CHAPTER

10

The Special Theory of Relativity

> Nature and Nature's laws lay hid in night:
> God said, "Let Newton be" and all was light.
>
> **Alexander Pope**

> It did not last: the Devil, shouting "Ho
> Let Einstein be" restored the status quo.
>
> **Sir John Collings Squire**

Physics changed around 1900. Physicists began investigating phenomena far-removed from the normal range of human experience, things like the structure of atoms and the precise speed of a light beam. They found that Newtonian physics (Chapters 3–5) and nineteenth-century electricity and magnetism (Chapters 8 and 9) were far off the mark in phenomena involving very high speeds, very strong gravitational forces, large astronomical regions, and the microscopic world. To deal with these new realms, they invented new theories called special relativity (this chapter), general relativity (Chapter 11), and quantum physics (Chapters 12 and 13).

All of these new theories reproduce, nearly exactly, the standard Newtonian predictions within the normal range of human perceptions. For example, special relativity correctly predicts new, non-Newtonian results for objects moving at speeds comparable to lightspeed, but also correctly predicts the normal Newtonian results for slower-moving objects such as cars and speeding bullets.

But despite this similarity within the normal range of human perception, the concepts behind these new theories are quite unlike the concepts behind Newtonian physics. For example, you learned in Chapter 5 that Newtonian physics describes the universe as a kind of giant predictable clockwork mechanism. But you'll find in Chapter 13 that, according to quantum physics, the universe is nothing like a clock, quite non-mechanical, and far from predictable. The new theories represent the most accurate knowledge known about the real physical universe, and they describe a different universe from what you would have expected on the basis of Newtonian or pre-Newtonian concepts. So expect your preconceptions about space, time, motion, gravity, matter, energy, and physical reality to be assaulted in the next four chapters. Each of these new theories has a non-intuitive oddness about it, as might be expected since they deal with phenomena beyond your normal range of perception.

In this chapter you'll learn about some unexpected effects that happen when objects move at high speeds, speeds comparable to lightspeed. You'll also learn that space and time aren't quite what you thought they were, and you'll learn something new and, for most people, amazing about energy. Einstein's "special theory of relativity" is based on two simple ideas and all of its odd conclusions are off-shoots of these. This theory has a reputation for being difficult, but this comes really from its strangeness rather than any inherent difficulty. Its conclusions violate common sense. The main requirement for understanding this theory is not intelligence but mental flexibility.

Einstein created two related theories of relativity. The "special" theory of relativity, discussed in this chapter, revolutionizes the way we think about space and time, and this leads to a further revolution in our concepts of mass and energy. The "general" theory of relativity, discussed in the next chapter, revolutionizes our concepts of space and time even further, and radically reformulates the way we look at gravity.

Following some historical context in Section 10.1, Section 10.2 discusses the older "Galilean" way of viewing the phenomena with which Einstein was concerned. Sections 10.3 and 10.4 cover the theory's two key laws: the principle of relativity and the principle of the constancy of lightspeed. Sections 10.5 and 10.6 present Einstein's prediction of the relativity of time. Section 10.7 presents two more predictions: the relativity of space and the relativity of mass. Section 10.8 presents Einstein's famous prediction of the equivalence of energy and mass, the aspect of special relativity that Einstein himself thought was most important, and discusses its profound significance.

10.1 EINSTEIN: REBEL WITH A CAUSE

I thought of that while riding my bicycle.

Einstein, on the Theory of Relativity, in the *Quotable Cyclist*

Common sense is nothing more than a deposit of prejudices laid down by the mind before you reach eighteen.

Einstein

My intellectual development was retarded, as a result of which I began to wonder about space and time (things which a normal adult has thought of as a child) only when I had grown up.

Einstein

If I were a young man again and had to decide how to make a living, I would not try to become a scientist or scholar or teacher. I would rather choose to be a plumber or a peddler, in the hope of finding that modest degree of independence still available under present circumstances.

Einstein, in a remark made near the end of his life.

The Scottish mathematician and physicist Lord William Thomson Kelvin stated in an address to physicists at the British Association for the Advancement of Science in 1900 that "There is nothing new to be discovered in physics now. All that remains is more and more precise measurement." Many scientists[1] of that day shared Kelvin's confidence that the known "grand unifying principles"—Newton's laws and the laws of thermodynamics and electromagnetism—were complete and permanent.

But the world soon changed. In 1900, Max Planck introduced a revolutionary new principle, the quantum of energy (Chapters 12 and 13). And a scant five years later, in 1905, a quite different but equally revolutionary theory was hatched in the brain of an obscure patent clerk in Bern, Switzerland: Albert Einstein (**Figures 10.1** and **10.2**).

Einstein was a rebel in more ways than one. In his midteens he got fed up with high school and dropped out. This surprised no one, for he had been a mediocre student and a daydreamer since beginning elementary school. Before that he had been a slow child, learning to speak only at 3 years of age. His high school teachers were glad to see him go, one of them informing Einstein that he would "never amount to anything" and another suggesting that he leave school because his presence destroyed student discipline. Einstein was delighted to comply. He spent the next few months as a model dropout, hiking and loafing around the Italian Alps.

After deciding to study engineering, he applied for admission to the Swiss Federal Polytechnic University in Zurich, but he failed his entrance exams. It seems he had problems with biology and French. To prepare for another try, he spent a year at a Swiss high school, where he flourished in this particular school's progressive and democratic atmosphere. He recalled later that it was here that he had his first ideas leading to the theory of relativity. The university now admitted Einstein. He was known as a charming but indifferent university student who attended cafes regularly (where he enjoyed discussing philosophy and science) and lectures sporadically (because he preferred to spend time in physics laboratories). He managed to

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[1] But perhaps not most scientists. Many physicists were dissatisfied with the theoretical foundations of physics and rejected Newtonian mechanics as the basis for physics in favor of electromagnetism.

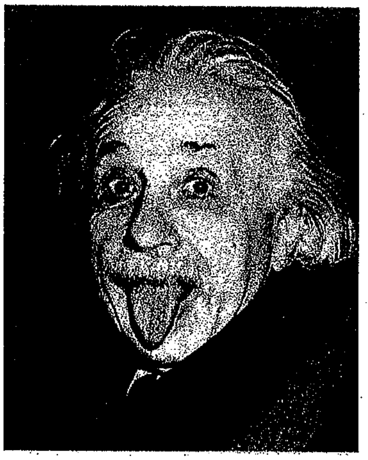

Figure 10.1
Never one to take himself too seriously, Einstein stuck his tongue out when asked to smile on his seventy-second birthday.

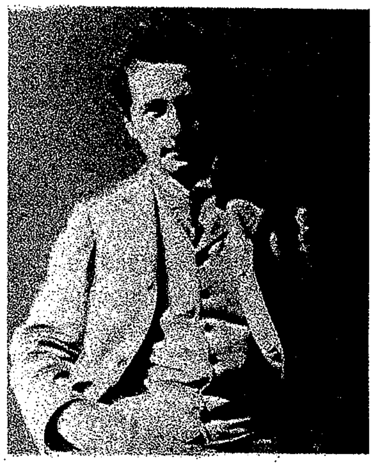

Figure 10.2
Albert Einstein during his student days in Zurich, a few years before he created his special theory of relativity.

pass the necessary exams and eventually graduate with the help of friends who shared their systematic class notes with the nonconforming Einstein.

Were I wrong, one professor would have been quite enough.
Einstein, when asked about a book in which 100 Nazi professors charged him with scientific error.

Following his graduation in 1900, Einstein applied for an assistantship to do graduate study, but it went to someone else. After looking unsuccessfully for a teaching position, in 1902 a friend helped him land a job as a patent examiner. Einstein often referred to his seven years at this job as "a kind of salvation" that paid the rent and occupied only 8 hours a day, leaving him the rest of the day to ponder nature. And ponder he did. One of the many remarkable aspects of the theory of relativity is that it was invented nearly single-handedly.

10.2 GALILEAN RELATIVITY: RELATIVITY ACCORDING TO NEWTONIAN PHYSICS

Here is a typical relativity question: Suppose that a train passenger, call her Velma, throws a baseball toward the front of the train. Both she and Mortimer, who is standing on the ground watching the passing train, measure the baseball's speed (**Figure 10.3**). Will they get the same answer? If not, how will their answers differ? Think about it.

This question concerns two observers who are moving differently. We say that Velma and Mort are in **relative motion** whenever they are moving at different speeds or in different directions. A **theory of relativity** is any theory that works out answers to questions concerning observers who are in relative motion. You can think of the train as being Velma's laboratory, or her **reference frame,** within which Velma measures things like the speed of the ball. You can think of the ground beside the tracks as a second reference frame, Mort's reference frame, for his measurements. The standard question that any theory of relativity asks is how measurements made in one reference frame compare with those made in another. Scientists have thought about questions like this since at least the time of Galileo.

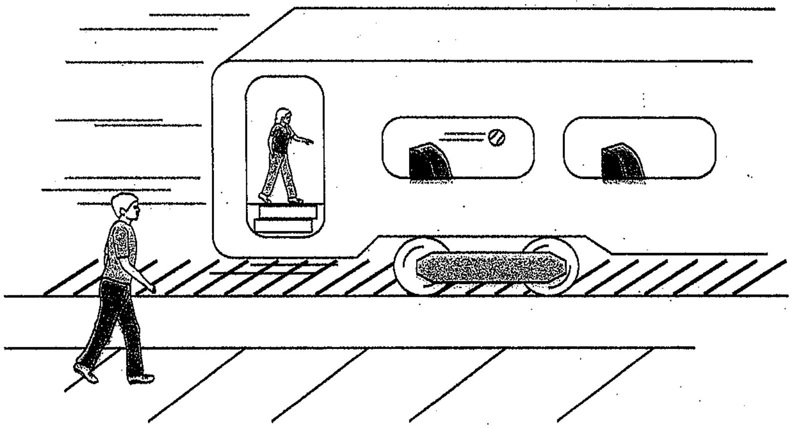

Figure 10.3
Velma throwing a ball, observed by Mort.

To be more specific, suppose that the train moves at 70 meters per second (150 miles per hour, a typical modern train speed). Suppose that Velma throws the baseball toward the front of the train at 20 m/s "relative to Velma" (as measured on the train, using meter sticks and clocks that are on the train). How fast does the baseball move "relative to Mort" (as measured on the ground)?

...Think about that.

Well, during each second, the baseball moves 20 meters toward the front of the train as measured by Velma. But as observed by Mort, the baseball moves an additional 70 meters during that same second, because the train itself moves 70 meters. So the ball must move at 90 m/s relative to Mort. Right? Because Galileo would have given the same answer four centuries ago, this straightforward and fairly intuitive form of relativity is called **Galilean relativity.**

CONCEPT CHECK 1 Velma's normal ball-throwing speed is 20 m/s. She is in a train moving eastward at 70 m/s and throws a ball toward the rear of the train. The velocity of the ball relative to Velma is (a) 50 m/s eastward; (b) 50 m/s westward; (c) 20 m/s eastward; (d) 20 m/s westward; (e) 70 m/s eastward; (f) 70 m/s westward.

CONCEPT CHECK 2 In the preceding question, the velocity of the ball relative to Mort, who is standing beside the tracks, is (a) 50 m/s eastward; (b) 50 m/s westward; (c) 20 m/s eastward; (d) 20 m/s westward; (e) 70 m/s eastward; (f) 70 m/s westward.

Let's turn to a similar example involving light beams instead of baseballs. You learned in Chapter 9 that light is an electromagnetic wave moving at 300,000 km/s, a speed that I will symbolize by the letter *c*. It's difficult to imagine such a high speed. A light beam travels from New York to Los Angeles in a hundredth of a second. Trains, jet planes, and even Earth satellites moving at 8 km/s are slowpokes by comparison.

Imagine that Velma pilots a really fast rocket ship past Earth at 75,000 km/s, or 0.25*c* (25% of lightspeed), and that she holds a source of light—a flashlight or a laser—pointed forward. Mort stands on Earth. What would be the speed of the light

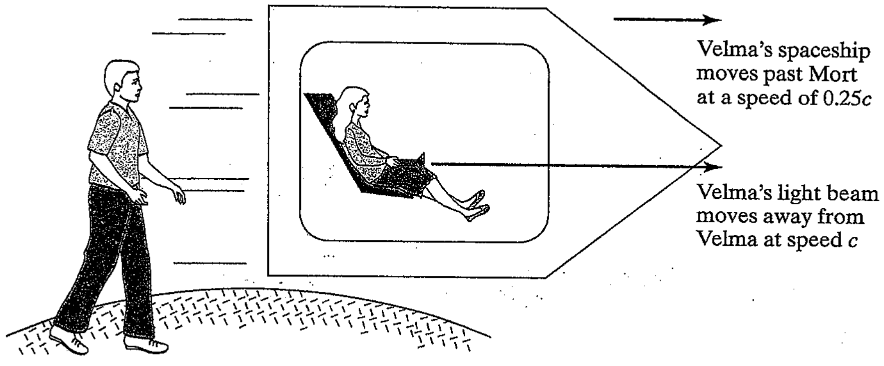

Figure 10.4
How fast is Velma's light beam moving, as observed by Mort?

beam—the moving tip of the beam—relative to Velma and relative to Mort? It seems plausible that Velma measures the light beam to move at speed c, since she's holding the light source. In fact, experiments with light beams emitted by moving sources show this to be true: Any light beam from a moving source moves at speed c relative to the source.[2] What speed would Mort measure for the same light beam (**Figure 10.4**)? Following the logic of the baseball example, the sensible answer would seem to be $1.25c$. After all, the light beam travels 300,000 km in each second as measured by Velma, and Velma travels 75,000 km in each second as measured by Mort, so it seems sensible that the light beam would travel 300,000 km + 75,000 km in each second, or 375,000 km/s, as measured by Mort.

This is the answer Galileo would have given, the answer given by Galilean relativity. It is the answer that all scientists would have given up through the end of the nineteenth century. It is indeed a most sensible answer. Nevertheless, it's experimentally wrong. Nature does not always comply with our notion of what is sensible! To see why there might be something wrong with this answer and to learn nature's answer, let's turn in the next two sections to Einstein's thoughts.

10.3 THE PRINCIPLE OF RELATIVITY

You ride in a smoothly moving unaccelerated jet airplane in level flight at unchanging velocity. The flight attendant pours you a cup of coffee. Where should you hold your cup: directly under the spout, or someplace else to take into account the motion of the airplane? In other words, does the coffee pour straight downward relative to the airplane? Try it sometime and see. Or try dropping a coin from one hand to the other in a moving vehicle (but not when you are driving): Is the catching hand directly beneath the dropping hand?

The answer is that the coffee pours straight downward relative to the plane.

You could experiment with many things, all within a smoothly moving reference frame: a falling ball, frictionless air coasters (Figure 3.6), electric currents, magnets,

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[2] Provided the source is not accelerating; see Section 10.4.

and more. Just as for the poured coffee, you would find that the results are the same as when the experiments are performed in a reference frame at rest on Earth.

Suppose you are a passenger on an airplane with no windows in the passenger compartment. You fall asleep and awaken later to find yourself alone in the compartment. Can you tell, without receiving information from the outside world,[3] whether your airplane is in level flight at unchanging velocity or parked on the ground? The answer is no. You could throw a ball, do handstands, pick up nails with magnets, and the like, and everything would be the same, regardless of whether your plane was in flight or parked.

This is another example of a symmetry principle (Chapters 1 and 9). It says that, no matter from what nonaccelerating reference frame you view the universe, the laws of physics are the same. I'll summarize this as:

The Principle of Relativity

Every nonaccelerated observer observes the same laws of nature. In other words, no experiment performed within a sealed room moving at an unchanging velocity can tell you whether you are standing still or moving.

Unless you look outside, you can't tell how fast you're going. It's a plausible idea and was the key to Einstein's thinking about relativity. It's called the "principle of relativity" because it says that all motion is just relative motion. When you say "the car moves at 25 km/hr westward," you really mean that "the car moves westward at 25 km/hr relative to the ground" or that "the car and the ground are in relative motion at 25 km/hr." You could just as well say that the car is standing still and the ground is moving eastward at 25 km/hr. You could even say that the ground is moving eastward at 1600 km/hr (which it is, relative to Earth's center, due to Earth's spin) and that the car is moving eastward at only 1575 km/hr. It is only the relative speed, the 25 km/hr, that really counts.

CONCEPT CHECK 3 What about acceleration—can this be detected without looking outside? (a) Yes, you can do simple experiments to tell you whether you are accelerating. (b) Yes, but the experiments must involve light beams. (c) No.

10.4 THE CONSTANCY OF LIGHTSPEED: STRANGE BUT TRUE

The Lord is subtle, but He is not malicious.

Einstein

Have you ever asked yourself what it would be like if you could keep up with a light beam? Some people do. The 16-year-old Einstein did, and his reflections on this question helped lead him to his theory of relativity. To Einstein, the possibility of moving along with a light beam seemed paradoxical, contradictory. The reason is that, to an observer moving along with a light beam, the light beam itself would be at rest. To this observer, the light beam would appear as an electromagnetic "wave" that was standing still! To Einstein, this seemed absurd. Here's why.

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[3] Information from the pilot would be from the outside world, because the pilot's information enters through the cockpit window and through radio receivers.

You could see that Einstein was motivated not by logic in the narrow sense of the word but by a sense of beauty. He was always looking for beauty in his work. Equally he was moved by a profound religious sense fulfilled in finding wonderful laws, simple laws in the universe. It was really a religious experience for him, of the most profound sort, even though he did not believe in a personal god.

Banesh Hoffmann, Mathematician and Author, in *Some Strangeness in the Proportion*

Our understanding of electromagnetic waves, such as light, is based on Maxwell's theory of electromagnetic fields (Chapter 9). Recall that Maxwell's theory predicts that any disturbance in an electromagnetic field, such as a disturbance caused by the motion of an electrically charged object, must propagate as a wave moving outward through the field *at speed c*. This particular speed, 300,000 km/s, is built into Maxwell's theory.

Einstein believed that Maxwell's theory should, like all other laws of nature, obey the principle of relativity. So Maxwell's predictions should be correct within every moving reference frame. Since speed *c* is built into Maxwell's theory, Einstein concluded that *every observer ought to observe every light beam to move at speed c*, regardless of the observer's motion. No matter how fast you move, a light beam should always pass you at speed *c*, relative to you. If every observer sees every light beam move at speed *c*, then nobody can even begin to catch up with a light beam, much less move along with a light beam.

It's a simple idea. But it's also pretty crazy, which is why it took Einstein to think of it. After all, if you run after a departing light beam, common sense tells you that from your perspective the speed of the departing light must be less than 300,000 km/s. And if you run toward an approaching light beam, common sense says that the speed of the approaching light must be greater than 300,000 km/s. Einstein's idea is so odd that other turn-of-the-century physicists who might have discovered it did not. It's the second important principle underlying Einstein's theory. I'll summarize it as:

The Principle of the Constancy of Lightspeed

The speed of light (and of other electromagnetic radiation) in empty space is the same for all nonaccelerated observers, regardless of the motion of the light source or of the observer.

I don't try to imagine a personal God; it suffices to stand in awe at the structure of the world, insofar as it allows our inadequate senses to appreciate it.

Einstein

Like the principle of relativity, this principle is valid only for nonaccelerated observers. The reason is that Maxwell's theory, like most laws of physics, is valid only for nonaccelerated observers.

To get a feel for it, we'll apply this principle to several "thought experiments," impractical experiments that could in principle be performed. Each experiment involves a light beam, which we take to be a laser beam but which could just as well be a flashlight beam.

Suppose Velma moves away from Mort at a quarter of lightspeed and holds a laser pointed forward, as in Figure 10.4. As noted in Section 10.2, she observes the beam to move away from her at speed *c*. What speed does Mort observe for the laser beam? Galilean relativity and our intuitions answer 1.25*c*, or 375,000 km/s. But Einstein's relativity predicts that the answer is *c*, or 300,000 km/s!

Another example: Mort has the laser and he shines it in the direction of Velma who is departing from him at a quarter of lightspeed (**Figure 10.5**). Mort observes the beam to move away from him at speed *c*, but what does Velma observe? Galileo, and common sense, now predict 0.75*c*, but Einstein predicts *c*.

To dramatize the oddness of this, imagine that Velma is moving away from Mort at a speed of 0.999 999*c*, just a hair slower than lightspeed (**Figure 10.6**). Mort switches on his laser and sees the light beam depart from him at speed *c*. As

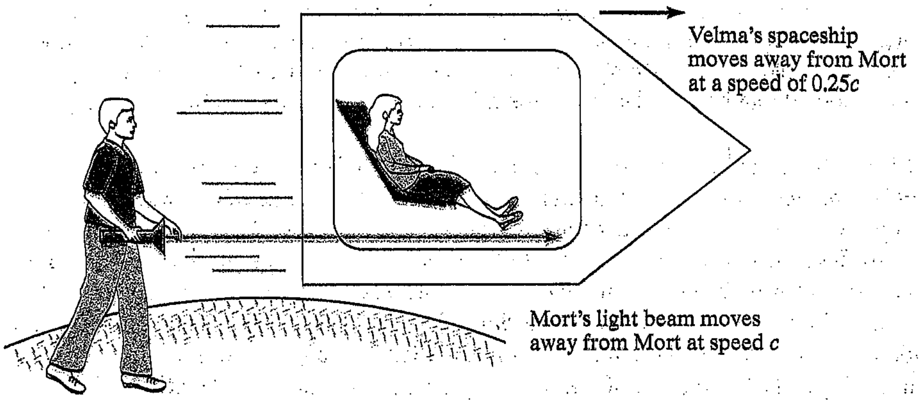

Figure 10.5
What is the speed of Mort's light beam relative to Velma?

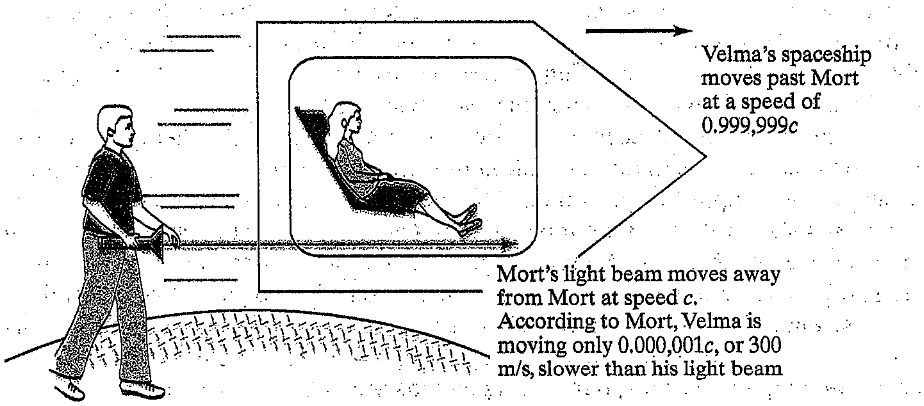

Figure 10.6
Now how fast is Mort's light beam moving, as observed by Velma?

observed by Mort, Velma moves only slightly slower than the light beam—he says that she nearly keeps up with the light beam. Galilean relativity predicts that Velma observes the light beam passing her at only 0.000 001*c*. This is just 300 m/s—the speed of fast jet airplanes. But Einstein's relativity says that she sees the light beam pass her at precisely 300,000 km/s, despite the fact that she is moving away from the light source at nearly lightspeed!

Maybe you've noticed that we don't allow Velma to have precisely speed *c*. If we imagined that she moves right at speed *c*, we'd get into the difficulty that Einstein noted: She would observe the light beam to be at rest. So an observer can move at nearly, but not precisely, speed *c* relative to another observer. Later, we'll see why.

How do we know that light goes the same speed for all observers? Strange though the constancy of lightspeed may seem, it's verified daily. However, most experiments involve fast-moving microscopic particles rather than spaceships. In one especially striking experiment in 1964, a subatomic particle moving at nearly lightspeed emitted electromagnetic radiation both forward and backward. Galilean relativity predicts that the forward-moving radiation should move much faster than *c* while the backward-moving radiation should move much slower than *c*, as measured in the laboratory. But measurement showed that both radiation beams move at speed *c* relative to the laboratory.

Maxwell and other nineteenth-century scientists had a more conventional view of light beams. As explained in Chapter 9, they believed that light was a wave in a material medium, just as water waves are waves in water. They called this medium **ether.** Nobody had observed ether. It couldn't be made of ordinary atoms, because light waves travel through outer space where there are essentially no atoms. Instead, ether was thought to be a continuous material substance filling the entire universe and made of some unknown nonatomic form of matter. The ether theory assumes that the "natural" speed of light, 300,000 km/s, is light's speed relative to the ether. Observers moving through the ether should then measure other speeds for light beams, speeds that should depend on the observer's speed through the ether. But as the principle of the constancy of lightspeed states, and as experiment shows, all observers measure the same speed for all lightbeams, so the ether theory must be wrong. Since Einstein, electromagnetic waves have been viewed as the vibrations of an electromagnetic field, which itself is not made of any material substance. As discussed in Chapter 9, this contrasts sharply with the materialist worldview of Newtonian physics.

The constancy of lightspeed is the key principle that gives the theory of relativity its odd quality. It's natural to question this principle. How do we know it's true? The answer is simple but profound: It's true because nature says so. Numerous experiments show that every light beam moves at speed *c*, regardless of the motion of the source or observer. Although this odd notion violates our preconceived beliefs, it is observation of nature, rather than preconceived beliefs, that determines truth in science. Our preconceived beliefs about motion are based on observations of objects moving far slower than lightspeed and are very nearly correct at such speeds. But at higher speeds, our preconceptions are radically incorrect.

The foundations of Einstein's theory are the principle of relativity and the constancy of lightspeed. Their role in the theory of relativity is identical to the role of Newton's laws in Newton's theory of force and motion: They form the logical basis of the theory, from which everything else is derived and which are themselves justified directly by observation. Physicists call this theory the **special theory of relativity.** The word *special* distinguishes this theory from another, related theory of Einstein's called the **general theory of relativity** (Chapter 11). The distinguishing feature of the general theory of relativity is that it allows accelerated observers, while the special theory allows only non-accelerated observers, so the general theory is a more general—broader—theory than the special theory. Strictly speaking, Earth itself is an accelerated reference frame, because it spins on its axis and because it rotates around the sun. But these accelerations are so small that the predictions of the special theory are excellent approximations for any Earth-based observer.

The remainder of this chapter explores five of special relativity's most important predictions: the relativity of time, the relativity of space, the relativity of mass, *c* as the speed limit, and $E = mc^2$.

▶ CONCEPT CHECK 4 Velma moves away from Mort at $0.75c$. She turns on two lasers, one pointed forward and the other backward. According to Galilean relativity, how fast should the forward and backward beams move, as observed by Mort? (a) $0.25c$ and $1.75c$. (b) $1.75c$ and $0.25c$. (c) $0.25c$ and $0.75c$. (d) $0.75c$ and $0.25c$. (e) c and c.

▶ CONCEPT CHECK 5 In the preceding question, Mort actually observes (a) 0.25*c* and 1.75*c*; (b) 1.75*c* and 0.25*c*; (c) 0.25*c* and 0.75*c*; (d) 0.75*c* and 0.25*c*; (e) *c* and *c*.

10.5 THE RELATIVITY OF TIME

The constancy of lightspeed suggests something is amiss in our intuitive conceptions of space and time. After all, speed measures how far an object moves through space divided by the time to move, so speed is intimately tied to space and time.

We feel that we understand what "time" is, but its meaning fades when we ponder it. Being enmeshed in time, we cannot study it from a distance, so our attempts to define it are usually circular, implicitly using the concept of time in order to define time. Einstein's insight into time was that it's physical—part of the physical universe. Just as one can measure the properties of a stone or of a light beam, one can measure the properties of time. And how should we measure the properties of time? With clocks! This reply is more profound than it appears. The only way we can measure time is with real, physical "clocks," by which we mean any phenomenon—a swinging pendulum, Earth's rotation around the sun—that goes through identical repetitions. Physically, the concept of a clock really defines time. So to investigate the properties of time, we must investigate clocks. How do clocks really behave? Einstein managed to predict the properties of clocks using as his starting point only the two principles of the special theory of relativity.

An ordinary spring-wound or battery-driven clock would be hard to study based only on Einstein's two principles because these clocks are so complex, involving springs, electric current, gears, and so forth. So Einstein invented a simple kind of clock, a simple thought experiment, really. His **light clock** (**Figure 10.7**) involves no mechanically moving parts; its only motion is the motion of a light beam. Two parallel mirrors face each other, one above the other, and a light beam bounces up and down (reflects) between them. Although it's not terribly practical for the clock maker, it's convenient to imagine that the mirrors are separated by 150,000 kilometers, because then the time for one complete round trip of the light beam is just 1 second. You know it's 1 second because the constancy of lightspeed says all light beams travel 300,000 kilometers in 1 second. We'll assume this light clock ticks at the end of each round-trip.

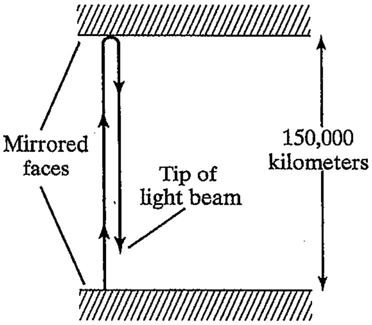

Figure 10.7
A light clock. A light beam bounces up and down between two mirrors. If the distance between mirrors is 150,000 km, then 1 second will elapse during one complete round-trip up and back down.

We begin investigating the properties of time by installing one light clock in Velma's spaceship moving eastward past Earth, and another in Mort's laboratory on Earth. Let's think about Velma's light clock. She sees her light beam bouncing straight up and down, covering 300,000 km per tick [**Figure 10.8(a)**]. Simple enough. But from Mort's point of view, the tip of Velma's light beam is not only moving up and down, it's also moving eastward because of Velma's eastward motion. So the tip of Velma's light beam, as seen by Mort, moves along diagonal paths. Figure 10.8b shows Mort's observations of Velma's spaceship at three instants: when the tip of her light beam is at the bottom mirror, when it has moved up to the top mirror, and when it is back at the bottom mirror.

Since the distance between the mirrors is 150,000 km, you can see from the figure that the distance along one of the two diagonals is greater than 150,000 km. This means that the total round-trip distance traveled by Velma's light beam, as measured by Mort, is greater than 300,000 km. There is nothing surprising or subtle about this; Galileo would have said the same thing. Now comes the part that Galileo (and our intuitions) wouldn't agree with: The constancy of lightspeed says that Mort observes

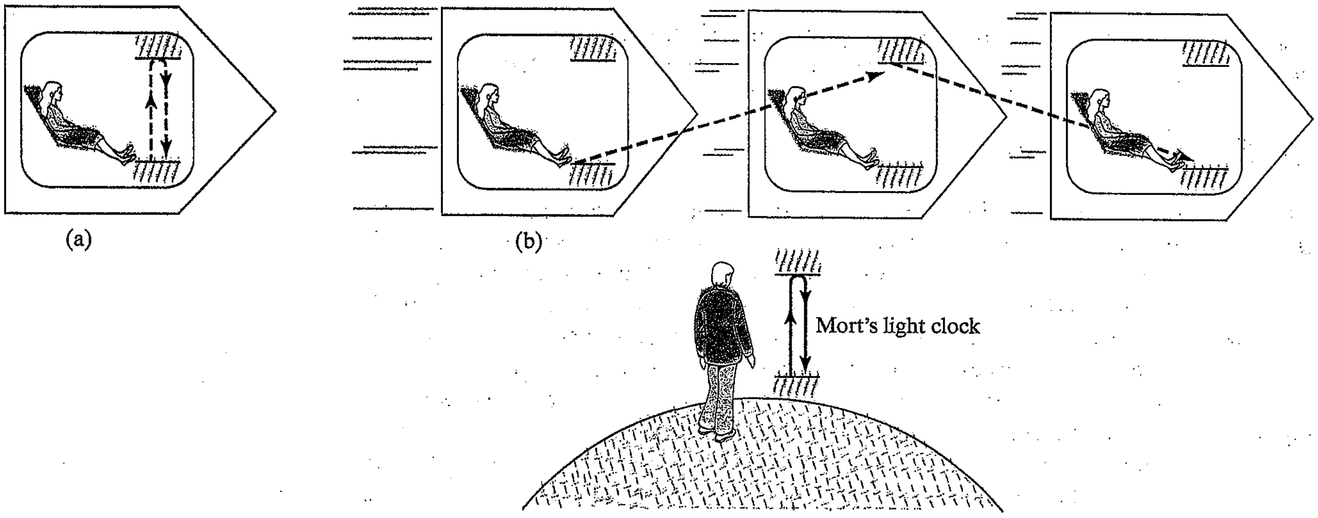

Figure 10.8

(a) Velma in her spaceship, observing her light clock. (b) Velma's spaceship and the light beam on Velma's light clock as observed by Mort using his own light clock. According to Mort's observations, the tip of Velma's light beam moves along the diagonal path shown by the dashed arrows.

Velma's light beam to move at just 300,000 km/s (Galileo would say that Mort observes Velma's light beam to move faster than 300,000 km/s, because of Velma's motion). Since Mort observes the round-trip distance to be greater than 300,000 km, it follows that according to Mort it takes more than 1 second for Velma's light beam to make the round-trip! So, as measured by Mort using his clock, more than 1 second elapses between Velma's ticks. According to Mort, Velma's clock runs slow.

Velma's second is different from Mort's second. The two observers measure different time intervals for the same event (one round-trip of Velma's light beam). *Time is relative to the observer*. It's simple, but hard to believe.

Let's turn things around. How does Mort's clock appear to the two observers? To Mort, his own clock's light beam travels 300,000 km in one round-trip and requires 1 second to do so. But from Velma's viewpoint, Mort's clock is moving westward, so the tip of Mort's light beam is moving along a diagonal and therefore the total round-trip distance traveled by Mort's light beam as observed by Velma is greater than 300,000 km. But because Velma observes Mort's light beam to move at 300,000 km/s, she must observe that more than 1 second elapses between Mort's ticks. According to Velma, it's *Mort's* clock that runs slow.

The rule is *moving clocks run slow:* Mort and Velma both observe that the other person's clock runs slow. This is not your normal situation caused by an inaccurate clock, in which if my clock runs slow according to your clock, then your clock must run fast according to my clock. This raises an interesting question: Whose clock is really running slow, and whose is really accurate? The answer is that Velma and Mort are both right! Velma observes that Mort's clock is slow, and Mort observes that Velma's clock is slow, and both observations are correct. This situation is not caused by inaccurate clocks; it is instead a property of time itself. There is no single "real" time in the universe, no "universal time"; there is only Mort's time and Velma's time and all the other possible observers' times.

As you might expect, there is a formula that quantitatively describes the **relativity of time.**[4] **Table 10.1** gives a few of the numerical results that can be calculated from this formula, and **Figure 10.9** is a graph based on the same formula. As you can see from the table, the effect is negligible even at orbiting satellite speeds (10 to 20 km/s). It's not until speeds of $0.1c$—a speed that would get you around the world in 1 second—that the effect amounts to even a half of 1%. But at large fractions of lightspeed, the effect becomes quite large: At 99.9% of lightspeed (not shown on the graph), Mort and Velma's seconds will be more than 22 seconds long as measured by the other observer. The relativity of time is also called **time dilation,** because a time interval of 1 second on a moving clock is expanded, or dilated, to more than 1 second as measured by an observer past whom the clock is moving.

Although we investigated the relativity of time by studying light clocks, the conclusion holds for every type of clock—every regularly repeating phenomenon. Einstein thought about light clocks only in order to learn what the two principles of his theory implied about time. Every clock must behave the way a light clock behaves because they all measure the same thing: time. And every phenomenon that occurs during an interval of time must also behave in this way. Think, for example, of an ice-cream cone melting. Suppose you can make ice-cream cones that melt in exactly 10 minutes and that both Velma and Mort have one of these cones. These cones are a kind of clock, a clock that ticks in 10 minutes.

> It requires a very unusual mind to undertake the analysis of the obvious.
>
> Alfred North Whitehead, Twentieth-Century Philosopher

CONCEPT CHECK 6 Mort and Velma have identical 10-minute ice-cream cones. Velma passes Mort at 75% of lightspeed. Use Table 10.1 to predict the times measured by Mort for his and Velma's cone to melt. (a) 10 minutes for Mort's cone, 10 minutes for Velma's cone. (b) 10.5 minutes and 10 minutes. (c) 10 minutes and 10.5 minutes. (d) 15 minutes and 10 minutes. (e) 10 minutes and 15 minutes.

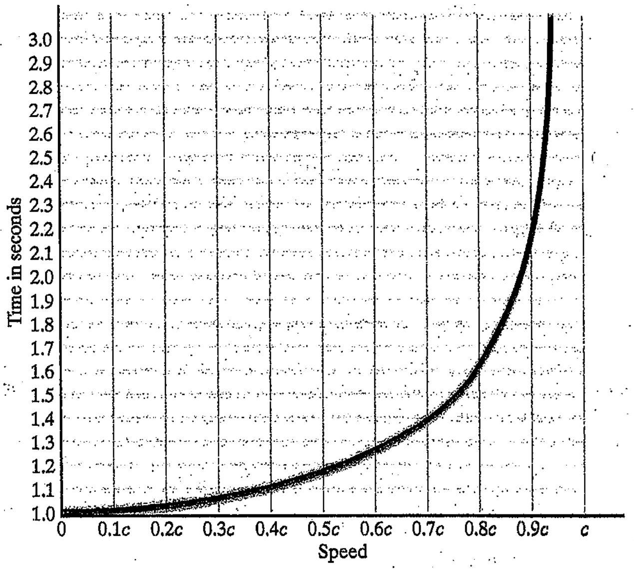

Figure 10.9
The relativity of time. The graph shows the duration of one clock tick (representing 1 second in the clock's reference frame) on a moving clock, for various speeds of the clock relative to the observer.

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[4] This formula can be derived from Figure 10.8 by using the Pythagorean theorem, which states that a right triangle's short side lengths a and b are related to its diagonal length c by $c^2 = a^2 + b^2$. The formula is $T = T_o/\sqrt{(1 - v^2/c^2)}$, where v is the relative speed, T_o is the time between two of Velma's ticks as observed by Velma ($T_o = 1$ second), and T is the time between two of Velma's ticks as observed by Mort.

Table 10.1

The relativity of time: some quantitative predictions

To give you a feel for these speeds: 0.3 km/s is a typical subsonic jet plane speed, 3 km/s is twice the speed of a high-powered rifle bullet, at 3000 km/s you could cross the United States in 1 second, and at 30,000 km/s you could circle the globe in 1 second. Clearly, relativistic effects are small until the speed becomes very large!

Relative speed (km/s)	Relative speed as a fraction of lightspeed (c)	Duration of one "tick" on a moving clock, as measured by an observer past whom the clock is moving (s)
0.3	10^{-6}	1.000 000 000 000 5
3	10^{-5}	1.000 000 000 5
30	10^{-4}	1.000 000 005
300	0.001	1.000 000 5
3000	0.01	1.000 05
30,000	0.1	1.005
75,000	0.25	1.03
150,000	0.5	1.15
225,000	0.75	1.5
270,000	0.9	2.3
297,000	0.99	7.1
299,700	0.999	22.4

Instead of ice-cream cones, they could have frogs. Suppose your local biology department hatches guaranteed 10-day frogs, having a 10-day lifetime. Biological life occurs in time, too, so these frogs can be thought of as a kind of clock. So if Velma passes Mort at 75% of lightspeed, he says that her frog lives 15 days but that his frog lives only 10 days (see Concept Check 6). And she says that his frog lives 15 days but that her frog lives only 10 days. So each observes their own frog to die first. And both observations are correct! Fantastic.

"But," you may ask, "whose frog really dies first?" If you are tempted to ask this, your unspoken belief is that there is one single, universal, "real" time. But there isn't. There is only Mort's time, and Velma's time, and every other individual observer's time.

How do we know that time flows differently for different observers? The relativity of time has been verified repeatedly in laboratories, by observing fast-moving subatomic particles. One experiment, similar to the frog example, involved a type of subatomic particle known as a "muon." Muons, unlike most ordinary matter, are not permanent objects. Instead, they have a "lifetime" after which they disintegrate spontaneously into other particles. The lifetime of a muon is only 2.2 microseconds (2.2 millionths of a second), as measured by you if the muon is at rest relative to you. But a muon moving rapidly past you lives much longer as measured by you, because of time dilation. For example, at 99% of lightspeed (muons often move this fast in high-energy physics labs), Table 10.1 says that its lifetime will be lengthened by a factor of 7.1, so it will not disintegrate until 7.1 × 2.2 = 15.6 microseconds have passed. This experiment has been done, and the moving muons were observed to have lifetimes that were lengthened by just the predicted amount.

10.6 TIME TRAVEL: YOU CAN'T GO HOME AGAIN

As you might have suspected, the next step is to investigate the life spans of Velma and Mort themselves. Suppose they are born at the same time[5] and that they have 80-year lifetimes. In other words, Velma observes her lifetime to be 80 years and Mort observes his lifetime to be 80 years. If Velma and Mort spend their lives moving at 75% of lightspeed relative to each other, then Table 10.1 informs us that Mort's descendants observe that Velma lives for 120 years, as measured by Mort's clocks. And Velma's descendants observe that Mort lives for 120 years, as measured by Velma's clocks. From Mort's viewpoint, Velma ages slowly; she ages by just a year during each of Mort's 1.5 years; he dies after 80 of his years; and she dies after 120 of Mort's years but having the physical appearance of a person who is only 80. According to Velma, all of this is reversed. And both of them are correct. Incredible.

▶ CONCEPT CHECK 7 When Velma observes herself to be 60 years old, she will observe Mort to be (a) 30; (b) 40; (c) 60; (d) 80; (e) 90.

This suggests a perplexing question. Suppose that Velma and Mort are born at the same time on Earth, as twins perhaps, and Velma then boards a spaceship, takes a fast trip to a far star, and returns to Earth. This scenario is different from the scenario in the preceding paragraph, because now Velma and Mort begin and end in the same reference frame. Once they are back together they must agree on who is older, because there is only a single time in any single reference frame. Which twin will be older, or will they be the same age?

The testimony of our common sense is suspect at high velocities.
Carl Sagan, Astronomer and Author

Let's think about that. Recall that the special theory of relativity applies only to nonaccelerated observers. But in the scenario for the two twins, Velma leaves Earth, speeds up enormously, turns around to get back to Earth, and then comes to rest on Earth. Since this trip necessarily involves three enormous accelerations, the special theory of relativity does not apply to Velma's observations. But the special theory does apply to Mort's observations, since he didn't accelerate. As you have seen, the theory predicts that he observes Velma to age slowly during her entire trip, because she is moving relative to him. For example, if she moves at $0.75c$, he should observe that 1.5 of his years elapse for every 1 of hers (Table 10.1). If Velma's trip takes 60 years as measured by Mort, he observes that only 40 of her years elapse. So he observes that when they get back together on Earth, he is 60 and she is 40! Her observations must agree with this, since the two are now in the same reference frame. This is how you can get to be 20 years younger than your twin brother.

How do we know that time travel is possible? This conclusion has been experimentally verified, but in a less-dramatic fashion. Atomic clocks were flown around the world on commercial jet flights and compared to clocks that remained at rest on Earth. Although the predicted time difference was only a fraction of a second, it was measurable using high-accuracy clocks. As predicted, the clock that went on the trip came back younger (it hadn't ticked as many times) than the clock that stayed home. And the quantitative difference in elapsed time was precisely as predicted. As you will see in a moment, such experiments demonstrate that time travel is possible, but only into the future.

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[5] You might wonder what "at the same time" means, since we are assuming that Mort and Velma are in different reference frames. To simplify matters, suppose that Mort and Velma are just passing each other. Then "at the same time" means that as either one comes into the world, he or she observes that the other is coming into the world too.

This suggests some astonishing possibilities. Suppose your mother leaves Earth for the star Vega, a sunlike star lying relatively close to our sun and a possible candidate for a planetary system. The distance to Vega is 26 light-years, meaning that it takes light 26 years to reach Vega from here. A **light-year** is the distance light travels in 1 year.

Suppose mom's spaceship averages a colossal 0.999c. She spends 3 years on a planet that is orbiting Vega and returns home. Since she travels at nearly lightspeed, each one-way trip takes slightly more than 26 years, as measured on Earth. So she is gone for slightly more than $26 + 3 + 26 = 55$ years, as measured on Earth. If you were 5 and mom was 30 when she departed, you would be 60 when she returned. But mom would no longer be 25 years older than you! Table 10.1 informs us that during the 52 "Earth-years" of space travel at 0.999c, she aged by only 1 year for every 22.4 years of "Earth time." So she aged by only $52/22.4 = 2.3$ years during the 52 Earth-years. Including the 3 years spent on Vega, she aged by only 5.3 years during the entire trip. So mom is 35.3 years old when she returns, and you are 60! This is how you can get to be older than your mother.

It's a form of **time travel.** Your mother took a trip to Earth's future. She could travel much further into the future, hundreds or thousands of years into the future, by moving faster, say at 0.9999c. But it's a one-way trip. You can't go home again to the past from which you departed.

Time dilation suggests that humans might travel to distant stars within a human lifetime. Suppose you travel to a star 200 light-years away, at 0.999c relative to Earth. Even though the trip takes a little over 200 years as measured on Earth clocks, it takes you only $200/22.4 = 9$ years as measured in your spaceship. When you arrive at the star, two centuries have elapsed on Earth. Even if you immediately hurry back to Earth, you time-travel four Earth centuries into the future during the round-trip but you age by only 18 years. On Earth, you will be a relic from four centuries earlier.

CONCEPT CHECK 8 It is physically possible for your mother to leave Earth after you were born and return (a) before you were born; (b) before she was born; (c) younger than you; (d) older than you; (e) younger than she was when she left; (f) older than she was when she left.

10.7 THE RELATIVITY OF SPACE AND MASS

What is space? Just as time means "what is measured by clocks" (Section 10.5), space means "what is measured by rulers."

What operations should Mort perform to measure, say, the width of a window? For a window at rest relative to Mort, the prescription is to place a measuring rod along the window and compare the ends of the window with the marks on the rod. If the window is moving past Mort, he should continue using a measuring rod that is fixed in his own reference frame, because he wants to know the width of the moving window as measured in his own reference frame. If the width being measured lies along the direction of motion, Mort must measure the positions of the two ends *simultaneously* because otherwise the window will shift positions during the lag between measurements and Mort won't measure the true width.

In order to ensure that the front-end and back-end measurements are simultaneous, Mort must use two clocks—one at each end. This means that the measurement of the width of a moving object is mixed up with the measurement of time; *time and space are tangled up with each other!* Since time is relative, it then comes as no surprise to learn that space is relative too. I won't go through the argument that proves this result; it's similar to the argument in Section 10.5 showing that moving clocks run slowly. More specifically, Einstein's theory predicts that Mort observes the window's width along its direction of motion to be shorter than does Velma who is traveling along with the window (**Figure 10.10**). This effect is called **length contraction.** There is no length contraction along directions perpendicular to the window's direction of motion.

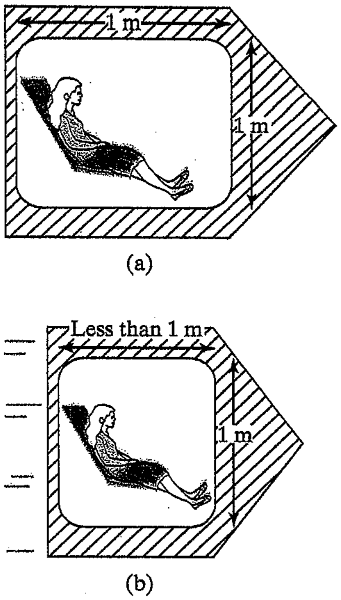

Figure 10.10
The window in Velma's spaceship as measured by (a) Velma and (b) Mort.

As with time dilation, length contraction works both ways: Just as Mort finds that Velma's window is contracted, Velma finds that Mort's window is contracted.

A quantitative analysis leads to a formula, graphed in **Figure 10.11**.[6] The figure graphs the predicted length of a 1-meter-long object such as a meter stick, held parallel to its motion, for various speeds of the object. Like time dilation, length contraction is barely detectable for speeds below about 0.1*c* but becomes large at higher speeds.

Length contraction is not simply something that happens to meter sticks. Since space is defined by meter sticks, it is space itself that is contracted. Just as Velma's time flow is different from Mort's time flow, we must speak of "Velma's space" and "Mort's space" rather than a single, universal space. Space is different for different observers. Space is relative.

CONCEPT CHECK 9 Velma measures her spaceship to be 100 m long and 10 m high. Is it possible for her spaceship to move fast enough past Mort for its length to be equal to its height, as observed by Mort? (a) Yes, by moving at about 0.9*c*. (b) Yes, by moving at about 0.99*c*. (c) Yes, by moving at about 0.1*c*. (d) No, because she would have to move at precisely lightspeed to accomplish this. (e) No, because objects do not change their shapes.

Einstein's new principle, the constancy of lightspeed, affects nearly everything in physics: time, space, and more, including Newton's law of motion (Chapter 4). This law states that an object's acceleration is equal to the net force exerted on the object divided by the object's mass, or in symbols

$$a = F/m$$

This implies that if you exert an unchanging force on an object, the object maintains an unchanging acceleration. Eventually, the object will be going at lightspeed and still accelerating. An observer riding on such an object could catch up with and pass a light beam.

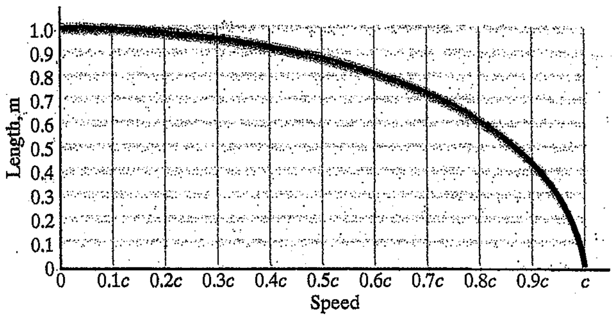

Figure 10.11
The relativity of space. The predicted length of a meter stick for various speeds of the meter stick relative to the observer.

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[6] The formula is $L = L_0 \sqrt{(1 - v^2/c^2)}$ where L_0 is the object's rest length (the length as measured by an observer for whom the object is at rest), and L is the length of the object when it is moving at speed v.

So Newton's law of motion is not consistent with the theory of relativity! Apparently, relativity alters Newton's law in such a way as to prevent objects from accelerating up to lightspeed. To describe this alteration of Newton's law of motion, let's imagine that Mort and Velma (who is moving past Mort) have identical 1 kilogram objects, 1 kilogram melons perhaps. If Mort pushes on his melon with, say, a 1 newton force, he will find that it accelerates at 1 m/s^2, just as Newton's law of motion predicts. If he now pushes on Velma's melon (which is moving past him) with a 1 newton force, Newton's law of motion predicts that Velma's melon accelerates at 1 m/s^2, but *relativity theory predicts*[7] *that Velma's melon accelerates at less than* 1 m/s^2. As was the case for other relativistic effects, this effect is negligibly small at normal speeds but large at speeds comparable to lightspeed.

From Mort's point of view, a 1 newton force applied to both melons produces a smaller acceleration in Velma's melon than in his own melon. From Mort's point of view, Velma's melon has more **inertia** than does his own melon (recall that a body's inertia is its resistance to acceleration). But this is the same as saying that Velma's melon has more mass, because the fundamental meaning of **mass** is "amount of inertia" (Chapter 4). In other words, Mort measures Velma's melon to have a larger mass than his own melon, even though they are identical melons. As usual, the effect works the other way around: Relative to Velma, her melon has a mass of 1 kg, but Mort's melon has a mass of more than 1 kg.

Thus, mass is relative: An object's mass increases with its speed, so different observers measure different masses for the same object. A quantitative analysis leads to a formula that predicts an object's mass for various speeds.[8] **Figure 10.12** is a graph of this formula, along with the previous graphs for time dilation and length contraction. The formulas for mass increase and time dilation have identical forms, so their graphs have identical shapes.

In Newtonian physics, "mass" (or inertia) means the same thing as "quantity of matter." But in relativity, an object's mass increases with its speed while its quantity of matter does not increase because it still contains the same atoms. So mass no longer means "quantity of matter." But we need a word for an object's quantity of matter. That word is **rest-mass,** the mass of an object as measured by an observer in a frame of reference in which the object is at rest. For example, Velma's and Mort's melons both have rest-masses of 1 kg, regardless of who observes them. This number, 1 kg, is a measure of its quantity of matter. An object's *mass*, however, is the amount of inertia it possesses and is different for different observers. The mass and rest-mass of a slow-moving object are essentially the same, but the mass of a high-speed object is significantly greater than its rest-mass.

How do we know that mass increases with speed? Relativistic mass increase is an everyday fact of life in high-energy physics labs. A subatomic particle can be accelerated to speeds so close to lightspeed that its mass is thousands of times greater than its rest-mass. One way to check this prediction is to bend a high-speed particle's path by applying electric or magnetic forces and measure the curvature of the resulting path. If high-speed particles really do have larger masses, their paths should curve less than they otherwise would, because their larger inertia tends to keep them moving straight ahead. Measurements show that such paths are less curved than they would be in the absence of relativistic mass increase and that the amount of curvature agrees with Einstein's predictions.

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[7] The reason is that accelerations of Velma's melon, as viewed by Mort, are reduced because distances are contracted and time intervals are expanded.

[8] The formula is $m = m_0/\sqrt{(1 - v^2/c^2)}$, where m_0 is the object's rest-mass (the mass as measured by an observer for whom the object is at rest), and m is the mass of the object when it is moving at speed v.

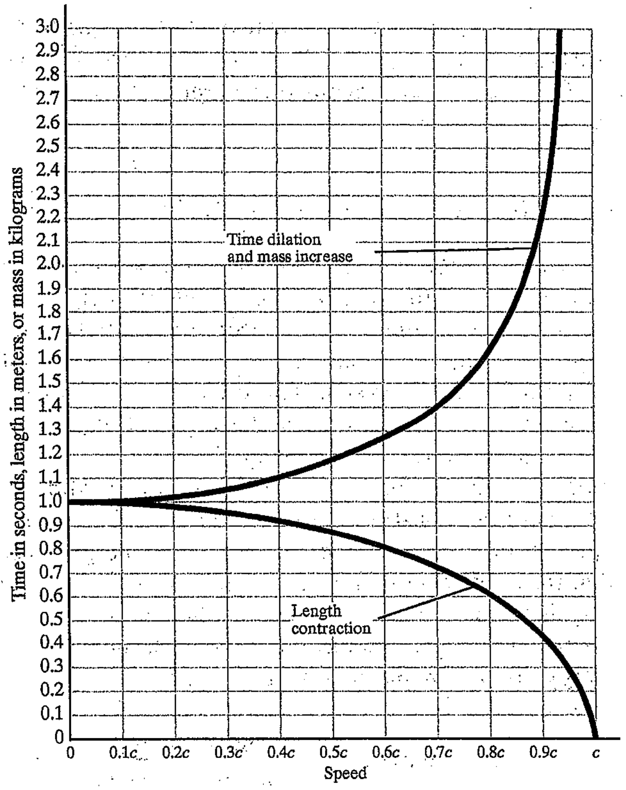

Figure 10.12
Relativistic mass increase, length contraction, and time dilation. The graph shows the duration of one clock tick (representing 1 second in the clock's reference frame) on a moving clock, the length of a moving meter stick, and the mass of a moving standard kilogram, for various speeds of the clock, meter stick, and kilogram relative to the observer.

Time, space, and mass are relative, but not everything is relative. In fact, the two basic principles of Einstein's theory tell us that the speed of any light beam is the same for every observer, and the same goes for the laws of physics.

Relativistic mass increase explains why you cannot accelerate objects up to lightspeed. At high speeds, an object's mass becomes very large, increasing without limit as the speed approaches *c* (Figure 10.12). Eventually, the force needed for further acceleration becomes so large that the object's surroundings cannot provide it. But there is something that moves as fast as lightspeed: light itself. In fact, light never moves slower than 300,000 km/s.[9] When you turn on a lightbulb, the light does not accelerate from zero up to lightspeed; instead, it moves at precisely lightspeed from the instant it is created. Light is quite different from any material object. When you put a material object down in front of you, it has rest-mass. Light beams must not have rest-mass, because if they did, then relativistic mass increase would make their mass infinite while moving at lightspeed. Anything that has no rest-mass and always moves at lightspeed, such as light and other forms of electromagnetic radiation, is classified as radiation. It's a useful distinction: **Matter** has rest-mass and always moves slower than lightspeed, while **radiation** has no rest-mass and always moves at lightspeed.

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[9] However, light travels through material substances such as water or glass at an average speed that is sometimes far less than lightspeed. When moving through matter, light momentarily vanishes when absorbed by an atom and is re-created when emitted by the atom. Whenever the light actually exists as light, it moves at 300,000 km/s.

CONCEPT CHECK 10 Which is a form of matter? (a) Red light. (b) The invisible waves drawn in **Figure 9.3**. (c) The invisible carbon dioxide gas emitted by automobiles. (d) The electron beam that creates the picture on a TV tube. (e) A gamma ray.

10.8 $E = mc^2$: ENERGY HAS MASS, AND MASS HAS ENERGY

As an object speeds up, its kinetic energy increases and, as you have just learned, its mass increases. So, at least in the case of kinetic energy, energy increase and mass increase go hand in hand. Working from the theory of relativity and the law of conservation of energy, Einstein found that mass is connected to every form of energy in this fashion. You can increase a system's mass by simply lifting it (giving it gravitational energy), warming it (giving it thermal energy), stretching it (giving it elastic energy), or giving it any other form of energy.

Does that surprise you? It surprises me. If you stretch a rubber band, you don't expect its mass to increase. It's still the same rubber band, after all. This is a new and surprising prediction: Any increase in a system's energy increases its mass, regardless of what form that energy increase might take.

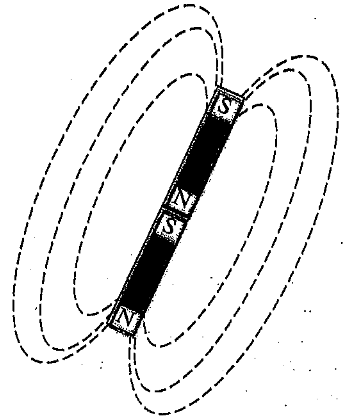
Einstein's analysis yields a simple formula that quantitatively relates the change in mass to the change in energy. The formula states that the change in mass equals the change in energy divided by the square of lightspeed:

$$\text{change in mass} = \frac{\text{change in energy}}{\text{square of lightspeed}}$$

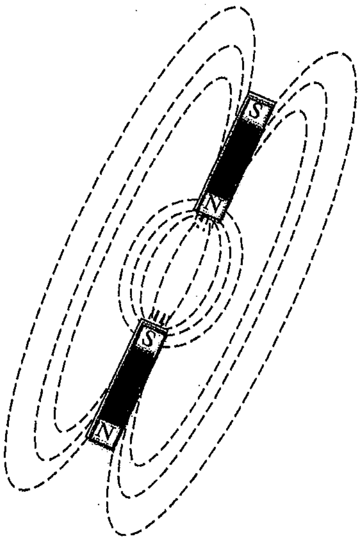
In the standard metric units, mass and energy are in kilograms and joules, and $c = 3 \times 10^8$ m/s, so $c^2 = 9 \times 10^{16}$ m^2/s^2. Note that, since the standard metric unit for use in physics formulas is meters rather than kilometers, you need to use 3×10^8 m/s for "c" rather than 300,000 km/s.

Here's an example. Suppose you stretch a large, strong rubber band by exerting an average force of 300 N through a distance of 0.6 m. Since work equals force times distance, you've done 300 N × 0.6 m = 180 J of work on the band. So the work–energy principle (Chapter 6) says you've added 180 J of energy to the band. This increases the band's mass by $180 / 9 \times 10^{16} = 2 \times 10^{-15}$ kg = 0.000 000 000 000 002 kg. Not much. The increase is small because c^2 is so large. This is why relativistic mass increase wasn't noticed before Einstein: In ordinary situations, it's too small to notice.

As a second example, suppose you have two bar magnets and that the north pole of one is joined to the south pole of the other so that they cling together [**Figure 10.13(a)**]. Since it takes work to pull them apart, the separated magnets of Figure 10.13b must have more energy than do the joined magnets. But more energy means more mass. So the total mass of the two combined magnets increases simply by pulling them apart! The separation process creates a magnetic field in the space between the two magnets [Figure 10.13(b)]. The excess energy in the separated magnets resides in this invisible and nonmaterial magnetic field. You encountered such "field energy" before, in the energy of electromagnetic radiation. But now you can see that fields also have *mass*. This mass is in the "empty" space between the magnets. The work done in separating the two magnets is only a few joules, so the mass difference is again tiny. Nevertheless, it's extraordinary that nonmaterial fields in empty space have mass.



(a) Less energy



(b) More energy

Figure 10.13
The separated magnets of (b) have more energy, and hence more mass, than do the two joined magnets of (a). The excess energy and mass in (b) reside in the invisible and nonmaterial magnetic field, indicated by dashed lines.

Turning to more dramatic examples, nuclear reactions entail nature's strongest forces, the forces acting within the atomic nucleus (Chapters 14 and 15). For now, all you need to know about **nuclear reactions** is that they are analogous to chemical reactions but they involve changes in nuclear structure rather than changes in electron orbits. For example, in nuclear power reactors and nuclear weapons, the element uranium undergoes a nuclear reaction known as **nuclear fission** in which the nucleus of each uranium atom is altered.[10] Fission is a little like combustion, but the forces involved are so strong that the thermal energy created is far larger than in any chemical reaction. So the rest-mass loss, after removing the thermal energy, is far larger. If a kilogram of uranium is fissioned, the rest-mass loss is about 0.001 kg (1 g), which is a 0.1% mass decrease and easily detected. This can be checked experimentally, and the results agree with Einstein's predictions.

When I think of matter, I like to think mostly of fields. We are fields rather than particles.

Freeman Dyson, Physicist

Nineteenth-century scientists believed matter was indestructible, in other words, that rest-mass was conserved in every physical process. This is certainly plausible. Since the days of the early Greek materialists (Chapter 2), most scientists have felt that matter is indestructible—that although its form might change, its total amount cannot change. Nineteenth-century chemists performing high-precision mass measurements concluded that rest-mass is conserved even in highly energetic chemical reactions. But Einstein's relativity contradicts the conservation of matter. Matter—that is, *rest*-mass—is *not* conserved in chemical reactions, in stretching a rubber band, and so forth. But these changes in rest-mass are so small that they are experimentally undetectable. In high-energy processes such as nuclear fission, however, the changes are easily detected, and the results show clearly that matter is not conserved.

Now take this reasoning one step further: Einstein believed that this result extended not just to *changes* in mass but to *all* of the mass of any system. In other words,

$$\text{total mass of any system} = \frac{\text{total energy of that system}}{c^2}$$

or, in symbols,

$$m = \frac{E}{c^2}$$

This implies Einstein's famous formula,

$$\text{total energy of any system} = (\text{system's total mass}) \times (c^2)$$
$$E = mc^2$$

So *all energy has mass, and all mass has energy*. Since energy means the capacity to do work, and mass means inertia, the practical meaning of $E = mc^2$ is that any system of mass m should be able to do mc^2 units of work, and any system of energy E has an inertia E/c^2.

How do we know that $E = mc^2$? If Einstein is right, there should be some physical process by which mc^2 units of work can be obtained from any object of mass m. Such processes, known as **matter–antimatter annihilation**, have been discovered. Here's how they work.

In addition to the protons, neutrons, and electrons that form ordinary matter, physicists have discovered three other material particles, known as "antiprotons," "antineutrons," and

If matter turns out in the end to be altogether ephemeral, what difference can that make in the pain you feel when you kick a rock?

John A. Wheeler, Physicist

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[10] Each uranium nucleus splits to form two nuclei of various lighter-weight elements.

The visible world is neither matter nor spirit but the invisible organization of energy.
Heinz Pagels, Physicist

> "antielectrons." If one of these "antiparticles" is brought close to its corresponding particle, the two particles vanish entirely, and high-energy radiation is created. It's an extreme example of the nonconservation of matter: Matter entirely vanishes, to be replaced by radiation. So any material object can be turned into radiation by annihilating all its protons, neutrons, and electrons—although it would be difficult to collect enough antiparticles to annihilate a macroscopic object. The energy of this radiation can then be used to do work. Furthermore, when the radiation's energy is measured, it is found to equal the total mass of the particles times c^2.

Science has found no "things," only events. The universe has no nouns, only verbs.
R. Buckminster Fuller, Architect and Futurist

There are no things, only processes.
David Bohm, Physicist

$E = mc^2$ is simple but subtle, and easy to misinterpret. Most of the confusion arises from confusion between *mass* (inertia) and *rest-mass* (matter). Following are two common misconceptions about $E = mc^2$.

It is sometimes said, incorrectly, that Einstein's relation means that "mass is not always conserved." It is true that *matter* (rest-mass) is not always conserved. But *mass* (inertia) is always conserved, because mass equals energy divided by c^2, and energy is always conserved.

It is sometimes said, incorrectly, that Einstein's relation means that "mass can be converted to energy." It's true that *rest*-mass—*matter*—can be converted to nonmaterial forms of energy such as radiation. But you just saw that mass is always conserved, so mass can never be converted to anything else! In proton-antiproton annihilation, for example, the mass of the pair is precisely equal to the mass of the created radiation. But *rest*-mass, or *matter*, is destroyed, and is converted to radiation. One must be careful with the word *mass*.

To summarize:

The Principle of Mass–Energy Equivalence

Energy has mass; that is, energy has inertia. And mass has energy; that is, mass has the ability to do work. The quantitative relation between the energy of any system and the mass of that system is $E = mc^2$.

Mass–energy equivalence represents another sharp break with the Newtonian worldview, which follows the Greek materialists in believing that interactions between indestructible atoms moving in empty space determine everything that happens in the physical universe. Let's think about the mass–energy relationship at the atomic level. Since all energy has mass, some of an atom's mass must be due simply to the kinetic energy of its parts (electrons, protons, and neutrons) and to the energy of its various electromagnetic and nuclear force fields. This suggests an intriguing question: *Is that all there is?* Are atoms made *only* of fields and motion? If so, atoms are not only mostly empty space, they are *entirely* empty space, made only of fields similar to the magnetic fields in Figure 10.13, and the motion of those fields!

High-energy physics (Chapter 17) has already provided part of the answer. It is now known that protons and neutrons are made of three smaller particles called "quarks." Because quarks exert enormous forces on each other, the energy in their force fields is enormous. In fact, calculations show that the energy of these fields is sufficient to explain 90% of the mass of the proton (or neutron)! Since essentially all of the mass of ordinary matter comes from protons and neutrons, this result implies that some 90% of the mass of ordinary matter comes from the nonmaterial energy of fields and motion!

The remaining 10% might arise in a similar way, although this is not yet confirmed. Our most accurate theory of physics (the "standard model," Chapter 17) suggests the existence throughout the universe of a field called the "Higgs field." If verified, the Higgs field will explain the still-unexplained 10%.

The fundamental theories of contemporary physics known as "quantum field theories" (Chapter 17) also suggest that all mass arises solely from nonmaterial fields. For example, Steven Weinberg, a leading high-energy theorist, states the following:

> [According to the physical theories developed during the 1920s] there was supposed to be one field for each type of elementary particle. The inhabitants of the universe were conceived to be a set of fields—an electron field, a proton field, an electromagnetic field—and particles were reduced to mere epiphenomena. In its essentials, this point of view has survived to the present day, and forms the central dogma of quantum field theory: *the essential reality is a set of fields* [Weinberg's emphasis] subject to the rules of special relativity and quantum mechanics; all else is derived as a consequence of the quantum dynamics of these fields.

In this **field view of reality,** there is no "there" there (to quote the poet Gertrude Stein), no "things" at all. Electrons and other material particles are only non-material fields in space, similar to the magnetic field in the space between the poles of a magnet. All mass is due only to the energy of fields. Since fields are "possible forces" (Chapter 8), and forces are interactions, this view implies that every "thing," *everything*, is interactions and motion. It's the interactions and motion themselves that are fundamental rather than the material particles that we had always supposed were doing the interacting and the moving. It's a view that stands Newtonian materialism on its head.

We are such stuff
As dreams are made on
Shakespeare, The Tempest

▶ CONCEPT CHECK 11 In which of the following processes does the system's mass change? (a) A bullet that speeds up while moving down a gun barrel. (b) A rubber band that is being stretched around a package. (c) Two positively charged objects that are moved closer to each other and placed at rest. (d) An electron and an antielectron, at rest, that spontaneously annihilate each other.

▶ CONCEPT CHECK 12 In the preceding question, in which processes does the system's *rest*-mass change?

Answers to Concept Checks and odd-numbered Conceptual Exercises and Problems can be found in the back of the book.

Review Questions

GALILEAN RELATIVITY

1. What is meant by relative motion, reference frame, a theory of relativity?
2. A train moves at 70 m/s. A ball is thrown toward the front of the train at 20 m/s relative to the train. How fast does the ball move relative to the tracks? What if the ball had instead been thrown toward the rear of the train?
3. A spaceship moves at $0.25c$ relative to Earth. A light beam passes the spaceship, in the forward direction, at speed c relative to Earth. According to Galilean relativity, how fast does the light beam move relative to the spaceship? Is this answer experimentally correct? If not, then what answer is correct?

THE PRINCIPLES OF RELATIVITY AND CONSTANCY OF LIGHTSPEED

4. How does travel in a jet airplane illustrate the principle of relativity? How must the airplane be moving in order to illustrate this principle?
5. State the principle of relativity in your own words. Does it apply to every observer? Explain.
6. State the principle of the constancy of lightspeed in your own words. Does it apply to every observer? Explain.
7. Use the principle of the constancy of lightspeed to explain why no observer can move at precisely speed c relative to any other observer.
8. What is the ether theory, and why did physicists ultimately reject it?
9. In Galilean relativity, space and time are absolute and lightspeed is relative. What is the situation in Einstein's relativity?
10. What distinguishes the special from the general theory of relativity?
11. List the basic "laws" of the special theory of relativity.

THE RELATIVITY OF TIME

12. How is time defined in physics?
13. Describe the light clock.
14. Velma passes Mort at a high speed. Both observers have clocks. What does each observer say about Velma's clock? What do they each say about Mort's clock?
15. One twin goes on a fast trip and returns. Does the special theory of relativity apply to the observations of both twins? Why, or why not?
16. One twin goes on a fast trip and returns. Have the two twins aged differently during the trip? If so, how do their ages differ?
17. Explain how you can travel to the future.

THE RELATIVITY OF SPACE AND MASS

18. What do we mean by "space" or "distance"?
19. What does "space is relative" mean?
20. Velma passes Mort at a high speed. Each of them holds a meter stick parallel to the direction of motion. What does each observer say about Velma's meter stick? What does each say about Mort's meter stick?
21. According to Einstein's theory, which of these are relative: time, lightspeed, rest-mass, length, mass?
22. Velma passes Mort at a high speed. Both observers carry a standard kilogram. What does Mort say about the mass of each of the standard kilograms? What does Velma say?
23. Mort exerts a 1 newton force on his standard kilogram. What acceleration does this give to the kilogram? What will he find if he exerts the same force on Velma's standard kilogram while Velma is passing him at a high speed?
24. What is the distinction, if any, between rest-mass, mass, and matter? Which ones increase with speed?
25. What is the distinction between matter and radiation?
26. Why can't material objects be sped up to lightspeed? Does anything move at lightspeed?

$E = mc^2$

27. What does $E = mc^2$ mean? Does it mean that mass can be converted to energy? Explain.
28. Is matter always conserved? Is mass always conserved? Is rest-mass always conserved? Is energy always conserved?
29. According to Einstein's relativity, is rest-mass precisely conserved in chemical reactions?
30. Describe an experiment in which a system's entire rest-mass vanishes. Is matter conserved here? Mass? Energy?

Conceptual Exercises

GALILEAN RELATIVITY

1. Two bicyclers, on different streets in the same city, are both moving directly north at 15 km/hr. Are they in relative motion?
2. According to the Galilean theory of relativity, does every observer measure the same speed for a light beam?
3. Velma moves toward Mort at half of lightspeed. Mort shines a searchlight toward Velma. What does Galilean relativity predict about the speed of the searchlight beam as observed by Velma?
4. Velma bicycles northward at 4 m/s. Mort, standing by the side of the road, throws a ball northward at 10 m/s. What is the ball's speed and direction of motion, relative to Velma? What if Mort had instead thrown the ball southward at 10 m/s?
5. A desperado riding on top of a train car fires a gun toward the front of the train. The gun's muzzle speed (speed of the bullet relative to the gun) is 500 m/s, and the train's speed is 40 m/s. What is the bullet's speed and direction of motion as observed by the sheriff standing beside the tracks? What does a passenger on the train say about the bullet's speed? What if the desperado had instead pointed his gun toward the rear of the train?

6. Velma is in a train moving eastward at 70 m/s. Mort, standing beside the tracks, throws a ball at 20 m/s eastward. What is the ball's speed and direction relative to Velma?
7. Velma is in a train moving eastward at 70 m/s. Mort, standing beside the tracks, throws a ball at 20 m/s westward. What is the ball's speed and direction relative to Velma?

THE PRINCIPLE OF RELATIVITY

8. Does the principle of relativity require that *every* observer observe the same laws of physics? Explain.
9. If you were riding on a train moving at constant speed along a straight track and you dropped a ball directly over a white dot on the floor, where would the ball land relative to the dot?
10. Suppose that you drop a ball while riding on a train moving at constant speed along a straight track. If you measure the ball's acceleration, will your result be greater than, less than, or equal to, the usual acceleration due to gravity?
11. Think of several ways that you could determine from inside an airplane whether the plane was flying smoothly or parked on the runway. Do each of these ways involve some direct or indirect contact with the world outside the airplane?
12. How fast are you moving right now? What meaning does this question have?
13. If you drop a coin inside a car that is turning a corner to the right, where will the coin land?
14. If you drop a coin inside a car that is slowing down, where will the coin land?

THE CONSTANCY OF LIGHTSPEED

15. Does every observer measure the same speed for a light beam? Explain.
16. A star headed toward Earth at 20% of lightspeed suddenly explodes as a bright supernova. With what speed does the light from the explosion leave the star? With what speed (as measured on Earth) does it approach Earth?
17. Is it physically possible for a person to move past Earth at exactly lightspeed? Explain.
18. Velma's spaceship approaches Earth at $0.75c$. She turns on a laser and beams it toward Earth. How fast does she see the beam move away from her? How fast does an Earth-based observer see the beam approach Earth?
19. A desperado riding on top of a freight-train car fires a laser gun pointed forward. What is this gun's "muzzle velocity"? Suppose the train is moving at 40 m/s (0.04 km/s). How fast does the tip of the laser beam move relative to the sheriff, who is standing on the ground beside the train? What answer would the Galilean theory of relativity have given to this question?

THE RELATIVITY OF TIME

20. Velma passes you at a high speed. According to you, she ages slowly. How does she age according to her own observations? How do you age according to her?
21. Suppose you have a twin brother. What could be done to make him older than you?
22. The center of our galaxy is about 26,000 light-years away. Could a person possibly travel there in less than 26,000 years as measured on Earth? Could a person possibly travel there in less than 26,000 years of his or her own time? Explain.
23. A woman conceives a child while on a fast-moving space colony moving toward a distant planetary system. How long should it take before the baby is born, as measured by the woman? Would an Earth observer measure the same amount of time?
24. A certain fast-moving particle is observed to have a lifetime of 2 seconds. If the same particle was at rest in the laboratory, would its lifetime still be 2 seconds, or would it be more, or less, than 2 seconds?
25. Does the special theory of relativity allow you to go on a trip and return older than your father?
26. Does the special theory of relativity allow your father to go on a trip and return younger than you?
27. Does the special theory of relativity allow you to go on a trip and return younger than you were when you left?
28. When you go on a very fast trip, must you always return older than you were when you left?
29. A satellite orbits Earth at 8 km/s. Find its speed as a fraction of lightspeed. Would an orbiting astronaut directly notice the effects of time dilation without using sophisticated measurement techniques?
30. Velma passes Earth at 50% of lightspeed. On her video player, she watches a taped video program that runs 1 hour. How long does the program run as measured by an Earth-based observer?
31. Your fantastic rocketship moves at 30,000 km/s. If you took off, moved at this speed for 24 hours as measured by you, and returned to Earth, by how much time would your clock differ from Earth-based clocks? Would you have aged more than, or less than, people on Earth? By how much?
32. Answer the preceding question assuming that your extraordinarily fantastic rocketship moves at 99% of lightspeed.
33. Mort and Velma have identical 10-minute ice-cream cones. Velma passes Mort at 75% of lightspeed. How long does Mort's cone take to melt as measured by Velma?
34. How fast must Velma move in order for her 10-minute ice-cream cone to melt in 30 minutes as measured by Mort?

THE RELATIVITY OF SPACE AND MASS

35. How fast must Velma move past Mort if Mort is to observe her spaceship's length to be reduced by 50%? If Velma is flying east to west across the United States (about 5000 km wide) at this speed, how wide will she observe the United States to be?
36. Mort's swimming pool is 20 m long and 10 m wide. If Velma flies lengthwise over the pool at 60% of lightspeed, how long and how wide will she observe it to be?
37. Mort's automobile is 4 m long as measured by Mort. What length does Velma measure for Mort's auto, as she passes him at 90% of lightspeed?
38. Velma, who is carrying a clock and a meter stick, passes Mort. Is it possible that Mort could observe length contraction of Velma's meter stick but observe no time dilation of her clock? If so, how?
39. Velma, who is carrying a clock and a meter stick, passes Mort. Is it possible that Mort could observe time dilation of Velma's clock but observe no length contraction of her meter stick? If so, how?

40. Velma drives a really fast rocket train northward past Mort, who is standing beside the tracks. Two posts are driven into the ground along the tracks. How does Mort's measurement of the distance between the posts compare with Velma's: longer, shorter, or the same?
41. If Velma passes Mort at a high speed, Mort will find her mass to be larger than normal. Will he also find her to be larger in size?
42. Velma's spaceship has a rest-mass of 10,000 kg, and she measures its length to be 100 m. She moves past Mort at $0.8c$. According to Mort's measurements, what are the mass and the length of her spaceship?
43. How fast must Velma move past Mort if Mort is to observe her spaceship's mass to be increased by 50%? How fast must she move if Mort is to observe her spaceship's length to be reduced by 50%?
44. A meter stick with a rest-mass of 1 kg moves past you. Your measurements show it to have a mass of 2 kg and a length of 1 m. What is the orientation of the stick, and how fast is it moving?
45. Use Figure 10.12 to estimate how fast Velma must move, relative to Mort, for Mort to observe that her body's mass is 50% larger than normal.

$E = mc^2$

46. When you throw a stone, does its mass increase, decrease, or neither? Can this effect be detected?
47. A red-hot chunk of coal is placed in a large air-filled container where it completely burns up. The container is a perfect thermal insulator—in other words, thermal energy is unable to pass through the container's walls. According to $E = mc^2$, does the total mass of the container and its contents change during the burning process? If so, does the mass increase, or decrease?
48. Referring to the previous question: Suppose that the container is not a thermal insulator—in other words, thermal energy passes through the walls. In this case, does the total mass of the container and its contents change during the burning process? If so, does the mass increase, or decrease?
49. An electron and an antielectron annihilate each other. In this process, is energy conserved? Is mass conserved? Is rest-mass conserved?
50. Two mousetraps are identical except that one of them is set to spring shut when the trigger is released, and the other is not set. They are placed in identical vats of acid. After they are completely dissolved, what, if any, are the differences between the two vats? Will the masses differ?
51. In a physics laboratory, an electron is accelerated to nearly lightspeed. If you were riding on the electron, would you notice that the electron's mass had increased? If you were standing in the laboratory, what would you notice concerning the electron's mass and energy?

Problems

Use the time-dilation formula $T = T_0/\sqrt{(1 - v^2/c^2)}$ (explained in footnote 4) to answer questions 1–6.

1. Time dilation depends on the quantity $\sqrt{(1 - v^2/c^2)}$, which in turn depends on the fraction v^2/c^2. Evaluate the fraction v^2/c^2 for each of the following speeds: 3 km/s (high-powered rifle bullet), 30 km/s (speed of Earth in its orbit around the sun), 3000 km/s (fast enough to cross the United States in about 1 second). Is time dilation a very significant, noticeable effect at these speeds?
2. Time dilation depends on the factor $\sqrt{(1 - v^2/c^2)}$. Evaluate this factor for each of the following speeds: 30,000 km/s (fast enough to circle the globe in 1 second), 150,000 km/s.
3. Velma passes Mort at 30,000 km/s. What fraction of lightspeed is this? What is the duration of one of Velma's seconds (a time interval that Velma observes to be 1 second in duration) as observed by Mort?
4. Velma passes Mort at 150,000 km/s. What fraction of lightspeed is this? What is the duration of one of Mort's seconds (a time interval that Mort observes to be 1 second in duration) as observed by Velma?
5. Velma passes Mort at a high speed. His clock, as observed by her, runs at half of its normal speed—for example, his clock advances by only 30 minutes during a time of 1 hour as recorded on her own clock. What must be the value of the quantity $\sqrt{(1 - v^2/c^2)}$? Find Velma's speed relative to Mort.
6. Velma passes Mort at a high speed. Her clock, as observed by him, runs at 25% of its normal speed—for example, her clock advances by only 15 minutes during a time of 1 hour as recorded on his own clock. What must be the value of the quantity $\sqrt{(1 - v^2/c^2)}$? Find Velma's speed relative to Mort.
7. You give 90 J of kinetic energy to a 1 kg stone when you throw it. By how much do you increase its mass?
8. A large nuclear power plant generates electric energy at the rate of 1000 MW. How many joules of electricity does the plant generate in one day? What is the mass of this much energy?
9. If you had two shoes, an ordinary shoe and an "antishoe" made of antiparticles, and you annihilated them together, by how far could you lift the U.S. population? Assume that each person weighs 600 N, that each shoe's rest-mass is 0.5 kg, and that all the energy goes into lifting.
10. **MAKING ESTIMATES** Show that, if all the energy released (transformed) in fissioning 1 kg of uranium were used to heat water, about 2 billion kg of water could be heated from freezing up to boiling. (Assume that the uranium's rest-mass is reduced by about 0.1%. Roughly 4 J of thermal energy is needed to raise the temperature of 1 gram of water by 1°C.) How many tonnes of water is this (a tonne is 1000 kg)? How many large highway trucks, each loaded to about 30 tonnes, would be needed to carry this much water?
11. Solar radiation reaches Earth at the rate of 1400 watts for every square meter directly facing the sun. Using the formula πR^2 for the area of a circle of radius R, find the amount of solar energy entering Earth's atmosphere every second. Earth's radius is 6400 km.
12. Use the answer to the preceding question to find how many kilograms of sunlight hit Earth every second.