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How the Stories of Motion and Light Became the Special Theory of Relativity, v1:

Pythagoras to Ptolemy

From the Greeks to Einstein

Volume I

From Pythagoras to Ptolemy

It may have once been the case that all roads lead to Rome, but for most of western philosophy, physical science, and mathematics, all roads lead *from* Greece. This volume is the first stop in our path towards Einstein's Special Relativity: our MOTION themes start with the Greeks, eventually centered on Plato and Aristotle. Likewise, but to a lesser degree, ideas about LIGHT frustrated the Greeks without much analysis. This volume will be different from subsequent ones, as its stories are of a number of people, not all of whom would be classified as scientists today. You'll see why. But we'll close this volume with the one of the earliest quantitative astronomers: Claudius Ptolemy.

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Chapter 3

The Most Important Mathematician You've Never Heard Of : Eudoxus and Greek Astronomy

"We shall try to note down everything which we think we have discovered up to the present time; we shall do this as concisely as possible and in a manner which can be followed by those who have already made some progress in the field. For the sake of completeness in our treatment we shall set out everything useful for the theory of the heavens in the proper order, but to avoid undue length we shall merely recount what has been adequately established by the ancients. However, those topics which have not been dealt with [by our predecessors] at all, or not as usefully as they might have been, will be discussed at length, to the best of our ability."

- Ptolemy, *Almagest, Book I, 1*

The passage above is the opening stanza of the last verse of Greek astronomy and is at the threshold of a strange 1500 year dance between the rigorously mathematical (Ptolemy) and achingly abstract (Aristotle) models of the universe. How we got there is the purpose of this chapter as it lays the ground work for two millennia of mutually supportive and mutually conflicting views of motion by the Earth, motion on the Earth, and motion in the heavens .

I'll bet that many of you have seen the solar system arrangement as imagined by Copernicus (surprises await in Chapter **??**) with the Sun in the center and all of the planets, including Earth, obediently orbiting it in perfect circles. What he challenged was the ancient, and universally-held idea, that it's the stationary Earth that's in the center of the universe, not the Sun. Fascination with that picture is prevalent in many decorated medieval manuscripts through the centuries and one of the earliest is shown in Figure 3.1. This is from a 10th century edition from the British Museum of a poem by the Greek poet, **Aratus** from about −275 called *Phaenomena* which was named for a book of the stars and constellations by the Greek mathematician, Eudoxus, of probably a century before. It was he who created that 2000 year old "geocentric" model of the universe—one in which the Sun, Moon, planets, and stars all orbit around the stationary Earth. We will see that the poem *Phaenomena* figures crucially in the history of astronomy two centuries after Aratus wrote it, so watch for it reappearing as we proceed.

I took some pains in the last chapter to underscore that models of MOTION ON THE EARTH belong in Aristotle's corner as he really invented the dynamics of motion. But while we tend to ascribe that geocentric model of the universe to him as well, he borrowed it lock stock and barrel from Eudoxus and Plato.

Figure 3.1: Aratus the poet lived about a century after Eudoxus (and hence, Aristotle) and turned his astronomy book into a poem. Later, Cicero translated it and this 10th century manuscript is an illustrated copy of that work. https://sarahjbiggs.typepad.com/.a/6a013488b5399e970c01bb07c8696d970d-pi

This "geocentric" picture became the authoritative, unquestioned dogma of the medieval and renaissance periods even though it made no numerical predictions and was known since Aristotle's time to be just wrong. The other game in town was precise and predictive and was the model of the Greek astronomer, Claudius Ptolemy, from the first century, CE.

The Greek world—indeed, the whole world—was radically and violently altered by Alexander the Great and between Aristotle and Cleopatra, astronomy become an experimental and quantitive science. The culmination of Greek astronomy came after Greek–everything became Roman–everything and just before the Roman Empire began its decline. One last Greek, in our long string of Greek philosophers, mathematicians, and scientists remained and we'll close our chapter with Ptolemy's "turn-the-crank" model for MOTION IN THE HEAVENS.

A game that many scientists play is to trace their scientific lineage back for centuries—their major professor's professor and so on (there's an app for that). I followed mine back through centuries and found that I descended from Copernicus![1] I'd like to think I've made him proud.

Sometimes it turns out that someone's student ends up in the history books. But not many students actually take over the known world by force!

When Plato died, the Macedonian King Philip II "encouraged" Aristotle to relocate to Macedonia in order to teach his 13 year old son, Alexander. He set up a school, taught Alexander (and perhaps the future general/king, Ptolemy) for three years, and then stayed for seven more before returning to Athens where he started his school, the Lyceum. By this time the teen-aged Alexander was already on the battlefield and with his father, had occupied the entirety of the Peloponnese. So Athens was once again ruled by outsiders—now connected to Aristotle!

After Philip II was assassinated,[2] and Alexander, soon to be "The Great," ascended to the throne and began his brutal lightening-fast, nine year conquest of the entire western world: modern Turkey, the middle east, Egypt, Arabia, and all the way across Afghanistan to India, leaving military oversight over Athens and the rest of Greece. While he stayed in touch with Aristotle, sending him samples from all over Asia, his teacher became distant, put off by Alexander's adaptation of Persian customs, dress, and persona.

Alexander died in Babylon in -323 under suspicious circumstances and, within a year, Aristotle himself died at the age of 63 at his mother's family estate outside of Athens. His Macedonian connections had become dangerous and his adopted

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[1] Everyone I know seems to come from Copernicus. A mark that what he started had legs?

[2] Assassination, murder, and betrayal were a family hobby.

city turned on him: impiety was charged, a death sentence issued, and so he fled to his mother's home uttering his famous remark about the city not sinning against philosophy for a second time. In his absence, the Lyceum stayed active under new management for another century.

Alexander's senior commanders divided up the sprawling kingdom among a dozen generals and aides and they did what came naturally: they fought among themselves for 40 years. In the end, three kingdoms and a dizzying array of city-states were established: the survivors were Macedonia and Greece, Seleucia (roughly modern-day Iraq), and Egypt.

Hundreds of thousands of Greeks migrated into the newly acquired territories establishing an international Greek-ness of culture, arts, and philosophy which was the beginning of the **Hellenistic Age**.[3] The entire western world became "Greek." Of the two dozen cities that Alexander created or conquered named for himself, the "Alexandria" that mattered most to him, and to us, was the new Egyptian port city of Alexandria.

Egypt became unusually secure under Alexander's former body guard and general (and rumored Aristotle student), **Ptolemy I Soter** (−367 to −282) who eventually fashioned himself, "Pharaoh." He adopted Egyptian customs,[4] and was an intellectual of sorts, creating the first state-supported national laboratory and library. The "Alexandrian Museum" was a national facility devoted to research and among its first recruits was the mathematician, Euclid, who while in residence, wrote *Elements*, the most-read book in history, besides the Bible. For 2500 years, from Copernicus to Thomas Jefferson, mastering *Elements* was the route to mathematical literacy.[5] For centuries the Museum was home to scores of Greek scholars, all supported by the dozen Ptolemy's from the I$^{\text{st}}$ to the final one, Cleopatra.

The Library of Alexandria probably contained all of the manuscripts of the classical and Hellenic philosophers, poets, playwrights, and physicians. There was a hunger for knowledge of all sorts and agents of Ptolemy's library director searched every ship that docked, stealing or copying any books on board and renting or stealing manuscripts from all of the major cities.

Among the scores of Alexandrian scientists are the astronomers Eratosthenes of Cyrene, Aristarchus of Samos, and especially Claudius Ptolemaeus who will figure into our story, while only Heraclides of Athens, Hipparchus of Nicaea, and Apollonius of Perga played major roles outside of Alexandria. The Greek Ptolemy dynasty lasted 300 years until the legendary feud involving "the" Cleopatra (a common name for female Ptolemy-family successors), Marc Antony, and Julius Caesar. The Library and Museum lasted into the first five centuries CE until the Muslim conquests of the near east, north Africa, and Spain when it was eclipsed by

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[3] Often the pre-Alexandrian Greek era is called "Hellenic."

[4] including that of rulers marrying their siblings

[5] Ptolemy found it rough-going and asked for an easier way to learn it, but was told by the author that "...there is no Royal Road to geometry," a sentiment still applicable today.

great Muslim libraries in Baghdad, Cairo, and Cordoba in Spain.

3.1 A Little Bit of Eudoxus

Recall that Philolaus was the source of Plato and Aristotle's knowledge of Pythagoreanism—for example, the "Pythagorean" cosmology came through him or probably originated from him. Was he a student of Pythagoras? Their overlaps are nearly right in order to imagine that relationship, but that's controversial. He's certainly the closest we get to the great man so it's not far-fetched to continue the teacher → student theme that began this chapter: Pythagoras → Philolaus → Archytas → Eudoxus. Lunar craters are named after each which is not the normal teacher-student legacy. (Set the context with the timeline in Figure **??** on page **??**.)

Eudoxus of Cnidus (circa -408 to around -355) was the son of a physician and became one himself, but we know of him as a gifted mathematician and astronomer. As we'll see, astronomy and medicine were connected through astrology and mathematics and astronomy have always been kin, so these seemingly disparate skills go together. Cnidus was a city founded by Sparta on the southern Aegean coast of modern Turkey and was where he started... and finished, between which times he traveled all over the Aegean to study and teach. As a young man he went to Tarentum to study mathematics with the pre-eminent Pythagorean mathematician (and much more) **Archytas of Tarentum** (-428 to -347) who seemed like a sensible guy:

> To become knowledgeable about things one does not know, one must either learn from others or find out for oneself. Now learning derives from someone else and is foreign, whereas finding out is of and by oneself. Finding out without seeking is difficult and rare, but with seeking it is manageable and easy, though someone who does not know how to seek cannot find. Archytas, *fragment*.

Let's learn a little bit about him in Figure Box 3.2 on page 14. After you've read about Archytas, return to this point ☟ and continue reading about his student, Eudoxus.

FIGURE BOX 3.2

The image on the left is a famous engraving (by an unknown artist...maybe late 18th century) suggesting an ancient sentiment due to Archytas, a friend and competitor of Plato. Among the most famous arguments in cosmology is whether the universe is infinite or finite in size and Archytas had the first of many similar inspirations that the universe cannot be finite: He did a thought experiment, imagining traveling to its presumed edge and attempting to thrust his stick beyond that limit. If he could extend it, then, well, that's not the edge...and so he'd have to go further, repeating the experiment without end. This is a good example of the kind of intuitive cleverness that seemed to be built into this great Greek mathematician, politician, and military leader. The very model of a modern major—Pythagorean— general. Archytas was a committed Pythagorean and a mathematician of great skill. But he also he was a civic leader and an elected military general, in spite of Tarentum law, re-elected seven times because he never lost a battle. (Did I mention that Greeks fought constantly?) When he did step down, the army started losing.

Archytas was reported to be an even-tempered, cultured man who led Tarentum through a period of democracy and that Aristotle apparently wrote more (lost) books about Archytas than he wrote about any other person. There is some evidence that he wrote a book on mechanics and that he enjoyed making mechanical toys for children—very un-Plato-like in spirit.

His mathematical skills were legendary and he solved an old problem with mystical roots: Apollo sent a plague to the city of Delos and a delegation was sent to Delphi to learn from the Oracle how to rid themselves of the pestilence. The instructions were to take their cubical altar to Apollo...and build a new one with double its volume. This is called the problem of "duplicating the cube" (also called the Delian Problem) and it required cleverness on Archytas' part, beyond just geometry, which caused Plato to disparage his effort. Archytas contributed to many branches of mathematics and Euclid's *Elements* includes some of his proofs.

All in all, Archytas was the most accomplished Pythagorean of all and in the spirit of the opening to this chapter, we're indebted to him for his products, but also one of his students. The most accomplished of all Greek mathematicians before Archemides, Eudoxus, from whom 2000 years of cosmology originated.

Now go back to page 13 and pick up where you left off.

He seemed to not be able to stay in one place. After his mathematics instruction, he went to Sicily to study medicine, then by the age of 23 he went to Athens and stayed briefly (and apparently, unhappily) with Plato's Academy (rooming 7 miles away, so a long commute to lectures). After less than a year, he was back on the road to home in order to raise funds...so that he could travel even further! He went to Egypt with what we'd call a scholarship and studied astronomy there for 16 months, shaving his head and learning from the priestly-cast astronomers, before leaving for the northern modern Turkish Black Sea coast and the Greek colony of Cyzicus. By this point he's lecturing on his own and established a popular school and an observatory. With data from his observing in the north and from Egypt, he published his first book, *Phaenomena*, which was a compendium of star locations and *On Speeds*, of their motions. Recall that this is the subject of Aratus' important poem.

Around −368, during his 30s, he moved his school to Athens, by which time Plato was 60 years old and Aristotle had left for Macedonia. It was here, as the legend goes, that Eudoxus was challenged by Plato to form a geometrical model of the heavens. The legend is unlikely as by this point, Eudoxus was the mathematical champion of the Greek-speaking world and more likely to issue challenges, than accept them. Plato's mathematical skill was no match for Eudoxus' whose work was memorialized in a number of Euclid's *Elements*. As we'll see below his model was born and in various guises, persisted until Galileo, Kepler, and Newton.

He first calculated/measured the length of a year of 365 days and 6 hours. and it's Eudoxus' astronomy and cosmology that are our concern here and so let's work up to that with a review of the problems that everyone in antiquity faced when trying to describe what we observe from Earth and then work through Plato's ideas that formed an almost linear line of inspiration: from Pythagoreans, to Plato, and to Eudoxus.

3.2 A Little Bit of the Sky

The biggest export of Greek astronomy before the Romans was Aristotle's model of the cosmos with its Earth-centered ("geocentric") description of MOTION BY THE EARTH and MOTION IN THE HEAVENS . It became popularized, petrified, and deified when it was officially incorporated into Church dogma after the work of Thomas Aquinas in the late 13th century. So from that point until the Baroque era, Aristotle reigned supreme. He was revolutionary and inventive in so many areas, so it's amusing that his cosmological model had the longest run and that it was almost entirely due to Eudoxus. We'll dig a little deeper into their ideas as both were influential. But Aristotle had predecessors.

The stars seem innumerable and for millennia people have found recognizable images of animals and deities in the stellar patterns, the constellations; particular bright stars were given names; and that region in the sky at night that corresponds to the path of the Sun had special constellations called the zodiac. Babylonians and Egyptians in particular took notes on when stars or parts of constellations rose, and when that event occurred, what stars were directly overhead, and what stars were disappearing in the west. Patiently, each night for hundreds of years these observations were recorded, to become useful during the Hellenistic period.

3.2.1 What Ancients Saw and What We Still See

There are very few objective experiences that we can share with people who lived thousands of years ago. But if you watch the Sun’s path across your sky during a day and across months you’ll see exactly what individuals saw over many millennia. Further, if you look at the night sky over a single and many nights you’ll experience exactly the same things as all of prior humanity. We can disagree about a lot, but every human has experienced the same MOTION IN THE HEAVENS. You might even generate some of the same “why“ questions as they did and the Greeks were always full of “why“ questions.

Now suppose you’re indeed a smart Greek with time on your hands and able to spend years just recording what the sky presents to you during the days and nights. A few things would stand out...and if you were a patient and persistent observer nuance would start to emerge. In *Greek Astronomy, Today* in Section 3.8.1 I’ll “set the record straight” with modern explanations for each of these scenes and motions but here we’ll just observe. Let’s go out tonight.

The celestial sphere. Let’s look up after sunset and watch the stars’ motions through a night. Figure 3.3 is what we’d see. Here we have an observer looking south with the eastern horizon on their left and the western horizon on their right. Directly overhead is the **zenith** which would be 90° from all points on the horizon. Let’s follow one particular, familiar constellation.

Virgo, the “maiden” is the largest constellation in the zodiac and is most evident in the spring. Its shape presents two “legs” and two “arms” seemingly attached to a “body.” The downward “hip“ is Spica, one of the brightest stars in the sky. The two outstretched arms reach to the spectacular Virgo Cluster of thousands of elliptical and spiral galaxies. Our interest is more modest.

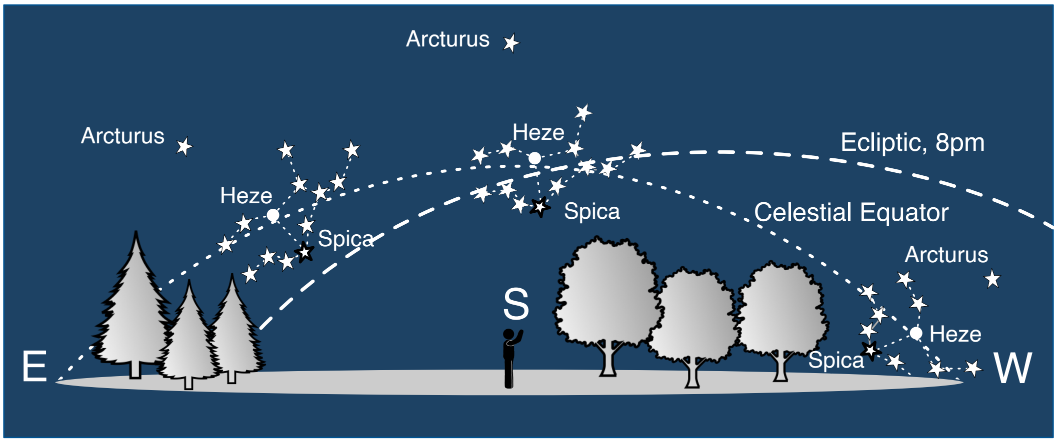

Figure 3.3: CAPTION

The naked-eye star, Heze, is joined at the other hip to Virgo, so to speak, and is actually two relatively modest stars appearing to us to be close together. What's useful for us is Heze's location because it traces out an important circular path. Figure 3.3 shows it as a dotted circle on March 19, 2024 from East Lansing, Michigan with three replicas of Virgo showing its positions from late in afternoon (invisible since the Sun is still up), to overhead about 9 PM, and then at about 2 AM when it sets. That dotted curve to which Heze appears to be attached is special, it starts directly in the east and ends directly in the west. Also pictured is Arcturus, the fourth brightest star in the sky which likewise follows another circular path which is parallel to Heze's. In fact, as you watch, you can imagine all of the stars in the sky following concentric, circular paths every night. Figure 3.4 (a) shows a time-lapse photograph of the northern sky where all of the circular star-trails are evident with the axis of all of those circles centered on the North Star, Polaris.

The most natural impression is that you're standing in the middle of an enormous 24 hour spinning sphere — the **Celestial Sphere**—with stars attached to its inside surface. If the Earth were to become transparent, you'd see the whole stellar panorama turning around you and its axis from Polaris to the other side below you in the southern hemisphere. Heze's path is special since that dotted line traces out the equator of that spinning sphere, the **Celestial Equator, CEq** as it's labeled in Figure 3.3.

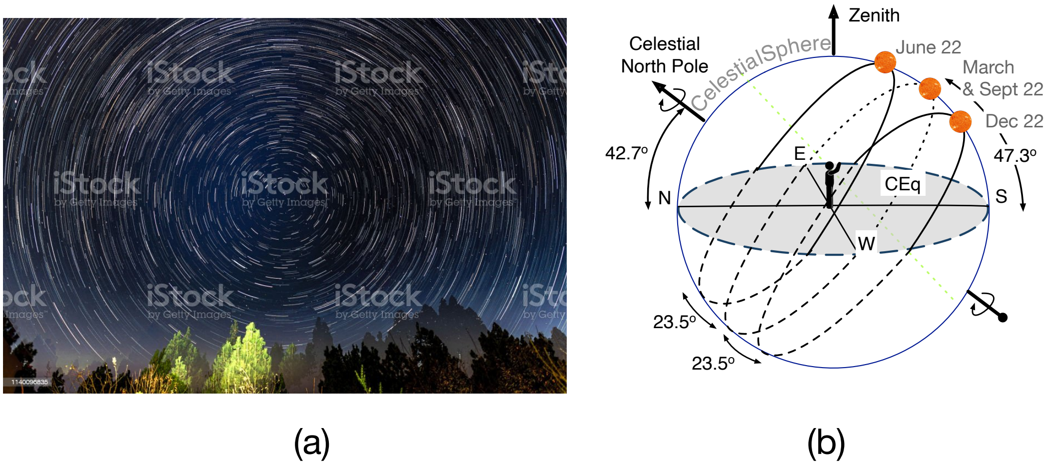

Figure 3.4: (a) A time-lapse photograph of the star images during a single night in the northern hemisphere are shown clearly demonstrating the circular "inside" of the Celestial Sphere. The pole is conveniently located (now) at the North Star, Polaris. (b) A perspective view a view of the Celestial Sphere from one's horizon, here for the latitude of 42.74° of East Lansing, Michigan, is shown. The three bands show the Sun's path in the sky at the Summer Solstice (top), Winter Solstice (bottom), and the Equinox (middle). Each of the bands around that central arc are 23.5° above and below it.

This picture is an old one identified with Aristotle, as we'll see. It's also a quantifiable picture. By his time, everyone knew that the Earth was spherical and that the some of the angular quantities in the sky matched angular quantities on the Earth's surface. For example, in Figure **??** the angle that the Celestial Pole makes with the northern horizon is identical to the observer's latitude. Greeks were spread between northern Africa (about 30° north of the equator) and the northern shores of the Black Sea (about 45° north), so the apparent position of the celestial pole was easily seen to be different when viewed from different locations. That means that the angle that the celestial equator makes with the southern horizon is (90° − the observer's latitude). Figure 3.4 is again drawn for East Lansing, Michigan. Here you can see three angles, all of which the Greeks determined. The latitude of 41.7° for East Lansing is shown as the altitude of the North Pole (celestial and Earth poles); The altitude of the Celestial Equator is $09^\circ - 41.7^\circ = 47.3^\circ$, which is also the altitude of the Sun at an equinox; and finally, the angular separation of the Sun's extreme altitudes is 23.5° up and down from the equinox Sun's path.

Of particular importance were the constellations in which the "Sun resides" during the time of an equinox.[6] During the times of the Greeks, that point in the sky was in the leading edge of the zodiacal constellation of Aries—the "First Point of Aries" became the origin of a coordinate system in order to document the location of stars and planets and became particularly important in the −200's by important astronomers.

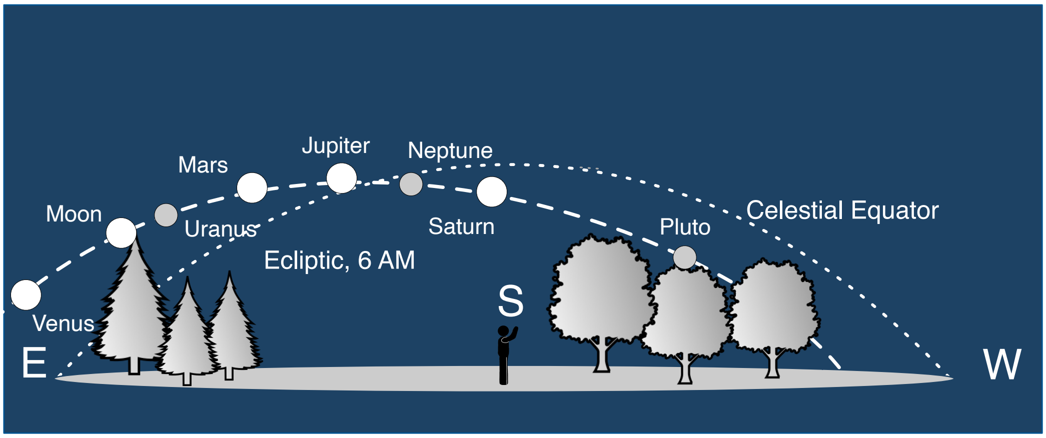

Figure 3.5: The position of the planets from East Lansing, Michigan at 6 AM. The dotted line is the Celestial Equator and the dashed line is the ecliptic. The gray circles indicate where planets that the Greeks could not have seen with the naked eye.

Planets' apparent motions. There are a few brighter objects which execute similar east-west motions through an individual night, are very bright, don't twinkle like stars, and occupy strange, un-star-like positions from night to night. Of course, these are the "planets," probably named by the Greeks from their word for "wanderer,"

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[6] Of course, they could not see the stars when the Sun is out, but they knew to look at the sky exactly 12 hours later and then extrapolate 180° around the zodiac to determined where that point of "residence" is.

planetai, Figure 3.5 shows the sky at 6 AM from East Lansing, Michigan in which many planets are above the horizon at once. The bright circles are naked eye planets and the gray circles are the rest of the complement, with Pluto added for nostalgia. The Sun is on that same dashed curve and is just below the eastern horizon (led by Venus). All of the planets are within ±7° of the dashed mean curve (except Pluto which is 17°, one of the reasons it's no longer considered a planet of ours). This common "lane" in which all of the solar system (and the Moon) objects reside is called the **ecliptic** and the central path is sometimes called the "mean Sun." At a different day and time, the Celestial Equator hasn't moved, but the ecliptic traces out a different curve relative to the horizon and you can see that in Figure 3.3, where it's represented again as a dashed curve. This must have been confusing!

The ecliptic is inclined to the Celestial Equator by 23.5°. The constellations of the zodiac are distributed around the sphere within that strip of the sky[7] and the center of it is the path of the Sun.

Finally, there are two kinds of "motions" spoken of for the planets, which is confusing.

- If you watch a planet during a single night, you'll see it move from east to west in line with the stars behind it. This is called "**prograde motion**."
- But there's another kind of "motion" which is not during a single night, but appears as a comparison from night to night. Suppose you look at Mars every night at 10 PM and take note of what stars are behind and around it. About every 26 months you'll see something strange happen. Suppose Star A and Star B are on either side of Mars. In some successive nights the arrangement of the three objects will go something like this cartoon facing the south (Mars' back and forth would actually take about four months):

Night #1 East ........*A*..........*M*.........*B* West
Night #2 East ........*A*........*M*...........*B* West
Night #3 East ........*A*......*M*.............*B* West
Night #4 East ........*A*....*M*...............*B* West
Night #5 East ........*A*..........*M*.........*B* West
Night #6 East ........*A*...........*M*........*B* West
Night #7 East ........*A*...............*M*....*B* West
Night #8 East ........*A*.............*M*......*B* West
Night #9 East ........*A*...........*M*........*B* West
Night #10 East ........*A*..........*M*.........*B* West
Night #11 East ........*A*........*M*...........*B* West
Night #12 East ........*A*......*M*.............*B* West
Night #13 East ........*A*....*M*...............*B* West

Each night Mars seems to be more east of the star pattern near it. But between nights 4 and 11 Mars appears more west and after a number of nights, then reverse course and continue its nightly progression eastward. This is called "**retrograde motion**"

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[7] There are 13 zodiac signs, but that's inconvenient for astrologers so they ignore one of them.

and it surely must have confused everyone. Certainly the common description of retrograde motion as a “motion” is confusing nomenclature since the “movement” is actually over many nights.

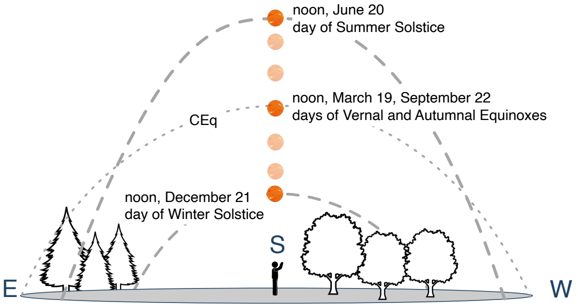

Figure 3.6: An observer looking south would see the Sun take very different paths through the year. Of course the Sun moves from east to west, but at various altitudes. This figure shows the situation for East Lansing, Michigan which is at a latidue of 42.74° above the Earth’s equator. On December 21st the Sun takes it’s lowest path and the days are the shortest and the Sun’s rising and setting is south of east and west. On June 20th, the Sun is nearly overhead with rising and setting north of east and west, so the days are long. Between those extremes the paths are different slightly each day. In the middle period on

Sun’s apparent motion. That smart Greek’s days (and ours) would be dominated by the Sun. If you’re in the northern hemisphere, in general you’d see it appear to rise over your eastern horizon, pass not quite overhead, and then disappear over your western horizon. Look at Figure 3.6 which plots the Sun’s trajectories through a year for East Lansing, Michigan which is at a latitude of 42.74°. On December 21st the Sun takes its lowest path, the days are the shortest because the Sun rises south of east and sets south of west. The lowest Sun path in the figure shows the situation at noon on December 21st, 2024 which is the day of the **Winter Solstice**. Every day after, you would notice that the Sun’s eastern rise is a little bit north from the day before and that it would set a little bit further north as well and so each day would be a little longer. Furthermore, at noon the point each day when it’s at its peak would be just a little higher than the previous day. Then on June 20th, the Sun has gone as far up as it will and is nearly overhead at noon, rising and setting quite a bit north of east and west, so that day is the longest of the year. Then the situation reverses and the Sun is lower every day until the next December. Between those extremes the paths are different slightly each day.

In that round trip, there’s one day on the way up and one day on the way down when the Sun rises precisely in the east and sets precisely in the west and at noon, it’s height above your horizon is exactly between those two extremes during late December and June. Also on those two days, the day and night durations are the same all over the world: 12 hours and so each is called an **equinox**.[8] These points happen in late March (called the Vernal Equinox)[9] and late September (the Autumnal Equinox).[10] Each **equinox** is a precise astronomical event and marks the

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[8] This derives from the Latin *aequus*, for "equal" and *nox*, for "night.”

[9] Latin for "spring” is *ver*.

[10] In 2023, the WS, VE, SS, and AE occur on December 22, 2023, 3:27 AM, March 20, 2023, 9:24 PM,

point when the Sun passes through the Celestial Sphere on its way up or down. In Figure 3.6, you can see that the trajectory of the Sun's path in the middle is dotted rather than dashed to highlight that this is a special day: the Sun's path is very close to the Celestial Sphere circle and crosses it at the precise time of that day defining both of the equinoxes. In 2024, those moments are March 19th at 11:06 PM EDT and September 22nd 8:44 AM EDT.

Equinoxes are distinct events throughout ancient history, across cultures. The Vernal (or Spring) Equinox was celebrated around the world: from the Mayans to the ancient Germanic tribes to the ancient Saxons the VE was celebrated as a time of renewal and rebirth. Structures like Stonehenge, the Mayan pyramids, the Egyptian Pyramid of Khafre, and others in Cambodia, Ireland, and New Mexico point out the VE. Understanding them, though, only became a goal among a few Hellenistic Greeks when "solar models" were invented by mathematically clever and imaginative astronomers. As our story unfolds, notice how the Sun figures into every corner of ancient astronomy—and yet, it was considered to be just another orbiting object.

Clearly associated with the Sun are the seasons and they aren't the same length—spring and summer are longer than fall and winter, but there are definite times of cold and warm weather in the two hemispheres. In 2023 in the northern hemisphere: after 89 days in 2022, winter ended; spring was 93 days long; Summer was 94; and Autumn was 89. The Athenian astronomers Meton and his student, Euctemon found 92, 93, 90, and 90 days in about −432, so this was a known problem. (The student also has a lunar crater named for him.)

The apparent motion of the Moon. Prominent for its size and its regularly changing features is our Moon. If looked at from overhead, it travels in a clockwise orbit, nearly circular, with a period of 27.322 days, changing its appearance through phases during that cycle.

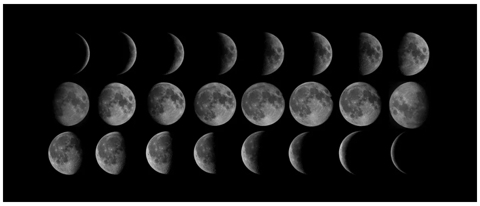

Figure 3.7: Views of the familiar faces of the Moon through a month, not showing the new Moon phase. Getty

Unlike the Sun and the stars, the Moon changes its appearance every single night. Sometimes it's "full" and a bright circle. Sometimes it's not there at night, but maybe visible during the daytime. Most times the bright part of the Moon is a crescent shape, culminating in a half-circle, and then back to crescent. Occasionally, the Moon gets in the way of the Sun and we have a solar eclipse. Sometimes the Earth blocks the Moon from the Sun and we have a lunar eclipse. Why these events don't happen every month was a puzzle. One thing doesn't change about the Moon and that's the face that we see—another puzzle.

June 21, 2023, 2:57 PM, and September 23, 2023, 6:49 AM, GMT

Puzzled about these observations? If you can't wait for Copernicus, Tycho, Kepler, and Galileo...then take a look at *Greek Astronom, Today* in Section 3.8.1 for our modern interpretation how it goes.

3.3 A Little Bit of Presocratic Astronomy

Pythagoras •Philolaus •Parmenides •Archytas
(Set the context with the timeline in Figure **??** on page **??**.)

In Chapter **??**, I briefly discussed the Presocratics' cosmologies with two ideas among them that were shared: all but two appeared to believe in a flat, and stationary Earth. The two who thought differently were Pythagoras and Parmenides.

Parmenides had a number of original ideas about the heavens—in particular, he may have been the first to conceive of the whole universe as being spherical (Pythagoras/Philolaus might also have determined this) and finite.

> "...like the mass of a well-rounded sphere, from one middle, equal in every respect."
> Parmenides

He was also apparently the first Greek to note that the Moon reflected the light of the Sun and must be spherical and he was even poetic about it:

> "[the moon is a body] shining by night, wandering around earth with borrowed light..." Parmenides

"Borrowed light" is a nice phrase. If the Moon "borrows" its light from the Sun and doesn't shine on its own, then the shape of the phases of the Moon lead to a spherical shape conclusion.[11] Ironic, isn't it that Parmenides can perhaps be credited with a scientific discovery—one that requires observation— when we tend to think of him as anti-scientific.

The Pythagorean team (probably more Philolaus than Pythagoras, so I'll call it collectively "Pythagorean/Philolaus") extrapolated their fondness for regular motions, musical tones, and numbers and built a cosmology that tried to put all of these commitments into one model. They were responsible for many "firsts" in Greek astronomy: they too hypothesized that the Universe is spherical, most credit them with establishing that the Earth is spherical (for metaphysical and symmetry reasons), they proposed a popular ordering of the planets (Earth, Moon, Sun, Mercury, Venus, Mars, Jupiter, and Saturn...surrounded by the stars), they hypothesized that the planets' speeds are inversely proportional to the size of their orbits, and they concluded that the "morning star" and "evening star" (our Venus) were not two different planets but the same one which is close to the Sun. And, crucially: they were the first to propose that the planets follow circular orbits around a center.

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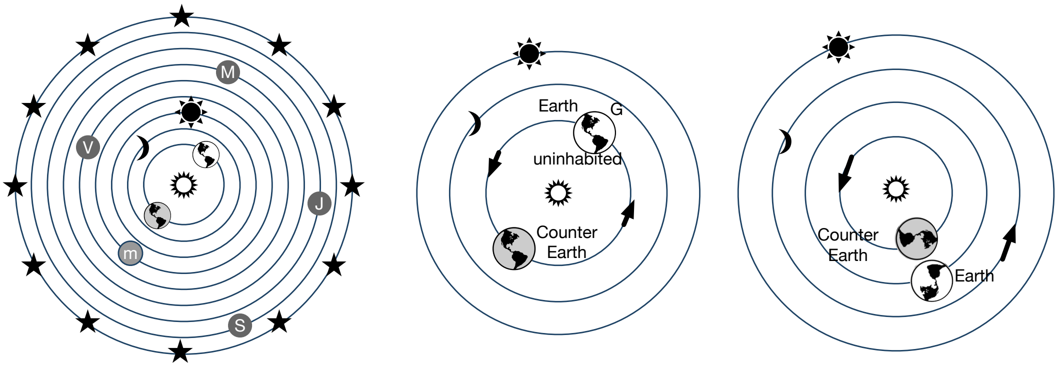
[11] It was traditional to credit Parmenides with extrapolating from a spherical Moon to declaring that the Earth, too, is spherical. But that's not authenticated and Pythagoreans' claim to a spherical Earth is perhaps more likely.

There was a first version of Pythagorean/Philolaus cosmology in which the Earth is at the center of the universe containing a “central fire” or “’Hestia,” in homage to the immobile goddess of the hearth. But that morphed into the cosmology of Chapter **??** with the “central fire” situated in the center of the universe, relegating Earth to be just another celestial object orbiting around it in circular orbits. Figure 3.8 (a) shows the whole system with the Earth, Moon, Sun, and the planets orbiting counterclockwise around the center and inside an outer shell of the stars. The Earth orbits the central fire once a day and the Sun, once a year. So the Earth daily catches up and passes the Sun accounting for day and night.



(a) (b) (c)

Figure 3.8: (a) shows the Pythagorean system with all of the heavenly bodies and the Earth orbiting the central fire in a counterclockwise sense. In (b) the Earth is shown in one of a number of interpretations of Philolaus’ system. Greece (G) is on the far side, leaving the side facing the fire without people. In this orientation it’s morning as the Earth is catching up with the slower-moving Sun. In (c) the counter earth is positioned so that it blocks the central fire.

We don’t see a “central fire” and there were two proposals as to why, shown in Figure 3.8 (b) and (c). The standard interpretation is the second one in which inhabitants of the Earth are shielded from the fire by the presence of a “counter earth” which strategically blocks it, see J. L. E. Dreyer, 1953. Without the counter earth there are only nine components to the universe and so Aristotle was critical of them for perhaps arbitrarily adding the counter earth just to make the total 10, as suggested in D. R. Dicks, 1970.

This is the first cosmology based on a *regular, circular* MOTION IN THE HEAVENS and a model in which MOTION BY THE EARTH is not zero. The idea of course has spawned 2000 years of astronomical research! Circles, everywhere.

3.3.1 Summary of the Astronomy of Parmenides, Pythagoras, and Philolaus

(Set the context with the timeline in Figure **??** on page **??**.)

- Parmenides (−514 to −450):

  - He was first to assert that the whole universe was spherical.
  - He was perhaps the first to recognize that the Moon does not shine by its own light, but reflected ("borrowed") light from the Sun. The Pythagoreans might also have realized that.
- Pythagoreans [Pythagoras (-575 to -500) especially including Philolaus (-470 to -385)]:
  - "They" were first to realize that the Earth is spherical.
  - "They" were first to hypothesize a particular ordering of the planets, perhaps with the their orbit size inversely proportional to their speeds.
  - "They" realized that the "morning" star and "evening" star were the same planet, Venus.
  - "They" were to propose a model in which the planets (including Earth and Sun) all orbited a central point (for them, the mysterious "central fire.") in perfectly circular orbits.
  - Their insistence on heavenly motions being uniform and circular outlived their specific model.

3.4 Act VII Plato and Exodus' Models

Plato •Eudoxus •Aristotle
(Set the context with the timeline in Figure **??** on page **??**.)

In Chapter 1 we touched on Plato's cosmology in *Timaeus* but that was a late development for him as his ideas about the nature of the cosmos grew over almost his whole career. His learning from Archytas in mathematics and the symmetry tendencies of the Pythagoreans launched him in the direction of building everything around circles, and then spheres.

Recall that the *Republic* was nominally a treatise on the nature of justice and how to build a just state which he proposes be totalitarian. When philosophy and political science students read it, they're probably surprised by its ending, which is a full-on Pythagorean cosmology, the "Myth of Er."

> "Once upon a time he died in war; and on the **tenth day**, when the corpses, already decayed, were picked up, he was picked up in a good state of preservation. Having been brought home, he was about to be buried on the twelfth day; as he was lying on the pyre, he came back to life, and, come back to life, he told what he saw in the other world." Plato, *Republic*

Socrates is trying to motivate why someone should live a good life and relates a cosmic carrot-and-stick story, not unfamiliar to other religious admonitions. Er is a soldier who was killed and does what all deceased do. . . they go to a place where their lives are evaluated, not by St. Peter at the Pearly Gates, but by four judges who tell him that he's got a job to do: after 10 days[12] his body will be retrieved from the battlefield and on day 12 he's to be resurrected from the dead, dramatically on his own pyre before it's lit. He's to tell others what he's seen which includes a strange

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[12] Why 10 days? some Pythagoreanism is maybe showing?

vision of a pillar of light that extends to the heavens which Plato describes as a spindle and whorl used for spinning wool. Figure 3.9 (a) shows a Roman woman spinning wool with the weighted whorl at the bottom which spins as she works. Figure 3.9 (b) is the umbrella-like structure (the whorl upside down) that Socrates describes:

(a) (b) (c)

Figure 3.9: The figure in (a) is a Roman sketch of a woman spinning wool using a spindle and whorl, which is the weight at the bottom with a hook. The image in (b) is Plato's description of the whorl actually hollowed out with nested layers of whirl-shaped half-spheres. The image in (c) is the cosmos that the onion-layered whorl represents with the Moon; Sun; and the first planet, Mercury attached to the first three of eight spheres. I've only included three in this cartoon.

> "Its shape was that of (whorls) in our world, but…it was as if in one great whorl, hollow and scooped out, there lay enclosed, right through, another like it but smaller, fitting into it **as containers** that fit into one another, and in like matter another…There were **eight of the whorls** in all, lying within one another…" Plato, *Republic*

The eight "containers" are hinted at in my sketch in Figure 3.9 (b) and the whole is abstracted as nested spheres in Figure 3.9 (c), where I've only shown three spheres (remember, "containers") for simplicity. Earth is no longer a "regular" planet but is in the center with concentric spheres of the Moon, Sun, the outer planets, and again, the stars on the furthest shell, which Socrates says is "speckled." So, Plato's first cosmology has MOTION BY THE EARTH as zero and MOTION IN THE HEAVENS is described as Pythagorean, but using spheres, not just circles. He also tells you how they move and the sounds that they emit as a Siren sits on each sphere and sings a tone. This is the world's first three dimensional cosmological model. But the it didn't match what the planets do and Plato actually tried to remedy it in the *Timaeus*. Given his penchant for not modeling appearances, this was an unusual move and suggests to me that getting it right was (briefly?) important to him.

The *Timaeus* is Plato's "origin story" and in the previous chapter I described the Craftsman's efforts to create matter using geometric three dimensional shapes. It's also his cosmology update from the *Republic* and quite different. Socrates teases the story out of the main character, Timaeus—a Pythagorean—and then uncharacteristically allows the speaker have the floor without much interruption. It's where Plato becomes mathematical, in a spooky, Pythagorean way.

Does this string of numbers mean anything to you: 1,2,3,4,9,8,27? Me neither, but they function as a part of the instructions to the Craftsman in order to build the universe following a numerology algorithm described in a nearly unintelligible paragraph:

> "And he began the division in this way. First he took **one portion** from the whole, and next a **portion double of this**; the **third half as much again as the second**, and **three times the first**; the **fourth double of the second**; the **fifth three times the third**; the **sixth eight times the first**; and the **seventh twenty-seven times the first**." Plato, **Timaeus**

Timaeus is tough to read (impenetrable in some places) and so I've unpacked the algorithm from the paragraph in Appendix A.3.1. The upshot is that the Craftsman has fashioned a universe with two rotating spheres. One of them he calls "the same" and represents the (unavoidable) rotating Celestial Sphere. The other he calls "the different" which is inclined to the first. Those numbers represent the relative sizes of the layers inside of that inclined sphere where the planets are arranged. His Er story didn't account for the ecliptic, and this "different" sphere set is that correction.

> "This whole fabric, then, he split lengthwise into **two halves**; and making the **two cross one another** at their centers in the form of the letter X, he bent each round into a circle and joined it up, making each meet itself and the other at a point opposite to that where they had been brought into contact." Plato, *Republic*

Figure 3.10 is a silly attempt to illustrate this. Figure 3.10 (a) is a person playing with a hula hoop, perfectly aligned so that the axis of the toy's rotational plane points through our person's head. This represents the axis and equator of the Celestial Sphere around the Earth. Figure 3.10 (b) shows just how good this person is at hula hoops: two are rotating, the original, and another that somehow our friend manages to get to rotate at an angle relative to the first one. Some serious hip-action would be required. This represents the ecliptic, inclined by that spacing corresponding to the latitude of the observer. Those strange numbers? Well, there would actually be seven hoops with diameters proportional to those numbers: 1–2–3–4–8–9–27. Figure 3.11 shows what this is really about.

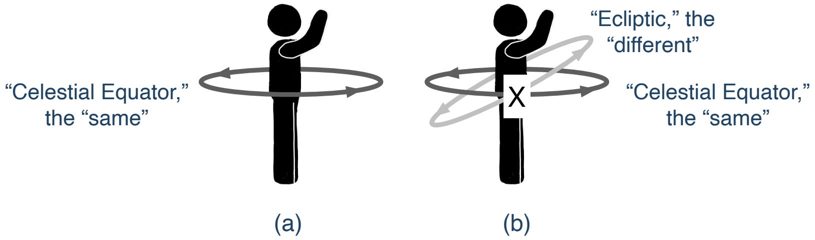


Figure 3.10: Pretty good hula hoops chops.

The celestial sphere and its axis I've called the NCP (north celestial pole) in the

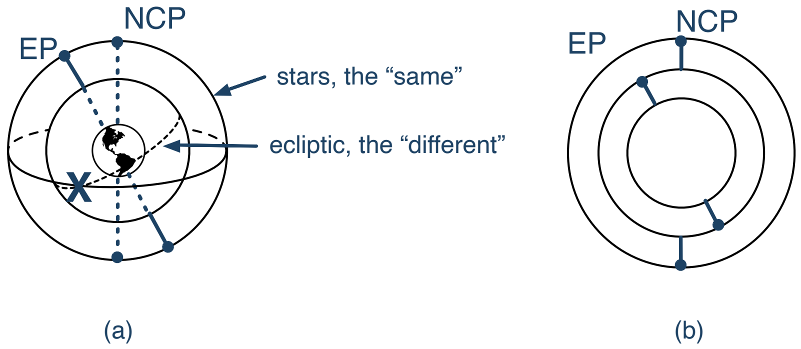


Figure 3.11: (a) shows the two spheres with their equators. One the Celestial Sphere (carrying the stars around the Earth each night, so an axis centered on the North Pole of the Earth) and the other is the ecliptic (in which the planets reside as they appear to go around the Earth) with the pole of that sphere, the North Ecliptic Pole. (b) takes away the three-dimensional view and will be a useful sketch for these kinds of constructions in what follows.

diagram. The other strip is the equator of the other, ecliptic, sphere (with axis labelled EP) which makes an “X” where it crosses in two places with the Same. (These are the points of the equinoxes, when the Sun on the ecliptic crosses the Celestial Equator.) Inside of this strip, the segments correspond to the locations of the Moon, Sun, Mercury, Venus, Mars, Jupiter, and Saturn. Of course, this is a little mad but Eudoxus took on the task of turning this story into a geometrical model.

3.4.1 Eudoxus' Model

By the time Eudoxus had returned to the Academy, he would have been familiar with the *Republic* and probably *Timeaus*. Once Plato had inserted the ecliptic path, he still needed to explain retrograde motion. And he knew it:

> “...as for the dances of these and how they relate to each other, the **backward-cycles and forward-progressions** of the circles to each other... to speak without visual representations of these same would be a **vain effort**.” Plato, *Timaeus*

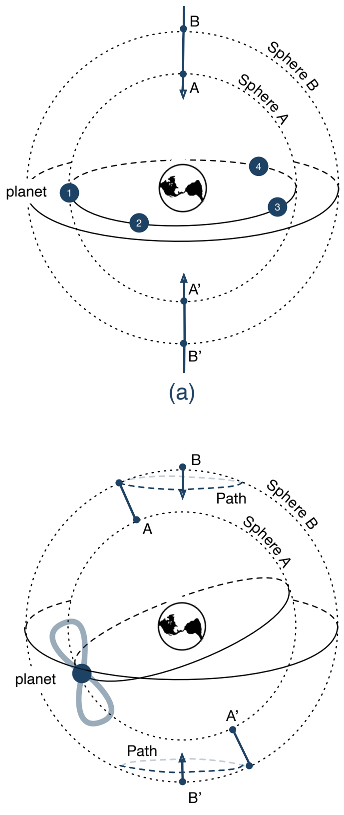
So, he realized the problem... but had no solution and just gives up (“vain effort”). He was out of his depth but Eudoxus was ready and came up with a brilliantly complex model and while it's not known what Plato thought of it, it's clear how Aristotle reacted: he made it his. It's intricate, so let's go to the box and work out the inner workings of the idea and then skip to the end. Look at Figure Box 3.12 on page 29. After you've read the material in that Box, return to this point ☟ and continue reading.

The figure in Box 3.12 describes the tool-kit that Eudoxus used to construct a full model of each planet in which they ride on the equators of coupled, spinning spheres. The two spheres shown in the box form the minimal number of moving parts unique to every planet and they are each embedded inside of two other spheres, one for the ecliptic whose equator includes the rough paths of the planets and the other is the Celestial Sphere which includes the motions of the stars around the Earth every nearly 24 hours. Let's take it slow in Figure 3.13.

The fundamental Eudoxus set was four spheres, centered on the Earth. Using the nomenclature from Figure 3.13 and Box 3.12, labeling them from the inside out:

A: the sphere to which the planet is attached,
B: the next sphere which precesses around that inner sphere (producing Eudoxian figure-eight)
C: the sphere that rotates around the ecliptic—that stretches out that Eudoxian figure 8 in Figure 3.12 to produce retrograde motion, and
D: the outer-most sphere that rotates daily showing the pattern of the starry Celestial Sphere.

FIGURE BOX 3.12



(a)

(b)

The model that Eudoxus created is an impressive bit of geometry mixed with inspired imagination. It's the famous "nested spheres" model that made it all the way to the Baroque as an explanation for the odd motions of the planets. In a very modern way, it's full of parameters that could be tweaked to make it fit the observations...some of which he made himself at the observatory he created in his school before he returned to Athens.

Imagine taking two hoops, one of which is slightly smaller than the other and is attached inside the larger one across their mutual diameters. Figure 3.12 (a) shows this with a "planet" attached to the equator of the inside hoop. Now if we spin that hoop around its axis AA' the planet will follow a circle from position 1 through 2, 3, 4 and so on. This spinning *observed from the outside* essentially defines a sphere, Sphere A, here centered on the Earth. If the two hoops are attached, and if the outer hoop spins around its axis, BB', creating the surface of Sphere B, then the motion of the planet will be the sum of the two speeds at the hoop pair equators. So if the outer hoop spins at the same rate as the inner hoop, but in the opposite direction, then the planet would appear to the Earth to remain stationary at position 1.

Now imagine that the axis of the inner hoop is attached at a point offset on the surface of the Sphere B as shown in Figure 3.12 (b). Now when Sphere B spins, it takes the AA' axis of Sphere A around with it tracing the path shown. In addition, if Sphere B spins while its following that path independently, the motion is a complicated figure eight pattern as shown. Eudoxus figured this out and named the shape a "hippopede" which is "horse fetter" in Greek. (A fetter is like a chain.) Now there are many variables at work which would alter the shape of the hippopede: the speeds of the two spheres and the angle at which AA' axis of Sphere A is inclined to the BB" axis of Sphere B.

Now go back to page 27 and pick up where you left off.

All of these separate motions are coupled. . . and that's just for one planet! By tuning the inner two spheres' rotation speeds and the inclination of their inner axes, the

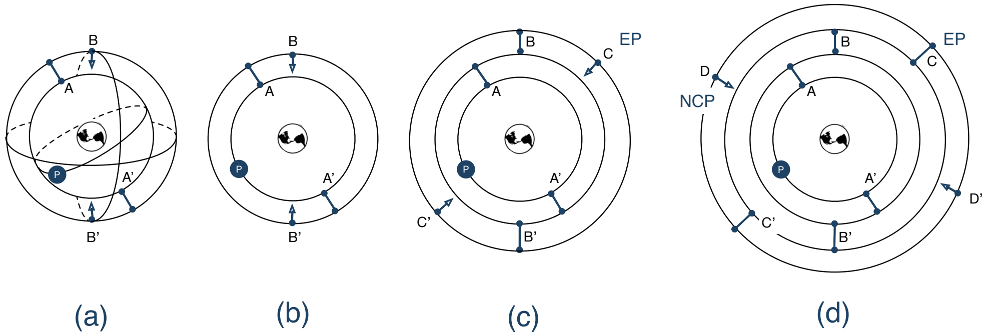


Figure 3.13: (a) is a slightly different rendering of Figure 3.12. (b) is an abstraction of (a) taking out some of the lines that suggest a solid sphere, for clarity. (c) includes the sphere of the ecliptic (EP for Ecliptic pole is shown) with axis of rotation CC′. Notice that it's attached to the outer sphere of Eudoxus' tool-kit pair. And (d) includes the sphere of the outer stars, the celestial sphere (NCP for the North Celestial Pole is shown) and the ecliptic shere is attached to it.

motions of the planet can be made to do the figure-eight dance at just the right time of year and with the right elongation in the sky—to make the planet appear to reverse direction and recover, and resume as viewed by the Earth. Each planet required four spheres and the Sun and Moon required three each, plus the Celestial Sphere: 27 spheres to do the job. This was a mammoth intellectual puzzle that Eudoxus created and then solved with those relatively simple pieces of interlocking spheres.

It still didn't quite do the job as well as it might and in the best tradition of what Thomas Kuhn would have called "Normal Science," **Callippus of Cyzicus** (−370 to −300) tried to make it better without starting over. He was a student of Plato's and worked with Aristotle and worried about the seasons' length problem and some finer points of the planets' motions. He added two additional spheres for the Sun and Moon and one each for Mercury, Venus, and Mars for a total of seven more. So now: 34 spheres. Was it all just an exercise in geometry? Perhaps. The Eudoxian program of research was abstract without numbers and so no predictive capability. It might indeed have been more of a story than a scientific model, like Plato, and like Aristotle's will be.

Around −370, Eudoxus also apparently created a star catalog in his book *Phenomena* of at least 47 stars which a century later were memorialized in the famous poem of that same name by Aratus that I introduced in the preface to this chapter. These entries were not numerical or with coordinates, but were story-like recording the times of the rise, set, and position overhead of constellations or stars near parts of constellations. For example, "As a guide the Ram and the knees of the Bull lie on it, the Ram as drawn lengthwise along the circle, but of the Bull only the widely visible bend of the legs. On it is the belt of the radiant Orion and the coil of the blazing Hydra, on it too are the faint Bowl, on it the Raven, on it the not very numerous stars of the Claws, and on it the knees of Ophiuchus ride. It is certainly not bereft of the Eagle: it has the great messenger of Zeus flying near by; and along it the Horse's head and neck move round."(Dennis Duke, 2008). What we know of Eudoxus' catalog come to us from Aratus and the later Hipparchus' critique of the poem and by extension, Eudoxus' work.

3.5 Act VIII Aristotle's Model

When it came to astronomy, Aristotle was downright derivative. Ironically, his model that became Church dogma wasn't his, and to make matters worse, it was flawed and largely ignored soon after he died. How it went from forgotten to dogma is the story of Chapter **??**, but let's see what he actually did and why. His astronomical writings were scattered throughout two large books, *On the Heavens* and *Meteorologies* and his solutions to known problems were a mixture of pure metaphysics, physics—often relying on his own rules of motion as authoritative,—and the observations of others. Aristotle didn't observe the heavens.

3.5.1 Properties of the Earth, Aristotle-style

Aristotle vigorously disagreed with the Pythagorean/Philolaus cosmology in which the Earth orbits the center of the universe and devised challenges defending a stationary Earth that any future moving-Earth proponent would have to meet squarely.

The Earth Pythagorean/Philolaus adherents proposed that the Earth is spherical, arguing largely from aesthetic grounds, namely that circles are good and spheres are good and so the Earth should be spherical as well. Oh, and that the universe is spherical and so must be the Earth.

Aristotle proposed multiple, more concrete reasons why. First, when one observes a lunar eclipse, one sees that the shape of the demarcation between light and dark is always convex. So if the Earth's shadow is the explanation for the eclipse, then the Earth must be at least circular, if not spherical. He knew from reports that people in the southern latitudes saw different stars on their horizon than those in the northern latitudes. He argued against those who insisted (still) that the Earth was flat by noting that the horizon looks flat, but that's simply because the Earth is large.[13]

He also had a physics reason. Since earthy material would naturally be aimed at the center of the universe then all earthy material would be drawn to a single point and highly compressed equally in all dimensions with the result: a sphere of earthiness. That sphere would be surrounded by a thick sphere of water. That would be surrounded by a sphere of air and then fire. So a spherical double-double-decker sandwich of the four terrestrial elements filling up the whole volume below

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[13] Nowhere in Aristotle is the famous alleged argument attributed to him that when ships begin to appear on the horizon that first the mast and then the hull are observed.

the Moon, the “sub-lunar realm.” This argument supported two other Aristotelian– imperatives: that the Earth finds itself in the center of the universe and that it’s stationary.

The Stellar Parallax Argument Finally, he makes a good argument for the stationary Earth which becomes the essential challenge to any future moving Earth cosmology.

Look at a point across your room with one eye closed and put your finger in front of you and notice what’s behind it on a wall or distant surface. Now switch eyes and notice that the what’s behind your finger now seems to have moved. If you open and close each alternate eye successively, the background will appear to jump from side to side relative to your finger. This is called “parallax” and it’s because your eyes are attractively located inches apart from one another on your face and enough so that the lines of sight from each are slightly different.

If the Earth is orbiting a center, then at one point of the year a particular star would appear as a line at a particular angle (like your right eye open). Then at the half-way-point around its orbit (six months later if the orbit is around the Sun), when the Earth is on the other side of that center (like your left eye open), look for that same star and it will be at a completely different angle. “**Stellar parallax**” or “annual parallax” is the name of this phenomenon and we’ll see it more than once in our story.

Nobody observed stellar parallax leaving only two explanations. Either the Earth doesn’t move around a center of revolution, or the stars are so far away that parallax isn’t visible. Nobody was prepared to imagine a universe that big, and so the conclusion was that MOTION BY THE EARTH is zero.[14]

He agreed with Parmenides and the Pythagoreans that the light from the Moon is reflected light, that the shape of the crescent of the Moon’s phases suggests that the it must be a sphere. From that and his spherical Earth hypothesis, he reasoned that all of the heavenly bodies are likely spherical, albeit made from different stuff.

For millennia, Aristotle has been held responsible for the theory of five elementary substances: in *On the Heavens* he added what he called the “first body” to the familiar earth, water, air, and fire. Much later this was renamed “the fifth element;” and later, the “**aether**;” and later than that, the Latinate, “quintessence.” In spite of almost all popular and even scholarly sources, Aristotle never identifies his first body as “aether” although he was surely aware that Plato used that term explicitly. History assigns Cicero from the first century BCE, as the source of Aristotle’s reference to “aether” with the assumption that famous Roman orator had access to now lost Aristotelean manuscripts. Or, given our repeated reminder that much of what we know of the Greeks is muddled...it’s possible that Aristotle never used the word. I’ll use “aether” as it will become a useful contrast with the 19th century “ether,” the direct experimental lead-in to Relativity. And, by the way: Aristotle is often said to

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[14] It took until the 19th century to actually observe stellar parallax because the universe really is that big.

have insisted that the Eudoxian spheres were crystalline, the "Crystalline Spheres" were indeed an assumption in Medieval and Renaissance times, but nowhere does Aristotle refer to this. (See, David E. Hahm, 1982)

Aristotle's aether is eternal, not composite, neither heavy nor light, and is the most divine of all of the heavenly objects. So it's not anything like the four Aristotelian elements, but for some reason he holds heavenly objects to some of the same physics as terrestrial objects.

The Sky The heavens differ from terrestrial objects in an obvious way: the night sky repeated, every night, while everything on the Earth seems less ordered. Sure falling objects executed their motions according to rules, but every object's behavior is different so the eternal permanence of the heavenly motion contrasts with the impermanence and changeability of MOTION ON THE EARTH. Furthermore, for Aristotle natural motions near the Earth were in straight lines—with a beginning and an end. But the motions of the heavenly bodies seem circular, and so, never-ending...eternal. Obviously, then, the deep sky is made of special, different stuff.

Aristotle's universe is a finite volume in space all the way to the outermost starry sphere, like that of the Pythagoreans. Furthermore, it's always been there and he speculates on and rejects an argument about the possible creation of the universe. So he disagrees with Plato. That for him would presume that before that event, there was already a notion of up and down and that bothered him. So, the universe is a finite volume in space, but of infinite extent in time.

3.5.2 Aristotle's Cosmology

The basic features of Aristotle's cosmology were the same as Plato's as were his ordering of the planets (and different from what Philolaus assumed for the Pythagorean model): Earth–Moon–Sun–Mercury–Venus–Mars–Jupiter–Saturn and the stars. Ever the mechanist, he worried about real material concerns: *how* do they *actually* move as a composite unit?

First, he knew that what was required was a model of the whole universe—Eudoxus' model was a template for each planet, not a whole cosmos— and so each of those sets of spheres needed to all be packaged together into one big onion of spheres, one set inside of another. And this became his problem: since he couldn't have Jupiter's motions affecting Saturns and Mars' motions, he needed to "mechanically" decouple each one.

Remember that I noted that if you had two connected Eudoxian spheres rotating at the same speeds, but in opposite directions, that their motions would cancel one another. Aristotle took that idea and intentionally inserted "rewinding spheres" to do that in such a way to preserve the spheres' connections to the ecliptic and celestial spheres but to isolate them.

Table 3.1 shows that for all of the planets but the Moon and Sun, four spheres were sufficient for Eudoxus. (The Sun and Moon didn't need the daily, celestial sphere rotation.) Callippus added spheres for the inner planets, Sun, Moon, and Mars. It was these 33 spheres that Aristotle then tried to turn into an actual seven-object, whole system.

Table 3.1: The number of spheres for each of the Eudoxian systems for the Moon, Sun, and planets—not including the outer sphere of the fixed stars— with the Aristotelian unwinding spheres counted separately in the last column.

Planet	**Eudoxus**	**Callipus**	**Aristotle**	**Unwinding**
Saturn	4	4	4	3
Jupiter	4	4	4	3
Mars	4	5	5	4
Sun	3	5	5	4
Venus	4	5	5	4
Mercury	4	5	5	4
Moon	3	5	5	
Total:	26	33	33	+22 = 55

> It is necessary, if all the spheres put together are going to account for the observed phenomena, that for each of the planetary bodies there should be other counteracting ["unrolling"] spheres, one fewer in number [than Calluppus]...for only thus is it possible for the whole system to produce the revolution of the planets." Aristotle, *Meteorologies*.

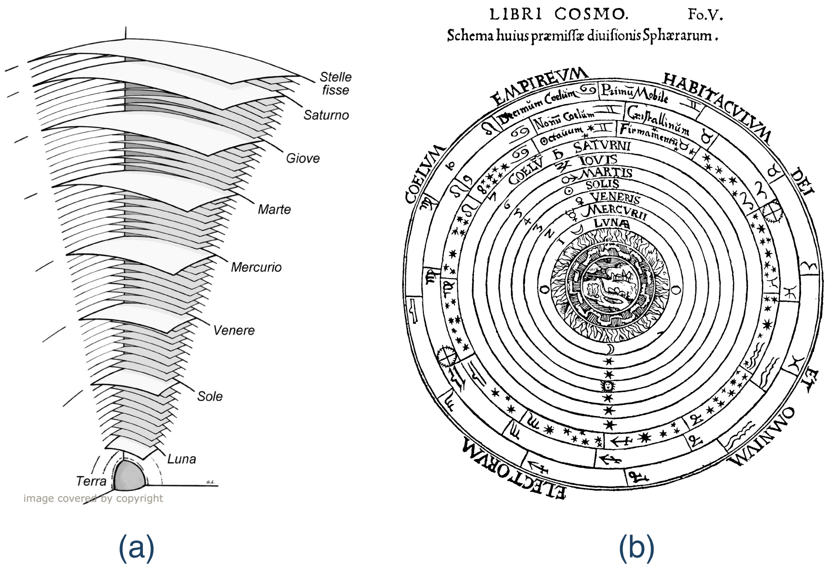

Figure 3.14: (a) Representation of the 55 spheres of Aristotle's model. Notice that Jupiter (Italian, Giove) has one too many layers and that the Moon (Luna) is depicted as having none. (Museo Galileo. (b) is a typical Medieval representation of the Aristotelean cosmology.

Figure 3.14 (a) shows a rendering of the 55 Aristotelean spheres (from https://brunelleschi.imss.fi.it/vitrum/evtr.asp?c=8252. (b) shows a typical Medieval picture of Aristotle's cosmology, the Prime Mover is noted (see below), and in the center, the four Aristotelean elements are drawn. But there's an interesting difference: the planetary order is not Aristotle's but from later.[15] Again, he was always fascinated with his own ideas about motion and for some reason, he assumed that bodies made of the completely unique aether still needed to follow his physics and causal rules. Why didn't he just say that aether spheres just naturally isolate themselves, one set from another?

In that same sticking-to-the-terrestrial-rules spirit, he seemed believe that the spheres needed a cause in order to execute their natural, circular motion and that drives his model into strange places. Just like *unnatural motion* for terrestrial objects required a contact pusher, inexplicably he decided that the *natural, circular motion* of his spheres *also needed contact pushers*. That creates an embarrassing regress problem. Every sphere had its very own pusher and so did the outer, star sphere, but how does that last pusher itself remain stationary in order to be able to move that last sphere? Another pusher? He complicated this by insisting that the pushers had themselves no substance, were outside of space and time, and were essentially pure intellect. He called them "unmoved movers" or "Prime Movers" and the idea was a soft toss to Thomas Aquinas 1600 years later to equate the Primer Mover with the Catholic deity.

Aristotle's astronomy is underwhelming and unsatisfying and it didn't solve the major issues endemic to an Earth-centered cosmology: since the model required each planet to be always the same distance from Earth, why do they vary in brightness? And a relatively new problem in his time: why are the seasons, autumn, winter, spring, and fall, all of different durations? These brought Aristotelean modeling to a halt. New ideas were required.

3.5.3 Summary of the Astronomy of Plato, Eudoxus, and Aristotle

(Set the context with the timeline in Figure **??** on page **??**.)

By the time that Aristotle was done, astronomy had converged on a qualitative, "picture-model" built by two philosophers and a mathematician.

- Plato (-427 to -348):
  - He placed the Earth is at the center of the universe.
  - He modeled the planets as attached to spinning spheres.
  - He proposed that the outer star-sphere spins around the Earth once a day.
  - He placed the sphere of the planets to be inclined to that of the stars so that they all orbit at an angle inclined to the Earth's equator—on the ecliptic.
- Eudoxus (-390 to -340)

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[15] Aristotle seems to have made at least one mistake and actually had two models, one of 47 and the other of 55 spheres. Nobody knows why.

  - He modeled each planet's motion as created by four spheres, with axes inclined to one another to replicate retrograde motion and motion relative to the stars. (The Sun and Moon only needed three spheres.)
  - He modeled each planet's model as separate from the others and he did not propose a whole solar system, just pieces.
  - Callipus added spheres for some of the planets in order to slightly tune some of the motions to better match observation.
  - He apparently created one of the first published star catalogues, memorialized in the poem by Aratus, *Phaenomena*.
- Aristotle (-384 to -322):
  - He adopted Eudoxus and Callipus' approach in order to model all of the planets by piecing together the Eudoxian sets of spheres, one inside of the other from Saturn to the Moon.
  - Since each is tied to the one beneath, Aristotle felt that additional spheres were needed in order to isolate the motions of the planets from one another. These were the rewinding spheres.
  - He insisted that the volume outside of the orbit of the Moon was made of a different element from the four elements that operated within. That fifth element, the aether, filled the remaining volume to the outer stars, providing the material of the heavenly bodies. Natural motion in the aether is perfectly circular.
  - He originated the idea that the universe was "full" of the aether—-no gaps or emptiness. This demand became necessary in all future Greek cosmologies.
  - Aristotle's physics guided (or handcuffed) speculation about any motion that the Earth might have had. The Earth had to be in the center of the universe, not spinning, nor orbiting any point.
  - He was very critical of the Pythagorean idea of an orbiting Earth for (his) physics reasons, but also because there was no apparent parallax which meant that the stars were so far away as to hide parallax (too far for anyone's taste) or that the Earth was stationary.

Modeling of this sort stopped after Aristotle as there were problems with any model in which the planets orbit in perfect circles with their common center on the Earth:

- The seasons would all have the same durations, but everyone knew that was not the case.
- The brightness of the planets would not change, but everyone knew that was not the case.
- The ordering of the planets was arbitrary.

3.6 A Little Bit of Hellenistic Astronomy

Euclid •Aristarchus •Eratosthenes •Archimedes •Apollonius •Hipparchus •Ptolemy
(Set the context with the timeline in Figure **??** on page **??**.)

There were two basic thrusts after the fanciful modeling of Plato, Eudoxus, Callippus, and Aristotle. Hellenistic astronomy became both observationally intense—data collection became sophisticated— and mathematically sophisticated, culminating with Claudius Ptolemy's enduring model in the second century, CE. Let's unwrap this extraordinary period of Alexandrian astronomy and set the stage for 1500 years of surprisingly authoritarian science.

3.6.1 A Moving Earth

Heraclides of Pontus (−387 to −312), from the southern coast of the Black Sea, was a contemporary of Plato and Aristotle. As the son in a wealthy family and an apparently smart young man, was able to emigrate to Athens where he became a favorite student of Plato's and was put in charge of the Academy when Plato went on his last, ill-fated trip to Syracuse. He also studied with Aristotle (who was 10 years his senior) and the Pythagoreans in Athens, so he was fully rounded in the three major pillars of classical Greek philosophy. Plato died in −348 and his successor, Speusippus, died in −339 and when Heraclides lost the election for the next leader, he returned north to Pontus. That's where he probably did his astronomy where he had two good ideas, neither of which went anywhere for 2000 years.

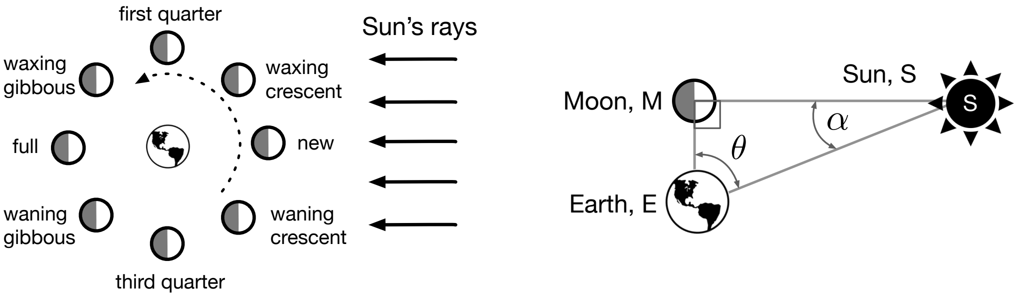
It should have bothered Aristotle that his model required the outside starry sphere to be rotating at an astonishing rate in order to make it all the way around each day. The obvious alternative was a spinning Earth and stationary stars and Heraclides proposed just that.

His other imaginative idea addressed a second interesting fact: Mercury and Venus have a different relationship to the Sun from all of the other heavenly bodies. They seem to cling to it, appearing and disappearing as the Sun rises and sets. It was Heraclides who first suggested that this special relationship could be explained by making those two inner plants satellites of the Sun. His cosmology was that the Earth is at the center of the universe, spinning on its axis, orbited by Sun as "normal," but the Sun in turn was itself a second center of rotation with Mercury and Venus orbiting it. Aristotle's grip was not universal, even in his own time.

3.6.1.1 The Greek Copernicus

While Heraclides could be thought of as ushering in the post-Athens, Hellenic era, it was **Aristarchus of Samos** (−210 to −230), a toddler when Heraclides died, who conceived the best model of the universe and a completely new way to deal with the cosmos: by measuring it. He studied with Strato of Lampsacus, who was the third director of Aristotle's Lyceum, and when Strato went to Alexandria to tutor and counsel Ptolemy II he brought Aristarchus along as his pupil. Strato returned to Athens, but Aristarchus stayed in Alexandria and did his mathematics and astronomy in that growing Greek-Egyptian intellectual center. He probably overlapped with the senior Euclid and surely learned all of Greek mathematics known to that time, conceivably from its most famous chronicler. He fashioned his single surviving text *On the Sizes and Distances of the Sun and the Moon* like Euclid's *Elements*: propositions followed by orderly proofs.

As the Moon orbits the Earth half of it is always illuminated, but we see phases as it makes its way around us. From our modern understanding, Figure 3.15 (a) shows the named phase states as we see them. When it's on the other side of the Earth from the Sun and we're in nighttime, we see it fully illuminated ("full Moon"). When it's between us and the Sun ("new Moon") we don't see it at night (after all, we're looking away from the Sun at night), but can sometimes see it during the day. In between, it shows us partially illuminated crescents. But look at the two quarter Moons. From Earth, at exactly that point we see the Moon split into two equal halves, one dark and one bright.



(a) (b)

Figure 3.15: The Moons phases and positions are shown in (a) relative to the Earth and Sun. From this vantage point, the Moon orbits counterclockwise. In (b) the particular position and phase that makes the Aristarchus calculation possible with the right angle shown occurring at just the first or third quarter when the Moon is half lit.

While Aristarchus didn't anticipate the Moon orbiting the Earth, he did realize that this quarter phase had a particular geometric arrangement with respect to the Sun and Figure 3.15 (b) shows his idea. At that moment, the angle between the Sun and the Earth is a right angle, $\angle EMS = 90^\circ$.

> "...when the Moon appears to us halved, the great circle which divides the dark and the bright portions of the Moon is in the direction of our eye...when the Moon appears to us halved, its distance from the Sun is less than a quadrant by one-thirtieth of a quadrant." Aristarchus, *On the Sizes and Distances of the Sun and the Moon.*

By "distance from the Sun" he means angle α in the diagram, $\angle MSE$. With a modest amount of modern trigonometry, it's possible from the angles to calculate the ratio of the distance of the Earth to the Sun to the distance of the Earth to the Moon in one line. Without modern trigonometry it's a straightforward exercise in geometry. Aristarchus did just that and found:

$$\frac{\text{Distance, Earth to Sun}}{\text{Distance, Earth to Moon}} = 19 - 20$$

where the range is his own estimate of how well he could determine the angle. Appendix A.3.2 completes this calculation and some other interesting measurements that he and others made. This are stunning in their originality and also in their simplicity. He also subsequently calculated three additional things about the universe, for a total of four groundbreaking conclusions:

1. the distance of the Earth to the Sun) $\approx 20\times$ distance of the Earth to the Moon
2. the diameter of the Sun $\approx 19\times$ the diameter of the Moon
3. the diameter of the Earth $\approx 2.85\times$ the diameter of the Moon
4. the distance of the Earth to the Moon $\approx 10\times$ the diameter of the Earth

His mathematics and methods are correct but he had some mistakes, crucially because α is very hard to measure and so his determination of $\theta = 87^\circ$ was wrong...it's actually closer to 89.853° which makes the distance of the Earth to the Sun) $\approx 390\times$ distance of the Earth to the Moon.[16]

But that's not all. Let's let Aristarchus' Italian/Greek contemporary **Archimedes of Syracuse** (-287 to -312) take over from here:

> "Aristarchus has brought out a book consisting of certain hypotheses, wherein it appears, as a consequence of the assumptions made, that the universe is many times greater than the "universe" [expected]...**His hypotheses are that the fixed stars and the sun remain unmoved, that the earth revolves about the sun on the circumference of a circle, the sun lying in the middle of the orbit,** and that the sphere of fixed stars, situated about the same centre as the sun, is so great that the circle in which he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the centre of the sphere bears to its surface." (emphasis, mine) Archimedes, *The Sand-Reckoner*.

Aristarchus was apparently the first to envision a Sun-centered ("heliocentric") universe and, oh by the way he also apparently adopted Heraclides' notion of a spinning Earth. Copernicus-in-training. Nobody knows how he came to this conclusion...even though it solves many of the problems (planets' brightness, for example). His model was largely ignored and the fact that Archimedes tossed that reference off so casually is indicative of what must have been an overwhelming concern for the parallax problem (which is a prejudice about the possible enormity of the universe) and Aristotle's authority when it came to terrestrial physics.

But there it is: the first modern-sounding MOTION BY THE EARTH and MOTION IN THE HEAVENS . Copernicus later took comfort in Aristarchus' idea.

This is an auspicious moment! Aristarchus'work ushers in the beginning of quantitative astronomy which was quickly taken up by his contemporary, **Eratosthenes** (-276 to -194), who became the Chief Librarian of the Alexandria Library

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[16] The point of First Quarter would be in the same part of the sky as the Sun, just before Sunset. Without modern tools, measuring that angle would essentially impossible, if not dangerous! James Evans, 1998 suggests that Aristarchus concocted the "one-thirtieth" as an extrapolation of the time that it takes for the Moon to reach the First Quarter as the largest angle that could come from a month of 30 days to orbit and one quarter of that for the phase. That's almost even more impressive reasoning.

just following Aristarchus' death. (He was also a geographer, mathematician, astronomer, and a poet. The nickname given to him was Pentathlos, implying a Greek pentathlon athlete of many talents.) With his access to Library data, Eratosthenes learned that at noon on the summer solstice (the first day of summer) in Syene, Egypt, the Sun's rays were known go right into a vertical well without hitting the sides. Syene (modern day Aswan) has a latitude of just about 24° which is at the northern tropic, the Tropic of Cancer which means at the Summer Solstice, the sun is directly overhead (the definition of the Tropic of Cancer) and so would not cast a shadow from a vertical stick in the ground. Meanwhile, Alexandria is directly north of Syene at the same longitude and so Eratosthenes reasoned that the Sun is so far away that it's okay to presume that its rays were parallel at both cities. Therefore, for a spherical Earth, the shadow of the Sun on a vertical stick in Alexandria would cast a shadow. He measured it rather than the 0° at Syene, it was 7.2° at Alexandria. That angle is $1/50$th of the 360° of a circle so that the circumference of the Earth must be 50 times the distance between the two cities, which is 833 km (in modern units). Fifty times 833 km is 42,000 km for Earth's circumference— only a few percent higher than a more modern value! Appendix A.3.2 shows this calculation.

Eratosthenes wasn't done. He also devised a way to measure the obliquity of the ecliptic—that angle 23.5° of inclination of the ecliptic from the Celestial Equator. And he made a star catalog of 650 stars. And he wrote a poem about himself. He reportedly went blind in his old age and chose to commit suicide as a result.

So for the first time, astronomers learned the size of the Earth and more could be learned: for example, using Aristarchus and Eratosthene's results, from Aristarchus' #3 above they could conclude that the diameter of the Moon is 4700 km, where the actual value is about 3500 km.

> ▻ I hope you can appreciate that Greek astronomers are no longer merely telling stories. They're measuring our universe.

3.6.2 Casting Aside Aristotle and Eudoxus

The next important step is another storyteller, but an important mathematician who had a good idea. **Apollonius of Perga** (-240 to -190) migrated from Turkey to Alexandria as a young man to study in the successor school of Euclid. "The Great Geometer" became his historical label and he's remembered for discovering the mathematics of "conic sections" (circles, parabolas, ellipses, and hyperbolas)—a subject beyond Euclid's geometry.

For our story we know of him as the geometer who puzzled over the seasons problem and found a way to modify the Eudoxian model to loosen the requirement of all spheres centered on the Earth. His discovery is shown in Figure 3.16 (a) in which E shows the location of the Earth, S is the location of the orbiting Sun, and D is a point in space—attached to no object— which is displaced from E. The distance $\overline{EC} = e$ is called the **eccentricity**. The Sun uniformly follows the dashed

eccentric circle, centered on D and not the Earth! Notice that the result is a Sun's path sometimes further from, and sometimes closer to the Earth. When it's further, it would take longer to go halfway around and so the seasons during that path segment would be longer.

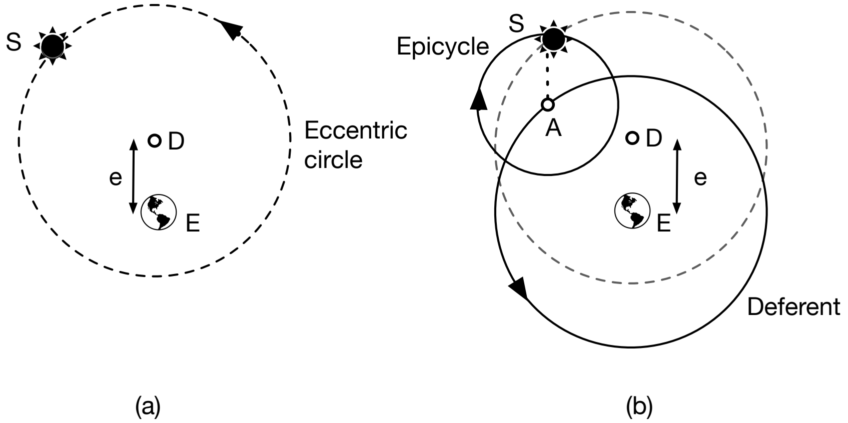

Figure 3.16: In both figures, E is the location of the Earth and S is the location of the Sun. In (a) an eccentric circle is shown for a proposed Sun orbit around the Earth. By putting the center at a spot in space displaced from the Earth by the eccentric, *e*, the seasons would appear on Earth to be of different durations. In (b) the equivalent (under the conditions described in the text) epicycle solution is shown with an overlay of the eccentric circle shown in a light dashed line for comparison. The deferent is centered on the Earth and the epicycle is centered on the rim of the deferent.

But there's more to this as Apollonius discovered a geometric equivalence also illustrated in Figure 3.16 (b). Here a circle, called the **deferent** is centered on the Earth but doesn't act as an orbital path for the Sun. Rather, the Sun rides on another circle, the clockwise rotating **epicycle** with its center (A) attached to the rim of the counterclockwise, rotating deferent. Notice that the rotational sense (here, clockwise) of the epicycle is opposite to that of the orbit of its center, A, on the deferent. Each of these models would cause Earth to experience more Sun during part of the year and less Sun the other parts, which would change the length of the seasons.

> The idea of an epicycle is not easy to grasp since we don't use them any more in planetary astronomy. But if you look up some night, you'll see an example of an epicycle. Think modern (for a moment): we know that the Earth goes around the Sun and that the Moon goes around the Earth. The Moon' s orbit can be thought of as an epicycle: the Earth's (nearly) circular orbit around the Sun would be the deferent and the Moon's orbit around the Earth is the epicycle. So looked at from the Sun, the Moon's orbit would be a slightly off-center orbit around the (orbiting) Earth. This particular epicycle is one in which in Figure 3.16 (b), E coincides with D. We're going to meet epicycles in a major way when we get to Ptolemy and Copernicus.

Numerical predictions were not the goal for Apollonius, but a more realistic frame-work was—and probably the geometry was also an attraction for him. So his ideas were one more step away from Aristotle toward a new way of doing science.

3.6.3 The Greatest Astronomer: Hipparchus

The most celebrated astronomer of antiquity was, yet another Greek about whom we don't have many biographical details. However, **Hipparchus of Nicea** (about -190 to about -120) was so accomplished that his feats were detailed in later Hellenistic astronomy texts and most completely two centuries later by Ptolemy. His mature astronomy work appears to have been done on the island of Rhodes a large island to the west of Cyprus and far from his home near Constantinople. There he built an observatory and created or improved on instruments for measuring positions of stars and planets. He was a serious observer of astronomical objects and events and a mathematician of significance. Finally, the world was ready for a complete astronomer...The Greatest Astronomer, he was later called.

Let's be clear: **astronomy was different after Hipparchus**. He dedicated himself to an entirely different purpose from the "picture-stories" of Plato and Aristotle. Hipparchus measured numerical features of the cosmos.

Hipparchus' Solar Model. Hipparchus figured out that if he used the eccentric model only a few parameters were required in order to determine, e and so the problem of the seasons' unequal durations could be solved geometrically, almost like being a cosmic surveyor. His model is shown in Figure 3.17 with the anchor for astronomical positioning, the Vernal Equinox (VE, ♈) (a convention used to this day). The Sun (☉) orbits the center of the eccentric orbit at C and the Earth is displaced by the eccentricity, e (which is usually quoted as the fraction of the distance $\overline{CE}$ to the radius, $\overline{CA}$). The dash-dot lines denote the axis from the Vernal Equinox (mid-March) and the Autumnal Equinox (AE, mid-September) and the Summer Solstice (SS, mid-June) and the Winter Solstice (WS, mid-December) and the four unequal quadrants delineate the four seasons. Here it's drawn for antiquity in which spring was the longest season and autumn was the shortest (while in our time summer is longest and winter is shortest). In astronomy, the furthest point of a celestial object's orbit from a reference is called the "**apogee**" and the closest approach, the "**perigee**." The figure shows the arrangement for antiquity, when the angle of the dotted line through E and C was about $\alpha = 65^\circ$. Today, it's greater than 90° which is why our summers are longer than antiquity's summers.

His result was that the eccentric is displaced from the Earth by about 1/24th (about 0.04) of its orbital radius so it's almost a circle centered on Earth, which is why the season durations are within a few days of one another.[17] Notice that our summer and spring is when the Sun is at apogee and fall and winter are at perigee.[18]

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[17] Had $e = 0$, then all four season would have been the same length and the Sun's orbit would have been Aristotle-like, centered on the Earth.

[18] Why the Sun is *furthest* away during the summer is a reasonable question and understanding that waited for Kepler and Newton.

Hipparchus could use his solar model to predict the location of the Sun at any time in the future and it was accurate and used for many hundreds of years.

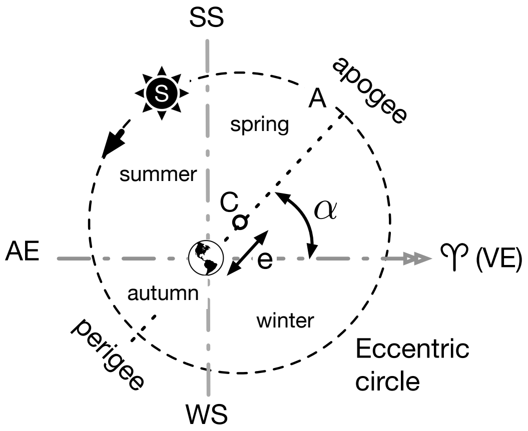

Figure 3.17: Hipparchus and Ptolemy's solar model showing the seasons in antiquity (today, winter is shorter and summer is longer). SS and WS are the Summer and Winter Solstices, VE (♈) and AE are the Vernal and Autumnal Equinoxes and the seasons are then defined as the four quadrants among them. The Earth (⊕) is displaced from the Sun (⊙) by the eccentricity, e, the distance in space from Earth to the center of the eccentric circle about which the Sun orbits. The dotted line is described in the text.

Hipparchus' Lunar Model. The Moon's motion is different and more complicated than the Sun's with at least three parameters required to determine its motion. He managed that as well, this time using an epicycle model. Finally that legend ascribed to Thales from 400 years before is made whole: Hipparchus could predict both solar and lunar eclipses!

In addition to his modeling of the Moon's motion, he found a way to determine the distance from the Earth to the Moon. With his version of trigonometry (see below), he found that the distance from the Earth to the Moon is 65.5 times the radius of the Earth and that's about right (it's about 60.336). (Newton used his result in his invention of his Law of Gravitation.) Hipparchus attempted the same thing for the distance to the Sun, but underestimated it by a factor of 50.

Hipparchus' Fixed Star catalog. Hipparchus began the first quantitative survey of the fixed stars—the ones thought to be on the inside of the Celestial Sphere. Prior to him, locations of bright stars were noted by identifying a rough relative position in words: that a the star in the "shoulder" of one in one constellation is rising when the star in the "sword" of another constellation is setting and that the star on the "right leg" of a third constellation appears right overhead when this happens. More stories. Hipparchus took a different approach.

His data were extensive and would have required impressive patience (night after night) and commitment to a multi-year research project. Ptolemy tells us that Hipparchus cataloged around 850 stars, their positions, and their brightnesses and they were in use for centuries afterwards. Others had made catalogs (Eudoxus and Eratosthenes), but his was different: he invented a coordinate system and assigned positional numbers to each star. Think about how your GPS specifies a location on the Earth: my phone tells me that the location of the Library of Alexandria is 31.20870° N, 29.90911° E. What that tells me is that the library is a little more than 31° north of the equator (the **latitude**) and about 30° east of some point that's world-wide agreed to be the observatory at Greenwich, England (the **longitude**). Hipparchus adopted the same thing, but applied to the stars—the underside, if you will, of that Celestial Sphere above us. (More about this and how his system is essentially identical to modern astronomy is discussed in *Greek Astronomy, Today* in Section 3.8.2.

A many-decade detective story unfolded in trying to figure out which (if any) of Hipparchus' data were included in Ptolemy's more extensive star catalog. And there's a clue. Remember Aratus' poem, *Phaenomena* from Figure 3.1 which was written as an ode to Eudoxus? The one book we have of Hipparchus' is his *Commentary on the Phaenomena of Eudoxus and Aratus* in which he severely criticized mistakes of fact in the poem regarding the relative positions of stars in the constellations. He included a set of positions for 22 stars of his own observation and these have been extensively compared with Ptolemy's catalog and the agreement is pretty good. Without that poem, and Hipparchus' grumpiness about a 200 year old poem,[19] we wouldn't have any corroborating information that Hipparchus really did create the first ever quantitative star catalog. Well, maybe until 2022! For that breaking story, look at *Greek Astronomy, Today* in Section 3.8.3.

Hipparchus' Trigonometry. The mathematical problems he had to solve for his solar and lunar models were surely the inspiration for a tool that marks the invention of trigonometry. Figure 3.18 shows his idea. A chord inside of a circle with radius R and center O is shown as the length $\overline{AB}$ where the chord subtends the angle θ. By hand Hipparchus divided carefully drafted circles into degrees based on 360° (which came from the Babylonians), but much finer: 21,600 segments which is the number of arc minutes in 360°. Then he painstakingly created "tables of chords" of varying lengths for each segment giving him a fairly precise lookup table of angles, radii, and chords. Given a radius, and the length of a cord, an angle could be looked up in the table. Or visa versa. It's equivalent to a table of trigonometric sines since as in the figure, if one divides the chord in two so that there are two right angles at point C, then the $\sin(\frac{\theta}{2}) = \frac{1}{2}\left(\frac{\overline{AB}}{R}\right)$.

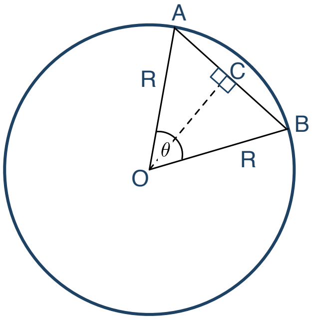

Figure 3.18: Showing how ancient "chords" related to a modern sin for a given angle θ.

Hipparchus' Discovery of the Precession of the Equinoxes.

The discovery for which he's most known was that the Earth's seasons might shift over time. He found this in two, complimentary ways. His first approach suggested the location against the zodiac of the summer solstice was 12 hours different from that recorded by Aristarchus, 145 years before. That inspired him to make a second, clever measurement to confirm that odd result.

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[19] He wrote other ill-tempered reviews of other people's writings.

He figured out how to determine the longitude of a star (the angular distance of the star relative to the Vernal Equinox) near the ecliptic and compare that to an earlier measurement from other astronomers. He focused on the bright star, Spica (the brightest in the constellation Virgo, or α Virginis) for which he had data from an Alexandrian astronomer, Timocharis in -294 and -283 almost two centuries before him. This could be done easily in principle. Just measure the angle between the Sun and the star, right? That is:

$$\text{Longitude, Spica} = (\text{longitude, Sun}) + (\text{arc-angle between Spica–Sun}).$$

He knew the longitude of the Sun from his Solar model which gave him the angle α from Figure 3.17. The arc-angle in longitude of Spica and the Sun is a different story since if the Sun is out, that's daytime (!) and so you can't see the star. But he was very clever. He made use of the fact that during a lunar eclipse, the Earth is directly between the Moon and the Sun...so they are 180° apart and at night, he would be looking away from the Sun, toward Spica. So measuring the arc of longitude of Spica relative to the eclipsed Moon gives him his answer:

$$\begin{gathered}\text{Longitude, Spica} = (\text{longitude, Sun}) + (\text{arc-angle between Spica–Moon}) \\ + (\text{arc-angle between Sun–Moon}).\end{gathered}$$

At an eclipse, the (arc-angle between Sun–Moon) is 180°! Using Timocharis' Spica-Moon measurement, the longitudinal difference of Spica was 8° west of the Autumnal Equinox while he determined 6°: the longitude of Spica had increased by 2° in 150 years. (He actually did this as an average of two different eclipses 11 years apart.) That's about 1° per 75 years (consistent with his other measurement). Ptolemy did a similar experiment 265 years later and compared it with Hipparchus' and got about 1° per 100 years.

So what's going on here? Hipparchus concluded that the zero-point of longitude (the Vernal Equinox, which is where the ecliptic crosses the Celestial Equator) must be moving somehow over very long times.

This we know now has a physical cause: the Earth's axis of rotation points at an angle that's not perpendicular to the plane of its orbit around the Sun. It's tilted by close to that 23.5° from Figure **??** and like a top, the mass of the Earth causes it to precess around the Celestial Pole. This wobble of the Earth *looks* like a wobble of the ecliptic and so the equinoxes will be in a different location as time marches on. How fast? We know now the precession rate is pretty close to Hipparchus' and Ptolemy's measurements: about 1° per 72 years. So to go all the way around, requires $72 \times 360^\circ = 25,920$ years.

3.6.4 Summary of the Astronomy of Aristarchus, Eratosthenes, Apollonius, and Hipparchus

(Set the context with the timeline in Figure **??** on page **??**.)

- Aristarchus (-310 to -230):
    - He made the first attempts to use geometry to measure distances among and sizes of the Earth, Moon, and Sun.
    - He proposed the first model of a Sun-centered cosmology, apparently without geometrical modeling.
- Eratosthenes (-276 to -194):
    - He measured the diameter of the Earth to impressive accuracy.
    - He measured the obliquity of the ecliptic—that 23.5° tilt of the ecliptic from the celestial equator.
    - He apparently created a star catalog of more than 600 stars. This would have been in words itemizing apparent locations of stars relative to constellation points.
- Apollonius (-240 to -190):
    - He was mathematician of the first rank and found a picture-way to model the Sun's motion around the Earth to create seasons of different lengths through the introduction of the deferent and eccentricity.
    - He also found a mathematically identical, but geometrically different form for planetary motion called epicycles. His proof of their equivalence was lauded as an important step by Ptolemy.
- Hipparchus (-190 to -120):
    - He built on Apollonius' deferent model and found a way to measure the actual eccentricity of the Sun's orbit and the longitude of the apogee. This was the first attempt to not only geometrically model the cosmos (or any physical mechanism) but to also quantitatively measure the shape parameters of the model.
    - He found a way to determine the distance to the Moon in terms of Earth radii, a value used by Newton much later.
    - His star catalog of more than 800 entries went beyond the stories that had been told previously: he invented a coordinate system that could be used by anyone to find the actual numerical positions of objects relative to an "origin" of essentially a celestial longitude and latitude.
    - He discovered that the Earth's seasons shift relative to the star's positions over time—the precession of the equinoxes. Understanding the physical cause of this phenomenon waited for Newton's explanation of the precession of the Earth's axis of rotation...slowly: about 1° per 75 years.

3.7 The End of Greek Astronomy: Ptolemy

While Aristotle's concentric spheres model lay dormant, it was to rise again in the middle ages and assume a strange parallel existence next to the model that made precise predictions. This is the model of Claudius Ptolemaeus, known for nearly two millennia as **Ptolemy of Alexandria** (100 to 170 CE). He created the most complete model of the cosmos before Copernicus and, refreshingly, his books survived intact thanks to Arab intellectuals' commitment to preserving and commenting on the works that they encountered from the Islamic conquest of the Near East, all of Northern Africa, and Spain.

Ptolemy wrote three books on astronomy for which we have original Greek and some Arabic translations. *Mathematical Composition* is the main work, now known by its Arabic title of *Almagest*, a corruption of the Arabic *Al* with the Greek word *megistē*, for "*the greatest*." The second is the *Handy Tables* which consists of two parts: the second part includes tables of his planets and stars of which we know from medieval versions 200 years after Ptolemy's life. The first part is the instruction manual on how to use the tables, surviving only in its Greek origin. *Almagest* is too complicated to have been absorbed by most and so the *Handy Table*s assured widespread use of Ptolemy's work. The third, *Planetary Hypotheses*, is an upgrade of the earlier *Almagest* and an attempt to build a plausible physical model of the purely mathematical *Almagest*. It was only appreciated and fully translated as two books in the 1960s!

Even though we finally have a complete set of one of our astronomer's works, ironically we know little about his life, except for a few references of his and a few later narratives by Roman and medieval scholars. Ptolemy almost certainly worked in Alexandria as his extensive observations come from that latitude. He's the first of our Greeks to have two names! "Claudius" indicates that he was a Roman citizen, probably during the time of Emperors Hadrian to Marcus Aurelius. "Ptolemaeus" indicates that his was of Greek ancestry.

Almagest is a huge subject. It is 700 pages long in a modern edition and more than a thousand pages are required to fully lay out the considerable mathematics of the book (N. M. Swerdlow and O. Neugebauer, 1984). It's not for the faint of heart. It's also pure mathematics and little philosophy and *not a physical model*.

Here's what it's like. I could imagine building a mechanical model of the economics principle of supply and demand. Suppose we have a playground teeter-totter with an arrow on the right end that points to a dial indicating high or low for prices of goods. Right side up, prices high, right side down, prices are low. If we start with the teeter-totter level and add weights to the right to represent *supply* of that product and weights to the left to represent *demand* for that product...we've got a mechanical model of the economy. When the supply, right-weight is larger than the left demand-weight, the arrow points down—prices fall. Likewise, when demand outweighs (sorry) supply, then the left side goes down and the arrow points up for higher prices.

This is a perfectly predictable model of the economy and through careful analysis of past economic history, one could tune the amounts of weight that would correspond to a prediction of prices and mark the dial with $ indicators. But, while it's a good model, *it's not a realistic representation of the economy.* *Almagest* is like that. It's a very complicated model of moving and spinning circles, lots of numbers to characterize the circles, scores of huge tables of numbers,[20] and could accurately predict positions of the heavenly bodies. But Ptolemy made no claim that the Sun, Moon, and planets actually performed the motions in his model.

Ptolemy's Philosophical Roots and Prerequisites for the Book: Books I and II of *Almagest* describe his working philosophy, defending it with standard arguments. But apart from the actual heavenly body motions, it's Aristotle, top to bottom. The mathematics required was Euclidean plane geometry and the use of Hipparchus' chord tables, except Ptolemy made them even more precise. He used the new "spherical geometry," and he developed it from scratch for the reader. With this introduction, he's ready to solve the world.

Ptolemy's Solar Model: Book III This was relatively easy and critically important. All of positional astronomy—to this day— depends on understanding where objects in the sky are relative to the Vernal Equinox, which in turn depends on the Sun's motion and position at any time. He didn't invent a solar model—he replicated Hipparchus' and was generous with his praise the original author.[21] So, Ptolemy's model of the Sun's is exactly the same: Figure 3.17. He repeated Hipparchus' determination of the eccentricity and agreed, but with higher precision: $e = 0.0415$ as compared with Hipparchus' $e = 0.04$.

Ptolemy's Lunar Model: Book IV and V. The motion of the Moon is difficult to grasp even today. Ptolemy's solution was ugly and also his biggest mistake: he could solve for eclipses (lunar and solar), but his model predicts that the Moon's apparent size would vary by a factor of two in a month, which obviously isn't the case. His solution is tortured and from our modern perspective, clearly an indication that there must have been something wrong. One has the impression of him just giving up and declaring successful eclipse predictions as a victory. He made careful tables of predictions of the eclipses—which were accurate— for any date, and washed his hands of the Moon problem.

Ptolemy's Model Fixed Star Catalog: Books VII and VIII. It was Ptolemy who told us of Hipparchus' catalog of the positions of 850 stars. He takes on the same task, but also includes the positions and apparent star brightness of 1022 objects from 48 constellations in his catalog and with this began almost two centuries of fights among historians. Did Ptolemy copy Hipparchus' 850 stars (shifting their longitudes by $2^\circ 40'$ to correct for the precession of the equinox over 265 years) or did he measure their positions as he claimed? Or had Hipparchus' catalog been wrong? The comparison of the Hipparchus' 22 stars' from his *Commentary* to Aratus' poem with their counterparts in Ptolemy's catalog is the key. There are translations problems since Greek numbers were written using Greek letters and sometimes mistakes happened in translation and transcription of centuries-old media. Stars were not always named, but a little story was told about each one to locate it within a constellation. So mistakes happened. This argument has largely subsided: within

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[20] Perhaps the first use of tables in any manuscript in history.

[21] He has been accused of plagiarizing Hipparchus, but that's not fair as he gave ample credit.

the uncertainties that can reasonably be attributed to each, most of Hipparchus' 22 stars do match their Ptolemaic counterparts and that each astronomer is likely vindicated. I'm sure you're glad that we've cleared that up.

The bottom line about Ptolemy's catalog is this: it represented an enormous effort over probably decades and was the best star chart all the way to Tycho de Brahe in the late 16th century (Copernicus used much of it). A remarkable achievement and legacy.

Ptolemy's Planetary Theories: Books IX through XIV. His planetary models (yes, there were three) were the target of the Muslim astronomers, Copernicus, Galileo, Tycho, Kepler, and Newton and it took all of them to bring Ptolemy down. Its accuracy is still impressive so something besides getting the right numbers was behind its downfall, an important part of our story later.

The end product of his planetary research is a chapter for each of the five planets including its geometrical model, the particular parameters built into each model, a description of how he determined each parameter from his observations, and then five deliverables: a set of tables of positional coordinates for each planet, for any day in the future. It was these tables that were reprised in his User's Manual, the *Handy Tables*.

He must have struggled mightily to make Aristotelean circular orbits work but he held accuracy to a higher standard than the Classical Greeks, for whom a nice picture-story was sufficient. In order to "get it right"—which meant, make predictions that worked— required him to make excursions from some of Aristotelian rules. For example, the eccentric model for the Sun and a strange epicyclic model of the Moon had heavenly bodies orbiting seemingly arbitrary points in space apart from the Earth! But as painful as the Moon solution was, getting the motions of the planets right was another story altogether.

3.7.1 Mars, Jupiter, and Saturn

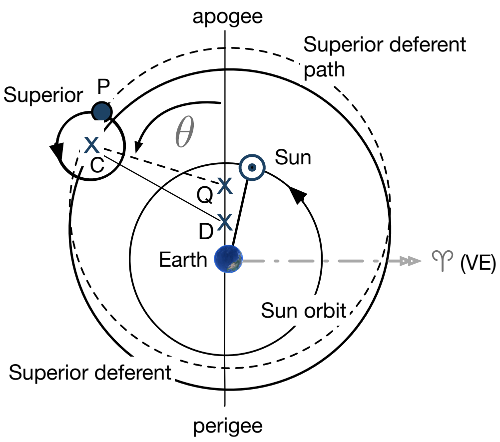
The prominent retrograde motion of especially Mars as well as Jupiter and Saturn added an entirely different set of complications from the naive epicycle model of Apollonius and Hipparchus. The simple epicycle picture of Figure 3.16 wouldn't do. Ptolemy had to insult Aristotle one more time and that particular solution offended Copernicus and his Arab predecessors. Let's look at his solution for the outer planets as they're a little simpler. Figure 3.19 shows his model that functions for Mars, Jupiter, and Saturn. Look at Figure Box 3.19 on page 50. After you've read the material in that Box, return to this point ☟ and continue reading.

The new wrinkle is the introduction of a third point in space, the **equant** (Q), displaced from the deferent point by the same amount as D is from E. A superior planet's epicycle's center P doesn't undergo uniform circular motion about the deferent center, D, *but about the equant*, Q. That is, the angle θ uniformly increases in time around the epicycle's path, so it appears to perform *non-uniform* rotation around D (its center) *and non-uniform around Earth.* The model constrains this movement such that the line from a superior planet to P, $\overline{\text{Superior-P}}$, is always parallel to the line connecting the Earth and the Sun, $\overline{\text{Sun-Earth}}$. Notice that this creates a special relationship among the Vernal Equinox, the Sun, and the planet.

So a superior planet orbits in its epicycle with center (P) following its deferent as originally imagined by Apollonius—except that as compared to Figure 3.16 the epicycle rotation is reversed from counterclockwise to clockwise. *That creates retrograde motion.* The Sun is shown with its orbit centered on the Earth (since its eccentric center is too small to explicitly show). So there are two centers of motion here—one for the Sun and another for Mars' deferent.

The dashed curve in the figure is the trajectory of Mars' deferent. So what Ptolemy knew was the various positions that Mars, Jupiter, or Saturn would have on the *dashed line*, but what he needed in order to build each model was its position on the deferent, the solid line. That's a formidable mathematical transformation.

FIGURE BOX 3.19



Ptolemy's model (not to scale) for a superior planet, Mars, Jupiter, or Saturn (P) and its relationship to the Sun (⊙). Here, "Superior" (◯) is on an epicycle with its center at C. C rotates clockwise around the circular deferent path with its center at the center, D. The Earth is not at the center of anything, except close to that for the Sun! The angular speed of P around D — the amount that the angle θ increases with time is constant, but about the point Q...not D. The dashed circle is the path that P actually takes which its offset from the deferent's center.

Each planetary "kit" looks like this for superior planets and slightly different for the inferior planets. Every deferent radius for all planets was chosen to be 60 in an arbitrary set of units. The necessary parameters were determined by Ptolemy separately for each planet, including: the epicycle radius, the separation of Earth from the deferent point (D), which is also the separation of D from the equant, Q, the orientation of the apogee to the Vernal Equinox direction, and the angular speed at which θ increases in time.

Now go back to page 49 and pick up where you left off.

> "...in a tour de force of possibly the most complex and extended calculation in all of ancient mathematics, he developed a method of successive approximation that allows

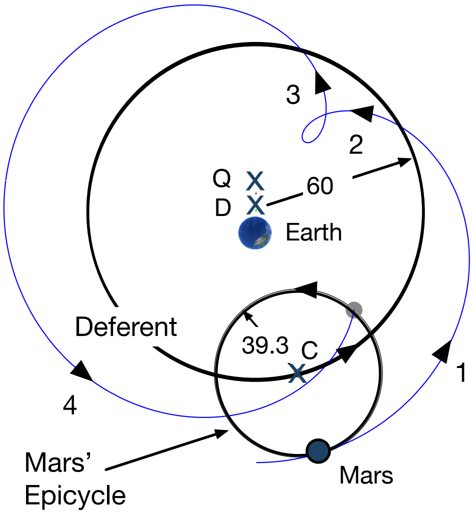


Figure 3.20: Mars (♂) is shown on its epicycle with its center, C, rotating around the deferent with its center at D. I've used Ptolemy's actual relative sizes for Mars. All deferents were in units of 60. Mars' epicycle's radius is 39.3/60 and the distance from Q to Earth is 12/60. One can see the strange loop motion described in the text.

> the numerical values of the eccentricity and the direction of the apsidal [direction of the apogee of Mars' orbit] line to be found to any degree of accuracy. Both the problem and the solution are remarkable...his solution shows a very high order of mathematical intuition...The number of astronomers after Ptolemy who understood and could apply the method must have been very small." [N. M. Swerdlow and O. Neugebauer, 1984, Vol 1, p307.]

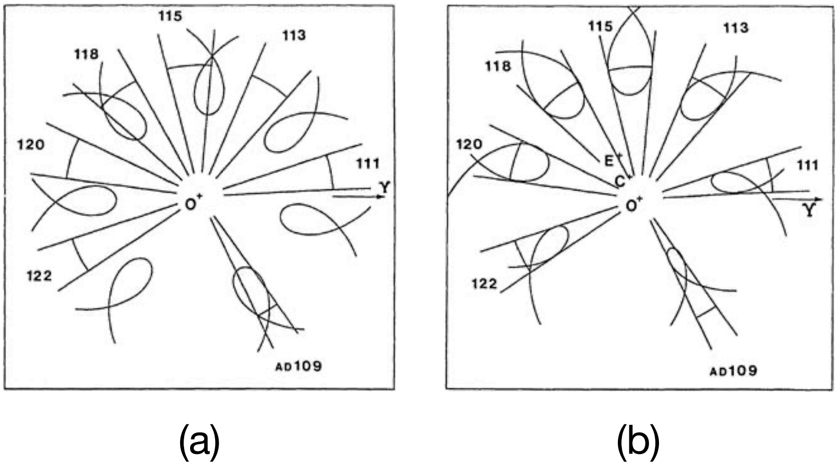
Let's pick on Mars. Mars orbits Earth (in our 20th century way of viewing things) about every 687 days, or 1.88 Earth years and undergoes retrograde motion about every 2.1 years, or a little more than one revolution around the Sun. The backwards appearance lasts a little more than two Earth months, or about 72 days. Ptolemy's model with the equant rather precisely describes Mars' retrograde motion as it forces a kind of loop-the-loop as viewed from Earth.

In Figure 3.20 I've calculated the Mars model to show its epicycle and eccentricity (separation among Earth, D, and Q) using parameters taken from *Almagest*. Mars' path is, well, unusual. There are 4 points identified on the actual path that Mars takes while riding on its epicycle. We start at position 1, and as the epicycle turns and as the deferent turns, Mars moves to position 2 where it starts to appear to slow making that loop which makes it appear to go backwards during 72 nights. Then it comes out of retrograde and continues its forward-appearing path at 3 and nearly completing it's 1.8 year long path at 4. In each Mars year, the location of the loop shifts a bit relative to the Vernal Equinox.

This is what's seen from Earth with a bonus: it also addresses the fact that in retrograde, the planets are brighter, here, because it would literally be closer to

Earth. Just how often and how fast would be determined by the parameters—Jupiter and Saturn's parameters are quite different.

It works very well as seen in Figure 3.21 from James Evans, 1984 (inspired by James Evans, 1998). This shows seven bands that should encompass the retrogrades of Mars as viewed from Earth for some of the years of Ptolemy's observations, from 109–122 CE. The loops are the Mars retrograde events relative to the Vernal Equinox (the trajectory between points 2 and 3 in Figure 3.20) and the wedges show predictions of where that should happen. In (a) predictions are for a straight epicycle model *without an equant* while (b) shows the same thing, but *including the equant*. This, and other successful measurements surely convinced Ptolemy that he was right. He needed the equant.



(a) (b)

Figure 3.21: Seven retrograde loops of Mars for times of Ptolemy's observations (a) without the equant and (b) with the equant.

> The relationship that Mercury and Venus have with the Sun was very problematic. Today we know that they orbit very close to the Sun but even now measuring their positions is challenging. The Sun's in the way! Observations had to be done just after sunrise and just before sunset...and carefully as to not blind one's self. So they presented a set of problems which couldn't be solved without separate models for each. And those solutions are strange, especially for Mercury with more moving centers of deferents.

Think about all of the major ways in which Ptolemy has violated Aristotelian imperatives. Is Earth at the center now? Of what? The outer planets and the Sun no longer orbit around it symmetrically. They also don't orbit at constant speeds except now around an uninhabited point in space, not around the Earth. It's torturously pieced together in ways that Aristotle could never have imagined—and that a modern physicist would not have tolerated. "Simplicity" is nice in physical models, not guaranteed, but when your model is so bizarre you'd tend to think that it's trying to tell you that the world is probably not that way. But this is the first time.

Going from pictures and stories to numerical prediction surely meant that when predictions worked, then it must be some part of the truth. The late 16th century's Johannes Kepler is from whom we learn the real solar system model and we'll have to wait 1400 years to Chapter **??** for him to appear and save the day.

Not always appreciated, was the fact that in *Almagest*, the outer planet's deferents were all taken to be the same radius and that the distances were all set by the epicycle's individual radii. He chose 60 "units" (always working within the Babylonian base-60 sexagesimal system we use today for time and angles) for that common deferent radius with the Mars:Jupiter:Saturn epicycle radii in proportions of approximately 7:2:1. This was because the planetary models in *Almagest* were not a system. Much like Eudoxus before him, he treated each planet separately and made no attempt to merge them, until much later in his life. Figure 3.22 shows Ptolemy's independent planetary pieces.

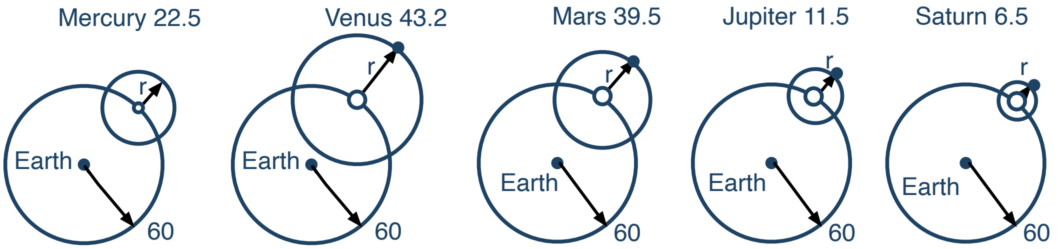
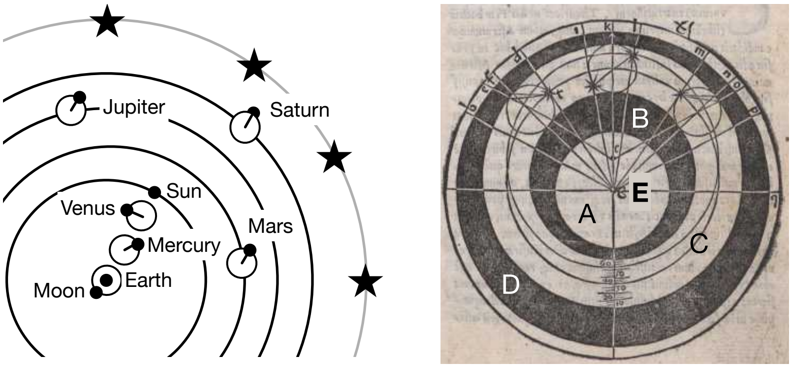


Figure 3.22: Each of the planets' epicycles are shown with their differing *r* values listed above as they ride on their deferents which each of the same radius. The units are arbitrary, so the relative epicycle radius to deferent is a measure of their relationship to the Earth. So the larger is r, the closer that planet is to Earth.

3.7.2 Ptolemy's Cosmology.

Just as it was important for Aristotle to build a multi-planet system out of Eudoxus' separate planets, it obviously seemed incomplete to Ptolemy also. So he later wrote *Planetary Hypotheses* which upgraded some of his measurements but also presented a whole cosmology of all of the heavenly objects. Figure 3.23 (a) shows it in a simplified format with an abstraction of the epicycles for each planet: the line in each epicycle shows the relationship of the planet to the center of its epicycle. Notice that for the outer planets, the epicycles are constructed for that line-direction in each is parallel to one another and parallel to a line connecting Earth to the Sun. For the inner planets, it's the *centers* of their epicycles that all lie on that parallel line connecting the Earth to the Sun.

> ▻ The Sun drives the whole machinery and the inner planets and outer planets have different models and constraints. But those clues weren't enough to resurrect the Aristarchus model with the Sun at the center. Such was still the strong pull of Aristotle's prejudices.



(a) (b)

Figure 3.23: The whole cosmology of Ptolemy. In (a) the planets, and Sun are arranged in a very particular way relative to the Sun. The lines in the circles for each planet represent the center of epicycle to the planet. For each of the outer planets, the epicycle-to-planet lines are *all parallel to one another and parallel to the line that connects the Earth to the centers of the inner planets, to the Sun.* The centers of the deferents for each inner planet and the Moon are all along one another and point at the Sun. **The Sun is always key.** In (b) an image from *Theoricae novae planetarum* by Georg Peurbach is shown which represents a slice through the Medieval idea of Ptolemy's 3-dimensional model for one planet. Notice the epicycle in various positions inside of the region labeled C. The other labels are described in the text. (Wikipedia, Georg Peurbach)

Recall in Section 3.5.2, I noted that that the classical planet ordering was Plato's and Aristotle's: Earth–Moon–Sun–Mercury–Venus–Mars–Jupiter–Saturn and the stars. Ptolemy made the executive decision to change that to Earth–Moon–Mercury–Venus–Sun–Mars–Jupiter–Saturn and because of his authority, it stuck. (Again, notice that the Sun sits between (our) inner and outer planets. Interestingly, whenever a Medieval or Renaissance rendering of Aristotle's cosmos was presented in books it was Ptolemy's not Aristotle's ordering that was used. Sometimes Ptolemy's name is included on an image, even though the picture might be Aristotle's equal-orbit, totally geocentric geometry. Ptolemy's and Aristotle's pictures get mixed up during Medieval and Renaissance depictions.

Planetary Hypotheses also presented a physical model for his cosmology. In it, there are solid aether spheres which carry the epicycles through...pathways in the solid aether around the Earth. This wasn't interpreted as an image until the early part of the 15th century when Georg Peurbach's 1454 *New Theories of the Planets* included the image shown in Figure 3.23 (b).[22] Think of this as a slice through a spherical aether unit required to support and guide a planet. The light volume labeled A would contain another such unit, and so on...so that together they would nest together like Russian dolls. It's what's in a unit that's hard to swallow. The light region, C, is a kind of hollowed-out shell within which an epicycle rolls around a diameter. It's off center since the planet follows the epicycle sometimes close to the

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[22] We'll meet Peurbach in the next chapter.

Earth, E, and sometimes away from it.

He imagined that the largest excursion of, say, Mercury's orbit in its epicycle, constrained inside of Mercury's C cavity, would just match the smallest excursion of Venus' orbit in its epicycle, within its C cavity. Then the largest excursion of Venus' orbit would just match the inner excursion of the Sun's and so on. He packed them together with minimal spacers of aether (D and B in Figure 3.23 (b)).

He demanded uniform motion of the spheres, but the shifting of their centers is a problem. Imagine a soccer ball spinning around an axis at a uniform rate. Can it spin around another axis parallel to the first one at a uniform rate? No! It's physically impossible and this truly offended many Muslim astronomers and mathematicians who attacked his physical model in no uncertain terms.

While his planetary orbits were independent of one another, their relative orbital sizes could be calculated as each is determined by the tight-fit. So if you knew the size of one of them, you could then establish the size of others, working your way from edge to edge of each "spherical space-shell."

He knew the distance from the Earth to the Moon (from studies like that of Aristarchus) and the Earth to the Sun and in this way he actually calculated the distance from Earth *to each planet and to the stars themselves*! For example he calculated that the maximum distance from the Earth to Venus was 1079 Earth radii. (Today, we know that the maximum Earth-Venus distance, across the Sun pretending that they are as far away from one another as possible is more like 25,000 Earth radii.) For fun, he predicted that the distance from the Earth to the Stars—*the size of the entire universe*—would be $20,000 \times E_R$, or 126,000 km. Both an astonishing feat—-calculating the size of the entire universe—and wildly wrong. His universe's size is smaller than the actual furthest separation of Earth and Venus in our world.

3.7.3 Summary of the Astronomy of Ptolemy

(Set the context with the timeline in Figure **??** on page **??**.)

- Ptolemy (85 to 165):
    - He wrote the mamoth book, *Mathematical Composition*, nicknamed by Islamic astronomers as *Almagest*, which became its label to this day (it's in the dictionary of your word processor). It was the definitive tool for predicting the positions of all of the heavenly bodies. The naive Copernican heliocentric model is mathematically identical to the epicyclic model of Ptolemy. No better, no worse than Ptolemy's.
    - He created a star catalog of more than a 1000 stars, including a subjective measure of each's brightness.
    - He continued Hipparchus' solar model with a separate, and corroborating measurement of the eccentric.
    - He adopted the epicycle model of Apollonius and found ways to assign measured parameters to the epicycle variables: the deferent radii he took

as constant and found epicycle speeds of rotation, radius, and orbital speeds on the deferents, separately for each planet.

- He wrote a "handbook" (*Handy Tables*) that would teach an astronomer, physician, or astrologer how to predict the positions of planets using his model, without having to absorb the considerable mathematics of *Amalgest*.
- He later wrote a complete cosmology that attempted to put all of the planets, epicycles and all, into one nested cosmological model. This allowed him to make predictions about the sizes of orbits.

3.7.4 The End of Greek Astronomy

Think about the conceptual leap that we've taken: we've gone from Aristotle who told picture-stories about the universe to Ptolemy who quantitatively modeled the entire universe! He used measurable parameters that located all of the heavenly bodies, predicted their motions, and proposed numerical distances to every object including the size of the entire universe. It's an astonishing feat and nobody successfully challenged it for 1400 years (although there were many attempts by the Muslim astronomy and mathematics community) which is a pretty good record.

He was the last Greek astronomer. Science would explore new frontiers, but the Greeks would no longer be the explorers. Rather western research[23] in MOTION BY THE EARTH and MOTION IN THE HEAVENS shifted to India and among the Muslim scholars who did some original work, and translated, preserved, and commented on Greek writings—especially Ptolemy.

3.7.5 One More Thing?

This was an unusual set of chapters and what follows will be considerably less sweeping and more focused. But the scene is now set for the full story of MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS. Here's a fascinating coda to our Ptolemy story. He was so close!

Imagine a very simple auto race with two cars. The track consists of two lanes, both circular around a common center. One lane, in which car M stays has a larger radius than the other lane in which car E is constrained, So it's not a fair race, since M has further to go in a revolution than E. But, this is an analogy.

From the stands you can watch the two cars go in their counterclockwise circuit and here not only does E have an advantage as the inside lane, but E is also faster than M. So naturally, *it will periodically lap and pass M*. When that happens, to the driver in E it looks like M is in front...and then seems to E to go backwards as it's lapped!

By now you realize that in this race analogy I can substitute E for Earth, M for Mars, and S for Sun and we've just described a simple solar system of two planets viewed from two different perspectives (the people watching the race, and E). It

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[23] There was a parallel research path in China, but it didn't influence the eventual progress Europe

should be, and is, possible to construct an algorithm (involving vectors) to translate the motions from one frame to the other. The spectator's view corresponds to a solar system of the sort that you have learned that Copernicus described: all of the planets orbiting the Sun in perfect circles and the other, is the solar system that Ptolemy discovered in which the Earth is stationary and the Sun and planets orbit it...but on epicycles.

> ▻ The Ptolemaic model is mathematically identical to the Copernican model in which the orbit of an outer planet (like Mars) has the same dimension as the deferent circle of the Ptolemaic model.

What Ptolemy accomplished was an extraordinary mathematical feat. In fact, it's much more complicated than our modern view! He took a long, intellectual journey to his model whereas if he'd taken Aristarchus' model with the Sun in the center and circular orbits of the planets...he would have had a much simpler task. But what was in his way?

It was Ptolemy's commitment to the Aristotelian edict that the MOTION BY THE EARTH is zero, wrongly supported by a misunderstanding of the physics of MOTION ON THE EARTH *that was in the way of creating the better model*. Unraveling this is the task of this book: getting, first, the MOTION ON THE EARTH right and then applying it to MOTION BY THE EARTH and MOTION IN THE HEAVENS. It didn't come easy.

3.8 Greek Astronomy, Today

3.8.1 Let's Set The Record Straight: How we now understand the sky

From our more advanced vantage point: every one of the above points in Section 3.2.1 is explained overall by a Sun-centered solar system (with some nuance) around which the Earth and other planets orbit.

Elliptical orbits. We know that our solar system is built of eight planets (Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune). Figure 3.24 (a) is familiar to all schoolchildren today. We know that their orbits are not circular, but slightly elliptical, with the Sun at a focal point and as such, when they are close to the Sun, they whip around it fast and when the are far from the Sun their motion is slower. They are nearly all in the same plane, which is shown in Figure 3.24 (b) where we take Earth's orbital plane to define the ecliptic (0°) so relative to that, Mercury's orbit is the most inclined at $\pm 7^\circ$ from the ecliptic. All of the other planets' orbits are within that 14° band. For those of you mourning the elimination of Pluto from the planetary family, it's inclination to the ecliptic is more like $\pm 17^\circ$, as are other dwarf planets in the outer edges of the solar system. The undisputed opinion now is that Pluto's existence is due to some event that is not of the same origin of the other planets. Hence, it's being voted off of the planetary island.

Figure 3.25 (a) shows a line-up of planets (in simulation) as they appeared in the eastern sky on June 24, 2022 just before dawn from East Lansing, Michigan. Notice

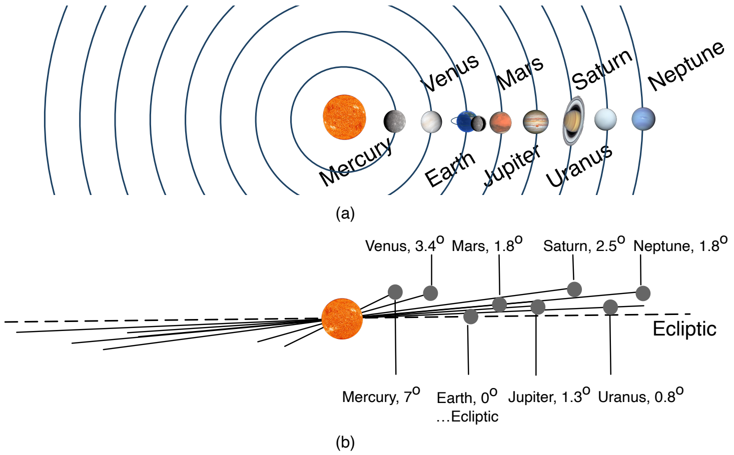

Figure 3.24: (a) is an abstract sketch of the solar system as we picture it today and and which we credit to Copernicus. "Abstract" because the alignment of the planets is for display purposes, actual relative radii of the orbits are not anything like shown, and the orbits are elliptical, not circular. (b) shows what the relative orbital planes are for each planet. The planets all have orbital planes inclined slightly to the overall ecliptic (the dashed horizontal line is the edge of the ecliptic plane). Notice that Mercury's is the one with the highest inclination of 7°. Pluto's is almost 17° up and down, indicative of its not belonging in the club of solar system planets.

Figure 3.25: The inclination of the Earth's spinning is oriented away from being perpendicular to the ecliptic in which the Earth's orbit is fixed. Also, the orbital plane of the Moon's orbit around the Earth is slightly inclined relative to the ecliptic as well.

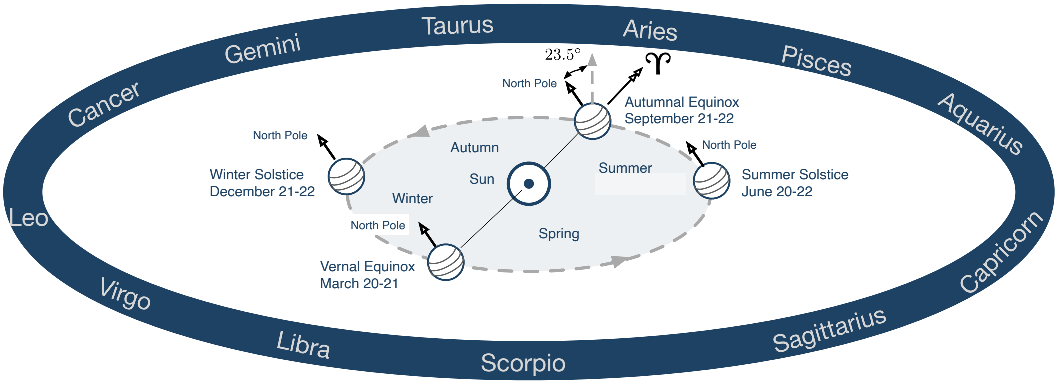

Figure 3.26: There's a lot in this image. The Sun (☉) is at the center and ecliptic is shown as the gray circle around which the Earth orbits. The 23.5° inclination is pictured showing how the solstices are inclined in our northern hemisphere's summer and winter. The Vernal Equinox (♈) is pointing at the zodiacal constellation of Aries, as it was in ancient times (today, it's in Pisces).

that the Sun is just peeking over the horizon and Mercury, Venus, the Moon, Mars, Jupiter, and Saturn are all nearly in a line along the ecliptic. Figure 3.25 (b) shows that the Moon's orbit is inclined to the ecliptic by about 5° which is why we don't see lunar and solar eclipses every month. (Hipparchus determined this angle.)

The Earth is tilted by that seemingly random 23.5° that figured so prominently in the stories above and in Figure 3.26 the Earth is shown at the four seasonal points of the two equinoxes and the two solstices. The shaded circle is inscribed by the ecliptic and is the plane with all of the planets, including Earth. Notice that the Earth is titled by that 23.5° as measured from the plane of the ecliptic and that its direction does not move throughout the year and points to the Celestial Pole. The Vernal Equinox is shown when the Sun is within the Aries constellation (as in anquity).

Now we can understand both cause of the seasons and why they are of different durations and Figure 3.26 tells the whole story. When the Earth's orbit is closest to the Sun, it's moving the fastest in its elliptical orbit, so it spends less time between the two equinoxes, here on the left side of its orbit. Notice that the tilt of the Earth's axis is away from the Sun, and so the full-force of the Sun's rays are directed, not to the northern hemisphere, but the southern. In fact, at the Tropic of Capricorn at a latitude of 23.5° South, the Sun would be overhead at the winter solstice. So less radiation intensity falling on the northern hemisphere, means it's cooler. So yes, the winter happens when the Earth is nearest to the Sun. On the other side, at the summer solstice, the Sun's rays are intense on the northern hemisphere as the Earth's tilt is now towards it and the Sun is overhead at noon on the summer solstice at the latitude of the Tropic of Cancer—where Syene is located at 23.5○ North.

Spinning Earth. The Earth has two motions, as do all of the planets. It orbits the

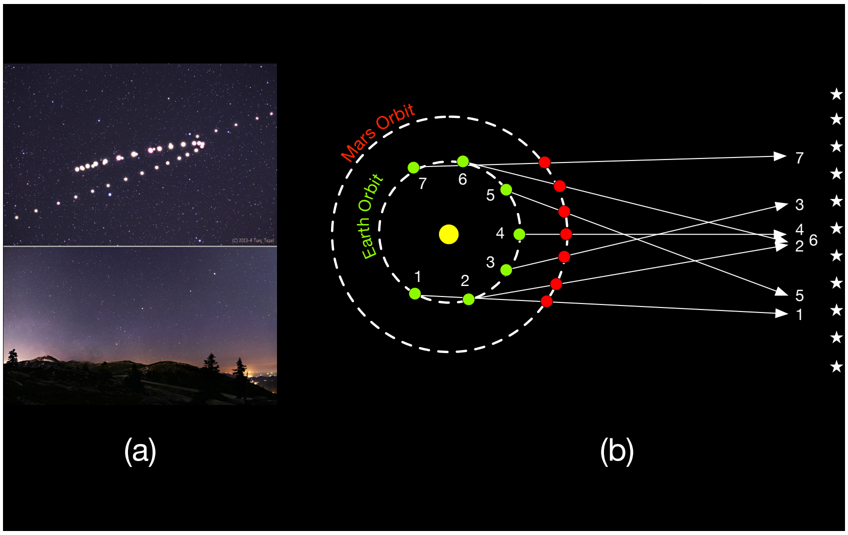

Figure 3.27: Retrograde motion by Mars. In (a) the sky in Turkey shows a photograph of Mars from December 5, 2013 in the upper right hand corner and then an overlayed photograph taken every five or six nights until August 8, 2014. The looping behavior in the middle is the retrograde motion. (b) shows how this happens (see the text for an explanation) https://twanight.org/gallery/tracing-the-red-planet/?preview=true

Sun in a nearly circular path in a counterclockwise sense when viewed from above the Sun's north pole. The Earth also spins on its own axis, also in a counterclockwise sense.[24] That the Earth spins on its axis explains the apparent motion of the Sun through our sky from E-W each day. The speed of the surface of the Earth due to its spinning is about 460 m/s (about 1000 mph) while the speed of the Earth's track along its orbit is 220 km/s (about 490,000 mph). We don't feel this motion since it is constant and we're held to the surface by the Earth's gravity. The same thing is true for the air and so we don't feel a wind as if the Earth were moving out from under the atmosphere.

Planets' orbits. The strange retrograde motion is easily explained in the heliocentric system. Earth and Mars, for example, have different "years" as they go around the Sun. Sometimes the Earth will lap Mars and leave it behind. That's the story and Figure 3.27 explains it. In (a), we see a time-lapse photograph of Mars in successive nights from December to August. Clearly Mars appears to "move" against the stars. (b) shows how. Each

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[24] only Venus among the planets spins in a clockwise sense while Uranus has a spin axis which is on its side, relative to the others. One explanation is that, like the Moon was created through some billions of years ago collision with the Earth, so to something massive might have struck the adolescent Venus and Uranus. Multiple hypotheses exist.

3.8.2 Hipparchus and Modern Celestial Coordinate Systems

(Dennis Duke, 2002) correctly argues that the coordinate system that Hipparchus seems to have originated and Ptolemy perpetuated is essentially identical to what is used today in astronomy, called the "equatorial system." Figure 3.28 (a) shows the situation. What Hipparchus did was measure the angle of a star relative to the North Celestial Pole and an angle along the ecliptic. If you look at Figure 3.26 you'll see that the Earth is surrounded by the 12 constellations of the zodiac. The Greeks (and Babylonians) divided the whole circular pattern into 12 signs, each of 30° each and his coordinate system referred to the constellation and then the number of degrees within that constellation. This is like the longitude on the Earth's surface—degrees around. The "zero" of this coordinate system is located at the position of the Vernal Equinox, which recall is where the Sun on the ecliptic crosses the Celestial Equator during the spring. The Sun was in the constellation Aries during these times (which is why the symbol for the Vernal Equinox is ♈, which is the symbol for that constellation. Today, the VE has moved to the constellation Pisces precisely because of the precision phenomenon that Hipparchus discovered.[25] (More about the Vernal Equinox below.) So in the *Commentary*, he wrote about the constellation Bootes (not among the 12 zodiac members):

> "Bootes rises together with the zodiac from the beginning of the Maiden to the 27th degree of the Maiden... Hipparchus, "

The "Maiden" is Virgo which is the 6th constellation ("sign") around from Aries (Figure 3.26). So the angle, α in the figure where the constellation Bootes rises is $(6-1) \times 30^\circ + 27^\circ = 177^\circ$.[26] A modern version of Bootes extends 202° to 237°, so it doesn't appear to match? Ah, but the precession of the equinoxes is worth 1°/72 years, so we need to add that factor times the number of years since Hipparchus recorded his measurement 2153 years ago—that's an additional 30° which makes that edge be 207°: Hipparchus is just right.

For the other coordinate, he measured from the North Celestial Pole *down to the object* of interest, χ in the figure. That's the "polar angle" and is the opposite of our Earth-faced latitude, which measures up from the equator.

The modern equatorial system uses the same idea. For the polar angle, a star or object's "latitude" coordinate is measured *up from the Celestial Equator*. This is called the "Declination, δ." So it's identical through a difference of 90°:

$$\chi = 90 - \delta.$$

This north-south polar angle measure is called "co-declination."

The modern longitude, called the Right Ascension, α, is measured also from the location of the Vernal Equinox, but typically recorded as a time, rather than an angle. This is natural, since the whole Celestial Sphere rotates 360° in 24 hours. So while the edge of Bootes is 202° for Hipparchus' units, it's $13^{\mathrm{h}}36.1^{\mathrm{m}}$.

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[25] The "Age of Aquarius" is next, as precession continues.

[26] Because Aries the first sign starts at 0°, so the 6th sign starts with 150°

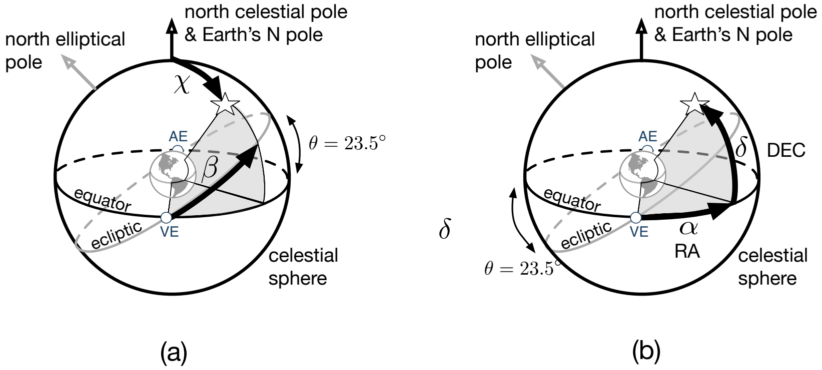

Figure 3.28: The Celestial Sphere is shown in both diagrams for two different coordinate systems that can be used to locate a star on the Sphere. In (a) the "longitudinal" coordinate (β) is along the ecliptic starting from the position of the Vernal Equinox along the ecliptic and the "latitude" coordinate (χ) is measured from the Celestial Pole to the star along a great circle. In (b) the longitude (α) is along the Celestial Equator from the Vernal Equinox (and so identical in angle to β) and the latitude is measured up from the Celestial Equator (δ). The coordinate system in (a) is called the Ecliptic Coordinate System and (b), the Equatorial Coordinate System. (b) is the standard modern system for star charts in which δ is called "declination" and α is called "Right Ascension" (and is recorded in modern tables in units of time, rather than angle where 24 hours equals 360°). A modern version of the Ecliptic Coordinate System uses $\lambda = 66.5^\circ - \chi$, but I represented it here from the pole because Ptolemy measured χ for "latitude." Hipparchus seems to have used both of these systems while Ptolemy used (a).

About the Vernal Equinox. I don't believe that there's any record of just how Hipparchus could have determined the location of the VE in the zodiac. After all, the Vernal Equinox for the Greeks was determined at noon on that day when the Sun is precisely between its altitude at the two solstices, and equivalently, when it rises and sets precisely in the east and the west. His accuracy was about 1/4 of a day for observations and I can think of two ways he might have done this.

He would surely already know roughly when the equinox was to happen and would start measuring the Sun's location, rise, and set for days before and days after the expected event. Then, later he could figure out precisely which day. But along with his altitude measurements, he might look at the east just before the Sun rises each of those days and precisely located which constellations were still visible before it becomes bright. Likewise, he would look just after sundown to see what constellations would be "coming out" as it gets dark.

He could also have noted when the equinox occurred, waited exactly 12 hours and then looked to see which constellation would be at the altitude of the Sun at noon.

In both of these, he would presumably conclude that it was Aries and the "First Point of Aries" became the nickname for where the Vernal Equinox is in the sky.

3.8.3 New Evidence for Hipparchus' Lost Star Catalog

When we're talking about millennia, "breaking news" needn't be "yesterday." So there is remarkable Breaking News when it comes to Hipparchus' star catalog. Parts of it might have been found.

In 2012 Jamie Klair, an undergraduate at the University of Cambridge was studying a multi-spectrum image of folio pages of an ancient Greek palimpsest[27] known as the *Codex Climaci Rescriptus* at St Catherine's Monastery on the Sinai Peninsula (now in Museum of the Bible's collection in Washington, D.C.). It was a summer project assigned by biblical historian at the University of Cambridge, Peter Williams, who continued the work and in 2017 he and French collaborators confirmed the observation and found more of it. They recently published it in (V. J. Gysembergh, 2022). In that image an under-text is slightly visible which he realized appeared to contain astronomical notations—actually a quotation from Eratosthenes. It appears that the original writings were erased in the 9th or 10th century and overwritten. But the multispectral imaging brings out the original impressions on 9 of the 146 pages.

By digitally bringing out the faint background writing, it's apparently astronomical data, coordinates, actually. Almost certainly from Hipparchus' observations. For example, one of the decoded and translated phrases in the hidden text is:

> Corona Borealis, lying in the northern hemisphere, in length spans $9^\circ 1/4$ from the first degree of Scorpius to $10^\circ 1/4$ in the same zodiacal sign (i.e. in Scorpius). In breadth it spans $6^\circ 3/4$ from 49° from the North Pole to $55^\circ 3/4$.

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[27] a document that has been reused by scrubbing out the original content

They noted that “length” is the east-west measure and “breadth” is the north-south measure. The north-south measure is as above, the co-declination and the east-west measure is again the Right Ascension, in angular units. Scorpio is the 8th constellation, so from the previous section, that’s $7 \times 30^\circ + 1 = 211^\circ$. Adding the 30° for precession since then would give a RA today of 240°. The edge of Corona Borealis is almost exactly that.

The stars in the 9 pages refer mostly to Ursa Major, Ursa Minor and Draco and the values are essentially those in Hipparchus’ *Commentary*. The general consensus is that this is the first concrete evidence for the long-lost Star Catalog of Hipparchus!

Appendix A

Appendices

A.1 Greeks Technical Appendix

A.1.1 Proof of Pythagoras' Theorem

A.1.2 Zeno's Paradox

A.2 Plato–Aristotle Technical Appendix

A.2.1 Socrates' Geometrical Problem

A.2.2 Logic and Electronics

A.2.3 Aristotle's Legacy in Physics and Engineering

This section is a little more detailed than normal, but the payoff is large! Aristotle left us a legacy which instantly became an active research project for ancient and medieval philosophers and eventually, present day philosophers, mathematicians, engineers, and scientists! He created a tool that guarantees how to properly analyze and judge conclusions reached through argument: Formal Logic. Read the next seven pages in detail for the whole story, skim them for a taste, or jump to the punch-line on page 73.

In everyday life, we all make arguments but have you ever thought about what makes you successful in defending your case? The facts need to be on your side but your stated reasoning should also be "logical." We all have a sense of what "logical" means, but it's surprisingly nuanced. Consider the following reasoning:

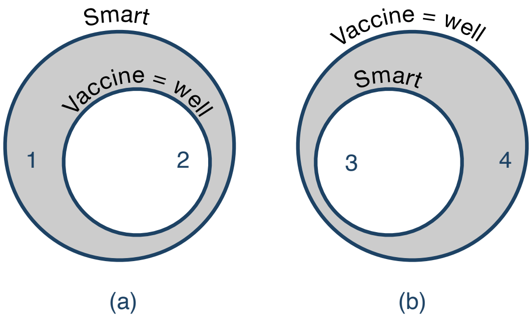
- Squirrels with superpowers can fly
- Rocky the Squirrel has superpowers
- Therefore, Rocky the Squirrel can fly.

This doesn't make sense because the first two sentences—the "premises"— are nonsense. And yet *it's a perfectly valid argument*! Appreciating the difference between a *valid* argument and a *true* argument leads us to Aristotle's amazing discovery that the rules of valid reasoning are due entirely to an argument's structure and arrangements of the sentences, not the specifics of the content. Your and my lives are now governed by Aristotle's invention of Formal Logic, his most important, lasting contribution.

Obviously, the distinction between *validity* and *truth* can be easy to spot. But the distinction between valid and invalid argument can be subtle. Think about these two arguments:

Table A.1: How to not reason logically.

A	B
Those who take the vaccine stay well. Those who take the vaccine are smart. Those who are smart stay well.	Those who take the vaccine stay well. Those who are smart take the vaccine. Those who are smart stay well.



(a) (b)

Figure A.1: A diagrammatic way to show that argument A in Table A.1 is invalid and that the conclusion of argument B is valid.

The argument in column A is invalid, not because the premises are ludicrous, but because of the form of the terms in the sentences. Read it very carefully with an eye on Figure A.1. Notice how the righthand and lefthand circles are different (not really Venn diagrams, but a cousin, called Euler Diagrams). The first premise in argument A is that if you take the vaccine you're going to be well. So in the lefthand diagram, everyone who took the vaccine is in region 2. The second premise in argument A says that those who took the vaccine are smart, but it doesn't rule out the logical possibility that some smart people didn't take the vaccine—region 1. So the conclusion, that if you're smart, you're well does not hold.

Argument B says things slightly differently. Again, smart=well. But then the second premise says that if you're smart, you took the vaccine, so all of the smart people are in region 2 and, they're vaccinated. That, of course leaves the possibility that there are people who took the vaccine, but aren't smart, region 4. That's good! But not the argument which leads to a valid conclusion: Those who are smart stay well (and because of the first premise, they also took the vaccine).

A.2.3.1 Greatest gift

Aristotle's greatest gift to us was his invention of Formal Logic which is a rigorous way to judge the validity of arguments. For example, he could tell you that the argument in column **A** is not valid and why and tell you how to construct arguments like column **B** which *are* logically valid. Every time. And sometimes surprisingly, independent of the actual subject-matter of the argument.

> Officially, Formal Logic is the field that studies reasoning and the various ways that conclusions can legitimately be drawn from premises.

This new-born subject is covered in a number of his books, including: *Categories*, *On Interpretation*, *Prior Analytics*, *Posterior Analytics*, *Topics*, and *On Sophistical Refutations* which collectively, were much later dubbed "*Organon*" which means "instrument" which suggest by that time, Logic was viewed as just a tool, as opposed to a part of philosophy. Now it's firmly the philosophical camp and even an important part of an entire branch of mathematics called Discrete Mathematics.

Logic became a research program almost as soon as he wrote it down (or lectured on it) and two millennia worth of people—to this day—study logical formalism, expanding it into new directions. It's studied by every student of physics and engineering in forms directly evolved from Aristotle.

A.2.3.2 Deduction and Induction

Broadly, there are two kinds of logic which you use every day. The first works according to strict rules which I think of it as the *algebra of reasoning* and you'll see why in a bit. Reason according to those rules, and you will reach correct conclusions. This is **Deductive Logic.**

The second kind of logic is less certain since it's not rule-bound and it delivers conclusions which can seem persuasive but aren't certain. This is **Inductive Logic**. From this point, when I refer to "logic" I'll mean deductive logic.

Among things that are obvious to us (and to everyday Greeks), Aristotle seemed to intuit as requiring bottom-up attention. He tightly defined terms and "obvious" ideas, dissected arguments finding rules along the way, and set down what it means to be clear with exquisite precision. Look at these two statements:

- All squirrels are brown.
- No squirrels are brown

1) Can these both be true at the same time? Of course not and this obvious idea has a name: *the law of contradiction.* Aristotle needed to be precise and actually provided multiple "proofs" to demonstrate this principle.

2) One of these must be true. . . there's nothing in-between, which is called the *law of the excluded middle*.

> "…there cannot be an intermediate between contradictories, but of one subject we must either affirm or deny any one predicate" Aristotle, *Metaphysics*.

Centuries of ink have been spilled over precisely understanding the implications of law of the excluded middle and how to symbolically state it unequivocally. But here's the first hint of our modern debt to him: his logic is two-valued, either true or false with no in-between. Hmm. Binary: True and false...one's and zero's.[1]

Last one:

- A squirrel is a squirrel.

This is called *the law of identity* and Aristotle didn't invent it and it sounds like Parmenides: "What **is**, **is**." These three ideas, collected together by him, are often called the Rules of Thought and were believed to be the bedrock for all of Logic. (That this was disputed in the 20th century shows that Logic is still a living-breathing subject.) Nobody ever thought this way before — so clearly—-and in Aristotle's patented approach to system-building, he lays it all out out exhaustively. As a master system-builder, he was the right man for the job.

His unique invention was to create an *algebra of language*. Here is a seminal moment in history, from the first book of his *Prior Analytics* (focus on the last sentences):

> "First then take a universal negative with the terms A and B. If no B is A, neither can any A be B. For if some A (say C) were B, it would not be true that no B is A; for C is a B. But if every B is A then some A is B. For if no A were B, then no B could be A. But we assumed that every B is A. Similarly too, if the premiss is particular. For if some B is A, then some of the As must be B. For if none were, then no B would be A. But if some B is not A, there is no necessity that some of the As should not be B; e.g. **let B stand for animal and A for man. Not every animal is a man; but every man is an animal.**" Aristotle, *Prior Analytics*.

I don't blame you if you get bogged down quickly in this quote. Look at the sentences that I've highlighted: he's using variables A and B, to stand for particular things, here in his example, A = man and B = animal. So his first sentence says for this particular case, "If no animal is a man, neither can any man be an animal." Instead of men and animals, you can plug in anything you want for A and B. It's the form of the argument, not the contents that determine whether the argument is valid.

Introducing variables as a placeholder for the subjects and objects in a statement is a seminal moment in the history of mathematics.

Amazing. Out of this, your mobile phone was born.

There are many different forms of arguments and for Aristotle, the **Syllogism** is just one of them. It's an argument written in a structure in which there are three

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[1] Things didn't stop there. Now there is a multi-valued logic with degrees of truth and falsity with many engineering applications. "Fuzzy Logic" is a legitimate decision-making tool in transportation control systems, earthquake prediction, even home appliance efficiency.

sentences with a subject and a predicate[2]: two premises and a conclusion and inside those sentences are three "terms."

Here is one of the syllogistic forms:[3]

- premise 1: If all A are B
- premise 2: and if all C are A
- conclusion: then, all C are B

> There are actually 256 possible argument-combinations of subjects and predicates and 24 were thought to yield valid deductions. Maybe you can see why studying Logic became a matter of intense research following Aristotle's death and into the first 100 years of both Arab and Western philosophers. There was lots of work to do.

Let's make a syllogistic argument about squirrels. I'll define C = squirrels, A = the group of all animals in trees, and B = brown animals. One kind of syllogism would have the form:

- All mammals in trees (A) are brown animals (B)
- and if all squirrels (C) are mammals in trees (A)
- then, all squirrels (C) are brown animals (B).

Before I moved to Michigan, the only squirrels I'd ever seen where brown. Now my yard is full of black squirrels. They're everywhere. Yet, my argument above seems to prove that squirrels are brown. So what went wrong?

My "Squirrels with superpowers" shined a bright light on the premises: they have to be legitimate. In scientific arguments, premises might be …hypotheses, in which case a deductive argument describes a way to test those ideas. Aristotle was well-aware of induction, deduction, and how they might go together.

Back to my squirrels proof. I reasoned inductively:

- (As a child) There's a brown squirrel
- (As an adult…many times) There goes another brown squirrel
- Wow…more brown squirrels and no other ones
- What is it with all of the brown squirrels?
- Gosh, all squirrels must be brown! (which was my premise)

Until I moved to Michigan. All it took to ruin my theory about squirrels was the observation of one black squirrel, much less an entire herd of them. Squirrels are not only brown, they're black. My proof founders on a false premise: "All mammals in trees (A) are brown animals (B)."

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[2] since his Categories are predicates, these topics were a part of his overall system

[3] Before 500 CE, Aristotle's original form was used:

- If A, then B
- If B, then C
- So, A is C

By the way, Sherlock Holmes is reputedly the Master of Deduction. Well, sorry. That's not true. If you look at his stories you'll see very, very few examples of deductive reasoning. He's the Master of Induction![4]

A.2.3.3 Your phone

Theophrastus (-371 to -287) was a favorite student of Aristotle's who led the Lyceum for 37 years after his teacher's death. Aristotle even willed him the guardianship of his children...and his library. While a devoted student, Theophrastus went beyond his teacher and expanded and modified some basic Aristotelian notions—extending a concept of motion to all 10 of the Categories, for example. He also moved the study of botany forward and worked extensively in Logic. Theodor Geisel (Dr. Seuss) used "Theophrastus" as a pen name.

He is probably the one who extended the form of argumentation into a new direction with the invention of "propositional logic" in which there are two items, rather than three of a syllogism. This is where the modern engineering action is. One form of such a proposition is called "Modus Ponens" (Latin for "method of affirming") which is an offshoot of the classical syllogism and is one of four possible "rules of inference." Modus Ponens goes like this:

- If A (the antecedent) is true, then B (the consequence) is true
- A is true
- Therefore, B is true.

Here, each line is a proposition (there can be more than two) with the first two being "premises" and the last, the "conclusion." The first sentence is a proposition which is conditional: the antecedent implies the consequence and it's "affirmed" if the next statement is true. B here is the consequence of A. Here's a concise way to present this:

- $A \rightarrow B$
- A
- $\therefore B$

The → symbol means "implies" and is associated with an "If...Then" kind of statement. The ∴ symbol means "therefore." It doesn't seem like much, but it's powerful and misunderstanding (or misusing) it is the source of many logical fallacies. Table A.2 shows an example:

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[4] Or more appropriately, the Master of Abduction. Look it up.

Table A.2: A typical logical fallacy involving public health.

A valid argument	A fallacy
• If a reactor leaks radiation (A), people nearby will get cancer (B). • The reactor leaks radiation (A). • Therefore, people nearby will get cancer. (B)	• If a reactor leaks radiation (A), people nearby will get cancer (B). • People nearby got cancer (B). • Therefore, the reactor leaks radiation (A).

The argument on the left is an example of Modus Ponens, while the argument on the right is a classic fallacy known as "Affirming the Consequent," a regularly exploited tool for those intentionally making invalid claims. Especially those who dispute public health strategies. Look at how the two columns are different. Remember, that in the proposition, B is the consequence of the antecedent, A and not the other way around. In the second row of the fallacious argument, the antecedent and consequence are reversed as compared with the valid argument. The fallacy is that people can get cancer from other causes than the proposition states.

Let's make a plan to picnic outdoors which requires us to keep an eye on the weather since if it's raining the ground would be wet and of course we wouldn't have a picnic if the ground is wet. We'd actually use Modus Ponens in our thought process and reason among ourselves:

- If it's raining, then the ground is wet
- It is raining
- and so the ground is wet.

Let's build a table—a picnic table (sorry)—that takes each line in the argument and makes it a column in a table. We could then ask a set of questions: Is it raining (Yes), is the ground wet (Yes)...was the proposition confirmed? Yes.

Table A.3: The picnic is cancelled because:

If A, then B	it's raining?	it's wet?	A	B	If A is true and B is true, then:
If it's raining, then the ground is wet	Y	Y	T	T	T

There are actually four complete ways in which the antecedent and consequence could appear:

- rain? Yes or No
- wet? Yes or No

So what about: suppose the ground is not wet (wet = F) then can it be raining? Well...no (rain = F). So if wet = F and rain = T, then the proposition would not be true since rain should imply wet. We can build up these four conditions into what is called Truth Table, which was invented in the early 20th century as an analyzing tool. Table A.4 describes the complete story:

Table A.4: All of the logical possibilities for two pieces of a conditional premise: raining and wetness. Here's a picnic table (sorry):

If A, then B	it's raining?	it's wet?	A	B	If A is true and B is true, then:
If it's raining, then the ground is wet	Y	Y	T	T	T
If it's raining, then the ground is not wet	Y	N	T	F	F
If it's not raining, then the ground is wet	N	Y	F	T	T
If it's not raining, then the ground is not wet	N	N	F	F	T

Sometimes these are hard to unravel. The first two lines are pretty obvious. It's asserted that when it rains that the ground is wet, so the second line is obviously false. The proposition requires "wet" with rain. The last line is pretty clear also. No rain, let's picnic since it will not be wet. The third one requires some thought. What does the if statement say about the ground if it's not raining? Nothing. You could be wet for other reasons so this does not falsify the proposition, so it's not F...and in a two-valued logic, the only alternative to F is T. Go lie down before we go on because it's about to get interesting and relevant.

Before getting to the punchline, let me make a couple of points:

- The → or if...then argument is one of six "connectives," all of which have truth tables like above. They are negation, conjunction ("AND"), disjunction ("OR"), conditional (that's the → conjuctive), biconditional, and exclusive OR.
- The Modus Ponens argument got its Latin name from the Medievals who seriously studied Logic. They identified it as one of four "Rules of Inference" which we use today: MP, Modus Tollens, Hypothetical Syllogism, and Disjunctive Syllogism.
- The Hypothetical Syllogism is just one form of the "regular" syllogism of our squirrel proof above. In fact, it can actually be proved to be the combination of two Modus Ponens arguments, one for A → B and the other for B → C. There's debate about whether Aristotle might have recognized his syllogism to have been an "hypothetical" in this sense with a deeper structure.
- In Appendix A.2 I've gone into some more detail logic gates as they're used in digital circuit design.

There are a handful of seminal discoveries about Logic that extend to our modern reliance on it. **Gottfried Wilhelm Leibniz** (1646–1716) refined binary arithmetic. In 1854, **George Boole** (1815–1864) invented the algebra of two-valued logic...how to combine multiple conjuctives into meaningful outcomes which can only be T or F, 1 or 0. In 1921 in his dense and very terse *Tractatus Logico-Philosophicus*, **Ludwig Wittgenstein** (1889–1951) invented the Truth Table, which can be used in logical proofs and complicated logical solutions to multi-variable inputs. Finally, in 1938 **Claude Shannon** (1916–2001) realized that Boole's algebra could be realized in electronic, "on-off" circuits. This was realized in the 1940's with vacuum tubes and then in the 1960's with transistors.

Notice that the picnic table can be thought of as a little machine: you input the four T-F possibilities in pairs for rain and wet and out comes the truth value of the proposition. Figure A.2 is a cartoon of such a machine.

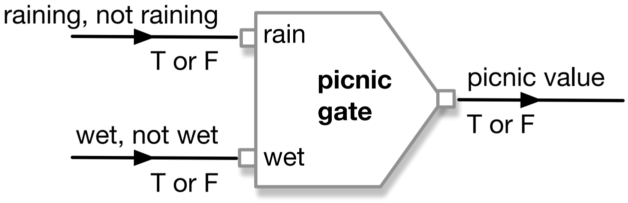

Figure A.2: A fake "picnic gate" machine that does the work of Table A.4

.

The image in this figure is maybe suggestive of digital component representations which are called "gates." There are electronic gates for eight functions, which are a practical expansion of the conjunctives mentioned above. Think about that. The whole of our digital world can be made with these eight gate functions.

What I wanted to show you is that your entire life now is based the ancient Greek Logic research program. For example, the 2022 iPhone 14 has 18 billion transistors in it and every one of them speaks through Aristotle to get their individual jobs done—or I should say their collective jobs done, since their language is forming and evaluating billions of logical two-term arguments in the same spirit as our raining-wet table.

A.2.3.4 The Punch Line:

Let's review what just happened:

We've found that Aristotle made a simple but profound discovery, namely that one could take a sentence, like "Fire engines are red or yellow" and turn it into essentially a mathematical statement, like "A are B or C" and then draw general conclusions about the combinations of general statements that don't involve the details. That sentence involving A, B, and C could also be a representation of the sentence, "All squirrels are either black or brown." This allowed him to then create a system of rules that could guarantee the validity of arguments, which, after all, are combinations of sentences.

The first kind of argument is now called the "categorical syllogism," and involves three variables and, like fire engines and squirrels, can be specific or more usefully, general, like:

All men are mortal.	A are B
Socrates is a man.	C is A
Therefore, Socrates is mortal	therefore, C is B

This evolved quickly into a rules guaranteeing validity of conclusions from a different form of argument involving two variables (an "hypothetical syllogism"):

If all men are mortal, then Socrates is a mortal	If A, then B.
All men are mortal	A is true.
Therefore, Socrates is mortal	therefore, B is true.

In fact there are variety of valid forms for each sort of argument but what's interesting in the second sort is that the truth value of arguments involving two variables can actually be created using electronic circuits using tables ("truth tables") of the different logical outcomes of the truth or falsity of the premises in an hypothetical syllogism. This was realized in 1938, built into vacuum tube circuits in the 1940's, and transistor digital electronics in the 1960's.

The first digital computers relied on thousands of vacuum tubes and filled whole rooms with hot, clunky racks of tubes and wires—your phone has 10s of thousands of times more processing power than these first early 1950s computers. When the transistor became commercially viable in the 1960s the digital world came alive.

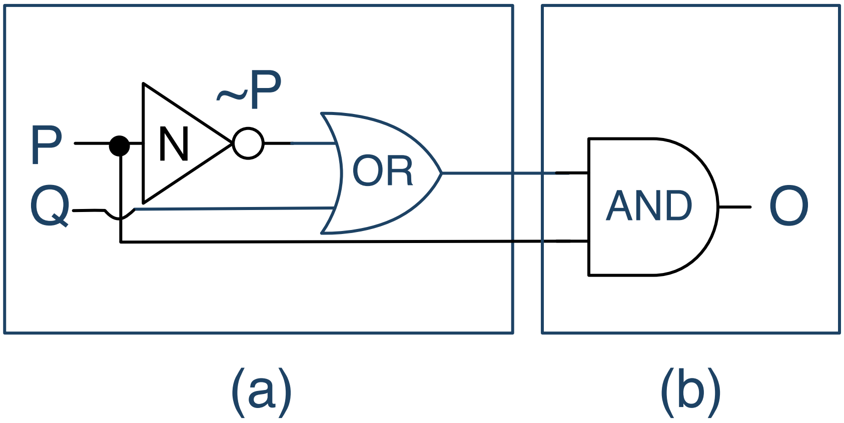

Figure A.3: (a) and (c) are the transistor-equivalents of the two logic gates, NOR and OR in (b) and (d). The little circuit to evaluate rain causing wetness...or not...is shown in (e).

In the spirit of overview, Figure A.3 shows two transistor arrangements and their modern "gate" symbol—please don't worry about the details! Just for flavor. (a) is the layout for a common transistor package that does the job of the logical gate symbol shown in (b). It's the NOR operation. A comes in, and NOT–A comes out. (c) is another transistor layout that has two inputs and produces the logical

OR combination, and (d) is the logical gate symbol for performing that operation. Finally, (e) is the digital gate solution for the Conditional argument from Table A.4—it's a real-life engineering representation of the fake "picnic gate" in Figure A.2.

With binary arithmetic, gates can be combined to do arithmetic functions, logical functions, and importantly, storage of bits. Digital memory consists of four so-called NAND gates, and so four transistors and is the basic cell of a computer 1-bit memory. It's a clever implementation of an input bit—to be stored—and an enable bit—which allows the output to change or not change.

All of these—and more—transistor components are actually imprinted in tiny silicon wafers in which a single transistor package might be only 20 nanometers in size. With the logical functions and the manufacturing techniques of today, my current Apple Watch has 32GB of random access memory (RAM) and so it can manage 32,000,000,000 Bytes of information, which is 25,6000,000,000 bits and so 102,400,000,000 individual transistors are inside my watch, just for the memory! The CPU and control circuitry would add millions of additional imprinted transistors and their gate-equivalents. All on m

A.2.4 Digital Gates

One more bit of insight makes really complicated electronic digital design possible and came from the very strange, yet enormously influential philosopher **Ludwig Wittgenstein** (1889-1951) who invented the concept of the "truth table," which we've already used in Table A.4. It's an orderly setup of all possible starting places (for two valued propositions) and their results when various operations are applied. Let's look at a three. True now is the bit 1 and False is the bit 0:

- The NOT operation: If I have an A then NOT–A creates the opposite of A. If we work in the zeros and ones world, then if A=1, then NOT–A = 0. The symbol for NOT is usually   so if A = 1, then   A = 0. (The   symbol is the common notation used by logicians. Engineers and physicists would write $\overline{\mathrm{A}}$ to represent the result of NOT–A.)
- The AND operation: This is between two states of, say, our A and B. In order for A AND B to be true, both A and B must be true—1— themselves. Otherwise, A AND B is false, or 0. The symbol for AND is $\wedge$ So A AND B = A $\wedge$ B.
- The OR operation: This is the combination that says A OR B is true if either A = 1 or B = 1 and false otherwise. The symbol for OR is $\vee$.

There are 5 other logical combinations. Table A.5 shows the truth table for AND and for OR. In the first set, the AND process, I've stuck to our T and F language, but the rest uses the zeros and ones language of engineering and binary arithmetic.

Table A.5: Truth tables for the AND and OR functions plus the construction of Modus Ponens. The **symbol for AND is** ∧, the **symbol for OR is** ∨, and the **symbol for NOT (negate) is** . Notice that (A) ∨ B is a construction out of AND and NOT of the conditional that's the first premise of Modus Ponens.

AND			OR			Combined function				= If A then B
A	B	A ∧ B	A	B	A ∨ B	A	B	A	(A) ∨ B	
T	T	T	1	1	1	1	1	0	1	= 1
T	F	F	1	0	1	1	0	0	0	= 0
F	T	F	0	1	1	0	1	1	1	= 1
F	F	F	0	0	0	0	0	1	1	= 1

Let's look at the first line so that you get the idea.

For AND:

- A is T and B is T and the AND of two T's is itself a T.

For OR:

- A= 1 and B = 1 and the OR of 1 ∨ 1 is 1.

Then the combination:

- repeating the A and B conditions from the first and second columns A= 1 and B = 1.
- taking the NOT of A, takes 1 into 0.
- combining that with the B in an OR results in A ∨ B = 0 ∨ 1 = 1

The last column shows that this is the same as the first line result of our picnic decision making in Table A.4. The rest of Table A.5 builds that combination for all possible A and B states, first by negating A and then combining that by "ORing" it with B. The last column shows the original "If A then B" premise that we worked out about raining and wetness. They formula and our reasoning lead to identical conclusions.

A.3 Greek Astronomy Technical Appendix

A.3.1 Plato's Timaeaus Cosmology—The Numerology

> "And he began the division in this way. First he took **one portion** from the whole, and next a **portion double of this**; the **third half as much again as the second**, and **three times the first**; the **fourth double of the second**; the **fifth three times the third**; the **sixth eight times the first**; and the **seventh twenty-seven times the first**. Next, he went on to fill up both the double and the triple intervals, cutting off yet more parts from the original mixture and placing them between the terms, so that within each interval there were two means, the one (harmonic) exceeding the

> one extreme and being exceeded by the other by the same fraction of the extremes, the other (arithmetic) exceeding the one extreme by the same number whereby it was exceeded by the other.” Plato, **Republic**

Okay the numbers seem arbitrary. But there’s an algorithm:

- one portion of the whole: ○, 1
- double of this: ○○, 2
- half as much again: ○ ○ ○, 3
- double of the second: ○ ○ ○○, 4
- three times the third: ○ ○ ○ ○ ○ ○ ○ ○ ○, 9
- eight times the first: ○ ○ ○ ○ ○ ○ ○, 8
- twenty-seven times the first: ○, 27

Now manipulate:

- The first four are the famous 1,2,3,4 and since they’re the special numbers, they have a job to do:
  - Square each of the first numbers—remember, 1 is not a number— (Greeks knew how to multiply): and you get 4 and 9.
  - Cube those same first two important numbers: and you get 8 and 27.

So all of the numbers in that excerpt are some manipulation of the numbers 2 and 3—he stopped at 3 because there are only three dimensions. Collecting all of the numbers, but now into even and odd strings (remember, 1 is neither even nor odd for Pythagoreans and apparently also, for Plato):

Then, Timaeus says that if you take the number strings you actually construct the intervals of the diatonic musical scale. More Music of the Spheres. Whew. Wait until we get to Kepler.

A.3.2 Some Aristarchus Measurements

A.4 Medieval Technical Appendix

A.5 Copernicus Technical Appendix

A.6 Brahe-Kepler Technical Appendix

A.7 Gilbert Technical Appendix

A.8 Galileo Technical Appendix

A.9 Descartes Technical Appendix

A.10 Brahe-Kepler Technical Appendix

A.11 Huygens Technical Appendix

A.12 Newton Technical Appendix

A.13 Young Technical Appendix

A.14 Faraday Technical Appendix

A.15 Maxwell Technical Appendix

A.16 Michelson Technical Appendix

A.17 Thomson Technical Appendix

A.18 Lorentz Technical Appendix

A.19 Einstein Technical Appendix

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