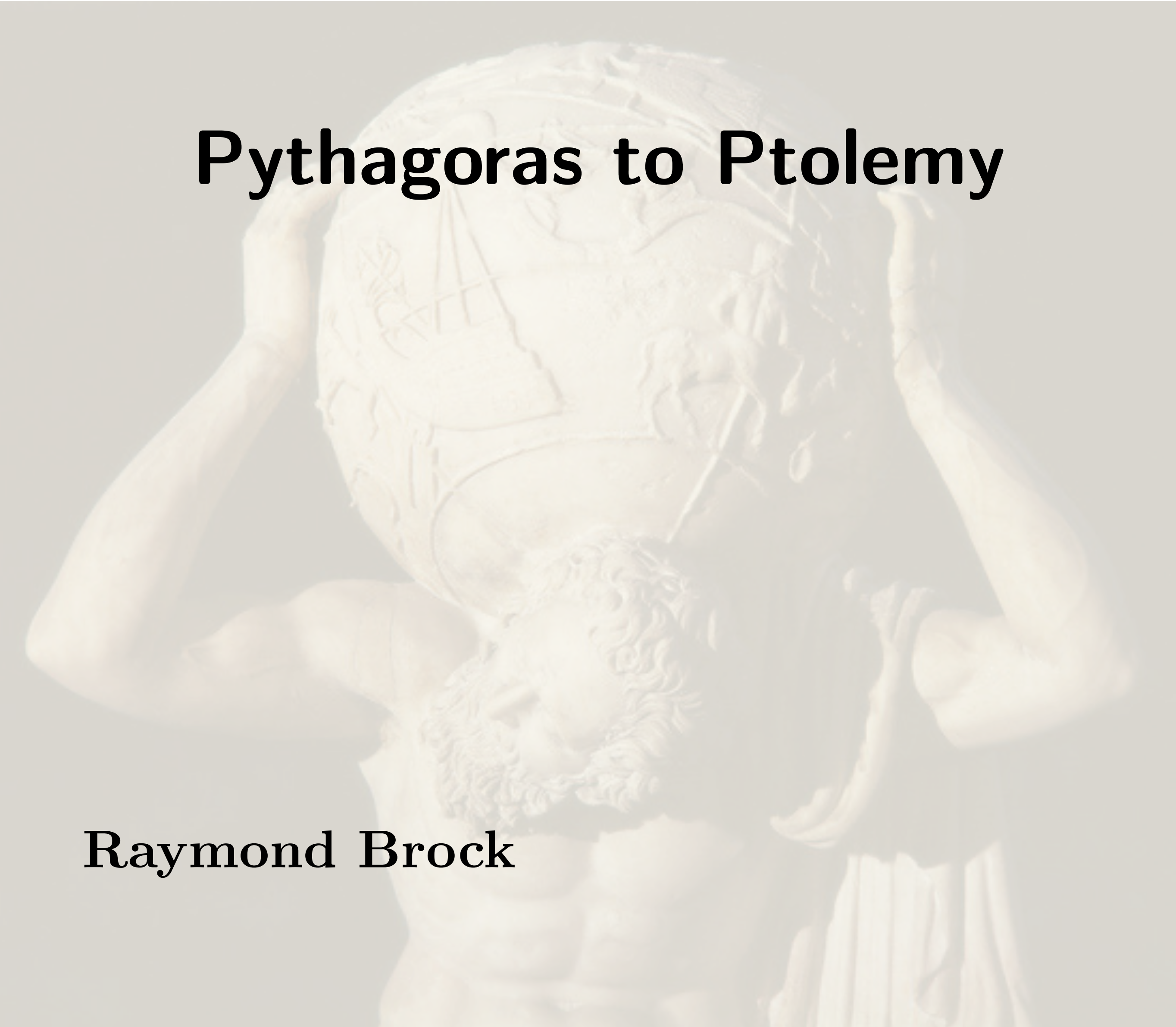
From the Greeks to Einstein

How the Stories of Motion and Light Became Einstein's Relativity



Pythagoras to Ptolemy

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Todo list

161 # Volume I

162 # Pythagoras to Ptolemy

It may have once been the case that all roads lead to Rome, but for most of western philosophy, physical science, and mathematics, all roads lead *from* Greece. This volume is the first stop in our path towards Einstein's Special Relativity: our MOTION themes start with the Greeks, eventually centered on Plato and Aristotle.
163 Likewise, but to a lesser degree, ideas about LIGHT frustrated the Greeks without much analysis. This volume will be different from subsequent ones, as its stories are of a number of people, not all of whom would be classified as scientists today. You'll see why. But we'll close this volume with the one of the earliest quantitative astronomers: Claudius Ptolemy.

Chapter 0

Series Preface: Read This!

"PREFACE PROBLEM: Nobody reads prefaces.
SOLUTION: Call the preface Chapter 1."

- **gause11**, *Are Your Lights On?*

"Why not just call it Chapter 0?"

- Raymond Brock, *...just now*

0.1 Why Do This?

Albert Einstein is usually imagined to be the very model of a modern major scientist. A brave genius, working entirely alone and, yes, it's certainly the case that it would be hard to be more unknown than the 26 year old Einstein. Yet he had an idea that cured a slow-motion nervous breakdown inside of the world's physics community. His Special Theory of Relativity brought two inconsistent theories together by healing a contradiction between them: either James Clerk Maxwell's triumphant model of LIGHT (electromagnetism) or Isaac Newton's mature model of MOTION (mechanics) seemed to be wrong or incomplete.

This series, *From the Greeks to Einstein* (let's give it a nickname, "G2E") follows parallel storylines of two very different theoretical clans: MOTION (in which there were three separate families: MOTION IN THE HEAVENS, MOTION BY THE EARTH, and MOTION ON THE EARTH) and LIGHT (where there were also three separate families: OPTICS, ELECTRICITY, and MAGNETISM). Those six different families separately developed, merging into a pair of conflicting theories: MOTION and LIGHT which Einstein tied together.

G2E's subtitle, *How the stories of motion and light became the Special Theory of Relativity*, emphasizes the theme of this work: stories. G2E is stories about people.

I've been a professional particle physicist for half a century and I've found that I suffer from an unusual affliction that affects my teaching and my research. Before I can teach something old or learn something new, I have to know its history. This isn't an especially efficient way to work but it's led to a fulfilling pastime and I suspect unusual classroom experiences. I've become so sure of this approach that I even tell stories in mathematically intense (calculate! calculate!), advanced graduate physics classes. This series is a written version of my teaching approach, structured around 20 or so scientists, their lives, their times, their colleagues, their projects, and their accomplishments.

0.1.1 Projects

In trying to reverse-engineer the emergence of innovative ideas in physics, I keep coming back to what *individuals* do. I'm keenly aware that when I choose to spend my limited time and group resources on a project it's both a commitment and an opportunity loss for what I decided *not* to work on. So it's personal and requires experienced scientific taste. For me: the model of the unit of behavior in science is what I'll call the Project which is a lot like how you might think of a project.

> There is a more standard, but disappointing "unit of behavior in science" called the "Paradigm" which came from Thomas Kuhn's historic 1962 *The Structure of Scientific Revolutions* (**kuhn96**). When we're working within a paradigm we're doing what Kuhn called "normal science," which at some point, accumulates contradictions, develops a crisis, a revolution occurs, and a new paradigm begins. Kuhn had trouble clearly explaining what a paradigm was—21 different uses of the word were identified! For example, is it Big, leading to historic Revolutions? Or could it be small...lots of paradigms in a scientist's lifetime. It was meant to be a collective world-view, a social thing, which was *also* a problem as it led to accusations of a distressing relativism in science.

I'll be didactic about Projects in my stories. By the way, in Kuhn's formulation, the passage of one paradigm to another is not progressive...just different. That was a problem for his model as, at least for professional scientists, science is certainly progressive and my working model is designed to be.

Simply put, each Project has inputs and outputs. In order to get a Project off the ground, one commits to these inputs:

1. **Numbers**. I'll have a set of factual commitments—numbers or parameters—about phenomena that I'll accept.

2. **Theories**. I'll commit to a set of theoretical concepts...accepted views of the world, so to speak.

3. **Techniques**. I'll have a commitment to set of best-practice mathematical and experimental skills and techniques.

4. **Norms**. I'll inherit and initially commit to a set of community norms and expectations about what Projects are worth exploring.

5. **Curiosity**. This defines a Project's goals. I'll be curious about some actual or imagined phenomenon. Maybe I just want to measure a parameter or do a "what if" theoretical calculation or build an amusing mathematical model. For the duration of the Project, I'll commit to it.

I've called these "commitments" because they are...until they aren't! What I mean is this: if I make a discovery of importance that affects what *other* scientists choose to work on, it usually involves my modification of, abandonment of, or invention of the input commitments that I respected at the outset of my Project. Analyzing those from past —Project to descendent, new Project — is interesting to me. If a Project is well-designed, we can identify each of these five commitments and as a pedagogical tool in our historical approach in G2E, that's exactly what I'll do:

> ▻ For almost 20 highlighted scientists I'll unpack the commitments (#1 through #4) plus what sparked their curiosity (#5) in their subsequently revolutionary Projects. We'll see how their work went from attention-getting to revolutionary in service to Einstein's eventual Special Theory of Relativity.

This approach necessarily brings both history into the stories and encourages a focus on the state of affairs during each person's working life. It also points at collaborators.

That Einstein picture of the completely isolated genius? They don't exist in the practice of productive science. There might very well be completely isolated geniuses, but if their isolation is complete they didn't influence anyone! (We'll see a few who only in retrospect were found to have been on the right track, but quiet about it.)

You see, an essential aspect of doing productive science is doing public science. Even the well-known "genius" scientists that we can all name had collaborators. They might have had real-time collaborators, or some of them really did work alone in their rooms but they all "collaborated" across time with people who came before them, relying on *their* previous projects to inform the inputs to their Projects. That's where the continuity and progress in science comes from: these real and virtual collaborations. This idea of collaborating with the past is even a little bit romantic which is maybe why physicists and astronomers enjoy the pedagogy of teaching physics so much.

But revolutions? They're a slow-walking event. If I'm to persuade you that my focus on unique individuals is helpful I should be able to identify when a revolution occurred. Revolutions aren't overnight, or when someone lays down their pen. The revolutionary nature of a Project reveals itself only in retrospect. Here's how this roughly goes: Someone completes an interesting Project, perhaps having measured surprising new numbers or conceived of a new model or invented a new technique. And if by using those new tools they solve some old problem or predict novel phenomena, then maybe that's attention-getting. But only when enough other scientists vote with their feet—and their precious time and resources— and adopt those new ideas as inputs to *their* Projects then, in retrospect, that original Project might be viewed as having been important—and should *everyone* in a community use those new tools? That's a revolution.

Both words in the familiar phrase, "Copernican Revolution" annoy many modern historians. "Copernican" because it singles out an individual as special. "Revolution" because it suggests that there are abrupt changes in the flow of intellectual history. In his *To Explain the World,* (**weinberg15**) chides (**shapin96**) for the first line of the latter's *Scientific Revolution*: "There was no such thing as the Scientific Revolution, and this is a book about it." Shapin is one of the voices of a movement that has recoiled against the idea of THE Scientific Revolution and certainly that a single person might be responsible. I've got a different take on this, especially since my career has actually straddled a bonafide revolution stimulated by special individuals, Weinberg, among them.

After chastising Shapin, Weinberg closed his introduction to his Copernicus chapter with the comment, "There was a scientific revolution, and the rest of this book is about it."

I agree. There have been Revolutionary Scientists *and* there have been Scientific Revolutions and the rest of this series is about them: Claudius Ptolemy, Nicolaus Copernicus, Tycho Brahe, Johannes Kepler, William Gilbert, Galileo Galilei, Rene Descartes, Christiaan Huygens, Isaac Newton, Thomas Young, Michael Faraday, James Clerk Maxwell, James Joule, Albert Michelson, J. J. Thomson, Hendrik Antoon Lorentz, and Albert Einstein.

Every chapter follows a similar template. The main bodies have major sections that center on one or two scientists: "A Little Bit About Copernicus" or "A Little Bit About Newton," or Kepler, or Maxwell, and so on. We'll learn about their lives, their contemporaries, and yes, we'll analyze their Projects—what they brought to their work and how they stimulated conceptual change as a result. The last major section will be "Copernicus Today" or "Newton Today" and so on. Each of our physicists left legacies; world-views; and in some cases, even technologies that we still use today. Finally, for many of the chapters there are technical appendices which go deeper into the mathematics than would be welcome in the main narrative of a series like this.

Chapter 1

It's All Greek To Me : The Greeks

"We are all Greeks. Our laws, our literature, our religion, our arts have their root in Greece."

- Percy Bysshe Shelley (1792-1822), *poet*

"There is a land called Crete, in the midst of the wine-dark sea, a fair, rich land begirt with water; and therein are many men past counting, and ninety cities."

- Homer, *The Odyssey*

Since this is a book on physics, and since you can only invent something once, I want to tell you how physics started. This is the first of three chapters on Greek philosophy and natural science and they will be different from the ones that follow as I'll talk about many Greeks, rather than focus on a few. In this chapter we'll learn about new habits of mind that evolved two centuries before Plato and drive us still.

About their nascent science, I'll ask four questions that will guide our whole project: what is the nature of motion by the Earth, what is the nature of motion on the Earth, what is the nature of the motions of the heavens, and what is the nature of light. In the text, you'll know which question is a focus because I'll tag the context with: MOTION or LIGHT. Within each there are more details: MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS as well as MAGNETISM, ELECTRICITY, and ELECTROMAGNETISM.[1]

The quotes above are a small sampling of how we modern scientists should look back at the Greeks. In many ways my field of particle physics is relentlessly Platonic (but don't tell anyone that I said that!). Plato (and to a lesser extent, Aristotle) continues to challenge us: *What* can we know? And, *how* do we know we know that something is true? And, of course, *how* do things move?

The next chapter will deal with them—but Plato was reacting to the thinkers who came before him, traditionally called "Presocratics," obviously meant to cover those who came before Socrates. Now, "Presocratics" is an all-purpose label that applies to people before Socrates, but also those who were contemporary to Socrates, and even some who where younger than Socrates. For all practical purposes, it essentially means: pre-Plato and this chapter is about the Presocratics.

I can identify four Greek Research Programs which still seem modern, but which were really first identified by the Presocratics. Each theme was seeded before Plato and Aristotle and then watered and then harvested. I'll highlight them as we move along. They are:

1. **Is the universe constructed of fundamental building blocks and might those fundamental entities behave together according to rules?** This is the nature of physics today: my field of particle physics is dedicated to finding and characterizing the fundamental entities that make up everything else. Quarks and Leptons are those entities. But just stockpiling particles is merely stamp-collecting. They have to interact with one another and so the rules are deeply important. We call them the four fundamental forces today.

2. **Is the universe inherently mathematical?** It's long been appreciated that the universe seems to operate according to rules that are mathematical or can be described as mathematical. Discoveries in physics and mathematics have each influenced the other. Why that relationship exists isn't understood and is yet so persuasive to some theoretical physicists, that they postulate—still— that the universe is not just mathematical, but *is mathematics*. I'll have a lot to say about this as it underpins not only MOTION and LIGHT but all of modern science.

3. **How can we reconcile permanence with change?** This is a tricky issue and one that bedeviled not only the Greeks, but much of

[1] This last one requires that we are into the mid 19th century to be relevant. Which is, a part of the story.

philosophy to the present day. Unraveling this tension is intimately connected to theories of knowledge: what can we know and what can we trust? The permanent part of physics today refers to the various "conservation laws"...the Conservation of Energy, for example. But our elementary particles move around, they mix together, they annihilate and are born out of the vacuum. All the time. Change and permanence, agonized over by the Presocratics and Plato, are firmly a part of our modern story.

4. **How is the Universe structured and what are the rules that govern its beginning and current state?** "Cosmology" is the Greek word for this study that mashes together their word *cosmos* for "the world" or "universe," and *logos*, the word for "study of." It's now a modern term and Cosmology is an entire discipline in physics and astronomy. It started with the Greeks and their ideas became, just like motion, mangled by Aristotle's authority. It took 2000 years to get it right.

The first three Research Programs are fleshed out in this and the next chapter. I'll reserve astronomy for Chapter 3 which is all about Greek cosmology.

Greeks reveled in drama and it's within the turmoil and bloodshed between the Persian Wars and Alexander the Great that western philosophy and nascent science had its beginnings. So we'll picture this as a play in eight acts. The curtain rises...on a catastrophe.

I imagine that it started out like any bright day on the northern coast of Crete. A lazy afternoon in this peaceful paradise.

Then, total darkness.

Without warning, the **loudest sound** ever experienced by humans was followed on the northern horizon by a hint of fire and smoke erupting tens of miles into the previously clear sky. Slowly the sun dimmed, and then the sky became black as six inches of ash fell all over the island like a dirty rain. In fact, debris fell as far as the whole of modern Turkey, northern Egypt, and the middle east. Following that sooty deluge, tidal waves fifty feet high engulfed the sea-side areas of Crete and destroyed everything for kilometers inland. That terrifying -1650 day...

Wait...Negative years? I'm sorry, but in my head the timeline of history is a number line with positive and negative numbers—years. Sure, it's a number line without a zero, but BC or BCE isn't separated from AD, or CE by a year 0 either. The names are too clumsy and so I prefer almost-straight-up arithmetic to enumerate years since it makes it a breeze to compare one year to another.

...in the capital city of Knossos was the consequence of a massive volcanic eruption on the island of Santorini, about 100 miles to the north. Look at your map application and navigate to 36°23′41.46" N 25°23′57.55" E. There you'll see a little Packman-like, backwards "C" feature in the Aegean Sea. That's the scar—the caldera from the "Minoan Eruption"—left behind by the opening act in what might have been the story of us in the West.

Our tragic Minoa—modern-day Crete—was a refined culture of master architects, mariners, and traders, an apparently relaxed and leisure-loving people. Their cities didn't seem to need much fortification—they seem to be secure among themselves and were rulers of the sea. They were literate and created the first *symbolic*, written language—two of them, actually. Their ancestors were pre-Bronze-Age migrants from the north, seasoned with Egyptian influence from about –3000.

Over the next thousand years, Minoans and Phoenicians became Mediterranean, international sea-going powerhouses trading across its entire breadth. Think about that: 1000 years of prosperity! Trading partners inclusive of hundreds of different cultures. After the volcano, they rebuilt but were never the same and were likely absorbed by a rougher crowd from the Greek mainland (which is called the "Peloponnese"). The Minoans are our literate ancient scientific ancestors, influencing the Greek culture even though they ceased to exist.

I like to think of those long-gone cultured Minoans as the polite part of our western scientific ancestors—the smart side of the family. But the famously disagreeable, Homeric Greeks came from that side of the family that you'd like to hide from your friends.

That "rougher crowd" were the Mycenaeans who evolved into the heroic Greeks of Homer's *Iliad*, made perhaps slightly more civilized by their Minoan acquisition. The centuries following were eventful and then blank: Iron-weapon-wielding northerners created chaos with the Mycenaeans and eventually initiated a multi-century dark age. What emerged around −800 included the still-standing Athens, Sparta, and Corinth; the singing and eventual writing of the Homeric sagas; and an explosive emigrant population prominently on the Aegean islands, western Ionian shores, and the southern boot of Italy. Established by −650, these colonies were active traders, especially in Melitus in Ionia. Figure 1.1 shows the Greek colonial expanse and details of the immediate Aegean and Italian city-states.

(a)

(b)

Figure 1.1: (a) The Presocratic and classical Greek colonial empire was vast, although I hesitate to use the word "empire" which implies cohesiveness since Greek colonies were only loosely connected to the mother ship. Eventually, the Egyptian port (to be called), Alexandria became the final storehouse of Greek learning, outside of Baghdad. All of this came at a price. Greeks were almost constantly at war. (b) The regions around Home Base show the eastern Ionian and western Italian Greek cities where the Presocratics lived. The inset in the lower right highlights the island of Santorini, the caldera left from the massive "Minoan Eruption" of approximately –1600.

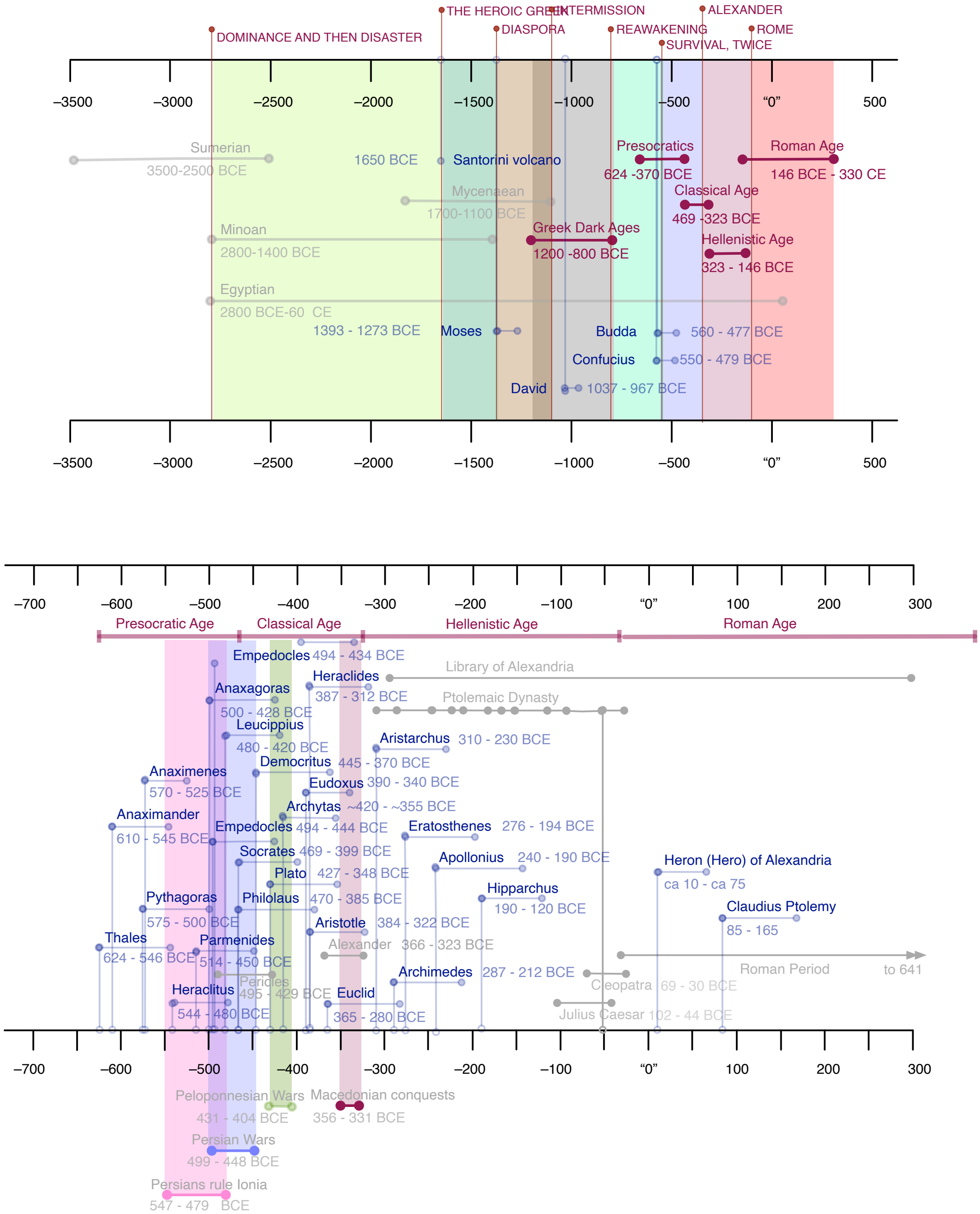

Figure 1.2: On the top, a Mediterranean timeline stretches from pre-biblical times to the end of the Roman empire. The bottom lays out the life spans of all of the Greeks you probably ever heard of...and the overlapping disasters that surrounded their lives.

1.1 A Little Bit of The Presocratic Greeks

Around 2800 years ago a proto-science began by people asking modern-sounding questions. We'll concern ourselves with our scientific parents: the Milesians (in Ionia, on the modern day west coast of Turkey) who invented the idea of substructure and natural rules, the Pythagoreans (in Italy) who emphasized the fundamental nature of mathematics, the Eleatics (in Italy) who fleshed out the tension between change and permanence, and the Pluralists (in Italy and Ionia), who found a rational alternative to the most persuasive and extreme of the Eleatics.

Brief relative (and rare) peace in the Ionian colonies, their positioning in the Mediterranean as a shipping crossroad, and the growth of large city-states led to a period suitable for growth of a new culture. And this was what emerged: The beginning of western philosophy. The time of the "Presocratics," literally those early philosophers who came before (or overlapped with) Socrates. These folks and their "Post-socratics (?)" asked modern-sounding questions of their surroundings.[2]

The timeline in Figure 1.2 shows roughly three distinct periods with names you might recognize. There are the Presocratics (from about –600 to about –430), the classic philosophers (from about –430 to about –250), and then the Hellenistic philosophers and scientists (from about -250 to +165). Notice that each of these periods overlap with war: Greeks fighting Persians, Greeks fighting Greeks (after the Persian wars, an over-confident Athens precipitated a dozen conflicts with Corinth and Sparta until the major Peloponnesian war), Macedonians fighting Greeks, and Greeks fighting the rest of the Mediterranean and Middle East. Notice that the whole of western history since the Magna Carta in 1215 would fit within a tick mark and a half in that top timeline.

1.1.1 ACT I: Is Nature Made From Stuff Governed By Rules?

Thales •Anaximander •Anaximenes •Pythagoras •Philolaus
(Set the context with the timeline in Figure 1.2 on page 20.)

Over my career I've published hundreds of scientific articles. Every publication has a common element: a bibliography with references to dozens or even more than a hundred other scientific works. Science doesn't happen in isolation as we're constantly building on, disputing, or confirming work of other scientists.

Take out a piece of blank paper. In many ways what your looking at is the bibliography of the first western philosopher, or even proto-scientist: **Thales of Miletus** (ca –624 to –547). Plato and Aristotle (and neo-Platonic philosophers who came centuries later) tell stories of him which form a lot of what we know. The fellow who invented history, Herodotus, also is a source.[3] Thales left no first-hand writings but stories about him abound.

[2] But the next century would see Ionia ruled by Persian-installed kings and tyrants.

[3] Herodotus was the first to tell about the past by trying to justify his assertions and find reasons for events. He's best known for his detailed history of the Greco-Persian Wars.

Here's one: my favorite *New Yorker* cartoon is a Robert Weber's 1981 image of professorial-looking, tweedy fellow with pipe on a NYC street corner asking a cop, "Excuse me, Officer. I'm an academic. Where am I?" That image of us academics didn't originate in a fancy magazine. Plato told the story that Thales was walking along looking at the stars and deep in thought and dropped straight into a well that he didn't see in his path. That embarrassment wasn't enough, as Plato also notes that a passing servant girl was on-hand to make fun of him in his reduced state.[4] But we also know that he was savvy enough to predict some weather changes and a possible bumper olive crop so he bought up all of the olive-presses in Miletus, and made a fortune selling them back.[5]

Maybe that happened. Here's another. It was suggested by Herodotus that Thales studied in Egypt, learned geometry and astronomy sufficiently to be able to predict an eclipse of the Sun on (our dating) May 28, −585 that pretty much stunned everyone, including causing a battle to pause. How did he do that?

Well, he couldn't have. That didn't happen. Available data wouldn't have allowed anyone to make such a prediction. It's trivial *now* to point back to the line of totality (the swath on Earth that would be dark) which would maybe have indeed been over the historical battle site, at that time. But a prediction? No.

Determining the veracity of stories like these is an example of a detective-story-approach to unraveling Thales and the other Presocratics: The eclipse fable suggests that Thales might have been an adult in −585 and thought by Herodotus to be a well-enough respected personage that his "predictions" might have mattered. So this story, while fiction, did contribute to the picture of the man called Thales and his reputation and his timeline. Thales was a real person.

What's not in dispute is that he initiated, or was a part of, a new way of asking questions *and* a new standard of what constitutes acceptable answers. Nobody thought like him and his immediate successors, and now we all do.

1.1.1.1 The World Before Thales & Co.

Why does it rain? Why are there earthquakes? Why are some people honest and others not? Why did my crop succeed and yours fail? Why is the Earth suspended under the sky? If you're Greek before about −500, there's a god for that. Why are there clouds? Yup, a god for that too. Why does the Sun shine? Another god. I tried to count all of the Greek gods, titans, minor deities, spirits, sea gods, agricultural gods, "rustic" gods, plus health and sleep gods. Oh, plus almost 30 mortals who earned promotion to god-like eternal life. It's hundreds. There is the varsity team—the 12 gods of Olympus and the 12 Titans. But the god-team bench is really deep.

[4] Plato's references to the Presocratics are often to make fun of them.

[5] He was also an astronomer of note and a mathematician with theorems to his credit. An all-around academic.

Take earthquakes. Currently, Greece ranks fifth or sixth in propensity for seismic activity. So Greeks were used to their ground moving. What everyone knew was that earthquakes happen because Poseidon (the god of the sea) is irritated. Without enough attention, he bangs his trident on the ground from Olympus and they get an earthquake. Or rain. If water falls from the sky it's also the case that another petulant god is unhappy, this time Zeus (the god of a lot, including the weather) using his lightening bolt symbol to make trouble.

There's a madness to this, but also a sort of understandable urge to assign every human experience to an outside influence. While Homer's tales include the gods as major actors, it was Homer's contemporary, Hesiod who thought that the history of the gods needed a rational and believable narative and his *Theogony* is basically the story of the world's origin including the genealogy of the gods. There's also a cosmology in these myths: the gods are themselves born... they've not always been around. And they have lives—outrageous ones.

That's interesting. They could have just "been there," outside of time like in other religions, but Greek myth seemed to require a logical, if not fanciful structure: Earth (Gaia) and Sky (Ouranos) were the first and their union is followed by scenes from *Animal House*... no, much worse. Infanticide, incest, fratricide, cannibalism, mutilation, and betrayal follow among the gods and the Titans, and between them and regular humans. Murders are the most light-hearted events in Hesiod's story.

The bottom line of Greek mythology is that everything happens for a reason. Why? Because a god is benevolent or unhappy or just doing their job.

1.1.1.2 Thales' Science and His Successors

GREEK RESEARCH PROGRAM #1 : Thales ushers in the first Greek Research Program, that the world is made of some fundamental substance that behaves according to natural laws.

Thales was the first that we know of to take a different approach. He's best known for asking what is the underlying, common structure of the universe, what Aristotle called on his behalf, the First Cause.[6] Thales reasoned that all of our universe depended on a single substance, and for him that substance is: water. After all, without water or moisture, things perish. Water is in the air and condenses and wets surfaces. It evaporates and reappears, sometimes revealing (creating?) soil underneath. Nothing lives without water and when things die, they become dry.

[6] Aristotle uses that word. But Aristotle was fond of Aristotle's philosophy and his reliance on "Cause" and "Substance" in his own work, motivated his description of the Presocratics' work. Those words were not available to the early Presocratics.

So as a single substance acting as the basis of all things, it's not too bad. This description of the world is **materialistic** and **monist** (the view that there is one underlying substance).

This concept is the first of three novel features of Presocratic proto-scientific thinking.

1. Thales suggested that humans could understand how the world works, including what causes the events and things that we experience. His suggestion is that the world is made of fundamental stuff guided by rules—laws of nature, so to speak—that govern how that stuff operates. The world needn't be a mystery.
2. Their "how" commitment searches for naturalistic reasons for events and existence. The previous "why" commitment was satisfied that "a god did it." For the "how" answers, the gods aren't involved. For example, the early Greeks inherited an ancient idea that the Earth is a flat disk with a dome of sky overhead, surrounded by a river (the Ocean or *Okeanos*) and the whole thing is held up by Atlas as a punishment handed out by Zeus. Thales agreed with the geographical part of this cosmology that the disk floats on water but earthquakes happen when the water sloshes. A wildly wrong explanation, but completely naturalistic. Poseidon is not involved.
3. Finally, the Presocratics jousted with one another: an idea or a research program from one, might be incorporated in another's account. Or, an idea or research program of one might be a focus of criticism resulting in an alternative account.

> This is not yet science, but science can't happen without at least these three commitments: **we can know about a rules-based universe**, **"how" cannot depend on the supernatural**, and **competition and collaboration are essential** to carry a project forward. All of this was new and now familiar.

Others who came after Thales adopted the same "research program" hypothesizing and defending an underlying substance for the world. Thales' "A" students, **Anaximander** (ca -610 to -545) and **Anaximenes** (ca -570 to -525) asked that question and answered it in different ways, but with the same basic motivation. Each of them had their own underlying substance idea.

Anaximander gave us one of the first maps, perhaps the sundial, and a full cosmology including a hockey-puck-like cylindrical Earth floating at the center of the universe. He watched the stars go around us and concluded that the Earth can't be falling. . . so it must be balanced at the center of the cosmos.

> Here, is our first reasoned theory of MOTION, in particular MOTION BY THE EARTH. He concluded that the Earth doesn't move, but for a reason: because of symmetry and balance.

Anaximenes went a step further and realized that what's important is *process*—things turn into other things. Cycles happen. Lawlike behavior is evident. Neither

Anaximander nor Anaximenes went along with Thales' contention that water could be the sole source of stuff—how can water be the source of its opposite, fire? That's not the point, though! They rejected his specifics, but bought into the project: While Anaximander chose something etherial and not itself one of the substances (the spooky "Apeiron"), Anaximenes chose air as the fundamental substance, but he had a scheme whereby air's various guises could account for the actual things we experience.

By this point, proto-scientific practice is pretty much up and running. They were naturalists, materialists, and the first Empiricists—using their powers of observation to study their world and attempt to explain it without recourse to a deity or a dogma.

1.1.2 ACT II: Pythagoreans in the West

It must be exhausting being a philosopher in your day job while also moonlighting as a deity and yet **Pythagoras of Samos** (ca -582 to -497) seemed to function as both, or so his followers asserted. Yes, that Pythagoras: of the triangle, although it's probably not what you think. What Pythagoras taught and what evolved out of the long Pythagorean school is difficult to parse today so it's not fair to attribute all of "Pythagoreanism" to that one person. The ideas that are attributed to him originated in Italy but evolved considerably becoming a dispersed movement that spread throughout the Hellenic world and beyond to the Renaissance hundreds of years later. Indeed by Plato's time, Pythagoras was already an enigma. As we'll see, Plato probably learned about him through Philolaus of Croton and Archytas of Tarentum, two acknowledged second generation Pythagoreans and mathematicians in their own right. So we have a nearly mythical figure: In the near-term there was Pythagoras, "so-called Pythagoreans" (as Aristotle called them), and Pythagoreanism. . . the seed-philosophy of mathematics that has lasted in some form to the present day. I'll mostly use the plural "them" rather that the singular, "him." "Pythagoras" is essentially the name of a movement and a culture and unreliably as a single individual.

His biographical details are from Roman-era writers and enthusiasts and it's difficult to know what's believable. There's general argreement that he grew up on the Aegean island of Samos and reportedly met the elderly Thales, and maybe studied with both Anaximander and Anaximenes. So suggested Heraclitus, from whom we do have actual written (critical) fragments about Pythagoras. He may have traveled around the Aegean with his merchant-marine father and probably lived in Egypt and maybe Babylon for at least two decades, absorbing language, philosophy, and mathematics. So, a well-traveled, probably comfortable young intellectual. The politics of Samos became tenuous and in spite of the fact that he'd established a following of students, at the age of 40, he relocated to the large Greek city of Croton in the "instep" of the boot of Italy. Some accounts suggest that he was accompanied by a number of loyal followers—the Pied Piper of Samos?—but most suggest that he moved by himself. In Italy he again established a following of reputedly as many as 600 (some say thousands) men and women in Italy and actually wielded some civic influence in Croton, serving as both an advisor and unwelcome busybody. He eventually founded a school that was to last 300 years, twice as long as my own Michigan State University has been around.[7] The ideas generated from that time evolved and so the border between the man and the movement is impossible to demarcate today.

This unusual school also functioned as a mystical, religious cult. Its members were regimented as to how to dress, what they could eat, what they may believe… and what secrets they must keep. They loved secrets. Pythagoras was its head and was by legend, supreme, teaching about his remembered past lives and reincarnations.

The legendary discovery moment came from thinking deeply about musical tones which they extrapolated to the proposition that numbers and mathematics are a fundamental fabric of the universe. Although they were not in competition with the Ionians, reliance only on a substance-based first principle wasn't sufficient for them. Rather they believed that their discoveries in mathematics revealed something fundamental about the world:

> "All things have form, all things are form; and all forms can be defined by numbers."
> Pythagoras

> "The Pythagorean … having been brought up in the study of mathematics, thought that things are numbers … and that the whole cosmos is a scale and a number."
> Aristotle *Metaphysics*

1.1.2.1 The Most Durable Discovery in History

GREEK RESEARCH PROGRAM #2 : Pythagoras ushers in the second Greek Research Program, that the world is mathematical. Or even that the world is mathematics.

Pythagoras left no writings, but stories/fables/tales reported by dozens of others abound. He claimed (or it was claimed for him) to have discovered integer relationships among the strings of a lyre[8] and the pleasant chords it could make. The lyre was probably a 7-string variety although he reportedly built a one-stringed tool ("kanon" or "monochord") to study its behavior (although that story is disputed). A quick taste of what the Pythagoreans left for us (and for Plato!):

When you pluck a string, clamped at the ends, you cause the string to vibrate with a fundamental frequency related to its length (and tension—think, a guitar). Call

[7] But both his and mine are mere babes, as compared with Oxford University, the University of Paris, or the Academy of Plato.

[8] and the tones from cups filled with different amounts of water which were noted for their pleasing sounds

that the "ground note." (A Pythagorean scale is different from how a piano is tuned, but I'll use piano as my analogy.) A piano's middle C is a natural ground note and has a frequency of 261 Hertz (Hz, are the units for "cycles per second," the number of repeated ups and downs of a wave). Pressing the lyre string at a half-way point and then plucking one of the two halves will cause the ground note to be repeated, but an octave higher. (On the piano, C above middle C is a frequency of 522 Hz, twice 261 Hz.) Pressing a lyre string at 2/3 of the length and plucking the long remaining string, causes the fifth above the ground to sound (for the ground of middle C, that would be G, or 392 Hz, 3/2 of middle C's frequency) and pressing 3/4 of the length, a fourth above that (A above middle C at 348 Hz, 4/3 times that of middle C's frequency).

Play those intervals on a lyre or chords on a modern piano and your ears will be happy. These are pleasant-sounding combinations while other combinations are not so sweet—we say dissonant. To the Pythagoreans, the difference between pleasant and dissonant was due to the integer ratios of the string lengths—what was important was not the strings, but the *numbers themselves*.[9]

> This revealed an **intimate link between numbers and the world**: integer ratios 2/1, 3/2, and 4/3 → to specific lyre string lengths → to pleasing your ear (your soul). This relationship made the numbers 1, 2, 3, and 4 very special to them. Your human well-being, connected to abstract numbers.

Lyres had been around for millennia, so surely this particular discovery was not news. But what Pythagoreans did was new. They elevated numbers to a significance that's *beyond just counting*. They **invented the concept of number itself**: from 2 oranges to the abstract concept of "2." This direct connection between a few integer numbers, their ratios, and special numbers with important meanings[10] influenced all that's "scientific" up to the present day: A brand new commitment...to an abstraction.

[9]It's a matter of current physiological research to understand why some combinations of tones are pleasing and others are dissonant.

[10]Notwithstanding "42" as the numerical explanation of everything in *Hitchhiker's Guide to the Galaxy*

When it comes to Pythagoreans, who did what, when is murky. In the lower timeline of Figure 1.2 between Pythagoras and Plato you'll see **Philolaus of Croton** (ca -470 to -385) who was the first Pythagorean to write about their program, although only fragments and references from others remain. Much of what Plato and Aristotle knew probably originated from his writings. (Plato only mentions "Pythagoras" and "Pythagorean" once each, but Aristotle was more expansive.) Plilolaus was a scholar in his own right and it's hard to discern what ideas were his and what came from Pythagoras himself, or even in Pythagoras' lifetime. What Plato and Aristotle knew of Pythagoreanism probably came from Philolaus or Archytas, another Pythagorean known well to Plato[a] Highly readable accounts are **ferguson2008** and **gerl70**.

[a]And, what we know of Philolaus might have come from the Pythagorean, Hippasus. The most unlucky Pythagorean. He is remembered as having constructed bronze disks who's thicknesses matched the lyre string ratios. When struck they would then create the same pleasing sounds as strings. He's also historically the poor guy who found the non-integer problem with the Pythagorean Theorem. Stay tuned.

This connection between integers and one's soul seemed to have been just the beginning. They also connected numbers with shapes and so geometry and by extension, to space itself. Keep them in mind: 1, 2, 3, and 4.

What can you do with them? Well, you can add them: $1+2+3+4=10$ which is not such a complicated thing, but these are special numbers after all and so their combinations must be special as well: "10" was important.

You can also make patterns with numbers—and a highly useful definition of modern mathematics (especially in physics)—is that it's the process of finding patterns. Figure 1.3 shows examples of Pythagorean patterns with integer numbers and an important Egyptian application.

Figure 1.3 (a) starts with one stone, and adds the first odd number, 3,[11] arranged around it turning $1+3$ into 4, *but it also laying them out as a pattern in space.* Numbers = geometry for the first time. This is a "square number" which follows the rule (in modern notation) of $1+3=4=2^2$. We can expand this into more square numbers and the next one is in Figure 1.3 (b) which shows that $1+3+5=9=3^2$. One can also take two stones and add the next even number around them in (c), say three above and one to the right, to get a "rectangular number."

Especially important is the arrangement shown in Figure 1.3 (d). Remember, $1,2,3,4$ are special. Lay out four stones, then layer three on top, then two, and finally one. You've now made a special triangle—-the tetraktys ("fourness")—with 4 stones on each of three sides. So it's an equilateral triangle and all four of the important numbers are contained in it. . . adding to 10. Maybe they liked bowling.[12]

[11]The number 1 was not a number for them: numbers meant a plurality. One is not "odd" nor is it "even." It's unique.

[12]There is a fable that a Pythagorean became ill at an inn while traveling but had no money to compensate the owner for his care while convalescing. The traveler told the owner to hang an image of the tetraktys and other Pythagorean travelers would compensate him far beyond his original costs. And they did. So it goes.

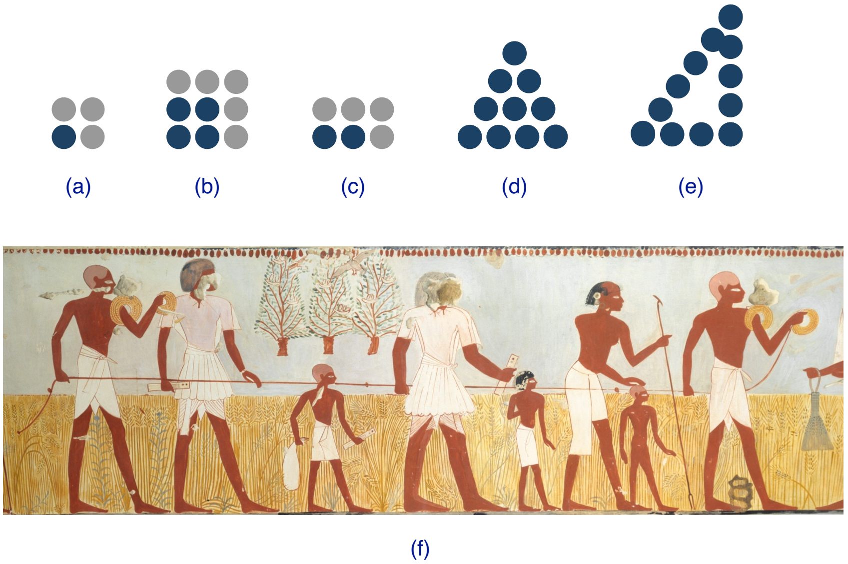

Figure 1.3: Dots represent stones that they would have used to signify numbers—precisely like the dots on dice. The image (f) is from the Tomb of Menna showing Egyptian workers getting ready to do surveying with a knotted rope. See the text for a description.

There's another connection between numbers and geometry—again, connected with the physical world. "1" was a special number, neither odd nor even (for them) and plays the role of a beginning. The source. A single isolated point is the starting point (no pun intended) for everything. "2" represents a line, which starts with a point and is constructed of points. "3" represents a triangle which delineates a flat plane and is constructed of lines, and "4" represents a tetrahedron, a three-dimensional solid constructed of triangles. That's it. Three dimensions to our physical space is all there is and so "4" represents completion and its encoding in the tetraktys (count the stones in any direction in the tetraktys and you'll count 1, 2, 3, and the base, 4) and that relationship with "10" tied it all together for them. (Of course today multidimensional spaces are a mathematical walk in the park. We know that our physical world consists of at least four dimensions. So stopping at "4" was premature!) There's more. "5" is special as it's the sum of the first even and first odd number. "6" is special since it's both the sum of the first three numbers and simultaneously, the product of the first three numbers. And so it goes.

Notice that there's another triangular pattern in Figure 1.3 (e). If you count the spaces between stones, you'll find that they delineate $3-4-5$ which is a familiar triangle to some of you, but a familiar triangle to thousands of years of Egyptian builders. This triad of numbers has practical value as it's a sure-fire way to make a right angle. Take a length of rope and tie 12 knots equally spaced from end to end. Then have a worker hold one end, another hold the third knot, and a third worker grasp the rope 4 more knots along. If the other end is then given to the first worker. The only way to make each of the three segments taunt is for there to be a right angle between the 3 and 4 knot segments. There are other such triads that make a right angle in this way, for example $6-8-10$. The ancient Babylonians and Egyptians knew of many of them and used them in surveying and building without realizing that this was an important thing. Figure 1.3 (f) is from the Tomb of Menna showing a knotted rope for surveying. As you know from high school, Pythagoreans figured out what this means in an abstract way.

There was a mystical quality to numbers and numerology was a thing and so the numbers also had special meanings for things beyond just "quantity." For example, 5 is the sum of the first even and odd numbers $2+3$ and since 2 symbolized female and 3 male, then 5 symbolized marriage. The first even number is 2 and squared is 4 and so that first square number, 4 symbolized *justice*. Likewise, the first odd number is 3 and its square is 9 and so it also symbolized *justice*. (Even today, we refer to a "square deal" as a proper deal.)

In fact, 10 was such an important number that in one version of Pythagoras' cosmology we have another early moment of MOTION BY THE EARTH. The Earth and all of the other celestial objects moved around something called the "central fire." This actually comes from Philolaus:

> "The first thing fitted together, the one in the center of the sphere, is called the hearth." Philolaus *Fragment 7*

The bodies are, from the inside-out, Earth, Moon, Sun, Venus, Mercury, Mars, Jupiter, Saturn, and the celestial sphere, but... wait. That adds up to 9. It must be 10 in order to be right, so they added the "counter-Earth" who's orbital mechanics are such to be perfectly positioned to block our view of the central fire since we don't see it.

> "... inasmuch as ten seemed to be the perfect number and to embrace the whole nature of numbers, they asserted that the number of bodies moving through the heavens were ten, and when only nine were visible, for the reason just stated they postulated the counter-earth as the tenth." Aristotle *Metaphysics*

That's a very modern interpretation of the use of mathematics in physics. You postulate the importance of a principle ("10 is magic"), you create a model of the universe (or some small part of it) built within the model, and then using the basic rules of the model (like arithmetic or something fancier) tweak it while still committing to the model. Here the counter-earth was such a tweak. That's actually how physicists work within models until they become unwieldy or are ruled out by experiment. I'll have more to say about a modern day view of Pythagoreanism *Presocratic Greeks, Today* in Section 1.2 and their cosmology in Chapter 3. It's a matter of much discussion (a polite way of saying, "argument") today. It gets worse when we add Plato to the mix.

The connection of music and integers led Pythagoras to assert that the regular harmonies of the cosmos were everywhere. The planets and stars all move and emit tones that ordinary humans can't hear since it forms a background to everyday noise. But Pythagoras? Ah, he's different. He's the only human who can hear the Harmonies of the Spheres. Being a deity has its privileges.

You're wondering about that theorem, I know you are. Look at Figure 1.4 and relive high school for a moment. Notice that Figure 1.4 (b) is the knot/stones-version of the Egyptian right-angle trick.

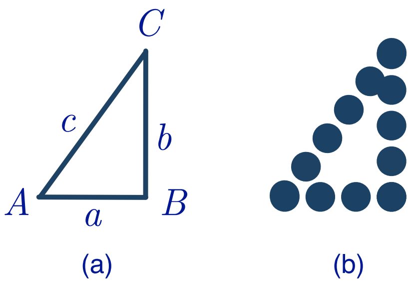

Figure 1.4: In (a) is a modern-day version of drawing a triangle, while in (b) is the same thing but with stones or knots delineating distances. The length c is the hypothenuse.

Maybe you remember the little song for a right-angled triangle: "…the square of the hypotenuse is equal to the sum of squares of the other two sides."

Or less lyrically,

$$c^2 = a^2 + b^2.$$

> There's no evidence that Pythagoras first proved this, and in fact, plenty of evidence that it was long known before him. (There are now about a hundred different proofs of the "Pythagorean Theorem." I offer a couple in the Technical Appendix, A.1.1.) The Egyptians had a real estate problem to solve: the Nile overflowed its banks every year and the fertile crop land alongside of it would be covered with water. That meant a problem: once the water receded, whose land was whose? Out of a need, geometry for Egyptians was a necessity. This was another job for the practical $3^2 + 4^2 = 5^2$. But the Babylonians were the champs. Not only did they keep accounting records, they did so in a base-60 number system...which must be 6 times better than our base-10 system, right? We've fragments that showed that they had worked out things like $119^2 + 120^2 = 169^2$ which admittedly doesn't come up every day.

There is a Pythagorean-Theorem story that tells you much of what you need to know about his cult. Remember, integers were the thing and so we feel sorry for the poor guy (historically, maybe Hippasus) who noted that a triangle with legs of 1 would have an hypotenuse that's Pythagorean-impossible since $1^2 + 1^2 = (\sqrt{2})^2$. This $\sqrt{2} = 1.4142135624\ldots$[13] never ends—the definition of an "irrational number"—it goes on forever and so decidedly not one of the mandated integers. Since he'd found a non-integer, for his trouble, as the story goes, he was thrown overboard from a ship in order that his little discovery not be revealed to the other cult members. Maybe this happened.

In the end, as sometimes happens with cults, Pythagoras' welcome in Croton wore out. His house was burned and he escaped, only to die in his escape…or not. We don't know. But what he and his colleagues created lived far beyond them.

[13]"dot dot dot," ... is mathematics-speak for "never ends."

1.1.3 ACT III: The Eleatics in the West

Heraclitus •Parmenides •Zeno
(Set the context with the timeline in Figure 1.2 on page 20.)

What happened next unsettled the young enterprise of philosophy and, after Plato and Aristotle, initiated millennia of philosophical controversy. We saw that the Ionians relied on their senses and took it for granted that events in the world changed in time. But you and I have both learned that our senses can be tricky and not always accurate. And, even if we see/hear/feel accurately, the targets of our perception themselves change. So if that's the case, then what about our "scientific" observations? Can we trust our senses to gather accurate impressions of our surroundings and base our theories on those impressions? This investigation traditionally pits two Presocratics against one another, the "Riddler" of Philosophy, **Heraclitus of Ephesus** (ca –540 to –480) and the first "Lawyer" of Philosophy, **Parmenides of Elea** (ca –514 to –450). The former was an Ionian from the big city of Ephesus, not far from Miletus. The latter, was from the colony of Elea in southern Italy. Pythagoras' territory.

Heraclitus was a loner, while Parmenides evolved a school of philosophy called the "Eleatics." You might not have heard of that, but you may recognize one of Parmenides' significant followers: Zeno…of Achilles and the Tortoise fame. Heraclitus (by himself) and Parmenides and his followers took up the subject of change. Heraclitus was decidedly on the side of, sure, things change. But he took it in an abstract direction. On the other side, Parmenides concluded that change is an illusion. He even *proved* that change is an illusion. At first glance, that seems strange, but his novel method of philosophizing was persuasive and as a consequence he created two branches of philosophy. And in the course of digging into the problematic nature of Change, set off a huge argument over centuries. Obviously, this is prior to any kind of physics-like analysis of MOTION!

GREEK RESEARCH PROGRAM #3a : The Problem: Tension between Change versus Permanence begins with Heraclitus and Paremenides.

1.1.3.1 The Riddler

Although we know few details of Heraclitus' life, he was apparently prominent in Ephesus. His father was said to have been an aristocrat, but Ionia was under Persian control during his life and suggestions that Heraclitus might consider a political life might be hard to picture. He wasn't a people-person. He would have been a child when Anaximenes died but he was critical of the Milesians and scathing in his criticism of his contemporary, Pythagoras. About 100 fragments of Heraclitus' work remain showing that his style was. . . unusual. He wrote very short tweets which have puzzled and delighted readers for thousands of years.

He was a monist as well: fire was his fundamental substance. And as interpreted by one of his aphorisms, he had a cosmology,

> "This world-order [*kosmos*], the same of all, no god nor man did create, but it ever was and is and will be: everliving fire, kindling in measures and being quenched in measures." Heraclitus

This is the first time that the word "cosmos" appears in Greek philosophy and he's clearly insisting that the cosmos always was the case and always will be the case. That's interesting since Plato deliberately labeled him inaccurately as naively saying that "everything changes" and that nothing is permanent.

You and I think of MOTION ON THE EARTH as moving from one place to another during some time, right? Remember, the Greeks were just beginning to do this analysis and moving from place to place was not their primary concern. Change by itself was and Heraclitus was the first to abstract *any* change as basically a form of motion, seeming to assert that universal "flux" was an important feature in the world.

> "It is not possible to step twice into the same river." Heraclitus

This is a famous paraphrase of a translation of his most famous of three "river aphorisms," The idea is that the river is always flowing and if you step into "the river" once, and then step into it a second time, it's a different river. So two rivers sort of functioning at the same time. It's a little different from this one:[14]

> "As the same thing in us are living and dead, waking and sleeping, young and old. For these things having changed around are those, and those in turn having changed around are these." Heraclitus

A young person is connected to their older self through the changes that they undergo. A is different from B, but linked because *A changes into B*. But, living and dead? This is a deep idea and seems to suggest that A and its opposite, B, are actually the same thing. In fact Change here has a job: it's a sort of glue that links together different things or different aspects of a thing. So apparent opposites are connected meaning that everything in the world is connected. One.

Plato used Heraclitus as a punching bag and said that connecting opposites like Heraclitus suggests gives us logical contradictions. Plato had an agenda. Aristotle was a little more forgiving and we'll see how he codified and categorized change, which will explicitly include our notion of loco-motion. But it seems that he had to go through Heraclitus to get there.

It's easy to be amused by Heraclitus' words, and for millennia, that's been a sport and I have more for you in *Presocratic Greeks, Today* in Section 1.2.1 below.

[14]While the most famous Heraclitus aphorism, there are at least three versions of it and some dispute as to its overall authenticity.

1.1.3.2 Nothing Gets Done: The Parmenides Problem

Parmenides took the extreme, opposite position, probably writing after Heraclitus. His argumentation is tightly logical so much so that it's possible to be swayed by the apparent inevitability of his arguments. If you can penetrate the denseness of it. I'll call his oddly persuasive but troubling conclusions the **Parmenides Problem**. It will seem to us like the Parmenides Problem will not go away.

He is the first in a long line of philosophers of both Metaphysics (the philosophy of the nature of being) and Epistemology (the philosophy of knowledge). He wrote a single book in verse (and according to Aristotle, not very well). It's a narrative story about his meeting with a goddess and how she teaches him about two kinds of knowledge.

There is the "first path" to knowledge: knowledge that is true by necessity. This "Way of Truth" is confined to your reasoning, not your senses. The second path to knowledge, that of perception, is "habit" and from "your heedless eye." This "Way of Seeming" is needed in order to get along in the world, but you can't trust it because you can be fooled. For that reason the "Way of Seeming" can't tell you what is true. So:

> ▻ **The Parmenides Problem**: True means permanent. So, anything that changes cannot be true.

Remember our own experiences: our senses can fool us and the objects of our perceptions can evolve between observations. What can you trust in the world if not your eyes? So he got rid of both issues. Truth can only refer to permanent things.

Accepting his premises, his logic seems oddly persuasive. In a nutshell, which could be on a T-shirt, I can sum up Parmenides in his two words (read it carefully…if nobody's around maybe even read this out loud): "**It is**." It's punchy. He also then reasons that "**It is** and it cannot, **not be**." *It cannot…* ***not be***. If something **is**, it can't be **not-is** at the same time. Further, if something **exists**, then **it is**. Consequently, if it **doesn't exist**, then it is **not-is**. So knowing what **is**, is to know what **exists**. So far, so good. Something can't exist and not exist simultaneously. (Can you see how this is against Heraclitus, who seemed to welcome A and not-A simultaneously?)

He goes further. If something exists (it **is**) then also it could never have been different in the past, nor will it be different in the future. For if it came into existence as **is**, then before that event it must have been: **not-is**. It changed. If it changes into something else in the future, then it goes from being **is** to then being **not**-is. How can something at one time be **not-is** and at another time be **is**? That can't happen! So if something **is**, it's always been **is**. In some sense, then the past and the present are one. Whew. Are you with me?

He's staked out clever ground in two new ways: His approach seems so logical that it launched philosophical analysis as an appropriate way to make arguments. And, he's defined what it is to be real: what's real must be true and therefore, it must be unchanging. The only place where truth can be realized is in your head. Where you reason.

Parmenides' sidekicks ran with this. Zeno took his arguments to the extreme and that's our connection with MOTION. Maybe you remember the story of how Achilles couldn't beat a tortoise in a race?

This is one of 10 of "Zeno's Paradoxes," **The Achilles**. Achilles, being the fastest human, is to race a tortoise, maybe the slowest animal, so he gives the tortoise a head start, halfway to the finish line. They both start but poor Achilles is faced with an impossible task. In order to traverse half of the distance to the tortoise's starting point, he has to traverse half of that half. Then half again of *that* half. In fact he needs to travel through an infinite number of paths, which is impossible so he can't catch the tortoise! There are three other paradoxes on motion (The Dichotomy, The Arrow and The Stadium), all designed to support Parmenidean conclusions about motion. In Technical Appendix A.1.2 I explain how we think of Zeno's paradoxes today as...well, not paradoxical.

Zeno gets this from Parmenides and since the reasoning seemed to be impenetrable, with an apparent gloss of a mathematical sheen lending a seeming validity, all of those races that you've seen with your lyin'eyes were apparently fooling you. I touch on two others in *Zeno and His Paradoxes*, Section 1.2.3 below.

We've now encountered examples of significant philosophical or scientific commitments. Sides were beginning to be drawn in natural philosophy that continue to this day: Can knowledge about the world be gained by thinking? Or must knowledge come from observation. The former is called Rationalism and the latter, Empiricism and physicists still argue about this. Clearly Pythagoras is in the first camp and so was Parmenides—distrust of the senses disqualified observation as a source of truth. And, geometrical argument seems like a good example of what must be true. The Ionians pioneered the second camp gleaning knowledge and theories about the universe by looking and hypothesizing from their observations.

Finally, the void. The vacuum. A state of actual nothing! By now you can imagine what Parmenides thinks of such an idea: it's impossible since it's the state of **non-being**. Another Eleatic, Melissus took this to the ultimate conclusion without the need of Zeno-like paradoxes. Just logic: anything that **is** cannot move since it would need a place to move to— it would need an open space where **nothing is** in order to relocate. But a place where **nothing is**. . . is nothing. But nothing can't be the case so there is no motion. Another MOTION problem.

Parmenides was the first to seriously question what can be known and by what means. Your senses deceive you all the time and so you can't depend on your observations for truth. But at the same time, your rational, logical thought—an argument assembled before Aristotle invented the actual rules of logic—is dependable. He then laid out a dispassionate argument that leaves one wondering what in the world is wrong with it.

> How do we get around this? In order to do science, or frankly, to live, one has to be able to hold a tentative, hypothetical idea in your head, but less than "True." But Parmenides was worried about that Truth with a capital "T" and so he couldn't abide an idea that is not true or even tentative as a stand-in for what's true and so his philosophy was sterile. Scientists don't deal in that kind of truth.

Well, this is embarrassing. My project here is an account of MOTION and now we've just encountered what seems to be a persuasive argument that MOTION is impossible. That's not progress, is it.

The Parmenides Problem is an important stepping-off point for Plato.

1.1.4 ACT IV: Antidotes to Parmenides?

Empedocles •Anaxagoras •Leucippus •Democritus
(Set the context with the timeline in Figure 1.2 on page 20.)

Parmenides' arguments were unsettling. The notion of a tightly logical argument was brand new, and yet even if its conclusions seemed nonsensical, you've got to struggle to find holes in his reasoning. But that didn't stop four intrepid souls. We still call them "Presocratics" but really they were "Co-socratics" (I made that up) since they all lived around the time of Plato's mentor. They're our last stop before Plato.

GREEK RESEARCH PROGRAM #3b : Attempts at solutions: Back to Monism for solutions to the Parmenides Problem?

1.1.4.1 Empedocles and Anaxagoras

One philosophical god was apparently not enough. **Empedocles of Sicily** (–494 to maybe –434) was another self-appointed deity. He was a contemporary to the Ionian, **Anaxagoras of Ionia** (–500 to maybe –428) who had a similar solution to the Parmenides Problem. Both took the position that the world is made of multiple entities and that *those* entities are what's permanent, but their *combinations* are multitude and accommodate change. In some ways, a modern approach.

Empedocles was a character. Legend has it that he dressed in a purple robe, with wreaths around his neck. He claimed to have performed miracles, raising folks from the dead, curing illness, and so on and he claimed to have been reincarnated from previous lives as a bird, a fish, a girl, a bush (really? shrubbery?) ...His brand was very Pythagorean he lived and worked in that same region of the Greek confederacy as the still functioning Pythagorean society, so there might have been some influence. He famously wore bronze-soled shoes everywhere. They figure into his legendary ascendence at the end. He was supposed to have leaped into the active volcano at Etna and disappeared but one of those distinctive shoes was left behind casting doubt on that last miracle. It seemed that the volcano spit the sandal out after consuming him. These stories come two centuries after his lifetime.

We only have fragments from Empedocles who wrote in verse, as seemed to be the custom in the west. It is from him that we get the familiar **Earth, Air, Fire, and Water** as basic elements (he called them "roots"). We will see that Plato and Aristotle took hold of this idea and ran with it all the way to Galileo's time. These four roots accommodate change by mixing with one another driven by two opposing forces, "Love" and "Strife." Again, a simplistic but modern-sounding notion of fundamental forces acting on the basic constituents of matter.

This is inspired. The roots are indivisible and have always existed, as have the two "forces" of Love (an attractive force) and Strife (a repulsive force). He also agreed that no-thing can come from nothing. So, we can check off both the Parmenides permanence and not-nothing boxes. But he also accommodates our senses, while warning of their fragility. What we observe is that things in our world are different from one another and that there are many of them. Some rocks are hard and some rocks are brittle. They're both rocks, so how do we build our observed rocks with only four roots?

> ▻ Empedocles contribution was that everything we observe is constructed of varying *proportions of the root elements*.

All rocks might be made of the same combinations of the roots, but a hard rock would have more of the Earth root than the brittle rock. With infinitely mixing proportions of the four roots, you can make the variety of the world. Sounds a little like a proto-chemistry.

Empedocles insisted that there was no purpose to the universe and that we're all subject to chance, postulating that we actually live in an undulating, repetitive cycle of a spherical universe in which Love and Strife compete for dominance.

His contemporary, Anaxagoras was from the other side of the West-East divide. He was an Ionian who ended up in Athens, establishing the first of a long string of Athenian philosophers. His arrival came during the classical period when the architecture; sculpture; literature; and yes, philosophy that we think of when we think "Greek" began.

Rather than only four substances, Anaxagoras presumes as many elements as there are things. Things. . . are themselves infinitely divisible. How do you acquire hair and bones? Well you eat foods that contain elements of. . . hair and bones. Everything is in everything. He insisted that the senses give us a window or a picture into aspects of reality that are not directly observable, but nonetheless, existing. Again, another modern idea from one of our "Co-socratics."

Notice that neither of our two characters explicitly address the issue of locomotion. This is a confusion that Aristotle promulgates, as we'll see. "Change" *per se* is broader than a thing moving from one place at one time to another place at a later time. So as you'll see in *Zeno and His Paradoxes*, Section 1.2.3 while Zeno works on that problem, he starts with the presumption that change is not possible and so by extension locomotion is impossible and hence the paradoxes try to persuade you of that. Our next two "Co-socratics" do find a way to explain locomotion which again, Aristotle rejects out of hand.

1.1.4.2 Atoms

I'll bet that you first learned the origin of the word "atom" in elementary school. "*Atomon*" is Greek for indivisible and the origin of that idea was again, the anxious need to find a way around the Parmenides Problem. You probably also learned that the inventor of atomism was **Democritus of Abdera** (about –445 to –370), originally from a region that's closer to Macedonia than it is to Athens, so a northerner. Here are three interesting things about Democritus. First, we classify him as a Presocratic, but that's really a misnomer. He's a "Post-socratic," younger than Socrates by more than 20 years. Secondly, he didn't invent the idea of atoms. He inherited it from **Leucippus of Miletus** (about –480 to –420). Finally, Plato doesn't mention him! He apparently burned Democritus' books. Aristotle knew him very well, maybe because of their shared northern roots.

Obviously, the idea of atoms is one with legs, albeit with ups and downs over the next two millennia, usually, unwelcome and only accepted when Einstein found two ways to demonstrate that there are indeed invisible chunks of matter. (That's a story that's not our current Einstein focus, but a large part of his miraculous 1905 year.)

However, the atoms (typically a mixture of Leucippus and Democritus' contributions) of classical Greece and our idea of atoms are very different. First, there are an infinite number of Greek atoms of all possible shapes. Some have hooks and can attach to others (think velcro), while some pairs have shapes that fit together. They move around and bounce off of one another, or they cling to one another forming compounds that eventually become the substances that we're familiar with. We know of them because of the sensible qualities that they bring to objects we can deal with using. . . our senses. For example, things that taste sweet are composed of smooth atoms while things that are acidic are composed of sharp-edged, angular atoms.

How is this an antidote for the Parmenides Problem? First, the atoms are permanent but second they are constantly in motion and all change is due to their arrangements, and re-arrangements.

But the real way in which this works is that both atomists insist that what's real are atoms. . . *and the void*. The void is the place where moving things can go *to*. So locomotion is possible. There. That does it for Parmenides. So, the atomists are happy to make room (so to speak) for MOTION ON THE EARTH.

> The void is an unpopular idea, and to this day we continually redefine what the vacuum is (or isn't). Our current understanding, again, my scientific playground, is that there is no place in the universe where there is nothing. The vacuum is full, but it's a quantum mechanical fullness that has no connection to any ideas before about 1950.

But, as I said, Plato ignored this singular, logical conclusion to the Parmenides Problem, which seems a cowardly way of dealing with an idea. As we'll see, Aristotle could not abide the void so he's no atomist either.

There's one more interesting fact about this pair's ideas and that's an idea that Plato would embrace, but with only partial credit to the right people. Everyday objects are not real things, and the attributes that we ascribe to visible, touchable, tasty, smelly, and loud objects of our sensible world are all based on convention. Democritus wrote:

> "By convention sweet and by convention bitter, by convention hot, by convention cold, by convention color; but in reality atoms and void." Democritus

Even though we can't see atoms, we know they're there because our minds tell us about what we can't see. A reality that's beyond our senses. Now this is a very modern idea and also a very Plato-idea and we'll see it emerge in a slightly different guise when we talk about Galileo and how he invented physics when he used this notion—now labeled "Platonic," but could be labeled Democritus-ian.

1.1.5 What's Important For Our Project

Our project is about MOTION and LIGHT. Does it make any sense to speak of either of them without numbers? MOTION implies speed (to us), immediately bringing to mind numbers: miles per hour, for example. LIGHT involves brightness, color, reflection and refraction... qualities that we can describe using words, but they're a stand-in for actual numbers as well: you'd evaluate a lightbulb's brightness by "lumens" and its color by "Kelvin" which are numbers. "Red" is a name for a particular frequency of light.

This is so much a part of our thinking now, that it almost seems trivial to mention it. Wouldn't it seem odd to think in any other way for almost everything, from cooking to taking a pain reliever to deciding when to buy new tires? Attaching numbers to the physical world is a gift of the Presocratics and in particular, the Pythagoreans. Trivial or not, before the Pythagoreans, numbers as more than just counting would have been a foreign concept, after them, well, numbers are *in* everything.

But their gifts were generous beyond just this. Lets quickly summarize what the Pre-, Co-, and Post-socratics have brought to the scientific table.

The invention of the scientific commitments that we use today came from them:

1. They eliminated the supernatural as an acceptable argument for why things in the world happen. We can know about the physical world.
2. They conceived of the notion that the universe is made of naturalistic stuff: the water, aperion, air first-guesses, to more intricate and even modern-sounding permanent entities that go together in proportions to build the stuff we experience.
   (a) They toyed with the idea that these entities had to obey rules that allowed for their interactions, and in some cases, motions.
3. They invented the notion that mathematics is tied both to geometry and to things in the world, essentially birthing modern mathematics. We literally have no other way to describe and predict the properties and behavior of the physics world.
4. Some Greeks realized that learning about the universe involved seeing, touching, and hearing what the universe of things does. But others noted that our senses are unreliable and so couldn't reliably deliver truth, if "truth" meant "permanent," setting up the problematic notion of Change. Taking a page from their high school geometry class, mathematics was a pretty good model of what is constant and true. But we only can deal with geometrical objects through reason. So: don't look at the world, *think* about the world. That's what I've called the Parmenides Problem: is change in the world an illusion?
5. Reactions to the Parmenides Problem led to at least two directions: primary substances mixed in proportion, Earth, Water, Air, and Fire... or atoms. It also confused everyone that followed and heavily motivated Plato and in a different way, Aristotle.

And, proto-science, and now science as we know it, is a social activity.

6. They argued. One philosopher added to or reacted to what another said. This created the necessary social structure and behavior necessary to support the scientific enterprise.

We're now ready for Plato.

1.2 Presocratic Greeks, Today

1.2.1 Tweeting With Heraclitus

Heraclitus is challenging because he's tough to analyze and because the available material is…pithy. The general view is that he really did write in these short aphorisms and that they aren't somehow surviving snippets of something larger.

The most famous of them, that tends to support his historical brand that "everything changes" is the River Analogy. The most famous version is due to Plato's rendition which he wrote in *Cratylus*:

> "Heraclitus, I believe, says that all things pass and nothing stays, and comparing existing things to the flow of a river, he says you could not step twice into the same

> river.” Plato

But there are actually three versions of the river tweet:

> “On those stepping into rivers staying the same other and other waters flow.” *Cleanthes*, a Greek Stoic from two centuries after Heraclitus’ life and almost a contemporary of Plato

> “Into the same rivers we step and do not step, we are and are not.” *Heraclitus Homericus*, a commentator from 500 years after Heraclitus’ life

> “It is not possible to step twice into the same river according to Heraclitus, or to come into contact twice with a mortal being in the same state.” Plutarch, from the Renaissance

The first is probably the most likely and doesn’t contradict the more popular version. However, this story illustrates the difficulty, once again, of the detective work involved in assigning credit (or blame) to the Presocratics.

I mentioned that he wasn’t a people-person, probably unsuited for political leadership (notice the disdain for his Italian contemporary, Pythagoras):

> “One is worth ten thousand to me, if he is the best.”

> “Eyes and ears are poor witnesses to people if they have uncultured souls.”

> “War is the mother of everything.”

> “The best of men choose one thing in preference to all else, immortal glory in preference to mortal good; whereas the masses simply glut themselves like cattle.”

> “It is not good for men to get all that they wish to get.”

> “What sense or thought do they have? They follow the popular singers, and they take the crowd as their teacher.”

> “Learning many things does not teach understanding. Else it would have taught Hesiod and Pythagoras, as well as Xenophanes and Hecataeus.”

> “Poor witnesses for men are the eyes and ears of those who have barbarian souls.”

> “The adult citizens of Ephesus should hang themselves, every one, and leave the city to children, since they have banished Hermodorus, a man pre-eminent among them, saying, Let no one stand out among us; or let him stand out elsewhere among others.”

His unity of opposites appears in multiple places:

> “Sea is the purest and most polluted water: for fish drinkable and healthy, for men undrinkable and harmful.”

> “Collections: wholes and not wholes; brought together, pulled apart; sung in unison, sung in conflict; from all things one and from one all things.”

> “Every pair of contraries is somewhere coinstantiated; and every object coinstantiates at least one pair of contraries.”

> “Good and ill are one.”

But, he’s also inspirational:

"Nature loves to hide."

"Sound thinking is the greatest virtue and wisdom: to speak the truth and to act on the basis of an understanding of the nature of things."

"Abundance of knowledge does not teach men to be wise."

"This world-order [kosmos], the same of all, no god nor man did create, but it ever was and is and will be: everliving fire, kindling in measures and being quenched in measures."

"The character of man is his guardian spirit."

"The sun is new every day."

. . . and amusing:

"And they pray to these images, as if one were to talk with a man's house, knowing not what gods or heroes are."

"Souls smell in Hell."

"Every beast is driven to the pasture with blows."

"Asses would rather have straw than gold."

1.2.2 Modern Day Pythagoreans

Want to liven a party? Raise the following question:

1. Is mathematics invented? Or,
2. Is mathematics discovered?

That is, are the theories, proofs, and concepts of mathematics the creation of human thought, or are they "out there" waiting to be revealed by thinking about them? "Platonists" would rally around #2. and we'll talk about that in the next chapter.[15]

Now if you want to rejuvenate your now yawning party-goers, narrow the question to:

3. Is mathematics invented in order to explain the physical universe? Or,
4. Is mathematics discovered to be already "in" the physical universe?

Number 3 suggests that mathematics is only a tool —a language—to describe the universe. Maybe it's a lucky break that we've invented it and that it seems to do pretty well. Perhaps another tool might have worked? For example, a musical score for guitar could be represented by standard musical notation. But it can also be represented by chord diagrams.

Number 4 suggests that the discovery of mathematical and especially numerical relationships and their match to what we observe in the universe represents an

[15]Want to start an argument? Try to defend any definition of what Pythagoreanism is. (You can also spice up the conversation by trying to defend what Platonism is, which is the next chapter.) I'm not a philosopher, but I do have a sense of how my interpretation of these two ideas fits my experience in modern physics research.

uncovering of its fundamental mathematical fabric. Here, Pythagoreans do find a place: their discovery was that #4 is how it goes. Numbers (and in modern language, patterns) are *in* physical objects.

Most rough-and-ready physicists would lean towards #3, but not everyone. I'm close to #4, but in a practical and not spooky way. (Some of my contemporaries are okay with spookiness when it comes to math and reality.)

We owe a debt to the Pythagoreans and while their application of "number" to the world is primitive, there are vestiges of their discovery that make science (and modern life) possible.

Mathematics describes the universe There is this nagging feeling that math and physical reality share a pretty special bond. Before the advent of Pythagoreanism, we saw that the Ionian approach to parting ways with deities was to ascribe a fundamental "stuff" as the basis of all physical things. Now, we don't depend only on that. We use math.

Take the weather. Before Pythagoreanism took hold, numbers meant "one apple," "two apples," and so on. Counting and nothing more. Before Pythagoras, I think that describing the weather using numbers might have seemed as strange as for us saying that the weather is "happy." While the ancient Pythagoreans didn't use numbers in most of the ways that we do, they might not be surprised that we are now comfortable to describe the properties of our weather more completely with numbers than with words. I just looked at the weather in Pythagoras' modern Crotone in Italy and it's not happy: it's 22° C (79° F), with a relative humidity of 76% and since the dew point is 71°, that's borderline uncomfortable. The barometric pressure is 1016 mb and rising and with a cloud cover of only 11%, and so visibility is 10 miles. This short narrative puts a picture in your mind of the weather conditions that words would do much less efficiently or accurately. I could take those numbers and recreate exactly those conditions in a lab. They are a natural measuring stick for us and that's due to our Pythagorean inheritance.

MIT cosmologist, Max Tegmark holds an extreme view that the numbers in our story aren't just *in* the weather, they *are the weather*. That is, if there's a one-to-one correspondence between a number and my interpretation of what the number means, then they're the same.

A taste from his controversial book, regarding the electric field:

> "If you can thus pair up every entity in our external physical reality with a corresponding one in a mathematical structure ('This electric-field strength here in physical space corresponds to this number in the mathematical structure,' for example), *then our external physical reality meets the definition of being a mathematical structure* (emphasis, mine)—indeed, that same mathematical structure. **tegmark2014**, page 280

That he's under attack suggests that physicists do have strong opinions about #3 versus #4, as much as they'd probably outwardly profess disinterest.

> "So the bottom line is that if you believe in an external reality independent of humans, then you must also believe that our physical reality is a mathematical structure. Nothing else has a baggage-free description. In other words, we all live in a gigantic mathematical object—one that's more elaborate than a dodecahedron, and probably also more complex than objects with intimidating names such as Calabi-Yau manifolds, tensor bundles and Hilbert spaces, which appear in today's most advanced physics theories. *Everything in our world is purely mathematical—including you.* (my emphasis)" **tegmark2014**, page 260

Or, in his technical publication **tegmark1998**,

> "Physical existence is equivalent to mathematical existence."

I've heard him ask what is a tree. To most it's a barky, green, leafy structure with roots and a hardness and so on. To him it's a collection of electrons and quarks and reflecting and absorbing light. In turn, the electrons are "-1, 1/2, 1, and 0.511." That is, the properties of trees are the collection of the properties of electrons and electrons are uniquely described as a negative electrical charge of -1 unit,[16] a quantum mechanical "spin" of 1/2, a "lepton number" of 1, and a mass of 0.511 MeV/c^2. Protons, neutrons, and quarks. . . and the light that's absorbed and emitted are also described completely and uniquely by a different set of numbers.

Now the labels that the numbers have are entirely human-defined. But no matter how an alien species might define the unit of electric charge, the electron (and proton) have ± 1 of it. So, to him what is a tree is defined by what are the properties of a tree, which are entirely defined by a small set of numbers.

Finally, this is a book about Einstein's Special Theory of Relativity and it can almost be completely thought of as discovering the importance of a single number: the speed of light, c. No number is more special than c.

Tegmark is not alone, but his is a very small club.

There are special numbers While I'd not be prepared to say that marriage is "5" and when justice is done, that "9" is involved, there are special numbers that our universe seems to have latched onto that both explain what we observe, and were some of these numbers different, we would not be here. I just referred to one such special number, the charge of an electron or a proton.

Many numbers in nature play a role that designates unique properties of substances or processes that substances undergo. There are static properties of matter which have conventionally-defined, critical numeric values. Here's one: 1836.153. This is the ratio of the mass of the proton divided by the mass of the electron. An alien species might not use the same units that we do, but whatever system they used would have to replicate this ratio. Otherwise, their big bang and chemistry would be completely different from ours. The formation of hydrogen atoms in the early

[16] The "fundamental electrical charge" is traditionally 1.6×10^{-19} Coulombs, usually denoted by "e." An electron's is $-1e$, a proton's is $+1e$, and a neutron's is $0e$.

universe would have occurred at a different temperature and our early universe would not have formed galaxies.

Another one: Water freezes at a particular temperature. What the number is depends on a conventional scale ($^\circ$ C or $^\circ$ F), but that there is a definitive event and that it can be quantified by a unique number of degrees makes it special. If that freezing point of water were slightly different, then the geological history of the Earth would have been different.

Inherent in a Pythagorean view of the physical universe is that the "numbers are in the thing" and that we can poke at nature with experiments and extract the mathematical essence that's embedded inside. Just like Pythagoras did... before anyone else.

1.2.2.1 Unreasonable?

Generally, we physicists don't generally lack in confidence. So in the interest of full-disclosure, here's a complete capitulation, a sort of a reluctant confession that we don't know why math and physics are so linked up:

Ask Mr Google to search just for the words "unreasonable effectiveness" and stand back. In less than a second, you'll be treated to a list of 12 million references to the Nobel Laureate, Eugene Wigner's 1960 article, *The unreasonable effectiveness of mathematics in the natural sciences.* It's actually a written version of a lecture he gave at NYU and it's among the most famous documents in physics. It's so ubiquitous, that Wiley Publishing is pleased for you to download it for free.

In that same vein, here's a word that you won't find physicists using: "miracle." The last paragraph in Wigner's article states:

> "Let me end on a more cheerful note. **The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift** which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning." [emphasis mine]

"A more cheerful note"? "The *Miracle*"? for heavens' sake? If that's his conclusion, can you get a sense of how his previous nine pages went?

There's a straight line from Pythagoras (and Pythagoreans... remember) to Plato and Platonism and to physics! But we don't understand this "unreasonableness," and sometimes it is kind of uncomfortable. Gloves come off when physicists and astronomers argue about multiverses, string theories, and measurement theory in quantum mechanics.

So, by now maybe you're a little more aware of the possibility that we may *all be a little bit Pythagorean*. Over and over we learn this.

1.2.3 Zeno and His Paradoxes

Parmenides had a following and his most devoted, and enthusiastic partner was the younger **Zeno of Elea** (ca –490 to ca –430). What he did was mess with everyone's mind about simple, common-sense experiences. He's remembered primarily for 10 paradoxes, two of which about motion I'll remind you of here as the most famous. He wants to show you that what you think you know, you don't, that common sense deceives. (Like in Quantum Mechanics and Relativity, where common sense left the building a long time ago.) I'll do them in reverse order. (By the way, how do we know of his arguments? Plato, again, in a dialog where Socrates deals with the young Zeno, playing himself. And Aristotle, who goes after Zeno.)

"The "Dichotomy." This is the famous race. In order to run the 100 meter dash, you've got to get to 50 meters. In order to get to 50 meters, you've got to get to 25 meters. See where I'm going (or maybe *not going*)? According to Zeno, there are an infinite number of distances that have to be traversed in order to move in space at all. So you can't get to 100 meters, in fact, you can't move at all. MOTION ON THE EARTH is impossible. Aristotle noticed that this is like the Achilles and the Tortoise paradox, except the conclusion of no motion is reserved to the Dichotomy.

Now this has been dissected for centuries. Ask Mr Google about "Zeno" and you'll see 36 million hits. The push-back begins with Aristotle, who argued persuasively, but in the end, inconclusively, that you can move through an infinite number of spaces if the time intervals become shorter and shorter while you do it. Aristotle hated infinity, so this must have been hard for him. But this presumes that Zeno was suggesting that the motion would take an infinite amount of time, but maybe it's because he was trying to cram an infinite number of steps into a finite period of time. So Aristotle's argument is not general enough.

The modern solution requires an understanding of how speed relates to time and space, a very modern set of ideas that are the heart of Relativity. I'll show you a complete explanation in Appendix A.1.2.

The Paradox of Infinite Divisibility. This paradox is the jumping-off point to an entirely different way of dealing with Heraclitus and Parmenides: If an object is made of parts, then one should be able to start cutting. . . into two parts, then four parts, and so on. At some point you reach some end: 1) If after an infinite number of slices, you find nothing. . . then the object was made of nothing—a **not-is**. 2) If after a finite number of slices, you find something. . . but it has zero size, then the object was made of something that has no size. Another kind of **not-is**. 3) If after a finite number of slicings, you find something that has finite size, like an element? Well, you're just not done slicing!

This is a modern thing as we are perfectly content to imagine that quarks that make of the proton and neutron have no size,[a] likewise the electron. But we have a field description of elementary particles and the forces among them, so we have a quantum mechanical push-back against Zeno here. But prior to the 20th century, a physics solution was not possible.

[a]Yes. Our word "particle" creates an image of a little billiard ball, doesn't it? In actuality, the size of quantum mechanical objects is so ill-defined as to have little meaning outside of an agreed-upon criteria involving waves.

You can see how this works. Zeno was apparently clever enough to waste the pixels on your computer screen in 36 million hits. . . all in service to the Parmenides two arguments: **Nothing changes** and **knowledge from perception cannot lead to truth**.

Chapter 2

Can’t Live With ’Em Or Without ’Em : Plato and Aristotle

“The safest general characterization of the European philosophical tradition is that it consists of a series of footnotes to Plato.”

- A.N Whitehead (1861-1947), *Process and Reality*

Bert and Ernie, Kirk and Spock, Mantle and Maris, Venus and Serena, Abbott and Costello...Plato and Aristotle. One can’t have one without the other and, just like the other pairs in that list, these last two are deep subjects. My need for Plato and Aristotle’s contributions to the study of motion are for two ideas: following Pythagorean inspiration, Plato and his collaborators built the first spherical working model of motion by the Earth and Aristotle expanded on it. They were both wrong.

And, while Plato didn’t concern himself with motion on the Earth (except in an almost impenetrable portion of his last book), Aristotle was all over motion on the Earth and invented its systematic study, informing—and infecting—science for 2000 years. It took until the 17th century before we could be all over with Aristotle. His models of motion on the Earth, motion by the Earth, and motion in the heavens became Medieval and Renaissance Church dogma, but are wrong in almost every respect. By pushing back scientists learned what was better and why.

So why is it that Plato’s shadow hangs around while Aristotle’s importance for physics disappeared more than 400 years ago? We still talk about Platonic worldviews in some fundamental branches of physics, but nobody talks about Aristotelian–anything. Plato put important questions in play that remain troubling: What can we know? How do we know when we're right? And, most importantly, what is the role of 'mathematics in the fabric of the universe?

It was the worst-kept secret sneak attack in history. Everyone knew that the Persians were coming as under King Darius' son Xerxes the Great's command, the invading infantry slowly marched along in parallel to the Persian navy counter-clockwise around the inside of the Aegean basin, subjugating the Ionians along the way. Anaximenes lived under that locally-sourced, Persian rule that drove Pythagoras to Italy. About 100 years before Socrates' execution following a 10 year advance in −480 the battle was joined with an amassed Persian force of at 150,000 soldiers and 600 warships. Athens was evacuated and the Persians destroyed the city.

The Greek confederation then organized itself: the wounded Athens mounted the naval campaign and Sparta, the foot soldier command. What followed was a series of military maneuvers, still studied today. Spartan heroism of King Leonidas with 300 Spartan troops and a total of 9,000 allied soldiers met and slaughtered the Persians at the pass at Thermopylae. The movie and the comic book series *300* might jog your memory (**nunnari2006**). While this was going on, the Athenian navy engaged and overwhelmingly defeated the much larger Persian naval force. Finally during the summer of −479, the Persians were defeated in a decisive land battle. Yet, war continued in one form or another for thirty more years until the Persians fled the Aegean leaving behind a Sparta with a greatly enhanced reputation. Proud Athens rebuilt after that disaster in −480 and under Pericles' leadership — throughout the decades of extended conflict, began its 75 year Golden Age when everything you think of as Greek in culture, art, architecture, and philosophy was intentionally created.

Ironically, even though Sparta could be credited as having been the major military force in the Greeks' victory, its isolated and belligerent nature simply did not equip it to lead during peacetime. In contrast, while Athens had been destroyed, its nature was to rebuild stronger, to politically organize, and to lead. All while doing what Greeks did best: fighting.

While the Golden Age was unrolling, Athens simultaneously managed to battle with: Sparta −465; Corinth and Sparta −459; Samos −440; Corinth again −433; Potidaea −433; Mageria −433; Sparta again −431 (Socrates was active as a soldier during this period), (Score: **Sparta 1, Athens 0**) Syracuse and Sparta −415, (Score: **Sparta 2, Athens 0**) ; Sparta now allied with Persia −414, (Score: **Sparta 3, Athens 0. Game, Set, Match**).

After that third war with Sparta,[1] Athens surrendered to Spartan general Lysander in −404. Plato was 23 years old and Socrates had five years to live.

Athens badly handled their unfortunate overreach and eventual defeat and in the final stages of the war they managed to: expel their leading general, execute six other military leaders, and flip from autocracy to democracy and back to autocracy. Socrates was on the autocracy side and it was the democrats who condemned him to drink the hemlock in −399.

Athens' subjugation by Sparta after the two Peloponnesian Wars was tumultuous — governance of the city jerked back and forth between oligarchs and democrats. In the same way that the Golden Age of Classical Greece emerged during war with the Persians, in the midst of the city's internal chaos, western philosophy began and was followed quickly by the first systematic attempts to understand MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS by our two lead actors. Yet the catalyst to all of this progress was interested in neither. Socrates' persistent question was: how to live a virtuous life, not how do things move. As his talented acolyte, Plato adopted the older man's voice and wrote truly engaging tales, but expressed his own ideas and, while his program was ostensibly one of ethics, the Socrates/Plato approach opened a new front in the battle with the Parmenides Problem which resonates in modern physics today. And, as so often happens in philosophy (and physics), the next productive steps were in opposition, launched by Aristotle, one of the most remarkable intellects in history and whose words we have are probably from lecture notes and not intended as his legacy literature. Yet in physics: Plato endured and Aristotle is gone.

2.1 Act V A Little Bit of Plato

Plato (−429 to −348) is actually a nickname, suggesting someone of broad shoulders or perhaps a wrestler. The name on his driver's license would have been Aristockes and his aristocratic family had been influential for generations. Plato was no democrat and grew up during the Peloponnesian War (−431 to −405) and the subsequent subjugation of Athens by the victorious Spartans. In many ways Plato's idea of the correct form of government was clearly informed by the collectivism and brutality of the Spartan way. (Set the context with the timeline in Figure 1.2 on page 20.)

> One of the signature events of his life was the story of his attempt to help form a government in Syracuse where he somehow got the idea that he could turn the tyrant Dionysius into a philosopher-king, since in Plato's opinion leaders should be philosophers. That got him imprisoned and even sold into slavery for a while (or so the story goes), until he was ransomed. He actually tried two more times, which brings to mind one's questionable mental state as per Einstein's observation much later about repeating the same mistake over and over and expecting a different outcome..

[1] who actually allied with Persia!

His life's direction was formed when he, like many young men in the newly democratic Athens, started to associate with **Socrates** (−470 to −399) who, after his (apparently distinguished) service as a foot soldier in the war, took philosophy on an entirely different course from investigating the nature of reality to how best to live a satisfactory life. We learn in school about Socrates' self-administered execution at the hands of the democratic Athenian politics—one of the reasons that Plato was distrustful of democracy. It was traditional to give the convicted criminal options on how they would like to do away with themselves and Socrates suggested that he be given free food for life. That was turned down and eventually death by poisoning was prescribed.

Plato's 35 books are all in the dialogue form, conversations between Socrates and a variety of fictional and real persons. Unlike Aristotle's largely academic writing (which might have been lecture notes), Plato's books are literature and valued for their style and lyricism. Plato himself is only mentioned twice and he never speaks directly. The assumption is that he's speaking through his mentor and that the ideas are his, and not meant to be those of the older man. (One book, *The Apologies*, might have been more personally Socrates as in that volume he defends himself against his accusers.) So the ideas are Plato's and the books comprise his philosophy as it evolves over his productive, long life. Almost all of his work follows a general theme, and what he seems to struggle with is what I've called the Parmenides Problem. Plato wants to contrast what we experience in our everyday world—objects (physical things) and ideas (like virtue, justice, beauty, what's good)—with abstract ideas that are the source of the variety of physical things and the imperfect values we associate with more aspirational ideas.

It might be reasonable to view the Socrates of Plato's dialogs as a literary invention, but he was known to broader Athens and even parodied in the *Clouds*, a vicious comedy by Aristophanes and figured in other writers' accounts, including in dialog form. But the world now knows of Socrates through Plato and he figures into every one of Plato's dialogues as "that guy" who irritates everyone, although in the later dialogues his role diminishes. His job is to ask simple-seeming questions (the "Socratic Method") of an assembled group of friends (or foes), often about an ethical matter. What's temperance? What is virtue? What is justice? The course of these sorts of innocent sounding conversations is repeated: the folks being questioned are maneuvered into impossible rhetorical cul-de-sacs, shown to be incapable of any kind of logical thinking, and more often than not, shown to not know things that they should have known. Meanwhile, Socrates rarely says what he thinks, in fact, he usually hides behind the assertion that he doesn't know either, but at least he knows that he doesn't know. Superior to a fault. These questions also often segue into something more than they seem, and many of them move to more weighty topics like how *do* you know what you know. That is, they form the beginning of serious Epistemology, one of the foundational philosophical disciplines.

Plato's output was large and I'll choose only a few topics that inform our scientific project. Unlike almost all of the previously considered Greek philosophers, we have complete writings. He famously started *The Academy*, a school that lasted more than 700 years whose star pupil was Aristotle, whom we will consider below. Bertrand Russell (in his Literature Nobel Prize winning, *A History of Western Philosophy*) appropriately sums up what we're about to dive into:

> "Aristotle's metaphysics, roughly speaking, may be described as Plato diluted by common sense... He is difficult because Plato and common sense do not mix easily." (**br1946**) *A History of Western Philosophy*

My focused concern is with two aspects of Plato's philosophy and then his physics and how they're related. I'll leave his modeling in astronomy to Chapter 3 when I will preview all of the Greek astronomy at once, but I'll consider his overall approach to astronomy here. Of concern then (and now) are Plato's Epistemology—what does it mean to know something (from the *Meno* and *Phaedo*), his Metaphysics—what is the nature of reality (from *Phaedo*, *Parmenides*, and *Republic*), and his physics (from *Republic*, *Timaeus* and Book X of the *Laws*).

2.1.1 What Is True Knowledge?

Plato was deeply influenced by our Parmenides Problem and took this on with a study of the broader question of what actually constitutes true knowledge. He thought deeply about this and his conclusions became grist for philosophical mills for the next 2500 years.[2] He decided that there are two hallmarks to knowing: that knowledge should be infallible and that it should be "of something that is." Typical was the exchange between Socrates and the 16 year old Theaetetus in the dialogue by that name. Socrates teases out of the boy his ideas of four kinds of knowledge, and demolishes every one of them. First up, what do we learn by *perception* as a source of knowledge? That's dispatched by Socrates, perception is infallible (since your internal evaluation of what you perceive is true to you), but perception is incapable of demonstrating that the objects of perception actually exist. So it fails on the second hallmark. Second up is *belief* as a source of knowledge? That results in a blistering dissertation on subjectivity. And, finally, third up is "true belief." Naive belief and even true belief are fallible, so failing on the first hallmark. Three outs. But what about *belief with a reason* to hold that belief, what in the context of *Theaetetus* is sometimes called "true belief plus an account" or, "Justified True Belief"? This is sometimes incorrectly described as Plato's own theory of knowledge, but Socrates makes hash of JTB and leaves the question in an unsatisfying state. Let's look at a couple of examples.

J+T+B was considered to be among the best efforts into nearly the present day and relies on the three aspects memorialized in its name. The B: one can't claim knowledge about something you don't believe. (I read that my calendar reports that today is Tuesday, but I believe it's Monday certainly doesn't qualify as knowledge of Monday.) The T: the fact must be true (if the fact is not true, then you cannot be

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[2] I'm grateful to philosopher, Professor Harold I. Brown for important discussions on this complex topic in Platonic philosophy.

said to have knowledge of it.) The J: whatever you claim about the fact, you need to be able to justify it.

Consider this claim: It is 3 o'clock, I believe it's 3 o'clock, because I looked at my watch and see that time displayed. B, T, and J are all in play and this seems a reasonable example of knowledge.

> But there are holes and weaknesses. What about instead of that J, how about J2: It is 3 o'clock, I believe it's 3 o'clock, because 3 is my favorite number. I'm right, since it really is 3 o'clock but that justification is silly and certainly doesn't qualify as knowledge of the time. How about this, J3: It is 3 o'clock, I believe it's 3 o'clock, because I looked at my watch and see that time displayed. But...I didn't know that my watch was broken and had stopped at precisely 3 o'clock. So it was just luck that my reading corresponded to the right time. So that's hard to accept as knowledge. In fact, it was only in 1963 that Edmund Gettier apparently found counterexamples to JTB which are now called "Gettier Cases."

Clearly Justification is the rub and many efforts have tried to turn J+T+B in to J+T+B+X. . . where X is some thing added to take care of the Gettier Cases. It's an ongoing problem. For scientific claims of knowledge, sometimes Justification weaknesses turn on problems with observation and even the senses so we're right back to the Parmenides Problem.

Plato had an answer and it turns out to be more than a theory of knowledge, but also a theory of what's real: fixing epistemological problems resulting in metaphysical commitments.

True knowledge for Plato can only come from permanent, unchanging things. Thanks, Parmenides. If something is true, it must be so forever, which means that it was never not true, nor will it ever become not true. He falls squarely in the Being camp, as opposed to the Becoming camp.

> Notice how this demand of permanence as the qualifying feature of true knowledge is an **unquestioned commitment**. There's no room for degrees of knowing—we all know things with varying levels of trust and this is especially true in science where not being able to question an assertion is actually now the very definition of "unscientific." I think that their insistence on permanence is a function of their being impressed with geometry and the fact that it was very early days in the brand new field of epistemology.

Plato differed from ardent Eleatics like Parmenides by insisting that knowledge is indeed possible, but there's a catch.

He proposed after *Theaetetus* that there are two worlds:

- The world of the Forms.
- The world of the senses.

2.1.2 The Forms

Plato's theory of the Forms is one of the most difficult ideas in philosophy but comprehending it is critical for an understanding of his projects, *but also for appreciating physics.* He gives abstract concepts an existence of their own and a job to do with a consequence that sort of grates on you.

Take high school (please): if you ever took a geometry class you were given a set of pieces out of which you could create new pieces with just a ruler and a compass. These pieces include things like points with no extent and lines with no thickness. You manipulated and proved theorems about perfect triangles and perfect circles. Let's focus on that last one.

Think of all of the "circular" things that you come in contact with in your everyday life. Coins, dials on appliances, buttons on your shirt, a camera lens, a cookie, maybe a rendition of something circular in an image or on a screen. You know that none of these circles are the circles of your geometry class. But all of these circular things share their property of *circularity*. They may woefully miss in the perfection of that high school circle in your mind—but through thinking about it, you know that your Oreo is circular, almost.

Plato would say that that unique abstract circle *actually exists* as a "Form." That there's a kind of reality—a realm— that's different from the reality that you think of when you drop that circular plate in the kitchen. That abstract realm is where the Forms exist.

That high school geometry-circle is such an abstract notion. But you can grasp that reality, you can apply it, engineers can use it, and you recognize it when you see it...only in your mind. But try an experiment: construct the best circular thing that you can and measure its diameter in a hundred points around the center at micron precision— while your rendition may be a good one, it's not that abstract high school one, is it. The Form of a circle is aspirational but can't be studied by measuring regular-life circular things, rather it can only be brought to life through your intellect. The Form of a circle has always been there (circles were not born) and that realm is outside of space and time. Can you get on board with abstract things being real?

Maybe Plato's assignment of "real" to mathematical abstractions is a little less odd than at first glance. But he went further than geometry and you might have experience with non-mathematical abstractions. Here's one: "We hold these truths to be self-evident, that all men are created equal..." What is a self-evident truth? If it's a "truth" then questioning it is a waste of effort, it's permanent in a Parmenides sort of way. If an idea is self-evident, then in some sense it's always been there, imprinted in us, while apparently accessible, but at the same time, distant.

You can't hold such a truth in your hand and you know it's not universal in our everyday life since, "all men are created equal" is untestable since the ones we know are tall, some are smart, and yes, some are disadvantaged. That they're "equal" is an abstraction—again, an aspirational idea of perfection—that we can hold in our minds but we know won't be realized in "our world." But a nation of 300 million "Platonists" swears by that truth.

What about realities outside of our plate-dropping reality? If one is a Christian, then you've been brought up to believe in such a reality—heaven (and hell) are outside of our everyday lives.

When I go to a furniture store I see hundreds of sofas. They're all different, but they all share. . . a "sofa-ness." They're all *participating (sharing) in the Form of the Sofa* which I can (only) know of in my mind. It's a perfect sofa.

With the forms, the Parmenides Problem is dealt with in a brand new way: there is a world of Being and a world of Becoming and they are connected, but in a hierarchical way. And, it's not just living room furniture that has Forms. There is a Form for everything: even Justice, Virtue, Beauty, and the Good. . . the latter of which is somehow a super Form.

2.1.3 The Republic

Plato's contribution to science is not any particular theory or practice, but as (**gerl70**) suggests it is more his philosophy of science that we value. This is laid out most explicitly in *Republic*, probably his most famous book, ostensibly a treatise on politics and good governance. It's here where he describes how a city should be ruled, certainly not by popular election, but by the training of a special category of people bred and educated in order to be rulers, the philosopher-kings, the guardians. Their lives would be scripted from early ages, living communally, and essentially the pool of potential candidates for leadership. Their educations would be scripted as well, relying on an intensive study of mathematics to create a habit of mind. The goal is for them to be completely comfortable with the most abstract concepts, including Justice and what's Good. Learning mathematics is a primary route to that appreciation. *Republic* includes a few analogies to try to get Plato's point across. Two are relevant for physics.

Analogy of the Divided Line.

Along with the Allegory of the Cave, the "Analogy of the Divided Line" is important for Plato and I think important for physics—as Galileo and modern physics will eventually enlighten for us. A rendition of the Divided Line is in Figure 2.1. What we can know is a hierarchy, from muddled to perfectly clear and divides into two broad "realms," one representing our *Becoming* world—The Visible Realm—which we occupy in everyday life, and the other representing the *Being* world—The Intelligible Realm—which is outside of space and time and only recognized through thought.

The Becoming realm is broken into two levels of which the objects of the first, and lowest segment are shadows and illusions of objects in our experience. The shaky knowledge we have about them are mere illusion and dreams. The objects of the

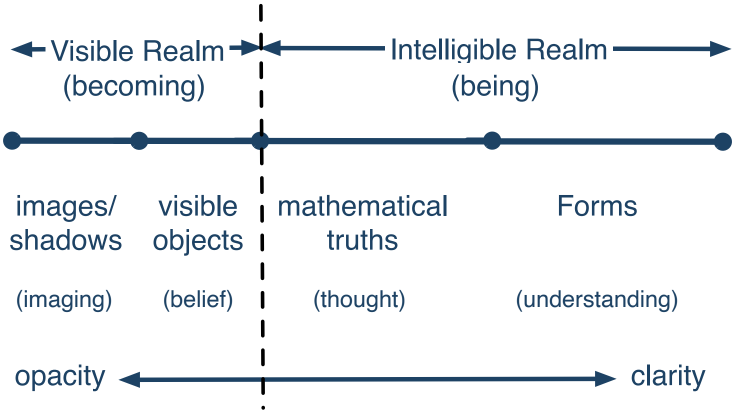

Figure 2.1: The line represents a kind of knowledge-hierarchy, from unclear to perfectly clear.

second stage are actual, everyday objects themselves, and the knowledge we have about them are opinion and belief gleaned through our (untrustworthy) senses. Taken together these two stages constitute our knowledge of our everyday world, where things change: the Visible Realm is where we we use our senses and dreams to navigate our lives.

The Intelligible Realm is only accessible through thought and reason and is likewise divided into two more sophisticated segments. The first of these includes knowledge gained through mathematics and hypotheticals (think high school geometry) about which we have knowledge through reasoning. And finally, the highest segment of the Intelligible Realm is of the Forms, the pinnacle of clarity, "beyond hypothesis" which is aspirational, not easily realizable.

Earlier I opined that "degrees of knowing" is a more modern way of thinking and the Divided Line actually sneaks up on just that. As we'll see when we study Galileo, there is a realm of the universe which is very hard to observe (on Earth) but which is our goal when we theorize about nature. So I'm not quite willing to pass this off as silly, while at the same time I don't agree with the realm of the Forms as an ethereal parallel universe that we cannot access but through rationality. Stay tuned.

Allegory of the Cave.

He famously tries to work out more of these distinctions in the *Republic* with the famous "Allegory of the Cave" and in the *Meno* with the idea of" Reminiscence." In the former, prisoners in a dark cave are shackled to the ground facing a wall. They can only look straight ahead and what they see are shadows of objects and puppets that are held in front of a fire behind them so that they project on the wall. If they see a sofa on the wall, it's because the Form of the sofa, which is behind them and out of sight, is projected as a shadow of the real Sofa in front of the fire.

Now, if one of the prisoners escapes her bonds and looks around she'll see the fire and the contrived circumstances. The light from the fire would hurt and she'd want to go back to her former spot. But if she were dragged out of the cave and into the sun, she's blinded but slowly she'd look around her and realize that there are actual things in the world and not just shadows. Notice that in the Allegory, she's moving from left to right in the Divided Line in Figure 2.1. She ventures back into the cave and tries to describe that true reality to her still captured colleagues. But in the dark she'd not see well and the prisoners would not allow her to persuade them to follow her into the sun since it apparently takes away one's sight. Plato even worries that the prisoners might kill the one who escaped.

Obviously, Plato is describing the daunting project that he's taken on as the enlightened former prisoner trying to explain what's Real and True to everyday people who don't want to accept it. The similarities to Neo's trip out of the realm of perceptions and into the realm of the real is not an accident as the movie *The Matrix* is full of philosophical allegories, and the Cave is one of them.

What we can learn in the realm of the Forms is true knowledge and a goal of mastering philosophy. What we can know of the world of appearances is simply opinion. The Forms inspired many in the centuries to follow, from Neo-Platonic Christian images to modern science. We'll come back to them in Galileo where finally, properly characterizing MOTION begins. By the way, Plato despised art. A painting of a mountain as nothing but an imitation (the painting) of an imitation (a sensible, actual example mountain) of the form of Mountain, which is the only real thing.

2.1.4 Mathematics For Plato from Republic

Plato's experience in Italy wasn't limited to a failed experiment in his theory of governance, but began as a deliberate project to study with Pythagoreans. Pythagoras had been gone for a century by that point, but two schools grew up around his legacy. The *acusmatici* viewed themselves as the guarantors of Pythagoras' the man's legacy as a complete system. Not only his mathematics, but the other aspects of the Brotherhood were preserved and defended without expansion or elaboration. On the other side were the *mathematici* who bought into reverence for the man, but intentionally expanded the mathematics to new areas of research, an unwelcome sin in the eyes of the *acusmatici* who eventually died out.

Recall that Plato and Aristotle probably learned most of Pythagoreanism from Philolaus, but Plato's mathematical inclinations came from a contemporary, one of the mathematici that Plato befriended and learned from, **Archytas of Tarentum** (ca −420 to −355) who is one of our characters in Chapter 3.3. Our title character in the next chapter is **Eudoxus of Cnidus** (−408 to −355), a student of Archytas and the most significant mathematician before Archimedes. Both influenced Plato and Aristotle's cosmology, and that subject kicked off two millennia of modeling and eventually, dogma. The mathematics required in the guardians' education came from Architas, arithmetic, geometry, astronomy, and harmonics. Plato didn't fully agree and added a fifth subject, solid geometry.

This is very much physics as we'll see. Maybe you can begin to understand Plato's elevation of mathematics—in the Greek life of his day, geometry and proportions—to the point of his famous sign above the door, "Let no one who is not a geometer enter." (Well, that sign only crops up in the 4th century AD, so it's probably a myth.) Geometry is venerated by Plato and all who follow for centuries.

This is hit directly in *Republic* where Socrates extracts from Glaucon[3] the reasoning behind requiring astronomy for guardian training. As usual, Socrates/Plato starts out with a theme which in the course of explaining it, evolves into a matter of serious philosophical interest. Glaucon tries to guess at why astronomy is important. Maybe because it's useful for recognizing seasons, or timing agricultural events. Practical things. That doesn't go over well and so he tries again: maybe astronomy is "good for the soul"... that looking at they sky takes us away from looking at everyday things. Again, not productive for Socrates. Here's where geometry comes in and where Plato earns an uncertain reputation for suggesting that "armchair astronomy" is the only way to go: doing astronomy without ever looking at the stars. Here's how I interpret this:

Back to the literal drawing board: Take out a ruler and the sharpest pencil you have and carefully draw the most precise triangle you can create and now get the best protractor you can buy and try to verify that the interior angles of that triangle all add up to 180°. No matter how careful you are, you'll fail to perfectly measure $180.000...^\circ$. In fact, Socrates/Plato would tell you to not bother since studying an everyday triangle won't help. The perfect 180° is in your head and its truth is one of reasoning and geometrical proof.

Socrates/Plato suggest that the same is true for astronomy.

> "We shall therefore treat astronomy, like geometry, as setting us problems for solution", I said, "and ignore the visible heavens, if we want to make a genuine study of the subject and use it to convert the mind's natural intelligence to a useful purpose." Socrates/Plato, *Republic*.

He says that you can look at the stars but discerning their actual motions cannot be done by measuring the apparent, but flawed motions of the imperfect stars and planets. You can only understand their motions by reasoning; astronomy without looking up! Like the triangle, you might get hints from the world of Becoming, but only through reasoning can you learn what the stars and planets do in the perfect world of Being.

Here is **another unquestioned commitment** by Plato. That the stars and planets would necessarily execute perfect motion is an assumption. Again, this is the very earliest days of astronomy and philosophy and it's built on a variety of prejudices.

Plato's "Doctrine of Reminiscence" is another idea that comes from the Forms. In

[3] Possibly, Plato's older half-brother's name.

the *Meno* Socrates demonstrates that a slave boy actually knows geometrical proofs without knowing that he knows them! By asking questions, in his Socrates-way. In the *Meno* the protagonist, Meno (a real, young aristocrat) asks Socrates if Virtue can be taught and of course Socrates begins by asking the young man to define what Virtue is and then dismembers his multiple attempts at an answer. The scene degenerates into Meno now becoming frazzled and paralyzed as the discussion evolves. As often happens more than the problem at hand emerges, including what's called "Meno's Paradox": the realization that if you know something, you don't need to ask about it but if you don't know it, then you don't know enough to ask. Of course this all leaves everyone unsatisfied. (It's surprising to me that anyone ever wanted to talk to Socrates.)

The discussion turns to a religious view that the soul has always existed and will exist after we die and that the soul knows all that there is to know before and after and therefore, we already know everything. . . we've just forgotten it. He then proceeds to demonstrate this idea by asking a slave boy the geometrical proof of how to double the area of a square. By asking him successive questions, he pulls the proof out of the boy. (You can see the proof in Technical Appendix A.2.1.

> In school, did you ever successfully work out a proof in geometry or mathematics? Don't you do a little victory dance inside, maybe with a knowing nod — Aha!— that solution seems like it was there all along and all you did was *reveal* it. That you almost *remembered it*. This is the basic characteristic of Deductive Reasoning. It doesn't lead to anything new, but reinforces—-(or recalls, suggests Plato)—that something that was already in the premises. I know I've had that feeling and I can understand why Plato chose a geometric proof to illustrate his idea, which is broader than just math for him, of remembrance. What Plato was really after was the fact that the Form of that geometric proof was there all along, in that Intelligible Realm, all the time.

2.1.4.1 The Soul

The "Soul" is a very Greek idea which functions at multiple levels for Plato, in one dialogue, he assigns three separate jobs to the Soul. For our purposes, he's impressed with the idea that some things are inanimate — like a rock — and that somethings appear to be animate. The very word "animate" gives you a sense of what he thought might be the distinguishing feature between animate objects: they can they move on their own. So in some ways, this is a question of MOTION ON THE EARTH (but he extends it to MOTION IN THE HEAVENS). He found the Soul a useful cause for all things that can move of their own accord — he would speak of "self-motion" — as imbued with Soul. It's not only humans, but birds, flowers, even planets which appear to be able to execute locomotion on their own that enjoy their very own Soul. We'll see that this idea actually figures into some of his astronomy, so in a backdoor sort of way. . . this is an example of MOTION BY THE EARTH! It is this very talented Soul that causes self-motion among animate objects, but also persists before and after death. We get a glimpse of the all-knowing Soul when we do a mathematical deduction, as Socrates illustrated with the slave boy.

2.1.5 Timaeus

Boy, the European medievals must have been confused about Plato. Until the early 12th century, the only Latin translation of any of his works was just one: *Timeaus*. It's notoriously difficult, convoluted, and ripe for repackaging by the "neo-Platonists" up to Augustine. In this difficult late dialogue, *Timaeus* the title character, a fictional Greek statesman and scientist from southern Italy (ah, as we'll see, surely a Pythagorean), who, at yet another get-together, is encouraged by Socrates to tell the origins story of the universe. A sort of Greek Carl Sagan. *Timaeus* is less a dialogue than a monologue and it covers a lot of ground without Socrates being his usual, obnoxious self. Obviously, Plato had a lot on his mind in this book.

Plato was so enamored of mathematics that through Timaeus' voice, he builds what he calls a "likely story" of cosmology by mixing geometricized ideas of the atomists with a relentlessly Pythagorean numerology (that he learned directly from Archytas?), a major focus in Chapter 3.3.

The universe was assembled (not created) through the actions of a "Craftsman"[4] who builds everything—animals, planets, stars—from a blueprint of eternal ideas—surely, the Forms and does so using existing materials at hand. He's an artisan, more than just a laborer and less than a creative deity.

The dialog begins with Socrates counting, "One, two, three,. . . " a portending of the strange, mystical use of numbers as the Craftsman does his job. I'll reserve the astronomy part of Timaeus for Chapter 3 and make reference here to only those parts of the dialogue that overlap with our project. That leaves most of *Timaeus* untouched.

The story begins with fables about Athens of 9000 years previously, among which are its war with Atlantis and the idea that Earth is periodically destroyed, erasing memories for everyone. . . but somehow, not the Egyptians. This leads to a discussion of how the universe began. Timaeus asks (with Parmenides looking over his shoulder?):

> What is that which *always is and has no becoming*, and what is that which is *always becoming and never is*? That which is apprehended by intelligence and reason is always in the same state, but that which is conceived by opinion with the help of sensation and without reason is always in a process of becoming and perishing and never really is." (emphasis, mine) Plato, *Timaeus*

Of course, this is a reference to the Forms ("always is and has no becoming") in contrast to particulars and everyday things ("process of becoming and perishing and never really is"). In sympathy to Parmenides' poem, Timaeus also tells about both kinds of knowledge. This is his stepping off point to the fact that the universe has "become" and so was not always around which implies a creation act or a cause,

[4] In Greek, the "Demiurge."

or in any case, a creator. That's the Craftsman's job who follows a plan which is an aspirational blueprint.

The universe isn't created out of nothingness (more Parmenides?) but rather the Craftsman works with the material at hand using the Forms as a blueprint and fashions it into an Earth-centric ("geocentric") model, which we'll talk about in the next chapter. Plato leaves the impression that the Craftsman does the best that he can — a best-effort universe! There is a difficult overall purposefulness and expectation that the Craftsman is ". . . .greatest and best and fairest and most perfect." This is the best possible world.

Suffice it to say that the Sun, Moon, and planets all take their familiar places according to a mathematical (even musical—Pythagoras, again) format and that Time itself is created along with the planets. In fact the motions of those most-nearly-perfect celestial bodies is the cause of time. The ancients told the days, months, and years by the motions of the Sun, planets, and stars and so it's maybe not a surprise that Time and those objects have a causal relationship to one another.

The Craftsman isn't omnipotent and is restricted to Empedocles' four elements — the materials at hand. The *Timaeus* outlines the way in which Fire, Water, Air, and Earth go together (again, in proportion) by assigning them solid shapes: Fire is made of tetrahedrons, air is made of octahedrons, water is made of icosahedrons, and finally Earth is made of cubes. The solids themselves are made of two kinds of constituent triangles; the isosceles and scalene triangles. The former is what results from cutting a square into two parts along diagonals and the latter is a triangle in which the hypothenuse is twice the length of the shortest side. Two scalene triangles, side by side, attaching the long sides. . . makes an isosceles triangle. So the atoms (in a modern sense) of the four elements are made of two elementary, triangular constituents (like modern atoms are made of electrons and nuclei): tetrahedrons (4 faces of equilateral triangles), octahedrons (8 faces of equilateral triangles), icosahedrons (20 faces of equilateral triangles), and cubes (12 equilateral faces).

Water then could be broken down into fire and air as an icosahedron can be decomposed into two octahedrons of air and one tetrahedron of fire. In fact, that water evaporates can be modeled in his scheme by noting that two water solids can geometrically be reduced to five air solids. He's used up 4 of the 5 known three dimensional solid forms, historically (but inaccurately) called the Platonic Solids. So, having bought into a theory, he did what any theoretical physicist would do. If the solids are fundamental and only 4 of them seem to immediately come to good use, then there must above a job for the fifth shape, the dodecahedron, and he assigned that to some measure of the universe itself as it has so many faces, it's close to being a sphere?

Plato refers to a form of air as "...the most translucent kind which is called by the name of aether...," but he sticks to the four elements of Empedocles. Aristotle does something similar, but with a twist. There is some ambiguity among the terms "aether," "quintessence," and "ether." In this book I'll use the term "ether" to refer the 19th century substance that all thought "carried" the propagation of light waves throughout the universe. "Aether" and "quintessence" are Greek references and are often used interchangeably. In Chapter 3 I'll use "aether" to refer to Aristotle's fifth element.

So, Plato is revealing his Pythagorean biases: The world is geometry—pure, abstract form. But he's just getting started as his Pythagoreanism knows no bounds as we'll see when we consider his cosmology in Chapter 3!

Platonism is not just confined to philosophy or mathematics. The Medici family in Renaissance Florence was instrumental in reacquiring Greek philosophical texts from the Byzantine empire by importing Greek-speaking academics. They set up a school dedicated to Greek philosophy and a school for the children of the court. One of those children was a ward of Lorenzo and would have learned of this approach to the world. So when Michelangelo later noted, "I saw the angel in the marble and carved until I set him free" he was expressing a very Platonic idea that he absorbed as a young ward of Lorenzo di Piero de' Medici, modestly, Lorenzo the Magnificent.

2.1.6 Platonic Legacy

We've covered a lot, but only a little of the large subject that is Plato. I view the history of physics as ebbing and flowing between Plato's and Aristotle's influence and out of that I have concluded that our recognizable scientific discipline—my life's work—didn't happen until the history of physics swerved in the direction toward Plato and away from Aristotle. So our discussion of the Forms and how the mathematical picture is illuminated by his conclusion that there are two sorts of reality is necessary in order to tell the whole story of MOTION. There is one negative legacy that's more complicated than it's normally presented: the idea of "Saving the Phenomenon," or "Appearances." This is the statement that is used to assign this idea to him:

> "This was the method I adopted: I first assumed some principle, which I judged to be the strongest, and then I affirmed as true whatever seemed to agree with this, whether relating to the cause or to anything else; and that which disagreed I regarded as untrue." Plato, *Pheado*

It's more complicated than that and people still argue about it. I suspect that there are four reasons that this seems to lead to that direction:

1. Aristotle seems to be critical of that way of thinking (see his statement from *On the Heavens* below on page 2.2)
2. There's the "armchair astronomy" admonition by Socrates in *Republic*, described above.

3. There's the fact that his student/colleague Eudoxus takes on the task of describing the motion of celestial bodies using only circles. This will be discussed in the next chapter.
4. And there's this quotation from *Phaedo*.

The person that was most responsible for making this direct connection to Plato was the neoPlatonist, Simplicius, who flourished in the 6th century (CE) (He reported that Plato proposed the problem of finding "by the supposition of what uniform, circular, and ordered motions the appearances of planetary movements could be saved.")

In any case, this methodology had legs. Can you see how *un*scientific this is? First create the theory, and then interpret the facts only to support the theory. This is especially the case in his astronomy.

As I've hinted, his positive legacy is critical and abstract. His ideas were reformulated a number of times and Neo-Platonism was a pre-medieval version that eventually found its way into Catholic Church doctrine, much through Augustine, only to be reassessed centuries later.

What can't be overstated is the influence that Plato had on our project of describing the universe using mathematics: "Platonism" is an enduring feature of fundamental physics. Johannes Kepler in the 16th century was among the first truly Platonic (or even Pythagorean) scientists and as I joked earlier, my particle physics subdiscipline is very Platonic.

Notice that MOTION has not been a feature of my discussion of Plato. In part, we think of Plato's ideas about motion as focused on astronomical topics, which we'll cover later in this chapter. But also his ideas as expressed in *Timaeus* (and to some extent in the *Laws*) are so esoteric as to be mostly unintelligible. There we learn that the Soul is responsible in part for "self-motion." It's all very unsatisfying.

"Unsatisfying" is a good stepping-off point as we consider Aristotle and his huge negative impact on physics. For someone so wrong, it's ironic that we can't ignore him.

2.2 Act VI A Little Bit of Aristotle

> "Aristotle is a Foal. When a foal has had enough milk, it's known that it kicks its mother." ascribed to Plato

While Plato's practical impact on physics was limited to abstract and esoteric notions, not so with **Aristotle of Stagira** (−384 to −322) an even bigger subject. He was a systems builder with practicality and abstraction as joint projects. The extent of his intellectual reach was incredible and not only did he further philosophical ideas, he invented whole fields of science and philosophy.

He was born in Stagira, near Macedonia north of Greece and was connected to

Macedonian royalty as the son of the king's physician. He emigrated to Greece to study at Plato's Academy at the age of 17... and then stayed for almost 20 years. While he was in residence, probably beginning his writing, the Macedonian King Philip II began his conquest of northern Greek cities, including Athens... which came under his control through concession, and only limited conflict. (Set the context with the timeline in Figure 1.2 on page 20.)

When Plato died in −348, Aristotle went to Assus in the northwestern area of modern-day Turkey, married, and began (or continued) an impressive series of biological, marine biological, and zoological researches which he wrote about in *The History of Animals* and *On the Parts of Animals*. He was a details-person and described animals and insects with minute detail through dissection and description, beginning the classification exercise that established the whole science of biology for centuries. He classified more than 500 different species into genus and species forming categories of likeness and habit of mammals, fish, reptiles, and insects. It was here that he established his insistence on observation as the source of knowledge, an evolution away from Plato that was obviously severe. Think of his approach as like taking a deck of cards that's all swirled together on a table, and ordering the them all by identifying and sorting for like features—suit, color, and number. That kind of organization came naturally to Aristotle, it's very modern, and it seems to have first been apparent to him as a scientific practice.

His range was remarkable, covering: Law, physical science, psychology, natural science, philosophy, logic, ethics, and the arts. Words that we have from him include: energy, dynamic, induction, demonstration, substance, attribute, essence, property, accident, category, topic, proposition, universal... His metaphysics informed the development of his science and confused the awakening Western world from about 1100 to 1600. And, everything was a part of his system, and so abandoning one piece that might not make sense would bring the whole system down. It was a philosophical game of Jenga. In particular, his astronomy, and especially his physics, didn't make sense and we'll see that the Medievals knew that it didn't make sense. But selectively adjusting it seemed impossible.

One positive thing, if only his followers had preserved it: we have Aristotle to thank for dampening enthusiasm for the unwelcome Platonic idea of "Saving the Phenomena":

> "... speaking of phenomena, they say things that do not agree with the phenomena... They are so fond of their first principles that they seem to behave like those who defend theses in dialectical arguments; for they accept any consequence, thinking they have true principles—as though principles should not be judged by their consequences... " Aristotle, *On the Heavens*

We have three Aristotelian issues to consider for our narrow project which together only sample a small sliver of his whole universe: what is real, how does change happen, and his physical science.

2.2.1 Aristotle and What's Real and What's Knowledge?

Unlike Plato, Aristotle rejected the idea of a super-sensible realm housing the ethereal Forms. He had a different job for his Form that linked it with actual substance, here on Earth, closer to our idea of the form of a physical object. His focus—which was refreshing after the Parmenides Problem and now the Plato Problem—was on *individual things* which we learn about through a personal experience with the world, not through some intellectual abstraction. What's real for him are *particular objects*.

> "If we did not perceive anything we would not learn or understand anything." Aristotle, *On the Soul*

Like I said, refreshing.

Substance—stuff—and Form work together to make the world. The oft-used metaphor of a house is instructive. In order to make a house you need stuff—wood, nails, and so on—and a plan, an organizing principle. Substance and Form. An individual thing is then matter which has been given a form and you can't separate them. An individual thing must have both.

For Aristotle, perceived facts are the necessary ingredients for knowledge. We organize them in our memories, looking for commonalities and differences. We categorize our facts into bins of like and unlike with relationships among them. We have an individual perception of things, collect facts, ruminate on them by comparing in our memory with our internal database, and categorize. This is classical Empiricism, as opposed to Plato's classical Rationalism. So far, so good. (Think about that deck of cards, now abstracted as a philosophical goal.)

2.2.2 Change and Cause

But we still can't get away from the Parmenides Problem and Aristotle also did battle with change and permanence. Let's race through how he thought about change and how it functioned in his physics.

For him, Change relieves a... tension. An actual thing, what **is**, has within it the potential to become something new. As long as it's not in that newer state—it's "deprived" —and it is obligated to go there. Inevitably. So everything is also in a Hericlitean flux, but in a very particular and interesting way. In sympathy, perhaps, with Parmenides, in order for something to change into something else, it had to **be** in the first place and taking that all the way back, takes him into an abstract place where there needed to have been an original Unmoved Mover. We'll not follow that line of thought.

What's important about change for Aristotle, which fits into his bigger system, is that in order to acquire knowledge of something that changes means you can identify the Cause of change. Because: *all change must be caused* and what can be caused comes from within a set of Aristotelian "Categories" (of being). The ten Categories is a complicated idea and we'll skim. They are: substance, quality, quantity, relation, time, place, position, state, activity, and passivity — his complete set of predicates that can be assigned in a statement. For example, what can you say about Galileo:

- Galileo was human (substance)
- Galileo was smart (quality)
- Galileo was 5 feet tall (quantity)
- Galileo was older than Kepler (relation)
- Galileo lived during the 16th and 17th centuries (time)
- Galileo lived in Florence (place)
- Galileo sometimes sat at his desk (position)
- Galileo sometimes wore shoes (state)
- Galileo sometimes wrote with a pen (activity)
- Galileo was sometimes ill (passivity)

A particular substance must be all of these things in order to be a thing. In order to exist. Like I said, you have to be impressed with Aristotle's ability to take a complex topic and break it into its constituents. Remember, he invented Logic.

> " Nature is a principle of motion and change, and it is the subject of our inquiry. We must therefore see that we understand what motion is; for if it were unknown, nature too would be unknown." Aristotle, *Physics*

Substances have "motions" but not the kind you're thinking of. They're very Greek motions and can be quite abstract. For Aristotle, *motion is anything that goes to* something. In this change a substance remains a substance, but Form adjusts, characterizing the natural evolution of a state in which a goal is not achieved into a state in which a goal is achieved. And that idea of a "goal" is very important and in part, where Aristotle's physics goes astray. So the substance of a seed changes as it evolves into a flower. But the form of the seed and its various guises changes. Stay with me.

Motions can be of any of the Categories of being, but usually are among just three of them:

- change of quality
- change of quantity
- change of place

For example:

- Galileo changed from a boy to a man. That's a change of quality.
- Galileo changed from a person who weighed 50 pounds to a person who weighed 150 pounds. That's a change of quantity.
- Galileo moved from Padua to Florence. That's a change of place.

That last one, a change of place, is our modern idea of "motion" which he called "locomotion." But for him, locomotion is no more fundamental than any other kind of motion and that's very Greek. But, again, he's thought deeply and by accident, all three kinds of motion have examples today:

- Modern Change of Place: We tend to think of locomotion as the only one of his categories to apply to change in physics: objects moving from this place to that place, during some time.

That's familiar. But two of his other "motions" have modern examples which he would not have known of:

- Modern Change of Quality: A phase transition like water boiling or freezing could be considered a change of quality.
- Modern Change of Quantity: Aristotle could not have imagined a nuclear or particle decay from one thing into three different things, like the decay of a neutron into a proton, electron, and neutrino.

As for goals, it's easiest to think of the nature of something and that involves potentiality and deprivation. An acorn becomes a oak tree. An acorn does not become a Galileo, so it has within it the potential only to be an oak from the beginning. That inevitability also is universal and directed and that even becomes an argument against infinity since there is no such thing as unrealized or unconstrained potential.

> "It is not what has nothing outside that is infinite, but what always has something outside it." Aristotle, *Physics*

Now we know what properties a thing must have in order to exist and we know what kinds of change can happen. Again, to have knowledge of a change one must understand the causes: in fact, four causes. They are the material cause, the efficient cause, the formal cause, and the final cause.

Take a that house:

- The material cause of the house is the wood, nails, and so on.
- The efficient cause of the house is the action of the carpenter.
- The formal cause of the house is the blueprint in the mind of the carpenter.
- The final cause of the house is the purpose for which it was made.

There is sometimes a discussion about whether these function as causation or explanation. Are they the four "becauses"? In any case, the last one of them is problematic for physics as the notion that everything moves for a purpose (that "goal" again) doesn't work in modern terms. This is called "teleological." (One can imagine an argument for Aristotle that there is some teleological logic to how plants and animals "move" from one kind to another... seeds to plants, kittens to cats, and so on.) Of the four (and there's a lot more detail in Aristotle than just enumerating them), Efficient Cause comes the closest to a modern physics cause. That's splitting hairs!

2.2.3 Aristotle's Physics

Aristotle inherited his ontology (the philosophy of being) from his teacher, who inherited it from Empedocles. That is the four elements of earth, air, fire, and water are supplemented by one more, "aether" which is outside of the earth-bound region of the universe. Like the reactions to Parmenides, Aristotle envisions "stuff" as mixtures of the four elements. But he goes further than just classification, as their makeup, Causes, and Categories all feed into his explanation for the sort of motion that we think of. So understanding locomotion is intimately tied to the entirety of the Aristotelean system.

With respect to our familiar MOTION, he was very much an empiricist and locomotion in particular fits his overall philosophy. Watch a high kick of a soccer ball or a towering home run in baseball or a shot in the shot-put. The projectile will race to the top of its trajectory and then appear to fall steeper and faster than its rise. Drop a feather and a crumpled up piece of paper and a metal key. Will they hit the ground at the same time?

In each of these everyday examples it seems like the heavier object will hit the ground first. That fits his philosophy, or maybe his philosophy grew from watching things fall since the heavier an object is, the more deprived it is of its most natural place: the Earth. So any object seeks its place by virtue of the amount of earthiness it has in its composition. Heaviness is an attribute and the natural motion associated with heaviness is down, toward the center of the Earth. *Lightness is also an attribute* for Aristotle (for us, that's just less heaviness). Natural motion for a Light object is up, toward the sky. So, below the orbit of the Moon, objects have two kinds of natural motion:

- Natural locomotion for heavy objects is down and natural motion for light objects is up. These Earth-bound motions — MOTION ON THE EARTH — both follow straight lines toward their preferred places. So firey things want to be at the edge of the Moon's orbit and earthy things want to be at the center of the universe (the Earth).

But MOTION IN THE HEAVENS of the Sun, Moon, planets, and stars don't move in straight lines and have no apparent pushing force, so they must be composed of different stuff from Earth, Water, Air, or Fire and have a different sort of natural motion:

- Cosmic objects are made of "aether" and have circular natural motion.

Like all motions, Earth-bound objects away from their natural places are deprived and realization of their potential is to . . . go there. Celestial objects move naturally in circles. To fulfill their essence.

There is another kind of locomotion which is un-natural, dubbed "violent," and for Aristotle what causes violent motion must be a contact force. So throwing a ball is violent and unnatural, since it's not directed down. When the ball is in contact with your hand, you're making it move. When it leaves your hand? Well, here Aristotle had trouble and everyone knew it. The contortions that he went through to explain projectiles are pretty contrived. But he was wedded to his system and in spite of his scorn for Plato's Saving the Phenomenon, he seems all about that here.

When the ball leaves your hand, it doesn't immediately head towards the center of the Earth. The medium of the air is critical in two ways:

1. The motion of the hand is (somehow) transferred to the air which (somehow) successively creates forces in steps. . . air moves the projectile, then another segment of air moves the projectile. . . and so on until the ability of the air to perform that critical contact-force job is used up. Somehow the forces of air meet some dissipative force. . . of the air(!), and it stops.
2. Then the object falls directly to the ground because the air stops it.

The air both moves it and stops it! Also, the projectile doesn't share both unnatural, forced motion and a component of natural, downward motion. There's a lot not to like about this. Even probably including Aristotle given his complicated explanation. Figure 2.2 is a 16th century depiction of Aristotle's projectile paths: straight line up, then straight line down.

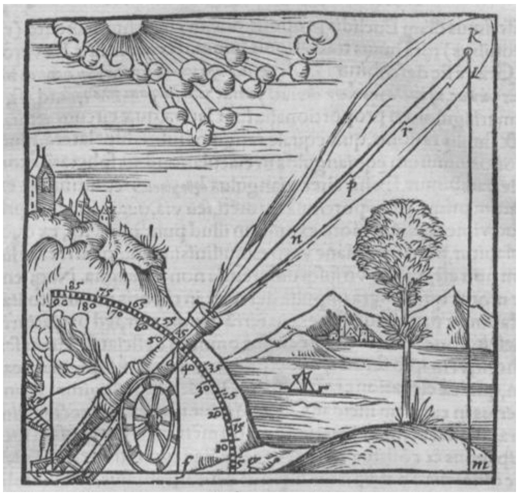

Figure 2.2: A drawing of Aristotlean projectile motion in a mathamatics text by Daniel Santbech in 1561.

Now he's not entirely consistent in his descriptions. In his *Physics*, he says:

> "Again, as it is, things thrown continue to move, though that which impelled them is no longer in contact with them, either because of"mutual replacement" as some say, or because the air which has been thrust forward thrusts them with a movement quicker than the motion by which the object thrown is carried to its proper place." Aristotle, *Physics*, IV, 8

Later in Book VIII he says:

> "Therefore, we must say that the original mover gives the power of being a mover. . . to air. . . naturally adapted for imparting and undergoing motion. . . The motion ceases when the motive force produced in one member of the consecutive series [of forces imparted by the air] is at each stage less, and it finally ceases when one member no longer causes the next member to be a mover but only causes it to be in motion.

> The motion of these last two—of the one as mover and of the other as moved—must cease simultaneously, and with this the whole motion ceases.

The first extract seems to make reference to an idea that's in *Thaeatus* called antiperistasis, in which Plato tries to explain respiration, suction, and falling bodies as displacing the air and back-filling it to avoid a vacuum. This either evolved too, or was also a suggestion by Aristotle that the air in front of a ball rushed around to the back and pushed the ball forward. I know. It makes no sense. The Medievals were very critical and modified the ideas.

Aristotle didn't know algebra, but we can most easily summarize his points with some simple proportions. The mathematical symbol for "proportional to" is $\sim$.

He would describe the locomotion of a projectile with these ideas:

- Heavier objects (made of more earth than other elements and so highly deprived of its natural place) would fall faster than light objects: $t \sim \frac{1}{W}$ where W is the weight, a stand-in for earthiness. Heavier objects would then fall faster than light objects —have a higher velocity.
- He had some sense of the resistance of air and so the velocity relates to weight and resistance as $v \sim \frac{W}{R}$ where R is some measure of the resistance that air or water or some medium asserts on the falling object.
- This leads to a convenient conclusion. If there is no resistance, then $R = 0$ and the speed that if falls would become infinite. But nothing can be infinite in Aristotle's philosophy, so there is no vacuum allowed... no medium with zero resistance.
- And finally, for violent motion, which requires an external force in contact with the object, $v \sim \frac{F}{R}$. No force, no speed. More force, more speed.

Each of the bullets describe exactly what you and I experience every day in a sport with a ball or just life. Aristotle is clearly a champion Empiricist.

There's more. If linear motion is the only natural motion then his Earth *must be stationary* otherwise, we'd would feel the effects of some tangential wind-force rotating the Earth. And we don't, so his Earth *does not rotate*. For objects in the heavens, since they move naturally but in circles, a different material is required, a fifth-element.

2.2.4 Summary of Aristotle and Locomotion

So to sum up the first real study of MOTION... ever.

1. MOTION ON THE EARTH is of two types:
    1. Natural motions are toward or away from the center of the Earth according to the degree of heaviness (among the four elements, Earth would dominate the others) or lightness (among the four elements, fire would

    dominate the others) that compose their substance. Natural motions are in straight lines. They represent the fulfillment of an object's potential.
    2. Unnatural, or violent motions are those which are not natural. They all require that an external force is applied throughout whatever trajectory a body experiences. Take away the force, and the motion would cease. These motions can be of any shape.
2. And MOTION BY THE EARTH?
    1. It's zero. The Earth is stationary because no forces can be detected that would be required to make it move. And, motion on the Earth doesn't suggest that the Earth is moving. Throw a ball up and it doesn't fall behind you, as he suggested would be the case if the Earth were moving. So he has an explanation as to why it must be stationary, but not a prediction. He's justifying his contention.
3. And MOTION IN THE HEAVENS?
    1. That motion is circular. Objects outside of the Moon's orbit are of an entirely different substance that what we experience: aether. Why? Since if they were of the same material that that of and on the Earth, its natural motion would be in straight lines.[5]

Aristotle's theories of MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS are relentlessly empirical: they are theories of what we all observe in our everyday lives. His theories of motion are wrong, relentlessly abstract, and hidebound to the rules by his overarching philosophy.

2.2.5 Plato and Aristotle on LIGHT

2.3 Plato and Aristotle, Today

2.3.1 Modern Day Platonists

> "I imagine that whenever the mind perceives a mathematical idea, it makes contact with Plato's world of mathematical concepts. . . When mathematicians communicate, this is made possible by each one having a direct route to truth, the consciousness of each being in a position to perceive mathematical truths directly, through this process of "seeing." ' Roger Penrose (1931-), theoretical physicist, Nobel Laureate

It's unlikely that anyone today would wonder about the application of Aristotelianism into the physics of MOTION ON THE EARTH, MOTION BY THE EARTH, nor MOTION IN THE HEAVENS but thousands of pages of writing (and links) have been devoted to the application of Platonism into modern physics, and especially in mathematics. Recall my party-question in the previous chapter: Is mathematics discovered or invented? Many mathematicians and physicists have concluded that it's discovered and that's the bumper-sticker version of modern Platonism: suitable for the 21st century.

In this *Plato and Aristotle, Today* section I'll describe a more modern version of

[5] some circular reasoning there, no pun intended

Platonism that might function in physics in two different aspects which I'll call "The Platonic Process in Physics" and "The Platonic Reality in Physics." It's about an evolved notion of the Forms.

2.3.2 The Platonic Process in Physics

The Forms were by far the Platonic idea with impact for all branches of philosophy, mathematics, and physics. His premise is that reality consists, not of only everyday stuff (that's the Ionian "monist" position that all of reality is made of matter) but that there is an additional reality-realm which consists of non-material entities outside of space and time. This is the premise of the movie *The Matrix* in which Morpheus gives Neo the choice of two pills: if he takes the blue pill, he's choosing to continue to live his life in an artificial but comfortable world in which we don't examine what's true and happily accept opinion as knowledge. If he takes the red pill, he's chosen the more difficult path: to live in the truth. The references to the *Allegory of the Cave* are obvious, but it's also the old biblical story of eating from the Tree of Knowledge.

Paying homage to Morpheus' red and blue pills, let's call our everyday, physical world, the **Blue World** (BW) and the ethereal, maybe more truthful world, the **Red World** (RW...in order to help us remember, think of it also as the "Real World."). And let me try to suggest that to be a modern physicist is to be partly a Platonist, working as if the BW and the RW both exist simultaneously. Stay with me.

Plato's classical RW is where the Forms reside in which they had two broad characteristics:

1. For Plato, forms exist in the RW which are permanent, outside of space and time, and represent the essences of all things and ideas. All objects in the BW — objects we would call physical objects — "participate" in the Forms. My example was the perfect sofa.
2. The RW contains the only true things and so acquiring Truth (with a capital "T") means somehow realizing the Forms in their natural, unusual habitat uniquely through our intellect.

So Plato's is both a story about ontology (the philosophy of what exists) and epistemology (the philosophy of what we can know).

The heated debates of the last 50 years about Platonism are largely about mathematics. In this literature it's not hard to find questions like whether the reality of a tree is different from the reality of $\sqrt{2}$. In some way, the latter is more permanent. And, of course, there are also the perfect objects of geometry...and maybe the rules of geometry. I think it's fair to generalize that there are three schools of thought in the Philosophy of Mathematics that can be labeled as:

- Intuitionalism, where mathematics is just the product of mental activity and a mathematical entity is constructed by the mind and lives solely in the mind. This is also sometimes called "structuralism" or "constructivism."

- Formalism, is probably the most popular camp in which there is no truth-value assigned to any mathematical property or entity. It's all just the study of logical consequences… dubbed "if-thenism." There's no commitment to anything beyond manipulating marks on paper according to the rules of the game.
- Platonism, suggests that mathematics is the study of abstract entities that have an existence that's as real as the external world targets of scientific experiment. So the question for Platonism is: do abstract mathematical things exist? Do abstract rules exist?

2.3.2.1 Quine–Putnam Indispensability Argument

I've had the misfortune… or fortune… of doing physics research for half a century after a masters degree in the philosophy of science. That means that I've never been able to avoid standing back and looking at what I do and what my colleagues do and categorizing and analyzing process, what counts as a valid argument, what counts as a valid scientific question, and what counts as an acceptable answer. And what about "reality"?

I'm intrigued with a particular strand of Platonism that's due to **Willard Quine** (1908- 2000) in the 1950s through 1990's, and **Hilary Putnam** (1926-2016), who later found common cause with Quine. Together, their ideas are called the **Quine–Putnam Indispensability Argument.** To an aw-shucks, country-physicist like myself, I interpret it to say:

1. Science (read "physics") works and interacts with real objects in the BW through experiments.
2. Mathematics works and interacts with abstract quantities and rules in the RW.
3. Physics cannot not work without mathematics, and so the two are *indispensable*. This is a partial answer to Wigner. "Unreasonable effectiveness" becomes "indispensability."
4. Given the impossibility of physics without mathematics, abstract mathematical-physics entities in the RW should enjoy the same level of reality as the objects of experiment in the BW.
5. So there are at least two realities: a physical reality and a mathematical reality.

The Quine–Putnam Indispensability Argument both rhymes with Wigner and demands a new definition of physics.

> "[talk of" mathematical entities is indispensable for science… therefore we should accept such talk… [which] commits us to **accepting the existence of the mathematical entities in question** [emphasis mine]." **putnam1971**, *Philosophy of Logic*.

Quine called himself a "reluctant Platonist" and I think that physics has joined that club. And as I'll show in Chapter **??**, Galileo was the charter member and he showed us all how to make progress in unraveling MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS once the club's Platonism was embraced.

A few random comments about the Quine–Putnam Indispensability Argument.

- Do I have to be a believer in order to do physics? No. You might be surprised how little philosophical thinking goes into a professional physics education. Long ago, the pain inherent in thinking too hard about, first quantum mechanics and then general relativity taught those of us who teach these subject to undergraduate and graduate students to not go there. "Shut up and calculate" is not just a funny phrase, it's actually an instruction that you must follow if you're going to make scientific progress. We physicists don't tend to analyze physics any more than a bird analyzes the dynamics of flight.
- Where does this leave mathematics and their philosophical problems? Well, first, we pretty much don't care! Second, Mathematical Platonism adherents think it's perfectly fine for there to be a plethora of mathematical realities. A multi-verse of mathematical worlds, if you will. Some of them have that special connection with physics...and some of them don't.
- I've concluded that we are relentlessly *both* Platonic and Pythagorean. We can't make progress nor explain the incredible success we've enjoyed without the rules of physics (the "laws") nor without the commitment to the numbers required to make predictions and then contact with experiment. The Platonic is joined with the Pythagorean, in contrast to Plato's Divided Line, the division is blurred and crossable.
- Is it just too unreasonable (sorry) to deal with this multiple reality stuff? A reasonable person might say that if I can touch it or kick it, then it's real. A pretty good working definition of "reality." Stay with me.

2.3.3 The Platonic Reality in Physics

What I described above is about a *process*. But there's also an "ontology." What are the objects of fundamental physics and do they live in the BW or the RW? Let's look at two objects and then go kick a rock.

2.3.3.1 Their Own Forms

There is no sofa that's identical to its form. Even two sofas designed and constructed in the same manufacturing facility will not be identical. Patterns on one will be slightly altered from the other. Tolerances on color or fabric structure or leg shape cannot be perfect. A BW sofa is not identical to it's RW Form. They're separated into the two Realms.

The 20th century has upended this very Platonic separation and Plato might have been intrigued with the result.

A molecule of hemoglobin in your blood contains 10,000 atoms of hydrogen, oxygen, nitrogen, and iron. Each of these atoms have protons, neutrons, and electrons. Isn't it remarkable that each of the many thousands of electrons in that single hemoglobin molecule are identical to one another?

Isn't it even more remarkable that each of those electrons in my blood is absolutely identical to an electron in an atom of hydrogen in the outer edges of the Andromeda Galaxy? Or to every electron that was flying around the early universe before Hydrogen atoms formed at 370,000 years after the big bang. (I might note that every hydrogen atom in your hemoglobin was in fact formed in the big bang.)

A prefect form of an electron — the ideal electron in the RW— is identical to its BW counterpart electron. No imperfection. No difference.

So the distinction between Forms and the objects in the BW that participate in the Forms evaporates as soon as we begin to deal with elementary particles. That is, when we begin to confront the universe as it is composed in the BW.

▻ Elementary particles in our everyday BW are their own Platonic Forms.

2.3.3.2 Are Wavefunctions BW Or RW Or Not Real At All?

Want some serious Plato? I give you Quantum Mechanics, the theory of the very small: atoms, electrons, nuclei, elementary particles, and quantum fields. Atoms and all of chemistry is precisely determined by a single equation called the Schrödinger Equation which can be solved to determine the "state" of an atom and make predictions about properties of matter. For example, the model of the optical spectra that result from electrons falling from high orbits to low ones by emitting unique colors of light is the first prediction of quantum atomic theory and was bang-on correct. Quantum mechanics is exquisitely precise and its predictions match experimental results to mind-boggling precision. It works better than any theory ever invented.

But Quantum Mechanics comes with a very strange substance that we cannot see, hear, touch, or measure. We arrive at predictions by calculating the evolution of the spooky entity called the "wave function," ψ. The wave function seems to me to be the very definition of a RW-existent, mathematical entity. Essential to the physics, but with an existence on paper only—a very Quine-Putnam idea.

We predict the results of an experiment involving atoms, molecules, or elections by mathematically evolving their wavefunctions using the Schrödinger Equation which takes $\psi(t_1)$ at some time, t_1 and tells you precisely how $\psi(t_2)$ will behave at time t_2 in the future. This works perfectly. Every time.

But here's the rub: *ψ is intrinsically undetectable*. It doesn't exist in the BW, but it does have a communicable existence as mathematical marks on paper. We make a connection in the BW by predicting the *probability* that a particle will be here. . . or there. . . .or over there. . . or on the Moon. That comes from the *square of the wavefunction*, ψ^2. Remember that party you un-livened up with the question about mathematics? Ask two physicists in attendance, "Is the wavefunction real?" Then stand back. That will liven it back up.

Let me repeat: we can calculate the value of ψ at any time or place in the future, but to connect with a measurement, we can only predict probabilities, no certainties are allowed. Ever. We cannot get from the equations of Quantum Mechanics to a measurement in the BW without passing through a RW Platonic manipulation of the mathematical entity, ψ.

If you ever needed a definition of a mathematical entity that behaves as if it has a reality only in the Intelligible Realm, the wavefunction, ψ, is the poster child for exactly that. For Quantum Mechanics to function, we must work wholly inside of a very strange mathematical RW which indispensably (in that Quine-Putnam sense) is very real. And Quantum Mechanics works better than any theory ever devised in any science.[6]

So every entity in physics is ultimately an elementary particle, which is its own Platonic Form and which is described by a mathematical entity which cannot be observed.

2.3.3.3 "I refute him thus!"

In a different context, it was the British writer of the *Dictionary* Dr. Samual Johnson claimed to be able to refute the Idealism of Bishop Berkeley that to be real was to be observed. He kicked a rock and declared, "I refute him thus!" Well, there's a lot inside of a rock.

It's quite natural to insist, "I know there's a real world out here because I can see and touch stuff!" Okay, let's talk about touching. That rock that you kicked with your foot is not a solid hunk of stuff. It's made of minerals in crystalline structures of definite chemical elements: atoms with electrons in their atomic shells which have complicated bonding with their "home" nucleus and across the crystals with neighboring atoms. Your foot is made up mostly water in cells and tissues, so, of course, different atoms in different arrangements.

These atoms of the "kick-er" and the "kick-ee" interact with one another as you bring your foot very, very close—molecularly close. There would be some deformation of the two materials (to your foot's disadvantage) since the rock's lattice is relatively rigid in comparison to the tissues of your foot. But what's going on? The electrons at the surface of your foot are repelled by the electrons in the outer orbits of the atoms at the surface of the rock. And to make it even more complicated, there's a region of quantum mechanical attraction and repulsion that is active between the whole molecules of the two materials called the "Van der Waals force." So *your kick is inherently a quantum mechanical process and is as real as the wavefunction* of the previous section, and the electrons and photons of the section before that. You think you kicked a solid thing that's a rock in the BW, *but what you did was cause a quantum mechanical interaction only describable in our RW*.

[6] Einstein famously washed his hands of Quantum Mechanics, immensely uncomfortable with its lack of certainty, related to the reliance on the wavefunction. And he was one of its inventors!

Again. As practicing physicists do we stay up at night worrying about the different realities that our description of nature presents to us? Or do we just keep on calculating...because it works. For almost all of us, it's the latter. We're actually all trained to be highly skilled "Quantum Mechanics" seemingly working in the BW of experiment, without concern for the philosophical niceties of the RW of the equations. This is the same as a skilled engine mechanic working under the hood of your car who doesn't need to know the material science or engineering of the digital electronics of the engine and control systems to solve BW problems.

But Plato is there. He's changed his mind about a few things, but when it comes to philosophical longevity— when it comes to physics—he outlasted Aristotle. By millennia. But Aristotle also had his moment. Take out your phone.

2.3.4 Aristotle's Legacy in Physics and Engineering

Aristotle invented the iPhone. Well, not exactly all of it, but he created the basic language that all electronics use to process instructions and communicate internally. This language allows digital components in integrated circuits to do arithmetic, compare number strings, turn peripherals on and off like pixels on a screen, and many other functions. All of this comes from seemingly endless strings of logical operations performed by mind-boggling numbers of individual digital "gates" of silicon which do very simple things.

You see, Aristotle invented that language and I think that's his modern legacy: Aristotle first conceived of the rules of **Formal Logic** which were so powerful, they instantly became active research projects for ancient and medieval philosophers for a thousand years. "Logic" is now the primary subject in whole fields: Philosophy of Logic, Discrete Mathematics, and Computer Engineering! If winning an argument is important and if you can reliably create valid arguments and always identify invalid ones, then you possess a superpower.[7] That was his goal. Making that superpower. For a more detailed introduction to the field of Formal Logic, see Technical Appendix A.2.3 Here I just want to hit some broad ideas.

2.3.4.1 Valid, Invalid, and Sound Arguments

In the courtroom, the board room, in science, and everyday life having the facts in hand is only part of a winning strategy to persuade others. Your argument has to be, we colloquially say, "logical." We all have a sense of what that means, but it can be nuanced. Let's look at two examples of arguments. Notice that I've added parentheses that demarcate important phrase chunks in each of the three lines.

Example 1.

- (All apples)(are fruit)
- (All red objects in that tree) (are apples)

[7] We'll see in Chapter **??** the re-discovery and use of Aristotle's Logic was arguably the major threat to the dogmatic Augustinian Catholic Church in the 12th century. An uneasy truce was pieced together by Thomas Aquinas by the 13th century.

- Therefore, (All red objects in that tree) (are fruit)

Example 1. hits you right, I'll bet. This is the kind of argument called a Syllogism consisting of two *premises* followed by a *conclusion*. Here's another one:

Example 2.

- (All elephants)(are English speakers)
- (All squirrels) (are elephants)
- Therefore, (All squirrels) (are English speakers)

Now Example 2. kind of hurts. These seem like very different arguments and you'd want to say that that this second one is absurd or wrong—more about that in a bit. But can you see that they share an important feature: they are both structured in the same way—they have the same **form**. Try this:

Example 3.

- (All A)(are B)
- (All C) (are A)
- Therefore, (All C) (are B)

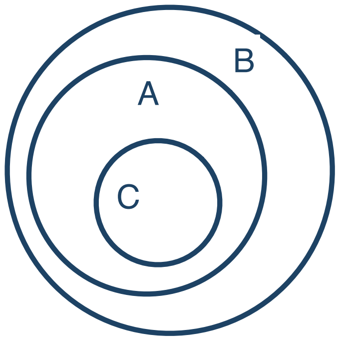

Figure 2.3: In a valid argument shows that one is forced to conclude that All C are B.

This shows the structure of both arguments. In both examples we can identify: A = apples/elephants, B = fruit/English speakers, and C = red objects in that tree/squirrels. Many substitutions will work for A, B, or C if the premises and conclusion are arranged like the above.

There's more: in any argument arranged as in Example 3. the conclusion is "forced" on you. The easiest way to see that is to look carefully at the "Euler Diagram" in Figure 2.3.

Can you see that in Figure 2.3 there are three circular areas, the biggest of which is B. All of region A is inside of the bigger region B so the first premise that (All A)(are B) is evident and that all of C is inside of A, so the second premise that (All C) (are A) is evident. So from the picture you forcefully conclude that (All C) (are B)—the conclusion of Example 1. You're worried about talking elephants. Stay tuned.

2.3.4.2 Greatest gift

Aristotle's unique invention that makes general rules possible for argumentation was to create what I think of as an *algebra of language*. Here is a seminal moment in history, from the first book of his *Prior Analytics* (focus on the last sentences):

> "...if every B is A then some A is B. For if no A were B, then no B could be A....e.g. **let B stand for animal and A for man. Not every animal is a man; but every man is an animal.**" (emphasis, mine) Aristotle, *Prior Analytics*.

Look at the sentences that I've highlighted: he's using variables A and B, to stand for things, here in his example, A = man and B = animal. Instead of men and animals, the variables could be squirrels or fruit. As long as the *form* is proper, we say that the argument is "valid."

Let's be clear—because Logic is all about clarity and bottom-up reasoning. We all use words that sometimes have specific meanings in specialize fields like Logic. Here are some that I'll make use of in this section. Some definitions for us:

- Here, we will use the term *statement* as a kind of a sentence which can be true or false. "Elephants are larger than squirrels." is a true statement. "All bachelors are talking squirrels" is a false statement.
- When a statement includes a "quantifier" (an example of which is "all"), a subject, a connective (often called a copula, a form of the verb "to be"), and a predicate we'll refer to these as *propositions*. (All apples are fruit.) is a true *proposition*.
- Not all sentences are *statements* or *propositions*. Our two here are aimed at logical argumentation.
- *Statements* and *propositions* can be true or false.
- We will use the term *Arguments* in two ways. In this subsection, a *Syllogistic argument* will stand as an ordered collection of *propositions* (here, the *premises* of the argument). As we saw, Syllogistic arguments are constructed as specific forms. (In the next section, we'll refer to a different kind of argument, a *Propositional argument*.)
- Syllogisms were Aristotle's first venture into Logical arguments and he identified 16 valid forms, but others after him found additional ones. Most likely it was the 13th century University of Paris scholar, William of Sherwood, who gave names and hints to identifying the 19 valid syllogisms (out of 256) and this particular one is called "BARBARA."[8]
- Syllogistic arguments consist of:
  - two propositions which are premises, which in the above examples are the first two sentences and
  - a single proposition which is a conclusion.
- A Syllogistic argument which is properly constructed according to one of the defined forms is simply *valid*, without regard to the terms (the A, B, or C).
- A Syllogistic argument constructed according to one of the defined forms which has true premises is called valid and *sound*. That is: If the premises are true, and the argument is properly formed, then the conclusions must be true in a sound argument.
- A Syllogistic argument which is not ordered according to one of the defined

[8] BARBARA wasn't a person, but a nemonic invented by Sherwood in order to remember the kinds of statements are in the premises and conclusion. Here the three are "All" statements, and hence his name, "A" statements. So they are "All x are y." E statements are of the form "No x is y" and for such a syllogism he invented the nemonic, CELARENT, with two E's and one A statement. He did this for each of the 19. Medieval analysis of Logic was exhaustive and probably exhausting. This dedication has carried on to this day.

forms is *invalid* and *unsound*.

Introducing variables as a placeholder for the subjects and objects in a statement is a seminal moment in the history of mathematics.

Amazing. Out of this beginning, your mobile phone was born.

Now, about talking elephants and talking elephant-squirrels. Elephants can't speak English and squirrels aren't elephants. So Example 2. is a *valid, but unsound argument* according to the rules of Logic that Aristotle invented. Why? Well, remind yourself of the "Euler Diagram" in Figure 2.3. Its conclusion is forced on you. Now consider this argument:

Example 4.

- (All elephants)(are English speakers)
- (All elephants)(are squirrels)
- Therefore, (All squirrels) (are English speakers)

This has the form:

Example 5.

- (All A)(are B)
- (All A)(are C)
- Therefore, (All C) (are B)

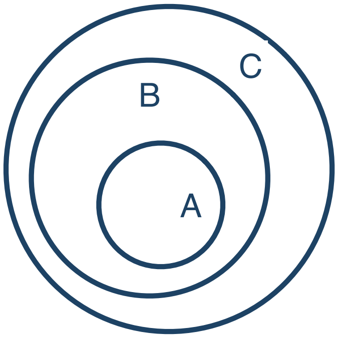

Figure 2.4: Here the invalid argument is clear. All of region A (elephants) are indeed included in region B (English speakers) but "all C (squirrels) are B (English speakers)" does not hold since there are regions in C (the squirrels region) that are *outside* of region B. Only some of region C are inside of region B.

Notice that between Example 3. and Example 5, that the order of A and C in the second premise are switched which is enough to make Example 4. invalid. So not only are the premises not true (so not sound), but it's also logically invalid and to get a sense of that, look at Figure 2.4. The caption explains why one is valid and the other not.

Aristotle covered this new-born subject in a number of his books, including: *Categories*, *On Interpretation*, *Prior Analytics*, *Posterior Analytics*, *Topics*, and *On Sophistical Refutations* which collectively, were much later dubbed "*Organon*" which means "instrument."

What I've chosen for my elephant-squirrel example is one of 256 possible syllogistic forms. Maybe you can see why studying Logic became a matter of intense research following Aristotle's death and into the first 1000 years of both Arab and Western philosophy. There was lots of work to do.

These arguments are examples of **deductive logic** which is often contrasted with **inductive logic**. In Deduction, if the form of the argument is according to the rules, then the argument is guaranteed to be valid. That's the sort of argumentation that was used in Socrates' discussion with the slave boy in the sense that the conclusion of a deductive argument is in some sense, already in the premises. Inductive logic is not reliable since it's not rule-bound and it delivers conclusions which can seem persuasive but aren't true.

Here's a personal, inductive argument about squirrels:

- (As a child) There's a brown squirrel
- (As an adult... many times) There goes another brown squirrel
- Wow... more brown squirrels and no other ones
- What is it with all of the brown squirrels?
- Gosh, I conclude that all squirrels are brown!

Induction not only can sound persuasive, it sounds scientific. And it is an important form of reasoning in science but it must be used with care. Aristotle knew of both kinds of logic.

Here's a problem with my induction about squirrels: Before I moved to Michigan, the only squirrels I'd ever seen were brown. Now my yard is full of black squirrels. They're everywhere. Many times in science a deduction uses premises which came from inductive reasoning, so even if the deduction form its proper, the argument might be unsound. Induction is always vulnerable to being questioned but the soundness of properly formed deductive arguments can only be challenged by questioning whether their premises are true. That's where a lot of the scientific action is.

From this point, when I refer to "logic" I'll mean deductive logic. By the way, Sherlock Holmes is reputedly the Master of Deduction. Well, sorry. That's not true. If you look at his stories you'll see very, very few examples of deductive reasoning. He's the Master of Induction![9]

2.3.4.3 Propositional Logic

Theophrastus (−371 to −287) was a favorite student of Aristotle's who led the Lyceum for 37 years after his teacher's death. Aristotle even willed him the guardianship of his children...and his library. While a devoted student, Theophrastus went beyond his teacher and expanded and modified some basic Aristotelian notions. He also moved the study of botany forward and worked extensively in Logic. Theodor Geisel (Dr. Seuss) used "Theophrastus" as a pen name.

He is probably the one who extended the idea of syllogistic argumentation into a new direction with the invention of "propositional logic" in which (for our examples here) there are two variables, rather than the three of a syllogism.[10] In the same spirit as our definitions above, we'll call these *Propositional arguments*. This is where the modern engineering action is.

Propositional arguments are different in form, and content from Syllogistic arguments. They involve a statement that is conditional: an "If this ....then that"

[9] Or more appropriately, the Master of Abduction, a, third kind of logic. Look it up.

[10] Propositional arguments can have any number of premises and variables.

statement. Let's contrast them. Here's a Syllogistic argument:

- (All apples)(are fruit)
- (All red objects in that tree) (are apples)
- Therefore, (All red objects in that tree) (are fruit)

Notice that the variables In Syllogisms are kinds of things (called classes in Logic).

Here's a Propositional argument which seems similar, but is very different:

- (If those red objects are apples) (then they are fruit.)
- (They are apples.)
- Therefore, (they are fruit.)

Here's how a Propositional argument is very different in an important way. The variables have a "truth-value," TRUE or FALSE.

Just as before it's useful to abstract the specific terms in the premises with general symbols and Table 2.1 does this on the left in words, and on the right using logical symbols. The → symbol means "implies" and is associated with an "If...then" kind of statement. The lone A is a standard way to say that "A is the case" or "A is true." Finally, the symbol ∴ means "therefore." It doesn't seem like much, but it's powerful. Establishing the truth-value of the conclusion of a Propositional argument

A Conditional in Words	A Conditional in Symbols
• If A is true, then B is true • A is true • Therefore, B is true.	• $A \rightarrow B$ • A • $\therefore$ B

Table 2.1: A Conditional argument and its concise symbolic equivalent.

can be straightforward, or complicated. The game is to analyze the argument, again, for formal validity and ask whether the truth value of the premises guarantees to the truth of the conclusion.

> An argument of this particular form (If A then B), (A), (therefore B) is called "Modus Ponens" (Latin for "method of affirming") and is one of six basic forms of propositional logical arguments. Another common propositional argument is "Modus Tollens," which also seems intuitive. For example: (If it is an apple) (then it is a fruit), (It is not an apple), (therefore it is not a fruit.)

2.3.4.4 Logical Fallacies

Propositional logic lays bare some logical fallacies which can be mistakes. Or logical fallacies can be used to convince people of the truth of a conclusion using an argument that appears to be valid, but is not. Look at the argument on the left in Table 2.2. Its validity is forced on you in the way that deductive arguments must do. A subtle change can take a valid argument and turn it into an invalid logical fallacy called "Affirming the Consequent," by switching the consequence for the

A Valid Modem Monens Argument	A Logical Fallacy
• If a reactor leaks radiation, people nearby will get cancer. • A reactor leaded radiation • Therefore, people nearby got cancer.	• If a reactor leaks radiation, people nearby will get cancer • People nearby got cancer • Therefore, the reactor leaked radiation.

Table 2.2: On the left, is a valid Modus Ponens argument. But on the right is a logical fallacy called Affirming the Consequent.

hypothesis in the second premise. Can you see that the argument on the right in the table is sneaky, and invalid? People get cancer from all sorts of causes and that someone got cancer does not mean that the reactor leaked radiation. Health care is often a target for this form of fallacy.

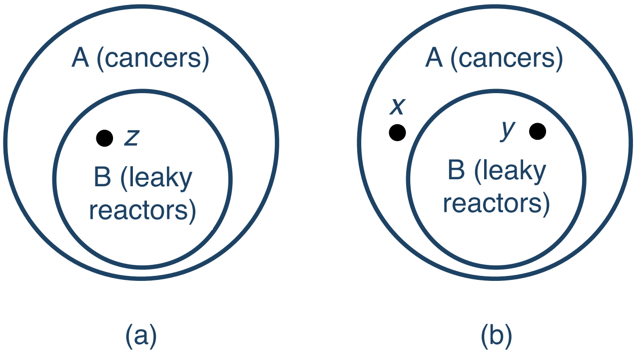

Figure 2.5: On the left is the valid argument that says that the placement of z with both a cancer and near a leaky reactor is the only result of the valid argument. But the right says that there is a cancer, but it could be either coincident with a leaky reactor (y) or have nothing to do with a reactor (x), and so the argument is invalid.

The objects in Figure 2.5 —which are not strictly Euler Diagrams— but similar to them— help to capture the argument. The conclusion of the valid and invalid arguments is apparent by the way the circles are arranged. The left diagram and the right diagram are the same since they represent the "If...Then" part of the argument. So within that arrangement, we can ask about validity by looking at entities that might fit the discussion. Look at entity "z" in the left diagram. It has the property B and since B is inside of A, it also has the property A. So given the argument, that the reactor leaked and entity z is inside that leaked region, it also is inside of the cancer region, completing the Modus Ponens true conclusion.

The diagram on the right has the same two regions, but now in the spirit of the invalid argument assert that entity y has the "attribute" of having cancer, so begin inside of region A. But this doesn't exhaust all of the possibilities for an entity having cancer. Entity x is also asserted to have the property of having cancer, but it doesn't support the conclusion that it overlaps with the leaky reactor region. So that second argument is not valid.

2.3.4.5 The Connection with Our Modern World

Aristotle's logical writing came from a deep level of analysis of language and thought. From the ground up. One might think that some ideas are just too trivial to write them down, but he wrote them down and defended his definitions even the most trivial bits. Here's one:

> "...there cannot be an intermediate between contradictories, but of one subject we must either affirm or deny any one predicate" Aristotle, *Metaphysics*

This is called the Law of the Excluded Middle. *A proposition is either true or it's negation is true.* There's no in-between. It's binary. This is a "two-valued" logic and Aristotle's structure was always built around that requirement: he didn't admit the (modern) idea of "degrees of truth" or "fuzzy logic." Trivial? Centuries of ink have been spilled over precisely understanding the implications of Law of the Excluded Middle and how to unequivocally state it symbolically. It's a simple idea that's deep and he had a number of such crisply defined notions so his Logic was really built from first principles.

What else can you think of that's strictly two-valued? How about binary arithmetic, where the only numbers are 0 and 1. How might you trivially represent 0 and 1? How about a pair of fixed voltages, say $V = 0$ and $V = 5$ volts.[11] There are a handful of seminal discoveries about Logic that extend to our modern usage. **Gottfried Wilhelm Leibniz** (1646–1716) refined binary arithmetic. In 1854, **George Boole** (1815–1864) invented the algebra of two-valued logic...how to combine multiple conjunctives into meaningful outcomes which can only be T or F, 1 or 0. In 1921 in his dense and terse *Tractatus Logico-Philosophicus*, **Ludwig Wittgenstein** (1889–1951) presented the Truth Table, which can be used in logical proofs (and circuit design). Finally, in 1938 **Claude Shannon** (1916–2001) realized that Boole's algebra could be realized in electronic, "on-off" circuits. This was put into practice in the 1940's with vacuum tubes and then in the 1960's with transistors.

2.3.4.6 Truth Tables

My goal here is to give you a hint about how important logical analysis has become, from following two of Aristotle's ideas: First, that statements and propositions can be written as abstract sentences with *variables* rather than with named things. And, that The Law of the Excluded Middle leads us to a *two-valued logic*.

Here's a statement: (It is raining.) This could be true (T) or false (F) depending on circumstances. But it's verifiable since we could determine T or F by looking out the window. We'll call that statement p. Here's another: (the grass is wet.), another verifiable statement which could be T or F and we'll call it q.

We can put these together into a compound statement using a "logical connective": (It is raining.) AND (The grass is wet). "AND" joins the two statements. We can

[11] the voltage range for transistor–transistor logic (TTL) logic used in many applications.

write this using the logical symbol, $\wedge$, which stands for AND, so our sentence—in general— can be abstracted in the Aristotle-variable-way as $p \wedge q$.

Our question of interest is: when will the compound statement, (It is raining.) AND (the grass is wet) be true? That is, what is the truth-value of "$p \wedge q$"...for the four possible T and F values that p and q might take on? Thought of a different way, if I asserted that compound statement, when am I telling the truth?

- If it is raining and the grass is wet, then $p = T$ and $q = T$ and I would be telling the truth if I said, "It is raining and the grass is wet."
- If it is raining and the grass is not wet. $p = T$ and $q = F$ then I would be lying if I said, "'It is raining and the grass is wet." (since $q = F$ means that the grass is dry).
- If It is not raining and the grass is wet. $p = F$ and $q = T$ then I would be lying if I said, "It is raining and the grass is wet."
- If it is not raining and the grass is not wet. $p = F$ and $q = F$ then I would be lying if I said, "It is raining and the grass is wet."

So of the four possible combinations of p and q, there is only one instance where the combination $p \wedge q$ is TRUE. This begs for an ordered way to present these possibilities and for each p and q, we can generate rows in a **Truth Table**. For AND, this is shown in Table 2.3. Notice that the entries in the last column correspond to the bullets just above and complete the possible p's and q's states.

Raining?	Wet?	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Table 2.3: The Truth Table for the AND connective.

Primitive logical connectives come by different names depending on one's discipline. They include: NOT, AND, OR, XOR ("exclusive OR"), NAND ("not-AND"), NOR (negate), XNOR ("exclusive NOR"), Implication, and Biconditional. They all have their own truth tables. And they're useful. What this means is that we can take many arguments and turn them into symbols using the connectives as "puzzle pieces."

Let's think about analyzing an everyday situation, like planning a picnic. Weather can be a problem for picnicking since wet grass can make the it unpleasant. So the morning of the planned outing, a picnic planner might muse something like:

- If it is raining, then the grass is wet
- It is raining
- And so the grass is wet.

Notice that this has the form of Modus Ponens and we're going to make a 21st century realization of it 2000 years after it was discovered. Here, $p =$ (It is raining.) and $q =$ (The grass is wet.). Let's set the stage and flesh out the single possibility for a valid Modus Ponens argument.

- (If it is TRUE that it is raining, then it will be TRUE that the grass is wet)
- AND (it is TRUE that it is raining)
- THEN (it is TRUE that the grass is wet)

But a Propositional argument contains phrases that have truth values, and in general, they are not necessarily all true. Recall the "am I lying" test from above: we could have $p =$ T or F and $p =$ T or F and only one combination of the four possible arrangements completes our valid raining-wet argument.

The entire set of possibilities can be compactly and completely captured in one big truth table and here I just present this result in Table 2.4. It's a picnic table (sorry). (In Appendix A.2.3 I build that whole table.) Notice that the AND operation between the third and first columns creates the third column's results, by comparing them using the rows of Table 2.3 as an instruction. The only combination that's true is the first one, the Modus Ponens argument itself. Validity of the argument is

Variables		Conditional	Conclusion
p	q	$(p \rightarrow q)$	$(p \rightarrow q)$ AND p
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

Table 2.4: The truth table for the Propositional argument above. The last column comes from comparing the third column with first column according the the T and F values in Table 2.3.

assured only if $p = T$ and $q = T$. Our connective, AND, figures prominently in this Propositional argument.

2.3.4.7 Modern Digital Arguments

Inspired by Aristotle, this "regular" conversation about the consequence of raining and the state of the grass can actually be embedded into a digital circuit using very basic digital packages[12] called "gates" (NOT, AND, OR, XOR, NAND, NOR, XNOR, and buffers). You'll recognize them as some of the logical connectives from above, plus one more that has a single input and just holds its value, called a buffer. The magic of the second half of the twentieth century is that particular combinations of transistors can produce digital packages corresponding to the gates which in turn can be soldered to a circuit board to make a decision-making circuit. With all of the

[12] You can go on Amazon and purchase integrated circuit packages of usually multiple gates in a single element that can be soldered onto a circuit board.

individual gates, an electrical engineer can piece them together to do a job. In the background, if not in the engineer's notebook, is the equivalent of a complicated truth table.

> Think about the decision-making that's required in order for an ATM machine to process your card, the keypad, your PIN, your request, and that you took out your bills. That each step was accomplished—and checked to have been done correctly— is actually a set of questions with T or F answers that a digital circuit is happy to perform for you.

Figure 2.6 is a cartoon of what this might mean. In the top figure, I show the engineering symbol for an AND gate. Below it, the black box could consist of a single digital gate element, or hundreds of digital gates, each receiving inputs from the outputs of other others. Here the box receives two binary inputs, each of which could be T or F.[13] and it outputs a result, r, either T or F. So there could be four possible inputs but one result. What's inside of the box are circuits of connected gates built on the logical structure of the problem.

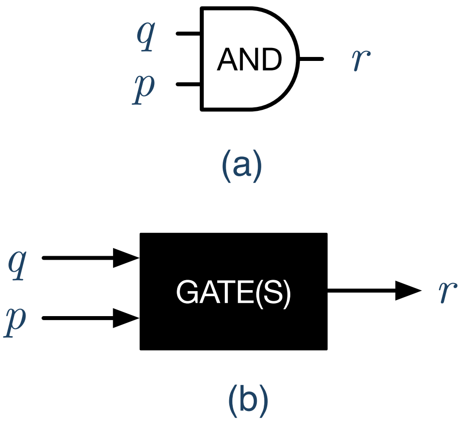

Figure 2.6: In (a) the engineering symbol for an AND gate is shown. The output of the AND gate, r, corresponds to the result of the truth table in Table 2.3. In (b) a black box of digital logic gates is suggested. The two inputs, p and q, are each either T or F and the output, r, is either T or F. This could be one gate or a thousand gates.

Our complete Modus Ponens picnic argument presented here as set of English statements could be recreated in a digital circuit (what might be inside the black box in Figure 2.6 (b)). For our particular example the circuit would consist of three gates (made from five transistors which would be so small that you cannot see them): an electronic circuit of the English sentences covering all of the possibilities of the argument.

I hope you can get a sense of how digital circuits are designed. There's a job to do, it's described in logical terms (p's and q's), a truth table (or equivalent) abstraction is done, and from (millions of) combinations of the seven digital gates that exist, a circuit design is created. Humans used to do this, indeed at the beginning of my career we laid out digital circuits by hand, but now computer aided design workstations do the work of creating schematics, simulating what electrical signals would do in the design, and preparing the instructions for printed circuit board (PCB) fabrication by specialized companies.

[13]which in practice, of course, is a 1 or 0 ("low" or "high") bit, and a the transistor level, a low and high voltage in a circuit

The first digital computers relied on thousands of vacuum tubes and filled whole rooms with hot, clunky racks of tubes and wires but when the transistor became commercially viable in the 1960s the digital world came alive. With binary arithmetic, gates can be combined to do arithmetic functions, logical functions, and importantly, storage of bits. A 1 bit digital memory consists of four so-called NAND gates—four transistors—and it's the basic cell of a computer's memory.

All of these—and more—transistor components can be imprinted in tiny silicon wafers in which a single transistor package might be only 20 nanometers in size or soldered to a circuit board as a package about half of size of a AA battery. With the logical functions and the manufacturing techniques of today, my current Apple Watch has 32GB of random access memory (RAM) and so it can manage 32,000,000,000 Bytes of information, which is 25,6000,000,000 bits and so 102,400,000,000 individual transistors are inside my watch, just for the memory! The CPU and control circuitry would add millions of additional imprinted transistors and their gate-equivalents. All on my wrist. All speaking "Aristotle."

Obviously, the 2500 year path from Classical Athens to cat videos on YouTube is full of breakthroughs and smart ideas. But it all started with Aristotle.

Chapter 3

The Most Important Mathematician You've Never Heard Of : Eudoxus and Greek Astronomy

"We shall try to note down everything which we think we have discovered up to the present time; we shall do this as concisely as possible and in a manner which can be followed by those who have already made some progress in the field. For the sake of completeness in our treatment we shall set out everything useful for the theory of the heavens in the proper order, but to avoid undue length we shall merely recount what has been adequately established by the ancients. However, those topics which have not been dealt with [by our predecessors] at all, or not as usefully as they might have been, will be discussed at length, to the best of our ability."

- Ptolemy, *Almagest, Book I, 1*

The passage above is the opening stanza of the last verse of Greek astronomy and is at the threshold of a strange 1500 year dance between the rigorously mathematical (Ptolemy) and achingly abstract (Aristotle) models of the universe. How we got there is the purpose of this chapter as it lays the ground work for two millennia of mutually supportive and mutually conflicting views of MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS .

I'll bet that many of you have seen the solar system arrangement as imagined by Copernicus (surprises await in Chapter 5) with the Sun in the center and all of the planets, including Earth, obediently orbiting it in perfect circles. What he challenged was the ancient, and universally-held idea, that it's the stationary Earth that's in the center of the universe, not the Sun. Fascination with that picture is prevalent in many decorated medieval manuscripts through the centuries and one of the earliest is shown in Figure 3.1. This is from a 10th century edition from the British Museum of a poem by the Greek poet, **Aratus** from about −275 called *Phaenomena* which was named for a book of the stars and constellations by the Greek mathematician, Eudoxus, of probably a century before. It was he who created that 2000 year old "geocentric" model of the universe—one in which the Sun, Moon, planets, and stars all orbit around the stationary Earth. We will see that the poem *Phaenomena* figures crucially in the history of astronomy two centuries after Aratus wrote it, so watch for it reappearing as we proceed.

I took some pains in the last chapter to underscore that models of motion on the Earth belong in Aristotle's corner as he really invented the dynamics of motion. But while we tend to ascribe that geocentric model of the universe to him as well, he borrowed it lock stock and barrel from Eudoxus and Plato.

Figure 3.1: Aratus the poet lived about a century after Eudoxus (and hence, Aristotle) and turned his astronomy book into a poem. Later, Cicero translated it and this 10th century manuscript is an illustrated copy of that work.
https://sarahjbiggs.typepad.com/.a/6a013488b5399e970c01bb07c8696d970d-pi

This "geocentric" picture became the authoritative, unquestioned dogma of the medieval and renaissance periods even though it made no numerical predictions and was known since Aristotle's time to be just wrong. The other game in town was precise and predictive and was the model of the Greek astronomer, Claudius Ptolemy, from the first century, CE.

The Greek world—indeed, the whole world—was radically and violently altered by Alexander the Great and between Aristotle and Cleopatra, astronomy become an experimental and quantitive science. The culmination of Greek astronomy came after Greek–everything became Roman–everything and just before the Roman Empire began its decline. One last Greek, in our long string of Greek philosophers, mathematicians, and scientists remained and we'll close our chapter with Ptolemy's "turn-the-crank" model for MOTION IN THE HEAVENS.

A game that many scientists play is to trace their scientific lineage back for centuries—their major professor's professor and so on (there's an app for that). I followed mine back through centuries and found that I descended from Copernicus![1] I'd like to think I've made him proud.

Sometimes it turns out that someone's student ends up in the history books. But not many students actually take over the known world by force!

When Plato died, the Macedonian King Philip II "encouraged" Aristotle to relocate to Macedonia in order to teach his 13 year old son, Alexander. He set up a school, taught Alexander (and perhaps the future general/king, Ptolemy) for three years, and then stayed for seven more before returning to Athens where he started his school, the Lyceum. By this time the teen-aged Alexander was already on the battlefield and with his father, had occupied the entirety of the Peloponnese. So Athens was once again ruled by outsiders—now connected to Aristotle!

After Philip II was assassinated,[2] and Alexander, soon to be "The Great," ascended to the throne and began his brutal lightening-fast, nine year conquest of the entire western world: modern Turkey, the middle east, Egypt, Arabia, and all the way across Afghanistan to India, leaving military oversight over Athens and the rest of Greece. While he stayed in touch with Aristotle, sending him samples from all over Asia, his teacher became distant, put off by Alexander's adaptation of Persian customs, dress, and persona.

Alexander died in Babylon in -323 under suspicious circumstances and, within a year, Aristotle himself died at the age of 63 at his mother's family estate outside of Athens. His Macedonian connections had become dangerous and his adopted

[1] Everyone I know seems to come from Copernicus. A mark that what he started had legs?

[2] Assassination, murder, and betrayal were a family hobby.

city turned on him: impiety was charged, a death sentence issued, and so he fled to his mother's home uttering his famous remark about the city not sinning against philosophy for a second time. In his absence, the Lyceum stayed active under new management for another century.

Alexander's senior commanders divided up the sprawling kingdom among a dozen generals and aides and they did what came naturally: they fought among themselves for 40 years. In the end, three kingdoms and a dizzying array of city-states were established: the survivors were Macedonia and Greece, Seleucia (roughly modern-day Iraq), and Egypt.

Hundreds of thousands of Greeks migrated into the newly acquired territories establishing an international Greek-ness of culture, arts, and philosophy which was the beginning of the **Hellenistic Age**.[3] The entire western world became "Greek." Of the two dozen cities that Alexander created or conquered named for himself, the "Alexandria" that mattered most to him, and to us, was the new Egyptian port city of Alexandria.

Egypt became unusually secure under Alexander's former body guard and general (and rumored Aristotle student), **Ptolemy I Soter** (-367 to -282) who eventually fashioned himself, "Pharaoh." He adopted Egyptian customs,[4] and was an intellectual of sorts, creating the first state-supported national laboratory and library. The "Alexandrian Museum" was a national facility devoted to research and among its first recruits was the mathematician, Euclid, who while in residence, wrote *Elements*, the most-read book in history, besides the Bible. For 2500 years, from Copernicus to Thomas Jefferson, mastering *Elements* was the route to mathematical literacy.[5] For centuries the Museum was home to scores of Greek scholars, all supported by the dozen Ptolemy's from the I$^{\text{st}}$ to the final one, Cleopatra.

The Library of Alexandria probably contained all of the manuscripts of the classical and Hellenic philosophers, poets, playwrights, and physicians. There was a hunger for knowledge of all sorts and agents of Ptolemy's library director searched every ship that docked, stealing or copying any books on board and renting or stealing manuscripts from all of the major cities.

Among the scores of Alexandrian scientists are the astronomers Eratosthenes of Cyrene, Aristarchus of Samos, and especially Claudius Ptolemaeus who will figure into our story, while only Heraclides of Athens, Hipparchus of Nicaea, and Apollonius of Perga played major roles outside of Alexandria. The Greek Ptolemy dynasty lasted 300 years until the legendary feud involving "the" Cleopatra (a common name for female Ptolemy-family successors), Marc Antony, and Julius Caesar. The Library and Museum lasted into the first five centuries CE until the Muslim conquests of the near east, north Africa, and Spain when it was eclipsed by

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[3] Often the pre-Alexandrian Greek era is called "Hellenic."

[4] including that of rulers marrying their siblings

[5] Ptolemy found it rough-going and asked for an easier way to learn it, but was told by the author that "...there is no Royal Road to geometry," a sentiment still applicable today.

great Muslim libraries in Baghdad, Cairo, and Cordoba in Spain.

3.1 A Little Bit of Eudoxus

Recall that Philolaus was the source of Plato and Aristotle's knowledge of Pythagoreanism—for example, the "Pythagorean" cosmology came through him or probably originated from him. Was he a student of Pythagoras? Their overlaps are nearly right in order to imagine that relationship, but that's controversial. He's certainly the closest we get to the great man so it's not far-fetched to continue the teacher → student theme that began this chapter: Pythagoras → Philolaus → Archytas → Eudoxus. Lunar craters are named after each which is not the normal teacher-student legacy. (Set the context with the timeline in Figure 1.2 on page 20.)

Eudoxus of Cnidus (circa -408 to around -355) was the son of a physician and became one himself, but we know of him as a gifted mathematician and astronomer. As we'll see, astronomy and medicine were connected through astrology and mathematics and astronomy have always been kin, so these seemingly disparate skills go together. Cnidus was a city founded by Sparta on the southern Aegean coast of modern Turkey and was where he started... and finished, between which times he traveled all over the Aegean to study and teach. As a young man he went to Tarentum to study mathematics with the pre-eminent Pythagorean mathematician (and much more) **Archytas of Tarentum** (-428 to -347) who seemed like a sensible guy:

> To become knowledgeable about things one does not know, one must either learn from others or find out for oneself. Now learning derives from someone else and is foreign, whereas finding out is of and by oneself. Finding out without seeking is difficult and rare, but with seeking it is manageable and easy, though someone who does not know how to seek cannot find. Archytas, *fragment*.

Let's learn a little bit about him in Figure Box 3.2 on page 98. After you've read about Archytas, return to this point ☟ and continue reading about his student, Eudoxus.

FIGURE BOX 3.2

The image on the left is a famous engraving (by an unknown artist...maybe late 18th century) suggesting an ancient sentiment due to Archytas, a friend and competitor of Plato. Among the most famous arguments in cosmology is whether the universe is infinite or finite in size and Archytas had the first of many similar inspirations that the universe cannot be finite: He did a thought experiment, imagining traveling to its presumed edge and attempting to thrust his stick beyond that limit. If he could extend it, then, well, that's not the edge...and so he'd have to go further, repeating the experiment without end. This is a good example of the kind of intuitive cleverness that seemed to be built into this great Greek mathematician, politician, and military leader. The very model of a modern major—Pythagorean— general. Archytas was a committed Pythagorean and a mathematician of great skill. But he also he was a civic leader and an elected military general, in spite of Tarentum law, re-elected seven times because he never lost a battle. (Did I mention that Greeks fought constantly?) When he did step down, the army started losing.

Archytas was reported to be an even-tempered, cultured man who led Tarentum through a period of democracy and that Aristotle apparently wrote more (lost) books about Archytas than he wrote about any other person. There is some evidence that he wrote a book on mechanics and that he enjoyed making mechanical toys for children—very un-Plato-like in spirit.

His mathematical skills were legendary and he solved an old problem with mystical roots: Apollo sent a plague to the city of Delos and a delegation was sent to Delphi to learn from the Oracle how to rid themselves of the pestilence. The instructions were to take their cubical altar to Apollo...and build a new one with double its volume. This is called the problem of "duplicating the cube" (also called the Delian Problem) and it required cleverness on Archytas' part, beyond just geometry, which caused Plato to disparage his effort. Archytas contributed to many branches of mathematics and Euclid's *Elements* includes some of his proofs.

All in all, Archytas was the most accomplished Pythagorean of all and in the spirit of the opening to this chapter, we're indebted to him for his products, but also one of his students. The most accomplished of all Greek mathematicians before Archemides, Eudoxus, from whom 2000 years of cosmology originated.

Now go back to page 97 and pick up where you left off.

He seemed to not be able to stay in one place. After his mathematics instruction, he went to Sicily to study medicine, then by the age of 23 he went to Athens and stayed briefly (and apparently, unhappily) with Plato's Academy (rooming 7 miles away, so a long commute to lectures). After less than a year, he was back on the road to home in order to raise funds...so that he could travel even further! He went to Egypt with what we'd call a scholarship and studied astronomy there for 16 months, shaving his head and learning from the priestly-cast astronomers, before leaving for the northern modern Turkish Black Sea coast and the Greek colony of Cyzicus. By this point he's lecturing on his own and established a popular school and an observatory. With data from his observing in the north and from Egypt, he published his first book, *Phaenomena*, which was a compendium of star locations and *On Speeds*, of their motions. Recall that this is the subject of Aratus' important poem.

Around −368, during his 30s, he moved his school to Athens, by which time Plato was 60 years old and Aristotle had left for Macedonia. It was here, as the legend goes, that Eudoxus was challenged by Plato to form a geometrical model of the heavens. The legend is unlikely as by this point, Eudoxus was the mathematical champion of the Greek-speaking world and more likely to issue challenges, than accept them. Plato's mathematical skill was no match for Eudoxus' whose work was memorialized in a number of Euclid's *Elements*. As we'll see below his model was born and in various guises, persisted until Galileo, Kepler, and Newton.

He first calculated/measured the length of a year of 365 days and 6 hours. and it's Eudoxus' astronomy and cosmology that are our concern here and so let's work up to that with a review of the problems that everyone in antiquity faced when trying to describe what we observe from Earth and then work through Plato's ideas that formed an almost linear line of inspiration: from Pythagoreans, to Plato, and to Eudoxus.

3.2 A Little Bit of the Sky

There are very few objective experiences that we can share with people who lived thousands of years ago. But if you watch the Sun's path across your sky and the night sky over many days you'll see the same things as all of prior humanity—consistency punctuated by usual events. We can disagree about a lot, but every human has experienced the same MOTION IN THE HEAVENS.

For millennia the skies seemed memorable and intimate. Cultures all over the world adopted the periodic motions in the sky as a to-do list for planting, religious observances, expectation of periodic floods, and other natural events. The heavens seem perfect and so it was natural to associate deities with the cycles (and picture their images in the star patterns and planets) but also to look to the heavens when unfortunate terrestrial events happened for correlation with unusual events like eclipses and conjunctions of planets with one another.

Remember that for Aristotle, everything changes, change is a "motion," and unnatural motions on the Earth are caused by something. And he wrote about the connection between the sky and the Earth. In his *Meteorology* he found it persuasive that large-scale but continually changing phenomena like the weather should be caused by the the continually, but predictably changing MOTION IN THE HEAVENS. Certainly, the Sun seems to influence life of plants and animals and the Moon's motion seemed to be connected with women's physiology (and later Ptolemy associated the tides with the Moon).

The Babylonians were the first to create a systematic program, with extensive cuneiform tablets with data and thousands of expected omens. In order to predict future Earth-bound events they created huge data-sets and invented an algorithmic approach to prediction. The Greeks made the program geometric. The former seems sterile, while that latter approach seems dynamic. It creates pictures, which is a very modern physics approach.

Horoscopic astrology became important and popular during the Hellenistic period and geometric tools were developed and deployed to better record astronomical events and match them to both personal lives and medical treatments. The distinction between astrologer and astronomer blurred and stayed entangled into the 17th century, each serving the other.

How to make sense of complicated MOTION IN THE HEAVENS? Many cultures tried, but the Greek geometrical approach was best suited to prediction and explanation. The problem was hard.

Let's imagine partnering with a Hellenistic Greek as we each observe the sky and note the puzzles that confront us both.

> Let's go out tonight at my home which has latitude and longitude of 42.7° N and 84.5° W. In what follows, I'll use "EL" to mean "East Lansing, Michigan" and we'll know that corresponds to that latitude. If your an ancient Greek, then my latitude is identical to that of Greek colonies in the south Black Sea. So around where Eudoxus worked for a while!

3.2.1 What Ancients Saw and What We Still See

Suppose you're indeed a smart Greek with time on your hands and able to spend years just recording what the sky presents to you during the days and nights. A few things would stand out...and if you were a patient and persistent observer, nuance would start to emerge. In *Greek Astronomy, Today* in Section 3.8.1 I'll "set the record straight" with full, modern explanations for each of these scenes and motions but here we'll just observe. k

The celestial sphere. Let's look up after sunset and watch the stars' motions through a night. Figure 3.3 is what we'd see on March 19, 2024 from EL. Here we have our ancient and modern partners looking south with the eastern horizon on their left and the western horizon on their right. Directly overhead is the **zenith** which would be 90° from all points on the horizon. Let's follow one familiar constellation.

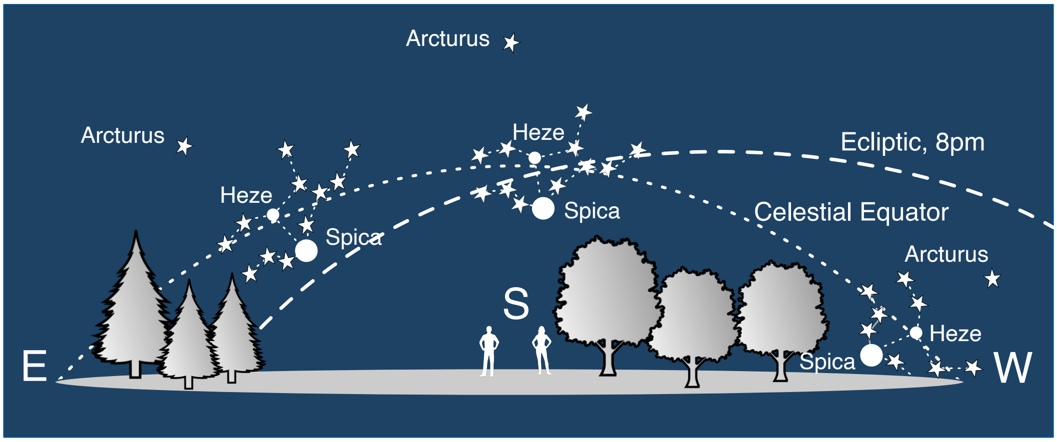


Figure 3.3: A time-lapse photograph of the star positions during a single night in the northern hemisphere are shown clearly demonstrating the circular "inside" of the Celestial Sphere. The pole is conveniently located (now) at the North Star, Polaris.

Virgo, the "maiden" is the largest constellation in the zodiac and is most evident in the spring. Its shape presents two "legs" and two "arms" seemingly attached to a "body." The downward "hip" is Spica, one of the brightest stars in the sky. The two outstretched arms reach to the spectacular Virgo Cluster of thousands of elliptical and spiral galaxies. Our interest is more modest.

The naked-eye star, Heze, is joined at the other hip to Virgo, so to speak, and is actually two relatively modest stars appearing to us to be close together. What's useful for us is Heze's location because it traces out an important circular path. Figure 3.3 shows it as a dotted circle with three replicas of Virgo showing its positions from late in afternoon (invisible since the Sun is still up), to overhead about 9 PM, and then at about 2 AM when it sets. That dotted curve to which Heze appears to be attached is special, it starts directly in the east and ends directly in the west. Also pictured is Arcturus, the fourth brightest star in the sky which likewise follows another circular path which is parallel to Heze's. In fact, as you watch, you can imagine all of the stars in the sky following concentric, circular paths every night. Figure 3.4shows a time-lapse photograph of the northern sky where all of the circular star-trails are evident with the axis of all of those circles centered at the **North Celestial Pole**, which for us now is very close the North Star, Polaris.

The most natural impression is that you're standing in the middle of an enormous 24 hour spinning sphere — the **Celestial Sphere**—with stars attached to its inside surface. If the Earth were to become transparent, you'd see the whole stellar panorama turning around you and its axis from Polaris to the other side poking out below you near the south pole. Heze's path is special since that dotted line traces out the equator of that spinning sphere, the **Celestial Equator, CEq**.

One of those nuances is that the stars' appearances are not repeatable night after night. The times that stars begin to appear on the eastern horizon changes each

Figure 3.4: A time-lapse photograph of the star positions during a single night in the northern hemisphere are shown clearly demonstrating the circular "inside" of the Celestial Sphere. The pole is conveniently located (now) at the North Star, Polaris.

night by four minutes early out of 24 solar hours, which is called "**heliacal rising**." This time advances through the year and the "ascendency" of stars in the east became milestones on a calendar that people could use to predict when events to happen. For example, when the bright star Sirius in the constellation Canis Major appears in the eastern sky just before dawn each year, Egyptians they knew that the Nile's flooding was coming.

Planets' apparent motions. There are other objects which execute similar east-west motions through an individual night; are brighter than stars; don't twinkle like stars; and occupy strange, un-star-like positions from night to night. Of course, these are the "planets," probably named by the Greeks from their word for "wanderer," *planetai*. Figure 3.5 shows a striking event in the sky at 2:30 AM on June 23rd, 2022 from EL in which four of the five naked-eye planets were all above the horizon at once. The bright circles are naked eye planets and the gray circles are the rest of the complement which require a telescope to see, but notice they too are all lined up with the others and the Moon. Pluto is added for nostalgia. The Sun too is about to rise following Venus on that same dashed curve. Obviously, their paths are somehow related.

All of the planets and Sun are within $\pm 7^\circ$ of the dashed mean curve (except Pluto which is 17°, one of the reasons it's no longer considered a planet of ours). This common "lane" in which all of the solar system (and the Moon) objects reside is called the **ecliptic** and the central path is sometimes called the "mean Sun." At a

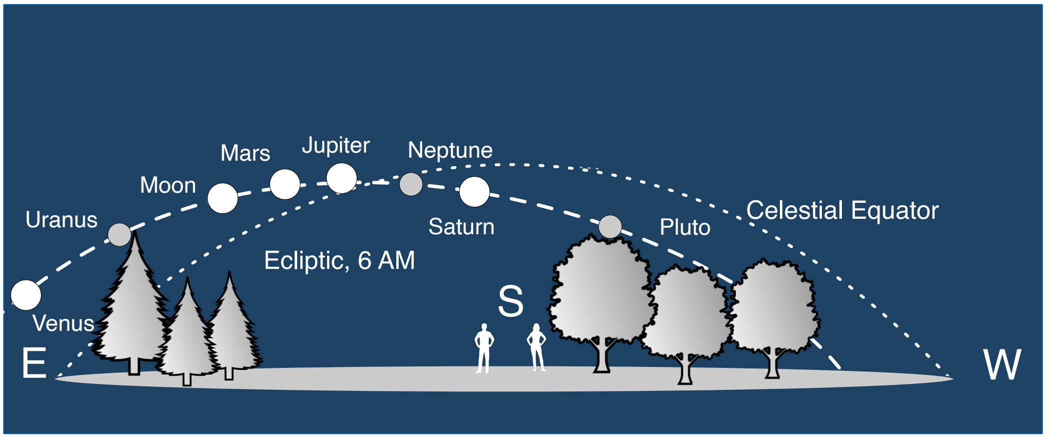


Figure 3.5: The position of the naked-eye planets (white circles) from EL at 2:30 AM on June 23rd, 2022. The dotted line is the Celestial Equator and the dashed line is the ecliptic. The gray circles indicate where planets that the Greeks could not have seen with the naked eye.

different day and time, the Celestial Equator won't have moved, but the ecliptic traces out a *different* curve relative to the horizon and you can see that in Figure 3.3, where it's represented again as a dashed curve, but for a different day, March 19, 2024. This must have been confusing!

The ecliptic plane is inclined to the Celestial Equator by 23.5°. The constellations of the zodiac are distributed around the sphere within that strip of the sky[6] and the center of it is the path of the Sun.

Finally, there are two kinds of "motions" spoken of for the planets, which is confusing.

- If you watch a planet during a single night, you'll see it move from east to west in line with the stars behind it. This is called "**prograde motion**."
- But there's another kind of "motion" which is not during a single night, but appears when one does a comparison from night to night. The planets have their own motions and if you look at, say, Mars every night at 10 PM and take note of what stars are behind and around it, at some point, you'll see something strange. Suppose Star A and Star B are on either side of Mars. In some successive nights the arrangement of the three objects will go something like this table below facing the south:

[6] There are 13 zodiac signs, but that's inconvenient for astrologers so they ignore one of them.

Night #1 East ........*A*..........*M*.........*B* West
Night #2 East ........*A*........*M*...........*B* West
Night #3 East ........*A*......*M*.............*B* West
Night #4 East ........*A*....*M*...............*B* West
Night #5 East ........*A*..........*M*.........*B* West
Night #6 East ........*A*............*M*.......*B* West
Night #7 East ........*A*...............*M*....*B* West
Night #8 East ........*A*.............*M*......*B* West
Night #9 East ........*A*...........*M*........*B* West
Night #10 East ........*A*..........*M*..........*B* West
Night #11 East ........*A*........*M*............*B* West
Night #12 East ........*A*......*M*..............*B* West
Night #13 East ........*A*....*M*................*B* West

Each night Mars seems to be more east of the star pattern near it—that separate motion of Mars. But between nights 4 and 11 Mars appears more west and after a number of nights, it then reverses course and continues its nightly progression eastward. This is called "**retrograde motion**" and it confused everyone. Certainly the common description of retrograde motion as a "motion" is confusing nomenclature since the "movement" is actually over many nights. This happens to Mars every 26 months and the retrograde loop takes about four months to complete.

Sun's apparent motion. That smart Greek's days (and ours) would be dominated by the Sun. If you're in the northern hemisphere, in general you'd see it appear to rise over your eastern horizon, pass not quite overhead, and then disappear over your western horizon. Look at Figure 3.6 which plots the Sun's trajectories through a year for EL during 2024. On December 21st, the Sun takes its lowest path, the days are the shortest because the Sun rises south of east and sets south of west. That lowest Sun path is on the day of the **Winter Solstice**—the shortest day of our year. Every day after, you would notice that the Sun's eastern rise is a little bit north from the day before and that it would set a little bit further north as well and so each day would be a little longer.

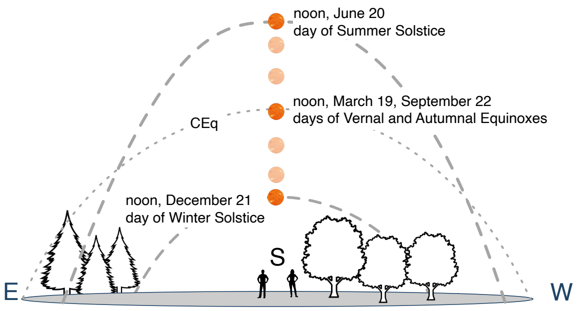


Figure 3.6: An observer looking south would see the Sun take very different paths through the year. Of course the Sun moves from east to west, but at various altitudes. This figure shows the situation for East Lansing, Michigan which is at a latidue of 42.74° above the Earth's equator. On December 21st the Sun takes it's lowest path and the days are the shortest and the Sun's rising and setting is south of east and west. On June 20th, the Sun is nearly overhead with rising and setting north of east and west, so the days are long. Between those extremes the paths are different slightly each day. In the middle period on

Furthermore, at noon the point each day when it's at its peak would be just a little higher than the previous day. Then on June 20th, the Sun has gone as far up as it will and is nearly overhead at noon, rising and setting quite a bit north of east and west, so that day is the longest of the year. It's the **Summer Solstice**. Then the situation reverses and the Sun is lower every day until the next December. Between those extremes the paths are different slightly each day.

In that round trip, there's one day on the way up and one day on the way down when the Sun rises precisely in the east and sets precisely in the west and at noon, it's height above your horizon is exactly between those two solstice extremes during late December and June. Also on those two days, the day and night durations are the same all over the world: 12 hours and so each of these special days is called an **equinox**.[7]These points happen in late March (called the **Vernal Equinox**)[8]and late September (the **Autumnal Equinox**).[9]Each **equinox** is a precise astronomical event and marks the point when the Sun on the ecliptic passes through the Celestial Sphere on its way up or down. In Figure 3.6, you can see that the trajectory of the Sun's path in the middle is dotted rather than dashed to highlight that the Sun's path that day is very close to the Celestial Sphere circle, crossing it at the precise moments of March 19th at 11:06 PM EDT (Vernal Equinox) and September 22nd 8:44 AM EDT (Autumnal Equinox).

Equinoxes are distinct events throughout ancient history, across cultures. The Vernal Equinox[10] was celebrated around the world: from the Mayans to the ancient Germanic tribes to the ancient Saxons as a time of renewal and rebirth. Structures like Stonehenge, the Mayan pyramids, the Egyptian Pyramid of Khafre, and others in Cambodia, Ireland, and New Mexico point out the VE. Understanding them, though, only became a goal among a few Hellenistic Greeks when "solar models" were invented by mathematically clever and imaginative astronomers. As our story unfolds, notice how the Sun figures into every corner of ancient astronomy—and yet, it was considered to be just another orbiting object.

This is a quantifiable picture. By the Hellenistic time (after Alexander's conquests), everyone knew that the Earth was spherical and that the some of the angular quantities in the sky matched angular quantities on the Earth's surface. Greeks were spread between northern Africa (about 30° north of the equator) and the northern shores of the Black Sea (about 45° north), so the apparent position of the celestial pole was easily seen to be different when viewed from different locations. For example, Figure 3.7 is a perspective view from EL corresponding to Figure 3.6 where the angle that the Celestial Pole makes with the northern horizon is identical to the observer's latitude. That means that the angle that the celestial equator makes with the southern horizon is (90° − the observer's latitude). You can see

[7]This derives from the Latin *aequus*, for "equal" and *nox*, for "night."

[8]Latin for "spring" is *ver*.

[9]In 2023, the WS, VE, SS, and AE occur on December 22, 2023, 3:27 AM, March 20, 2023, 9:24 PM, June 21, 2023, 2:57 PM, and September 23, 2023, 6:49 AM, GMT

[10]sometimes colloquially referred to as the Summer Equinox

three angles, all of which the Greeks determined. The latitude of 42.7° for East Lansing is shown as the altitude of the North Pole (celestial and Earth poles); The altitude of the Celestial Equator is $09^\circ - 42.7^\circ = 47.3^\circ$, which is also the altitude of the Sun at an equinox; and finally, the angular separation of the Sun's extreme altitudes is 23.5° up and down from the equinox Sun's path.

Of particular importance to the Greeks and all concerned later with astrology were the constellations in which the "Sun resides" during the time of an equinox.[11] During the times of the Greeks, the special point in the sky when spring would begin was when the Sun passed through the leading edge of the zodiacal constellation of Aries—the "First Point of Aries" and it became the origin of a coordinate system in order to document the location of stars and planets and became particularly important to astronomers in the −200's.

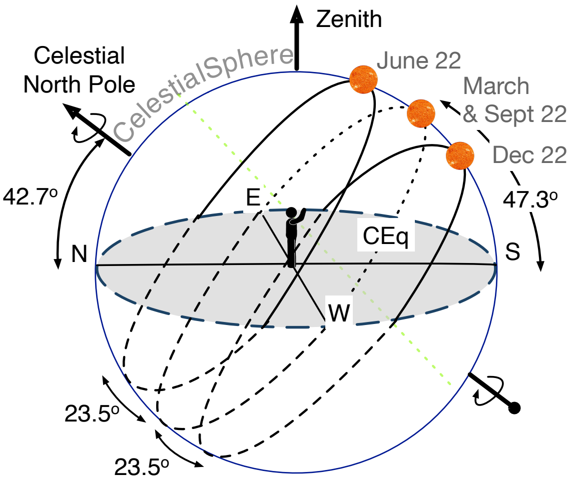


Figure 3.7: A perspective view a view of the Celestial Sphere from one's horizon, here for the latitude of 42.74° of East Lansing, Michigan, is shown. The three bands show the Sun's path in the sky at the Summer Solstice (top), Winter Solstice (bottom), and the Equinox (middle). Each of the bands around that central arc are 23.5° above and below it.

Clearly associated with the Sun are the seasons and they aren't the same length—spring and summer are longer than fall and winter, but there are definite times of cold and warm weather in the two hemispheres. In 2023 in the northern hemisphere: after 89 days in 2022, winter ended; spring was 93 days long; Summer was 94; and Autumn was 89. The Athenian astronomers Meton and his student, Euctemon found 92, 93, 90, and 90 days in about −432, so this was a known problem. (The student also has a lunar crater named for him.) Then, as today, we start spring at the Vernal Equinox, summer at the Summer Solstice, fall at the Autumnal Equinox, and winter at the Winter Solstice.

The apparent motion of the Moon. Prominent for its size and its regularly changing features is our Moon. If looked at from overhead, it travels in a clockwise orbit, nearly circular, with a period of 27.322 days, changing its appearance through phases during that cycle. Unlike the Sun and the stars, the Moon changes its

[11]Of course, they could not see the stars when the Sun is out, but they knew to look at the sky exactly 12 hours later and then extrapolate 180° around the zodiac to determined where that point of "residence" was.

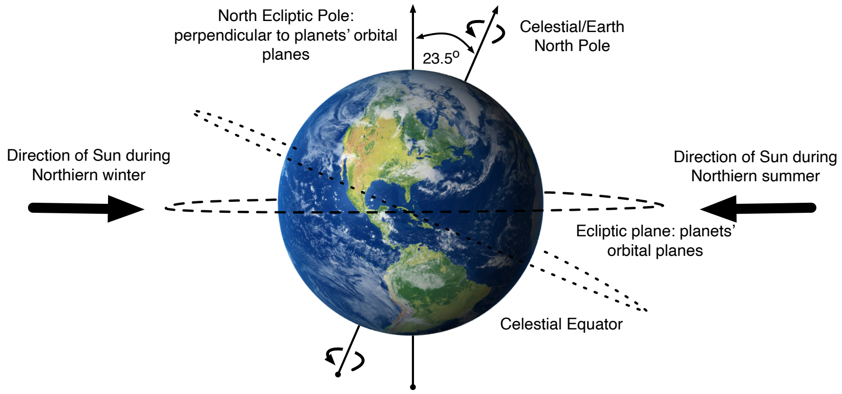


Figure 3.9: The facts of the matter are: The Earth and all of the planets orbit the Sun in a plane, the ecliptic plane; the Earth spins on an axis which is 23.5° inclined from the vertical to that plane. The Celestial Sphere then is also inclined and the stars appear to revolve at that inclination. The Sun's rays on the left are spread out over the Earth's surface in the northern hemisphere and we have winter and when the Earth is on the other side, six months later, the Suns rays (on the right) are more concentrated over the surface and we have summer.

appearance every single night. Sometimes it's "full" and a bright circle. Sometimes it's not there at night, but maybe visible during the daytime. Most times the bright part of the Moon is a crescent shape, culminating in a half-circle, and then back to crescent. Occasionally, the Moon gets in the way of the Sun and we have a solar eclipse. Sometimes the Earth blocks the Moon from the Sun and we have a lunar eclipse. Why these events don't happen every month was a puzzle. One thing doesn't change about the Moon and that's the face that we see—another puzzle.

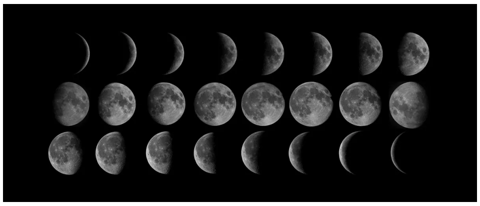


Figure 3.8: Views of the familiar faces of the Moon through a month, not showing the new Moon phase. Getty

The accumulated puzzles from our simple observations include at least these:

1. Why are the seasons of different durations (this has historically been called "the first anomaly")?
2. Why do the planets undergo retrograde motion (this has been historically called the "second anomaly")?
3. What is the nature of the spherical shell that seems to carry the stars around in celestial circles?
4. What is the reason for the appearance of the 23.5° inclination of the CEq and the ecliptic?
5. Why are the planets sometimes bright and sometimes dim?
6. Why don't lunar and solar eclipses happen every month?

Puzzled about these observations? If you can't wait for Copernicus, Tycho, Kepler, and Galileo...then take a look at *Greek Astronom, Today* in Section 3.8.1 for our modern interpretation how it goes. Figure 3.9 is a taste of the solutions of many of the puzzles.

3.3 A Little Bit of Presocratic Astronomy

Pythagoras •Philolaus •Parmenides •Archytas
(Set the context with the timeline in Figure 1.2 on page 20.)

In Chapter 1, I briefly discussed the Presocratics' cosmologies with two ideas among them that were shared: all but two appeared to believe in a flat, and stationary Earth. The two who thought differently were Pythagoras and Parmenides.

Parmenides had a number of original ideas about the heavens—in particular, he may have been the first to conceive of the whole universe as being spherical (Pythagoras/Philolaus might also have determined this) and finite.

> "...like the mass of a well-rounded sphere, from one middle, equal in every respect." Parmenides

He was also apparently the first Greek to note that the Moon reflected the light of the Sun and must be spherical and he was even poetic about it:

> "[the moon is a body] shining by night, wandering around earth with borrowed light..." Parmenides

"Borrowed light" is a nice phrase. If the Moon "borrows" its light from the Sun and doesn't shine on its own, then the shape of the phases of the Moon lead to a spherical shape conclusion.[12] Ironic, isn't it that Parmenides can perhaps be credited with a scientific discovery—one that requires observation— when we tend to think of him as anti-scientific.

The Pythagorean team (probably more Philolaus than Pythagoras, so I'll call it collectively "Pythagorean/Philolaus") extrapolated their fondness for regular motions, musical tones, and numbers and built a cosmology that tried to put all of these commitments into one model. They were responsible for many "firsts" in Greek astronomy: they too hypothesized that the Universe is spherical, most credit them with establishing that the Earth is spherical (for metaphysical and symmetry reasons), they proposed a popular ordering of the planets (Earth, Moon, Sun, Mercury, Venus, Mars, Jupiter, and Saturn...surrounded by the stars), they hypothesized that the planets' speeds are inversely proportional to the size of their orbits, and they concluded that the "morning star" and "evening star" (our Venus) were not two different planets but the same one which is close to the Sun. And, crucially: they were the first to propose that the planets follow circular orbits around a center.

[12] It was traditional to credit Parmenides with extrapolating from a spherical Moon to declaring that the Earth, too, is spherical. But that's not authenticated and Pythagoreans' claim to a spherical Earth is perhaps more likely.

There was a first version of Pythagorean/Philolaus cosmology in which the Earth is at the center of the universe containing a "central fire" or "'Hestia," in homage to the immobile goddess of the hearth. But that morphed into the cosmology of Chapter 1 with the "central fire" situated in the center of the universe, relegating Earth to be just another celestial object orbiting around it in circular orbits. Figure 3.10 (a) shows the whole system with the Earth, Moon, Sun, and the planets orbiting counterclockwise around the center and inside an outer shell of the stars. The Earth orbits the central fire once a day and the Sun, once a year. So the Earth daily catches up and passes the Sun accounting for day and night.

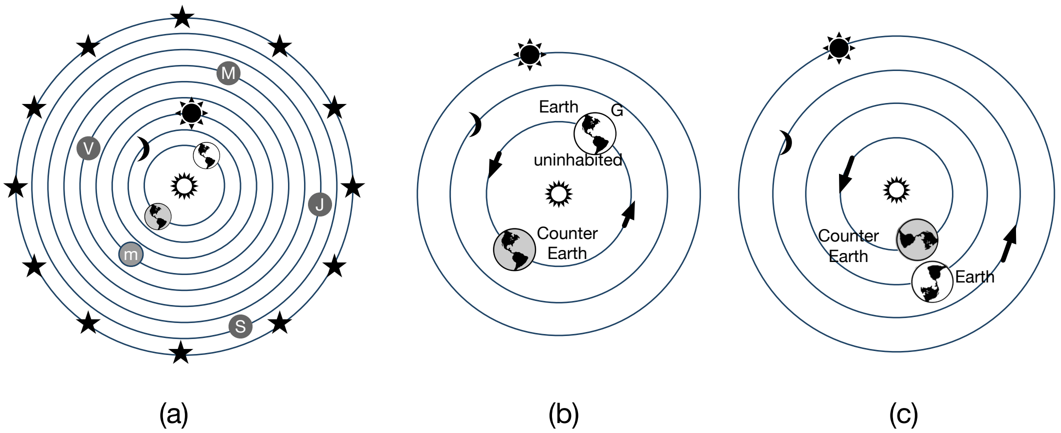


Figure 3.10: (a) shows the Pythagorean system with all of the heavenly bodies and the Earth orbiting the central fire in a counterclockwise sense. In (b) the Earth is shown in one of a number of interpretations of Philolaus' system. Greece (G) is on the far side, leaving the side facing the fire without people. In this orientation it's morning as the Earth is catching up with the slower-moving Sun. In (c) the counter earth is positioned so that it blocks the central fire.

We don't see a "central fire" and there were two proposals as to why, shown in Figure 3.10 (b) and (c). The standard interpretation is the second one in which inhabitants of the Earth are shielded from the fire by the presence of a "counter earth" which strategically blocks it, see J. L. E. Dreyer, 1953. Without the counter earth there are only nine components to the universe and so Aristotle was critical of them for perhaps arbitrarily adding the counter earth just to make the total 10, as suggested in D. R. Dicks, 1970.

This is the first cosmology based on a *regular, circular* MOTION IN THE HEAVENS and a model in which MOTION BY THE EARTH is not zero. The idea of course has spawned 2000 years of astronomical research! Circles, everywhere.

3.3.1 Summary of the Astronomy of Parmenides, Pythagoras, and Philolaus

(Set the context with the timeline in Figure 1.2 on page 20.)

- Parmenides (−514 to −450):

  - He was first to assert that the whole universe was spherical.
  - He was perhaps the first to recognize that the Moon does not shine by its own light, but reflected ("borrowed") light from the Sun. The Pythagoreans might also have realized that.
- Pythagoreans [Pythagoras (−575 to −500) especially including Philolaus (−470 to −385)]:
  - "They" were first to realize that the Earth is spherical.
  - "They" were first to hypothesize a particular ordering of the planets, perhaps with the their orbit size inversely proportional to their speeds.
  - "They" realized that the "morning" star and "evening" star were the same planet, Venus.
  - "They" were to propose a model in which the planets (including Earth and Sun) all orbited a central point (for them, the mysterious "central fire.") in perfectly circular orbits.
  - Their insistence on heavenly motions being uniform and circular outlived their specific model.

3.4 Act VII Plato and Exodus' Models

Plato •Eudoxus •Aristotle
(Set the context with the timeline in Figure 1.2 on page 20.)

In Chapter 1 we touched on Plato's cosmology in *Timaeus* but that was a late development for him as his ideas about the nature of the cosmos grew over almost his whole career. His learning from Archytas in mathematics and the symmetry tendencies of the Pythagoreans launched him in the direction of building everything around circles, and then spheres.

Recall that the *Republic* was nominally a treatise on the nature of justice and how to build a just state which he proposes be totalitarian. When philosophy and political science students read it, they're probably surprised by its ending, which is a full-on Pythagorean cosmology, the "Myth of Er."

> "Once upon a time he died in war; and on the **tenth day**, when the corpses, already decayed, were picked up, he was picked up in a good state of preservation. Having been brought home, he was about to be buried on the twelfth day; as he was lying on the pyre, he came back to life, and, come back to life, he told what he saw in the other world." Plato, *Republic*

Socrates is trying to motivate why someone should live a good life and relates a cosmic carrot-and-stick story, not unfamiliar to other religious admonitions. Er is a soldier who was killed and does what all deceased do... they go to a place where their lives are evaluated, not by St. Peter at the Pearly Gates, but by four judges who tell him that he's got a job to do: after 10 days[13] his body will be retrieved from the battlefield and on day 12 he's to be resurrected from the dead, dramatically on his own pyre before it's lit. He's to tell others what he's seen which includes a

[13]Why 10 days? some Pythagoreanism is maybe showing?

strange vision of a pillar of light that extends to the heavens which Plato describes as a spindle and whorl used for spinning wool. Figure 3.11 (a) shows a Roman woman spinning wool with the weighted whorl at the bottom which spins as she works. Figure 3.11 (b) is the umbrella-like structure (the whorl upside down) that Socrates describes:

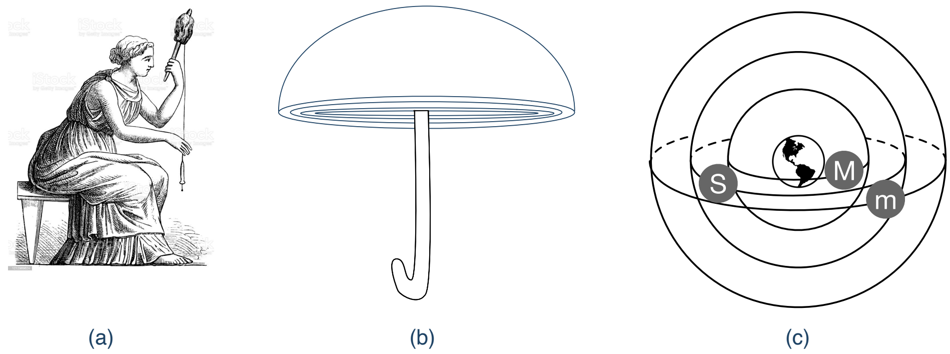


Figure 3.11: The figure in (a) is a Roman sketch of a woman spinning wool using a spindle and whorl, which is the weight at the bottom with a hook. The image in (b) is Plato's description of the whorl actually hollowed out with nested layers of whirl-shaped half-spheres. The image in (c) is the cosmos that the onion-layered whorl represents with the Moon; Sun; and the first planet, Mercury attached to the first three of eight spheres. I've only included three in this cartoon.

> "Its shape was that of (whorls) in our world, but…it was as if in one great whorl, hollow and scooped out, there lay enclosed, right through, another like it but smaller, fitting into it **as containers** that fit into one another, and in like matter another…There were **eight of the whorls** in all, lying within one another…" Plato, *Republic*

The eight "containers" are hinted at in my sketch in Figure 3.11 (b) and the whole is abstracted as nested spheres in Figure 3.11 (c), where I've only shown three spheres (remember, "containers") for simplicity. Earth is no longer a "regular" planet but is in the center with concentric spheres of the Moon, Sun, the outer planets, and again, the stars on the furthest shell, which Socrates says is "speckled." So, Plato's first cosmology has MOTION BY THE EARTH as zero and MOTION IN THE HEAVENS is described as Pythagorean, but using spheres, not just circles. He also tells you how they move and the sounds that they emit as a Siren sits on each sphere and sings a tone. This is the world's first three dimensional cosmological model. But the it didn't match what the planets do and Plato actually tried to remedy it in the *Timaeus*. Given his penchant for not modeling appearances, this was an unusual move and suggests to me that getting it right was (briefly?) important to him.

The *Timaeus* is Plato's "origin story" and in the previous chapter I described the Craftsman's efforts to create matter using geometric three dimensional shapes. It's also his cosmology update from the *Republic* and quite different. Socrates teases the story out of the main character, Timaeus—a Pythagorean—and then uncharacteristically allows the speaker have the floor without much interruption. It's where Plato becomes mathematical, in a spooky, Pythagorean way.

Does this string of numbers mean anything to you: 1,2,3,4,9,8,27? Me neither, but they function as a part of the instructions to the Craftsman in order to build the universe following a numerology algorithm described in a nearly unintelligible paragraph:

> "And he began the division in this way. First he took **one portion** from the whole, and next a **portion double of this**; the **third half as much again as the second**, and **three times the first**; the **fourth double of the second**; the **fifth three times the third**; the **sixth eight times the first**; and the **seventh twenty-seven times the first**." Plato, **Timaeus**

Timaeus is tough to read (impenetrable in some places) and so I've unpacked the algorithm from the paragraph in Appendix A.3.1. The upshot is that the Craftsman has fashioned a universe with two rotating spheres. One of them he calls "the same" and represents the (unavoidable) rotating Celestial Sphere. The other he calls "the different" which is inclined to the first. Those numbers represent the relative sizes of the layers inside of that inclined sphere where the planets are arranged. His Er story didn't account for the ecliptic, and this "different" sphere set is that correction.

> "This whole fabric, then, he split lengthwise into **two halves**; and making the **two cross one another** at their centers in the form of the letter X, he bent each round into a circle and joined it up, making each meet itself and the other at a point opposite to that where they had been brought into contact." Plato, *Republic*

Figure 3.12 is a silly attempt to illustrate this. Figure 3.12 (a) is a person playing with a hula hoop, perfectly aligned so that the axis of the toy's rotational plane points through our person's head. This represents the axis and equator of the Celestial Sphere around the Earth. Figure 3.12 (b) shows just how good this person is at hula hoops: two are rotating, the original, and another that somehow our friend manages to get to rotate at an angle relative to the first one. Some serious hip-action would be required. This represents the ecliptic, inclined by that spacing corresponding to the latitude of the observer. Those strange numbers? Well, there would actually be seven hoops with diameters proportional to those numbers: 1–2–3–4–8–9–27. Figure 3.13 shows what this is really about.

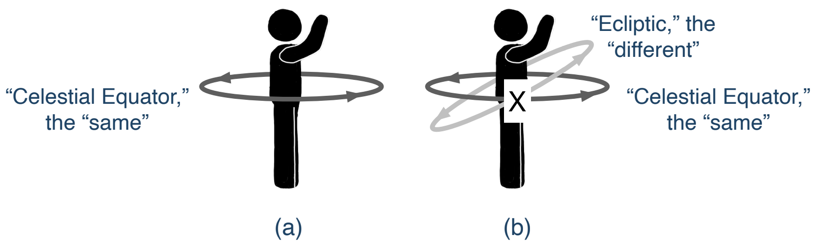


Figure 3.12: Pretty good hula hoops chops.

The celestial sphere and its axis I've called the NCP (north celestial pole) in the

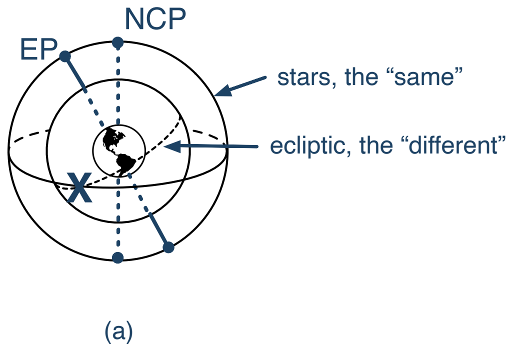


Figure 3.13: (a) shows the two spheres with their equators. One the Celestial Sphere (carrying the stars around the Earth each night, so an axis centered on the North Pole of the Earth) and the other is the ecliptic (in which the planets reside as they appear to go around the Earth) with the pole of that sphere, the North Ecliptic Pole. (b) takes away the three-dimensional view and will be a useful sketch for these kinds of constructions in what follows.

diagram. The other strip is the equator of the other, ecliptic, sphere (with axis labelled EP) which makes an "X" where it crosses in two places with the Same. (These are the points of the equinoxes, when the Sun on the ecliptic crosses the Celestial Equator.) Inside of this strip, the segments correspond to the locations of the Moon, Sun, Mercury, Venus, Mars, Jupiter, and Saturn. Of course, this is a little mad but Eudoxus took on the task of turning this story into a geometrical model.

3.4.1 Eudoxus' Model

By the time Eudoxus had returned to the Academy, he would have been familiar with the *Republic* and probably *Timeaus*. Once Plato had inserted the ecliptic path, he still needed to explain retrograde motion. And he knew it:

> "...as for the dances of these and how they relate to each other, the **backward-cycles and forward-progressions** of the circles to each other...to speak without visual representations of these same would be a **vain effort**." Plato, *Timaeus*

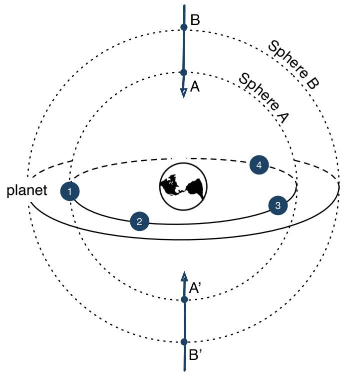
So, he realized the problem...but had no solution and just gives up ("vain effort"). He was out of his depth but Eudoxus was ready and came up with a brilliantly complex model and while it's not known what Plato thought of it, it's clear how Aristotle reacted: he made it his. It's intricate, so let's go to the box and work out the inner workings of the idea and then skip to the end. Look at Figure Box 3.14 on page 115. After you've read the material in that Box, return to this point ☟ and continue reading.

The figure in Box 3.14 describes the tool-kit that Eudoxus used to construct a full model of each planet in which they ride on the equators of coupled, spinning spheres. The two spheres shown in the box form the minimal number of moving parts unique to every planet and they are each embedded inside of two other spheres, one for the ecliptic whose equator includes the rough paths of the planets and the other is the Celestial Sphere which includes the motions of the stars around the Earth every nearly 24 hours. Let's take it slow in Figure 3.15.

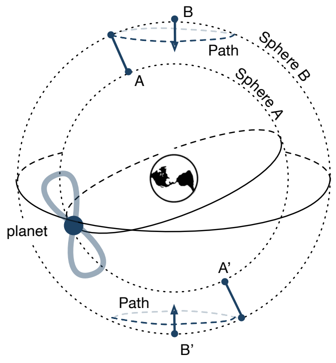
The fundamental Eudoxus set was four spheres, centered on the Earth. Using the nomenclature from Figure 3.15 and Box 3.14, labeling them from the inside out:

- A: the sphere to which the planet is attached,
- B: the next sphere which precesses around that inner sphere (producing Eudoxian figure-eight)
- C: the sphere that rotates around the ecliptic—that stretches out that Eudoxian figure 8 in Figure 3.14 to produce retrograde motion, and
- D: the outer-most sphere that rotates daily showing the pattern of the starry Celestial Sphere.

FIGURE BOX 3.14



(a)



(b)

The model that Eudoxus created is an impressive bit of geometry mixed with inspired imagination. It's the famous "nested spheres" model that made it all the way to the Baroque as an explanation for the odd motions of the planets. In a very modern way, it's full of parameters that could be tweaked to make it fit the observations...some of which he made himself at the observatory he created in his school before he returned to Athens.

Imagine taking two hoops, one of which is slightly smaller than the other and is attached inside the larger one across their mutual diameters. Figure 3.14 (a) shows this with a "planet" attached to the equator of the inside hoop. Now if we spin that hoop around its axis AA' the planet will follow a circle from position 1 through 2, 3, 4 and so on. This spinning *observed from the outside* essentially defines a sphere, Sphere A, here centered on the Earth. If the two hoops are attached, and if the outer hoop spins around its axis, BB', creating the surface of Sphere B, then the motion of the planet will be the sum of the two speeds at the hoop pair equators. So if the outer hoop spins at the same rate as the inner hoop, but in the opposite direction, then the planet would appear to the Earth to remain stationary at position 1.

Now imagine that the axis of the inner hoop is attached at a point offset on the surface of the Sphere B as shown in Figure 3.14 (b). Now when Sphere B spins, it takes the AA' axis of Sphere A around with it tracing the path shown. In addition, if Sphere B spins while its following that path independently, the motion is a complicated figure eight pattern as shown. Eudoxus figured this out and named the shape a "hippopede" which is "horse fetter" in Greek. (A fetter is like a chain.) Now there are many variables at work which would alter the shape of the hippopede: the speeds of the two spheres and the angle at which AA' axis of Sphere A is inclined to the BB" axis of Sphere B.

Now go back to page 113 and pick up where you left off.

All of these separate motions are coupled. . . and that's just for one planet! By tuning the inner two spheres' rotation speeds and the inclination of their inner axes, the

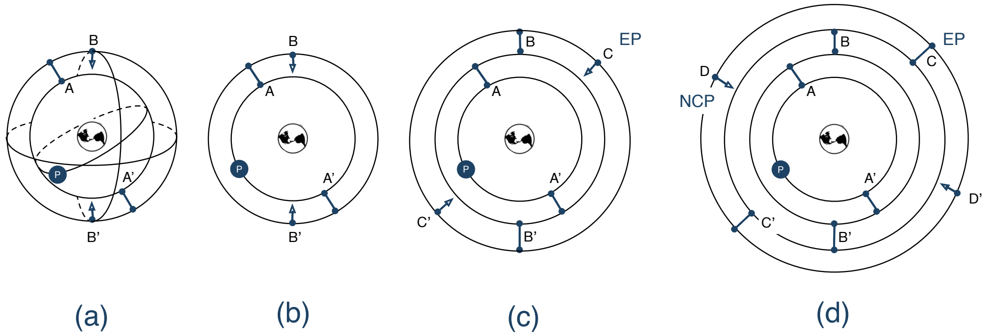


Figure 3.15: (a) is a slightly different rendering of Figure 3.14. (b) is an abstraction of (a) taking out some of the lines that suggest a solid sphere, for clarity. (c) includes the sphere of the ecliptic (EP for Ecliptic pole is shown) with axis of rotation CC′. Notice that it's attached to the outer sphere of Eudoxus' tool-kit pair. And (d) includes the sphere of the outer stars, the celestial sphere (NCP for the North Celestial Pole is shown) and the ecliptic shere is attached to it.

motions of the planet can be made to do the figure-eight dance at just the right time of year and with the right elongation in the sky—to make the planet appear to reverse direction and recover, and resume as viewed by the Earth. Each planet required four spheres and the Sun and Moon required three each, plus the Celestial Sphere: 27 spheres to do the job. This was a mammoth intellectual puzzle that Eudoxus created and then solved with those relatively simple pieces of interlocking spheres.

It still didn't quite do the job as well as it might and in the best tradition of what Thomas Kuhn would have called "Normal Science," **Callippus of Cyzicus** (−370 to −300) tried to make it better without starting over. He was a student of Plato's and worked with Aristotle and worried about the seasons' length problem and some finer points of the planets' motions. He added two additional spheres for the Sun and Moon and one each for Mercury, Venus, and Mars for a total of seven more. So now: 34 spheres. Was it all just an exercise in geometry? Perhaps. The Eudoxian program of research was abstract without numbers and so no predictive capability. It might indeed have been more of a story than a scientific model, like Plato, and like Aristotle's will be.

Around −370, Eudoxus also apparently created a star catalog in his book *Phenomena* of at least 47 stars which a century later were memorialized in the famous poem of that same name by Aratus that I introduced in the preface to this chapter. These entries were not numerical or with coordinates, but were story-like recording the times of the rise, set, and position overhead of constellations or stars near parts of constellations. For example, "As a guide the Ram and the knees of the Bull lie on it, the Ram as drawn lengthwise along the circle, but of the Bull only the widely visible bend of the legs. On it is the belt of the radiant Orion and the coil of the blazing Hydra, on it too are the faint Bowl, on it the Raven, on it the not very numerous stars of the Claws, and on it the knees of Ophiuchus ride. It is certainly not bereft of the Eagle: it has the great messenger of Zeus flying near by; and along it the Horse's head and neck move round."(Dennis Duke, 2008). What we know of Eudoxus' catalog come to us from Aratus and the later Hipparchus' critique of the poem and by extension, Eudoxus' work.

3.5 Act VIII Aristotle's Model

When it came to astronomy, Aristotle was downright derivative. Ironically, his model that became Church dogma wasn't his, and to make matters worse, it was flawed and largely ignored soon after he died. How it went from forgotten to dogma is the story of Chapter **??**, but let's see what he actually did and why. His astronomical writings were scattered throughout two large books, *On the Heavens* and *Meteorologies* and his solutions to known problems were a mixture of pure metaphysics, physics—often relying on his own rules of motion as authoritative,—and the observations of others. Aristotle didn't observe the heavens.

3.5.1 Properties of the Earth, Aristotle-style

Aristotle vigorously disagreed with the Pythagorean/Philolaus cosmology in which the Earth orbits the center of the universe and devised challenges defending a stationary Earth that any future moving-Earth proponent would have to meet squarely.

The Earth Pythagorean/Philolaus adherents proposed that the Earth is spherical, arguing largely from aesthetic grounds, namely that circles are good and spheres are good and so the Earth should be spherical as well. Oh, and that the universe is spherical and so must be the Earth.

Aristotle proposed multiple, more concrete reasons why. First, when one observes a lunar eclipse, one sees that the shape of the demarcation between light and dark is always convex. So if the Earth's shadow is the explanation for the eclipse, then the Earth must be at least circular, if not spherical. He knew from reports that people in the southern latitudes saw different stars on their horizon than those in the northern latitudes. He argued against those who insisted (still) that the Earth was flat by noting that the horizon looks flat, but that's simply because the Earth is large.[14]

He also had a physics reason. Since earthy material would naturally be aimed at the center of the universe then all earthy material would be drawn to a single point and highly compressed equally in all dimensions with the result: a sphere of earthiness. That sphere would be surrounded by a thick sphere of water. That would be surrounded by a sphere of air and then fire. So a spherical double-double-decker sandwich of the four terrestrial elements filling up the whole volume below

[14]Nowhere in Aristotle is the famous alleged argument attributed to him that when ships begin to appear on the horizon that first the mast and then the hull are observed.

the Moon, the "sub-lunar realm." This argument supported two other Aristotelian–imperatives: that the Earth finds itself in the center of the universe and that it's stationary.

The Stellar Parallax Argument Finally, he makes a good argument for the stationary Earth which becomes the essential challenge to any future moving Earth cosmology.

Look at a point across your room with one eye closed and put your finger in front of you and notice what's behind it on a wall or distant surface. Now switch eyes and notice that the what's behind your finger now seems to have moved. If you open and close each alternate eye successively, the background will appear to jump from side to side relative to your finger. This is called "parallax" and it's because your eyes are attractively located inches apart from one another on your face and enough so that the lines of sight from each are slightly different.

If the Earth is orbiting a center, then at one point of the year a particular star would appear as a line at a particular angle (like your right eye open). Then at the half-way-point around its orbit (six months later if the orbit is around the Sun), when the Earth is on the other side of that center (like your left eye open), look for that same star and it will be at a completely different angle. "**Stellar parallax**" or "annual parallax" is the name of this phenomenon and we'll see it more than once in our story.

Nobody observed stellar parallax leaving only two explanations. Either the Earth doesn't move around a center of revolution, or the stars are so far away that parallax isn't visible. Nobody was prepared to imagine a universe that big, and so the conclusion was that MOTION BY THE EARTH is zero.[15]

He agreed with Parmenides and the Pythagoreans that the light from the Moon is reflected light, that the shape of the crescent of the Moon's phases suggests that the it must be a sphere. From that and his spherical Earth hypothesis, he reasoned that all of the heavenly bodies are likely spherical, albeit made from different stuff.

For millennia, Aristotle has been held responsible for the theory of five elementary substances: in *On the Heavens* he added what he called the "first body" to the familiar earth, water, air, and fire. Much later this was renamed "the fifth element;" and later, the "**aether**;" and later than that, the Latinate, "quintessence." In spite of almost all popular and even scholarly sources, Aristotle never identifies his first body as "aether" although he was surely aware that Plato used that term explicitly. History assigns Cicero from the first century BCE, as the source of Aristotle's reference to "aether" with the assumption that famous Roman orator had access to now lost Aristotelean manuscripts. Or, given our repeated reminder that much of what we know of the Greeks is muddled...it's possible that Aristotle never used the word. I'll use "aether" as it will become a useful contrast with the 19th century "ether," the direct experimental lead-in to Relativity. And, by the way: Aristotle is often said to

[15]It took until the 19th century to actually observe stellar parallax because the universe really is that big.

have insisted that the Eudoxian spheres were crystalline, the "Crystalline Spheres" were indeed an assumption in Medieval and Renaissance times, but nowhere does Aristotle refer to this. (See, David E. Hahm, 1982)

Aristotle's aether is eternal, not composite, neither heavy nor light, and is the most divine of all of the heavenly objects. So it's not anything like the four Aristotelian elements, but for some reason he holds heavenly objects to some of the same physics as terrestrial objects.

The Sky The heavens differ from terrestrial objects in an obvious way: the night sky repeated, every night, while everything on the Earth seems less ordered. Sure falling objects executed their motions according to rules, but every object's behavior is different so the eternal permanence of the heavenly motion contrasts with the impermanence and changeability of MOTION ON THE EARTH. Furthermore, for Aristotle natural motions near the Earth were in straight lines—with a beginning and an end. But the motions of the heavenly bodies seem circular, and so, never-ending...eternal. Obviously, then, the deep sky is made of special, different stuff.

Aristotle's universe is a finite volume in space all the way to the outermost starry sphere, like that of the Pythagoreans. Furthermore, it's always been there and he speculates on and rejects an argument about the possible creation of the universe. So he disagrees with Plato. That for him would presume that before that event, there was already a notion of up and down and that bothered him. So, the universe is a finite volume in space, but of infinite extent in time.

3.5.2 Aristotle's Cosmology

The basic features of Aristotle's cosmology were the same as Plato's as were his ordering of the planets (and different from what Philolaus assumed for the Pythagorean model): Earth–Moon–Sun–Mercury–Venus–Mars–Jupiter–Saturn and the stars. Ever the mechanist, he worried about real material concerns: *how* do they *actually* move as a composite unit?

First, he knew that what was required was a model of the whole universe—Eudoxus' model was a template for each planet, not a whole cosmos— and so each of those sets of spheres needed to all be packaged together into one big onion of spheres, one set inside of another. And this became his problem: since he couldn't have Jupiter's motions affecting Saturns and Mars' motions, he needed to "mechanically" decouple each one.

Remember that I noted that if you had two connected Eudoxian spheres rotating at the same speeds, but in opposite directions, that their motions would cancel one another. Aristotle took that idea and intentionally inserted "rewinding spheres" to do that in such a way to preserve the spheres' connections to the ecliptic and celestial spheres but to isolate them.

Table 3.1 shows that for all of the planets but the Moon and Sun, four spheres were sufficient for Eudoxus. (The Sun and Moon didn't need the daily, celestial sphere rotation.) Callippus added spheres for the inner planets, Sun, Moon, and Mars. It was these 33 spheres that Aristotle then tried to turn into an actual seven-object, whole system.

Table 3.1: The number of spheres for each of the Eudoxian systems for the Moon, Sun, and planets—not including the outer sphere of the fixed stars— with the Aristotelian unwinding spheres counted separately in the last column.

Planet	**Eudoxus**	**Callipus**	**Aristotle**	**Unwinding**
Saturn	4	4	4	3
Jupiter	4	4	4	3
Mars	4	5	5	4
Sun	3	5	5	4
Venus	4	5	5	4
Mercury	4	5	5	4
Moon	3	5	5	
Total:	26	33	33	+22 = 55

> It is necessary, if all the spheres put together are going to account for the observed phenomena, that for each of the planetary bodies there should be other counteracting ["unrolling"] spheres, one fewer in number [than Calluppus]...for only thus is it possible for the whole system to produce the revolution of the planets." Aristotle, *Meteorologies*.

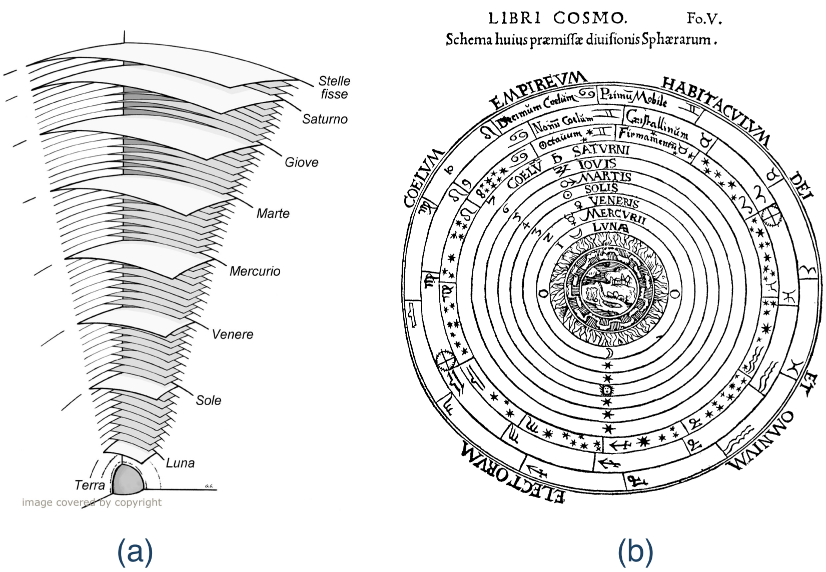


Figure 3.16: (a) Representation of the 55 spheres of Aristotle's model. Notice that Jupiter (Italian, Giove) has one too many layers and that the Moon (Luna) is depicted as having none. (Museo Galileo. (b) is a typical Medieval representation of the Aristotelean cosmology.

Figure 3.16 (a) shows a rendering of the 55 Aristotelean spheres (from https://brunelleschi.imss.fi.it/vitrum/evtr.asp?c=8252. (b) shows a typical Medieval picture of Aristotle's cosmology, the Prime Mover is noted (see below), and in the center, the four Aristotelean elements are drawn. But there's an interesting difference: the planetary order is not Aristotle's but from later.[16] Again, he was always fascinated with his own ideas about motion and for some reason, he assumed that bodies made of the completely unique aether still needed to follow his physics and causal rules. Why didn't he just say that aether spheres just naturally isolate themselves, one set from another?

In that same sticking-to-the-terrestrial-rules spirit, he seemed believe that the spheres needed a cause in order to execute their natural, circular motion and that drives his model into strange places. Just like *unnatural motion* for terrestrial objects required a contact pusher, inexplicably he decided that the *natural, circular motion* of his spheres *also needed contact pushers*. That creates an embarrassing regress problem. Every sphere had its very own pusher and so did the outer, star sphere, but how does that last pusher itself remain stationary in order to be able to move that last sphere? Another pusher? He complicated this by insisting that the pushers had themselves no substance, were outside of space and time, and were essentially pure intellect. He called them "unmoved movers" or "Prime Movers" and the idea was a soft toss to Thomas Aquinas 1600 years later to equate the Primer Mover with the Catholic deity.

Aristotle's astronomy is underwhelming and unsatisfying and it didn't solve the major issues endemic to an Earth-centered cosmology: since the model required each planet to be always the same distance from Earth, why do they vary in brightness? And a relatively new problem in his time: why are the seasons, autumn, winter, spring, and fall, all of different durations? These brought Aristotelean modeling to a halt. New ideas were required.

3.5.3 Summary of the Astronomy of Plato, Eudoxus, and Aristotle

(Set the context with the timeline in Figure 1.2 on page 20.)

By the time that Aristotle was done, astronomy had converged on a qualitative, "picture-model" built by two philosophers and a mathematician.

- Plato (−427 to −348):
  - He placed the Earth is at the center of the universe.
  - He modeled the planets as attached to spinning spheres.
  - He proposed that the outer star-sphere spins around the Earth once a day.
  - He placed the sphere of the planets to be inclined to that of the stars so that they all orbit at an angle inclined to the Earth's equator—on the ecliptic.
- Eudoxus (−390 to −340)

[16] Aristotle seems to have made at least one mistake and actually had two models, one of 47 and the other of 55 spheres. Nobody knows why.

  - He modeled each planet's motion as created by four spheres, with axes inclined to one another to replicate retrograde motion and motion relative to the stars. (The Sun and Moon only needed three spheres.)
  - He modeled each planet's model as separate from the others and he did not propose a whole solar system, just pieces.
  - Callipus added spheres for some of the planets in order to slightly tune some of the motions to better match observation.
  - He apparently created one of the first published star catalogues, memorialized in the poem by Aratus, *Phaenomena*.
- Aristotle (−384 to −322):
  - He adopted Eudoxus and Callipus' approach in order to model all of the planets by piecing together the Eudoxian sets of spheres, one inside of the other from Saturn to the Moon.
  - Since each is tied to the one beneath, Aristotle felt that additional spheres were needed in order to isolate the motions of the planets from one another. These were the rewinding spheres.
  - He insisted that the volume outside of the orbit of the Moon was made of a different element from the four elements that operated within. That fifth element, the aether, filled the remaining volume to the outer stars, providing the material of the heavenly bodies. Natural motion in the aether is perfectly circular.
  - He originated the idea that the universe was "full" of the aether—-no gaps or emptiness. This demand became necessary in all future Greek cosmologies.
  - Aristotle's physics guided (or handcuffed) speculation about any motion that the Earth might have had. The Earth had to be in the center of the universe, not spinning, nor orbiting any point.
  - He was very critical of the Pythagorean idea of an orbiting Earth for (his) physics reasons, but also because there was no apparent parallax which meant that the stars were so far away as to hide parallax (too far for anyone's taste) or that the Earth was stationary.

Modeling of this sort stopped after Aristotle as there were problems with any model in which the planets orbit in perfect circles with their common center on the Earth:

- The seasons would all have the same durations, but everyone knew that was not the case.
- The brightness of the planets would not change, but everyone knew that was not the case.
- The ordering of the planets was arbitrary.

3.6 A Little Bit of Hellenistic Astronomy

Euclid •Aristarchus •Eratosthenes •Archimedes •Apollonius •Hipparchus •Ptolemy
(Set the context with the timeline in Figure 1.2 on page 20.)

There were two basic thrusts after the fanciful modeling of Plato, Eudoxus, Callippus, and Aristotle. Hellenistic astronomy became both observationally intense—data collection became sophisticated— and mathematically sophisticated, culminating with Claudius Ptolemy's enduring model in the second century, CE. Let's unwrap this extraordinary period of Alexandrian astronomy and set the stage for 1500 years of surprisingly authoritarian science.

3.6.1 A Moving Earth

Heraclides of Pontus (-387 to -312), from the southern coast of the Black Sea, was a contemporary of Plato and Aristotle. As the son in a wealthy family and an apparently smart young man, was able to emigrate to Athens where he became a favorite student of Plato's and was put in charge of the Academy when Plato went on his last, ill-fated trip to Syracuse. He also studied with Aristotle (who was 10 years his senior) and the Pythagoreans in Athens, so he was fully rounded in the three major pillars of classical Greek philosophy. Plato died in -348 and his successor, Speusippus, died in -339 and when Heraclides lost the election for the next leader, he returned north to Pontus. That's where he probably did his astronomy where he had two good ideas, neither of which went anywhere for 2000 years.

It should have bothered Aristotle that his model required the outside starry sphere to be rotating at an astonishing rate in order to make it all the way around each day. The obvious alternative was a spinning Earth and stationary stars and Heraclides proposed just that.

His other imaginative idea addressed a second interesting fact: Mercury and Venus have a different relationship to the Sun from all of the other heavenly bodies. They seem to cling to it, appearing and disappearing as the Sun rises and sets. It was Heraclides who first suggested that this special relationship could be explained by making those two inner plants satellites of the Sun. His cosmology was that the Earth is at the center of the universe, spinning on its axis, orbited by Sun as "normal," but the Sun in turn was itself a second center of rotation with Mercury and Venus orbiting it. Aristotle's grip was not universal, even in his own time.

3.6.1.1 The Greek Copernicus

While Heraclides could be thought of as ushering in the post-Athens, Hellenic era, it was **Aristarchus of Samos** (-210 to -230), a toddler when Heraclides died, who conceived the best model of the universe and a completely new way to deal with the cosmos: by measuring it. He studied with Strato of Lampsacus, who was the third director of Aristotle's Lyceum, and when Strato went to Alexandria to tutor and counsel Ptolemy II he brought Aristarchus along as his pupil. Strato returned to Athens, but Aristarchus stayed in Alexandria and did his mathematics and astronomy in that growing Greek-Egyptian intellectual center. He probably overlapped with the senior Euclid and surely learned all of Greek mathematics known to that time, conceivably from its most famous chronicler. He fashioned his single surviving text *On the Sizes and Distances of the Sun and the Moon* like Euclid's *Elements*: propositions followed by orderly proofs.

As the Moon orbits the Earth half of it is always illuminated, but we see phases as it makes its way around us. From our modern understanding, Figure 3.17 (a) shows the named phase states as we see them. When it's on the other side of the Earth from the Sun and we're in nighttime, we see it fully illuminated ("full Moon"). When it's between us and the Sun ("new Moon") we don't see it at night (after all, we're looking away from the Sun at night), but can sometimes see it during the day. In between, it shows us partially illuminated crescents. But look at the two quarter Moons. From Earth, at exactly that point we see the Moon split into two equal halves, one dark and one bright.

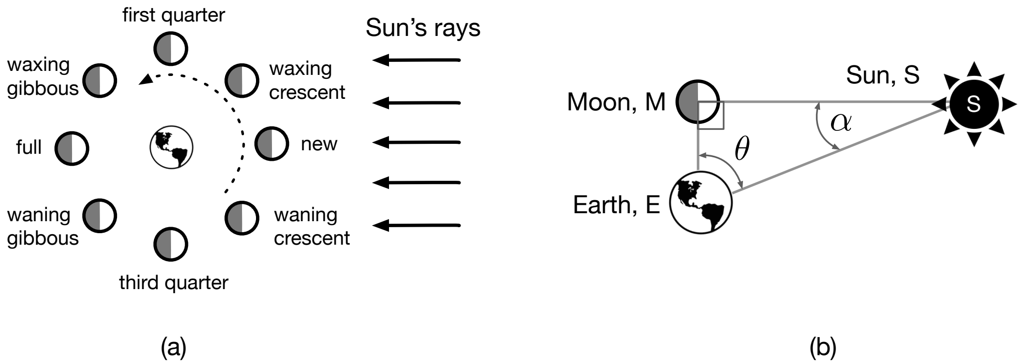


Figure 3.17: The Moons phases and positions are shown in (a) relative to the Earth and Sun. From this vantage point, the Moon orbits counterclockwise. In (b) the particular position and phase that makes the Aristarchus calculation possible with the right angle shown occurring at just the first or third quarter when the Moon is half lit.

While Aristarchus didn't anticipate the Moon orbiting the Earth, he did realize that this quarter phase had a particular geometric arrangement with respect to the Sun and Figure 3.17 (b) shows his idea. At that moment, the angle between the Sun and the Earth is a right angle, $\angle EMS = 90^\circ$.

> "...when the Moon appears to us halved, the great circle which divides the dark and the bright portions of the Moon is in the direction of our eye...when the Moon appears to us halved, its distance from the Sun is less than a quadrant by one-thirtieth of a quadrant." Aristarchus, *On the Sizes and Distances of the Sun and the Moon.*

By "distance from the Sun" he means angle α in the diagram, $\angle MSE$. With a modest amount of modern trigonometry, it's possible from the angles to calculate the ratio of the distance of the Earth to the Sun to the distance of the Earth to the Moon in one line. Without modern trigonometry it's a straightforward exercise in geometry. Aristarchus did just that and found:

$$\frac{\text{Distance, Earth to Sun}}{\text{Distance, Earth to Moon}} = 19 - 20$$

where the range is his own estimate of how well he could determine the angle. Appendix A.3.2 completes this calculation and some other interesting measurements that he and others made. This are stunning in their originality and also in their simplicity. He also subsequently calculated three additional things about the universe, for a total of four groundbreaking conclusions:

1. the distance of the Earth to the Sun) $\approx 20\times$ distance of the Earth to the Moon
2. the diameter of the Sun $\approx 19\times$ the diameter of the Moon
3. the diameter of the Earth $\approx 2.85\times$ the diameter of the Moon
4. the distance of the Earth to the Moon $\approx 10\times$ the diameter of the Earth

His mathematics and methods are correct but he had some mistakes, crucially because α is very hard to measure and so his determination of $\theta = 87^\circ$ was wrong...it's actually closer to 89.853° which makes the distance of the Earth to the Sun) $\approx 390\times$ distance of the Earth to the Moon.[17]

But that's not all. Let's let Aristarchus' Italian/Greek contemporary **Archimedes of Syracuse** (-287 to -312) take over from here:

> "Aristarchus has brought out a book consisting of certain hypotheses, wherein it appears, as a consequence of the assumptions made, that the universe is many times greater than the "universe" [expected]...**His hypotheses are that the fixed stars and the sun remain unmoved, that the earth revolves about the sun on the circumference of a circle, the sun lying in the middle of the orbit,** and that the sphere of fixed stars, situated about the same centre as the sun, is so great that the circle in which he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the centre of the sphere bears to its surface." (emphasis, mine) Archimedes, *The Sand-Reckoner*.

Aristarchus was apparently the first to envision a Sun-centered ("heliocentric") universe and, oh by the way he also apparently adopted Heraclides' notion of a spinning Earth. Copernicus-in-training. Nobody knows how he came to this conclusion...even though it solves many of the problems (planets' brightness, for example). His model was largely ignored and the fact that Archimedes tossed that reference off so casually is indicative of what must have been an overwhelming concern for the parallax problem (which is a prejudice about the possible enormity of the universe) and Aristotle's authority when it came to terrestrial physics.

But there it is: the first modern-sounding MOTION BY THE EARTH and MOTION IN THE HEAVENS . Copernicus later took comfort in Aristarchus' idea.

This is an auspicious moment! Aristarchus'work ushers in the beginning of quantitative astronomy which was quickly taken up by his contemporary, **Eratosthenes** (-276 to -194), who became the Chief Librarian of the Alexandria Library

[17] The point of First Quarter would be in the same part of the sky as the Sun, just before Sunset. Without modern tools, measuring that angle would essentially impossible, if not dangerous! James Evans, 1998 suggests that Aristarchus concocted the "one-thirtieth" as an extrapolation of the time that it takes for the Moon to reach the First Quarter as the largest angle that could come from a month of 30 days to orbit and one quarter of that for the phase. That's almost even more impressive reasoning.

just following Aristarchus' death. (He was also a geographer, mathematician, astronomer, and a poet. The nickname given to him was Pentathlos, implying a Greek pentathlon athlete of many talents.) With his access to Library data, Eratosthenes learned that at noon on the summer solstice (the first day of summer) in Syene, Egypt, the Sun's rays were known go right into a vertical well without hitting the sides. Syene (modern day Aswan) has a latitude of just about 24° which is at the northern tropic, the Tropic of Cancer which means at the Summer Solstice, the sun is directly overhead (the definition of the Tropic of Cancer) and so would not cast a shadow from a vertical stick in the ground. Meanwhile, Alexandria is directly north of Syene at the same longitude and so Eratosthenes reasoned that the Sun is so far away that it's okay to presume that its rays were parallel at both cities. Therefore, for a spherical Earth, the shadow of the Sun on a vertical stick in Alexandria would cast a shadow. He measured it rather than the 0° at Syene, it was 7.2° at Alexandria. That angle is 1/50th of the 360° of a circle so that the circumference of the Earth must be 50 times the distance between the two cities, which is 833 km (in modern units). Fifty times 833 km is 42,000 km for Earth's circumference— only a few percent higher than a more modern value! Appendix A.3.2 shows this calculation.

Eratosthenes wasn't done. He also devised a way to measure the obliquity of the ecliptic—that angle 23.5° of inclination of the ecliptic from the Celestial Equator. And he made a star catalog of 650 stars. And he wrote a poem about himself. He reportedly went blind in his old age and chose to commit suicide as a result.

So for the first time, astronomers learned the size of the Earth and more could be learned: for example, using Aristarchus and Eratosthene's results, from Aristarchus' #3 above they could conclude that the diameter of the Moon is 4700 km, where the actual value is about 3500 km.

> ▻ I hope you can appreciate that Greek astronomers are no longer merely telling stories. They're measuring our universe.

3.6.2 Casting Aside Aristotle and Eudoxus

The next important step is another storyteller, but an important mathematician who had a good idea. **Apollonius of Perga** (−240 to −190) migrated from Turkey to Alexandria as a young man to study in the successor school of Euclid. "The Great Geometer" became his historical label and he's remembered for discovering the mathematics of "conic sections" (circles, parabolas, ellipses, and hyperbolas)—a subject beyond Euclid's geometry.

For our story we know of him as the geometer who puzzled over the seasons problem and found a way to modify the Eudoxian model to loosen the requirement of all spheres centered on the Earth. His discovery is shown in Figure 3.18 (a) in which E shows the location of the Earth, S is the location of the orbiting Sun, and D is a point in space—attached to no object— which is displaced from E. The distance $\overline{\mathrm{EC}} = e$ is called the **eccentricity**. The Sun uniformly follows the dashed

eccentric circle, centered on D and not the Earth! Notice that the result is a Sun's path sometimes further from, and sometimes closer to the Earth. When it's further, it would take longer to go halfway around and so the seasons during that path segment would be longer.

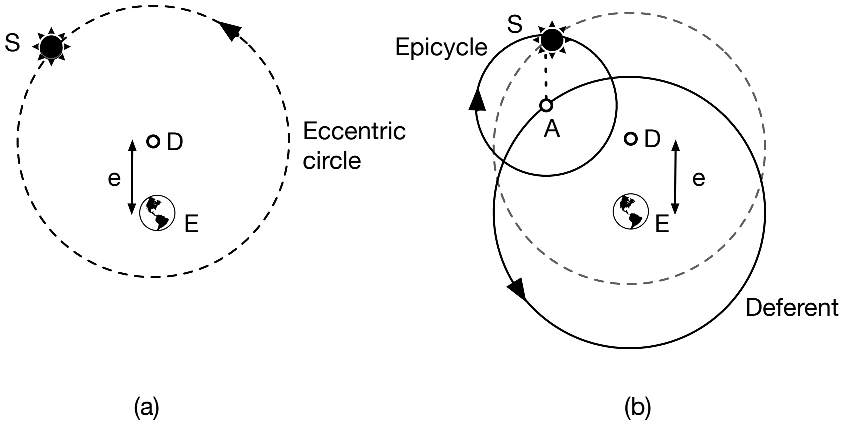


Figure 3.18: In both figures, E is the location of the Earth and S is the location of the Sun. In (a) an eccentric circle is shown for a proposed Sun orbit around the Earth. By putting the center at a spot in space displaced from the Earth by the eccentric, *e*, the seasons would appear on Earth to be of different durations. In (b) the equivalent (under the conditions described in the text) epicycle solution is shown with an overlay of the eccentric circle shown in a light dashed line for comparison. The deferent is centered on the Earth and the epicycle is centered on the rim of the deferent.

But there's more to this as Apollonius discovered a geometric equivalence also illustrated in Figure 3.18 (b). Here a circle, called the **deferent** is centered on the Earth but doesn't act as an orbital path for the Sun. Rather, the Sun rides on another circle, the clockwise rotating **epicycle** with its center (A) attached to the rim of the counterclockwise, rotating deferent. Notice that the rotational sense (here, clockwise) of the epicycle is opposite to that of the orbit of its center, A, on the deferent. Each of these models would cause Earth to experience more Sun during part of the year and less Sun the other parts, which would change the length of the seasons.

The idea of an epicycle is not easy to grasp since we don't use them any more in planetary astronomy. But if you look up some night, you'll see an example of an epicycle. Think modern (for a moment): we know that the Earth goes around the Sun and that the Moon goes around the Earth. The Moon' s orbit can be thought of as an epicycle: the Earth's (nearly) circular orbit around the Sun would be the deferent and the Moon's orbit around the Earth is the epicycle. So looked at from the Sun, the Moon's orbit would be a slightly off-center orbit around the (orbiting) Earth. This particular epicycle is one in which in Figure 3.18 (b), E coincides with D. We're going to meet epicycles in a major way when we get to Ptolemy and Copernicus.

Numerical predictions were not the goal for Apollonius, but a more realistic frame-work was—and probably the geometry was also an attraction for him. So his ideas were one more step away from Aristotle toward a new way of doing science.

3.6.3 The Greatest Astronomer: Hipparchus

The most celebrated astronomer of antiquity was, yet another Greek about whom we don't have many biographical details. However, **Hipparchus of Nicea** (about -190 to about -120) was so accomplished that his feats were detailed in later Hellenistic astronomy texts and most completely two centuries later by Ptolemy. His mature astronomy work appears to have been done on the island of Rhodes a large island to the west of Cyprus and far from his home near Constantinople. There he built an observatory and created or improved on instruments for measuring positions of stars and planets. He was a serious observer of astronomical objects and events and a mathematician of significance. Finally, the world was ready for a complete astronomer...The Greatest Astronomer, he was later called.

Let's be clear: **astronomy was different after Hipparchus**. He dedicated himself to an entirely different purpose from the "picture-stories" of Plato and Aristotle. Hipparchus measured numerical features of the cosmos.

Hipparchus' Solar Model. Hipparchus figured out that if he used the eccentric model only a few parameters were required in order to determine, e and so the problem of the seasons' unequal durations could be solved geometrically, almost like being a cosmic surveyor. His model is shown in Figure 3.19 with the anchor for astronomical positioning, the Vernal Equinox (VE, ♈) (a convention used to this day). The Sun (⊙) orbits the center of the eccentric orbit at C and the Earth is displaced by the eccentricity, e (which is usually quoted as the fraction of the distance $\overline{CE}$ to the radius, $\overline{CA}$). The dash-dot lines denote the axis from the Vernal Equinox (mid-March) and the Autumnal Equinox (AE, mid-September) and the Summer Solstice (SS, mid-June) and the Winter Solstice (WS, mid-December) and the four unequal quadrants delineate the four seasons. Here it's drawn for antiquity in which spring was the longest season and autumn was the shortest (while in our time summer is longest and winter is shortest). In astronomy, the furthest point of a celestial object's orbit from a reference is called the "**apogee**" and the closest approach, the "**perigee**." The figure shows the arrangement for antiquity, when the angle of the dotted line through E and C was about $\alpha = 65^\circ$. Today, it's greater than 90° which is why our summers are longer than antiquity's summers.

His result was that the eccentric is displaced from the Earth by about 1/24th (about 0.04) of its orbital radius so it's almost a circle centered on Earth, which is why the season durations are within a few days of one another.[18] Notice that our summer and spring is when the Sun is at apogee and fall and winter are at perigee.[19]

[18] Had $e = 0$, then all four season would have been the same length and the Sun's orbit would have been Aristotle-like, centered on the Earth.

[19] Why the Sun is *furthest* away during the summer is a reasonable question and understanding that waited for Kepler and Newton.

Hipparchus could use his solar model to predict the location of the Sun at any time in the future and it was accurate and used for many hundreds of years.

Hipparchus' Lunar Model. The Moon's motion is different and more complicated than the Sun's with at least three parameters required to determine its motion. He managed that as well, this time using an epicycle model. Finally that legend ascribed to Thales from 400 years before is made whole: Hipparchus could predict both solar and lunar eclipses!

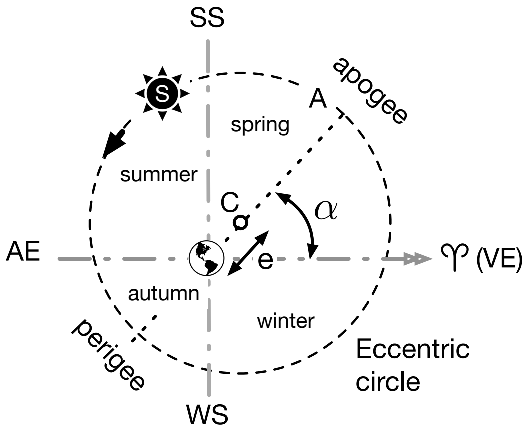


Figure 3.19: Hipparchus and Ptolemy's solar model showing the seasons in antiquity (today, winter is shorter and summer is longer). SS and WS are the Summer and Winter Solstices, VE (♈) and AE are the Vernal and Autumnal Equinoxes and the seasons are then defined as the four quadrants among them. The Earth (⊕) is displaced from the Sun (⊙) by the eccentricity, *e*, the distance in space from Earth to the center of the eccentric circle about which the Sun orbits. The dotted line is described in the text.

In addition to his modeling of the Moon's motion, he found a way to determine the distance from the Earth to the Moon. With his version of trigonometry (see below), he found that the distance from the Earth to the Moon is 65.5 times the radius of the Earth and that's about right (it's about 60.336). (Newton used his result in his invention of his Law of Gravitation.) Hipparchus attempted the same thing for the distance to the Sun, but underestimated it by a factor of 50.

Hipparchus' Fixed Star catalog. Hipparchus began the first quantitative survey of the fixed stars—the ones thought to be on the inside of the Celestial Sphere. Prior to him, locations of bright stars were noted by identifying a rough relative position in words: that a the star in the "shoulder" of one in one constellation is rising when the star in the "sword" of another constellation is setting and that the star on the "right leg" of a third constellation appears right overhead when this happens. More stories. Hipparchus took a different approach.

His data were extensive and would have required impressive patience (night after night) and commitment to a multi-year research project. Ptolemy tells us that Hipparchus cataloged around 850 stars, their positions, and their brightnesses and they were in use for centuries afterwards. Others had made catalogs (Eudoxus and Eratosthenes), but his was different: he invented a coordinate system and assigned positional numbers to each star. Think about how your GPS specifies a location on the Earth: my phone tells me that the location of the Library of Alexandria is 31.20870° N, 29.90911° E. What that tells me is that the library is a little more than 31° north of the equator (the **latitude**) and about 30° east of some point that's world-wide agreed to be the observatory at Greenwich, England (the **longitude**). Hipparchus adopted the same thing, but applied to the stars—the underside, if you will, of that Celestial Sphere above us. (More about this and how his system is essentially identical to modern astronomy is discussed in *Greek Astronomy, Today* in Section 3.8.2.

A many-decade detective story unfolded in trying to figure out which (if any) of Hipparchus' data were included in Ptolemy's more extensive star catalog. And there's a clue. Remember Aratus' poem, *Phaenomena* from Figure 3.1 which was written as an ode to Eudoxus? The one book we have of Hipparchus' is his *Commentary on the Phaenomena of Eudoxus and Aratus* in which he severely criticized mistakes of fact in the poem regarding the relative positions of stars in the constellations. He included a set of positions for 22 stars of his own observation and these have been extensively compared with Ptolemy's catalog and the agreement is pretty good. Without that poem, and Hipparchus' grumpiness about a 200 year old poem,[20] we wouldn't have any corroborating information that Hipparchus really did create the first ever quantitative star catalog. Well, maybe until 2022! For that breaking story, look at *Greek Astronomy, Today* in Section 3.8.3.

Hipparchus' Trigonometry. The mathematical problems he had to solve for his solar and lunar models were surely the inspiration for a tool that marks the invention of trigonometry. Figure 3.20 shows his idea. A chord inside of a circle with radius R and center O is shown as the length $\overline{\text{AB}}$ where the chord subtends the angle θ. By hand Hipparchus divided carefully drafted circles into degrees based on 360° (which came from the Babylonians), but much finer: 21,600 segments which is the number of arc minutes in 360°. Then he painstakingly created "tables of chords" of varying lengths for each segment giving him a fairly precise lookup table of angles, radii, and chords. Given a radius, and the length of a cord, an angle could be looked up in the table. Or visa versa. It's equivalent to a table of trigonometric sines since as in the figure, if one divides the chord in two so that there are two right angles at point C, then the $\sin(\frac{\theta}{2}) = \frac{1}{2}\left(\frac{\overline{\text{AB}}}{R}\right)$.

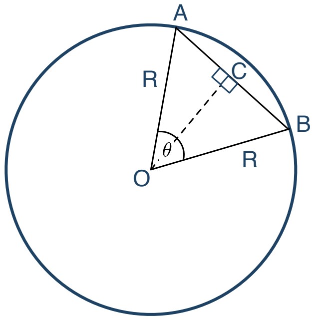


Figure 3.20: Showing how ancient "chords" related to a modern sin for a given angle θ.

Hipparchus' Discovery of the Precession of the Equinoxes.

The discovery for which he's most known was that the Earth's seasons might shift over time. He found this in two, complimentary ways. His first approach suggested the location against the zodiac of the summer solstice was 12 hours different from that recorded by Aristarchus, 145 years before. That inspired him to make a second, clever measurement to confirm that odd result.

[20]He wrote other ill-tempered reviews of other people's writings.

He figured out how to determine the longitude of a star (the angular distance of the star relative to the Vernal Equinox) near the ecliptic and compare that to an earlier measurement from other astronomers. He focused on the bright star, Spica (the brightest in the constellation Virgo, or α Virginis) for which he had data from an Alexandrian astronomer, Timocharis in -294 and -283 almost two centuries before him. This could be done easily in principle. Just measure the angle between the Sun and the star, right? That is:

$$\text{Longitude, Spica} = (\text{longitude, Sun}) + (\text{arc-angle between Spica–Sun}).$$

He knew the longitude of the Sun from his Solar model which gave him the angle α from Figure 3.19. The arc-angle in longitude of Spica and the Sun is a different story since if the Sun is out, that's daytime (!) and so you can't see the star. But he was very clever. He made use of the fact that during a lunar eclipse, the Earth is directly between the Moon and the Sun...so they are 180° apart and at night, he would be looking away from the Sun, toward Spica. So measuring the arc of longitude of Spica relative to the eclipsed Moon gives him his answer:

$$\begin{aligned}\text{Longitude, Spica} = (\text{longitude, Sun}) &+ (\text{arc-angle between Spica–Moon}) \\ &+ (\text{arc-angle between Sun–Moon}).\end{aligned}$$

At an eclipse, the (arc-angle between Sun–Moon) is 180°! Using Timocharis' Spica-Moon measurement, the longitudinal difference of Spica was 8° west of the Autumnal Equinox while he determined 6°: the longitude of Spica had increased by 2° in 150 years. (He actually did this as an average of two different eclipses 11 years apart.) That's about 1° per 75 years (consistent with his other measurement). Ptolemy did a similar experiment 265 years later and compared it with Hipparchus' and got about 1° per 100 years.

So what's going on here? Hipparchus concluded that the zero-point of longitude (the Vernal Equinox, which is where the ecliptic crosses the Celestial Equator) must be moving somehow over very long times.

This we know now has a physical cause: the Earth's axis of rotation points at an angle that's not perpendicular to the plane of its orbit around the Sun. It's tilted by close to that 23.5° from Figure **??** and like a top, the mass of the Earth causes it to precess around the Celestial Pole. This wobble of the Earth *looks* like a wobble of the ecliptic and so the equinoxes will be in a different location as time marches on. How fast? We know now the precession rate is pretty close to Hipparchus' and Ptolemy's measurements: about 1° per 72 years. So to go all the way around, requires $72 \times 360^\circ = 25,920$ years.

3.6.4 Summary of the Astronomy of Aristarchus, Eratosthenes, Apollonius, and Hipparchus

(Set the context with the timeline in Figure 1.2 on page 20.)

- Aristarchus (−310 to −230):
  - He made the first attempts to use geometry to measure distances among and sizes of the Earth, Moon, and Sun.
  - He proposed the first model of a Sun-centered cosmology, apparently without geometrical modeling.
- Eratosthenes (−276 to −194):
  - He measured the diameter of the Earth to impressive accuracy.
  - He measured the obliquity of the ecliptic—that 23.5° tilt of the ecliptic from the celestial equator.
  - He apparently created a star catalog of more than 600 stars. This would have been in words itemizing apparent locations of stars relative to constellation points.
- Apollonius (−240 to −190):
  - He was mathematician of the first rank and found a picture-way to model the Sun's motion around the Earth to create seasons of different lengths through the introduction of the deferent and eccentricity.
  - He also found a mathematically identical, but geometrically different form for planetary motion called epicycles. His proof of their equivalence was lauded as an important step by Ptolemy.
- Hipparchus (−190 to −120):
  - He built on Apollonius' deferent model and found a way to measure the actual eccentricity of the Sun's orbit and the longitude of the apogee. This was the first attempt to not only geometrically model the cosmos (or any physical mechanism) but to also quantitatively measure the shape parameters of the model.
  - He found a way to determine the distance to the Moon in terms of Earth radii, a value used by Newton much later.
  - His star catalog of more than 800 entries went beyond the stories that had been told previously: he invented a coordinate system that could be used by anyone to find the actual numerical positions of objects relative to an "origin" of essentially a celestial longitude and latitude.
  - He discovered that the Earth's seasons shift relative to the star's positions over time—the precession of the equinoxes. Understanding the physical cause of this phenomenon waited for Newton's explanation of the precession of the Earth's axis of rotation...slowly: about 1° per 75 years.

3.7 The End of Greek Astronomy: Ptolemy

While Aristotle's concentric spheres model lay dormant, it was to rise again in the middle ages and assume a strange parallel existence next to the model that made precise predictions. This is the model of Claudius Ptolemaeus, known for nearly two millennia as **Ptolemy of Alexandria** (100 to 170 CE). He created the most complete model of the cosmos before Copernicus and, refreshingly, his books survived intact thanks to Arab intellectuals' commitment to preserving and commenting on the works that they encountered from the Islamic conquest of the Near East, all of Northern Africa, and Spain.

Ptolemy wrote three books on astronomy for which we have original Greek and some Arabic translations. *Mathematical Composition* is the main work, now known by its Arabic title of *Almagest*, a corruption of the Arabic *Al* with the Greek word *megistē*, for "*the greatest*." The second is the *Handy Tables* which consists of two parts: the second part includes tables of his planets and stars of which we know from medieval versions 200 years after Ptolemy's life. The first part is the instruction manual on how to use the tables, surviving only in its Greek origin. *Almagest* is too complicated to have been absorbed by most and so the *Handy Table*s assured widespread use of Ptolemy's work. The third, *Planetary Hypotheses*, is an upgrade of the earlier *Almagest* and an attempt to build a plausible physical model of the purely mathematical *Almagest*. It was only appreciated and fully translated as two books in the 1960s!

Even though we finally have a complete set of one of our astronomer's works, ironically we know little about his life, except for a few references of his and a few later narratives by Roman and medieval scholars. Ptolemy almost certainly worked in Alexandria as his extensive observations come from that latitude. He's the first of our Greeks to have two names! "Claudius" indicates that he was a Roman citizen, probably during the time of Emperors Hadrian to Marcus Aurelius. "Ptolemaeus" indicates that his was of Greek ancestry.

Almagest is a huge subject. It is 700 pages long in a modern edition and more than a thousand pages are required to fully lay out the considerable mathematics of the book (N. M. Swerdlow and O. Neugebauer, 1984). It's not for the faint of heart. It's also pure mathematics and little philosophy and *not a physical model*.

Here's what it's like. I could imagine building a mechanical model of the economics principle of supply and demand. Suppose we have a playground teeter-totter with an arrow on the right end that points to a dial indicating high or low for prices of goods. Right side up, prices high, right side down, prices are low. If we start with the teeter-totter level and add weights to the right to represent *supply* of that product and weights to the left to represent *demand* for that product...we've got a mechanical model of the economy. When the supply, right-weight is larger than the left demand-weight, the arrow points down—prices fall. Likewise, when demand outweighs (sorry) supply, then the left side goes down and the arrow points up for higher prices.

This is a perfectly predictable model of the economy and through careful analysis of past economic history, one could tune the amounts of weight that would correspond to a prediction of prices and mark the dial with $ indicators. But, while it's a good model, *it's not a realistic representation of the economy.* *Almagest* is like that. It's a very complicated model of moving and spinning circles, lots of numbers to characterize the circles, scores of huge tables of numbers,[21] and could accurately predict positions of the heavenly bodies. But Ptolemy made no claim that the Sun, Moon, and planets actually performed the motions in his model.

Ptolemy's Philosophical Roots and Prerequisites for the Book: Books I and II of *Almagest* describe his working philosophy, defending it with standard arguments. But apart from the actual heavenly body motions, it's Aristotle, top to bottom. The mathematics required was Euclidean plane geometry and the use of Hipparchus' chord tables, except Ptolemy made them even more precise. He used the new "spherical geometry," and he developed it from scratch for the reader. With this introduction, he's ready to solve the world.

Ptolemy's Solar Model: Book III This was relatively easy and critically important. All of positional astronomy—to this day— depends on understanding where objects in the sky are relative to the Vernal Equinox, which in turn depends on the Sun's motion and position at any time. He didn't invent a solar model—he replicated Hipparchus' and was generous with his praise the original author.[22] So, Ptolemy's model of the Sun's is exactly the same: Figure 3.19. He repeated Hipparchus' determination of the eccentricity and agreed, but with higher precision: $e = 0.0415$ as compared with Hipparchus' $e = 0.04$.

Ptolemy's Lunar Model: Book IV and V. The motion of the Moon is difficult to grasp even today. Ptolemy's solution was ugly and also his biggest mistake: he could solve for eclipses (lunar and solar), but his model predicts that the Moon's apparent size would vary by a factor of two in a month, which obviously isn't the case. His solution is tortured and from our modern perspective, clearly an indication that there must have been something wrong. One has the impression of him just giving up and declaring successful eclipse predictions as a victory. He made careful tables of predictions of the eclipses—which were accurate— for any date, and washed his hands of the Moon problem.

Ptolemy's Model Fixed Star Catalog: Books VII and VIII. It was Ptolemy who told us of Hipparchus' catalog of the positions of 850 stars. He takes on the same task, but also includes the positions and apparent star brightness of 1022 objects from 48 constellations in his catalog and with this began almost two centuries of fights among historians. Did Ptolemy copy Hipparchus' 850 stars (shifting their longitudes by $2^\circ 40'$ to correct for the precession of the equinox over 265 years) or did he measure their positions as he claimed? Or had Hipparchus' catalog been wrong? The comparison of the Hipparchus' 22 stars' from his *Commentary* to Aratus' poem with their counterparts in Ptolemy's catalog is the key. There are translations problems since Greek numbers were written using Greek letters and sometimes mistakes happened in translation and transcription of centuries-old media. Stars were not always named, but a little story was told about each one to locate it within a constellation. So mistakes happened. This argument has largely subsided: within

[21] Perhaps the first use of tables in any manuscript in history.

[22] He has been accused of plagiarizing Hipparchus, but that's not fair as he gave ample credit.

the uncertainties that can reasonably be attributed to each, most of Hipparchus' 22 stars do match their Ptolemaic counterparts and that each astronomer is likely vindicated. I'm sure you're glad that we've cleared that up.

The bottom line about Ptolemy's catalog is this: it represented an enormous effort over probably decades and was the best star chart all the way to Tycho de Brahe in the late 16th century (Copernicus used much of it). A remarkable achievement and legacy.

Ptolemy's Planetary Theories: Books IX through XIV. His planetary models (yes, there were three) were the target of the Muslim astronomers, Copernicus, Galileo, Tycho, Kepler, and Newton and it took all of them to bring Ptolemy down. Its accuracy is still impressive so something besides getting the right numbers was behind its downfall, an important part of our story later.

The end product of his planetary research is a chapter for each of the five planets including its geometrical model, the particular parameters built into each model, a description of how he determined each parameter from his observations, and then five deliverables: a set of tables of positional coordinates for each planet, for any day in the future. It was these tables that were reprised in his User's Manual, the *Handy Tables*.

He must have struggled mightily to make Aristotelean circular orbits work but he held accuracy to a higher standard than the Classical Greeks, for whom a nice picture-story was sufficient. In order to "get it right"—which meant, make predictions that worked— required him to make excursions from some of Aristotelian rules. For example, the eccentric model for the Sun and a strange epicyclic model of the Moon had heavenly bodies orbiting seemingly arbitrary points in space apart from the Earth! But as painful as the Moon solution was, getting the motions of the planets right was another story altogether.

3.7.1 Mars, Jupiter, and Saturn

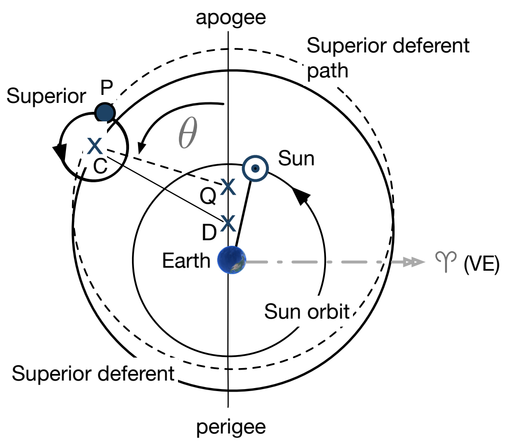
The prominent retrograde motion of especially Mars as well as Jupiter and Saturn added an entirely different set of complications from the naive epicycle model of Apollonius and Hipparchus. The simple epicycle picture of Figure 3.18 wouldn't do. Ptolemy had to insult Aristotle one more time and that particular solution offended Copernicus and his Arab predecessors. Let's look at his solution for the outer planets as they're a little simpler. Figure 3.21 shows his model that functions for Mars, Jupiter, and Saturn. Look at Figure Box 3.21 on page 136. After you've read the material in that Box, return to this point and continue reading.

The new wrinkle is the introduction of a third point in space, the **equant** (Q), displaced from the deferent point by the same amount as D is from *E*. A superior planet's epicycle's center P doesn't undergo uniform circular motion about the deferent center, D, *but about the equant*, Q. That is, the angle θ uniformly increases in time around the epicycle's path, so it appears to perform *non-uniform* rotation around D (its center) *and non-uniform around Earth*. The model constrains this movement such that the line from a superior planet to P, $\overline{\text{Superior-P}}$, is always parallel to the line connecting the Earth and the Sun, $\overline{\text{Sun-Earth}}$. Notice that this creates a special relationship among the Vernal Equinox, the Sun, and the planet.

So a superior planet orbits in its epicycle with center (P) following its deferent as originally imagined by Apollonius—except that as compared to Figure 3.18 the epicycle rotation is reversed from counterclockwise to clockwise. *That creates retrograde motion.* The Sun is shown with its orbit centered on the Earth (since its eccentric center is too small to explicitly show). So there are two centers of motion here—one for the Sun and another for Mars' deferent.

The dashed curve in the figure is the trajectory of Mars' deferent. So what Ptolemy knew was the various positions that Mars, Jupiter, or Saturn would have on the *dashed line*, but what he needed in order to build each model was its position on the deferent, the solid line. That's a formidable mathematical transformation.

FIGURE BOX 3.21



Ptolemy's model (not to scale) for a superior planet, Mars, Jupiter, or Saturn (P) and its relationship to the Sun (☉). Here, "Superior" (◯) is on an epicycle with its center at C. C rotates clockwise around the circular deferent path with its center at the center, D. The Earth is not at the center of anything, except close to that for the Sun! The angular speed of P around D — the amount that the angle θ increases with time is constant, but about the point Q...not D. The dashed circle is the path that P actually takes which its offset from the deferent's center.

Each planetary "kit" looks like this for superior planets and slightly different for the inferior planets. Every deferent radius for all planets was chosen to be 60 in an arbitrary set of units. The necessary parameters were determined by Ptolemy separately for each planet, including: the epicycle radius, the separation of Earth from the deferent point (D), which is also the separation of D from the equant, Q, the orientation of the apogee to the Vernal Equinox direction, and the angular speed at which θ increases in time.

Now go back to page 135 and pick up where you left off.

"...in a tour de force of possibly the most complex and extended calculation in all of ancient mathematics, he developed a method of successive approximation that allows

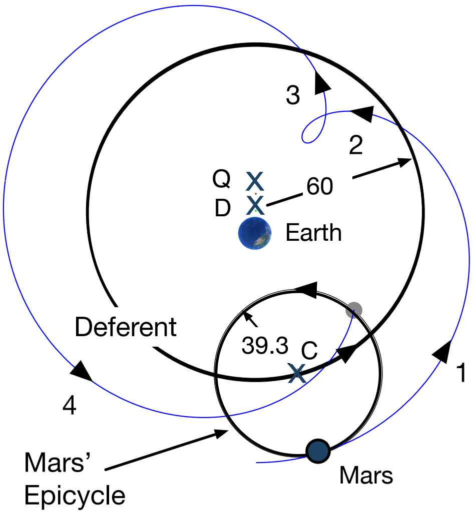


Figure 3.22: Mars (♂) is shown on its epicycle with its center, C, rotating around the deferent with its center at D. I've used Ptolemy's actual relative sizes for Mars. All deferents were in units of 60. Mars' epicycle's radius is 39.3/60 and the distance from Q to Earth is 12/60. One can see the strange loop motion described in the text.

> the numerical values of the eccentricity and the direction of the apsidal [direction of the apogee of Mars' orbit] line to be found to any degree of accuracy. Both the problem and the solution are remarkable...his solution shows a very high order of mathematical intuition...The number of astronomers after Ptolemy who understood and could apply the method must have been very small." [N. M. Swerdlow and O. Neugebauer, 1984, Vol 1, p307.]

Let's pick on Mars. Mars orbits Earth (in our 20th century way of viewing things) about every 687 days, or 1.88 Earth years and undergoes retrograde motion about every 2.1 years, or a little more than one revolution around the Sun. The backwards appearance lasts a little more than two Earth months, or about 72 days. Ptolemy's model with the equant rather precisely describes Mars' retrograde motion as it forces a kind of loop-the-loop as viewed from Earth.

In Figure 3.22 I've calculated the Mars model to show its epicycle and eccentricity (separation among Earth, D, and Q) using parameters taken from *Almagest*. Mars' path is, well, unusual. There are 4 points identified on the actual path that Mars takes while riding on its epicycle. We start at position 1, and as the epicycle turns and as the deferent turns, Mars moves to position 2 where it starts to appear to slow making that loop which makes it appear to go backwards during 72 nights. Then it comes out of retrograde and continues its forward-appearing path at 3 and nearly completing it's 1.8 year long path at 4. In each Mars year, the location of the loop shifts a bit relative to the Vernal Equinox.

This is what's seen from Earth with a bonus: it also addresses the fact that in retrograde, the planets are brighter, here, because it would literally be closer to

Earth. Just how often and how fast would be determined by the parameters—Jupiter and Saturn's parameters are quite different.

It works very well as seen in Figure 3.23 from James Evans, 1984 (inspired by James Evans, 1998). This shows seven bands that should encompass the retrogrades of Mars as viewed from Earth for some of the years of Ptolemy's observations, from 109–122 CE. The loops are the Mars retrograde events relative to the Vernal Equinox (the trajectory between points 2 and 3 in Figure 3.22) and the wedges show predictions of where that should happen. In (a) predictions are for a straight epicycle model *without an equant* while (b) shows the same thing, but *including the equant*. This, and other successful measurements surely convinced Ptolemy that he was right. He needed the equant.

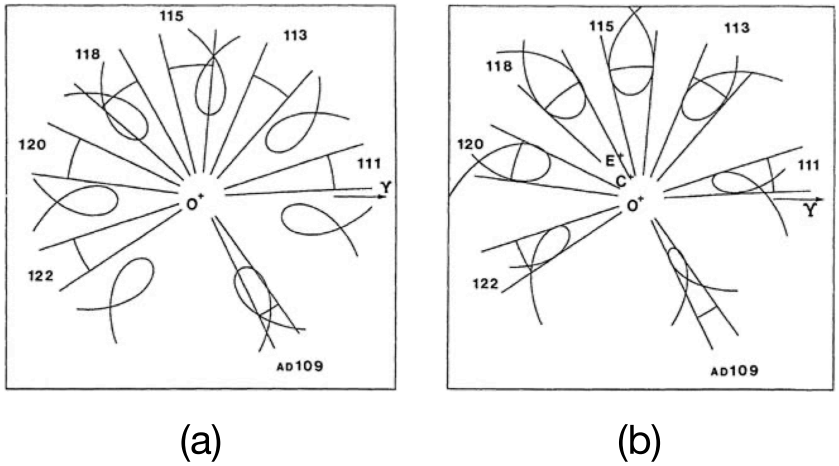


Figure 3.23: Seven retrograde loops of Mars for times of Ptolemy's observations (a) without the equant and (b) with the equant.

The relationship that Mercury and Venus have with the Sun was very problematic. Today we know that they orbit very close to the Sun but even now measuring their positions is challenging. The Sun's in the way! Observations had to be done just after sunrise and just before sunset...and carefully as to not blind one's self. So they presented a set of problems which couldn't be solved without separate models for each. And those solutions are strange, especially for Mercury with more moving centers of deferents.

Think about all of the major ways in which Ptolemy has violated Aristotelian imperatives. Is Earth at the center now? Of what? The outer planets and the Sun no longer orbit around it symmetrically. They also don't orbit at constant speeds except now around an uninhabited point in space, not around the Earth. It's torturously pieced together in ways that Aristotle could never have imagined—and that a modern physicist would not have tolerated. "Simplicity" is nice in physical models, not guaranteed, but when your model is so bizarre you'd tend to think that it's trying to tell you that the world is probably not that way. But this is the first time.

Going from pictures and stories to numerical prediction surely meant that when predictions worked, then it must be some part of the truth. The late 16th century's Johannes Kepler is from whom we learn the real solar system model and we'll have to wait 1400 years to Chapter **??** for him to appear and save the day.

Not always appreciated, was the fact that in *Almagest*, the outer planet's deferents were all taken to be the same radius and that the distances were all set by the epicycle's individual radii. He chose 60 "units" (always working within the Babylonian base-60 sexagesimal system we use today for time and angles) for that common deferent radius with the Mars:Jupiter:Saturn epicycle radii in proportions of approximately 7:2:1. This was because the planetary models in *Almagest* were not a system. Much like Eudoxus before him, he treated each planet separately and made no attempt to merge them, until much later in his life. Figure 3.24 shows Ptolemy's independent planetary pieces.

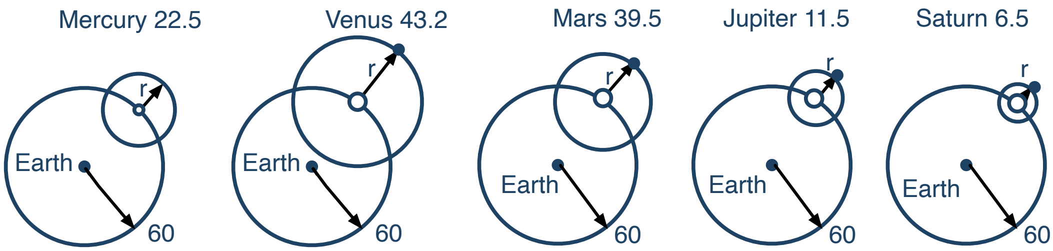
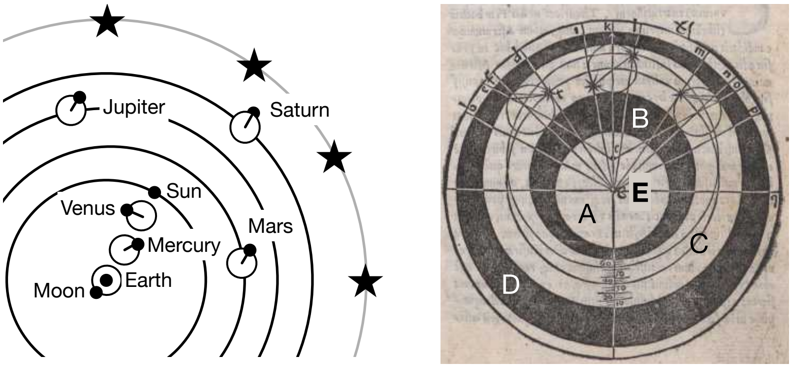


Figure 3.24: Each of the planets' epicycles are shown with their differing *r* values listed above as they ride on their deferents which each of the same radius. The units are arbitrary, so the relative epicycle radius to deferent is a measure of their relationship to the Earth. So the larger is r, the closer that planet is to Earth.

3.7.2 Ptolemy's Cosmology.

Just as it was important for Aristotle to build a multi-planet system out of Eudoxus' separate planets, it obviously seemed incomplete to Ptolemy also. So he later wrote *Planetary Hypotheses* which upgraded some of his measurements but also presented a whole cosmology of all of the heavenly objects. Figure 3.25 (a) shows it in a simplified format with an abstraction of the epicycles for each planet: the line in each epicycle shows the relationship of the planet to the center of its epicycle. Notice that for the outer planets, the epicycles are constructed for that line-direction in each is parallel to one another and parallel to a line connecting Earth to the Sun. For the inner planets, it's the *centers* of their epicycles that all lie on that parallel line connecting the Earth to the Sun.

> ▻ The Sun drives the whole machinery and the inner planets and outer planets have different models and constraints. But those clues weren't enough to resurrect the Aristarchus model with the Sun at the center. Such was still the strong pull of Aristotle's prejudices.



(a) (b)

Figure 3.25: The whole cosmology of Ptolemy. In (a) the planets, and Sun are arranged in a very particular way relative to the Sun. The lines in the circles for each planet represent the center of epicycle to the planet. For each of the outer planets, the epicycle-to-planet lines are *all parallel to one another and parallel to the line that connects the Earth to the centers of the inner planets, to the Sun.* The centers of the deferents for each inner planet and the Moon are all along one another and point at the Sun. **The Sun is always key.** In (b) an image from *Theoricae novae planetarum* by Georg Peurbach is shown which represents a slice through the Medieval idea of Ptolemy's 3-dimensional model for one planet. Notice the epicycle in various positions inside of the region labeled C. The other labels are described in the text. (Wikipedia, Georg Peurbach)

Recall in Section 3.5.2, I noted that that the classical planet ordering was Plato's and Aristotle's: Earth–Moon–Sun–Mercury–Venus–Mars–Jupiter–Saturn and the stars. Ptolemy made the executive decision to change that to Earth–Moon–Mercury–Venus–Sun–Mars–Jupiter–Saturn and because of his authority, it stuck. (Again, notice that the Sun sits between (our) inner and outer planets. Interestingly, whenever a Medieval or Renaissance rendering of Aristotle's cosmos was presented in books it was Ptolemy's not Aristotle's ordering that was used. Sometimes Ptolemy's name is included on an image, even though the picture might be Aristotle's equal-orbit, totally geocentric geometry. Ptolemy's and Aristotle's pictures get mixed up during Medieval and Renaissance depictions.

Planetary Hypotheses also presented a physical model for his cosmology. In it, there are solid aether spheres which carry the epicycles through...pathways in the solid aether around the Earth. This wasn't interpreted as an image until the early part of the 15th century when Georg Peurbach's 1454 *New Theories of the Planets* included the image shown in Figure 3.25 (b).[23] Think of this as a slice through a spherical aether unit required to support and guide a planet. The light volume labeled A would contain another such unit, and so on...so that together they would nest together like Russian dolls. It's what's in a unit that's hard to swallow. The light region, C, is a kind of hollowed-out shell within which an epicycle rolls around a diameter. It's off center since the planet follows the epicycle sometimes close to the

[23]We'll meet Peurbach in the next chapter.

Earth, E, and sometimes away from it.

He imagined that the largest excursion of, say, Mercury's orbit in its epicycle, constrained inside of Mercury's C cavity, would just match the smallest excursion of Venus' orbit in its epicycle, within its C cavity. Then the largest excursion of Venus' orbit would just match the inner excursion of the Sun's and so on. He packed them together with minimal spacers of aether (D and B in Figure 3.25 (b)).

He demanded uniform motion of the spheres, but the shifting of their centers is a problem. Imagine a soccer ball spinning around an axis at a uniform rate. Can it spin around another axis parallel to the first one at a uniform rate? No! It's physically impossible and this truly offended many Muslim astronomers and mathematicians who attacked his physical model in no uncertain terms.

While his planetary orbits were independent of one another, their relative orbital sizes could be calculated as each is determined by the tight-fit. So if you knew the size of one of them, you could then establish the size of others, working your way from edge to edge of each "spherical space-shell."

He knew the distance from the Earth to the Moon (from studies like that of Aristarchus) and the Earth to the Sun and in this way he actually calculated the distance from Earth *to each planet and to the stars themselves*! For example he calculated that the maximum distance from the Earth to Venus was 1079 Earth radii. (Today, we know that the maximum Earth-Venus distance, across the Sun pretending that they are as far away from one another as possible is more like 25,000 Earth radii.) For fun, he predicted that the distance from the Earth to the Stars—*the size of the entire universe*—would be $20,000 \times E_R$, or 126,000 km. Both an astonishing feat—- calculating the size of the entire universe—and wildly wrong. His universe's size is smaller than the actual furthest separation of Earth and Venus in our world.

3.7.3 Summary of the Astronomy of Ptolemy

(Set the context with the timeline in Figure 1.2 on page 20.)

- Ptolemy (85 to 165):
  - He wrote the mamoth book, *Mathematical Composition*, nicknamed by Islamic astronomers as *Almagest*, which became its label to this day (it's in the dictionary of your word processor). It was the definitive tool for predicting the positions of all of the heavenly bodies. The naive Copernican heliocentric model is mathematically identical to the epicyclic model of Ptolemy. No better, no worse than Ptolemy's.
  - He created a star catalog of more than a 1000 stars, including a subjective measure of each's brightness.
  - He continued Hipparchus' solar model with a separate, and corroborating measurement of the eccentric.
  - He adopted the epicycle model of Apollonius and found ways to assign measured parameters to the epicycle variables: the deferent radii he took

as constant and found epicycle speeds of rotation, radius, and orbital speeds on the deferents, separately for each planet.

- He wrote a "handbook" (*Handy Tables*) that would teach an astronomer, physician, or astrologer how to predict the positions of planets using his model, without having to absorb the considerable mathematics of *Amalgest*.
- He later wrote a complete cosmology that attempted to put all of the planets, epicycles and all, into one nested cosmological model. This allowed him to make predictions about the sizes of orbits.

3.7.4 The End of Greek Astronomy

Think about the conceptual leap that we've taken: we've gone from Aristotle who told picture-stories about the universe to Ptolemy who quantitatively modeled the entire universe! He used measurable parameters that located all of the heavenly bodies, predicted their motions, and proposed numerical distances to every object including the size of the entire universe. It's an astonishing feat and nobody successfully challenged it for 1400 years (although there were many attempts by the Muslim astronomy and mathematics community) which is a pretty good record.

He was the last Greek astronomer. Science would explore new frontiers, but the Greeks would no longer be the explorers. Rather western research[24] in MOTION BY THE EARTH and MOTION IN THE HEAVENS shifted to India and among the Muslim scholars who did some original work, and translated, preserved, and commented on Greek writings—especially Ptolemy.

3.7.5 One More Thing?

This was an unusual set of chapters and what follows will be considerably less sweeping and more focused. But the scene is now set for the full story of MOTION BY THE EARTH, MOTION ON THE EARTH, and MOTION IN THE HEAVENS. Here's a fascinating coda to our Ptolemy story. He was so close!

Imagine a very simple auto race with two cars. The track consists of two lanes, both circular around a common center. One lane, in which car M stays has a larger radius than the other lane in which car E is constrained, So it's not a fair race, since M has further to go in a revolution than E. But, this is an analogy.

From the stands you can watch the two cars go in their counterclockwise circuit and here not only does E have an advantage as the inside lane, but E is also faster than M. So naturally, *it will periodically lap and pass* M. When that happens, to the driver in E it looks like M is in front...and then seems to E to go backwards as it's lapped!

By now you realize that in this race analogy I can substitute E for Earth, M for Mars, and S for Sun and we've just described a simple solar system of two planets viewed from two different perspectives (the people watching the race, and E). It

[24] There was a parallel research path in China, but it didn't influence the eventual progress Europe

should be, and is, possible to construct an algorithm (involving vectors) to translate the motions from one frame to the other. The spectator's view corresponds to a solar system of the sort that you have learned that Copernicus described: all of the planets orbiting the Sun in perfect circles and the other, is the solar system that Ptolemy discovered in which the Earth is stationary and the Sun and planets orbit it...but on epicycles.

> ▻ The Ptolemaic model is mathematically identical to the Copernican model in which the orbit of an outer planet (like Mars) has the same dimension as the deferent circle of the Ptolemaic model.

What Ptolemy accomplished was an extraordinary mathematical feat. In fact, it's much more complicated than our modern view! He took a long, intellectual journey to his model whereas if he'd taken Aristarchus' model with the Sun in the center and circular orbits of the planets...he would have had a much simpler task. But what was in his way?

It was Ptolemy's commitment to the Aristotelian edict that the MOTION BY THE EARTH is zero, wrongly supported by a misunderstanding of the physics of MOTION ON THE EARTH *that was in the way of creating the better model*. Unraveling this is the task of this book: getting, first, the MOTION ON THE EARTH right and then applying it to MOTION BY THE EARTH and MOTION IN THE HEAVENS. It didn't come easy.

3.8 Greek Astronomy, Today

3.8.1 Let's Set The Record Straight: How we now understand the sky

From our more advanced vantage point: every one of the above points in Section 3.2.1 is explained overall by a Sun-centered solar system (with some nuance) around which the Earth and other planets orbit.

Elliptical orbits. We know that our solar system is built of eight planets (Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune). Figure 3.26 (a) is familiar to all schoolchildren today. We know that their orbits are not circular, but slightly elliptical, with the Sun at a focal point and as such, when they are close to the Sun, they whip around it fast and when the are far from the Sun their motion is slower. They are nearly all in the same plane, which is shown in Figure 3.26 (b) where we take Earth's orbital plane to define the ecliptic (0°) so relative to that, Mercury's orbit is the most inclined at $\pm 7^\circ$ from the ecliptic. All of the other planets' orbits are within that 14° band. For those of you mourning the elimination of Pluto from the planetary family, it's inclination to the ecliptic is more like $\pm 17^\circ$, as are other dwarf planets in the outer edges of the solar system. The undisputed opinion now is that Pluto's existence is due to some event that is not of the same origin of the other planets. Hence, it's being voted off of the planetary island.

Figure 3.27 (a) shows a line-up of planets (in simulation) as they appeared in the eastern sky on June 24, 2022 just before dawn from East Lansing, Michigan. Notice

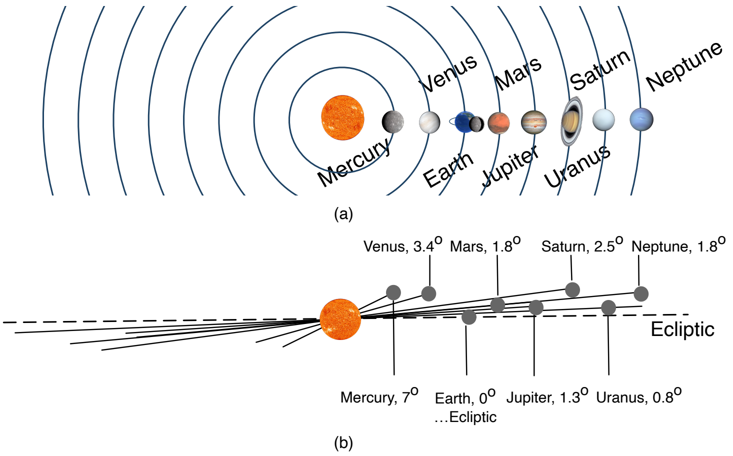


Figure 3.26: (a) is an abstract sketch of the solar system as we picture it today and and which we credit to Copernicus. "Abstract" because the alignment of the planets is for display purposes, actual relative radii of the orbits are not anything like shown, and the orbits are elliptical, not circular. (b) shows what the relative orbital planes are for each planet. The planets all have orbital planes inclined slightly to the overall ecliptic (the dashed horizontal line is the edge of the ecliptic plane). Notice that Mercury's is the one with the highest inclination of 7°. Pluto's is almost 17° up and down, indicative of its not belonging in the club of solar system planets.

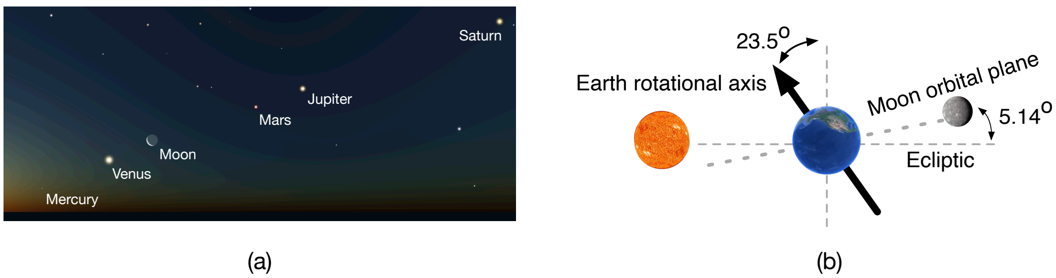


Figure 3.27: The inclination of the Earth's spinning is oriented away from being perpendicular to the ecliptic in which the Earth's orbit is fixed. Also, the orbital plane of the Moon's orbit around the Earth is slightly inclined relative to the ecliptic as well.

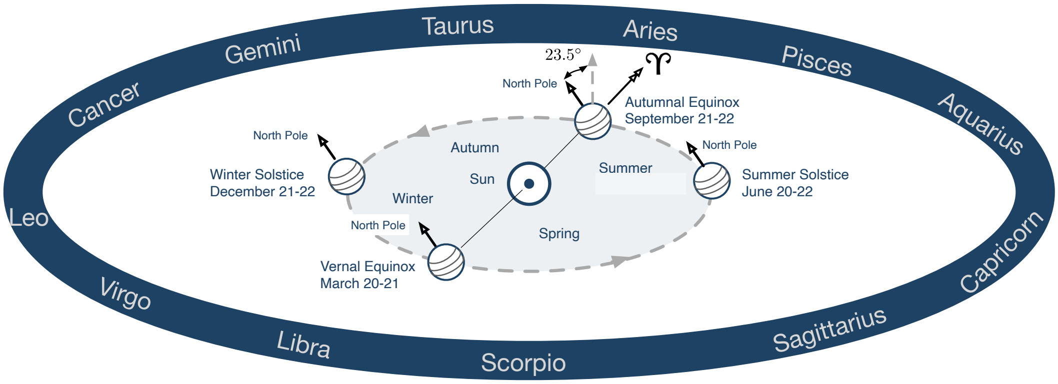


Figure 3.28: There's a lot in this image. The Sun (⊙) is at the center and ecliptic is shown as the gray circle around which the Earth orbits. The 23.5° inclination is pictured showing how the solstices are inclined in our northern hemisphere's summer and winter. The Vernal Equinox (♈) is pointing at the zodiacal constellation of Aries, as it was in ancient times (today, it's in Pisces).

that the Sun is just peeking over the horizon and Mercury, Venus, the Moon, Mars, Jupiter, and Saturn are all nearly in a line along the ecliptic. Figure 3.27 (b) shows that the Moon's orbit is inclined to the ecliptic by about 5° which is why we don't see lunar and solar eclipses every month. (Hipparchus determined this angle.)

The Earth is tilted by that seemingly random 23.5° that figured so prominently in the stories above and in Figure 3.28 the Earth is shown at the four seasonal points of the two equinoxes and the two solstices. The shaded circle is inscribed by the ecliptic and is the plane with all of the planets, including Earth. Notice that the Earth is titled by that 23.5° as measured from the plane of the ecliptic and that its direction does not move throughout the year and points to the Celestial Pole. The Vernal Equinox is shown when the Sun is within the Aries constellation (as in anquity).

Now we can understand both cause of the seasons and why they are of different durations and Figure 3.28 tells the whole story. When the Earth's orbit is closest to the Sun, it's moving the fastest in its elliptical orbit, so it spends less time between the two equinoxes, here on the left side of its orbit. Notice that the tilt of the Earth's axis is away from the Sun, and so the full-force of the Sun's rays are directed, not to the northern hemisphere, but the southern. In fact, at the Tropic of Capricorn at a latitude of 23.5° South, the Sun would be overhead at the winter solstice. So less radiation intensity falling on the northern hemisphere, means it's cooler. So yes, the winter happens when the Earth is nearest to the Sun. On the other side, at the summer solstice, the Sun's rays are intense on the northern hemisphere as the Earth's tilt is now towards it and the Sun is overhead at noon on the summer solstice at the latitude of the Tropic of Cancer—where Syene is located at 23.5○ North.

Spinning Earth. The Earth has two motions, as do all of the planets. It orbits the

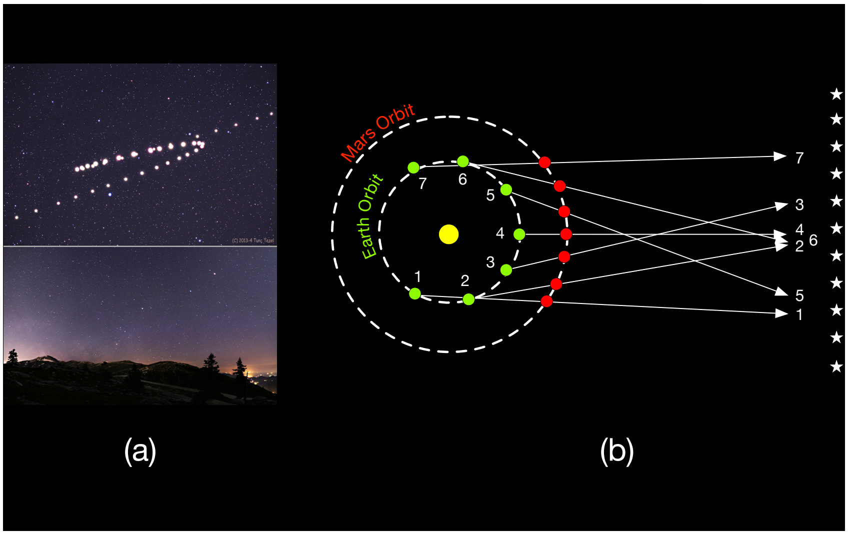


Figure 3.29: Retrograde motion by Mars. In (a) the sky in Turkey shows a photograph of Mars from December 5, 2013 in the upper right hand corner and then an overlayed photograph taken every five or six nights until August 8, 2014. The looping behavior in the middle is the retrograde motion. (b) shows how this happens (see the text for an explanation) https://twanight.org/gallery/tracing-the-red-planet/?preview=true

Sun in a nearly circular path in a counterclockwise sense when viewed from above the Sun's north pole. The Earth also spins on its own axis, also in a counterclockwise sense.[25] That the Earth spins on its axis explains the apparent motion of the Sun through our sky from E-W each day. The speed of the surface of the Earth due to its spinning is about 460 m/s (about 1000 mph) while the speed of the Earth's track along its orbit is 220 km/s (about 490,000 mph). We don't feel this motion since it is constant and we're held to the surface by the Earth's gravity. The same thing is true for the air and so we don't feel a wind as if the Earth were moving out from under the atmosphere.

Planets' orbits. The strange retrograde motion is easily explained in the heliocentric system. Earth and Mars, for example, have different "years" as they go around the Sun. Sometimes the Earth will lap Mars and leave it behind. That's the story and Figure 3.29 explains it. In (a), we see a time-lapse photograph of Mars in successive nights from December to August. Clearly Mars appears to "move" against the stars. (b) shows how. Each

[25]only Venus among the planets spins in a clockwise sense while Uranus has a spin axis which is on its side, relative to the others. One explanation is that, like the Moon was created through some billions of years ago collision with the Earth, so to something massive might have struck the adolescent Venus and Uranus. Multiple hypotheses exist.

3.8.2 Hipparchus and Modern Celestial Coordinate Systems

(Dennis Duke, 2002) correctly argues that the coordinate system that Hipparchus seems to have originated and Ptolemy perpetuated is essentially identical to what is used today in astronomy, called the "equatorial system." Figure 3.30 (a) shows the situation. What Hipparchus did was measure the angle of a star relative to the North Celestial Pole and an angle along the ecliptic. If you look at Figure 3.28 you'll see that the Earth is surrounded by the 12 constellations of the zodiac. The Greeks (and Babylonians) divided the whole circular pattern into 12 signs, each of 30° each and his coordinate system referred to the constellation and then the number of degrees within that constellation. This is like the longitude on the Earth's surface—degrees around. The "zero" of this coordinate system is located at the position of the Vernal Equinox, which recall is where the Sun on the ecliptic crosses the Celestial Equator during the spring. The Sun was in the constellation Aries during these times (which is why the symbol for the Vernal Equinox is ♈, which is the symbol for that constellation. Today, the VE has moved to the constellation Pisces precisely because of the precision phenomenon that Hipparchus discovered.[26] (More about the Vernal Equinox below.) So in the *Commentary*, he wrote about the constellation Bootes (not among the 12 zodiac members):

> "Bootes rises together with the zodiac from the beginning of the Maiden to the 27th degree of the Maiden... Hipparchus, "

The "Maiden" is Virgo which is the 6th constellation ("sign") around from Aries (Figure 3.28). So the angle, α in the figure where the constellation Bootes rises is $(6-1) \times 30^\circ + 27^\circ = 177^\circ$.[27] A modern version of Bootes extends 202° to 237°, so it doesn't appear to match? Ah, but the precession of the equinoxes is worth 1°/72 years, so we need to add that factor times the number of years since Hipparchus recorded his measurement 2153 years ago—that's an additional 30° which makes that edge be 207°: Hipparchus is just right.

For the other coordinate, he measured from the North Celestial Pole *down to the object* of interest, χ in the figure. That's the "polar angle" and is the opposite of our Earth-faced latitude, which measures up from the equator.

The modern equatorial system uses the same idea. For the polar angle, a star or object's "latitude" coordinate is measured *up from the Celestial Equator*. This is called the "Declination, δ." So it's identical through a difference of 90°:

$$\chi = 90 - \delta.$$

This north-south polar angle measure is called "co-declination."

The modern longitude, called the Right Ascension, α, is measured also from the location of the Vernal Equinox, but typically recorded as a time, rather than an angle. This is natural, since the whole Celestial Sphere rotates 360° in 24 hours. So while the edge of Bootes is 202° for Hipparchus' units, it's $13^{\mathrm{h}}36.1^{\mathrm{m}}$.

[26] The "Age of Aquarius" is next, as precession continues.

[27] Because Aries the first sign starts at 0°, so the 6th sign starts with 150°

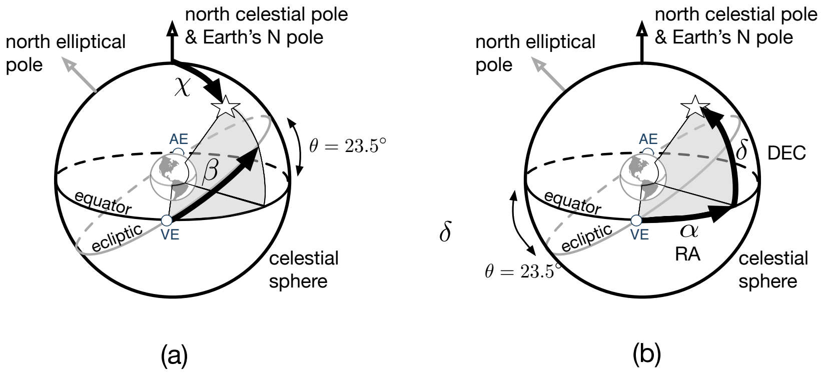


Figure 3.30: The Celestial Sphere is shown in both diagrams for two different coordinate systems that can be used to locate a star on the Sphere. In (a) the "longitudinal" coordinate (β) is along the ecliptic starting from the position of the Vernal Equinox along the ecliptic and the "latitude" coordinate (χ) is measured from the Celestial Pole to the star along a great circle. In (b) the longitude (α) is along the Celestial Equator from the Vernal Equinox (and so identical in angle to β) and the latitude is measured up from the Celestial Equator (δ). The coordinate system in (a) is called the Ecliptic Coordinate System and (b), the Equatorial Coordinate System. (b) is the standard modern system for star charts in which δ is called "declination" and α is called "Right Ascension" (and is recorded in modern tables in units of time, rather than angle where 24 hours equals 360°). A modern version of the Ecliptic Coordinate System uses $\lambda = 66.5° - \chi$, but I represented it here from the pole because Ptolemy measured χ for "latitude." Hipparchus seems to have used both of these systems while Ptolemy used (a).

About the Vernal Equinox. I don't believe that there's any record of just how Hipparchus could have determined the location of the VE in the zodiac. After all, the Vernal Equinox for the Greeks was determined at noon on that day when the Sun is precisely between its altitude at the two solstices, and equivalently, when it rises and sets precisely in the east and the west. His accuracy was about 1/4 of a day for observations and I can think of two ways he might have done this.

He would surely already know roughly when the equinox was to happen and would start measuring the Sun's location, rise, and set for days before and days after the expected event. Then, later he could figure out precisely which day. But along with his altitude measurements, he might look at the east just before the Sun rises each of those days and precisely located which constellations were still visible before it becomes bright. Likewise, he would look just after sundown to see what constellations would be "coming out" as it gets dark.

He could also have noted when the equinox occurred, waited exactly 12 hours and then looked to see which constellation would be at the altitude of the Sun at noon.

In both of these, he would presumably conclude that it was Aries and the "First Point of Aries" became the nickname for where the Vernal Equinox is in the sky.

3.8.3 New Evidence for Hipparchus' Lost Star Catalog

When we're talking about millennia, "breaking news" needn't be "yesterday." So there is remarkable Breaking News when it comes to Hipparchus' star catalog. Parts of it might have been found.

In 2012 Jamie Klair, an undergraduate at the University of Cambridge was studying a multi-spectrum image of folio pages of an ancient Greek palimpsest[28] known as the *Codex Climaci Rescriptus* at St Catherine's Monastery on the Sinai Peninsula (now in Museum of the Bible's collection in Washington, D.C.). It was a summer project assigned by biblical historian at the University of Cambridge, Peter Williams, who continued the work and in 2017 he and French collaborators confirmed the observation and found more of it. They recently published it in (V. J. Gysembergh, 2022). In that image an under-text is slightly visible which he realized appeared to contain astronomical notations—actually a quotation from Eratosthenes. It appears that the original writings were erased in the 9th or 10th century and overwritten. But the multispectral imaging brings out the original impressions on 9 of the 146 pages.

By digitally bringing out the faint background writing, it's apparently astronomical data, coordinates, actually. Almost certainly from Hipparchus' observations. For example, one of the decoded and translated phrases in the hidden text is:

> Corona Borealis, lying in the northern hemisphere, in length spans $9^\circ 1/4$ from the first degree of Scorpius to $10^\circ 1/4$ in the same zodiacal sign (i.e. in Scorpius). In breadth it spans $6^\circ 3/4$ from 49° from the North Pole to $55^\circ 3/4$.

[28] a document that has been reused by scrubbing out the original content

They noted that “length” is the east-west measure and “breadth” is the north-south measure. The north-south measure is as above, the co-declination and the east-west measure is again the Right Ascension, in angular units. Scorpio is the 8th constellation, so from the previous section, that’s $7 \times 30^\circ + 1 = 211^\circ$. Adding the 30° for precession since then would give a RA today of 240°. The edge of Corona Borealis is almost exactly that.

The stars in the 9 pages refer mostly to Ursa Major, Ursa Minor and Draco and the values are essentially those in Hipparchus’ *Commentary*. The general consensus is that this is the first concrete evidence for the long-lost Star Catalog of Hipparchus!

4460 # Volume II

4461 # Medievals to Copernicus

It may have once been the case that all roads lead to Rome, but for most of western philosophy, physical science, and mathematics, all roads lead *from* Greece. This volume is the first stop in our path towards Einstein's Special Relativity: our MOTION themes start with the Greeks, eventually centered on Plato and Aristotle.
4462 Likewise, but to a lesser degree, ideas about LIGHT frustrated the Greeks without much analysis. This volume will be different from subsequent ones, as its stories are of a number of people, not all of whom would be classified as scientists today. You'll see why. But we'll close this volume with the one of the earliest quantitative astronomers: Claudius Ptolemy.

Chapter 0

Series Preface: Read This!

"PREFACE PROBLEM: Nobody reads prefaces.
SOLUTION: Call the preface Chapter 1."

- **gause11**, *Are Your Lights On?*

"Why not just call it Chapter 0?"

- Raymond Brock, *...just now*

0.1 Why Do This?

Albert Einstein is usually imagined to be the very model of a modern major scientist. A brave genius, working entirely alone and, yes, it's certainly the case that it would be hard to be more unknown than the 26 year old Einstein. Yet he had an idea that cured a slow-motion nervous breakdown inside of the world's physics community. His Special Theory of Relativity brought two inconsistent theories together by healing a contradiction between them: either James Clerk Maxwell's triumphant model of LIGHT (electromagnetism) or Isaac Newton's mature model of MOTION (mechanics) seemed to be wrong or incomplete.

This series, *From the Greeks to Einstein* (let's give it a nickname, "G2E") follows parallel storylines of two very different theoretical clans: MOTION (in which there were three separate families: MOTION IN THE HEAVENS, MOTION BY THE EARTH, and MOTION ON THE EARTH) and LIGHT (where there were also three separate families: OPTICS, ELECTRICITY, and MAGNETISM). Those six different families separately developed, merging into a pair of conflicting theories: MOTION and LIGHT which Einstein tied together.

G2E's subtitle, *How the stories of motion and light became the Special Theory of Relativity*, emphasizes the theme of this work: stories. G2E is stories about people.

I've been a professional particle physicist for half a century and I've found that I suffer from an unusual affliction that affects my teaching and my research. Before I can teach something old or learn something new, I have to know its history. This isn't an especially efficient way to work but it's led to a fulfilling pastime and I suspect unusual classroom experiences. I've become so sure of this approach that I even tell stories in mathematically intense (calculate! calculate!), advanced graduate physics classes. This series is a written version of my teaching approach, structured around 20 or so scientists, their lives, their times, their colleagues, their projects, and their accomplishments.

0.1.1 Projects

In trying to reverse-engineer the emergence of innovative ideas in physics, I keep coming back to what *individuals* do. I'm keenly aware that when I choose to spend my limited time and group resources on a project it's both a commitment and an opportunity loss for what I decided *not* to work on. So it's personal and requires experienced scientific taste. For me: the model of the unit of behavior in science is what I'll call the Project which is a lot like how you might think of a project.

> There is a more standard, but disappointing "unit of behavior in science" called the "Paradigm" which came from Thomas Kuhn's historic 1962 *The Structure of Scientific Revolutions* (**kuhn96**). When we're working within a paradigm we're doing what Kuhn called "normal science," which at some point, accumulates contradictions, develops a crisis, a revolution occurs, and a new paradigm begins. Kuhn had trouble clearly explaining what a paradigm was—21 different uses of the word were identified! For example, is it Big, leading to historic Revolutions? Or could it be small...lots of paradigms in a scientist's lifetime. It was meant to be a collective world-view, a social thing, which was *also* a problem as it led to accusations of a distressing relativism in science.

I'll be didactic about Projects in my stories. By the way, in Kuhn's formulation, the passage of one paradigm to another is not progressive...just different. That was a problem for his model as, at least for professional scientists, science is certainly progressive and my working model is designed to be.

Simply put, each Project has inputs and outputs. In order to get a Project off the ground, one commits to these inputs:

1. **Numbers**. I'll have a set of factual commitments—numbers or parameters—about phenomena that I'll accept.

2. **Theories**. I'll commit to a set of theoretical concepts...accepted views of the world, so to speak.

3. **Techniques**. I'll have a commitment to set of best-practice mathematical and experimental skills and techniques.
4. **Norms**. I'll inherit and initially commit to a set of community norms and expectations about what Projects are worth exploring.
5. **Curiosity**. This defines a Project's goals. I'll be curious about some actual or imagined phenomenon. Maybe I just want to measure a parameter or do a "what if" theoretical calculation or build an amusing mathematical model. For the duration of the Project, I'll commit to it.

I've called these "commitments" because they are...until they aren't! What I mean is this: if I make a discovery of importance that affects what *other* scientists choose to work on, it usually involves my modification of, abandonment of, or invention of the input commitments that I respected at the outset of my Project. Analyzing those from past —Project to descendent, new Project — is interesting to me. If a Project is well-designed, we can identify each of these five commitments and as a pedagogical tool in our historical approach in G2E, that's exactly what I'll do:

> ▻ For almost 20 highlighted scientists I'll unpack the commitments (#1 through #4) plus what sparked their curiosity (#5) in their subsequently revolutionary Projects. We'll see how their work went from attention-getting to revolutionary in service to Einstein's eventual Special Theory of Relativity.

This approach necessarily brings both history into the stories and encourages a focus on the state of affairs during each person's working life. It also points at collaborators.

That Einstein picture of the completely isolated genius? They don't exist in the practice of productive science. There might very well be completely isolated geniuses, but if their isolation is complete they didn't influence anyone! (We'll see a few who only in retrospect were found to have been on the right track, but quiet about it.)

You see, an essential aspect of doing productive science is doing public science. Even the well-known "genius" scientists that we can all name had collaborators. They might have had real-time collaborators, or some of them really did work alone in their rooms but they all "collaborated" across time with people who came before them, relying on *their* previous projects to inform the inputs to their Projects. That's where the continuity and progress in science comes from: these real and virtual collaborations. This idea of collaborating with the past is even a little bit romantic which is maybe why physicists and astronomers enjoy the pedagogy of teaching physics so much.

But revolutions? They're a slow-walking event. If I'm to persuade you that my focus on unique individuals is helpful I should be able to identify when a revolution occurred. Revolutions aren't overnight, or when someone lays down their pen. The revolutionary nature of a Project reveals itself only in retrospect. Here's how this roughly goes: Someone completes an interesting Project, perhaps having measured surprising new numbers or conceived of a new model or invented a new technique. And if by using those new tools they solve some old problem or predict novel phenomena, then maybe that's attention-getting. But only when enough other scientists vote with their feet—and their precious time and resources— and adopt those new ideas as inputs to *their* Projects then, in retrospect, that original Project might be viewed as having been important—and should *everyone* in a community use those new tools? That's a revolution.

Both words in the familiar phrase, "Copernican Revolution" annoy many modern historians. "Copernican" because it singles out an individual as special. "Revolution" because it suggests that there are abrupt changes in the flow of intellectual history. In his *To Explain the World,* (**weinberg15**) chides (**shapin96**) for the first line of the latter's *Scientific Revolution*: "There was no such thing as the Scientific Revolution, and this is a book about it." Shapin is one of the voices of a movement that has recoiled against the idea of THE Scientific Revolution and certainly that a single person might be responsible. I've got a different take on this, especially since my career has actually straddled a bonafide revolution stimulated by special individuals, Weinberg, among them.

After chastising Shapin, Weinberg closed his introduction to his Copernicus chapter with the comment, "There was a scientific revolution, and the rest of this book is about it."

I agree. There have been Revolutionary Scientists *and* there have been Scientific Revolutions and the rest of this series is about them: Claudius Ptolemy, Nicolaus Copernicus, Tycho Brahe, Johannes Kepler, William Gilbert, Galileo Galilei, Rene Descartes, Christiaan Huygens, Isaac Newton, Thomas Young, Michael Faraday, James Clerk Maxwell, James Joule, Albert Michelson, J. J. Thomson, Hendrik Antoon Lorentz, and Albert Einstein.

Every chapter follows a similar template. The main bodies have major sections that center on one or two scientists: "A Little Bit About Copernicus" or "A Little Bit About Newton," or Kepler, or Maxwell, and so on. We'll learn about their lives, their contemporaries, and yes, we'll analyze their Projects—what they brought to their work and how they stimulated conceptual change as a result. The last major section will be "Copernicus Today" or "Newton Today" and so on. Each of our physicists left legacies; world-views; and in some cases, even technologies that we still use today. Finally, for many of the chapters there are technical appendices which go deeper into the mathematics than would be welcome in the main narrative of a series like this.

Chapter 5

Nicolaus Copernicus: Not What You Think!

"If the Lord Almighty had consulted me before embarking on creation thus, I should have recommended something simpler."

- attributed to Alfonso X, *King of Castile during the late 13th century*

I'll bet that as a child, Nicholas Copernicus enjoyed gingerbread and that he and his friends would have played in the ruins of a castle that once dominated his walled home town of Toruń.

Do I know these things for certain? Well, no and that's disappointing and in contrast with what we know of his Renaissance artist-contemporaries. There was no scientific biographer to write the lives of the mathematicians and astronomers of that same period, so we are *still* in detective mode trying to piece together the life and scientific efforts of one of the most renown of astronomers of that, or any time.

What does this have to do with ruined castles and gingerbread? "Gingerbread," because his home town of Toruń in the Kingdom of Poland was the European origin of that pastry, already more than two centuries established by the time he would have grown up. That he could have afforded the confectionary is certain, as his was an affluent household. That castle ruin was a proud symbol of the town's rebuke of the overlord Teutonic Knights and a sign of what was to become for a mature Nicholas. The inferences of a detective.

> Our most famous of astronomers left only two scientific documents, 17 letters, a suggestion to remodel Poland's coinage, and an tract demanding payment from a friend to whom he'd loaned money (don't loan money to friends). Out of the two scientific documents, the solar system's re-arrangement was established in the first short, informal document which summarized his plans with agonizingly little detail. The manuscript's historical title is *Nicolai Copernici de hypothesibus motuum coelestium a se constitutis commentariolus*, and it's usually called just *Commentariolus*, or "little commentary," but there's no reason to think that its author gave it a title. It's some 30 modern pages long and I'll spend a lot of time on it. Its date is uncertain and historians of science argue about how he came to his conclusions. The second scientific document, *De revolutionibus orbium coelestium* (*On the Revolutions of the Heavenly Spheres*), which I'll refer to as *Revolutionibus*, came three decades later, and was a major work. The detail in its 400 modern pages is excruciating, it's full of arithmetic mistakes, lacking references to his antecedents and sources, and overpowering in its complexity. There are a 1000 calculations just for the superior planets' descriptions in that final, printed book and so somewhere (!) there must have been many thousands of pages of notes, notebooks, and scraps...all lost. Talk about an agony for historians.
>
> Copernicus' work begins an era in the history of science in which Greek notions MOTION BY THE EARTH and MOTION IN THE HEAVENS were seriously challenged for the first time in 1400 years. It's the stepping-off point towards Isaac Newton's mechanics and astrophysics, which in turn, is our last stop in mechanics before Special Relativity.
>
> Copernicus' overall conclusions are quite clear, but how he got there requires imagination—that detective story. Georg Rheticus, his young colleague, supposedly wrote a lost biography, and so detective work and even fictional accounts (**banville76** and **sobel11**) have attempted to fill the gaps. Copernican scholarship is immense—a full profession for many historians— and I'll try to bring out the consensus views to get to where we're going: a universe in which the Earth becomes a planet, the order and periods of the planets are measured, and the Sun is in command. Dare I say, a revolution.

In Chapter **??** we followed the spread of humanism which paralleled inspired science and a growing independent attitude towards Aristotle's theories of MOTION ON THE EARTH. And we saw that attitudes to his MOTION BY THE EARTH and MOTION IN THE HEAVENS were criticized earlier and persistently in Arabic science and that in the early 15th century that western astronomy began to find its way in Europe.

5.1 Northern Europe and The Knights

A "very remote corner of the earth..." is how Nicolaus Copernicus (1473–1543) described the troubled region of his Baltic, eastern Poland home. Hard to argue with that. It's cold. It's not Italy. It's not exactly a crossroad of international, humanist thought. The Prussian region(s) were a mixture of a dominant German (hist native language) and less so, Poles, both under the thumb of the strange monastic, militant sect of The Teutonic Knights.

> The Teutonic Knights (or Teutonic Order), founded in 1190 in Palestine, was a brotherhood that originally built and managed German hospitals during the Third Crusade. As the epitome of German knighthood, following their elected "Grand Masters," its disciplined members evolved to forcibly converted others to Christianity. After the Third Crusade's inconclusive end, they returned to Europe as a papal and imperially-sanctioned military force with a mission to spread Christianity.
>
> The pagan inhabitants of Old Prussia on the Baltic Sea in present-day northern Poland, Lithuania, and Latvia, became the target. To the Vatican, forest and animal worship had to change and when Polish kings couldn't convert the inhabitants, the Knights were deployed to the "Northern Crusade." Successful, they were awarded territories (as in Figure 5.1), creating their state.

The merged kingdoms of Poland and the Duchy of Lithuania were Europe's largest nation and when Constantinople fell in 1453, European trade pivoted to the heavily trafficked Polish Vistula River, along which Copernicus lived as a child in the prosperous town of Toruń.

After a tumultuous 200 years under Teutonic rule, its townspeople successfully enlisted protection from the Polish crown ad after two wars, Toruń was absorbed into Poland proper. The Second Treaty of Toruń in 1466 divided Prussian lands, with "Royal Prussia" to the west of the Vistula belonging to Poland and to the east the Knights were confined to "East Prussia" (eventually, "Ducal Prussia"), as nominally a Polish fief. The Knights' ruined Toruń castle is still rubble today, the same that young Nicolaus surely played within.

Between the two Prussias was the triangle-shaped ecclesiastical state of Warmia (in German, Ermland)[1] the size of Rhode Island. Warmia had been a diocese of Prussia within the Teutonic State, but it was also a political entity with an elected "prince-bishop"—literally both the political *and* spiritual head. Copernicus lived his entire professional life in Warmia, split between his day job as a canon of the diocese and his avocation of changing the world's view of itself.

Eastern Prussia was personally dangerous for Copernicus and his duties to the citizens of Warmia were time-consuming. That he could find the concentration to work alone on complex mathematics and concepts is impressive.

[1] I'll use the Polish names for cities in Warmia, (in Latin, Varmia) but often the German names are in the Copernican literature and I'll mention them at each first visit.

Figure 5.1: Copernicus' Europe with locations where he lived in white and important astronomy regions and cities noted. The inset shows the two Prussias with Warmia in-between. Frombork is at the very top of Warmia on a bay of the Baltic Sea.

5.2 Reviewing the Ptolemaic System

Copernicus' Project was both reliant on and in opposition to much of Ptolemy's modeling. Let's review the Greek-Egyptian astronomer's high-points.

Recall that Aristotle proposed that all of the heavenly bodies were centered on, and circled the Earth in perfect circular orbits, moving at constant angular speeds. But that's not what's observed in at least two ways and so these behaviors were called "anomalies." The first anomaly is that the Sun's presumed motion around the Earth is sometimes fast and sometimes slow—not uniform and so the seasons are not of equal length. The second anomaly is that the planets exhibit that apparent backwards, retrograde motion (the Sun and Moon do not). Ptolemy's Project was to create a precise model of the anomalies that could be used to accurately predict the future positions and coincidence events of all of the heavenly objects. As we saw in Chapter 3, Ptolemy's primary planet building-block included two basic geometrical constructions. The first was an off-center orbit around the Earth, which is called an eccentric and was his choice for the path of the Sun. The second was his system of epicycles which, with some variations, served as a template for the planets and the Moon an is shown in Figure 5.2:

The **deferent** is a large circle of radius, *R*, with its center, *D*, near the Earth, but separated from it by a distance called the eccentricity, *e*. The deferent for every one of Ptolemy's planets has the same diameter, which he chose to be equal to 60 in his units. This was shown Figure 3.24.

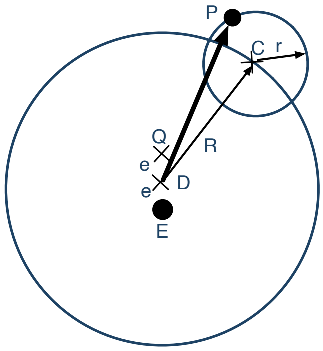


Figure 5.2: The basic construction of a deferent and epicycle.

The **epicycle** is a circle of radius *r* on which each planet, *P*, is attached, riding at constant anger speed around the epicycle's center, *C*. The radius of each epicycle is different its center, *C*, follows the deferent path around the *D*, bringing the rotating planet with it in its loop-the-loop path.

On the other side of the deferent center *D* is another location further displaced from the Earth by a second amount of *e*, the controversial **equant**, *Q*. The rotation of the deferent is forced to be uniformly circular motion about *Q*, rather than its geometrical center, *D*, and certainly not about the Earth.

Each planet's template is independent of the others, so in *Almagest* they functioned like puzzle pieces for a puzzle that's never assembled. They stand alone and apart, each built from typically three measurements to give *e*, the radius *r*, and the speeds of the deferent and epicycle as resulting numerical parameters.[2]

In *Planetary Hypotheses*, he outlined his cosmology and Figure 3.25 shows how the superior and inferior planets all have arrangements that align in various ways with the Sun.

The Sun doesn't have an epicycle but rather follows an eccentric route where its center is simply displaced from the Earth by an "eccentric." The whole arrangement of epicycles and eccentrics when forced together by Ptolemy later, didn't sit well with Copernicus who later noted:

> "...their experience was just like some one taking from various places hands, feet, a head, and other pieces, very well depicted...a monster rather than a man would be put together from them." Copernicus, Dedication of *De revolutionibus orbium coelestium* to Pope Paul III

Ptolemly's cosmology was confused and required rotational motions that included inconsistent rotational motions as described in Chapter 3. It was despised by the Muslim astronomers and Copernicus was offended by the equant, although he subscribed to the idea that the planets were embedded in solid spheres — "orbs"— made of aether.

[2]That's just for the "longitudinal" motions. Each planet's epicycle and deferent planes are different to account for the latitude differences for each.

5.2.1 Letting the Cat Out of the Bag

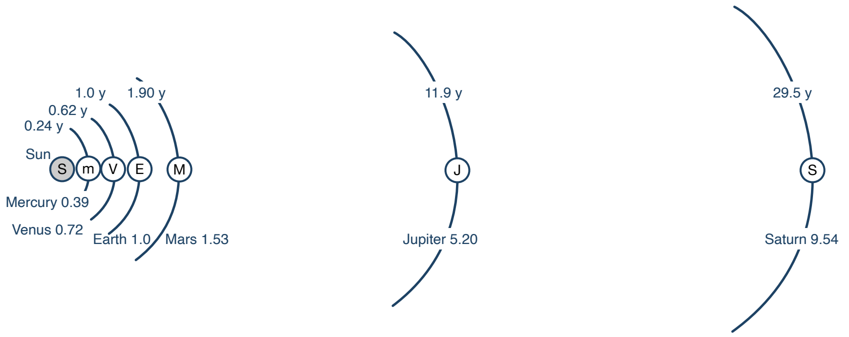


Figure 5.3: An approximation to the grade-school version of the Copernican system of planets all centered on the Sun. The layout is to proportion of distances from the Sun in AU (see the text) and are listed with the planet's names. Their "years" around the Sun are also shown at the top.

Trying to think like Ptolemy is difficult since we've all been taught the basic geometry of the Copernican solar system, so let me remind you of the conclusion to our story and then the discussion of how he got there will be easier to follow. Figure 5.3 shows the solar system (without moons) in rough proportion to distances from the Sun relative to the distance of the Earth which are now called Astronomical Units, or AU.[3] These distances are shown with their values in AU and the "sidereal" period—the "year" of a planet's trip around the Sun in Earth-years—is shown above for each.[4] There's a lot more to say about this in a bit.

It's useful to show the Copernican motions side-by-side with those of the Ptolemaic layout and Figure 5.4 does that. While it looks complicated, just follow the numbers:

- The right image is an overlay of snapshots of Mars' motion (the circle with "M") around the Earth (E) at four successive times denoted by M1, M2, M3, and M4. The arrows are the line-of-sight from Earth to the planet and the relative location of the mean Sun (circles with S at those same times, 1–4) is also shown. (For time 1 Mars is behind the Sun, so would be invisible from Earth.) The dash-dot curve is the path of Mars, showing the loop that models retrograde motion at time 3. The dashed circles are the epicycles carrying Mars which are centered on the deferent at *C*.
- The left image is the Copernican system, following Mars at those same M1–M4 times, plus the Earth (now at E1– E4 times) as they both go around the now stationary Sun. The arrows show the same thing: the line-of-sight from Earth to Mars and you can see that they are parallel to those lines in the right

[3]One AU is the average distance from the center of the Earth to the center of the Sun, so 1 AU = 149,597,871 km (92,955,807 miles).

[4]The word "sidereal comes from the Latin, sidereus, or "star." So the sidereal year is the time to go around the Sun relative to the stars.

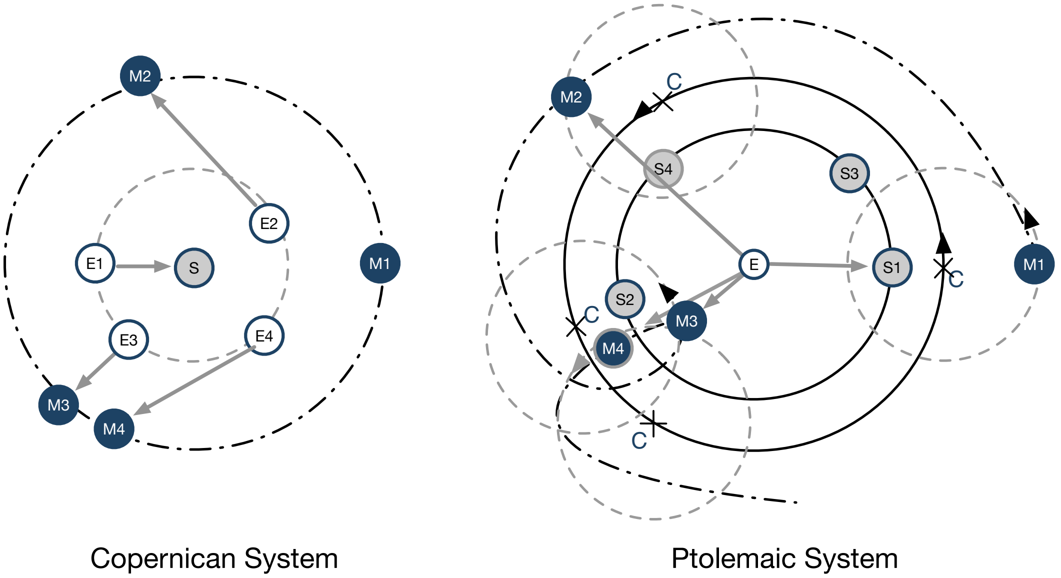


Figure 5.4: Four successive times for Mars' trip around the Sun (the Copernican model in the left-hand column) or the Earth (the Ptolemaic model in the right-hand column). The circumstances are described in the text.

hand column. That makes sense since each model must preserve the same appearance for someone on the Earth looking at Mars. While it's not drawn, notice that a line from Earth to the Sun on both sides is also parallel at all times.

- The Sun in the right image makes more than one revolution which is because (in Copernican terms) Mars takes more than an Earth-year to go around the Sun. That's reflected in the left image as Mars doesn't make it all the way around by the time Earth completes its year at E4.
- Finally, notice that when the planet is in retrograde motion in the right side at M3, at the end of the loop-the-loop that Ptolemy invented, Mars is also closest to Earth in the Copernican system.
- Notice that the dash-dot-path of M in the Copernican system follows a circle that's the same size as the deferent in the Ptolemic system and that the size of the Earth's orbit in the Copernican system is the same size as the epicycle in the Ptolemaic.

Ptolemy's model gave accurate position results (and still does with updated parameters) and Copernicus' model gives accurate results, but no better. Why did other astronomers take the Copernican Project seriously, indeed, why was Copernicus apparently. . . a Copernican?[5] How he reached his conclusions—at a very early

[5]Philosophers of Science like to distinguish what they call the Context of Discovery as distinct from the Context of Justification. For most of the 20th century, it was deemed improper for philosophy to pay attention to the Context of Discovery. Only the logical reconstruction of results matter. History

age—is another detective story. I've come to my own version which I'll tell here.

5.3 A Little Bit of Copernicus

Starting Copernicus' story at the end is standard since it's legendary. At the age of 70, he suffered a debilitating stroke and just before he passed away Bishop Tiedemann Giese, his dear friend of four decades, later wrote that he placed his friend's enormous, newly printed book—his life's work— in his dying hands. Giese seems a reliable source—he started his career with Copernicus as one of the few ordained Warmia canons and was by then the Bishop of Kulm.[6] It's a poignant end to a life of consequence and is echoed in the story of another Catholic official, Fr. Georges Lemaître, who'd mathematically anticipated the big bang and learned only shorty before his death in 1967 of the new experimental result that was the primary confirmation of *that* physicist-cleric's audacious cosmological theory.

The most famous story of MOTION BY THE EARTH and MOTION IN THE HEAVENS of all begins in Toruń on the banks of the Vistula River, a 1000 km long heavily used waterway carrying iron, salt, grain, and yes, gingerbread to the rest of Europe. Toruń was one of its most prosperous ports—Toruńian merchants and agents even had homes in London. The city escaped serious damage during WWII and is today a protected example of a 15th century medieval city.

We know the stately, peaked Gothic home on St. Anna Lane (now Copernicus Street) where Nicolaus was born to Niklas (MIkolaj in Polish) Koppernigk (1450–1483),[7] and Barbara, née Watzenrode. Niklas senior was a prosperous merchant who moved to Toruń in 1456 as a mature man and a fierce opponent of the Knights. Barbara came from an established merchant family. Newly an alderman, Niklas moved his family to a more prestigious home in City Center. One can only imagine what manner of commercial bustle, seasonal festivals, and publicly-administered, severe justice would have been a part of a youngster's growing up. The large house across from City Hall were converted into a department store in 1906.

Mikołaj Koperniks' (he latinized his name to Nicolaus Copernicus when went to the university) birth is recorded as 4:48 PM on Friday, February 19th, 1473. That's fake, a horoscope cast by a supporter when he was already a famous European mathematician. He was, nonetheless, born at the launch of the High Renaissance (Leonardo's ***Annunciation*** was completed the year before) and just as the world became large: Columbus sailed to the North American continent when Copernicus was 19 years old. Printing had only been invented 23 years before his birth and

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became more important in the 1960s and that's what we're doing here. But Copernicus (or actually, his friend Tiedemann Giese, to whom he willed his papers) made hard for those concerned with the Context of Discovery is that there are no papers.

[6]Copernicus willed his papers to Giese but they're lost, so we know his results, but we've no documented path to them.

[7]The family name might have come from the German term for metal, kopper, or the Polish word for dill, koper, either of which might match his originally pedant family.

commercial printing came to Cracow with the first production an astronomical almanac in the year of his birth.

Niklas died when Nicolaus was 10 years old and while not destitute, Barbara appealed to her brother for help. Lucas Watzenrode (1447–1512) was an ordained canon of Warmia and he took charge, as was apparently his nature (he was reported to never having been seen smiling and was once referred to as a "harsh, sinister man"), parceled out his nieces and nephews to a convent, marriage to a businessman, and the two nephews to school. The older Andreas had a difficult life and yet seemed to always follow in his younger brother's footsteps. He was made a canon in Warmia with his brother, but eventually suffered from leprosy and died at an unknown time and location in Italy, having been forced to leave the cannonry. Nicolaus helped to support his sister's children until the end of his life.

5.3.0.1 Copernicus' Childhood and University Education

Nicolaus probably attended primary school at St. John's Church, not far from home. The hard-to-please Uncle Lucas saw something in Nicolaus and he would have then studied at either of two highly regarded cathedral schools, in Kulm or Włocławek (both about 15 miles from home). . . so he would have left Toruń around 1485, never to permanently return.

Uncle Lucas was promoted as the Prince-Bishop of Warmia in 1489 which came with the responsibility for the civic and spiritual needs of the nearly self-sustaining province and the authority to direct his nephew's education and employment.

5.3.0.2 University of Cracow

> "There is in Cracow a famous university, which boasts many most eminent and highly -educated men, in which all sorts of proficiencies are practiced, such as the study of speaking, poets, philosophy, and physics. But the science of astronomy stands highest there, and in all Germany there is no school that would be more renowned, as I know from the accounts of many persons." Hartmann Schedel of Nuremberg

In 1491, Nicolaus and his brother enrolled at the University of Cracow[8] where their uncle had previously studied. Cracow was the capital of Poland, home of King Casimir IV Jagiellon and a cosmopolitan, humanist, European center.

The University was unusually endowed with chairs in both astronomy and astrology, so the theoretical and practical were both covered and scores of its graduates were employed in courts all over Europe. His class in the Arts had about 350 students, half of whom were from outside Poland and about a third left without a degree. . . and Nicolaus was one of those—after four years he moved on.[9]

[8] now, the Jagiellonian University of Krakow

[9] Uncle Lucas also left Cracow without a degree, taking his next step at the University of Cologne where he did graduate before going to the University of Bologna. **goddu10** suggests that having a paid appointment as canon *and* graduating with a degree would have violated the Warmia Chapter's rules unless he studied for an advanced degree at Cracow. If Bologna was in his and Lucas' plans,

Books were expensive and so manuscripts were probably read out loud to students in lectures (starting before daybreak). He certainly would have studied Peurbach's *Theoricae novae planetarum* and likely Buridan's studies of Aristotle's MOTION ON THE EARTH and MOTION BY THE EARTH. His personal copy of Euclid's *Elements* was printed in Venice in 1482 and among four books that he kept for his life, paying for wooden bindings of two sets of tables and inserting 16 blank pages (which became historically significant as we'll see) in the binding for his notes.

The University of Cracow had a number of distinguished astronomy/astrology professors, including some who studied in a chain of influence from Peurback and Regiomontanus and through contacts, they had advance copies of *Epitome*. Graduates were employed in courts all over Europe. One of the faculty reportedly concerned himself with planetary ordering, so there might have been a spark struck with Nicolaus. By the time he left, he was a professional astronomer with deep training

Copernicus left Cracow in 1495 and what he did next is of some conjecture. The most likely path is that he left Cracow for the canonry cathedral in Frombork on the Baltic Sea (see Figure 5.1), the northern-most part of Warmia, a non-trivial 400 mile trip so surely his uncle instructed him to go. Frombork was the Chapter home of 16 Warmia canons, the administrators of the whole Warmia diocese — and political state of its own: they managed the merchant, agriculture, military, peasant classes, and an economy requiring constant oversight. It was his eventual profession.

The job of canon was an odd profession and didn't require ordination and there's no evidence that Copernicus took Holy Orders and so he could not say mass. [10] A canon was expected to have a home inside of Frombork's walls and was given funds sufficient to own a horse, a servant, and a house outside of the walls. The Prince-Bishop's formidable castle was in Lidzbark Warminski (in German, Heilsberg), a two day journey.

One of the canons died and Lucas nominated Nicolaus to the post, a lifetime, lucrative job. An advanced degree from "some preeminent stadium," was required. So Copernicus left for Bologna, Italy in 1496, with a pending clerical church appointment in his rear view mirror. This was a 1000 mile, harrowing, three week journey through Cracow and Torun, to Venice and on to Bologna. He would he would have passed through Vienna and one can imagine his thoughts as he surely stayed in Peurbach and Regiomontanus' famous astronomy city.

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then he needed to obtain enough training to get into an Italian university, but without a degree so as to not violate the rules. So he might have delayed a degree until he absolutely needed to have one, which came in Italy many years later. This suggests that a Church appointment was planned early on.

[10] Yet canons were expected to observe a priestly vow of celibacy which, as we'll see, got him into some hot water with subsequent management.

5.3.0.3 Italy

Copernicus lived in four different Italian cities at two different universities, graduating from a third. Starting in 1496 he attended the University Bologna (Lucas' alma mater) where he studied canon (and perhaps secular) law. During that time, we know that he visited Rome for an extended visit to deliver lectures on mathematics during the Jubilee Year of 1500 — which must have been a city-wide, wild scene as that periodic celebration was organized for the scandalous Pope Alexander VI of Borgia infamy. I like the Rome story since it coincides within a few months of the time that Michelangelo had moved from Bologna to Rome to create *Pietà*. In fact, Michelangelo left Bologna for Rome in the same year that Copernicus arrived.

Bologna (law) and Padua (medicine) had the best faculties in all of Europe. The University of Bologna was the first university in the west with almost 100 faculty graduating five popes who shamelessly supported it and so where Copernicus lived for the next four years was a cosmopolitan center of intellectuals and boisterous student life. He had to sheepishly ask Uncle Lucas for more money suggesting that they didn't avoid distractions. While he was in Bologna, his appointment as canon was finalized.

Astronomy was still on his mind and he actually rented rooms from and did observations with Domenico Maria da Novara (1454-1504), Bologna's young astronomy professor who was apparently a student of Regiomontanus and studied at the Platonic stronghold of Florence. By this time *Epitome* had been printed and Nicolaus absorbed it and began to think for himself.

> Copernican literature is full of speculation about when and how Copernicus came to his heliocentric conclusion. To me these speculations sometimes seem to turn on searching for that *that one event*, *that one person*, *that one idea*... the ah-hah moment. I'm not convinced of this approach but I am impressed with some historical analysis in **westman11** who delved deeply into the Bologna astrology community during Copernicus' residence. It was vigorous in no small part because of Giovanni Pico della Mirandola's (1486–1493) loud denigration of the entire astrological enterprise. If one can't be certain of the order of the planets, then how could one possibly believe any astrological claim? As **barker13b** point out, "If these locations are wrong, then so are the powers, and the intensities of the powers, assigned to each planet." Remember that the relative ordering of Mercury, Venus, and the Sun had been an ongoing back-and-forth since the classical Greeks. Ptolemy made an executive decision about planetary ordering, not a scientific one. Copernicus had to know of Pico's very public objections.

He left Bologna after four years, again, without a degree. Were he to take up his new job in Warmia, schooling was over and he hatched a plan. Back to the north the brothers went, another 1000 mile trip, arriving in 1501 in order to appear before the Warmian Cathedral Chapter where they asked to go *back* to Italy so that Copernicus could study medicine in Padua in the Venetian Republic. The report from the Chapter read, he "promised to study medicine with the intention of advising our most reverend bishop in the Future, as well as member of our chapter, as a healing physician."[11]

There's a legitimate connection: in order to be a professional medieval physician, one must be proficient in astronomy and astrology. If the body's humors were not right or if some other disease was apparent, blood-letting was the cure. But from which part of the body the physician would extract the blood depended on the time of year and what part of the zodiac was rising. So medicine would be the perfect excuse to continue astronomy. The course of study for a medical diploma was three years, but his approval for another educational program granted by the Chapter? Only two.

Once those two years were up, he was out of excuses and needed to return so it was the time to collect a university diploma. Not from Bologna. Not from Padua, but from Ferrara, situated between Padua and Bologna, because it was much cheaper.[12] The tradition was that examiners were hired by the student who also had to hold a banquet for everyone which could cost as much as a year of tuition. So on May 31, 1503, Copernicus took the examinations for doctor of canon law at the University of Ferrara, where nobody knew him, and returned north to his new home, never to leave again.[13]

5.3.0.4 Being a Canon in Warmia

Nicolaus didn't return to Frombork, but rather to the Prince-Bishop's castle at Lidzbark as an advisor and counsel to his uncle taking at least a couple of diplomatic trips inside of Prussia and Poland. He acted as a personal physician for his uncle and others in the castle, successfully treating Lucas for a serious illness in 1507. He was a respected physician his whole life. He also must have had some time on his hands.

He probably learned some Greek in Padua and was proud of it, presumably to help him with Greek astronomical manuscripts. As a frivolous project, he translated into Latin pieces of an obscure Greek collection of stories called *The Universal History* from a seventh century Byzantine writer, Theophylactus Simocatta. They ranged from bawdy to serious and he published his version in book-form with a dedication to Lucas.[14] Lawrence Corvinus (c. 1465-1527), a friend and academic poet arranged for its printing in 1509 and wrote an introductory poem in which he indicated a not-warm acknowledgement to Lucas ("revered for his grave demeanor") but a glowing description of the author:

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[11] About Andreas, the Chapter wrote, "Andreas also seemed qualified to engage in studies."

[12] Without taking classes or enrolling, in Europe one could be examined and graduate from a university where you didn't do your work. Einstein did that.

[13] Andreas made another trip to Rome on Chapter business and then presumably once last time after being asked to leave because of his leprosy.

[14] It's not a very good translation. Copernicus' home-schooling in Greek has been taken apart many times. It's riddled with errors.

> "He discusses the swift course of the moon and the alternating movements of its brother as well as the stars together with the wandering planets — the Almighty's marvelous creation — and he knows how to seek out the hidden causes of phenomena by the aid of wonderful principles."

The Moon's "brother" was Earth. . . as distinct from the stars and the wandering planets. . . and he seemed to recognized that Nicolaus was doing something new, seeking out "the hidden causes. . . by means of wonderful principles." Somewhere between his Bologna time in 1496 and that publication date of 1509, Copernicus had begun to hatch his Project and this poem dates its earliest time.

5.4 Copernicus' Project

Copernicus' theory of his universe was described in the two books mentioned above. The first one is the brief summary, *Commentariolus*, and the second is *De revolutionibus orbium coelestium* from literally the last day of his life and decidedly, not brief. *Commentariolus* marks the earliest time that he could have reached his conclusions. It was probably a letter sent to colleagues and subsequently copied and passed around. *De hypothesibus motuum coelestium a se constitutis commentariolus* is surely not Copernicus' title and it's been known as *Commentariolus* since the 17th century. Almost all current versions of it originate from Tycho Brahe's[15] undated copy from about 70 years after Copernicus' death. So when was *Commentariolus* written?

That's tough since there is no copy of that manuscript written in his hand. The latest that it could have been written comes from lucky circumstantial evidence: In the papers of a Cracow professor of medicine, there was a note dated May 1, 1514 that mentions in translation, "[a]. . . six-folio theory declaring that the earth moves and the sun is in fact at rest. . . ". So early 1514 is the latest time that *Commentariolus* could have been written and the poetic preface to his Greek translation, is the earliest.

So the frame of Copernicus' intellectual development and his heliocentric evolution is roughly 1508 – 1514. The first is about four years into his six year stay in Lidzbark and the second, corresponds to his first four years when he was installed in Frombork. So it's reasonable to conclude that his years in Padua might have been a pivotal time for him.

5.4.1 What Did Copernicus Bring to the Project?

It must have been challenging to straddle eras as in some ways Copernicus had one foot in the Renaissance and the other in the Baroque. His Renaissance commitments would have come from his schooling and private study in Italy and probably included:

[15] We'll meet Tycho in the next chapter and yes, he's another one of those luminaries who's referred to by his first name.

1. Circles were the perfect trajectory for any heavenly body. So *his cosmology was Aristotelean*.
2. The planets (and Moon and Sun) traveled on the equators of rotating spheres of solid, ethereal matter... dubbed "crystalline." So he had a working commitment to Aristotle's aether as the underlying substance.
3. He accepted that the mathematical machinery of the planets was eccentrics and epicycles and so *his astronomy was Ptolemaic*.
4. He had somehow learned of the mathematical successes of the Maragha School and used some of their tools. Nobody understands how that knowledge seeped into his working awareness, but most think that his Padua years were a likely place where he might have heard of them or seen even some drawings.
5. He relied on the *Alfonsine Tables* almost exclusively.
    - Critically, he knew two pieces of data that I think figured crucially in his modeling. He knew how long each planet took between maximum retrograde positions and he knew the radius of each planet's epicycle in Ptolemy's relative units. These data had been known for 1200 years.
6. He inherited the flexibility of the early modern era that questioning Aristotle's physics was fair game.
7. He accepted that the Sun was a planet and that the Earth was at the center of the universe, just as Ptolemy fleshed out Aristotle's cosmology.

Rather than a single ah-hah moment, I can envision a progressively productive awareness of the virtues of a heliocentric model so the conceptual change for him is the modification of commitment #7 above.

5.4.2 What Came Out of Copernicus' Project?

1. The Earth is a planet.
2. This Sun is not a planet nor is it directly in the center of the universe.
3. His model in *Commentariolus* was identical to that of Ibn al-Shatir's for the Moon, Mercury, and the superior planets, but was Sun-centered.
4. He modified that heliocentric model later, still relying on Ibn al-Shatir for the Moon and Mercury but substituting an eccentric in exchange for an epicycle for the superior planets in *Revolutionibus*. This is both new and old.
5. He found two methods which definitively order the planets forcing fixed orbital radii for each.
6. He determined the duration of the "year" for each planet.
7. He determined the radius of each planet's orbit relative to that of the Earth.
8. He explained retrograde motion as a fact of Earth's orbital motion.
9. He was so persuaded of his conclusions (I think about the ordering of the planets) that he decided that the fixed star sphere was much further away than anyone had ever imagined.

5.4.3 Commentariolus

In his humanistic frame of mind, at the beginning of *Commentariolus* he paid great attention to "the ancients," including Pythagoras as if early Greeks and early Neo-Platonic writers were his advisors or teachers. And while he seemed not to take the explicit Pythagorean cosmology seriously, he certainly knew that treating the Earth as a moving and/or rotating planet was not unheard of.

> I pondered long upon this uncertainty of mathematical tradition in establishing the motions of the system of the spheres...I therefore took pains to read again the works of all the philosophers on whom I could lay hand to seek out whether any of them had ever supposed that the motions of the spheres were other than those demanded by the mathematical schools. I found first in Cicero that Hicetas [a 5th century BC Syracusian] had realized that the Earth moved. Afterwards I found in Plutarch that certain others had held the like opinion...
>
> Accordingly, let no one suppose that I have gratuitously asserted, with the Pythagoreans, the motion of the earth; strong proof will be found in my exposition of the circles.
> Copernicus *Commentariolus*

He would have been aware of the writings of Nicolaus of Cusa (1401–1464), who made any number of minority proposals, including that the Sun was the center of the universe and that the planets' orbits were not perfect circles.[16] and maybe Roman architect, Vitruvius (from the late first century).[17] And, he might have been aware of some Arabic writers who also dabbled in heliocentricity.

About half-way through the *Commentariolus*, he reveals in an off-handed way the (correct) order of the planets and that the amount of time that it takes for Saturn, Jupiter, Mars, Venus, and Mercury to circle the sun. How did he do that before 1514? I can imagine that it came in two stages. The first could be done with almost no geometry and only a little research within the Alfonsine Tables. I'll call this "Ordering of the Planets, the First Way," (Section 5.4.5).

Then probably later, with a lot more thought, including that original contribution by Regiomontanus, he could have confirmed that hypothesis in an entirely different way, which I'll call, "Ordering of the Planets, the Second Way," (Section 5.4.6). I know from my experience, that two distinctly different ways to reach the same scientific conclusion (whether in theory or in experiment) is confidence-building. You know you're on to something.

The first way would give the periods of the planets and strongly hint at their ordering and the second way would predict their order and give the distances of each from the Sun, confirming the first way.

[16] His idea of "learned ignorance" insisted that there are things we just can't know and made explicit reference in the paragraphs above.

[17] Who wrote in his The Ten Books on Architecture that "The planets Mercury and Venus nearest the rays of the sun, move round the sun as a center."

▻ So the idea that planets might go around the Sun was "in the air" and I think that stimulated the Project's main task: "If the Earth moved, what would be the consequences?"

5.4.4 Maybe Some Early Confidence?

Without any introduction, he starts in by highlighting and criticizing the ancients:

> "CALLIPPUS and EUDOXUS, who endeavored to solve the problem by the use of concentric spheres, were unable to account for all the planetary movements;...Yet the planetary theories of PTOLEMY and most other astronomers, although consistent with the numerical data, seemed likewise to present no small difficulty. For these theories were not adequate unless certain equants were also conceived; it then appeared that a planet moved with uniform velocity neither on its deferent nor about the center of its epicycle. Hence a system of this sort seemed neither sufficiently absolute nor sufficiently pleasing to the mind...."

So he's declared his unhappiness with constant circular motion only about the equant and not the Earth or the deferent center. He has either inherited Muslim astronomers' disgust, or come to it naturally himself.

> "Having become aware of these defects, I often considered whether there could perhaps be found a more reasonable arrangement of circles, from which every apparent inequality would be derived and in which everything would move uniformly, as a system of absolute motion requires...if some assumptions (which are called axioms) were granted me. They follow in this order."
> Copernicus, emphasis, mine *Commentariolus*

So here we have the no-older-than 40 year old Copernicus noting that he "often" thought about another model and declares seven "axioms"...which really are not that. They address both MOTION BY THE EARTH and MOTION ON THE EARTH and here they are verbatim with my comments:

1. "There is no one center of all the celestial circles or spheres." [This is a little obscure. It suggests that not all of the spheres have the same center, which in his model is the case...there are eccentrics for him as well as Ptolemy.]
2. "The center of the earth is not the center of the universe, but only of gravity and of the moon's orbit." [He's quietly changed the nature of the Moon from one of the planets to now a satellite that orbits the Earth—indeed, as on its own "epicycle" relative to the Sun.]
3. "All the planets revolve about the sun as their mid-point, and therefore the sun is the center of the universe."[This is sort of a working hypothesis as is #6. Apart from #1, the rest are actually derived from #3 and #6!]
4. "The ratio of the earth's distance from the sun to the height of the firmament is so much smaller than the ratio of the earth's radius to its distance from the sun that the distance from the earth to the sun is imperceptible in comparison with the height of the firmament." [He refers to the outer shell of the (fixed) stars as the "firmament." He's now prepared to go where others were reluctant: that the universe is so large, that parallax cannot be observed.]
5. "Whatever motion appears in any motion of the firmament, but from the earth's motion. The earth together with its circumjacent elements performs a complete

> rotation on its poles in a daily motion, while the unmoved firmament and highest heaven abide unchanged." [Now he's doing physics...or rather, avoiding physics. There are two points in #5. First, that the stars (firmament) appear to move is due to the Earth's rotation. The stars are fixed. Second, all of the "stuff" surrounding the Earth—air, clouds, water, birds—move with the moving Earth together. Anti-Aristotle, but pro-Oresme.]
>
> 6. "What appear to us as motions of the sun arise not from its motion but from the motion of the earth and our sphere, with which we revolve about the sun like any other planet. The earth has, then, more than one motion." [The Earth goes around the Sun, and not the other way around.]
> 7. "The apparent retrograde and direct motion of the planets arises not from their motion but from the earth's. The motion of the earth alone, therefore, suffices to explain so many apparent inequalities in the heavens." [He's solved retrograde motion in a natural way by realizing that viewing a moving planet from a moving platform—explained by Ptolemy as epicycles—is just because the Earth is also moving.]
>
> Copernicus *Commentariolus*

5.4.5 Ordering of the Planets, the First Way

Among the major astronomical events that were always recorded in Tables are oppositions and conjunctions, the first of which is shown (from the modern heliocentric perspective) in Figure 5.5.

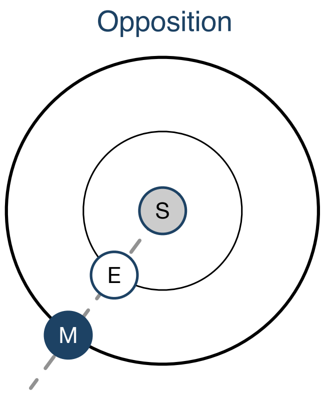


Figure 5.5: In an opposition the Sun, Earth, and a planet all line up in a row with the Earth in the middle.

In Figure 5.4 at the first times (E1 and M1) you can see examples of conjunction in both the Copernicus and Ptolemaic systems (and opposition for both at the third times (M3 and S3)) when the planet is on its closest point in the loop-the-loop in its ancient epicycle modeling.

Lets focus on Opposition. The time span from opposition to opposition was measured over and over from early Greeks to beyond Copernicus' time: how many days, months, or years does it take for a planet to reach the point of apparent closest approach when it's brightest, which is when the epicycle is doing its job, as in Figure 5.4, M3 on the right.

With a simple diagram and two numbers from antiquity and the presumption of heliocentricity, he — or anyone in the previous 1700 years —could have made a major discovery simply by asking a simple question about, say, Jupiter, "What would the relationship between Earth and Jupiter be in successive oppositions look if both orbited the Sun as planets?"

Let's define some travel times and terms and then look at the Earth-Jupiter case.

1. The number of days in an Earth year (specifically, the time to go around the

Sun as fixed relative to the stars) I'll call E, which he knew to be 365 days.[18] This is called the Earth's **sidereal year** since it's measured against the fixed stars.

2. Likewise, the number of days for Jupiter to go around the Sun I'll call S. That's the **planet's sidereal year** and that's what he wants to find out.

Think about driving. Your speedometer tells you your speed with respect to the Earth—that's analogous to a sidereal "speed." Likewise, a car that you just passed has a speedometer reading of its own. But suppose you want to know how fast you're going relative to the other car, not the Earth? You'd need to know the two speedometer readings and subtract them, right?

But what about the reverse problem: you know your speedometer reading (your speed relative to the Earth) and you know the *speed of the other car relative to you*...and you want to know the speedometer reading of that car you just passed...relative to the road. If you were a police car, that's a calculation that your radar system would do.

3. The number of days for a planet's orbit to repeat itself *relative to Earth* is called a **synodic year**. Both are moving platforms and this period has nothing to do with the Sun. Opposition is easiest repeatable observable to use as a way to mark the beginning and end of a year so let's call the synodic year P, the time between oppositions.

Copernicus knew the number of days that it takes for Jupiter, Earth, and the Sun to be in opposition is 399 days (more than an Earth year). But in Copernicus' Project, he faced the police-radar problem: from the 399 days between oppositions, how long it takes for Jupiter to go around the Sun? Copernicus' (I'm imagining young) insight was that if both Earth and Jupiter are orbiting the Sun, then **Jupiter's sidereal year could be calculated**.

With that in mind, lets think about the synodic year by looking at Figure Box 5.6 on page 176. After you've read the material in that Box, return to this point and continue reading.

In the *Commentariolus*, he referred (somewhat offhandedly) to the superior planets, and for Jupiter, rounding 11.75 years to 12 and reports on Mars and Saturn. Later in the document, he reports on Mercury and Venus.

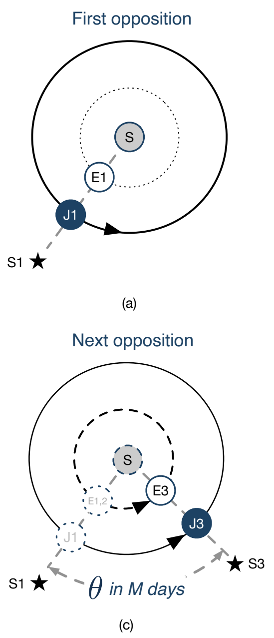
> "Saturn, Jupiter, and Mars have a similar system of motions, since their deferents completely enclose the great circle [He called the Earth's orbit the "great circle."] and revolve in the order of the signs about its center as their common center. **Saturn's deferent revolves in 30 years, Jupiter's in 12 years, and Mars' in 29 months**; it is as though the size of the circles delayed the revolutions."
> Copernicus *Commentariolus*

Table 5.1 shows his results and modern comparisons.

[18] ...and so did Copernicus, although for other purposes, he worried about the precision of that value

- The first column (geocentric) are the synodic years as understood by Ptolemy and everyone after (to Copernicus) determined from opposition measurements.
- The second column (geocentric) is called the "zodiacal year and refers to when a planet returns to a point against the zodiac as observed from the Earth. Because of the Ptolemaic model tying the inferior planets to the Sun, Mercury and Mars move with the rising and setting Sun together, they are the same. (See Figure 3.25 and recall that Mercury and Venus are tied along a line to the Sun. So where the Sun goes, they go.) Notice that this "year" is not very helpful in understanding the ordering of the planets. That was a 1300 year problem.
- The fourth column (heliocentric) is the numbers reported in the *Commentariolus*. These are the first sidereal periods every predicted.
- The fifth column (heliocentric) are refined and are in *Revolutionibus*.
- The last two columns (heliocentric) are the synodic and sidereal (the "regular" year) values from today.

FIGURE BOX 5.6



In (a) we see Jupiter (J1) and Sun in opposition. An observer on Earth (E1) can see J1 against a particular star (S1), a fixed reference point on the stellar background. One year later, the Earth has gone around 360° in 365 days and is at E2, while the planet has advanced only a little (J2) as in (b). As the Earth keeps orbiting, eventually it finds itself back in opposition with the planet with E3 and J3 in (c) but because it's more than 365 days to achieve that arrangement, we would see the planet against a different star, S3. That extra arc for Earth to catch up is θ in (c) and it's the same angle for the planet between J(1) and J(3), but about a larger orbit. The angle is the fraction of an Earth year that extra number of days represents.

What Copernicus must have figured out is that given that shared arc and the number of days extra, the full path length for the planet could be calculated. Let's put in some numbers. The synodic year for an Earth-Jupiter-Sun opposition is 399 days. So the extra number of days that Earth had to travel to catch up is $P - E = 399 - 365 = 34$ days which means that the fraction of Earth's orbit spend catching up is $\dfrac{34}{365} = 0.093$ and the angle of that arc is $\theta = 0.093 \times 360^\circ = 33.5^\circ$. Since Jupiter traveled that short arc in $P = 399$ days, and that arc is 0.093 of *it's* 360° year, so its sidereal year is: $S = \frac{P}{0.093} = \frac{399}{0.093} = 4{,}290$ days $= 11.75$ years. The consequence is rather astounding...solving a 2000 year old problem. Not bad for a young Nicolaus.

Now go back to page 174 and pick up where you left off and see that consequence.

Things to notice about the geocentric numbers: The Ptolemaic synodic periods are all over the map and are no guide. Zodiacal periods are not so different from the sidereal periods for the superior planets, since measuring against the zodiac is the same thing. But the inferior planets' values are theory-driven to be the forced period of 1 year.

These are firsts! Nobody had ever found a way to order the planets and measure their "years" before Copernicus. Notice how Earth's year is nestled nicely between that of Venus and Mars. It's easy for me to imagine him figuring this out with only minimal data, and realizing that he'd done something brand new: This is a

Table 5.1: The sidereal years for all of Copernicus' planets reported here in Earth years. He made some changes between *Commentariolus* and *Revolutionibus*, but his accuracy is impressive. For Mercury, he said "three months, that is 88 days" and for Venus he said "nine months." He made an arithmetic mistake in *Commentariolus*, fixed in *Revolutionibus*.

	Ptolemaic		*Comm.*	*Rev.*	Modern	Modern
Planet	**Synodic**	**zodiacal**	**sidereal**	**sidereal**	**synodic**	**sidereal**
Mercury	0.32	1	0.24	0.24	0.32	0.24
Venus	1.60	1	0.75	0.62	1.60	0.62
Earth	0.00	0	1	1.00	1.00	1.00
Mars	2.14	1.88	2.42	1.90	2.14	1.90
Jupiter	1.09	11.86	12	12.00	1.09	11.90
Saturn	1.04	29.46	30	30.00	1.04	29.50
Uranus					1.01	84.00
Neptune					1.01	164.80

powerful moment and only happens every once in a while in the history of science. We'll see a few more.

Now in my imagination, his Project gained a measure of excitement for him and he was in need of some supporting data for his now intriguing model. That second way of determining planetary ordering sealed the deal.

5.4.6 Ordering of the Planets, the Second Way

In 1587 Sigismund III Vasa, the son of King John III of Sweden and Catherine Jagiellon was the natural choice for the Polish monarchy and also, as a Swedish duke, a hereditary future monarch of Sweden. He was militantly Catholic, while Sweden was staunchly Lutheran and while those mixed connections kept Sweden and Poland out of Europe's Thirty Years' War, it didn't last and war eventually broke out between the Sweden and Poland in 1600.

What's the connection with Copernicus, you're wondering. Among the spoils of war were all of Copernicus' books which were removed from Frombork by Swedish soldiers and now reside in *The Copernicana Collection* at the Uppsala University Library.

Preserved in this collection and bound between Copernicus' copy of the *Alfonsine Tables* from 1492 and Regiomontanus 1490 edition of the *Tabulae directionum* is a cryptic page of notes certified as in his hand that Swerdlow liked to call "U". Considerable effort since the 1970s has gone into interpreting what they mean with in-print battles breaking out over interpretation. I think that the consensus is that these are the key to understanding Copernicus' second way of ordering the planets.

Copernicus realized an important thing about appearances of relativity moving objects, called "Galilean Relativity." Namely, you can't tell the difference if the objects are moving at constant speeds.

> “...every apparent change in place occurs on account of the movement either of the thing seen or of the spectator, or on account of the necessarily unequal movement of both. No movement is perceptible relatively to things moved equally in the same direction - I mean relatively to the thing seen and the spectator... As the ship floats along the calm, all external things seem to have the motion that is really that of the ship, while those within the ship feel that they and all its contents are at rest....” Copernicus *Revolutionibus*

This realization is by way of explaining a shift of the geometrical arrangement of the planets in *Almagest* from centering on the Earth to the Sun. It wasn’t a whim, but actually a complicated two-step geometrical process.

5.4.6.1 The *Epitome* Connection

Regiomontanus’ *Epitome* was only published in 1496, twenty years after his death and Copernicus owned a copy. While the *Epitome* was meant as a guide to *Almagest*, it was a sophisticated treatment of Ptolemy’s work, including more than a few original contributions.

It’s apparent that Copernicus spent time understanding *Epitome’s* Chapter 12 as it’s there that he must have intuited some important ideas. The Regiomontanus influence seems so crucial, that in some ways I think of him as almost a collaborator of Copernicus, albeit without their having overlapped by decades. In Figure 5.7 (a)

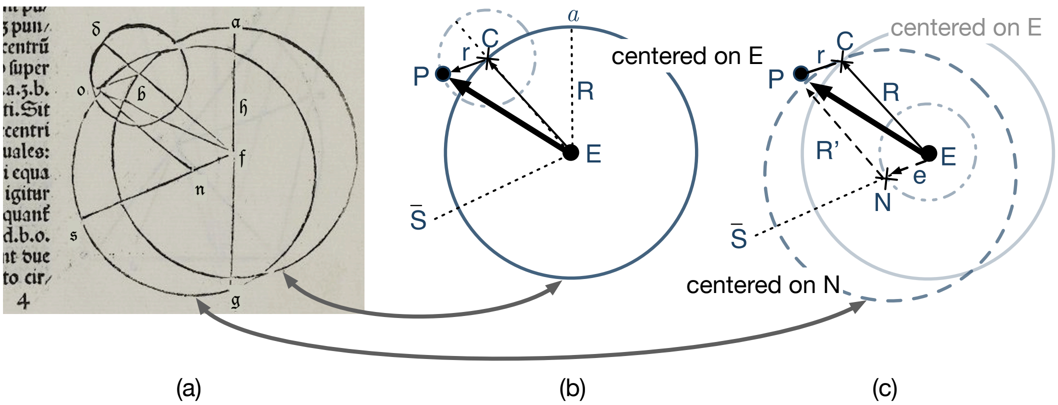


Figure 5.7: On the left is a section of a page in *Epitome* for superior planets. The center and right figures break the superimposed two scenarios in the left into their own images.

I’ve shown a complicated diagram that I’ve lifted out of Chapter 12 of *Epitome*. Regiomontanus packed more than one diagram into a single drawing here which I find that hard to parse and so I’ve separated out the two different images that are overlayed in (a), emphasizing the line of sight from Earth to a planet (his $\overline{fo}$) with a bold arrow and changed Regiomontanus’ labels in order be consistent with our previous images. Within Figure 5.7 (a) you can see Regiomontanus locating the planet (o) riding on an epicycle, centered on its deferent, which is itself, centered on point f.

I've extracted that in Figure 5.7 (b), replacing f with E (for Earth). The planet P is riding on the epicycle (dash-dot-dot circle) with radius r, centered at C, which rides on the deferent (dark, solid circle), centered on E with radius $\overline{EC} = R$. The bold arrow $\overrightarrow{EP}$ is the line-of-sight from Earth to the planet and the dotted line parallel to r toward $\overline{S}$ is the direction to the (mean) Sun.

The triangle $\triangle ECP$ in (b) shows a way to map out a path from Earth to the planet: draw the arrow $\overrightarrow{EC}$ and then another $\overrightarrow{CP}$ to go from $E \to C \to P$.[19]

But Regiomontanus pointed out that there's a *second* vector path:[20] Without altering the line of sight to the planet—that bold arrow $\overrightarrow{EP}$. In Figure 5.7 (c) I've shown how he demonstrates in (a) that P can also be reached by completing a parallelogram, ▱. This requires picking out a point in space that he (and I) have called N and that alternative route is constructed by drawing an arrow from $\overrightarrow{EN}$, followed by $\overrightarrow{NP}$, so a second triangle, $\triangle$, to go from $E \to N \to P$. Copernicus uses this parallelogram construction many times in his work.

The other piece that Regiomontanus embedded in Figure 5.7 (a) is recalling from Apollonius and Hipparchus (Figure **??**) that one can represent the path of a planet on an epicycle equivalently as a planet following a path without an *without* an epicycle. Such a path is around an off-center orbit—called the "eccentric." In Figure 5.7 (c) I've separated that situation out from the composite in (a). Here the eccentric (dashed circle) is centered on that new point, N. (The original deferent is still shown as the light, solid circle.)

If one traces out P's path in Figure 5.7 (c), while the epicycle has been mathematically transformed away, the planet's trajectory around E is identical to that epicycular-driven path in (b). I've added a *different* circle (also dash-dot-dot) centered on E, which is not in Regiomontanus' original drawing.of $ECPN$. Notice that that circle is identical to the epicycle with now a radius $\overline{EN}$, identical to r because of the parallelogram construction. I think that N and the transformation presented an important clue to Copernicus:

> ▻ Copernicus must have recognized that in Regiomontanus' transformation a line from Earth to N extends precisely to the Sun.

This construction has four consequences.

1. The line r is always parallel to and has the same length as e.
2. The other arms of the parallelogram are R and R' and they are parallel and the same lengths.
3. The Earth, E is still stationary and as P orbits, now on the eccentric, N orbits E.

[19] Regiomontanus is actually doing vector addition.

[20] This follows from Apollonius' proof mentioned in Chapter 3 that motivated Hipparchus and Ptolemy.

4. All Regiomontanus needs for his transformation to work is to preserve the ratios of

$$\frac{R'}{e} = \frac{R}{r} \tag{5.1}$$

Regiomontanus did one more thing in Chapter 12. His epicycle-eccentric tradeoff had been known by Ptolemy, yet inexplicably Ptolemy couldn't seem to make it work for the inferior planets. Regiomontanus did that. He had a similar geometrical scheme that could trade off the epicycles for eccentrics that would work for Venus and Mecury, and so all of the planets. Figure 5.8 shows his model for an inferior planet like Venus. Notice that the direction to the Sun is along the line $\overline{EC}$, which is different from the Sun's direction for the superior planets as in Figure 5.7.

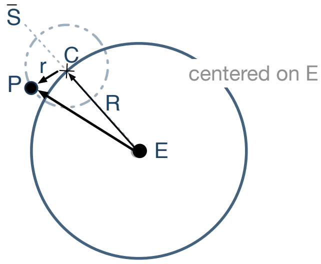


Figure 5.8: Regiomontanus model for an inferior planet, analogous to Figure 5.7 (b).

This now complete planetary reconstruction was meaningful to Copernicus and he seized on it and took notes shown in U in his own hand, reproduced in Figure 5.9. He left a maddeningly obscure puzzle which has been convincingly interpreted by Noel Swerdlow in **swerdlow73** and **swerdlow17** (where decades later he had to defend his original 1973 conclusions).

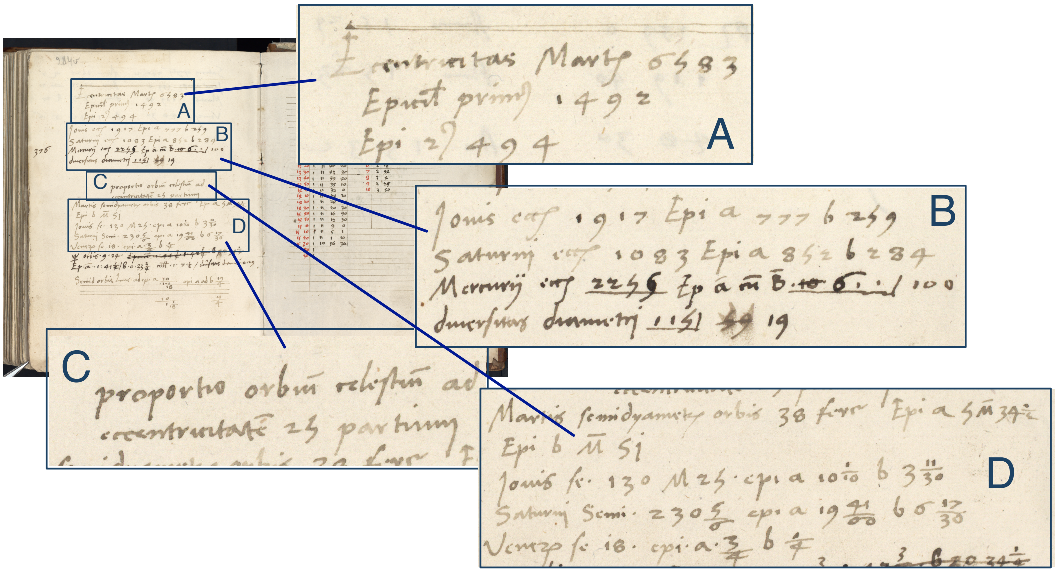


Figure 5.9: CAPTION

5.4.6.2 Three Big Steps

The invention of the heliocentric system seems to hang on that one page of scratch paper he'd had bound in his copy of the *Alfonsine Tables*. I've drawn boxes around some of the key points and we'll skim the surface. The top half of the page in the open book seem to provide input to the bottom half of the page and the bottom half of the page seem to be the source of some of the numbers he stated in *Commentariolus* since they are rounded as compared with U. So, importantly, it was written before *Commentariolus*.

He uses geocentric parameters about the epicycles from the *Alfonsine Tables*. In the first box, A, he wrote,

> Eccentricity of Mars 6583
> First epicycle 1492
> Second epi[cycle] 494
> Copernicus, translated in **swerdlow73** *Uppsala notes*

Why two epicycles? Stay tuned for that.

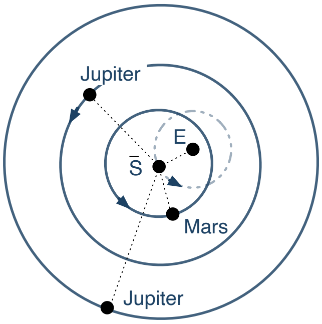


Figure 5.10: The first step in Copernicus' transformation of Regiomontanus' model makes Earth stationary with the Sun orbiting Earth and the other planets orbiting the Sun, shown here for Mars and Saturn.

The First Big Step. The path shift in Regiomontanus' diagram in Figure 5.7 (c) brings point N into the image as a corner of the parallelogram and since the line from Earth through N always points toward the position of the mean Sun, Copernicus moved the Sun to N where it falls on the rim of (my) added E-centered, dash-dot-dot circle with radius $\overline{EN} = e = r$, which is now $= \overline{ES}$. The epicycle circle has shifted to be centered on the Earth. So, P is orbiting N, which in turn is orbiting E as is shown in Figure 5.10.

Remember that for Ptolemy the radius of the epicycle for each planet was different and the radius of the deferent for each planet was the same. Copernicus writes those out in Figure 5.9 box A: " Eccentricitas Martis 6583" or "Eccentricity of Mars 6583." Recall that in Figure 3.24 the sizes of the epicycles are shown from *Almagest* for the common deferent of 60, with Ptolemy's Mars epicycle radius of 39.5. Copernicus scaled the 60 up to 10,000 for the superior planets (it makes the decimals easier to deal with) and so he worked with an epicycle radius of $r = \frac{39.5}{60} \times 10,000 = 6583$. He did this for each of the superior planets and in box B, you can also out: "Eccen of Jupiter 1917," "Eccen of Saturn 1083," and "Eccen of Mercury 2256."[21]

The Second Big Step. But what he did next was inspired. In Figure 5.9 box C he writes "Proportion of the heavenly spheres to an eccentricity of 25 parts." He scaled every planet's $\overline{ES}$ radius to be the same number, arbitrarily chosen as "25." Now imagine overlaying all of them centered on E: you'd have the set of relocated (formerly epicycle) dash-dot-dot circles each of radius $e = r = 25$ on top of one another and each P is now in a circular orbit of varying radii centered on S. Since

[21] He left out Venus, and Mercury as they presented computational challenges based on the sine tables that he had available

the parallelogram ratio in Equation 5.1 must be maintained, changing the radii (of the original epicycles) to be the same means that the originally equal R radii of the deferents, now must each scale to different values.

For example, let's take the new radius of the scaled Mars deferent to be R_M, then the parallelogram-ratio from Equation 5.1 becomes:

$$\frac{R'}{e} = \frac{R_M}{r}$$

$$\frac{R'}{6583} = \frac{R_M}{25} \text{ and solving for } R_M \text{ gives}$$

$$R_M = R'\frac{25}{6583} = R' \times 0.0038 \tag{5.2}$$

As noted above, to keep the numbers manageable, instead of $R' = 60$ for each, he arbitrarily assigned $R' = 10,000$ and so Equation 5.2 becomes $R_M = 38$ and in box D, you can make out, "Martis semidyameter orbis 38 were Epi, or "Semidiameter of the sphere of Mars about 38 Epi."

Likewise, he further calculates the rest of the planets in box D: "Jupiter 130;25 epi," Semi of Saturn 230 5/6 epi," Se of Venus 18 epi," and "Mercury 9;24."[22]

Third Big Step. The constructions to this point are still geocentric as in Figure 5.10. But one more inspired idea and another argument among historians. By all accounts, probably under the influence of Peurbach's *New Theories of the Planets*, Copernicus believed in the reality of the crystalline shells on which the planets were embedded. But as Figure 5.10 shows, the spheres of Mars and the Sun collide and that wouldn't do.

So he made a "coordinate system transformation" and shifted the positions of the formerly stationary-Earth, orbiting-Sun to become an orbiting-Earth, stationary-Sun. Now everything orbits the Sun and the Earth becomes a planet and a real "solar system" is born. The crystalline shells continue to do their job, and they are all circling the Sun.

Adding in the other planets and his calculation for each is shown in Table 5.2. The agreement with modern values is pretty good.[23] Notice that the radii of the "big" circle for each planet exactly follows the ordering of the planets that he found using the synodic period calculation. **These are two entirely different methods** that result in three brand new conclusions:

1. The order of the planets are: Mercury, Venus, Earth, Mars, Jupiter, and Saturn. This conclusion is supported by the following two measurements:
2. The sidereal periods for each planet's trip around the Sun, as compared with Earth's, are respectively: 0.24, 0.62, 1.0, 1.90, 12.0, and 30.0.

[22] Tthe 9;24 notation means units of 9 with 24/60th as a fraction. Also, I've glossed over the fact that for the inferior planets, the ratio is different.

[23] Deviations from modern are understandable: Mercury is hard to observe and one has to wait a long time to observe much motion out of Saturn, three decades. So his imprecision is understandable for his outer-most planet.

Table 5.2: Radii of the planets as reported in *Commentariolus* for Copernicus' scaled values of Ptolemy's epicycles in the second column, his scaling to the Earth-Sun radius of 25 in the next, those values as compared with the Earth's in the fourth, and modern values for that in AU in the last.

Planet	epicycle, r	scaled planets	r/r_{Earth}	Modern, AU
Mercury	2256	5.64	0.2	0.4
Venus	7191	17.98	0.7	0.7
Earth	10000	25	1.0	1.0
Mars	6583	38	1.5	1.5
Jupiter	1917	130	5.2	5.2
Saturn	1083	231	9.2	9.6

3. The distances from the Sun for each planet as compared with the Earth's (fixed at 25), are respectively: 5.64, 17.98, 25, 38, 130, and 231.

Remember, I'm guessing that he did that First Way calculation as perhaps a lark...exploring a new Project. It was a simple calculation and when it resulted in something interesting, then I hypothesize he found another, more complicated way to approach it. This sequence, I recognize as a very modern approach to a Scientific Project as I described in the Preface:

- Copernicus started a project by asking a question: what would be the consequences of a heliocentric universe?
- With that assumption, he came up with a prediction through a very simple calculation and found that he could predict the sidereal years' durations for each of the planets and that they naturally ordered themselves.
- That must have been encouraging and inspired by the work of some other scientist, he found an entirely different way to approach the question and with a more complicated set of calculations he found he could predict the sizes of orbits of all of the planets. That too suggested an ordering which was identical to his simple, different calculation.
- Then he realized that he has probably found something important and, like a modern scientist, he "published," in this case, through a letter to colleagues via *Commentariolus*.
- Like a modern Project, the initial results were promising but his competitor could make very precise predictions and so now harder work was required in order to refine the system that he had roughed out.

He's remarkably laid-back about this in *Commentariolus*, while I'm excited about it!

5.4.7 Why Two Epicycles?

> Eccen[tricity] of Jupiter 1917 Epi[cycle] a 777 b 259
> Eccen[tricity] of Saturn 1083 Epi[cycle] a 852 b 284
> Eccen[tricity] of Mercury 2256 Epi[cycle] a plus b 100
> Copernicus, translated in **swerdlow73** *Uppsala notes*

Copernicus appeared to have two separate workflows in his Project. The first was the Regiomontanus-inspired evolution from Geocentric to Heliocentric. Remember that Ptolemy needed the epicycle to contend with retrograde motion, but as Copernicus noted in his seventh postulate on page 173, by making the Earth an orbiting planet explained retrograde motion. In addition, Copernicus was focused on ridding any model of an equant and retaining uniform circular motion and even though he had the Sun at the center and the Earth as a planet, he still had a problem.

The reality of the situation is that planets do *not* execute circular orbits, but rather elliptical ones which are not uniform. We'll watch something like the equant return in Chapter **??** where we finally get it right: *non uniform elliptical motion is how it goes*. But one of his Project commitments that he could not shake off was that he tried to make circles do the job of ellipses and he needed a tool to encourage slight deviations from circular motion (the so-called "first anomaly" to account for the different length of the seasons). To do that he went to the trick introduced by the Maragha Observatory's Ibn al-Shatir's models for the superior planets, the Moon, and Mercury: two epicycles got rid of the eccentric for Ibn al-Shatir, which of course was in an Earth-centered system, but the idea still worked. Remember that multiple epicycles can draw *any* contour if you use enough of them and ellipses are a trivial curve to construct with epicycles. In *Commentariolus*, Copernicus literally copied Ibn al-Shatir's model and essentially modeled ellipses without realizing it. He also deployed the Tusi Couple to explain latitudes of the planets.

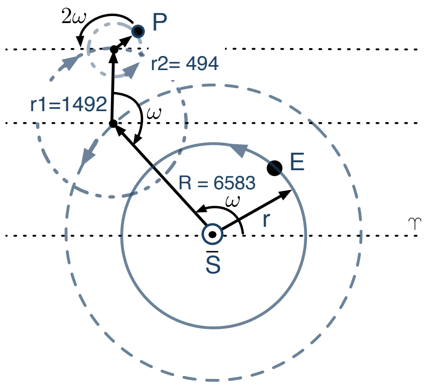


Figure 5.11: The two-epicycle model that Copernicus employed in *Commentariolus* to rid himself of equants. The radii are in the A snippet from U.

Figure 5.11 shows a rendering of such a planetary model as described in *Commentariolus*:

Three interesting things: It's amusing to realize that where Ptolemy needed an epicycle (retrograde motion), Copernicus didn't and where Ptolemy used an eccentric without epicycles (Sun's motion), Copernicus used them. The biggest mystery of all: where did he learn of Tusi and Ibn al-Shatir's tools? The best guess is that in Padua he might have heard a speech, seen a drawing, or had some conversations. But he makes no mention of his use of their ideas in either *Commentariolus* nor in *Revolutionibus*. It's the kind of thing that drives historians crazy.

He closes *Commentariolus* with the briefest of summaries:

> "And so altogether, Mercury moves on seven circles, Venus on five, the earth on three and the moon moves about it on four, and finally Mars, Jupiter, and Saturn on five each. Therefore, taken as a whole, 34 circles are sufficient to represent the entire structure of the heavens and the entire ballet of the planets." Copernicus *Commentariolus*

The simple description I did only dealt with the longitude motions of the planets. Their latitudinal motions are complicated and each different. Figure 3.26 shows that every planet orbits in a different plane. The 34 circles that he needed came from:

- The Earth has three.
- The Moon has three.
- Mars, Jupiter, and Saturn all have five.
- Venus has five.
- Mercury has seven.

So his model is neck and neck in competition with Ptolemy's for the number of epicycles required in order to match observation. Copernicus' project bore fruit by no later than 1514. But there was an enormous task ahead of him of getting it right and at least as precise as Ptolemy. That took 30 years.

And there was his day job.

5.4.8 Copernicus As Canon

In 1510, Copernicus moved to Frombork on an inlet bay of the Baltic and took advantage of the standard setup: a salary for life, support for a house outside of the city walls, two servants, and three horses. What supported that life-long lifestyle for 16 canons? Peasants. And management had to come from within the ranks of the 16 canons.

Lucas died in 1512 and the year before the Chapter selected him to the role of Chancellor, a big job which he held four times during his career (1511, . While the Prince-Bishop would have been the "President" of the diocese, the Chancellor would have been the Secretary of the Treasury, Attorney General, Secretary of Defense, Secretary of Homeland Security, Director of the Office of Management and Budget, and the Chief Archivist. If a letter was required from the Chapter to a king, the Chancellor wrote it. So it was a busy time to be Chancellor especially since King Sigismund resisted the Chapter's nominee and so negotiation was required. Eventually the canons' choice of one of their own was approved.

Notwithstanding the administrative burdens, Copernicus began to make observations with a handful of standard instruments. By 1513, he'd constructed a concrete patio to support a large triquetrum,[24] which was essential into the 17th century for determining the position of a planet or star, specifically, the angular position from

[24] This was a standard instrument which could be quite large. It was used to measure the angle of a sighted object from the zenith, the position directly overhead. Another angle often used is the altitude but they two can be easily calculated from the other. Imagine taking a pair of scissors and standing one of the blades perfectly perpendicular to a surface and letting the other blade adopt an angle...say pointing to a star. The two legs are the same length and so their outer points would be two on a circle of radius equal to each blade. If one would measure the distance between the two blade points, it would be a chord of that circle and so using the chord tables of old, or the trigonometric tables of Copernicus' time, that angle from the perpendicular could be calculated.

the zenith, the point directly overhead. Then, he moved again, this time purchasing a three story, cylindrical tower in the northwest corner of the Cathedral campus. It was large enough to house a servant-cook, living quarters for himself, and on the top floor, a workroom. It had windows almost all around and he constructed a viewing platform to complete his view. So he had two places to observe the sky. By that point he had completed his term as Chancellor, but inherited the responsibility of the bakery, mills, and brewery. He kept observing and undoubtedly calculating. And surely, worrying. His Project had expanded into an almost impossible task.

From no later than 1514 he would have been convinced that it was promising but he would have been aware that it was in competition with *Almagest* in two ways. First, putting Earth at the center or making Earth as a planet with the Sun at the center were two entirely different philosophical views. While Ptolemy's *Almagest* Project wasn't to make a model of how the world actually was—remember, it was just a calculation device—Copernicus wanted to know how the world was actually put together. So there was a philosophical competition.

But there was also a practical competition. If *Almagest* gave more reliable results for positions of the planets than Copernicus' model, then the philosophical competition wasn't even going to get started. So he had to make predictions at the same level of precision as Ptolemy, he remarked that precisions of $1/6$ th of a degree was his goal, which would have been better than in *Almagest* in many instances. (Hold your little finger out in front of you, and it would cover about one degree against the stars.)

Gerard's translation of *Almagest* was only printed in Venice in 1515 and between *Epitome* and that (troubled) first Latin text of *Almagest*, he had work to do. He surely reworked the *Almagest* as his copy had many notations in the margins. By that point, his astronomical measurements had shown him what others had also found: *Almagest* was not accurate in many places, either because of outright mistakes or because small errors from 150 CE, had over 1300 years' time, magnified into measurable discrepancies. So he had to check the parameters and results.

He decided early that the background stars would be his "coordinate system grid" and so he had to precisely determine the stars' locations. And he had to: adapt the still-evolving spherical geometry of astronomy and geography to a Sun-centered perspective, deal not only with the relatively straightforward longitudinal planetary motions, deal with the details of the planets' latitudes (which recall vary throughout the year within the ecliptic), model the Moon's motion (which Ptolemy clearly did badly), work on Mercury's and Venus' special challenges, correctly model the seasons, and check the precession of the equinoxes (which the Muslims, Ptolemy, and Copernicus all did incorrectly). And he had to create a planetary model for an orbiting Earth and make Tables for everyone to use.

This was a lifetime's worth of work.

5.4.8.1 Copernicus As An Administrator

Warmia had nearly 100,000 inhabitants most of whom tended the vast fields as peasants paying the Chapter rent[25] but at the same time planting and harvesting the crops, which in turn, were owned by the Chapter. Servitude comes to mind since if a peasant escaped, they would be chased and returned and maybe punished. It was a large operation with extensive records and after his term as Chancellor was completed, he was elected "Administrator" which meant that he was then in charge of the whole of the peasant-farm operation.

> "Bertolt Faber of Schonewalt took possession of 1½ parcels, sold by Peter Preus, who is very old. As regards these parcels. Bartolt will give the overlord [the Chapter] ½ mark as rent for the half-parcel. But as regards the other parcel, the Chapter graciously donated 1 mark to the aforesaid Peter for life."
>
> "Merten of Lesser Cleberg, father of five sons and holder of ½ parcels, complained about the small extent of his land. Therefore, with permission he bought 1½ additional parcels from Nichs Ruche. Nichs took possession of two other parcels that were ceded to him by Merten Micher, who is very old and incapacitated, having lost his sons and wife."
>
> "Jacob Wayner, who with his wife ran away last year, has now been brought back by the overseer."
> Copernicus *Chapter records as translated by (***rosen92***)*

Such was Copernicus' life as Administrator of Benefices between 1516 and 1519 and then again in 1521. He had to relocate to an abandoned Teutonic Order castle 90 miles south of Frombork in Olsztyn (see the map in Figure 5.1) and then constantly travel around Warmia doing the work of overseer, executive farmer, accountant, and manager of all of agriculture and the diocese's income.

His financial dealings led him to discover that the Warmia coinage system was chaotic and close to collapse. A coin was to contain the amount of silver stamped on the face, but coins were alloyed with copper to improve their durability and the amount of copper was unregulated in general, and in particular by the Teutonic Knights who bought up coins, melted them down, and re-minted them into corrupted versions, worth much less than advertised. Copernicus wrote a pamphlet, and as his practice, passed it around to friends and was persuaded to translate it into Latin. His thesis was that only the King should regulate minting rather than the dozen or so cities that made their own and the Knights who had turned counterfeit into a business. He wrote the tract in 1517 and sent it to the Prussian Council in 1519.

It was an eventful time. In the Autumn of 1517, a young professor at the Wittenberg University wrote up 95 objections to Catholic indulgences and by 1518 Martin Luther's "95 Theses" spread throughout Europe.

But his day job only got harder.

[25] although they could "sell" and trade land among them, but only with Chapter approval

5.4.8.2 War

Life for the peasants wasn't just naturally difficult. They had to contend with repeated raids from Eastern Prussia by the Teutonic Knights. In 1516, and on behalf of the Chapter, Giese as then-Chancellor wrote to King Sigismund:

> "...when robbers attacked a citizen of Elblag and cut off his hands, we sent a small detachment into Teutonic Prussia, caught one of the robbers, a nobleman, and retrieved his booty. He was taken into custody along with his horses and weapons. The grand master of the Teutonic Order has demanded their return. Also the robbers have intensified their activities. The chapter begs the king to protect them from their enemies."

The King threatened the Grand Master, but the Knights unconvincingly insisted that he wasn't involved. That 37th Grand Master was a pivotal figure. Albrecht von Hohenzollern had been elected in 1511 at the age of 20 and in spite of the fact that his mother was the King's sister, he had every ambition to regain the glory and the territories of the Knights at their height. Lucas had been a formidable foe, but his successor was no match. Albrecht was eventually to convert to Lutheranism which was a complete about-face from a devote Catholic with heredity links to the Holy Roman Emperor.

Warmia is surrounded on three sides by Eastern Prussia and raids were constant into the diocese's territory. No sooner had Copernicus returned to Frombork and presumably anticipating time for observing, when in 1520 the Albrecht's Teutonic Knights attacked the city, burning it—and Copernicus' outside home—to the ground. He escaped into the walled cathedral campus protected by a small contingent of the King's soldiers.

Nothing in his education or experience prepared him to be a wartime leader. The canons were spread around the diocese and the Prince-Bishop's castle was under siege and the Chapter replaced his Administrator-successor with Copernicus only after a short time. So while the canons retreated into many Warmian cities, Copernicus headed back to the lightly guarded castle at Olsztyn to resume his former duties. But under dire conditions.

Three hundred years of documents and records of the Chapter were housed in Olsztyn and Copernicus took it upon himself to preserve and catalog them all by hand-copying much of them. Were they to be overrun, the history of the diocese would disappear. In the meantime, while gathering as many arms, ammunition, and food as he could from the outside, he wrote feverishly to the King for help, promising to die if necessary in defense of the city and castle. "For we are desirous to do what befits noble and honest persons, who are completely devoted to Your Majesty, even if we had to perish." (**sobel11**) By this point all of the sheltered canons had left the city but for Copernicus and one colleague. With the few Polish soldiers dispatched to them by Sigismund, they met the invaders but a year after the war started, Albrecht demanded surrender.

Help came in a strange fashion as the Ottomans Empire invaded Hungary in 1521 and Emperor Charles V demanded that the Poles and Knights turn their attention to protecting Europe. Albrecht withdrew and a cease-fire was negotiated and Copernicus went to work trying now to piece together the results of the Knight's rampage through the peasant's farms. Through his three year term as Administrator and even while sheltering in Olsztyn, he continued to make observations and record them. And he must have continued—somehow—to calculate and write while literally under siege.

In that summer of 1521, he returned to the Chapter home where now Giese was Chancellor but still surrounded by unruly Knights who'd not left. Eventually a peace conference was called with emissaries of the King, Giese, and the Prince-Bishiop. But, the Bishop was too ill to attend and so, of course, Copernicus was delegated to negotiate peace. Deep into the summit, but six months later, Bishop Ferber finally arrived and Copernicus was free to return to Frombork, only to find himself reelected as Chancellor.

—-bishop for 10 months. 1523 jan through October. —-1526 King burns homes in cracow. Ferber banishes Lutherans from Warmia —-1538 conciliatory with Dantiscus about Anna. —-1533 Dasntisus bishop Kulm —-1537 Danstiscus bishop Warmia

5.4.8.3 The Essential Push

Copernicus' life was surrounded by multiple layers of political and clerical administrators and of course sometimes he was one, having learned from Lucas, probably the most skilled leader in his lifetime. It was a couple of years before the Knight's invasion that Luther's 95 Theses set off the thunder that rocked Europe for a century of war and upheaval. How Church administrators handled the rise of Protestantism ranged from tolerant to violent and it's amusing that the fate of Copernicus' public results turned on tolerance from a surprising Warmian source.

Lucas' successors affected Copernicus in a variety of ways. Bishop Fabian Luzjanski died in 1523, two years after the end of the Polish–Teutonic War and the Treaty of Cracow. While hostilities ceased, the treaty gave Grand Master Albrecht latitude and he disbanded the Knights and took his role as Duke seriously enough to establish an hereditary secular Duchy: so East Prussia → "Ducal Prussia." As a sign of the times, he did so under the guidance of Martin Luther whom he visited in Wittenberg, commencing with his conversion to Protestantism and Duke Albrecht was the first European ruler to establish Lutheranism as the state religion. It must have been difficult for King Sigismund I to acquiesce to his nephew's conversion, but the treaty mandated that Ducal Prussia was still vassal to the Kingdom of Poland and that must have sufficed. Yet a year later, Sigismund was directing the burning of Lutheran homes in Cracow and Luzjanski's successor, Maurycy Ferber was banishing all Lutherans from Warmia.

Just when one might have thought that the 50 year old Copernicus could get a breather following the war, but Luzjanski death in 1532 was followed by a 10 month period without a replacement. Again, Copernicus found himself to be called to a new duty, now as the interim Prince-Bishop of Warmia for almost a year. Lucas had probably envisioned this terminal trajectory for his nephew, but Copernicus must have refused ordination which made a bishopric impossible for him. Something always seemed to get in the way of his observing, calculating, and writing.

Johannes von Höfens (1485 – 1548) was a poet of note and diplomat and favored by Sigismund for a flattering poem in 1512. He signed his poetry as Johannes Dantiscus, honoring his home city of Danzig and has since been known as just "Dantiscus." He was knighted and served as a diplomat in Spain for many years, but what he really wanted was to be a canon in Warmia. And that turned out to be difficult because when openings occurred either the Vatican and or the Chapter refused him three times between 1515 and 1529, when he finally succeeded. However, he remained in Spain to complete his mission and in the meantime, was appointed Bishop of Kulm, a neighboring Warmian dioceses. So, canon in Warmia, and Bishop in Kulm. But he didn't forget the snubs.

Prince-Bishop Ferber had been unwell for two years following two strokes and was tended to by Copernicus and royal physicians. He designated Giese as his understudy but Sigismund intervened in favor of Dantiscus who assumed the role in 1537 and set about to even scores. First he managed to arrange for Giese to be appointed Bishop in Kulm. So another one ruling one dioceses and canon in Warmia. Dansiscus gave up his canonry as leverage against Giese ever becoming Warmian Prince-Bishop.

But he wasn't done. Three of the Warmian canons maintained relationships with women who ostensibly did cooking and cleaning—one of them openly had a family with children and he'd openly opposed Dantiscus' appointments. Copernicus also maintained a live-in, long-time relationship with Anna Schilling, his housekeeper who was married but separated from her husband. Giese and Copernicus had spurned multiple invitations from Dantiscus for personal and professional visits and so his retaliation was the exile for Giese, and a new-found obsession with out-of-wedlock arrangements (he'd fathered at least two illegitimate children in Spain and Lucas had a son in Braunsberg) and he demand that Copernicus and two other canons send their female companions away. It was ugly. They complied in principle, but Dantiscus' spies found that contacts were still maintained as Anna at first stayed in Frombork. But by 1539, the women were gone and under observation from their priests, in Anna's case, in Danzig.

While 1539 was ugly for personal reasons, it was the year that a young Lutheran moved the immovable: Copernicus finished the book that he'd promised 25 years before in *Commentariolus*.

In the midst of the bishopric intrigue, Copernicus seemed to face some resistance to his Sun-centered ideas, enough so that Geise tried to write in his favor by finding Biblical acceptance. Incredibly, through all of the turmoil in war and in his household, he'd continued to observe, calculate, and write. But he clearly became concerned about his reputation. By 1533 he was 60 years old and feeling nervous, even though he wasn't without supporters. The Medicean Pope Clement VIII had suffered the indignity of the Sack of Rome, been imprisoned, and watched helplessly as Henry VIII of England divorced Catherine of Aragon and married Anne Boleyn. But he still entertained and open mind toward art and science. His secretary and diplomat Johann Albrecht Widmanstetter, gave him a personal seminar on Copernicus' ideas and was rewarded for his effort with a gift.

This is notable for two reasons. First, that someone in Rome would know enough Copernicanism to be able to deliver a seminar means that his ideas had spread widely and in some detail. Second, of course, that the Pope was eager to hear about it underscored that Copernicus' position in the Church was not threatened at all. Widmanstetter went on to advise Nicholas Schönberg, who as Cardinal of Capua had traveled to Poland and with Widmanstetter's guidance had became enamored of Copernicus' ideas and wrote to him in 1536 an encouraging and flattering letter,

> "Some years ago word reached me concerning your proficiency, of which everybody constantly spoke..."At that time I began to have a very high regard for you, and also to congratulate our contemporaries among whom you enjoyed such great prestige. For I had learned that you ... had also formulated a new cosmology. In it you maintain that the Earth moves; that the Sun occupies the lowest...and that the Earth... revolves around the Sun in the period of a year. I have also learned that you have written an exposition of this whole system of astronomy, and have computed the planetary motions and set them down in tables, to the greatest admiration of all. Therefore with the utmost earnestness I entreat you, most learned Sir, unless I inconvenience you, to communicate this discovery of yours to scholars... I have instructed Theodoric of Reden to have everything copied in your quarters at my expense and dispatched to me. If you gratify my desire in this matter, you will see that you are dealing with a man who is zealous for your reputation and eager to do justice to so fine a talent. Farewell." . Cardinal Schönberg *Letter to Copernicus, reproduced in Revolutionibus*

The Catholic Church was clearly not Copernicus' foe, but supportive at the highest levels. However, Copernicus' reticence was significant and he seemed to have ignored the Cardinal. It appeared that he'd never publish. He seemed to (be trying? to) be content with his canonical duties and a busy life as a physician.[26]

5.4.9 Rheticus

The Lutheran problem became more and more serious in Warmia and throughout Poland and the severe reaction that eventually became the Counter Reformation following the Council of Trent from 1545 to 1563. The Catholic Church that resulted and that Galileo famously contended with was a very different organization from the one that supported Copernicus. However during his lifetime, he saw that change. Warmia was not safe for Lutherans, but that seemed to not have bothered a zealous young mathematics professor from Wittenberg.

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[26]even treating Albrecht in his castle in Ducal Prussia, who had mellowed in his Lutheran life

Appendix A

Appendices

A.1 Greeks Technical Appendix

A.1.1 Proof of Pythagoras' Theorem

A.1.2 Zeno's Paradox

A.2 Plato–Aristotle Technical Appendix

A.2.1 Socrates' Geometrical Problem

A.2.2 Logic and Electronics

A.2.3 Aristotle's Legacy in Physics and Engineering

This section is a little more detailed than normal, but the payoff is large! Aristotle left us a legacy which instantly became an active research project for ancient and medieval philosophers and eventually, present day philosophers, mathematicians, engineers, and scientists! He created a tool that guarantees how to properly analyze and judge conclusions reached through argument: Formal Logic. Read the next seven pages in detail for the whole story, skim them for a taste, or jump to the punch-line on page 201.

In everyday life, we all make arguments but have you ever thought about what makes you successful in defending your case? The facts need to be on your side but your stated reasoning should also be "logical." We all have a sense of what "logical" means, but it's surprisingly nuanced. Consider the following reasoning:

- Squirrels with superpowers can fly
- Rocky the Squirrel has superpowers
- Therefore, Rocky the Squirrel can fly.

This doesn't make sense because the first two sentences—the "premises"— are nonsense. And yet *it's a perfectly valid argument*! Appreciating the difference between a *valid* argument and a *true* argument leads us to Aristotle's amazing discovery that the rules of valid reasoning are due entirely to an argument's structure and arrangements of the sentences, not the specifics of the content. Your and my lives are now governed by Aristotle's invention of Formal Logic, his most important, lasting contribution.

Obviously, the distinction between *validity* and *truth* can be easy to spot. But the distinction between valid and invalid argument can be subtle. Think about these two arguments:

Table A.1: How to not reason logically.

A	B
Those who take the vaccine stay well. Those who take the vaccine are smart. Those who are smart stay well.	Those who take the vaccine stay well. Those who are smart take the vaccine. Those who are smart stay well.

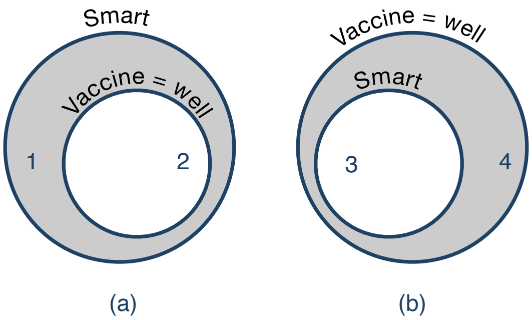

Figure A.1: A diagrammatic way to show that argument A in Table A.1 is invalid and that the conclusion of argument B is valid.

The argument in column A is invalid, not because the premises are ludicrous, but because of the form of the terms in the sentences. Read it very carefully with an eye on Figure A.1. Notice how the righthand and lefthand circles are different (not really Venn diagrams, but a cousin, called Euler Diagrams). The first premise in argument A is that if you take the vaccine you're going to be well. So in the lefthand diagram, everyone who took the vaccine is in region 2. The second premise in argument A says that those who took the vaccine are smart, but it doesn't rule out the logical possibility that some smart people didn't take the vaccine—region 1. So the conclusion, that if you're smart, you're well does not hold.

Argument B says things slightly differently. Again, smart=well. But then the second premise says that if you're smart, you took the vaccine, so all of the smart people are in region 2 and, they're vaccinated. That, of course leaves the possibility that there are people who took the vaccine, but aren't smart, region 4. That's good! But not the argument which leads to a valid conclusion: Those who are smart stay well (and because of the first premise, they also took the vaccine).

A.2.3.1 Greatest gift

Aristotle's greatest gift to us was his invention of Formal Logic which is a rigorous way to judge the validity of arguments. For example, he could tell you that the argument in column **A** is not valid and why and tell you how to construct arguments like column **B** which *are* logically valid. Every time. And sometimes surprisingly, independent of the actual subject-matter of the argument.

> Officially, Formal Logic is the field that studies reasoning and the various ways that conclusions can legitimately be drawn from premises.

This new-born subject is covered in a number of his books, including: *Categories*, *On Interpretation*, *Prior Analytics*, *Posterior Analytics*, *Topics*, and *On Sophistical Refutations* which collectively, were much later dubbed "*Organon*" which means "instrument" which suggest by that time, Logic was viewed as just a tool, as opposed to a part of philosophy. Now it's firmly the philosophical camp and even an important part of an entire branch of mathematics called Discrete Mathematics.

Logic became a research program almost as soon as he wrote it down (or lectured on it) and two millennia worth of people—to this day—study logical formalism, expanding it into new directions. It's studied by every student of physics and engineering in forms directly evolved from Aristotle.

A.2.3.2 Deduction and Induction

Broadly, there are two kinds of logic which you use every day. The first works according to strict rules which I think of it as the *algebra of reasoning* and you'll see why in a bit. Reason according to those rules, and you will reach correct conclusions. This is **Deductive Logic.**

The second kind of logic is less certain since it's not rule-bound and it delivers conclusions which can seem persuasive but aren't certain. This is **Inductive Logic**. From this point, when I refer to "logic" I'll mean deductive logic.

Among things that are obvious to us (and to everyday Greeks), Aristotle seemed to intuit as requiring bottom-up attention. He tightly defined terms and "obvious" ideas, dissected arguments finding rules along the way, and set down what it means to be clear with exquisite precision. Look at these two statements:

- All squirrels are brown.
- No squirrels are brown

1) Can these both be true at the same time? Of course not and this obvious idea has a name: *the law of contradiction.* Aristotle needed to be precise and actually provided multiple "proofs" to demonstrate this principle.

2) One of these must be true... there's nothing in-between, which is called the *law of the excluded middle*.

> "…there cannot be an intermediate between contradictories, but of one subject we must either affirm or deny any one predicate" Aristotle, *Metaphysics*.

Centuries of ink have been spilled over precisely understanding the implications of law of the excluded middle and how to symbolically state it unequivocally. But here's the first hint of our modern debt to him: his logic is two-valued, either true or false with no in-between. Hmm. Binary: True and false...one's and zero's.[1]

Last one:

- A squirrel is a squirrel.

This is called *the law of identity* and Aristotle didn't invent it and it sounds like Parmenides: "What **is**, **is**." These three ideas, collected together by him, are often called the Rules of Thought and were believed to be the bedrock for all of Logic. (That this was disputed in the 20th century shows that Logic is still a living-breathing subject.) Nobody ever thought this way before — so clearly—-and in Aristotle's patented approach to system-building, he lays it all out out exhaustively. As a master system-builder, he was the right man for the job.

His unique invention was to create an *algebra of language*. Here is a seminal moment in history, from the first book of his *Prior Analytics* (focus on the last sentences):

> "First then take a universal negative with the terms A and B. If no B is A, neither can any A be B. For if some A (say C) were B, it would not be true that no B is A; for C is a B. But if every B is A then some A is B. For if no A were B, then no B could be A. But we assumed that every B is A. Similarly too, if the premiss is particular. For if some B is A, then some of the As must be B. For if none were, then no B would be A. But if some B is not A, there is no necessity that some of the As should not be B; e.g. **let B stand for animal and A for man. Not every animal is a man; but every man is an animal.**" Aristotle, *Prior Analytics*.

I don't blame you if you get bogged down quickly in this quote. Look at the sentences that I've highlighted: he's using variables A and B, to stand for particular things, here in his example, A = man and B = animal. So his first sentence says for this particular case, "If no animal is a man, neither can any man be an animal." Instead of men and animals, you can plug in anything you want for A and B. It's the form of the argument, not the contents that determine whether the argument is valid.

Introducing variables as a placeholder for the subjects and objects in a statement is a seminal moment in the history of mathematics.

Amazing. Out of this, your mobile phone was born.

There are many different forms of arguments and for Aristotle, the **Syllogism** is just one of them. It's an argument written in a structure in which there are three

[1]Things didn't stop there. Now there is a multi-valued logic with degrees of truth and falsity with many engineering applications. "Fuzzy Logic" is a legitimate decision-making tool in transportation control systems, earthquake prediction, even home appliance efficiency.

sentences with a subject and a predicate[2]: two premises and a conclusion and inside those sentences are three "terms."

Here is one of the syllogistic forms:[3]

- premise 1: If all A are B
- premise 2: and if all C are A
- conclusion: then, all C are B

> There are actually 256 possible argument-combinations of subjects and predicates and 24 were thought to yield valid deductions. Maybe you can see why studying Logic became a matter of intense research following Aristotle's death and into the first 100 years of both Arab and Western philosophers. There was lots of work to do.

Let's make a syllogistic argument about squirrels. I'll define C = squirrels, A = the group of all animals in trees, and B = brown animals. One kind of syllogism would have the form:

- All mammals in trees (A) are brown animals (B)
- and if all squirrels (C) are mammals in trees (A)
- then, all squirrels (C) are brown animals (B).

Before I moved to Michigan, the only squirrels I'd ever seen where brown. Now my yard is full of black squirrels. They're everywhere. Yet, my argument above seems to prove that squirrels are brown. So what went wrong?

My "Squirrels with superpowers" shined a bright light on the premises: they have to be legitimate. In scientific arguments, premises might be …hypotheses, in which case a deductive argument describes a way to test those ideas. Aristotle was well-aware of induction, deduction, and how they might go together.

Back to my squirrels proof. I reasoned inductively:

- (As a child) There's a brown squirrel
- (As an adult…many times) There goes another brown squirrel
- Wow…more brown squirrels and no other ones
- What is it with all of the brown squirrels?
- Gosh, all squirrels must be brown! (which was my premise)

Until I moved to Michigan. All it took to ruin my theory about squirrels was the observation of one black squirrel, much less an entire herd of them. Squirrels are not only brown, they're black. My proof founders on a false premise: "All mammals in trees (A) are brown animals (B)."

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[2] since his Categories are predicates, these topics were a part of his overall system

[3] Before 500 CE, Aristotle's original form was used:

- If A, then B
- If B, then C
- So, A is C

By the way, Sherlock Holmes is reputedly the Master of Deduction. Well, sorry. That's not true. If you look at his stories you'll see very, very few examples of deductive reasoning. He's the Master of Induction![4]

A.2.3.3 Your phone

Theophrastus (-371 to -287) was a favorite student of Aristotle's who led the Lyceum for 37 years after his teacher's death. Aristotle even willed him the guardianship of his children...and his library. While a devoted student, Theophrastus went beyond his teacher and expanded and modified some basic Aristotelian notions—extending a concept of motion to all 10 of the Categories, for example. He also moved the study of botany forward and worked extensively in Logic. Theodor Geisel (Dr. Seuss) used "Theophrastus" as a pen name.

He is probably the one who extended the form of argumentation into a new direction with the invention of "propositional logic" in which there are two items, rather than three of a syllogism. This is where the modern engineering action is. One form of such a proposition is called "Modus Ponens" (Latin for "method of affirming") which is an offshoot of the classical syllogism and is one of four possible "rules of inference." Modus Ponens goes like this:

- If A (the antecedent) is true, then B (the consequence) is true
- A is true
- Therefore, B is true.

Here, each line is a proposition (there can be more than two) with the first two being "premises" and the last, the "conclusion." The first sentence is a proposition which is conditional: the antecedent implies the consequence and it's "affirmed" if the next statement is true. B here is the consequence of A. Here's a concise way to present this:

- $A \rightarrow B$
- A
- $\therefore B$

The $\rightarrow$ symbol means "implies" and is associated with an "If...Then" kind of statement. The $\therefore$ symbol means "therefore." It doesn't seem like much, but it's powerful and misunderstanding (or misusing) it is the source of many logical fallacies. Table A.2 shows an example:

[4] Or more appropriately, the Master of Abduction. Look it up.

Table A.2: A typical logical fallacy involving public health.

A valid argument	A fallacy
• If a reactor leaks radiation (A), people nearby will get cancer (B). • The reactor leaks radiation (A). • Therefore, people nearby will get cancer. (B)	• If a reactor leaks radiation (A), people nearby will get cancer (B). • People nearby got cancer (B). • Therefore, the reactor leaks radiation (A).

The argument on the left is an example of Modus Ponens, while the argument on the right is a classic fallacy known as "Affirming the Consequent," a regularly exploited tool for those intentionally making invalid claims. Especially those who dispute public health strategies. Look at how the two columns are different. Remember, that in the proposition, B is the consequence of the antecedent, A and not the other way around. In the second row of the fallacious argument, the antecedent and consequence are reversed as compared with the valid argument. The fallacy is that people can get cancer from other causes than the proposition states.

Let's make a plan to picnic outdoors which requires us to keep an eye on the weather since if it's raining the ground would be wet and of course we wouldn't have a picnic if the ground is wet. We'd actually use Modus Ponens in our thought process and reason among ourselves:

- If it's raining, then the ground is wet
- It is raining
- and so the ground is wet.

Let's build a table—a picnic table (sorry)—that takes each line in the argument and makes it a column in a table. We could then ask a set of questions: Is it raining (Yes), is the ground wet (Yes)...was the proposition confirmed? Yes.

Table A.3: The picnic is cancelled because:

If A, then B	it's raining?	it's wet?	A	B	If A is true and B is true, then:
If it's raining, then the ground is wet	Y	Y	T	T	T

There are actually four complete ways in which the antecedent and consequence could appear:

- rain? Yes or No
- wet? Yes or No

So what about: suppose the ground is not wet (wet = F) then can it be raining? Well...no (rain = F). So if wet = F and rain = T, then the proposition would not be true since rain should imply wet. We can build up these four conditions into what is called Truth Table, which was invented in the early 20th century as an analyzing tool. Table A.4 describes the complete story:

Table A.4: All of the logical possibilities for two pieces of a conditional premise: raining and wetness. Here's a picnic table (sorry):

If A, then B	it's raining?	it's wet?	A	B	If A is true and B is true, then:
If it's raining, then the ground is wet	Y	Y	T	T	T
If it's raining, then the ground is not wet	Y	N	T	F	F
If it's not raining, then the ground is wet	N	Y	F	T	T
If it's not raining, then the ground is not wet	N	N	F	F	T

Sometimes these are hard to unravel. The first two lines are pretty obvious. It's asserted that when it rains that the ground is wet, so the second line is obviously false. The proposition requires "wet" with rain. The last line is pretty clear also. No rain, let's picnic since it will not be wet. The third one requires some thought. What does the if statement say about the ground if it's not raining? Nothing. You could be wet for other reasons so this does not falsify the proposition, so it's not F...and in a two-valued logic, the only alternative to F is T. Go lie down before we go on because it's about to get interesting and relevant.

Before getting to the punchline, let me make a couple of points:

- The → or if...then argument is one of six "connectives," all of which have truth tables like above. They are negation, conjunction ("AND"), disjunction ("OR"), conditional (that's the → conjuctive), biconditional, and exclusive OR.
- The Modus Ponens argument got its Latin name from the Medievals who seriously studied Logic. They identified it as one of four "Rules of Inference" which we use today: MP, Modus Tollens, Hypothetical Syllogism, and Disjunctive Syllogism.
- The Hypothetical Syllogism is just one form of the "regular" syllogism of our squirrel proof above. In fact, it can actually be proved to be the combination of two Modus Ponens arguments, one for A → B and the other for B → C. There's debate about whether Aristotle might have recognized his syllogism to have been an "hypothetical" in this sense with a deeper structure.
- In Appendix A.2 I've gone into some more detail logic gates as they're used in digital circuit design.

There are a handful of seminal discoveries about Logic that extend to our modern reliance on it. **Gottfried Wilhelm Leibniz** (1646–1716) refined binary arithmetic. In 1854, **George Boole** (1815–1864) invented the algebra of two-valued logic...how to combine multiple conjuctives into meaningful outcomes which can only be T or F, 1 or 0. In 1921 in his dense and very terse *Tractatus Logico-Philosophicus*, **Ludwig Wittgenstein** (1889–1951) invented the Truth Table, which can be used in logical proofs and complicated logical solutions to multi-variable inputs. Finally, in 1938 **Claude Shannon** (1916–2001) realized that Boole's algebra could be realized in electronic, "on-off" circuits. This was realized in the 1940's with vacuum tubes and then in the 1960's with transistors.

Notice that the picnic table can be thought of as a little machine: you input the four T-F possibilities in pairs for rain and wet and out comes the truth value of the proposition. Figure A.2 is a cartoon of such a machine.

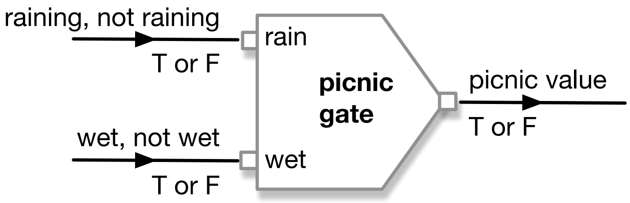

Figure A.2: A fake "picnic gate" machine that does the work of Table A.4

.

The image in this figure is maybe suggestive of digital component representations which are called "gates." There are electronic gates for eight functions, which are a practical expansion of the conjunctives mentioned above. Think about that. The whole of our digital world can be made with these eight gate functions.

What I wanted to show you is that your entire life now is based the ancient Greek Logic research program. For example, the 2022 iPhone 14 has 18 billion transistors in it and every one of them speaks through Aristotle to get their individual jobs done—or I should say their collective jobs done, since their language is forming and evaluating billions of logical two-term arguments in the same spirit as our raining-wet table.

A.2.3.4 The Punch Line:

Let's review what just happened:

We've found that Aristotle made a simple but profound discovery, namely that one could take a sentence, like "Fire engines are red or yellow" and turn it into essentially a mathematical statement, like "A are B or C" and then draw general conclusions about the combinations of general statements that don't involve the details. That sentence involving A, B, and C could also be a representation of the sentence, "All squirrels are either black or brown." This allowed him to then create a system of rules that could guarantee the validity of arguments, which, after all, are combinations of sentences.

The first kind of argument is now called the "categorical syllogism," and involves three variables and, like fire engines and squirrels, can be specific or more usefully, general, like:

All men are mortal.	A are B
Socrates is a man.	C is A
Therefore, Socrates is mortal	therefore, C is B

This evolved quickly into a rules guaranteeing validity of conclusions from a different form of argument involving two variables (an "hypothetical syllogism"):

If all men are mortal, then Socrates is a mortal	If A, then B.
All men are mortal	A is true.
Therefore, Socrates is mortal	therefore, B is true.

In fact there are variety of valid forms for each sort of argument but what's interesting in the second sort is that the truth value of arguments involving two variables can actually be created using electronic circuits using tables ("truth tables") of the different logical outcomes of the truth or falsity of the premises in an hypothetical syllogism. This was realized in 1938, built into vacuum tube circuits in the 1940's, and transistor digital electronics in the 1960's.

The first digital computers relied on thousands of vacuum tubes and filled whole rooms with hot, clunky racks of tubes and wires—your phone has 10s of thousands of times more processing power than these first early 1950s computers. When the transistor became commercially viable in the 1960s the digital world came alive.

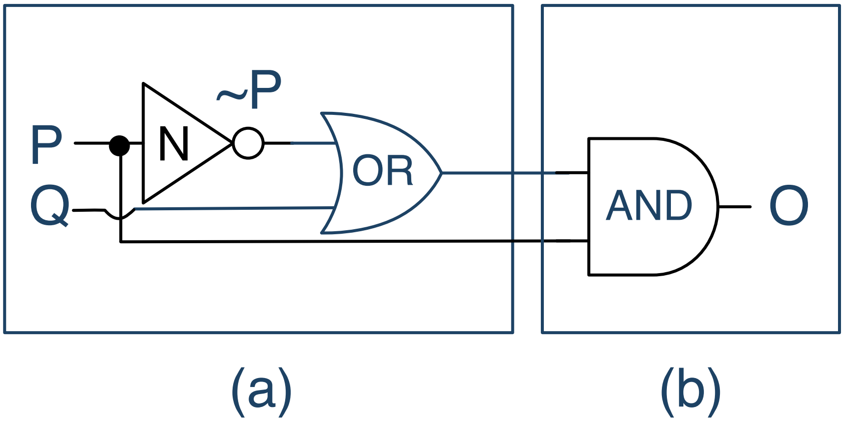

Figure A.3: (a) and (c) are the transistor-equivalents of the two logic gates, NOR and OR in (b) and (d). The little circuit to evaluate rain causing wetness...or not...is shown in (e).

In the spirit of overview, Figure A.3 shows two transistor arrangements and their modern "gate" symbol—please don't worry about the details! Just for flavor. (a) is the layout for a common transistor package that does the job of the logical gate symbol shown in (b). It's the NOR operation. A comes in, and NOT–A comes out. (c) is another transistor layout that has two inputs and produces the logical

OR combination, and (d) is the logical gate symbol for performing that operation. Finally, (e) is the digital gate solution for the Conditional argument from Table A.4—it's a real-life engineering representation of the fake "picnic gate" in Figure A.2.

With binary arithmetic, gates can be combined to do arithmetic functions, logical functions, and importantly, storage of bits. Digital memory consists of four so-called NAND gates, and so four transistors and is the basic cell of a computer 1-bit memory. It's a clever implementation of an input bit—to be stored—and an enable bit—which allows the output to change or not change.

All of these—and more—transistor components are actually imprinted in tiny silicon wafers in which a single transistor package might be only 20 nanometers in size. With the logical functions and the manufacturing techniques of today, my current Apple Watch has 32GB of random access memory (RAM) and so it can manage 32,000,000,000 Bytes of information, which is 25,6000,000,000 bits and so 102,400,000,000 individual transistors are inside my watch, just for the memory! The CPU and control circuitry would add millions of additional imprinted transistors and their gate-equivalents. All on m

A.2.4 Digital Gates

One more bit of insight makes really complicated electronic digital design possible and came from the very strange, yet enormously influential philosopher **Ludwig Wittgenstein** (1889-1951) who invented the concept of the "truth table," which we've already used in Table A.4. It's an orderly setup of all possible starting places (for two valued propositions) and their results when various operations are applied. Let's look at a three. True now is the bit 1 and False is the bit 0:

- The NOT operation: If I have an A then NOT–A creates the opposite of A. If we work in the zeros and ones world, then if A=1, then NOT–A = 0. The symbol for NOT is usually so if A = 1, then A = 0. (The symbol is the common notation used by logicians. Engineers and physicists would write $\overline{A}$ to represent the result of NOT–A.)
- The AND operation: This is between two states of, say, our A and B. In order for A AND B to be true, both A and B must be true—1— themselves. Otherwise, A AND B is false, or 0. The symbol for AND is $\wedge$ So A AND B = A $\wedge$ B.
- The OR operation: This is the combination that says A OR B is true if either A = 1 or B = 1 and false otherwise. The symbol for OR is $\vee$.

There are 5 other logical combinations. Table A.5 shows the truth table for AND and for OR. In the first set, the AND process, I've stuck to our T and F language, but the rest uses the zeros and ones language of engineering and binary arithmetic.

Table A.5: Truth tables for the AND and OR functions plus the construction of Modus Ponens. The **symbol for AND is** $\wedge$, the **symbol for OR is** $\vee$, and the **symbol for NOT (negate) is** . Notice that (A) $\vee$ B is a construction out of AND and NOT of the conditional that's the first premise of Modus Ponens.

AND			OR			Combined function				=
A	B	A $\wedge$ B	A	B	A $\vee$ B	A	B	A	(A) $\vee$ B	If A then B
T	T	T	1	1	1	1	1	0	1	= 1
T	F	F	1	0	1	1	0	0	0	= 0
F	T	F	0	1	1	0	1	1	1	= 1
F	F	F	0	0	0	0	0	1	1	= 1

Let's look at the first line so that you get the idea.

For AND:

- A is T and B is T and the AND of two T's is itself a T.

For OR:

- A= 1 and B = 1 and the OR of 1 $\vee$ 1 is 1.

Then the combination:

- repeating the A and B conditions from the first and second columns A= 1 and B = 1.
- taking the NOT of A, takes 1 into 0.
- combining that with the B in an OR results in A $\vee$ B = 0 $\vee$ 1 = 1

The last column shows that this is the same as the first line result of our picnic decision making in Table A.4. The rest of Table A.5 builds that combination for all possible A and B states, first by negating A and then combining that by "ORing" it with B. The last column shows the original "If A then B" premise that we worked out about raining and wetness. They formula and our reasoning lead to identical conclusions.

A.3 Greek Astronomy Technical Appendix

A.3.1 Plato's Timaeaus Cosmology—The Numerology

> "And he began the division in this way. First he took **one portion** from the whole, and next a **portion double of this**; the **third half as much again as the second**, and **three times the first**; the **fourth double of the second**; the **fifth three times the third**; the **sixth eight times the first**; and the **seventh twenty-seven times the first**. Next, he went on to fill up both the double and the triple intervals, cutting off yet more parts from the original mixture and placing them between the terms, so that within each interval there were two means, the one (harmonic) exceeding the

one extreme and being exceeded by the other by the same fraction of the extremes, the other (arithmetic) exceeding the one extreme by the same number whereby it was exceeded by the other." Plato, **Republic**

Okay the numbers seem arbitrary. But there's an algorithm:

- one portion of the whole: ○, 1
- double of this: ○○, 2
- half as much again: ○ ○ ○, 3
- double of the second: ○ ○ ○○, 4
- three times the third: ○ ○ ○ ○ ○ ○ ○ ○ ○, 9
- eight times the first: ○ ○ ○ ○ ○ ○ ○ ○, 8
- twenty-seven times the first: ○, 27

Now manipulate:

- The first four are the famous 1,2,3,4 and since they're the special numbers, they have a job to do:
  - Square each of the first numbers—remember, 1 is not a number— (Greeks knew how to multiply): and you get 4 and 9.
  - Cube those same first two important numbers: and you get 8 and 27.

So all of the numbers in that excerpt are some manipulation of the numbers 2 and 3—he stopped at 3 because there are only three dimensions. Collecting all of the numbers, but now into even and odd strings (remember, 1 is neither even nor odd for Pythagoreans and apparently also, for Plato):

Then, Timaeus says that if you take the number strings you actually construct the intervals of the diatonic musical scale. More Music of the Spheres. Whew. Wait until we get to Kepler.

A.3.2 Some Aristarchus Measurements

A.4 Medieval Technical Appendix

A.5 Copernicus Technical Appendix

A.6 Brahe-Kepler Technical Appendix

A.7 Gilbert Technical Appendix

A.8 Galileo Technical Appendix

A.9 Descartes Technical Appendix

A.10 Brahe-Kepler Technical Appendix

A.11 Huygens Technical Appendix

A.12 Newton Technical Appendix

A.13 Young Technical Appendix

A.14 Faraday Technical Appendix

A.15 Maxwell Technical Appendix

A.16 Michelson Technical Appendix

A.17 Thomson Technical Appendix

A.18 Lorentz Technical Appendix

A.19 Einstein Technical Appendix

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