finger in front of you and notice what's behind it on a wall or distant surface. Imagine a line extending from your eye, through your finger, and ending on the far wall. Now switch eyes and notice that the what's behind seems to have moved. Again, a line from eye-finger-wall, but a different line from the other eye. These two lines are at crossed directions as can be clearly seen if you open and close each alternate eye successively. The background will appear to jump from side to side relative to your finger. This is called "parallax" and it's because your eyes are inches apart enough that the lines of sight from each are slightly different: that is, those lines are at different angles.

If the Earth is orbiting a center, and not itself the center, then at one point of the year a particular star would appear as a line at a particular angle. Set up a structure that points at that star. Then much later, when the Earth is on the other side of that center, look for that same star and it will be at a completely different angle relative to the structure you erected when you first looked. Stellar parallax, is the name of this phenomenon.

Nobody saw it and there are really only two explanations. Either the Earth does not move around a center of revolution, or the Earth is fixed in space. For the Pythagoreans, that center was the Central Fire and for others, the Sun. Oh, wait, There's a third explanation: the stars are so far away that parallax is not visible.

Probably you know the real answer: it's the third one and it took until the 19th century to actually observe stellar parallax. The stars really are that far away. But

Aristotle and most others of that time thought that distance to be impossibly far away, that the universe could not be that large, and so they concluded that the lack of observed stellar parallax was the reason that the Earth must be stationary.

This is one of two reasons why for Aristotle— and nearly everyone who followed for nearly 2000 years afterwards, MOTION BY THE EARTH was zero. The Earth must be stationary.

3.4.5.2 Details of Aristotle's Model

Ever the mechanist, Aristotle wanted a mechanical model of the heavens and wasn't content to just live with a mathematical description. He took over the Eudoxus/Callippus spheres and worried about how to actually imagine real material objects.

But Aristotle was about "how" do they move?

3.4.5.3 The Physics of Aristotle's Model

Ut iaculis interdum leo varius tincidunt. Phasellus imperdiet ullamcorper feugiat. Aenean commodo ullamcorper sodales. Aliquam ut facilisis erat. In pulvinar urna sem, viverra porta enim pulvinar eu. Sed condimentum, tellus ut tristique feugiat, augue turpis consectetur mi, vitae interdum lectus leo in nulla. Nulla ac magna mauris. Vivamus pellentesque libero at tortor volutpat ultrices. Donec mattis velit ac mauris accumsan commodo.

Donec ut fringilla dui, ut maximus lacus. Duis scelerisque quam quis ornare sagittis. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos himenaeos. Duis ex justo, dignissim quis dictum sit amet, cursus eu mi. Maecenas posuere risus erat, vitae tempor lacus aliquam nec. Vivamus euismod sodales lobortis. Vivamus sit amet metus velit. Duis viverra, ligula vehicula ullamcorper interdum, est felis sollicitudin ante, nec convallis nisi neque nec ipsum. Quisque auctor sem sed diam lobortis venenatis. Curabitur bibendum porttitor dolor, quis congue erat tempus vel. Mauris viverra metus vel pulvinar semper. Morbi sodales posuere ultricies.

3.5 More of Plato's, Aristotle's, and Ancient Astronomy's Stories

3.5.0.1 Modern Day Platonists

> Linking numbers and geometrical patterns to features of the world is another modern concept. The model that we trust for the fundamental constituents of matter is the Quark Model. It is based on a complex idea of mathematics and produces geometrical patterns similar to the dots above, the location of which designates properties of the quarks.

Our best description of the universe at the quantum level is described by a mathematical function called the "wave function." This is a solution to a basic equation that describes its behavior in space and time and when squared, gives the probability that a microscopic entity (like and electron) will be at a particular place at a particular time. It works very, very well this wave function idea. It's also completely undetectable. It's certainly "in" the objects of nature.

Yes, the wavefunction (and its relativistic cousin, the relativistic quantum field) that describes the world, from chemistry to the Large Hadron Collider, is precisely calculable, but completely off-limits to any detection. It gives you probabilities, not certainty. Now, if that's not an example of mathematical fabric of the universe, then I don't know what is! The wavefunction's "reality" is that it's just a mathematical function!

3.5.0.2 Indebtedness to Eudoxus

Appendices

Appendix A

Greeks Technical Appendix

A.1 Zeno's Paradox

A.2 Proof of Pythagoras' Theorem

A.3 Socrates' Geometrical Problem

Appendix B

Plato–Aristotle Technical Appendix

B.1 Digital Gates

One more bit of insight makes really complicated electronic digital design possible and came from the very strange, yet enormously influential philosopher **Ludwig Wittgenstein** (1889-1951) who invented the concept of the "truth table," which we've already used in Table **??**. It's an orderly setup of all possible starting places (for two valued propositions) and their results when various operations are applied. Let's look at a three. True now is the bit 1 and False is the bit 0:

- The NOT operation: If I have an A then NOT–A creates the opposite of A. If we work in the zeros and ones world, then if A=1, then NOT–A = 0. The symbol for NOT is usually   so if A = 1, then   A = 0. (The   symbol is the common notation used by logicians. Engineers and physicists would write $\overline{A}$ to represent the result of NOT–A.)
- The AND operation: This is between two states of, say, our A and B. In order for A AND B to be true, both A and B must be true—1— themselves. Otherwise, A AND B is false, or 0. The symbol for AND is $\wedge$ So A AND B = $A \wedge B$.
- The OR operation: This is the combination that says A OR B is true if either A = 1 or B = 1 and false otherwise. The symbol for OR is $\vee$.

There are 5 other logical combinations. Table B.1 shows the truth table for AND and for OR. In the first set, the AND process, I've stuck to our T and F language, but the rest uses the zeros and ones language of engineering and binary arithmetic.

Table B.1: Truth tables for the AND and OR functions plus the construction of Modus Ponens. The **symbol for AND is** $\wedge$, the **symbol for OR is** $\vee$, and the **symbol for NOT (negate) is** . Notice that (A) $\vee$ B is a construction out of AND and NOT of the conditional that's the first premise of Modus Ponens.

AND			OR			Combined function				=
A	B	A $\wedge$ B	A	B	A $\vee$ B	A	B	A	(A) $\vee$ B	If A then B
T	T	T	1	1	1	1	1	0	1	= 1
T	F	F	1	0	1	1	0	0	0	= 0
F	T	F	0	1	1	0	1	1	1	= 1
F	F	F	0	0	0	0	0	1	1	= 1

Let's look at the first line so that you get the idea.

For AND:

- A is T and B is T and the AND of two T's is itself a T.

For OR:

- A= 1 and B = 1 and the OR of 1 $\vee$ 1 is 1.

Then the combination:

- repeating the A and B conditions from the first and second columns A= 1 and B = 1.
- taking the NOT of A, takes 1 into 0.
- combining that with the B in an OR results in A $\vee$ B = 0 $\vee$ 1 = 1

The last column shows that this is the same as the first line result of our picnic decision making in Table **??**. The rest of Table B.1 builds that combination for all possible A and B states, first by negating A and then combining that by "ORing" it with B. The last column shows the original "If A then B" premise that we worked out about raining and wetness. They formula and our reasoning lead to identical conclusions.

Sources