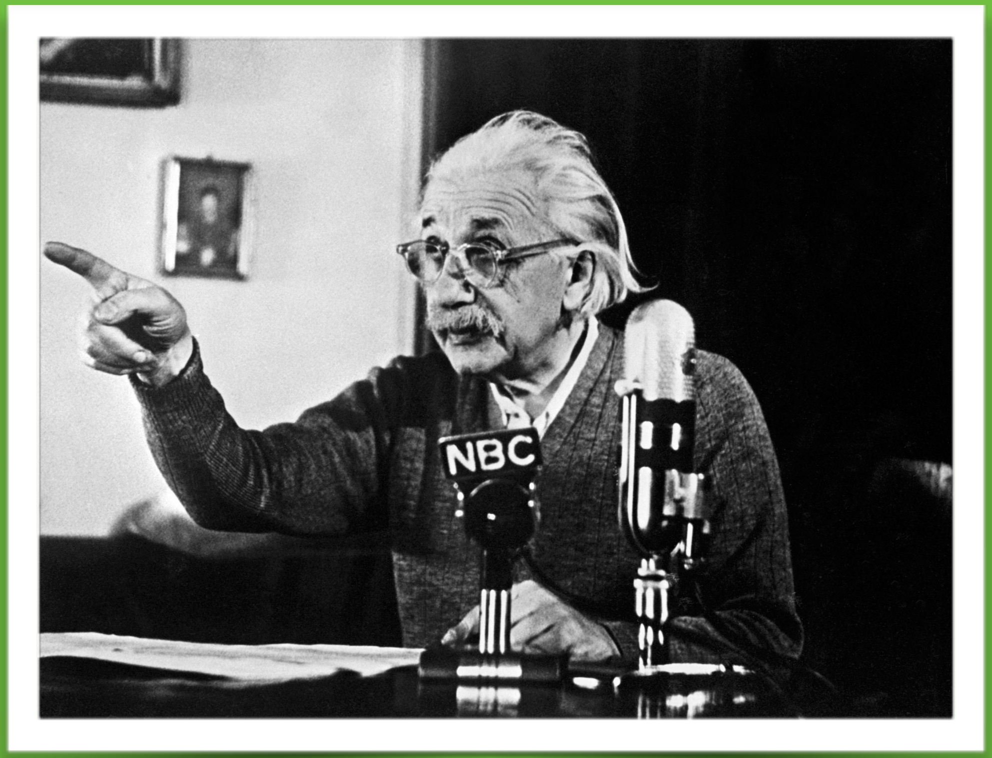
hi

Day 17, 15.03.2018

Einstein’s Theory of Special Relativity, 4



RIP

1942-2018

housekeeping

Gotta come to class

question about anything? I'll make a movie for you:

Special Relativity:

Hobson_Relativity.pdf is chapter 10 out of Hobson

Also, chapter 2 in Oerter is good.

need this and next lecture for HW! So HW7 due Sunday, rather than Friday

MasteringAstronomy registration expiration now set to March 15.

honors project began

https://qstbb.pa.msu.edu/storage/Homework_Projects/honors_project_2018/

contains the first instructions: the plan & tutorial

MinervaInstructions1_2018.pdf

dates:

complete first part, March 16

analyze data and complete writeup, April 20

Postulates of Special Relativity

1. All laws of physics – mechanical and electromagnetic – are identical in co-moving inertial frames.

   taking Galileo seriously, and then adding Maxwell

   called "The Principle of Relativity"

2. The speed of light is the same for all inertial observers.

   taking Maxwell seriously…that "*c*" in M.E.'s is a constant.

because c

the speed of light is constant in all inertial frames:

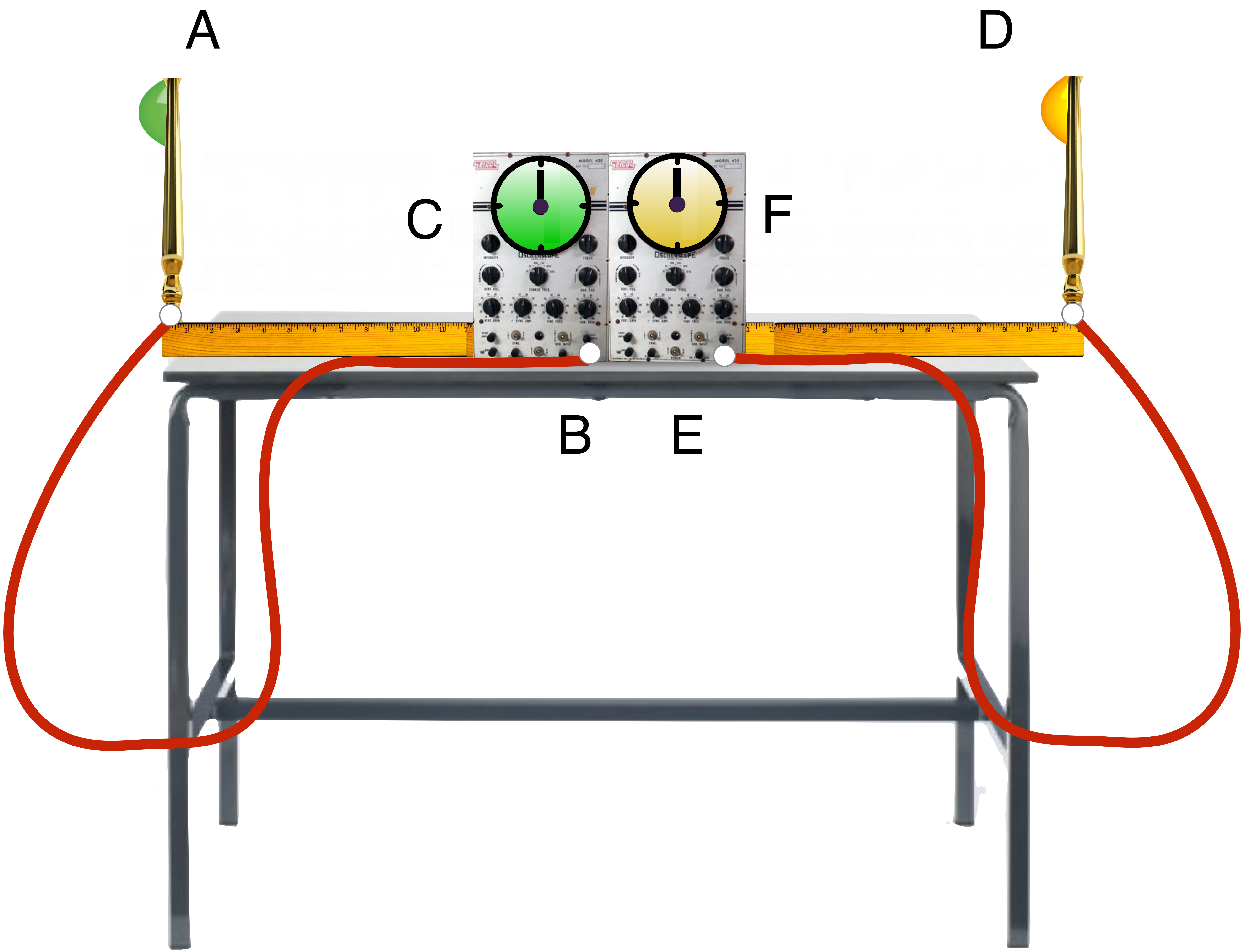
$c = 3 \times 10^8$ m/s = 300 million m/s = 1,080 million km/h

c = 671 million mph

constant of nature:

speed of light

value:	$c = 2.99792458 \times 10^8$ m/s
units:	m/s or ft/s or km/h
usage:	Speed of light in relativity or approximately $c = 3.0 \times 10^8$ m/s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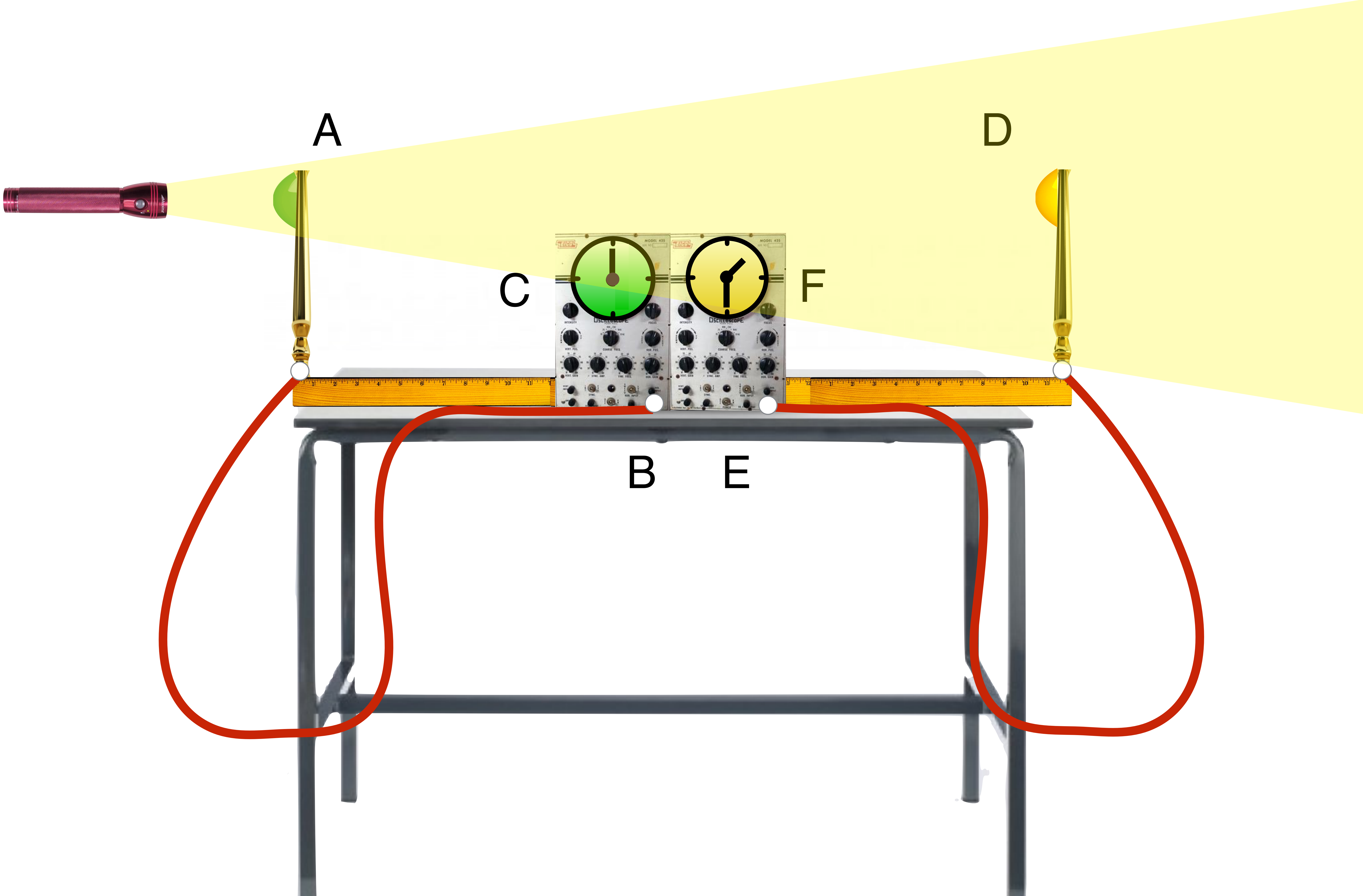
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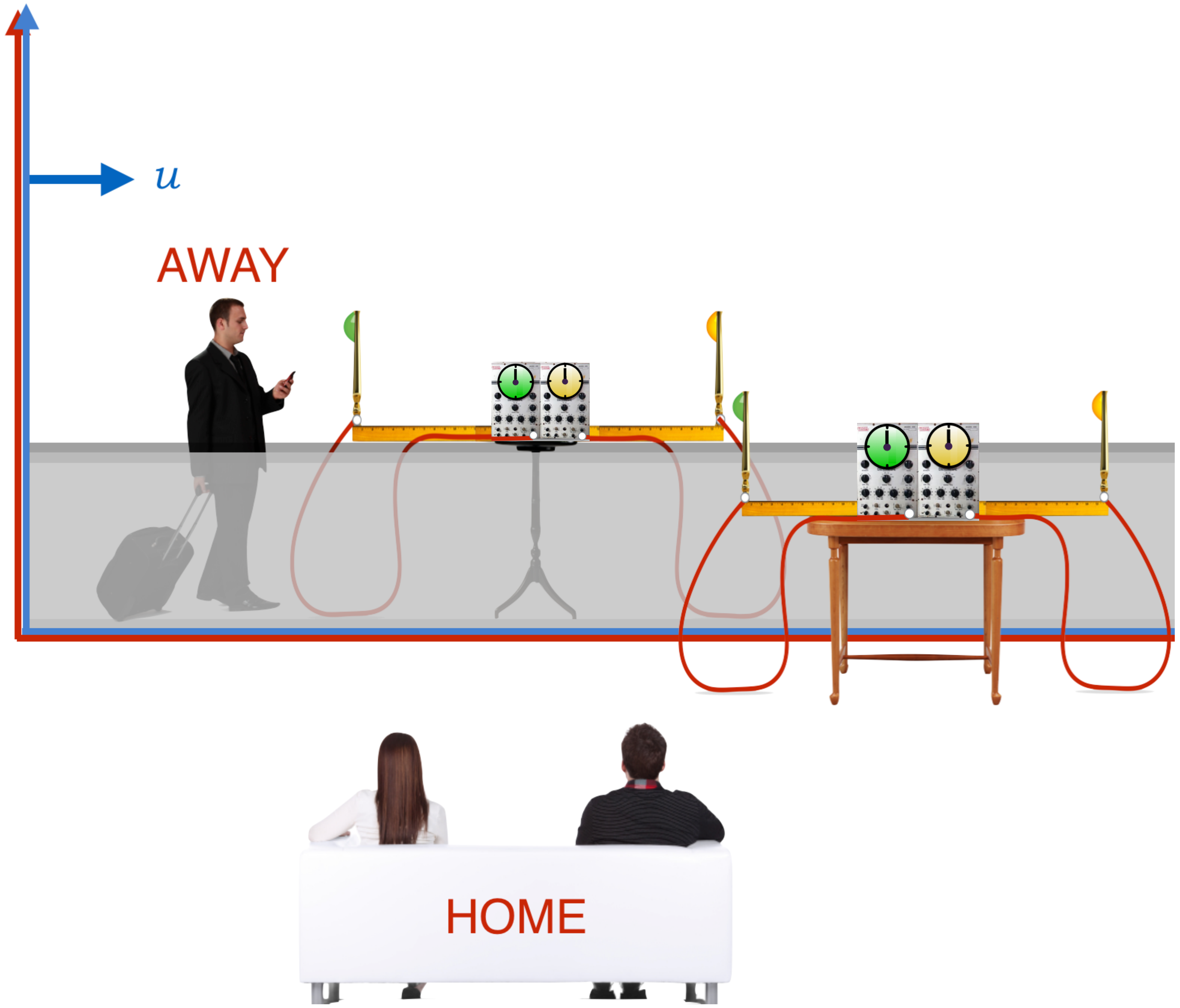
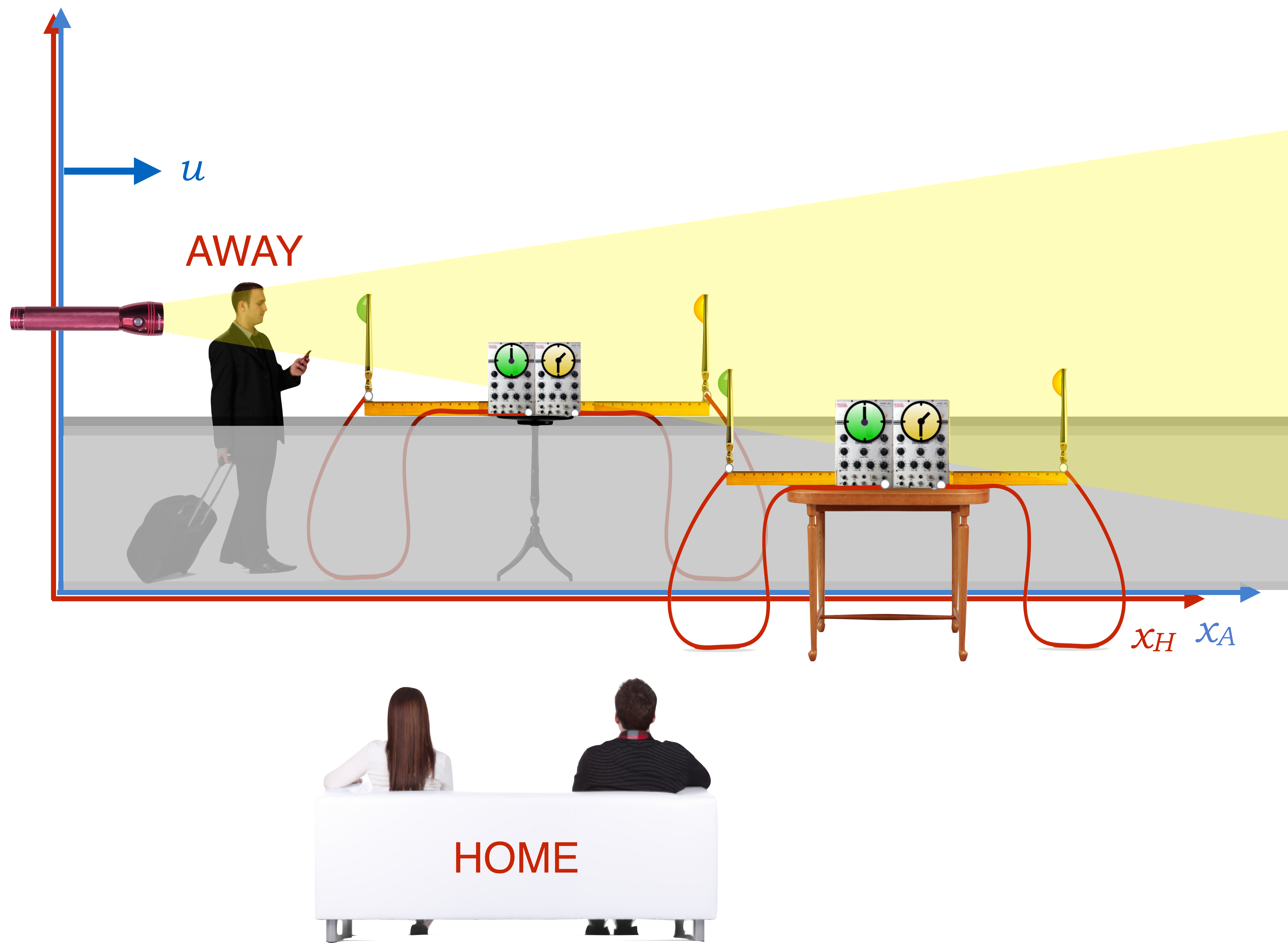
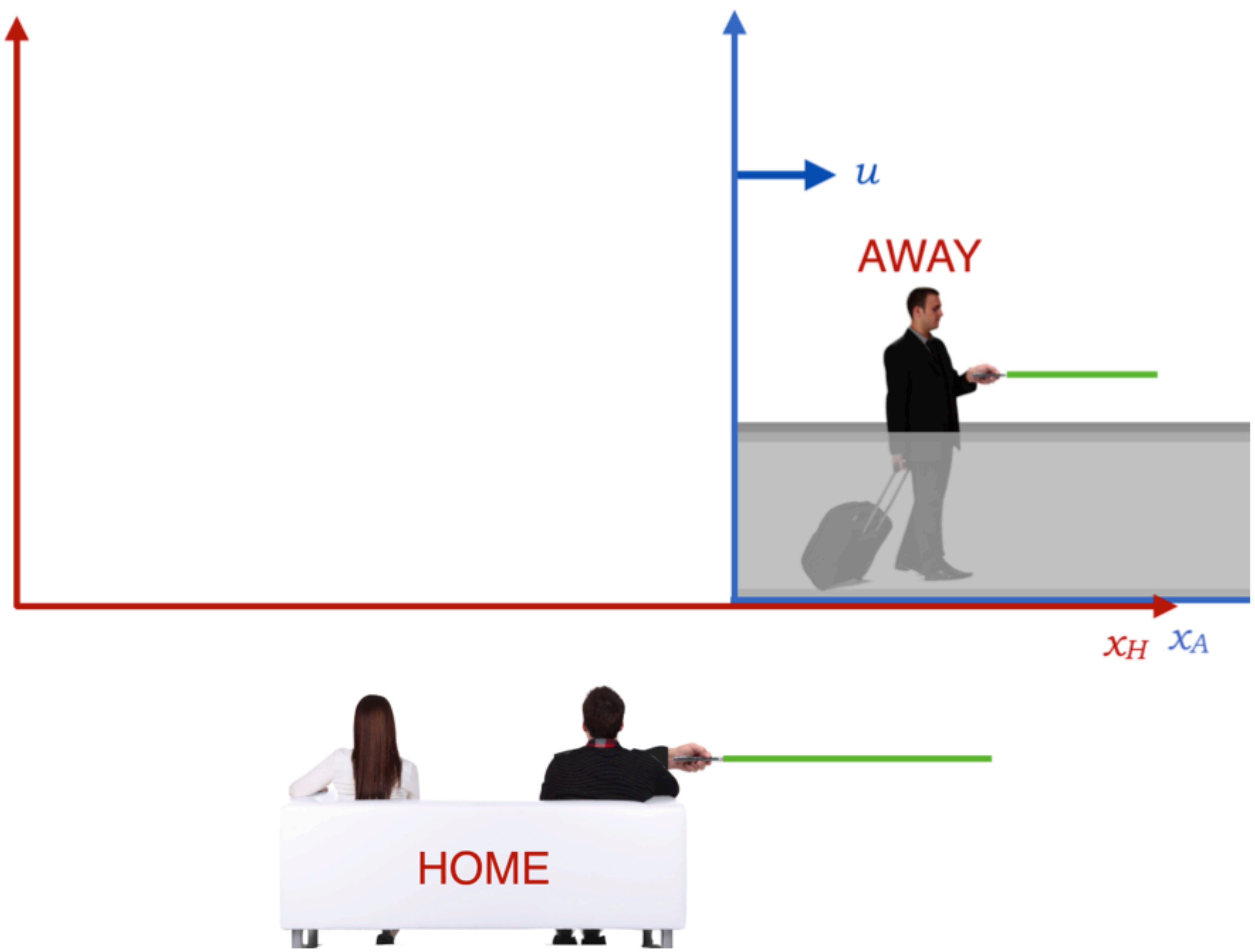


(a) (b)



u
AWAY
x_H
x_A
HOME



laser beams' speed is c relative to airport, sidewalk, downtown, cars on the highway, planes overhead, moon, Alpha Centauri, Milky Way center, ...

collecting these two consequences

of the two simple postulates

"Time Dilation": $$t_H = \gamma t_A$$

Moving clocks appear to run slower as seen by a relatively stationary observer

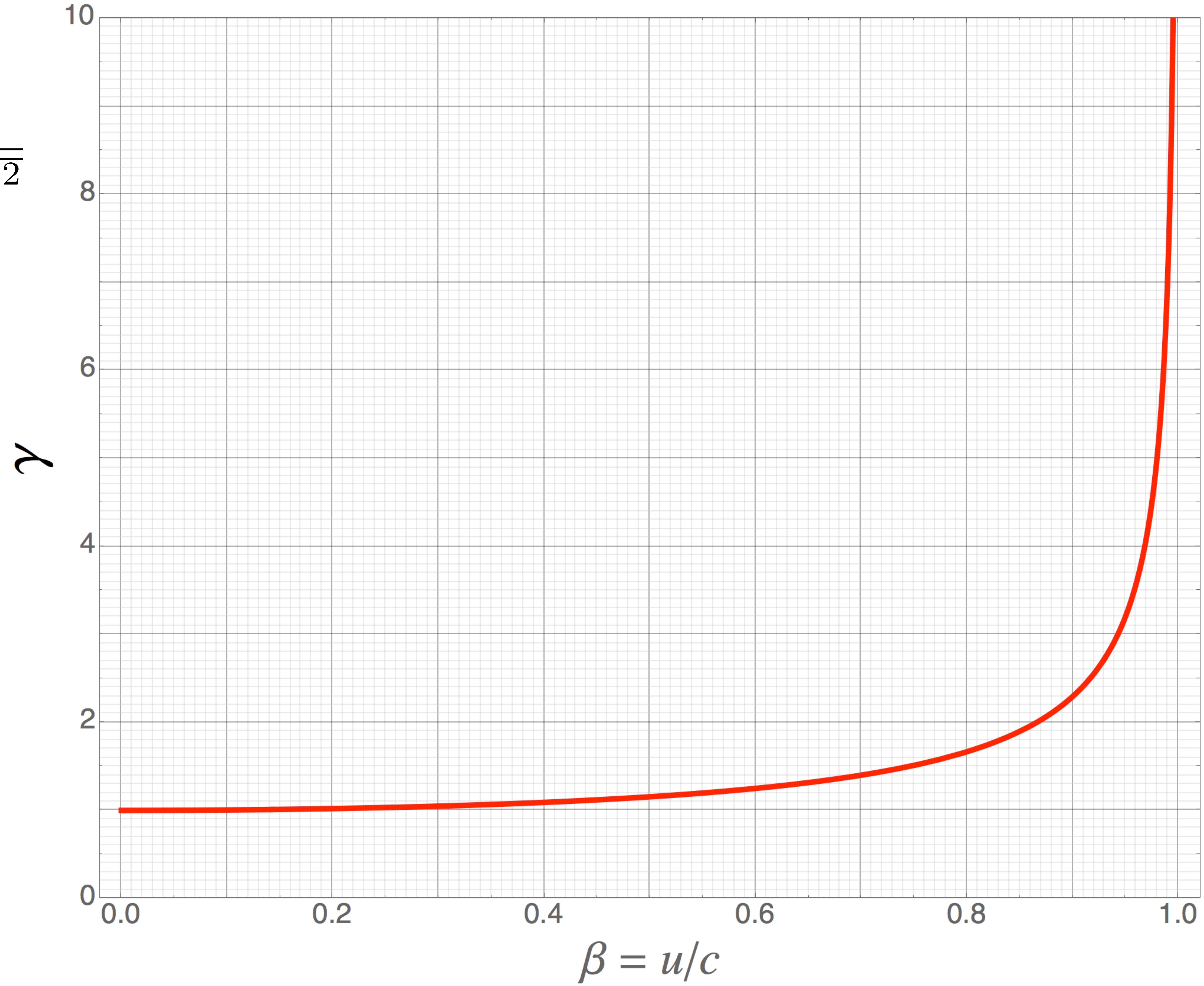
"Length Contraction": $$L_H = \frac{L_A}{\gamma}$$

Moving lengths appear shorter to a relatively stationary observer

“relativistic gamma”

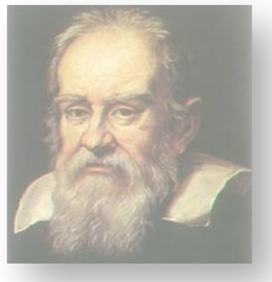
$$\gamma = \frac{1}{\sqrt{1-(\frac{u}{c})^2}}$$

$$\beta = u/c$$



Einstein?

mixes space and time coordinates



$$x_H = x_A + ut$$

$$t_H = t_A = t$$

Principle of Relativity

2 Postulates:

1. **All laws of physics – mechanical and electromagnetic – are identical in co-moving inertial frames.**

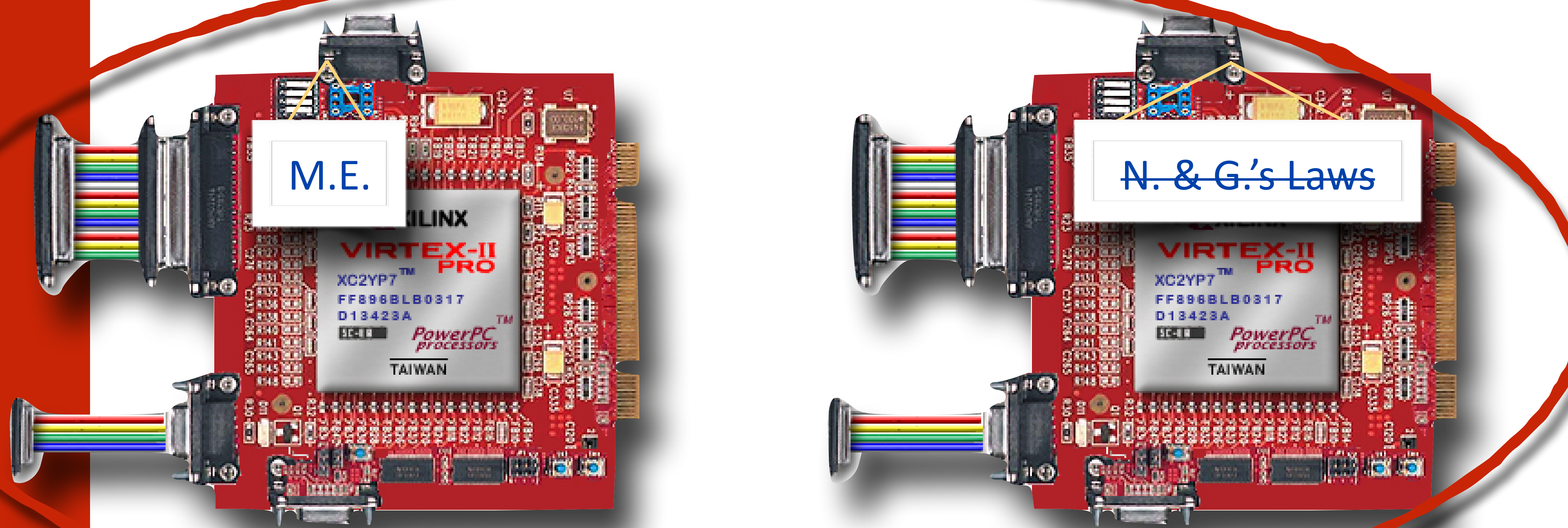
2. **The speed of light is the same for all inertial observers.**

"inertial frame":

constant velocity

good all along!

had to change!



`is Relativity`

the case?

relatives

this is an electron, e: 

this is a cousin of an electron…the “muon,” μ: 

they are exactly alike except that

$m(\mu)$ = 209 x $m(e)$

and in about 1.5 microseconds:

muon’s lives are short and sweet

‘‘muons’’: μ

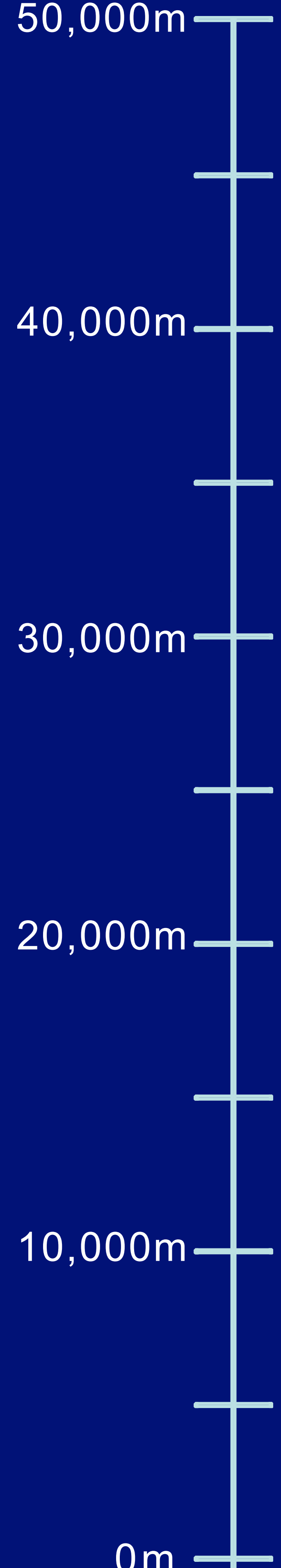
are unstable particles which are easily made in an accelerator lab and shown to have a half life of 1.56 μs…

1.56×10^{-6} seconds

Under Newton’s view, even if the muon goes at the speed of light, then it lives for only $(3 \times 10^{8}) \times (1.5 \times 10^{-6}$ seconds$) = 450$ m

stand-up cosmic

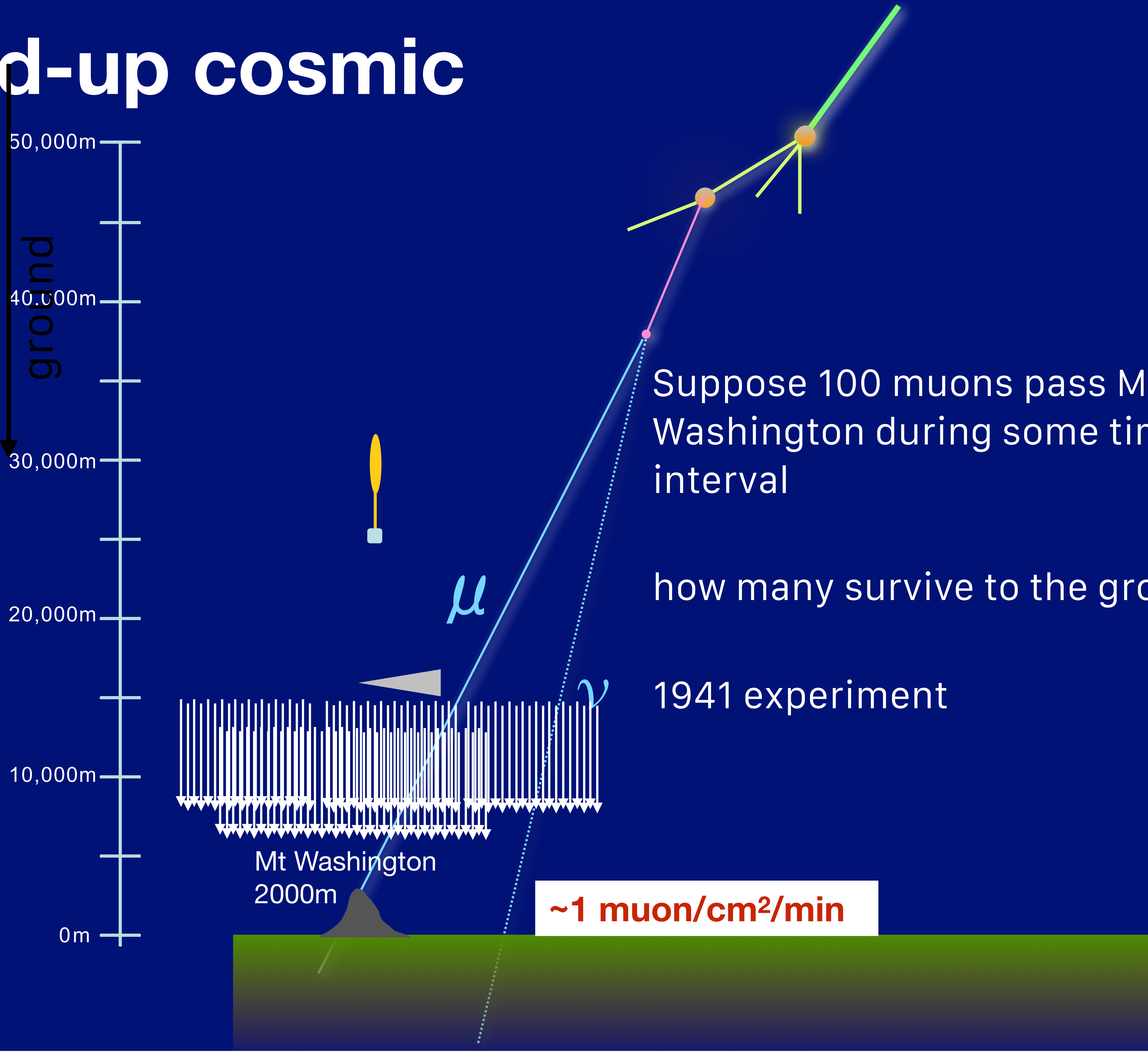
~20 particles/cm/s



Mount Washington Observatory
New Hampshire

D. H. Frisch and J. H. Smith, "Measurement of the Relativistic Time Dilation Using μ-Mesons," American Journal of Physics, 31 (5): 342–355, 1963).

stand-up cosmic



Suppose 100 muons pass Mt Washington during some time interval

how many survive to the ground?

1941 experiment

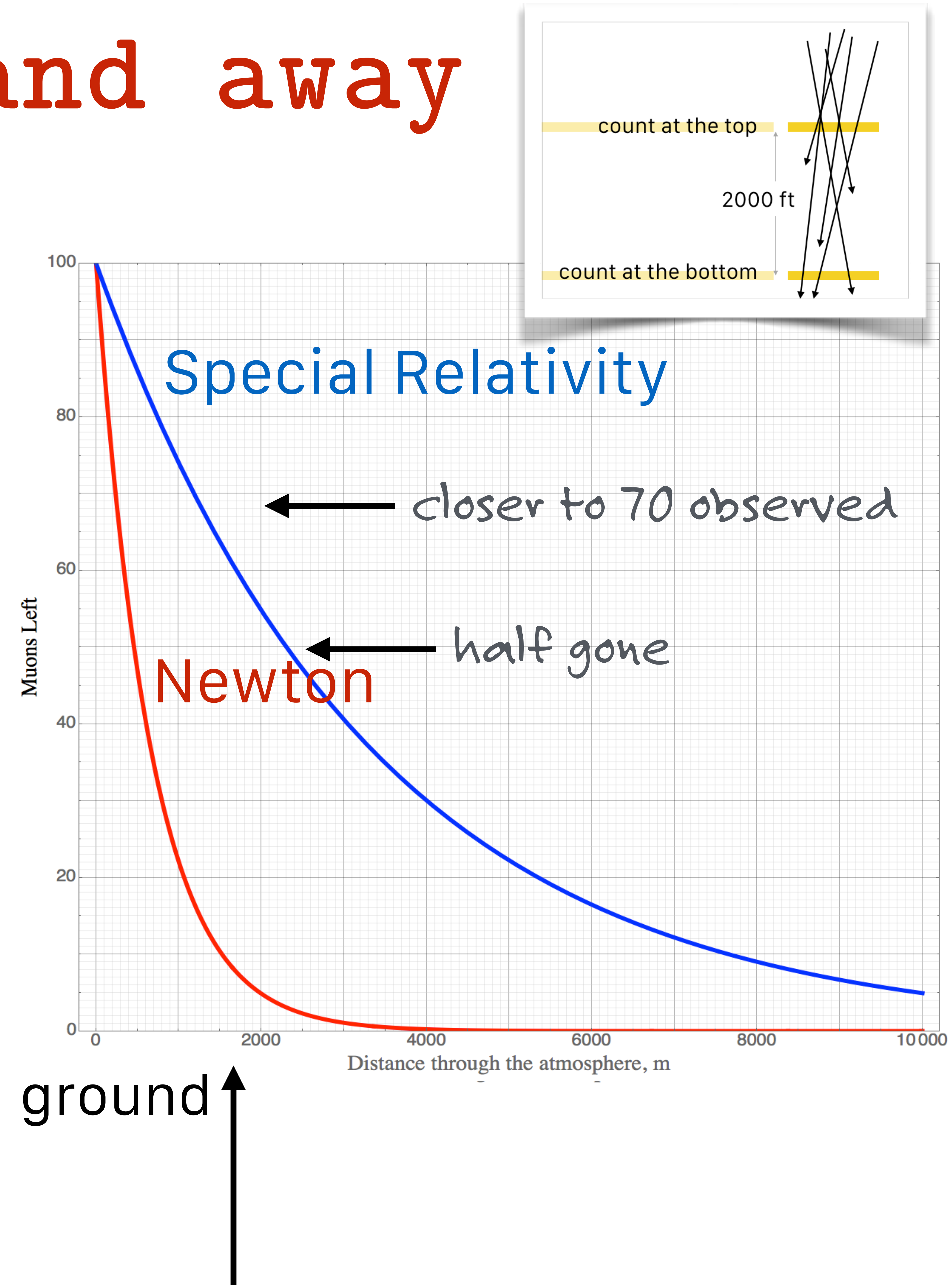
home and away

But relativity says:

for the muon moving with $\beta = 0.99$

as observed by the mountain, its clock appears to slowed to

γ x 1.6 microseconds



how can it decay and not decay?

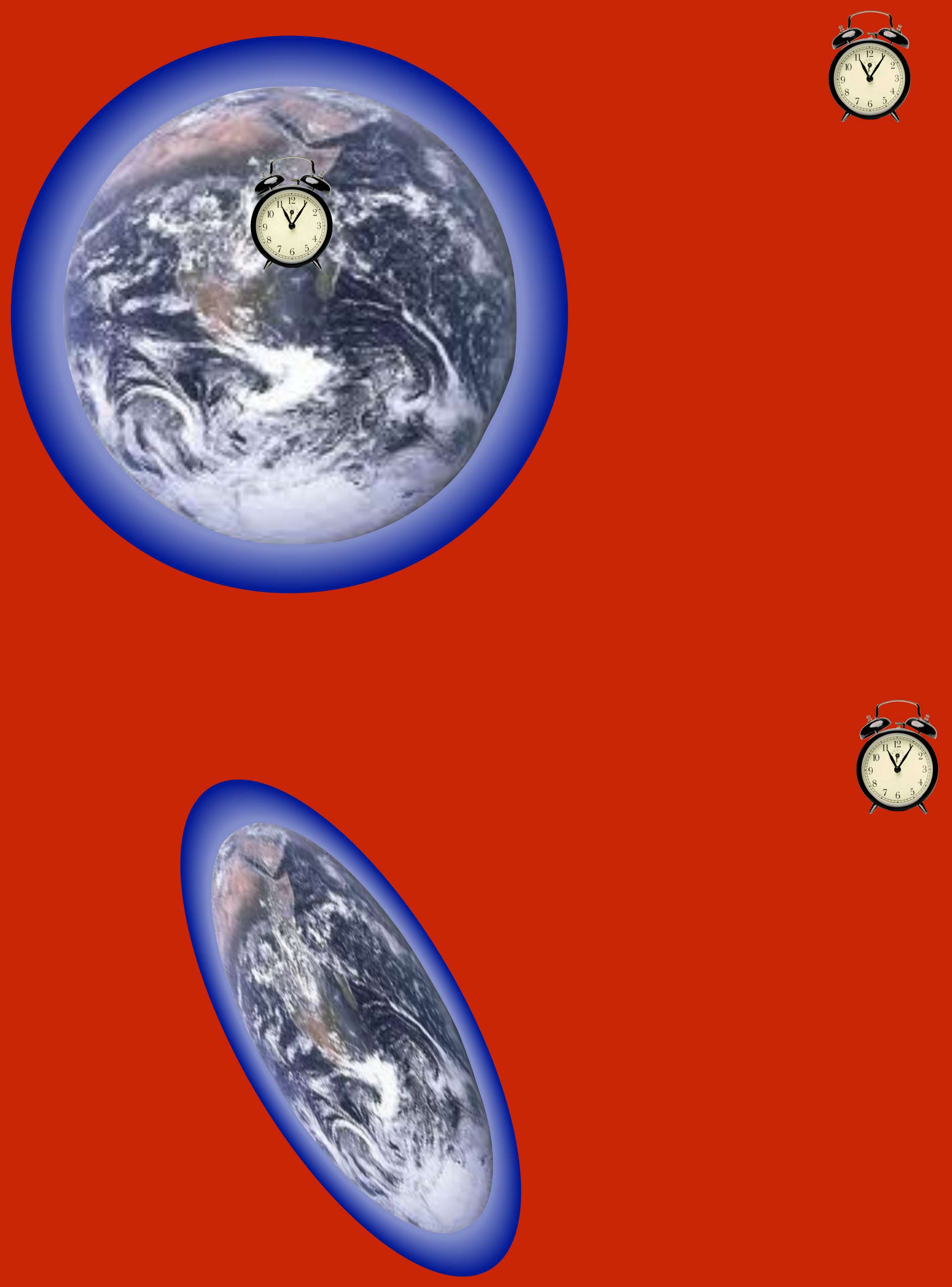
reciprocity

while it decays in 1.5μs in its rest frame...

it sees the atmosphere coming toward it at nearly c

which, to the muon, is Length Contracted

shorter by the same factor that the lifetimes differed



This has been measured many times:

an atomic clock was carefully carried around the world in 1972 and carefully calibrated and compared with ground-based clocks

J. Hafele and R. Keating

There are a number of corrections: accelerations, decelerations, the rotation of the orbit, the fact that the earth is not inertial - but relativity was absolutely correct

Predicted Effect	Flying East	Flying West
GTR (Gravitation)	+ 144 ± 14 ns	+ 179 ± 18 ns
STR (Velocity)	- 184 ± 18 ns	+ 96 ± 18 ns
Total	- 40 ± 23 ns	+ 275 ± 21 ns
measured:	- 59 ± 10 ns	+273 ± 7 ns

redone twice more in airplanes and rockets/satellites

We trust Special Relativity

we no longer do exp … irm or disconfirm it

it’ … ow

`combine speeds!`

Galileo, nope.

Einstein, yup.

the airport, redux

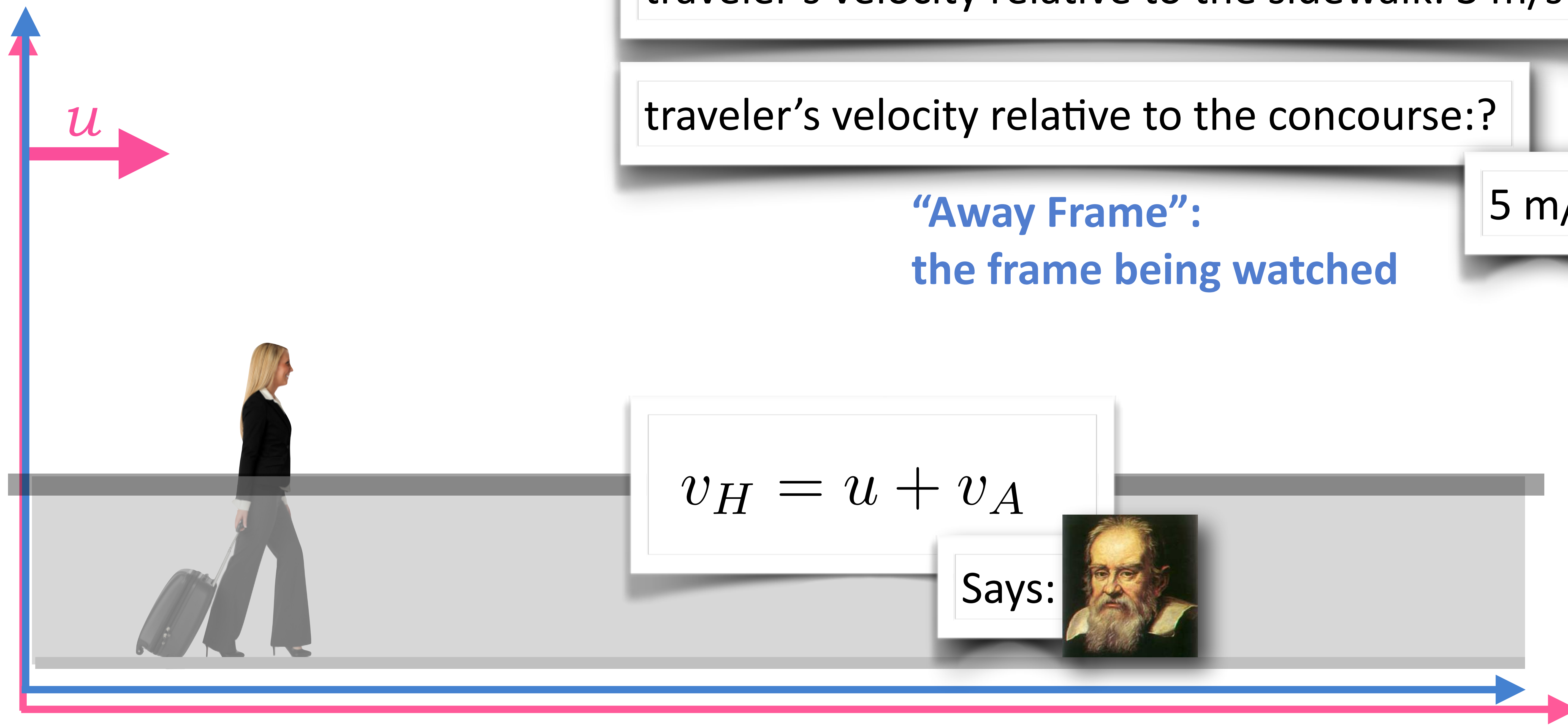
sidewalk velocity: 2 m/s

traveler’s velocity relative to the sidewalk: 3 m/s

traveler’s velocity relative to the concourse:?

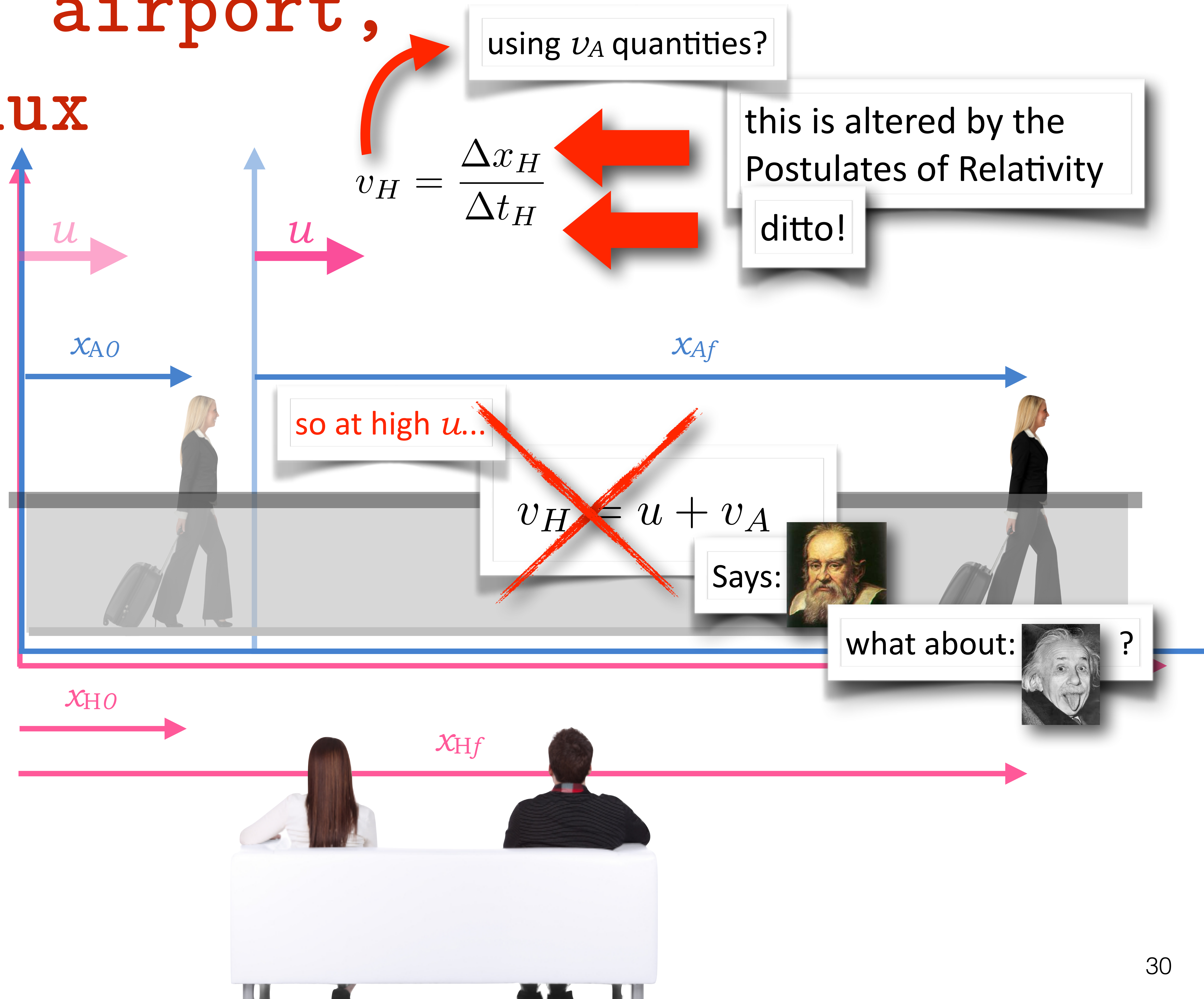
5 m/s

“Away Frame”:
the frame being watched

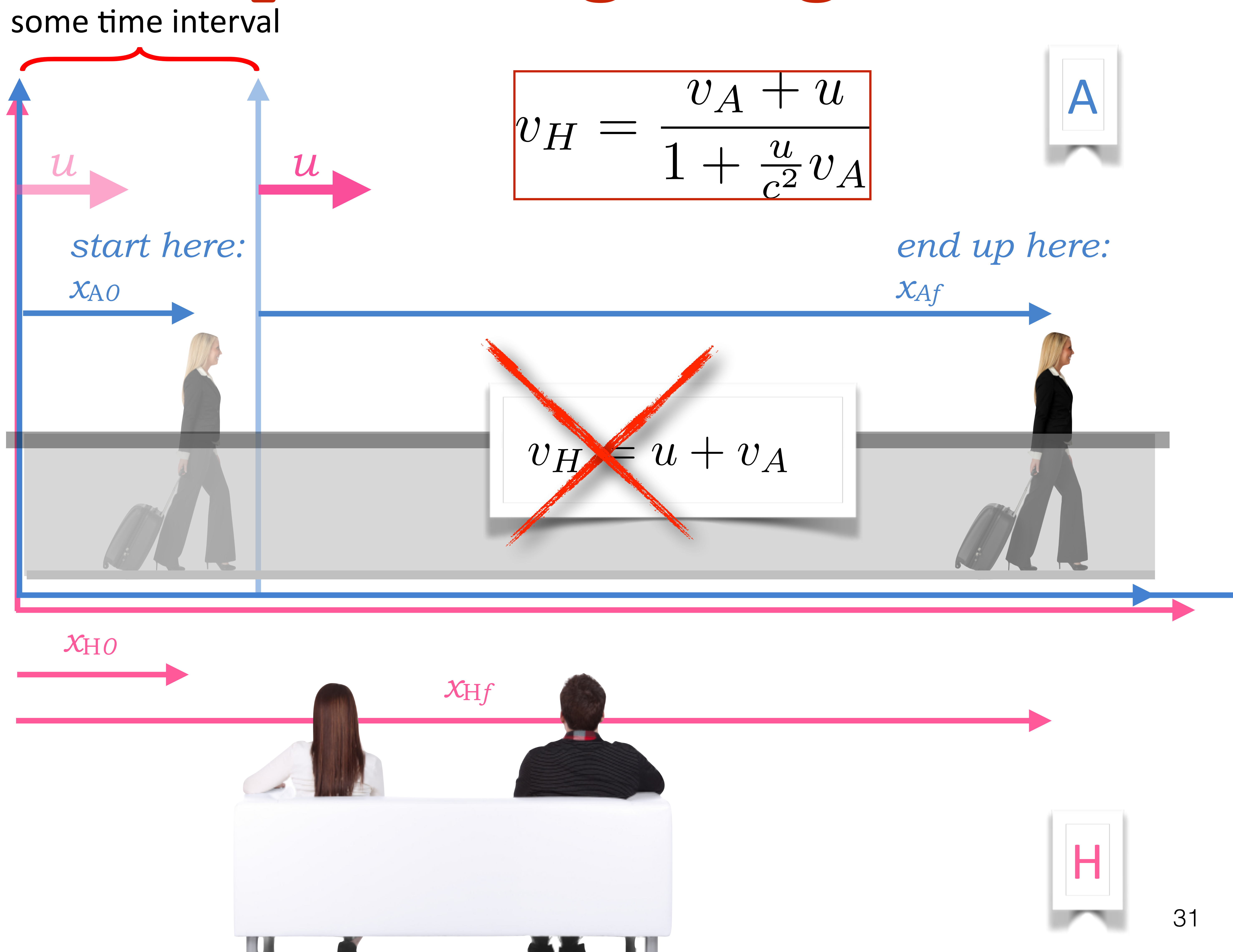


“Home Frame”:
watching a moving frame

the airport, redux



the airport, going fast



write it down.

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

relativistic velocity transformation

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

Look at this formula carefully...

Suppose u/c is very small...like normal life.

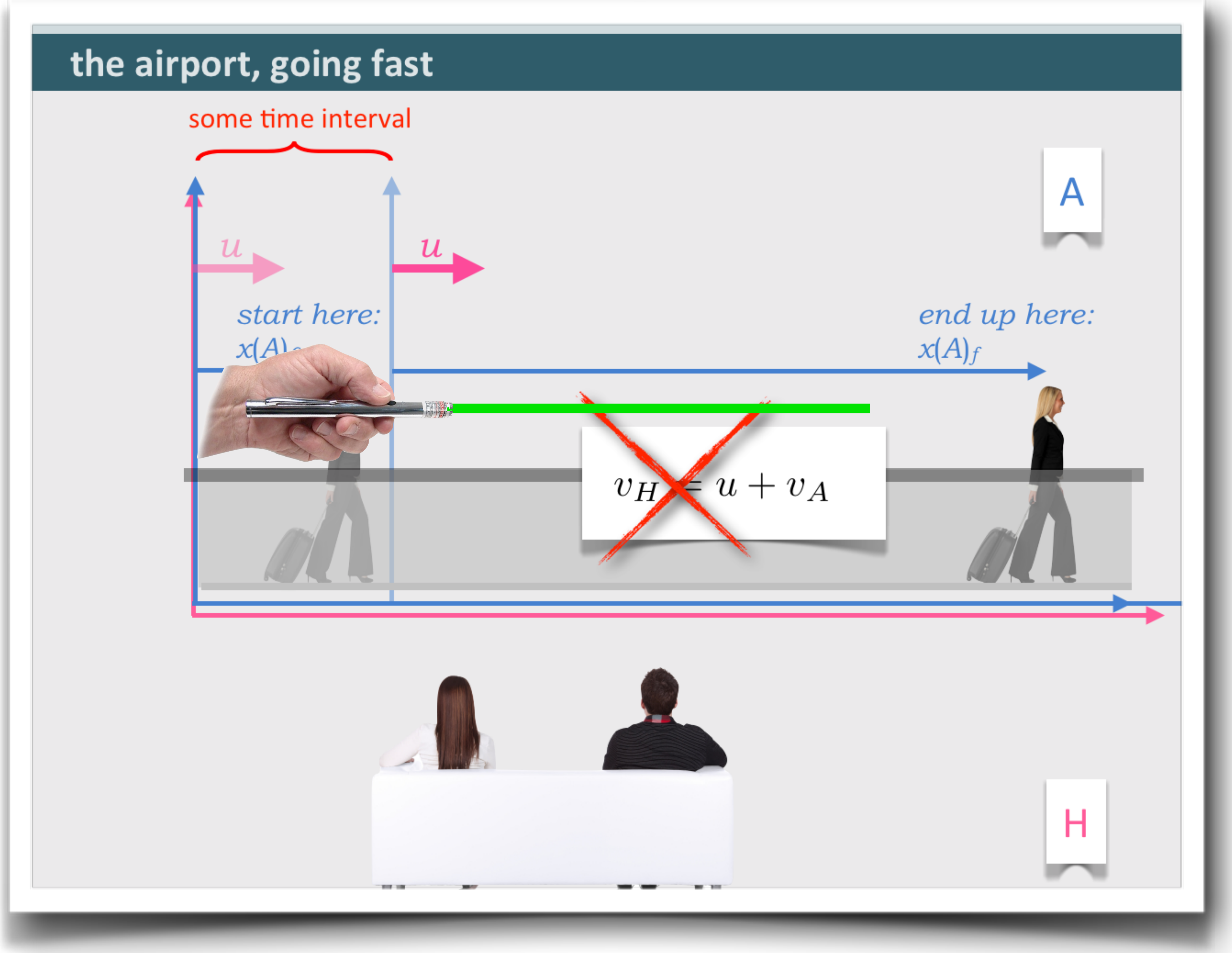
work it out

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

$\frac{u}{c^2}$ <<1...so

$$v_H \to u + v_A$$

and the old-time, non-relativistic airport sidewalk formula emerges



Suppose it's not a traveler, but light.

$v_A = c$ work it out

$$v_H = \frac{c + u}{1 + \frac{u}{c^2} c} = \frac{c + u}{(c + u)} c = c$$

The Second Postulate is preserved!

`nothing`

can accelerate to a speed faster than that of light

be careful

There are 3 velocities going on here.

$$v_H = \frac{v_A + u}{1 + \frac{u}{c^2} v_A}$$

u is the frame velocity

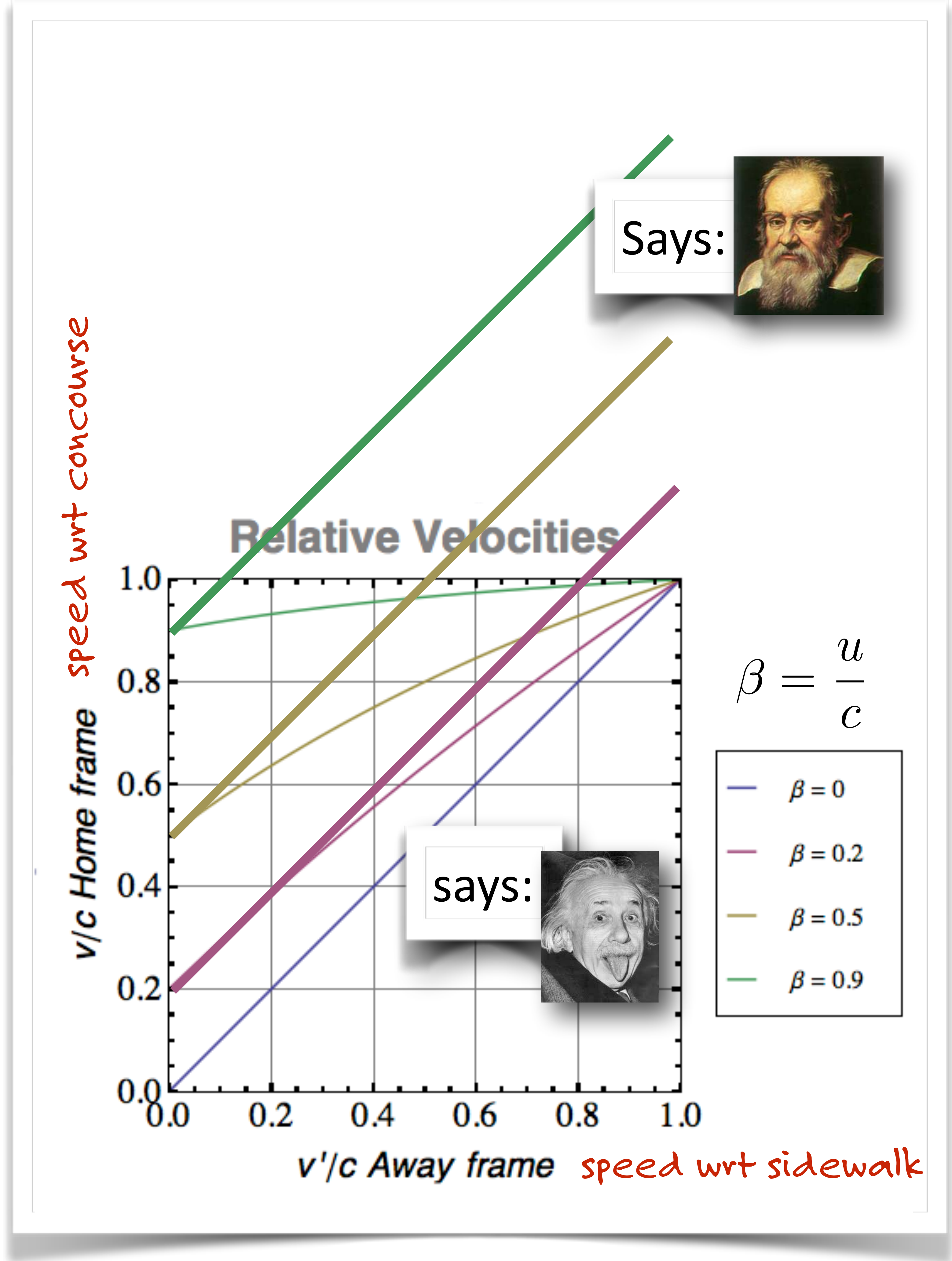
...same, A relative to H or H relative to A (sidewalk)

v_A is the velocity (traveler)

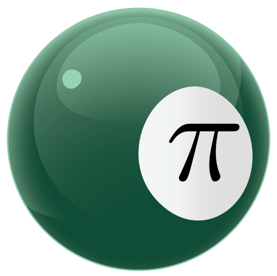
of something measured relative to the A frame

v_H is the velocity (traveler)

of something measured relative to the H frame



another particle

the "pion," π 

not like the muon or electron

mass $m(\pi)$ = 1.4 x $m(\mu)$

unstable

a pion decays into a muon

the pion travels at $u = 0.5\text{c}$ in the lab (H)

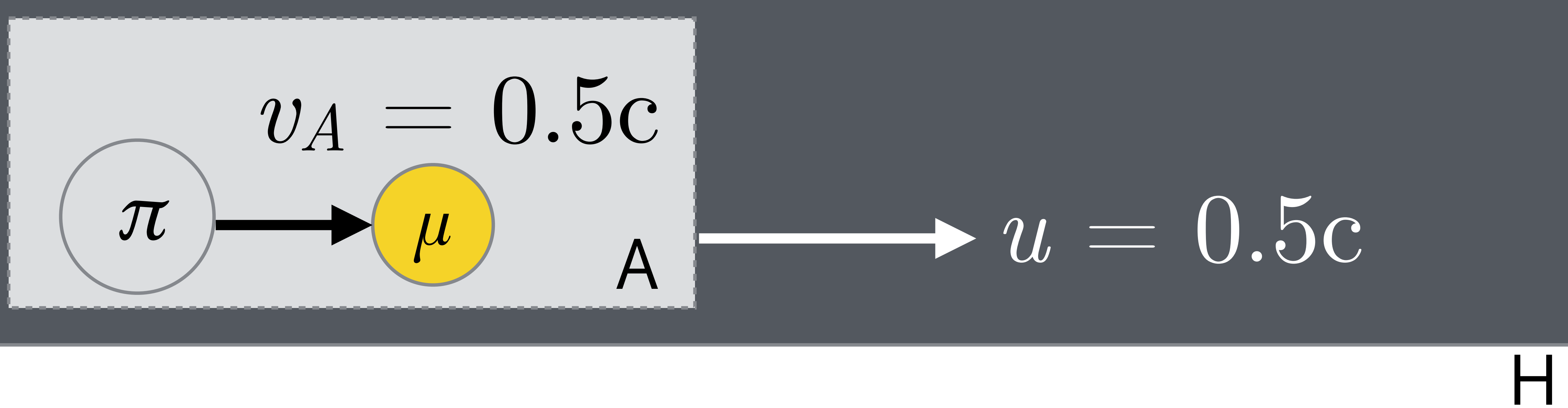
the muon travels right at $v_\text{A} = 0.5\text{c}$ in the pion's rest frame

What is the speed of the muon in the lab?

How far does it travel in the lab before decaying?

What is the speed if muon travels left at $v_\text{A} = -\,0.5\text{c}$ in the pion's rest frame?

What if the muon travels left at $v_\text{A} = -\,0.75\text{c}$ in the pion's rest frame?



Energy

push on something

Einstein said:

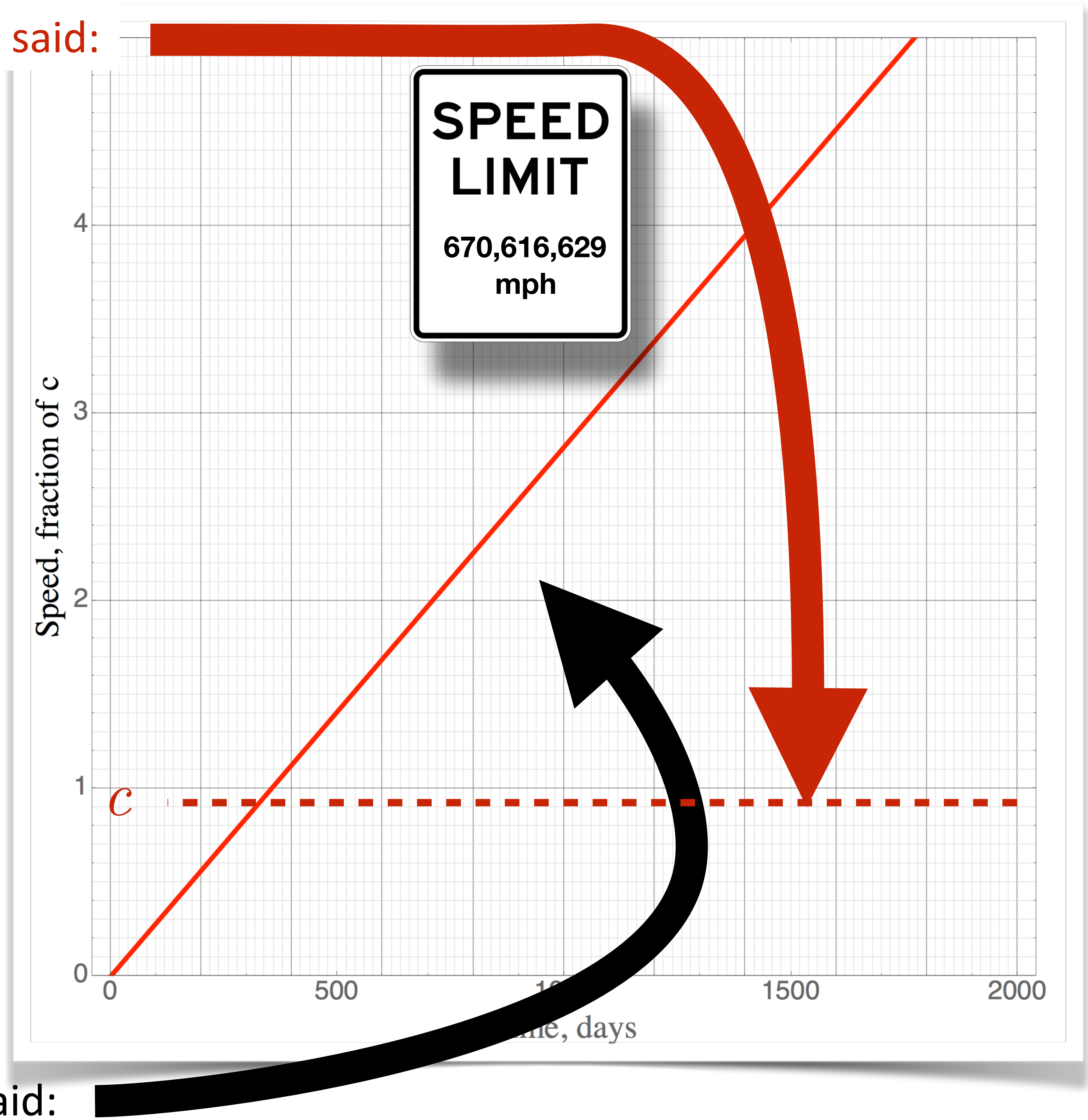
constant force to create a constant acceleration of

$1g$

Galileo/Newton said

speed increases:

$$v = gt$$

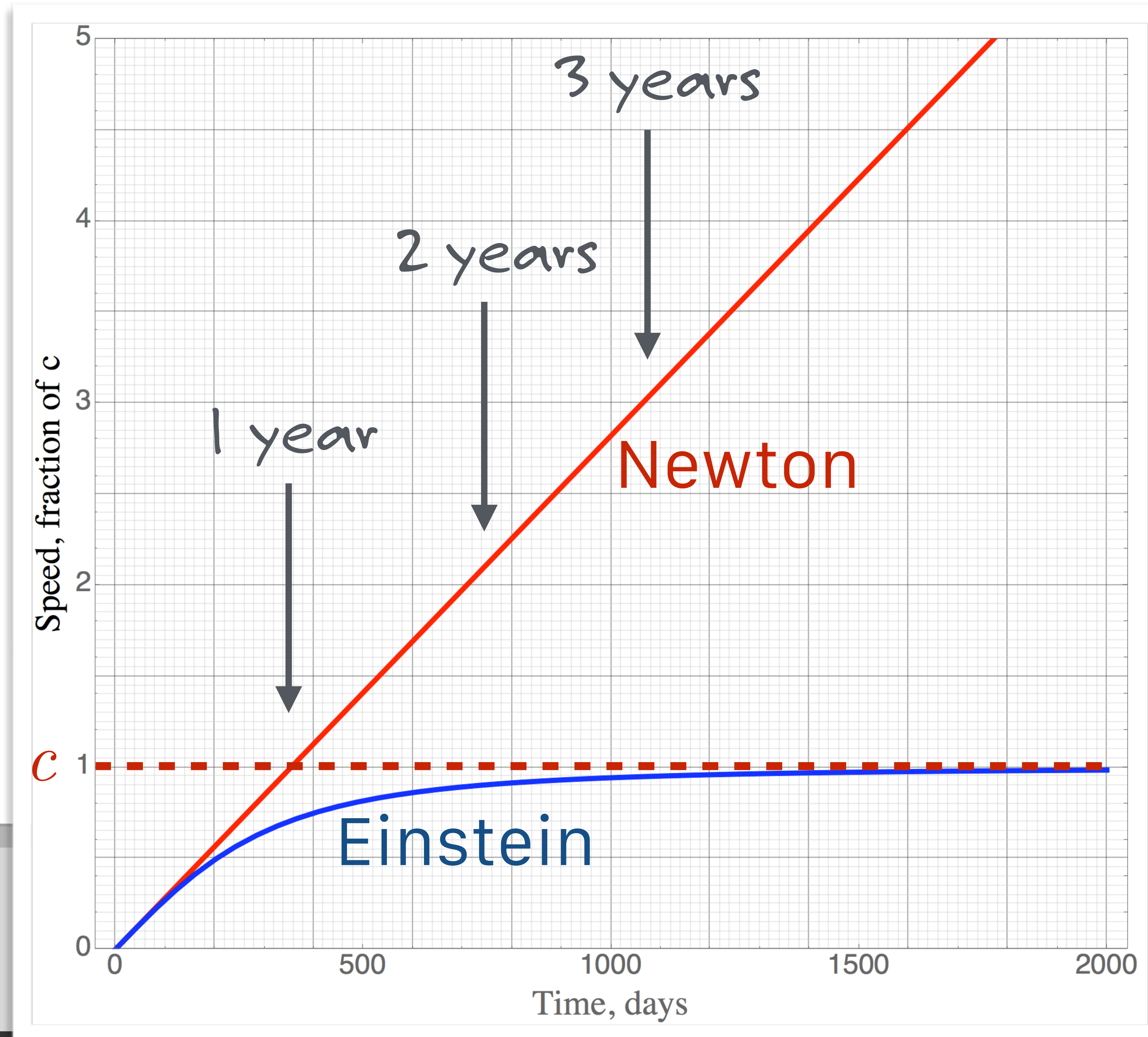


Newton said:

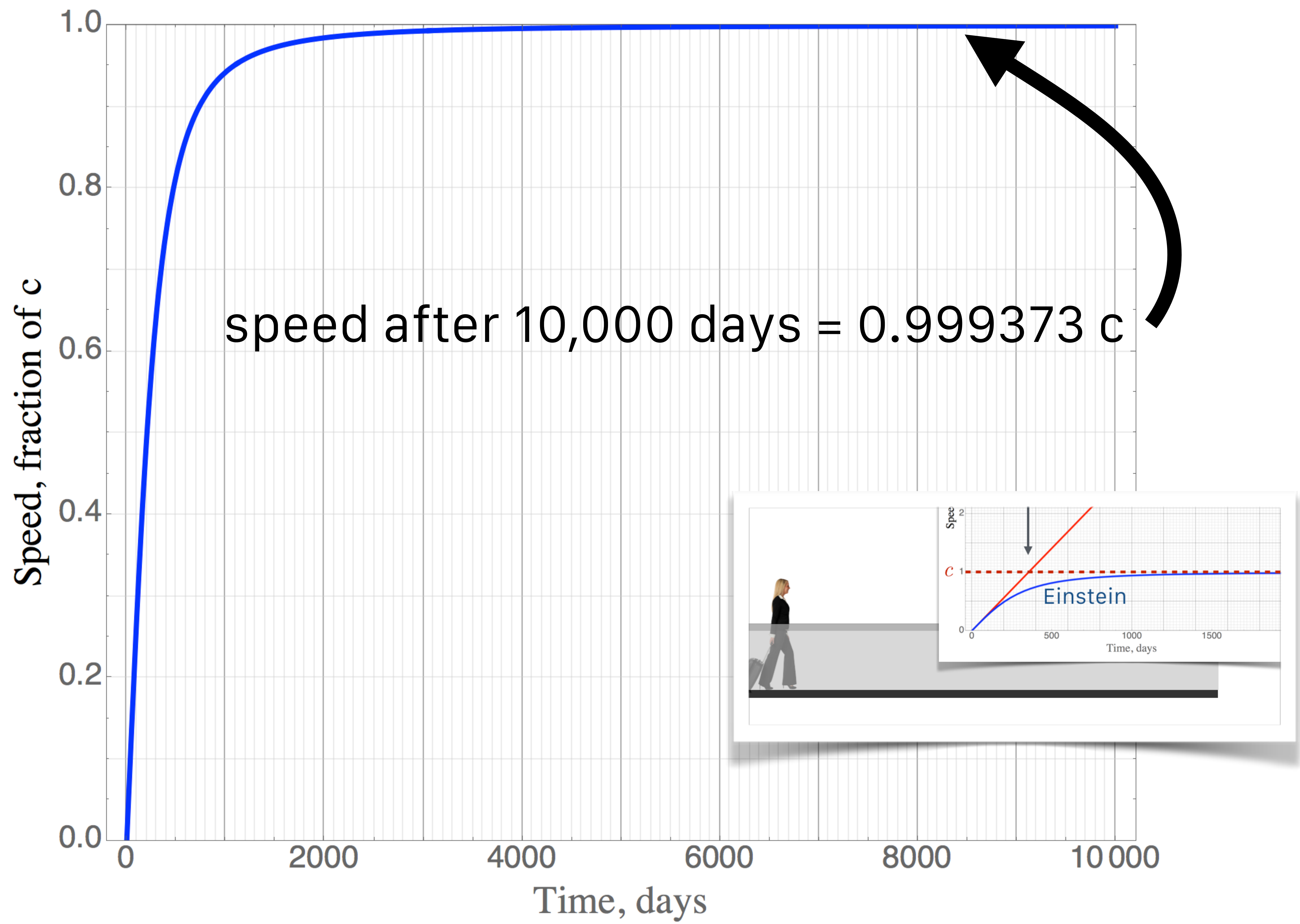
traveler transformation, 1g acceleration

speed not linear

in no frame can she be observed to go above c



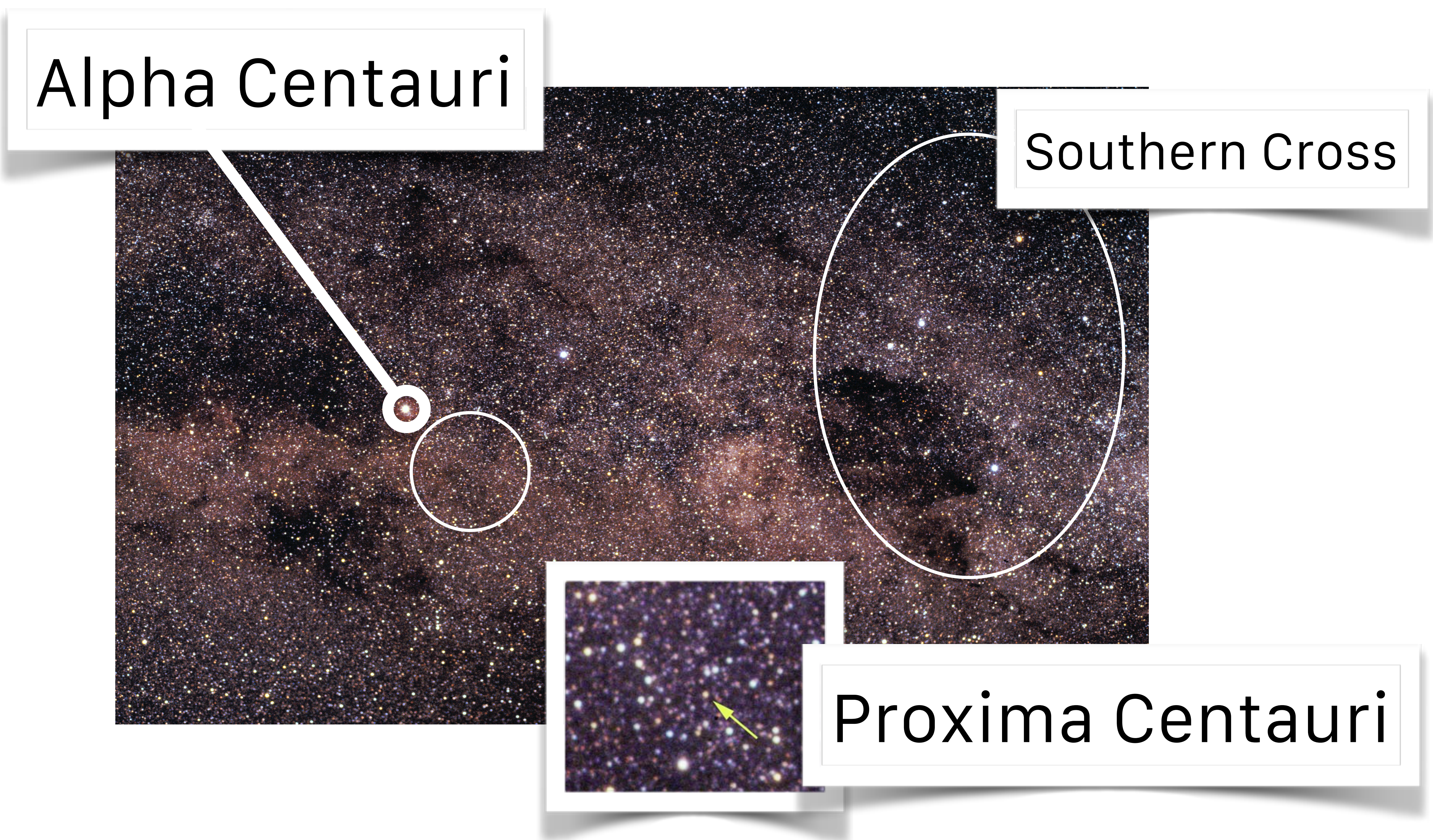
never get there

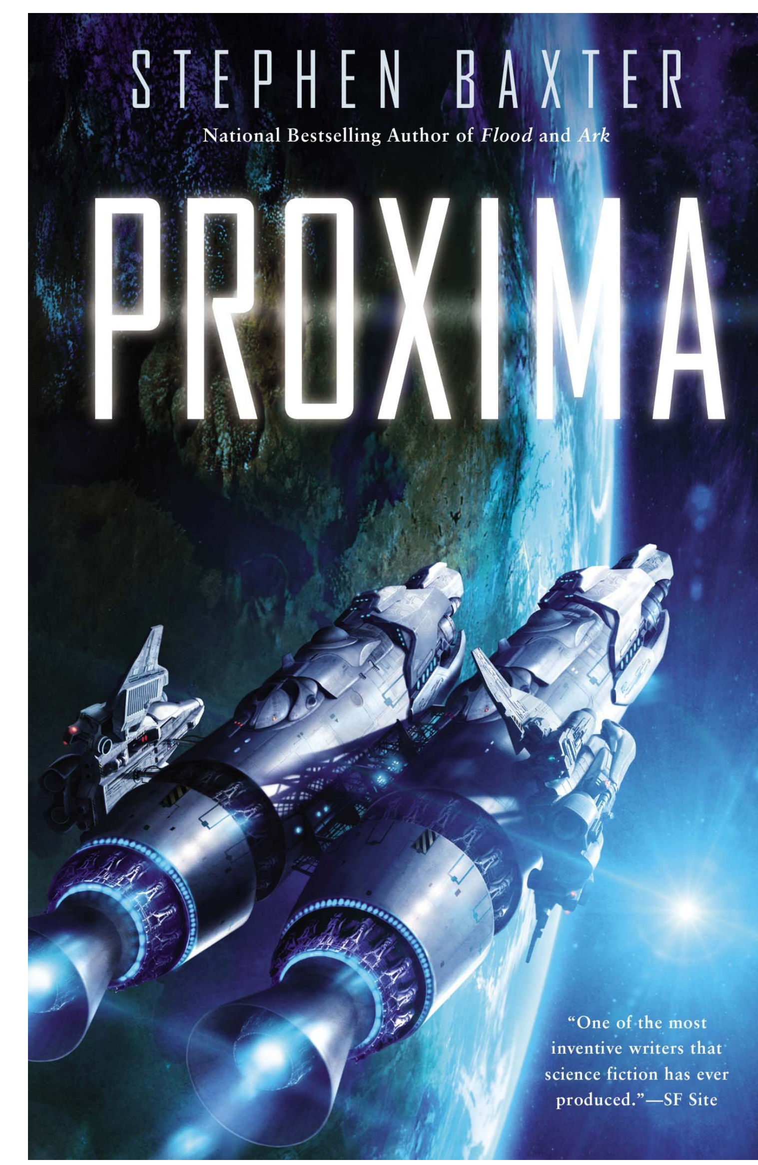


BTW

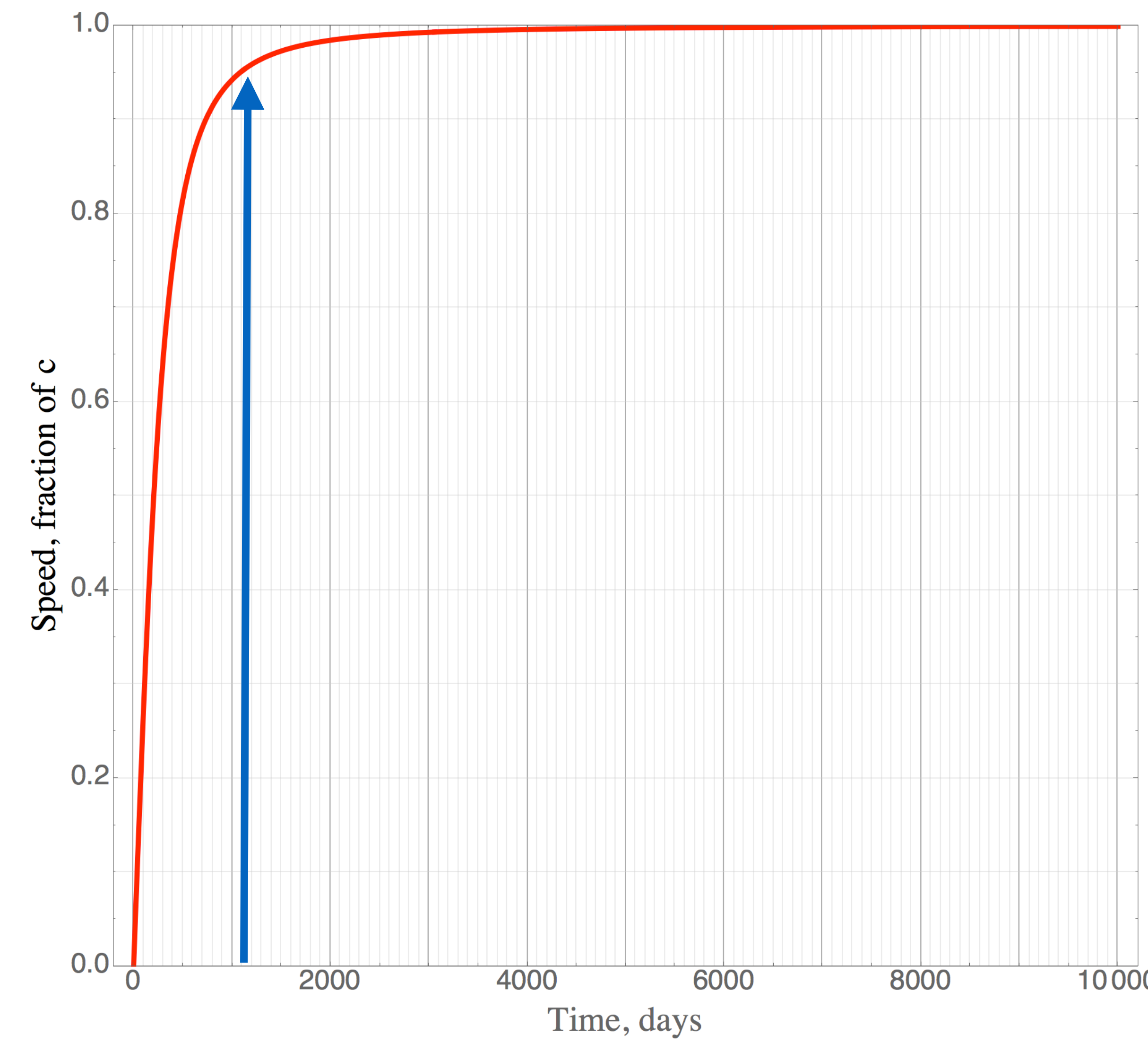
nearly every science fiction story ever

Closest star to Earth: Proxima Centauri: 4.23 light years





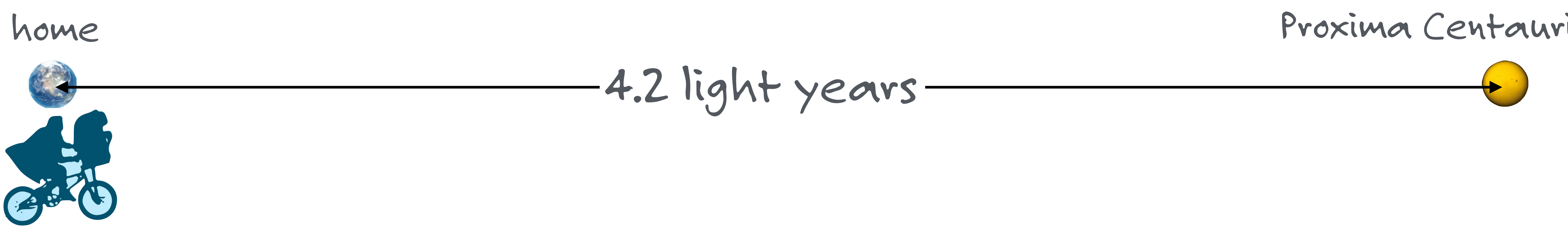
let's go



accelerate at 1g for 2 light years

cruise for 0.2 light years

decelerate at -1g for 2 light years

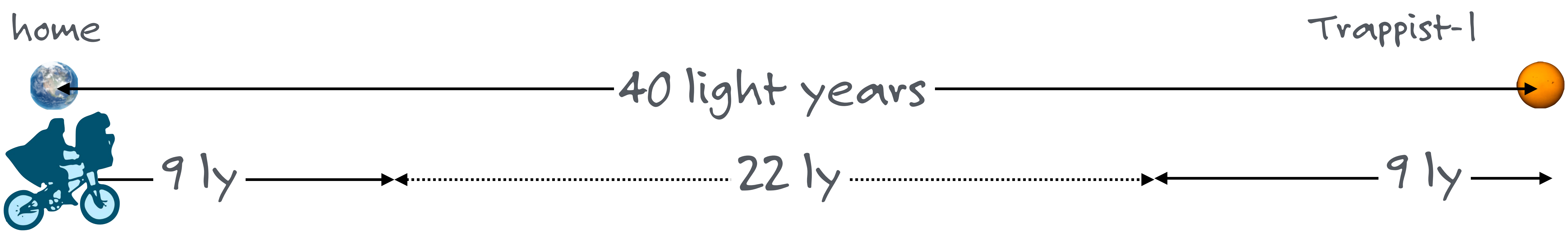


acceleration time, relative to Earth	2.8 years
top speed, relative to Earth	0.9453 c
acceleration time, relative to ship	1.7295 years
whole trip time, relative to Earth	5.8695 years
whole trip time, relative to ship	3.5428 years

let's go there

40 light years away…star is "Trappist-1" which is a dwarf

How about traveling there? Again, assume 1g acceleration

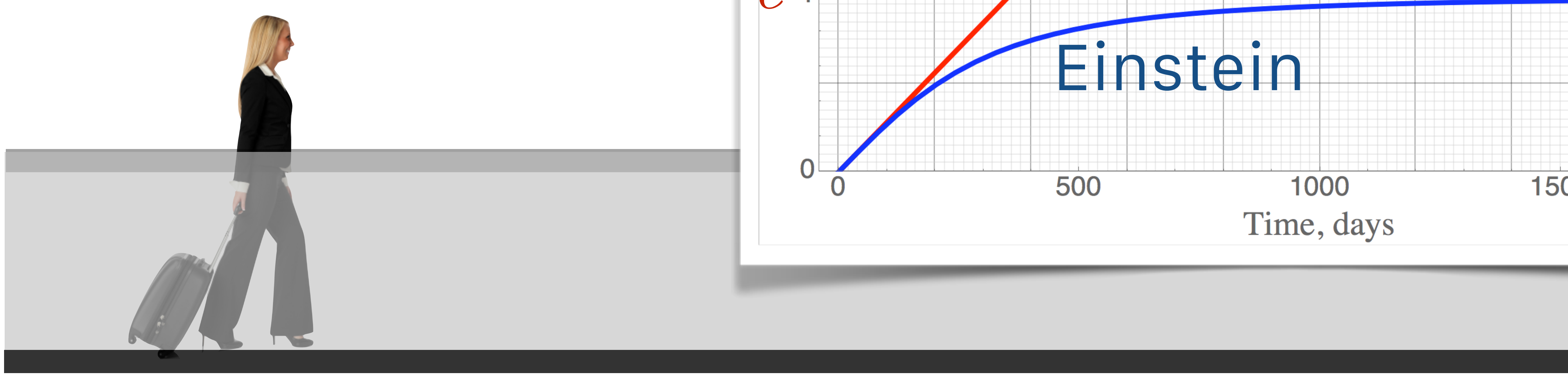


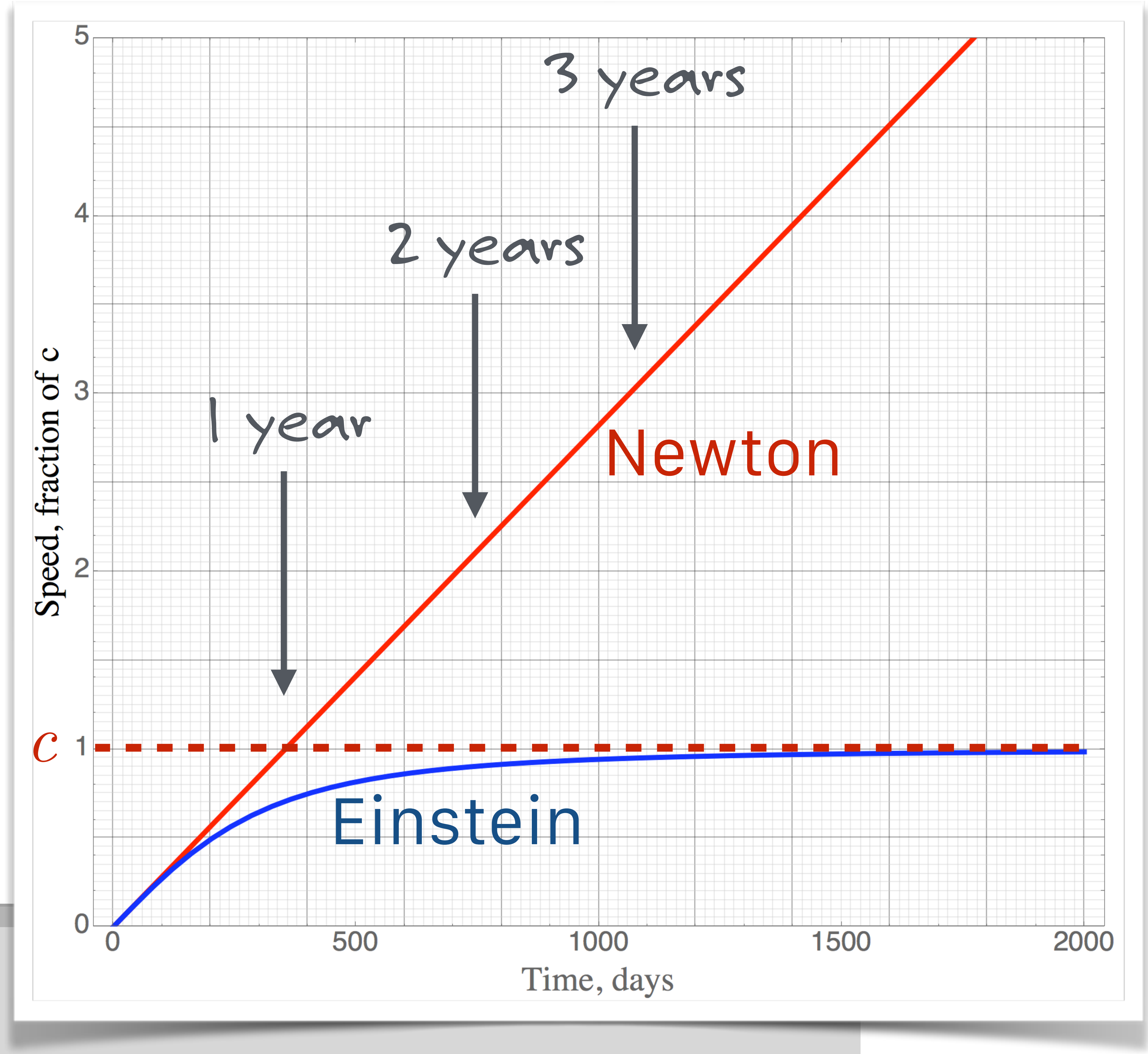
acceleration time, relative to Earth	9.9 years
top speed, relative to Earth	0.9953 c
acceleration time, relative to ship	2.97 years
whole trip time, relative to Earth	41.9 years
whole trip time, relative to ship	8 years

traveler transformation, 1g acceleration

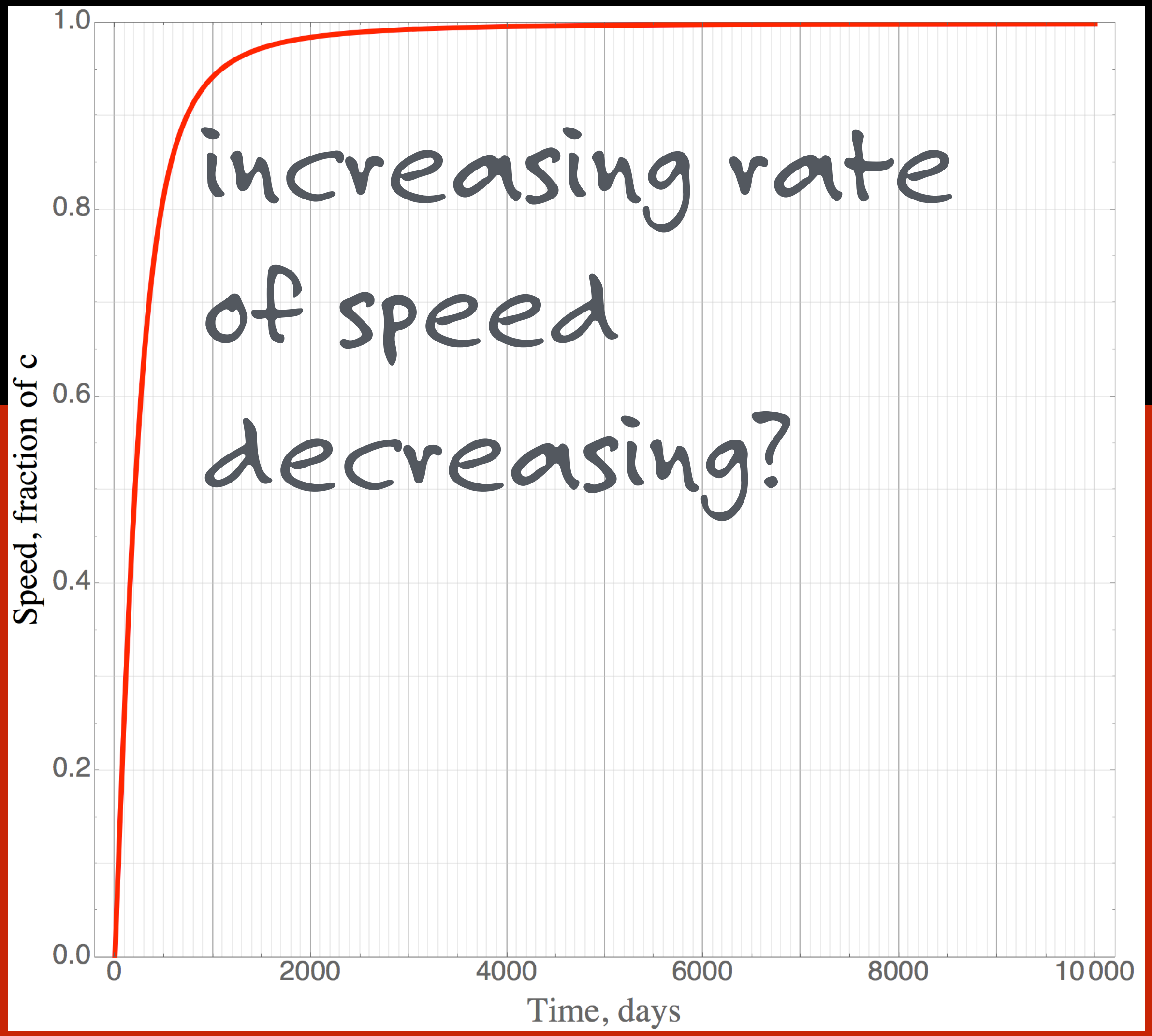
speed not linear

in no frame can she be observed to go above c





doesn’t this look like a reluctance to being accelerated?



Well.

What quantity is a measure of the reluctance to being accelerated?

Inertia.

If this reluctance increases...inertia seems to increase

and…what's the measure of a body's inertia?

mass

classical dynamical quantities

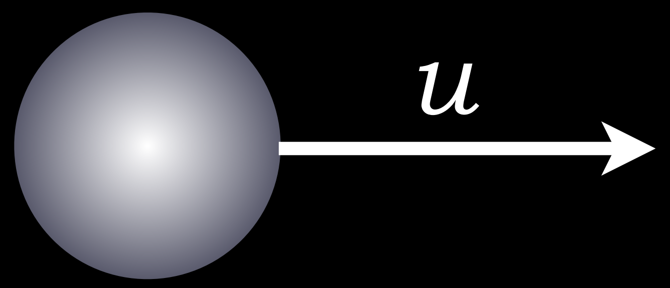
momentum, $p = mv$

Kinetic Energy, $K = 1/2mv^2$

and force $F = ma$

These have to change!

New, relativistic quantities reduce to these when u/c is very small



Momentum in relativity

got to be different from Newton

$$p_H = m\frac{\Delta x_H}{\Delta t_H}$$

want to preserve the idea of momentum conservation

Relativistic Momentum: $p = m\gamma u$

relativity and energy

through the back door…

there’s a “real” derivation, but too much mathematics

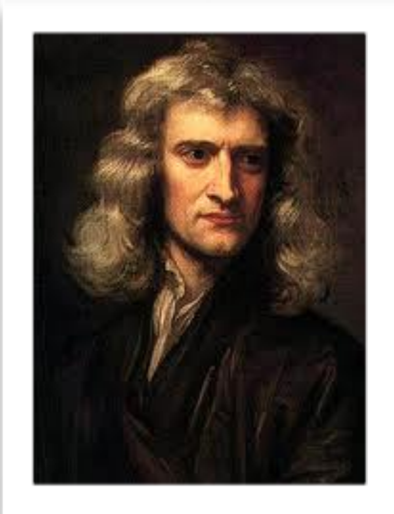
quick aside

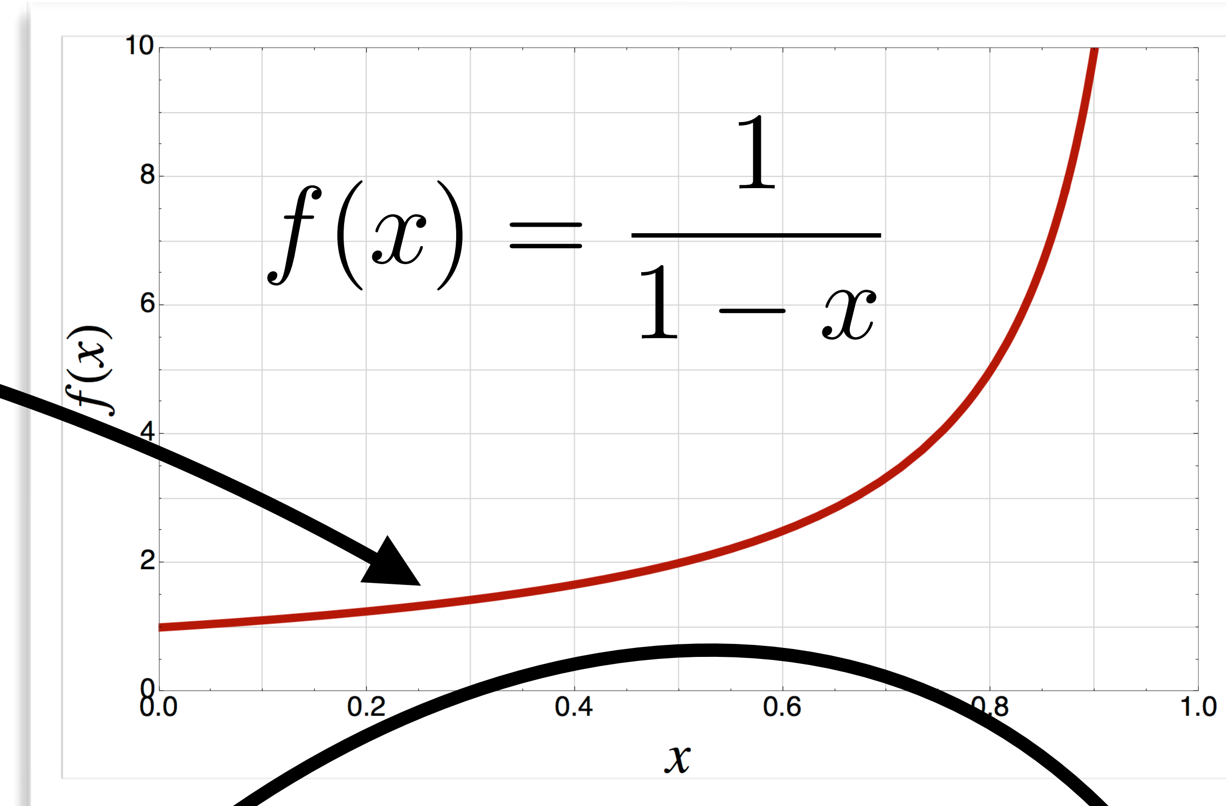
approximating functions

see manuscript math refresher chapter

somewhere in your life: the Binomial Series

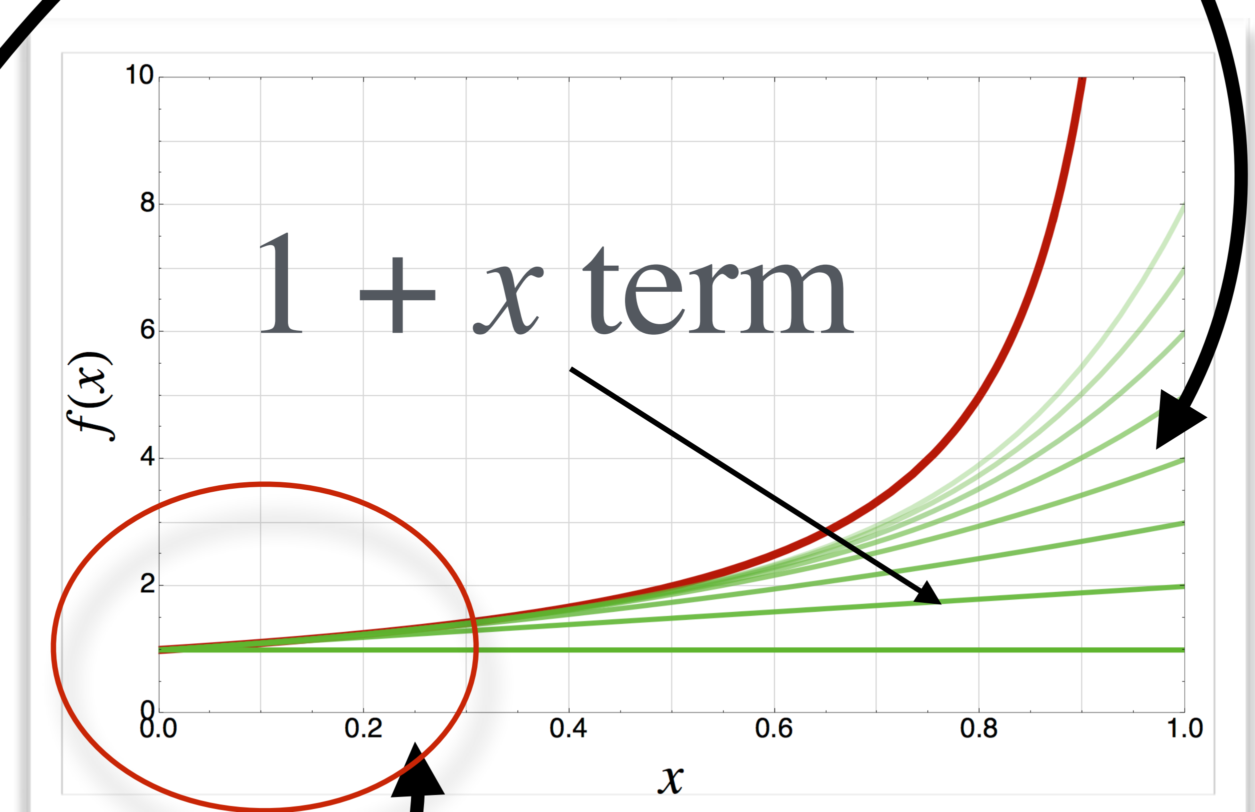
Binomial Series...useful to approximate functions.





$$f(x) = \frac{1}{1-x}$$

$$f(x) = \frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + O\left(x^{11}\right)$$



Suppose that x << 1, then the function could be approximated by a couple of terms...

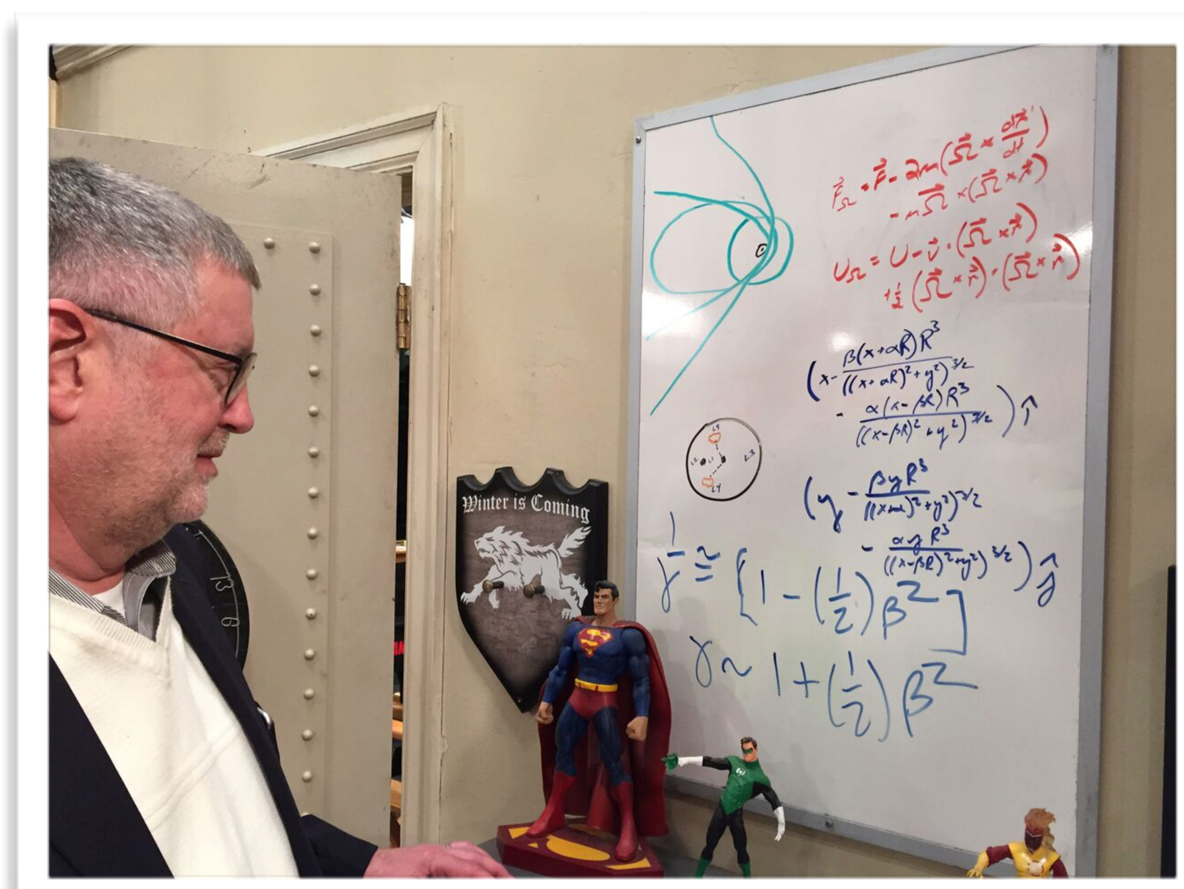
$$f(x) = \frac{1}{1-x} = 1 + x$$

what equation comes to mind?

when you're on the spot?

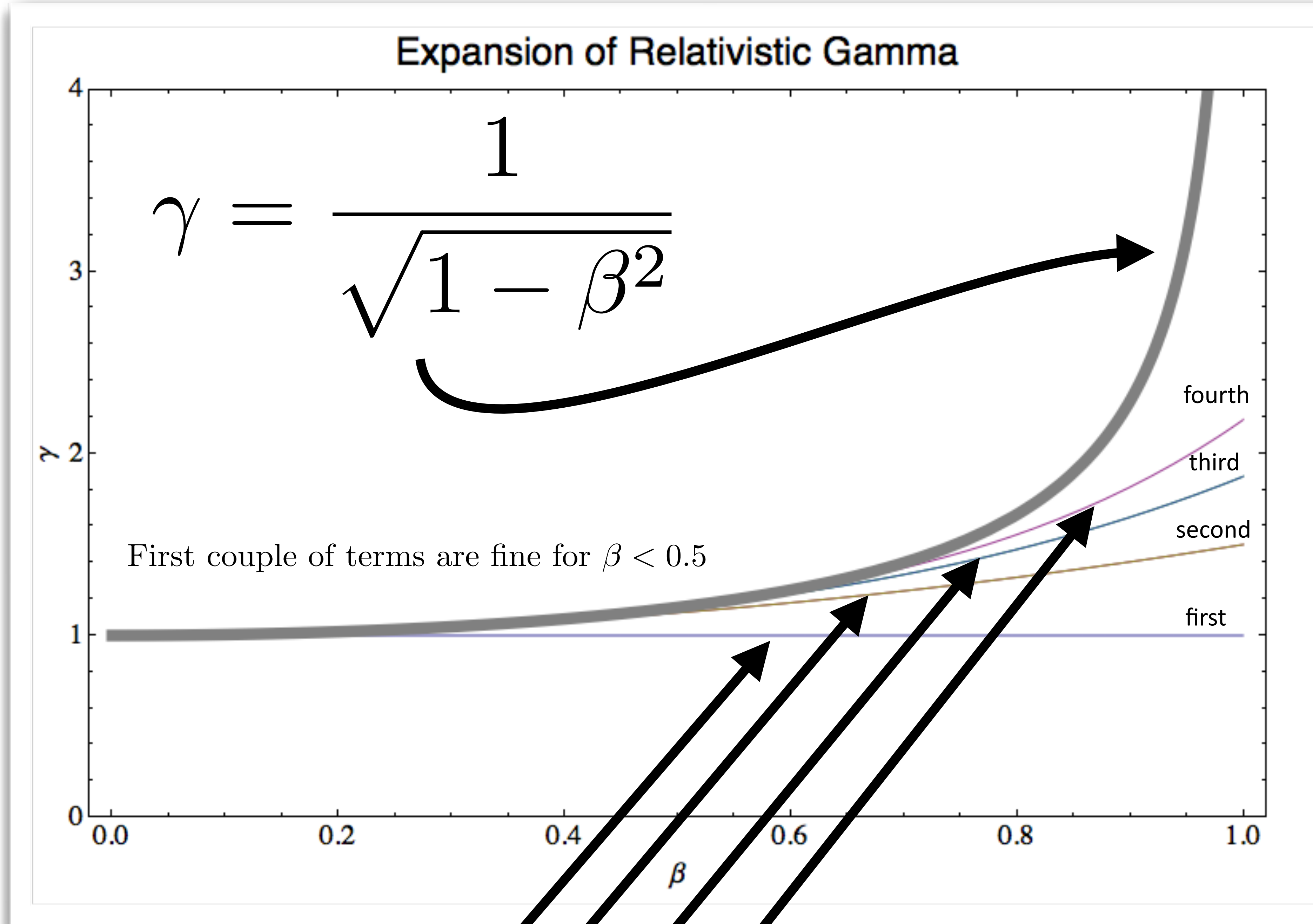
Why the binomial expansion of the relativistic gamma function, of course. Because, Relativity.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \sim 1 + \frac{\beta^2}{2} + \frac{3\,\beta^4}{8} + \frac{5\,\beta^6}{16} + \frac{35\,\beta^8}{128} + \frac{63\,\beta^{10}}{256} + \frac{231\,\beta^{12}}{1024} + \frac{429\,\beta^{14}}{2048} + O[\beta]^{15}$$

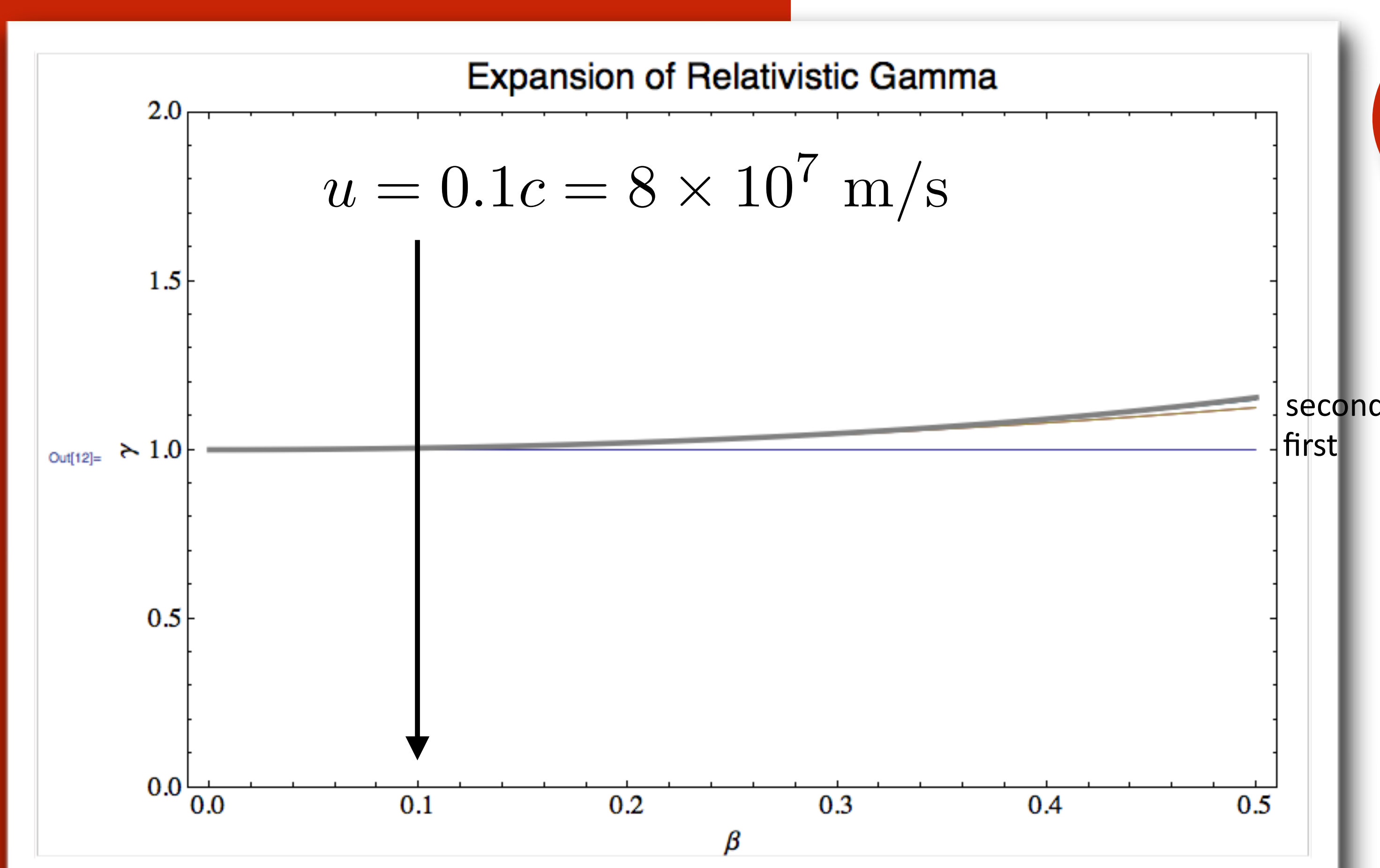


how well?

look at 8 terms in the expansion



$$\gamma \sim 1 + \frac{\beta^2}{2} + \frac{3\,\beta^4}{8} + \frac{5\,\beta^6}{16} + \frac{35\,\beta^8}{128} + \frac{63\,\beta^{10}}{256} + \frac{231\,\beta^{12}}{1024} + \frac{429\,\beta^{14}}{2048} + O[\beta]^{15}$$



Season 9, Episode 12

The Sales Call Sublimation, January 7, 2016

so let's use this and look for familiar things

slow moving objects but not completely classical

$$\gamma \sim 1 + \frac{\beta^2}{2} + \frac{3\,\beta^4}{8} + \frac{5\,\beta^6}{16} + \frac{35\,\beta^8}{128} + \frac{63\,\beta^{10}}{256} + \frac{231\,\beta^{12}}{1024} + \frac{429\,\beta^{14}}{2048} + O[\beta]^{15}$$

sing along

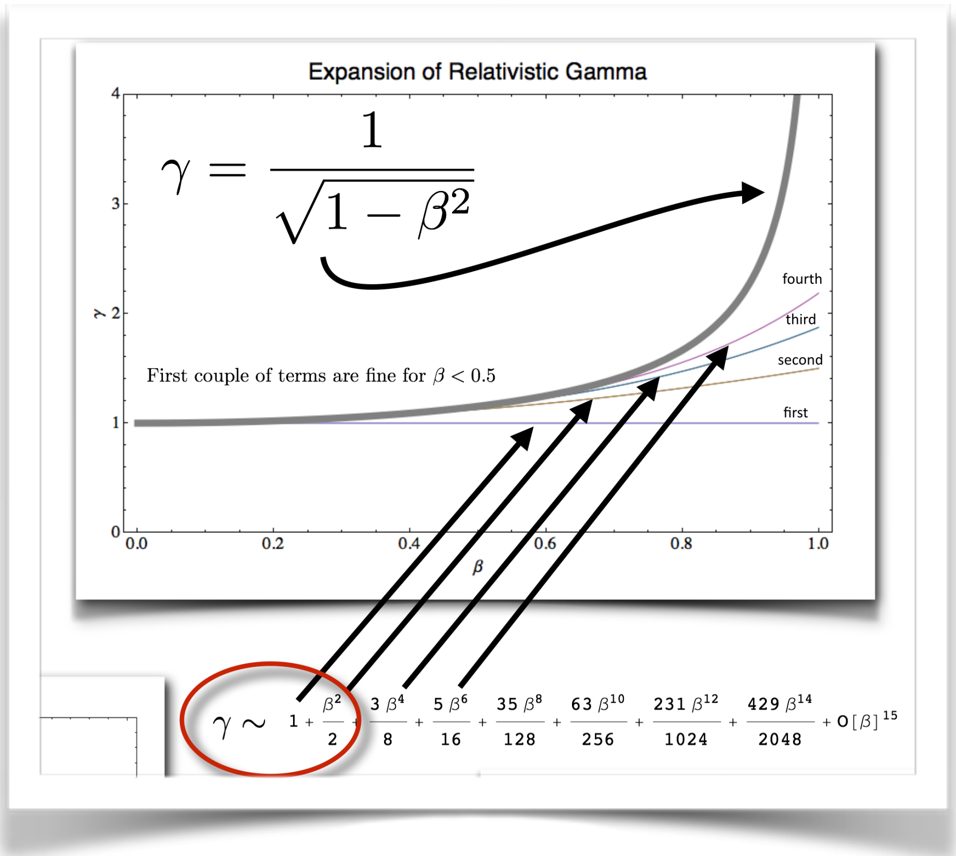
for β small:

now copy the approximate forms, but insert $\beta = u/c$

$$\gamma \sim 1 + \left(\frac{1}{2}\right)\beta^2$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

now, write along with me:



sing along

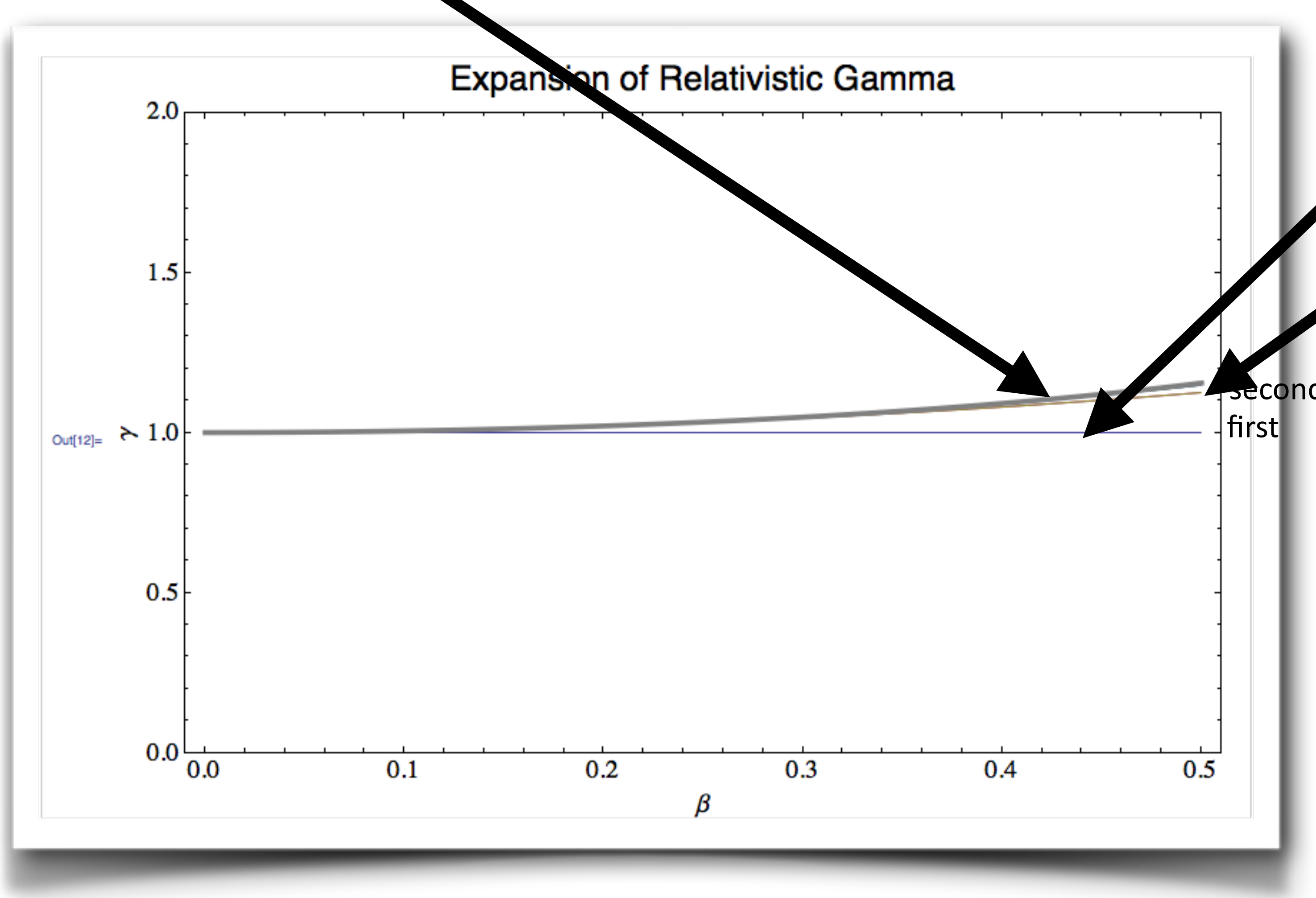
for β small:

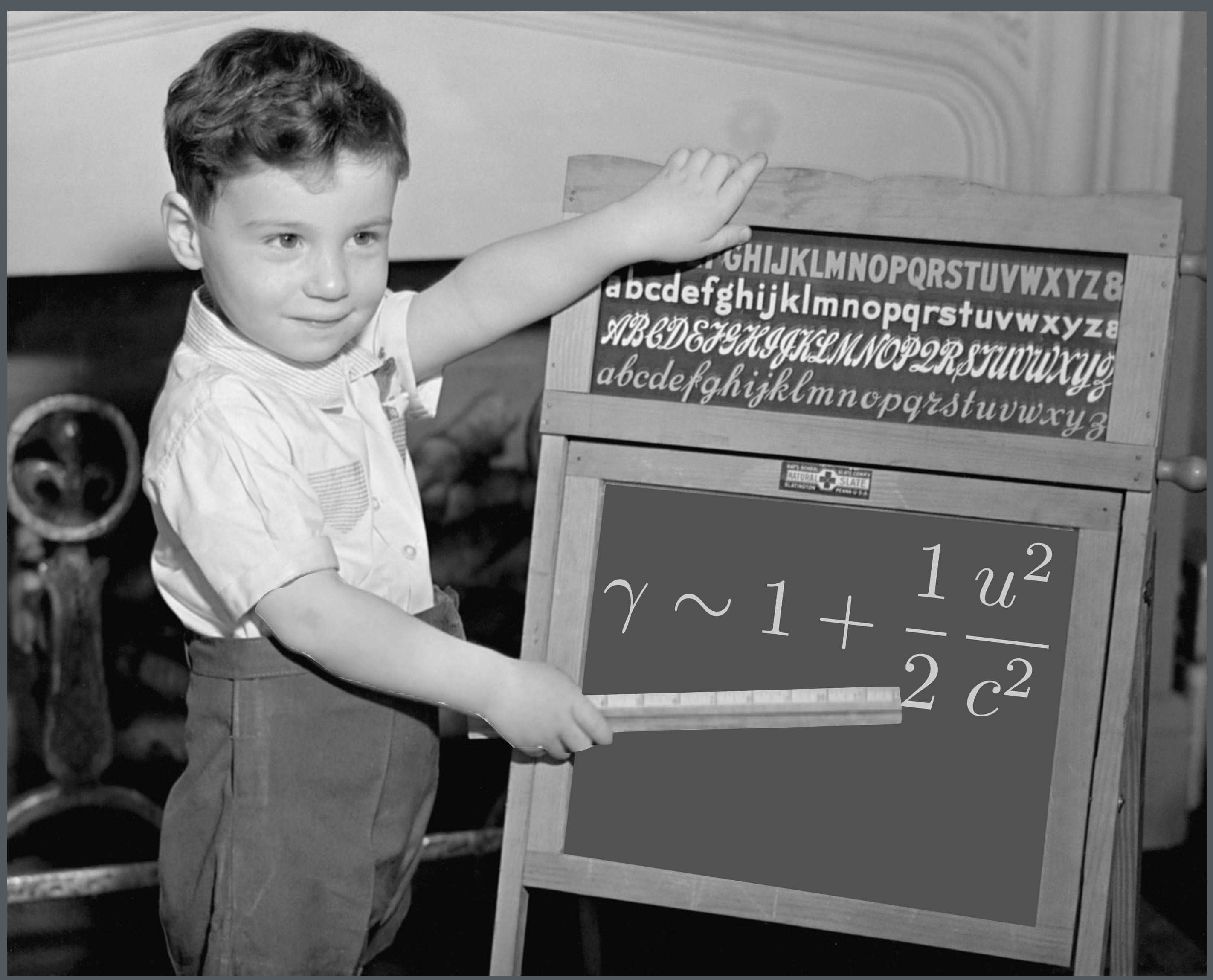
now copy the approximate forms, but insert $\beta = u/c$

$$\gamma \sim 1 + \left(\frac{1}{2}\right)\beta^2$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\gamma \sim 1 + \left(\frac{1}{2}\right)\frac{u^2}{c^2}$$





now let’s play