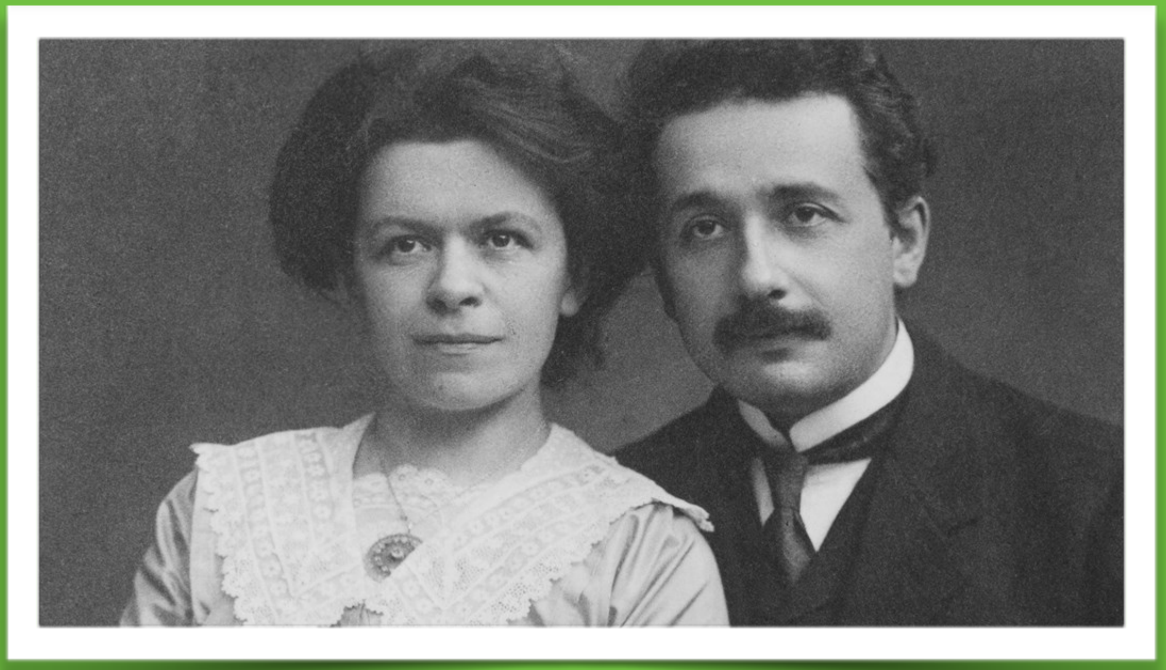
hi

Day 18, 20.03.2018

Einstein’s Theory of Special Relativity, 5



housekeeping

Gotta come to class

question about anything? I'll make a movie for you:

Special Relativity:

Hobson_Relativity.pdf is chapter 10 out of Hobson

Also, chapter 2 in Oerter is good.

need this and next lecture for HW! So HW7 due Sunday, rather than Friday

MasteringAstronomy registration expiration now set to March 15.

honors project began

https://qstbb.pa.msu.edu/storage/Homework_Projects/honors_project_2018/

contains the first instructions: the plan & tutorial

- MinervaInstructions1_2018.pdf

dates:

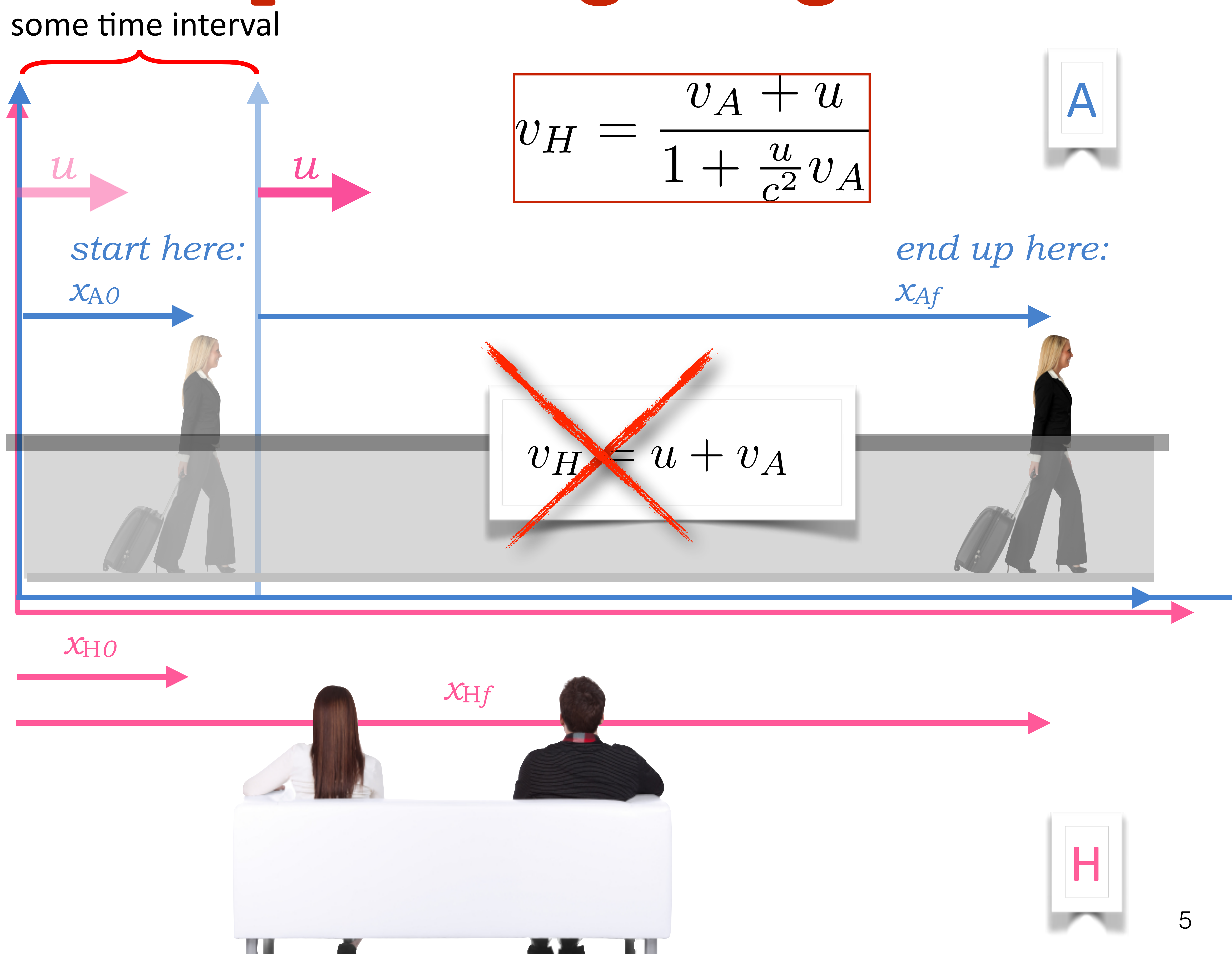
- complete first part, March 16
- analyze data and complete writeup, April 20

is Relativity

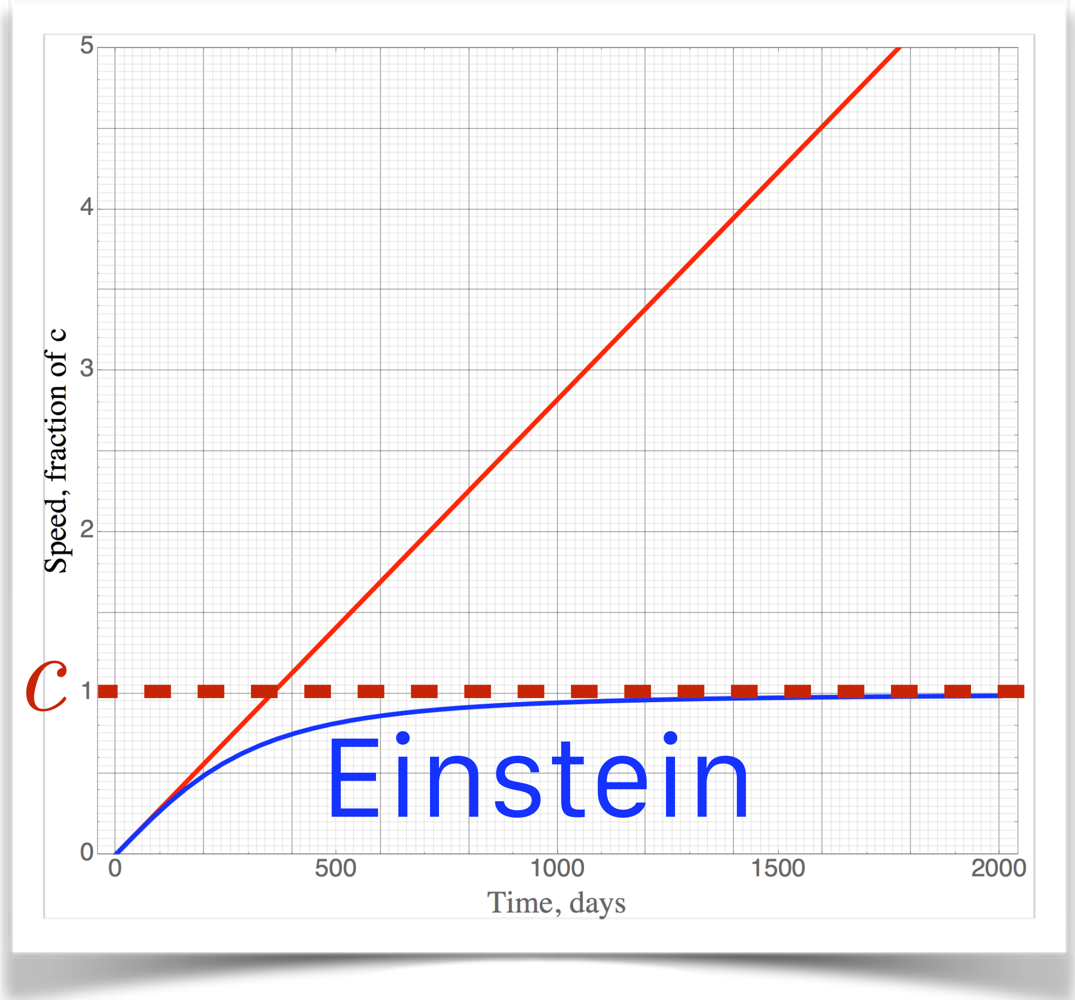
the case?

I showed you two classic tests

the airport, going fast



Energy



a resistance to acceleration

inertia

the measure of inertia

mass

classical dynamical quantities

momentum, $p = mv$ $\quad p = m\gamma u$

Kinetic Energy, $K = 1/2mv^2$

and force $F = ma$

These have to change!

New, relativistic quantities reduce to these when u/c is very small

relativity and energy

through the back door…

there’s a “real” derivation, but too much mathematics

we took a quick aside

approximating functions

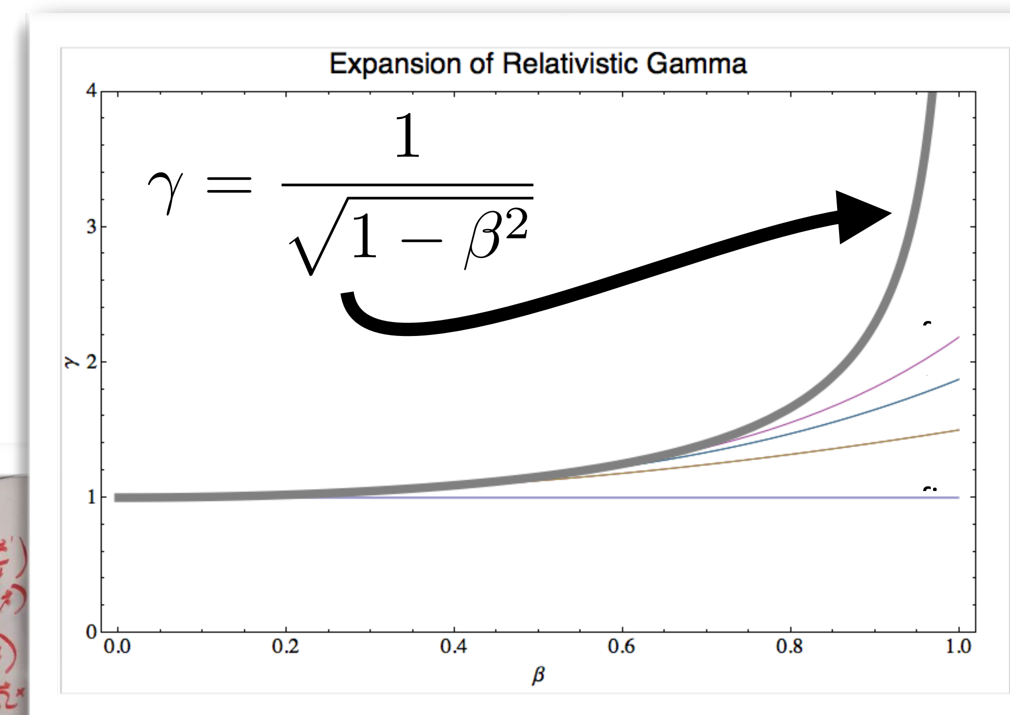
see Lesson 3

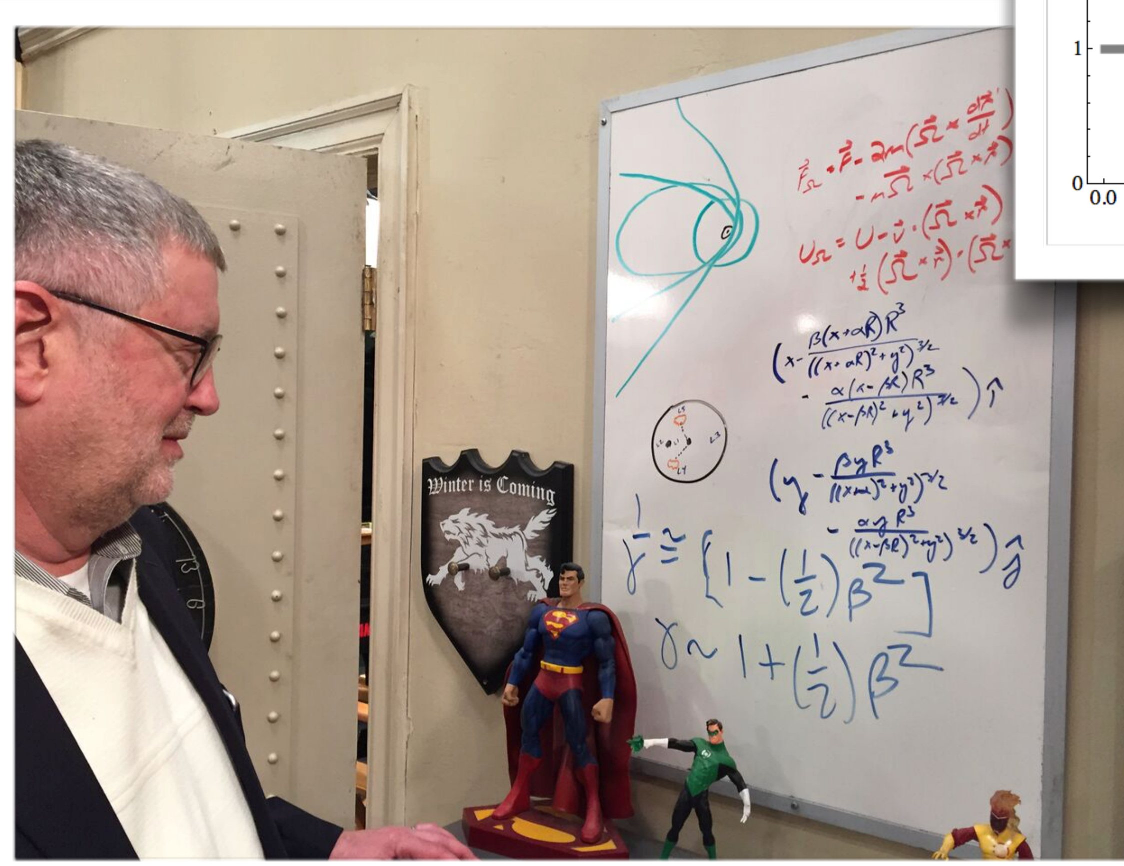
what equation comes to mind?

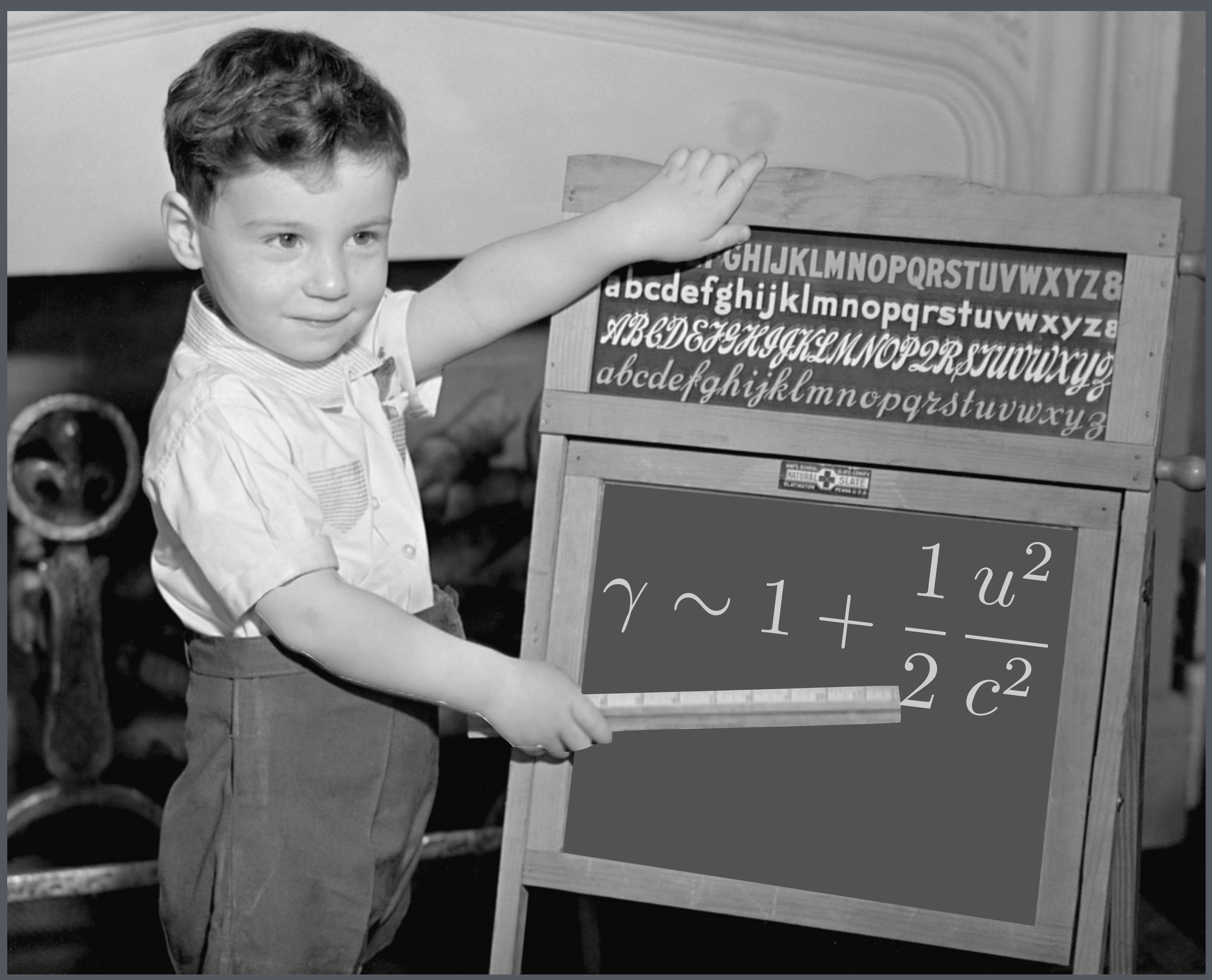
when you're on the spot?

Why the binomial expansion of the relativistic gamma function, of course. Because, Relativity.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \sim 1 + \frac{\beta^2}{2} + \frac{3\,\beta^4}{8} + \frac{5\,\beta^6}{16} + \frac{35\,\beta^8}{128} + \frac{63\,\beta^{10}}{256} + \frac{231\,\beta^{12}}{1024} + \frac{429\,\beta^{14}}{2048} + O[\beta]^{15}$$







now let’s play

jargon alert: **rest mass**

refers to:	mass of an object in its own rest frame (related to Rest Energy, the mass-energy of an object in its own frame)
entomology:	“rest” implying...not moving
example:	the rest mass of the electron is 9.109 x 10^{-31} kg

~~"convert mass into energy"~~

no.

Mass is energy and energy is mass.

$\$ = € * 1.06$ both currency, can both buy stuff

just a conversion factor...

$$E_m = mc^2$$

both energy, can both do work

just a conversion factor...

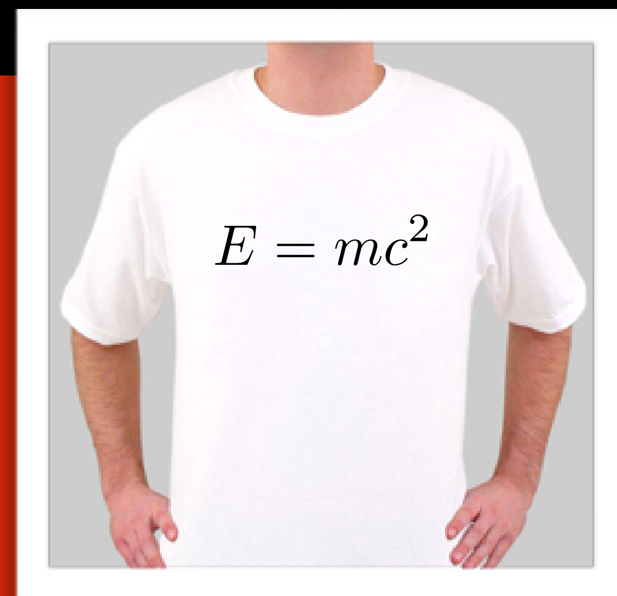
a big conversion factor

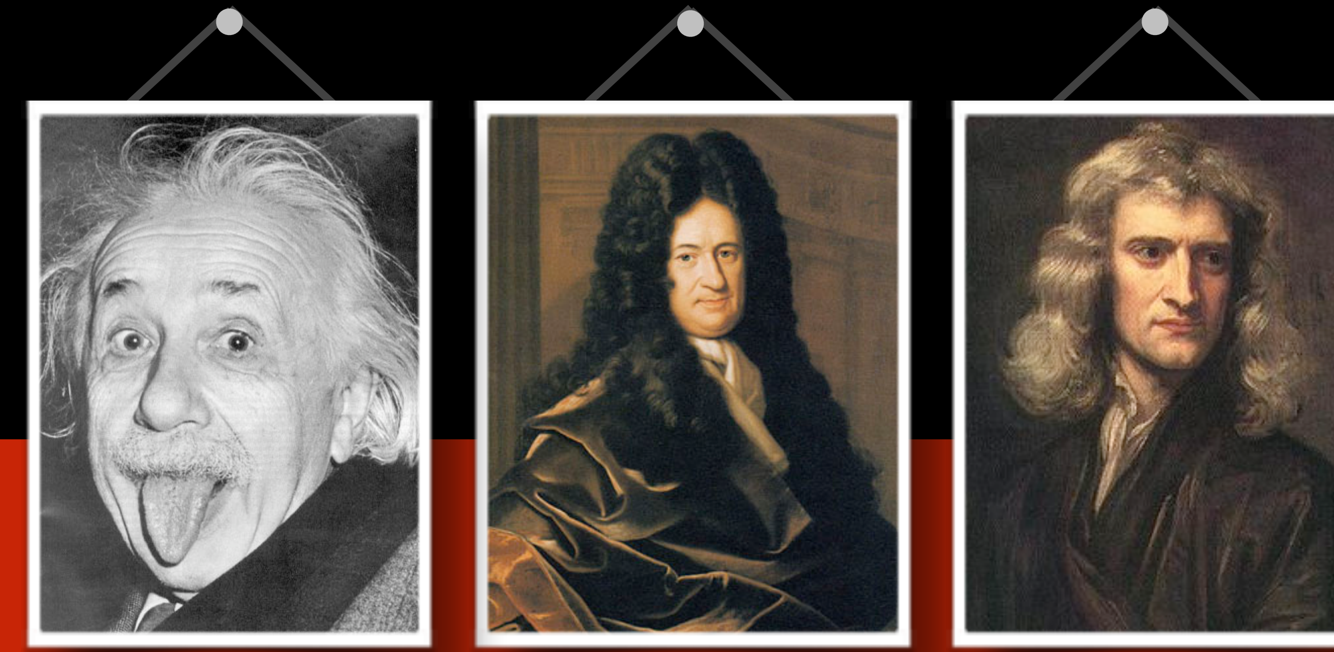
I *could* speak of the energy of mass... and the mass of energy

and I will.

Energy

cheat shirt:





lots of pent-up energy in an apple

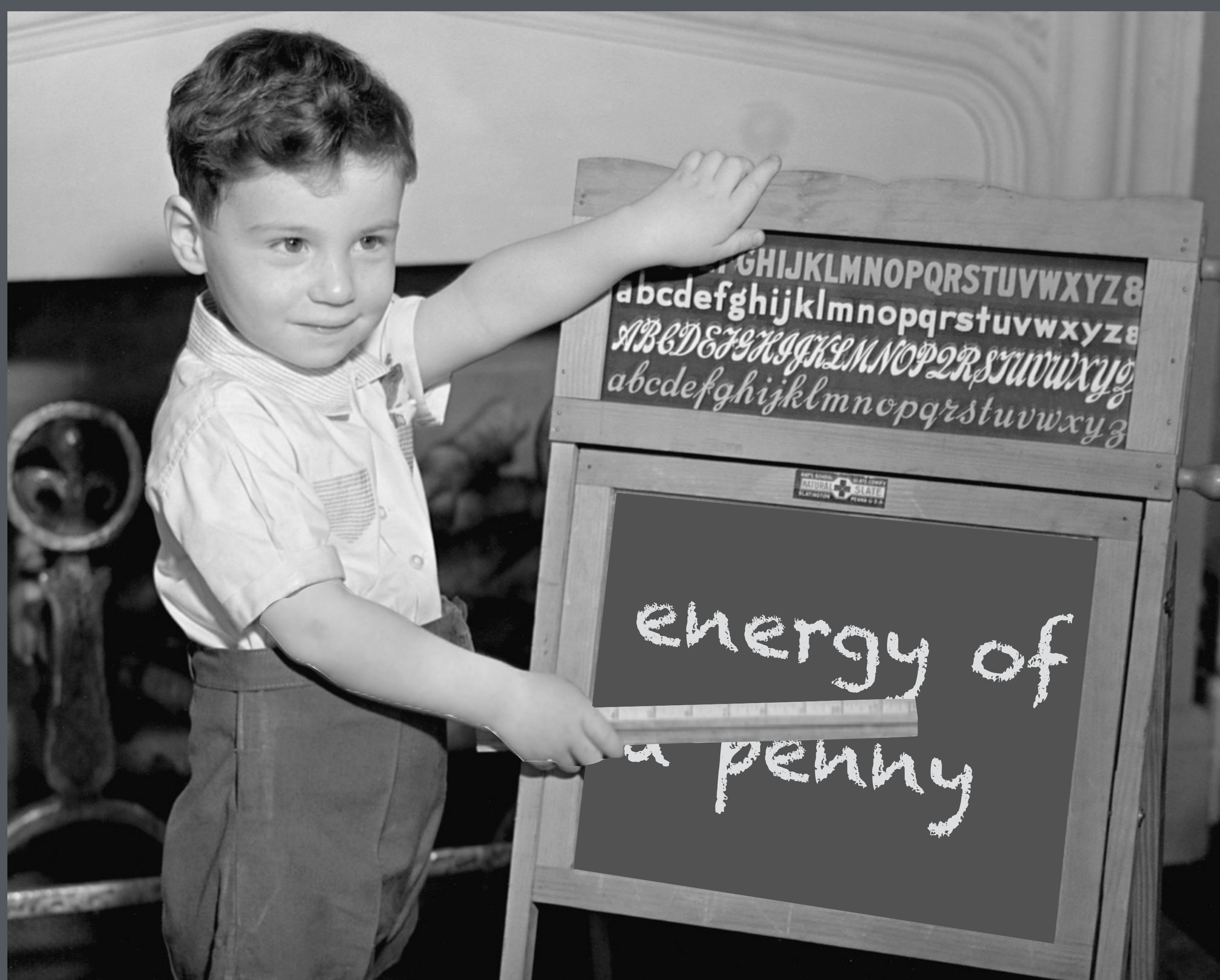
mass of the apple = 100 gm = 0.1 kg

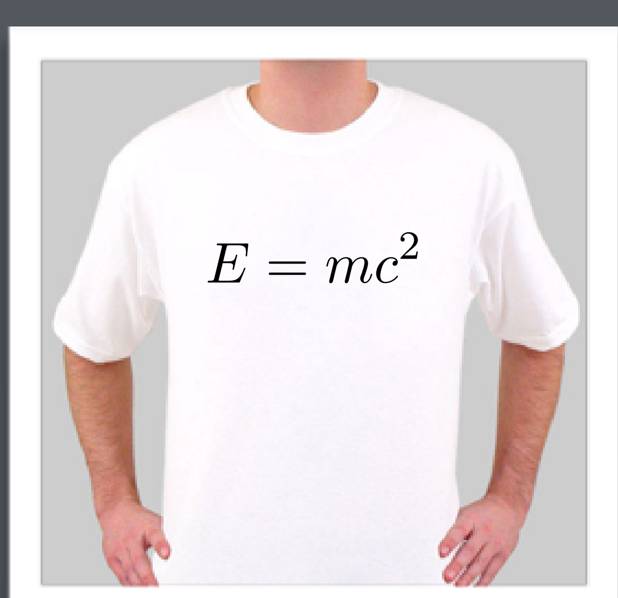
$c^2 = 9 \times 10^{16}\ m^2/s^2$

$$E_m = mc^2$$

$$= (0.1)(3 \times 10^8)^2$$ = Mass energy = 9,000,000,000,000,000 Joules!

Motion energy = 1 Joule





cheat shirt

the mass of a penny is

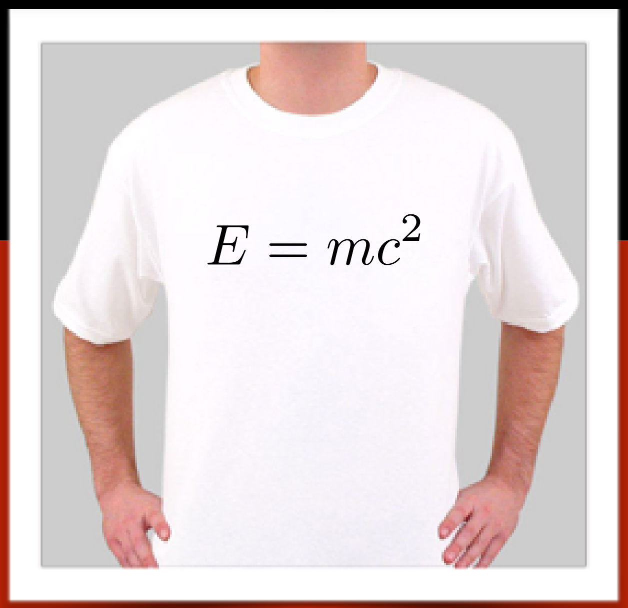
$$3\text{gm} = 3 \times 10^{-3}\text{kg}$$

The speed of light squared is:

$$c^2 = 9 \times 10^{16}\text{m}^2/\text{s}^2$$

How many Joules of energy is trapped in that mass?

that is...what’s the rest energy of a penny?



cheat shirt

the mass of a penny is

$$3\text{gm} = 3 \times 10^{-3}\text{kg}$$

The speed of light squared is: $c^2 = 9 \times 10^{16}\text{m}^2/\text{s}^2$

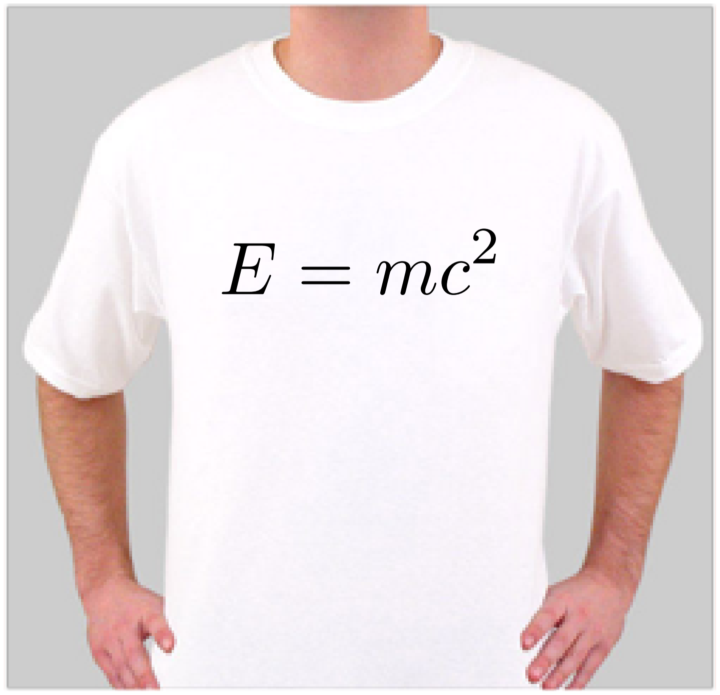
$$E_m(\text{penny}) = 27 \times 10^{13}\text{J}$$

Aircraft Carrier Nimitz: 91,400 tons at 32 knots:

$$K(\text{Nimitz}) = 1.1 \times 10^{10}\text{J}$$

the new energy

energy related to mass.



The Tee Shirt Equation!

Mass in rest frame, - "**rest mass**"

$$m$$

Energy related to mass, - "**rest energy**"

$$E_m = mc^2$$

Energy related to motion, "**kinetic**,"

$$K$$

Total energy of anything, I'll call E_T

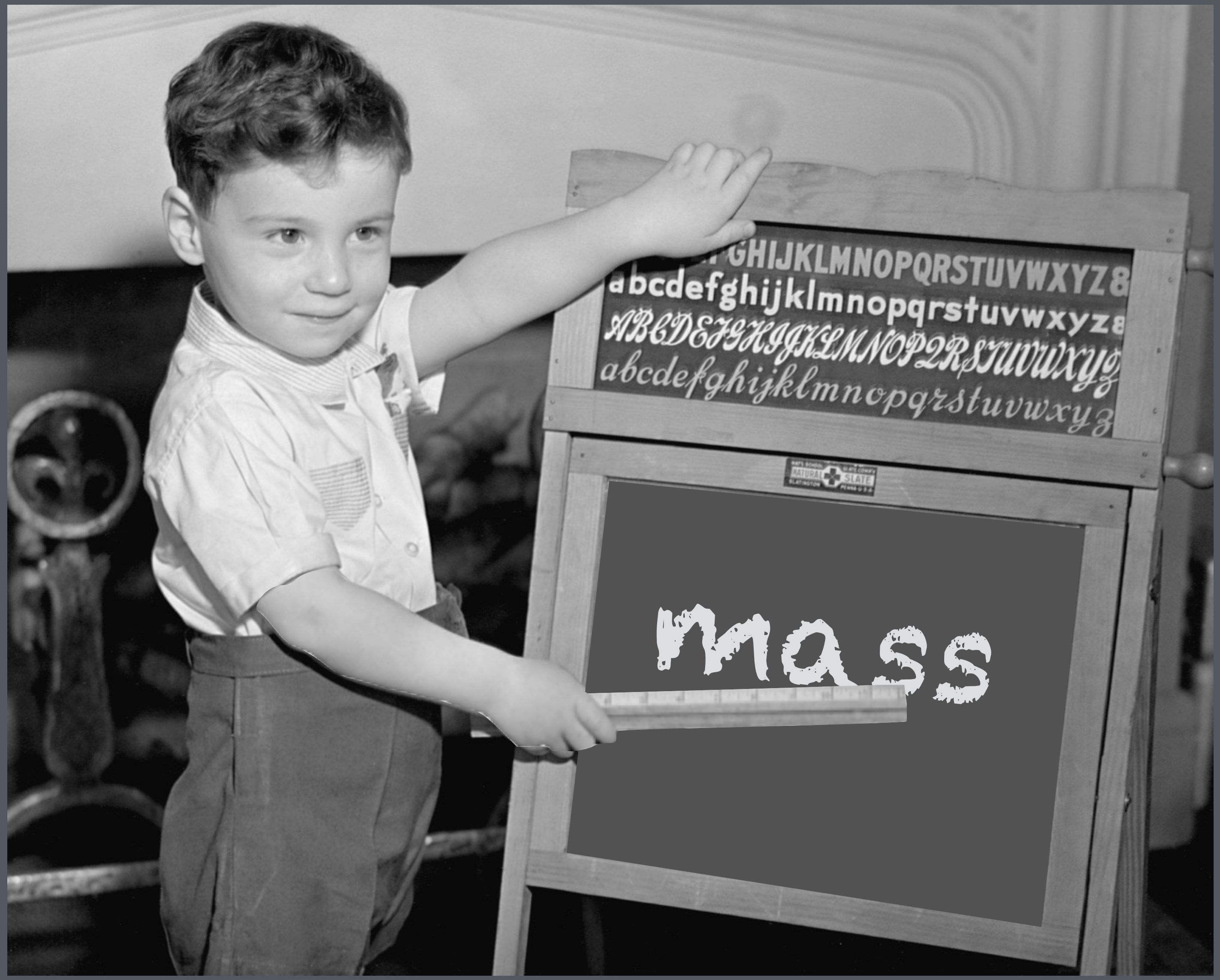
$$E_T = E_m + K$$

what "mass" really is: "trapped energy"

down this rabbit hole

if an object has mass it has energy

if an object has energy it has mass



play some more

look at the total energy

and increase of mass with velocity

$$E_T = m\gamma c^2$$

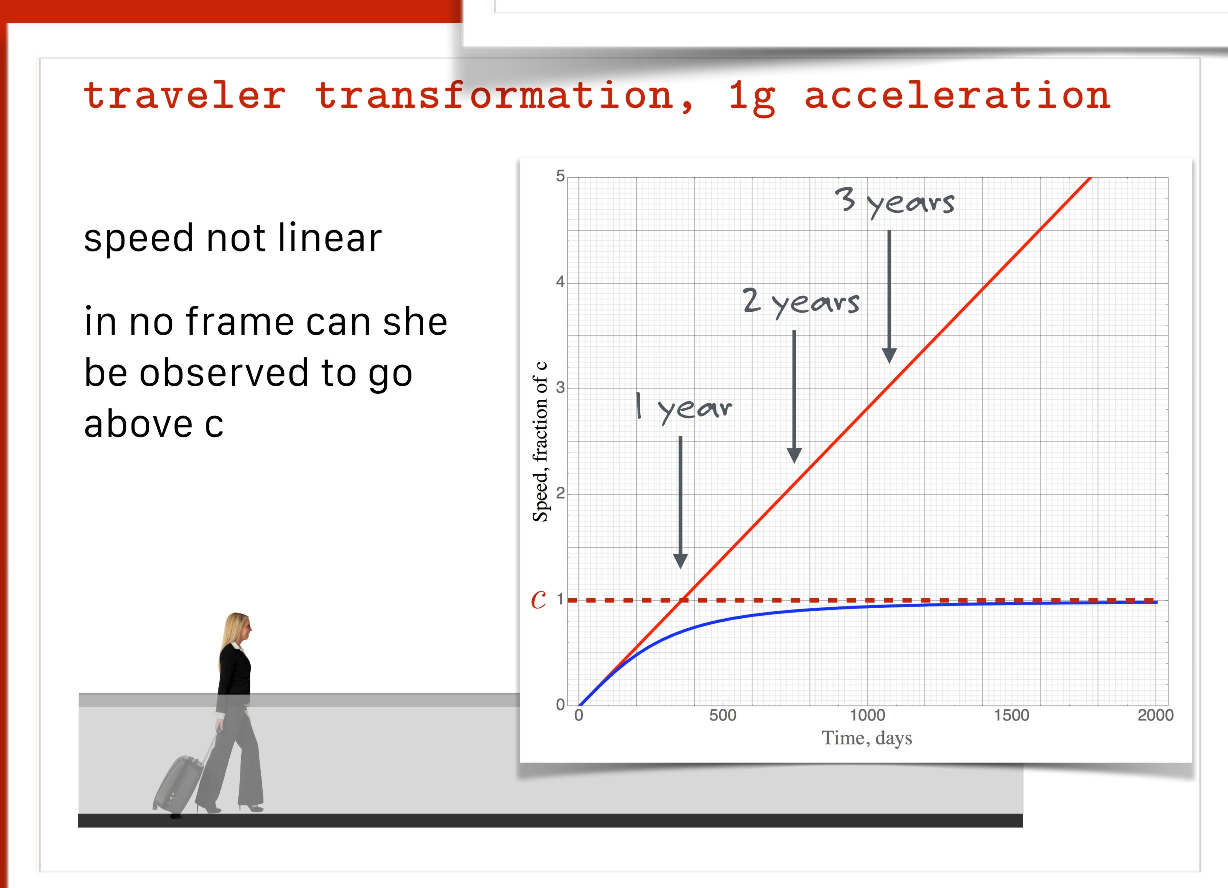
One way to interpret this is to associate gamma and m.

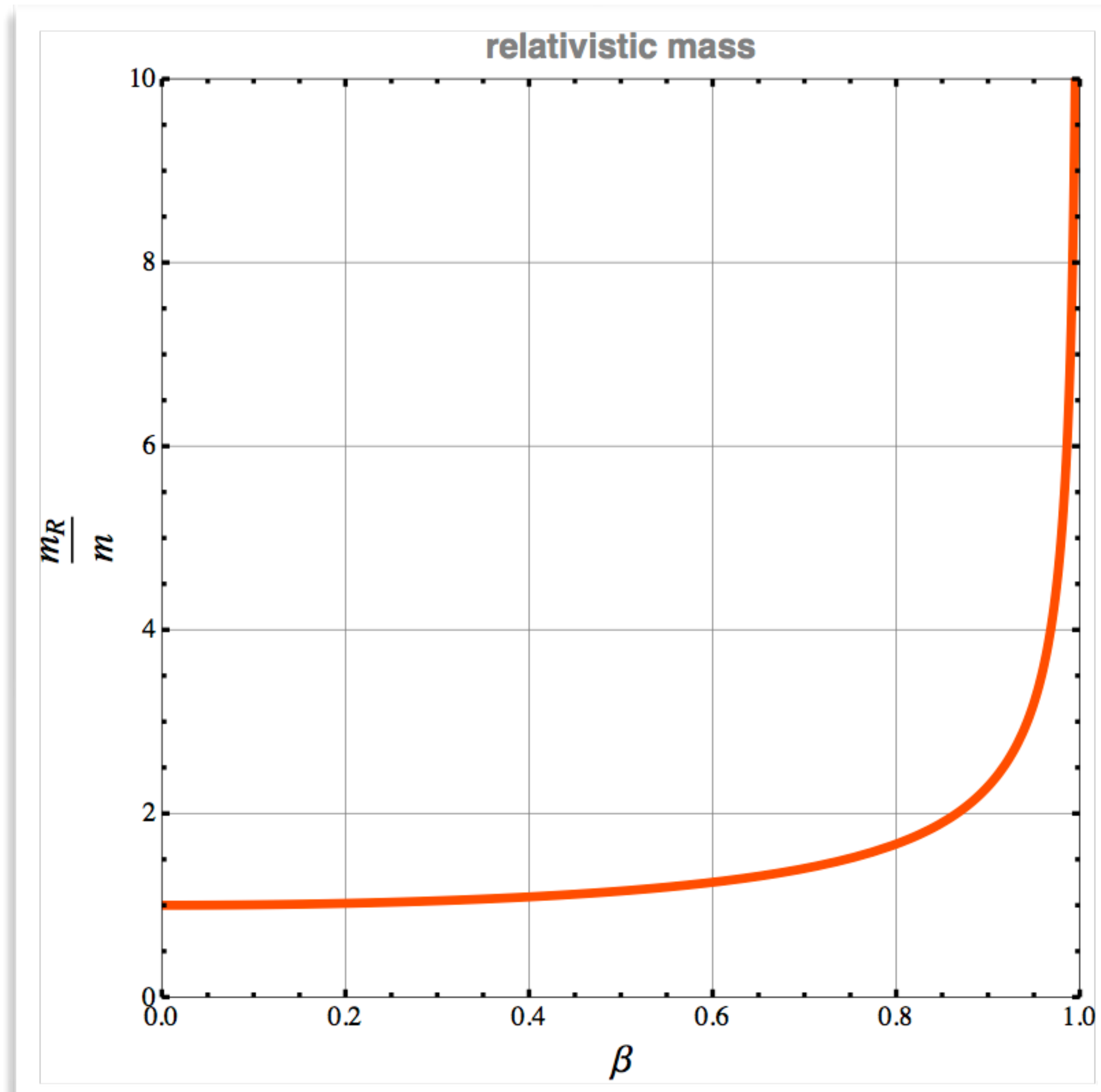
$$E_T = (m\gamma)c^2 \quad \text{so} \quad E_T = m_R c^2$$

...and speak of a speed-dependent "**relativistic mass**."

$$m_R = \gamma m \quad \text{so} \quad \frac{m_R}{m} = \gamma$$

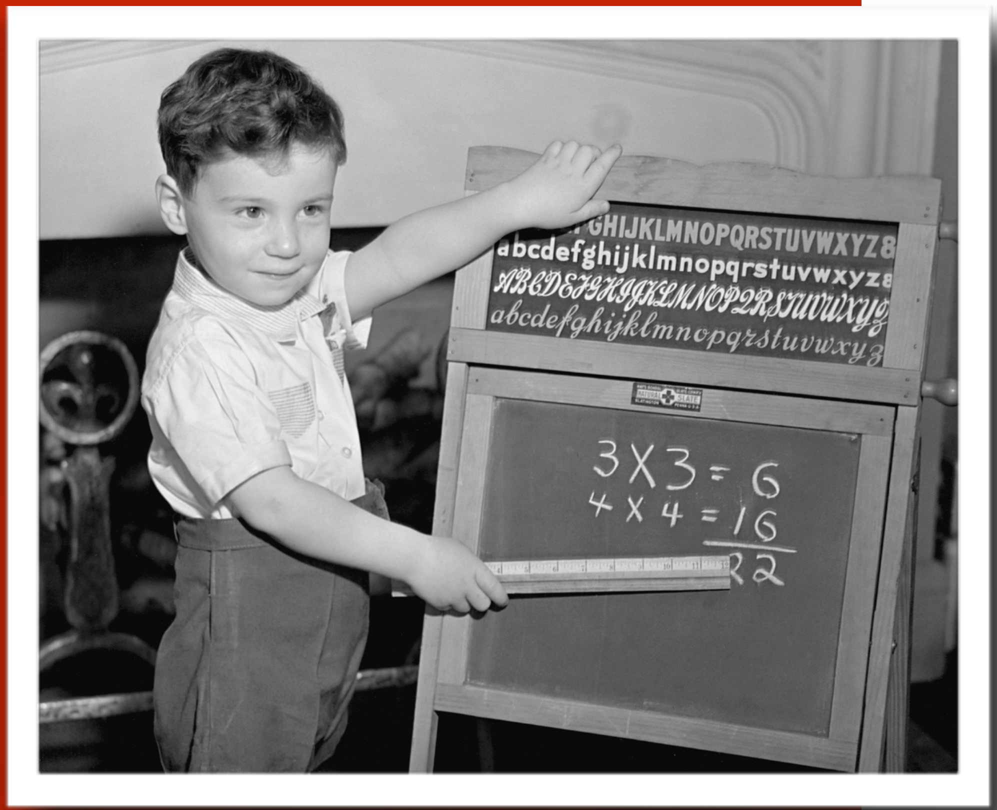
remember the inertia issue?

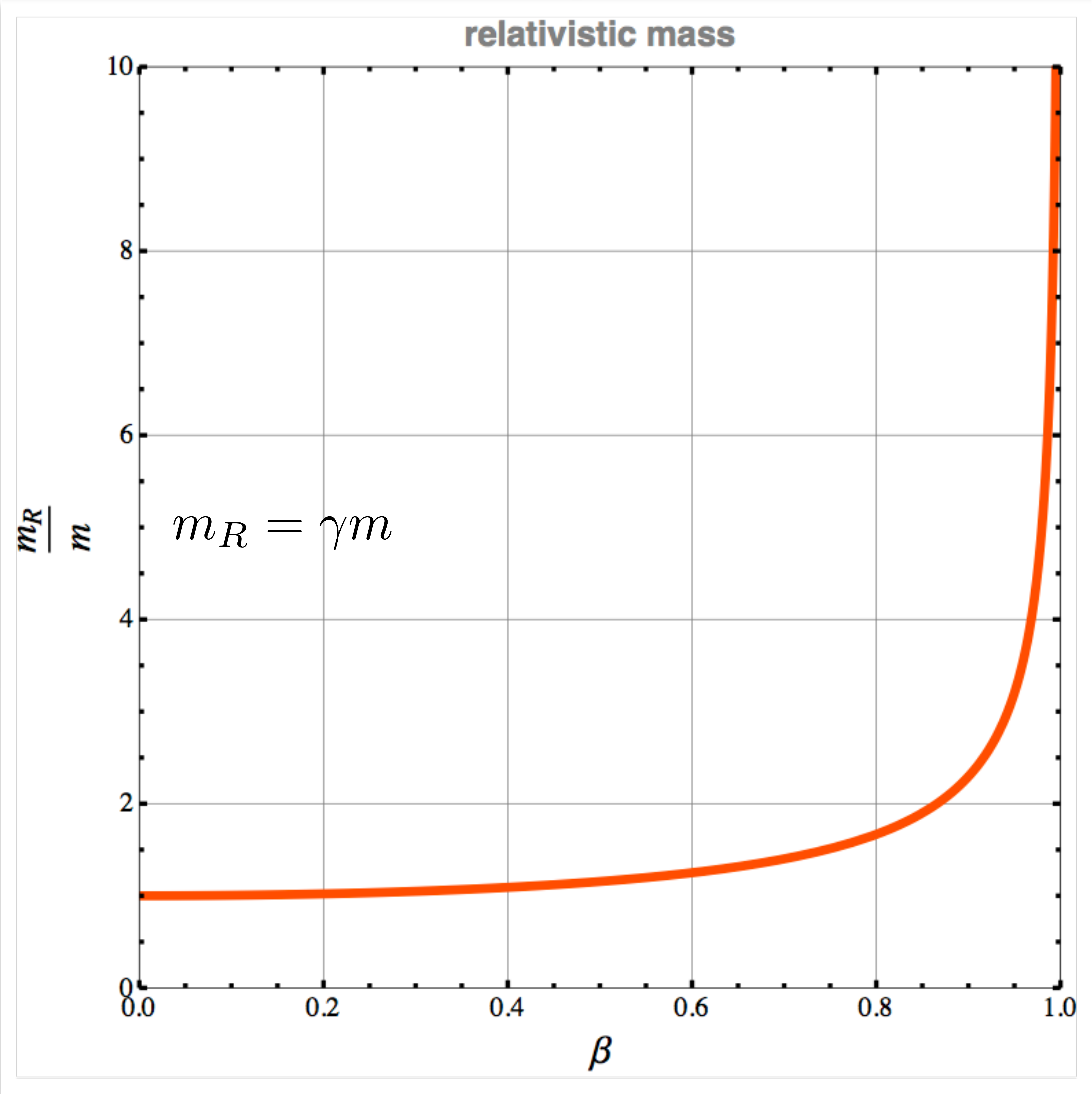




if a proton is going at 0.95% of the speed of light and has mass of 1p

how massive does it appear to be?





let's look at the kinetic energy

energy of motion...Kinetic Energy

\+ energy of rest...associated with mass

Total energy of an object

$$E_T = E_m + K$$

$$K = E_T - E_m$$

$$K = m\gamma c^2 - mc^2$$

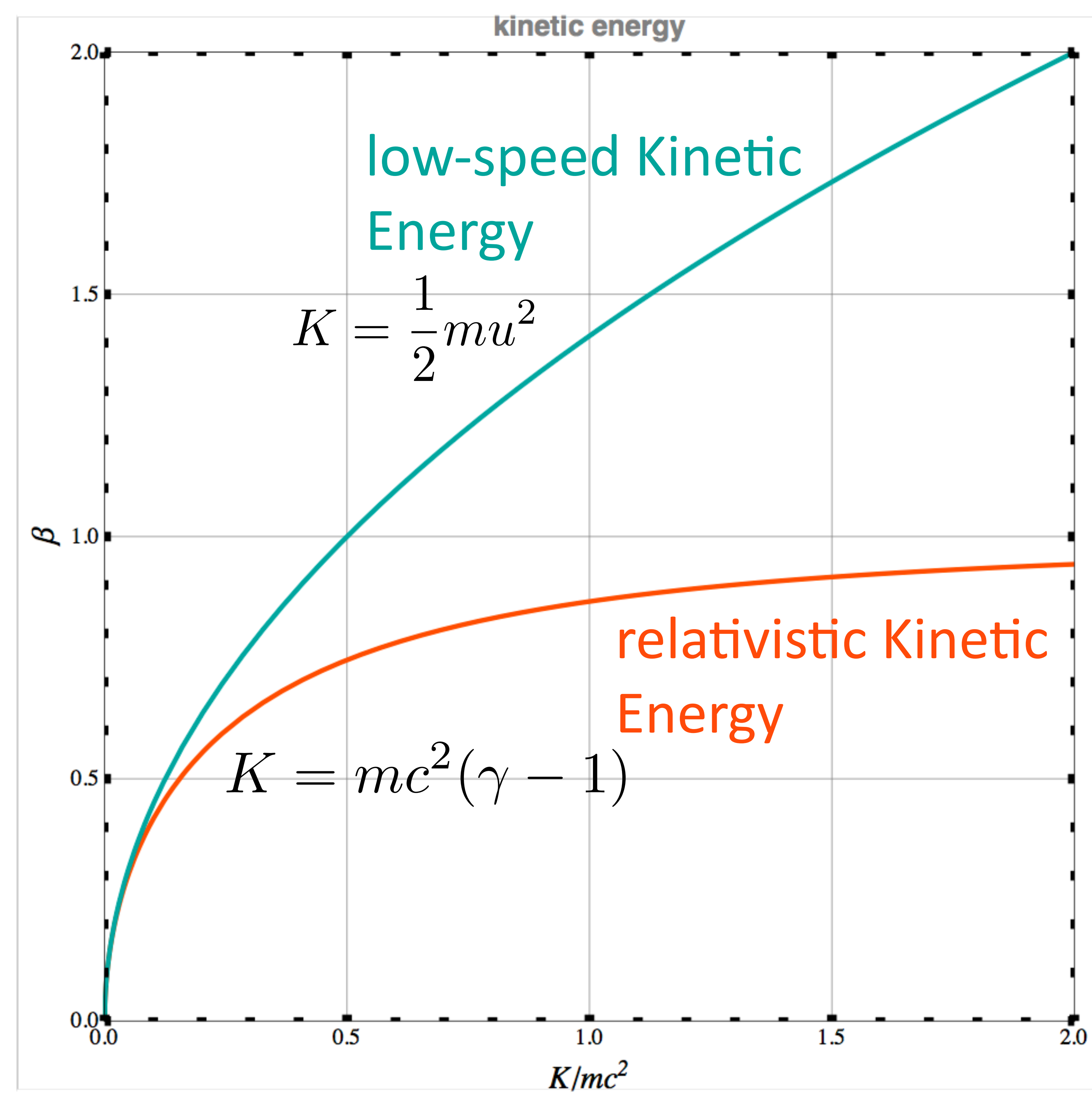
$$K = mc^2(\gamma - 1)$$

mass energy: $E_m = mc^2$

total energy: $E_T = m\gamma c^2$

kinetic energy?

Fully relativistic now



You might want to remember this:

energy of motion…Kinetic Energy

\+ energy of mass…Rest Energy

Total energy of an object

there aren't any other kinds of energy

from this point on:

if I refer to the rest mass*…I'll say so

otherwise, "mass" is this velocity-dependent quantity

*(This is not how we speak in polite particle physics circles…where "mass" is a constant always.)
But, I think for non-specialists this is more clear.

a useful invariant

$$E_m = mc^2$$

$$E_T = m\gamma c^2$$

$$p = m\gamma v$$

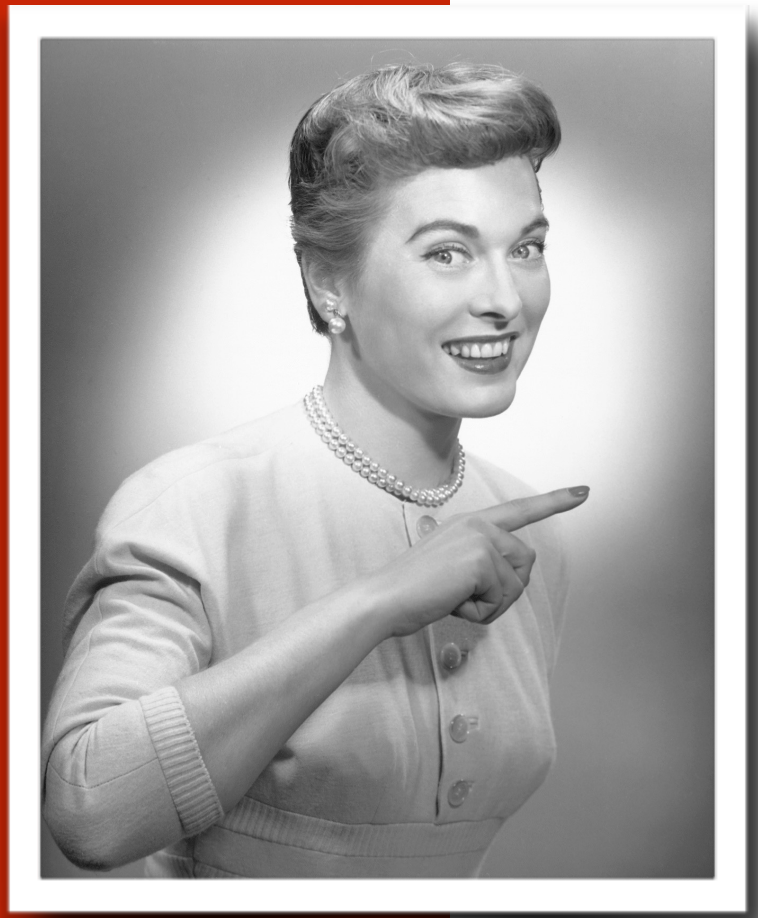
and an important formal linkage

fun fact...just with a little algebra...

$$E_m^2 = E_T^2 - p^2c^2$$

$$m^2c^4 = E_m^2 = E_T^2 - p^2c^2$$

no velocity dependences, just a number...



"Energy-momentum relation"...

$$E_T^2 = (mc^2)^2 + (pc)^2$$

practical

Energy/momentum relations:

“rest mass”... m

the mass of an object in its own frame

“relativistic mass”... $m_R = m\gamma$

the mass of a moving object

“Energy”... $E_T = m\gamma c^2$

the total Energy of a moving object

“rest Energy”... $E = mc^2$

the mass-energy of an object in its own frame

Kinetic Energy... $K = mc^2(\gamma - 1)$

the energy due to motion

Relativistic momentum... $p = m\gamma u$

momentum for each component of space

Energy-momentum relation... $E_T^2 = \left(mc^2\right)^2 + (pc)^2$

an alternative, useful expression

isn’t anything constant?

glad you asked

three things are always, always constant

"Invariants"

c

mc^2

$$m^2c^4 = E_T^2 - p^2c^2$$

↕ ↕

$$s^2 = (ct)^2 - x^2$$

kinship:

t and E

x and p

Einstein preferred "Invariant Theory" to "Relativity"

Cousin Quantities!

- **Space and time** are not separate entities, but linked as "spacetime"
- **Electric and magnetic fields** are not separate entities, but linked as "electromagnetism"
- **Energy and momentum** are not separate entities, but linked as "4-momentum"

real electrons

HV transmission lines feed substations?

138,000 V is common (BWL for example)

Assume that arc is at 138,000V, so electrons have that energy

...which would be the Kinetic Energy

an exercise in "electron volts"

What's the rest energy?

What's the rest mass?

What's the speed of the electrons?

What's the momentum of one of the electrons?

What's the relativistic mass of one of the electrons?

What's the total energy of one of the electrons?

This will be on video and figure into homework *completely inelastic collision*

1 | 12 | 2

a collision from earlier

where mechanical energy was not conserved.

and they stick together

But we certainly would have said: $m_1 + m_2 = M_{12}$

Now...energy conservation is different:

$$E_{(\text{before})} = E_{(after)}$$

$$[E_{(\text{Object 1})}] + [E_{(\text{Object 2})}] = [E_{(\text{Object 12})}]$$

$$E_{m(1)} + K_1 + E_{m(2)} + K_2 = E_{m12} + K_{12}$$

brand new thing!

a collision from earlier

where mechanical energy was not conserved.

completely inelastic collision

But before we certainly would have said:

$$m_1 + m_2 = M_{12}$$

1 | 12 | 2

and they stick together, and stop

systems' energy of masses + KE's = system's energy of mass + KE

brand new thing!

$$m(1)c^2 + m(2)c^2 + K(1) + K(2) = M(12)c^2 + K(12)$$

$$M(12)c^2 = m(1)c^2 + m(2)c^2 + K(1) + K(2)$$

$$M(12)c^2 > m(1)c^2 + m(2)c^2$$

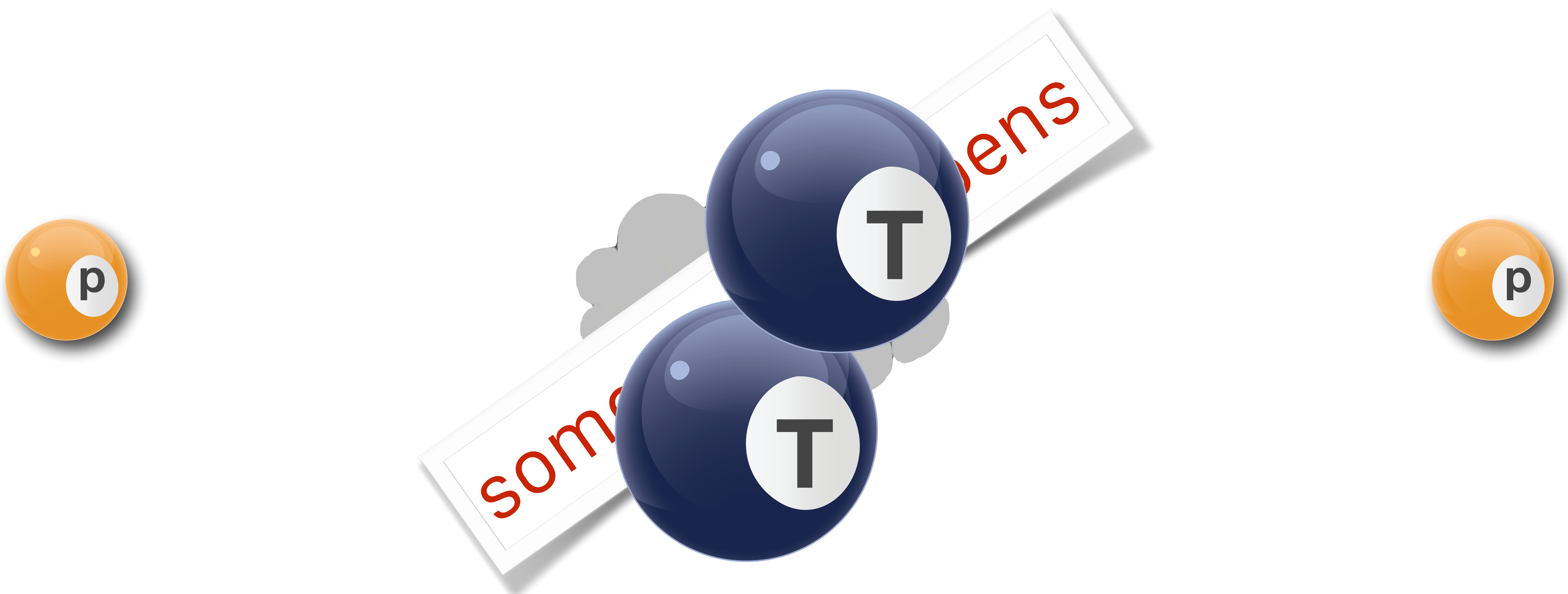
But now, the mass of the stuck-together system is more than the masses of the projectiles...

The energy of motion has become energy of mass.

this is how

we can take two protons, crash them together, and produce 2 “top quarks”...

each of which has the mass of 170 protons



conserved quantities:

3 of them now:

Energy Conservation:

both mass-energy and kinetic energy are counted

Momentum Conservation

energy-momentum conservation

Energy Conservation in a collision:

$$A + B \rightarrow C + D$$

[MassEnergy$_0$(A) + KE$_0$(A)] + [MassEnergy$_0$(B) + KE$_0$(B)] =

[MassEnergy(C) + KE(C)] + [MassEnergy(D) + KE(D)]

$$[m(A)c^2 + K(A)] + [m(B)c^2 + K(B)] =$$

$$[m(C)c^2 + K(C)] + [m(D)c^2 + K(D)]$$

particle colliding beam

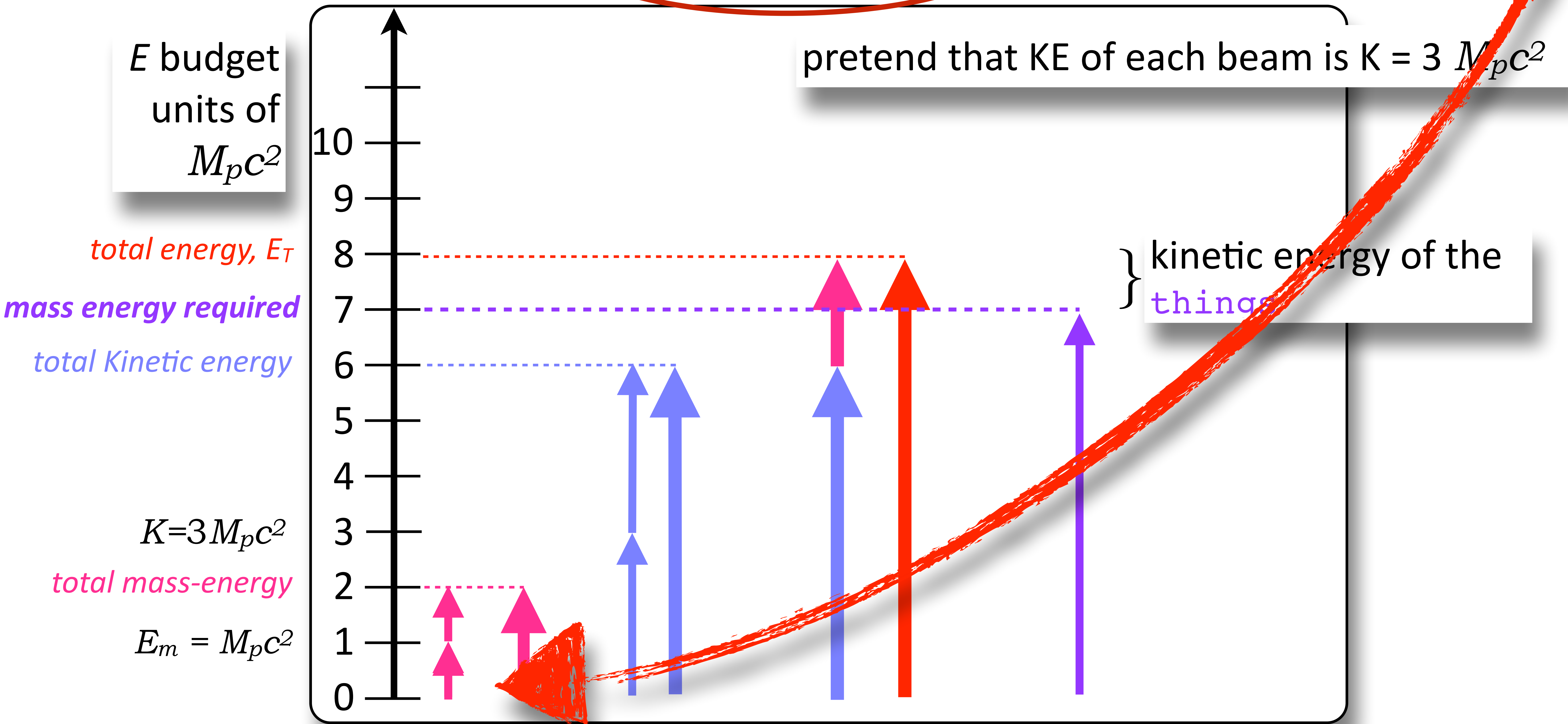
E_T = mass energy + kinetic energy + E_T = mass energy + kinetic energy

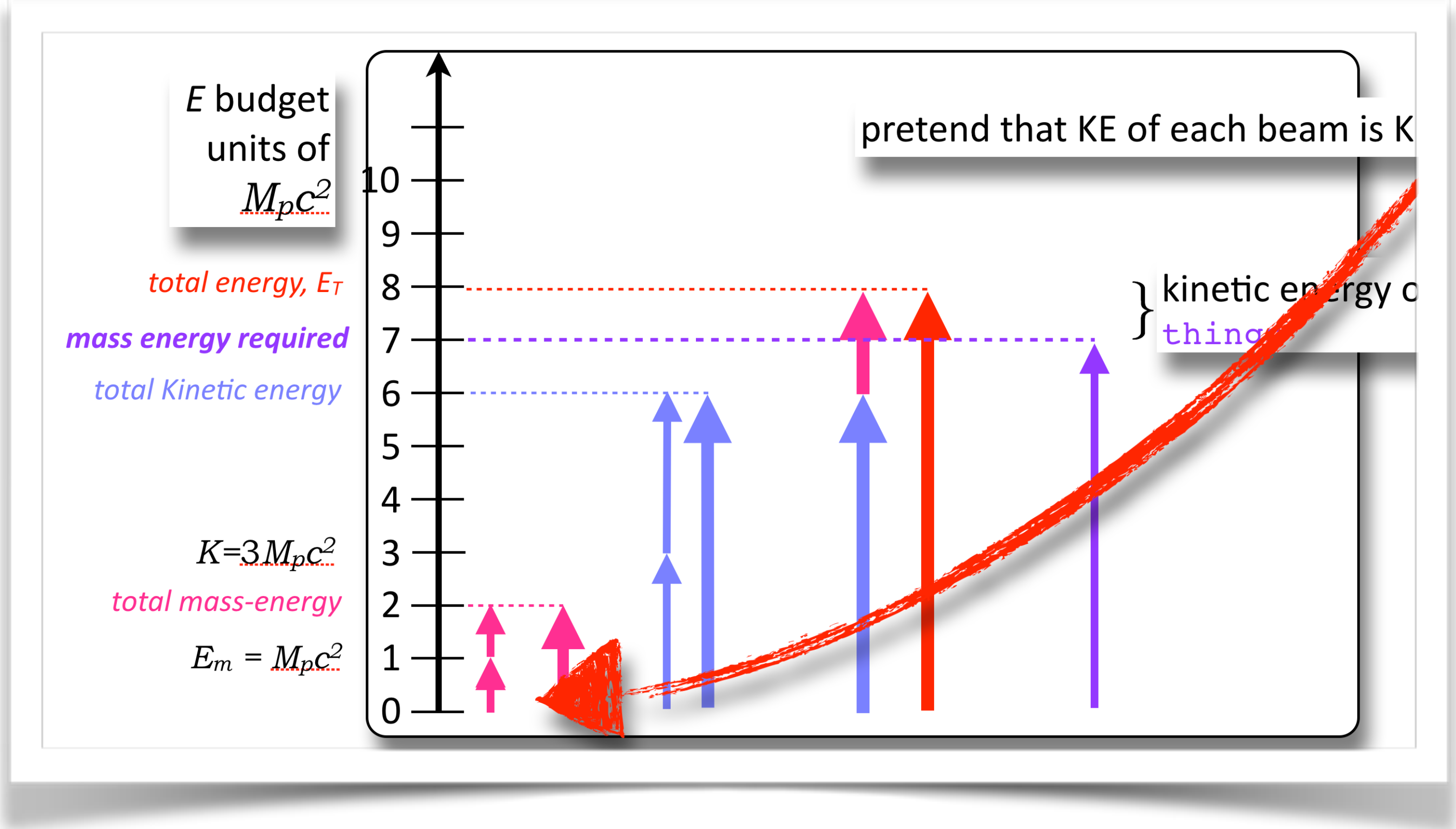
1 proton's mass = M_p

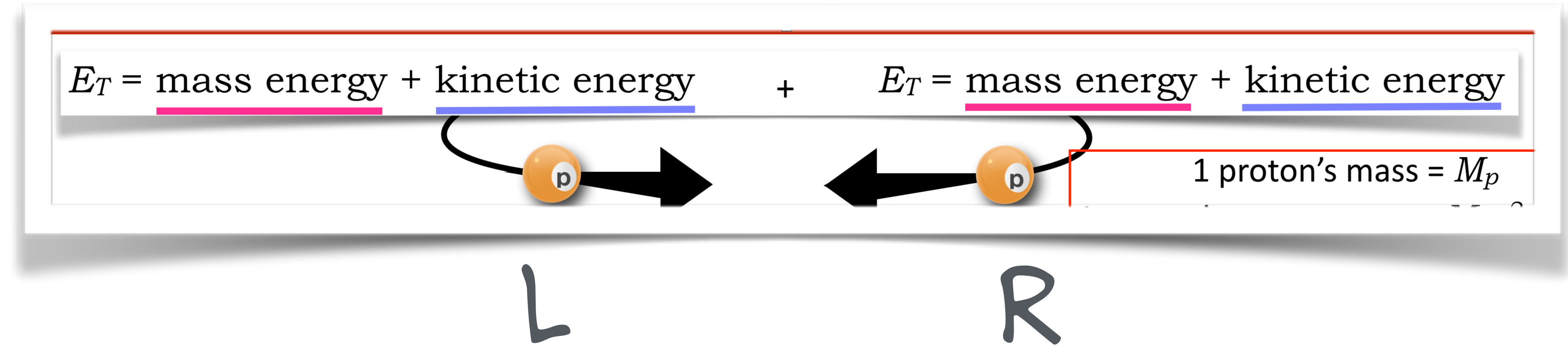
1 proton's mass energy = $M_p c^2$

Use head-on collisions to make objects more massive than protons.

Make Two things that each have $M(\text{thing}) = 3.5 \bullet M_p$

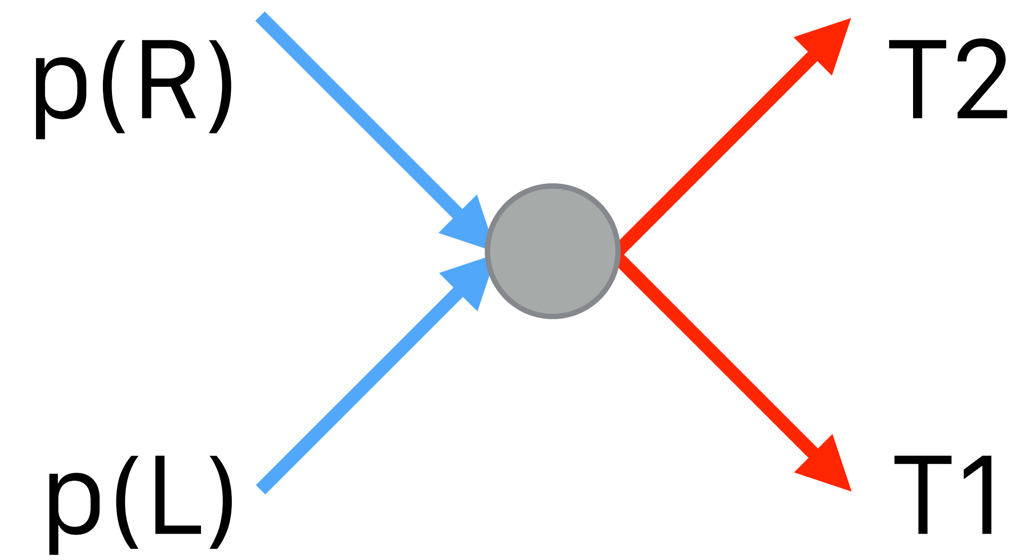






the reaction p(L) + p(R) → T1 + T2

the Feynman Diagram



Energy equation:

$$E_{T,0}(L) + E_{T,0}(R) = E_T(\text{thing1}) + E_T(\text{thing2})$$

1 + 3 + 1 + 3 = 3.5 + 3.5 + ?

8 = 7 + 1

what about the

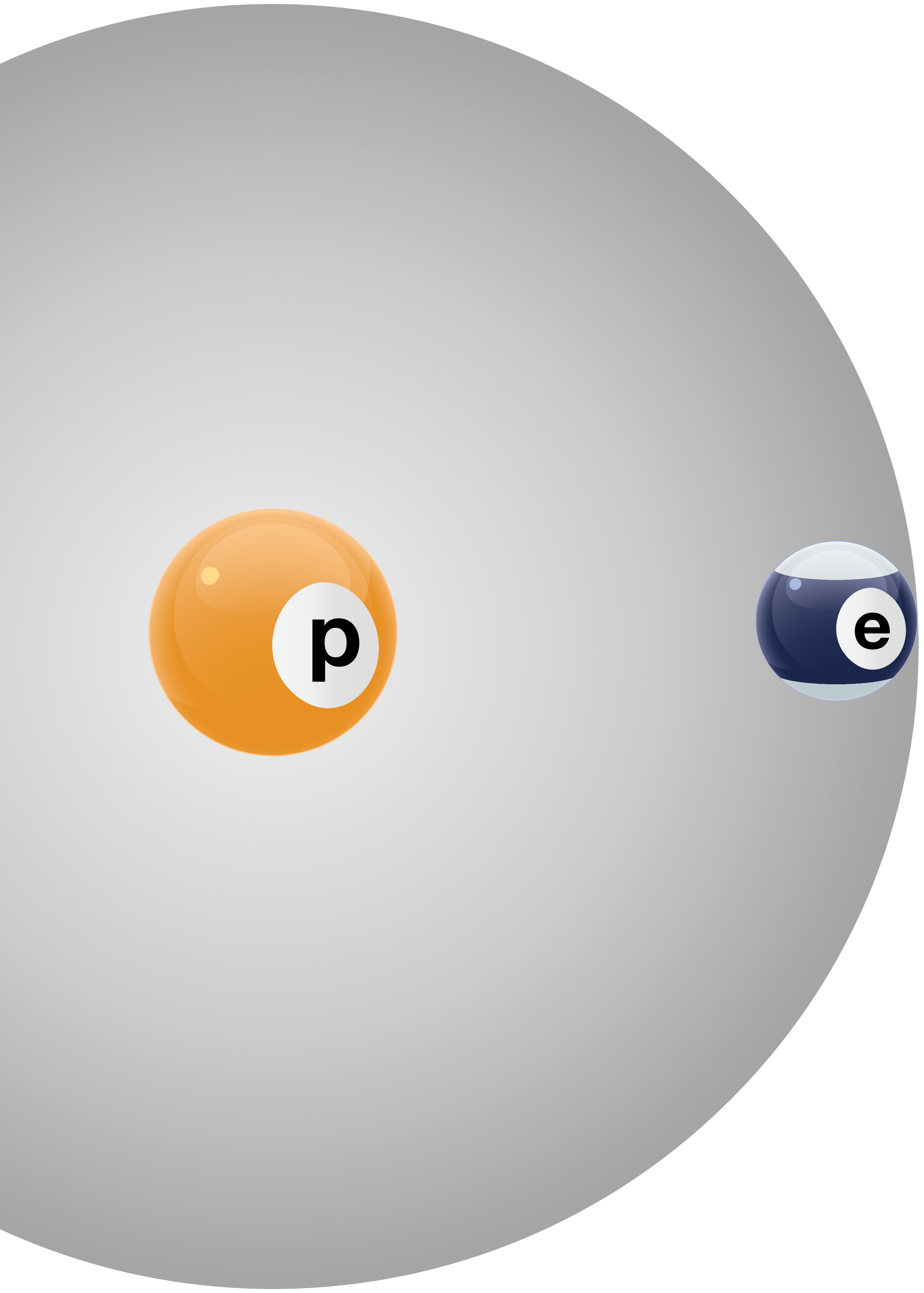
“energy of mass” and “mass of energy” crack?

suppose we have a bound system

What holds the electron to the proton?

Hydrogen Atom

Last week:
the electrostatic force, or the Electric field, right?



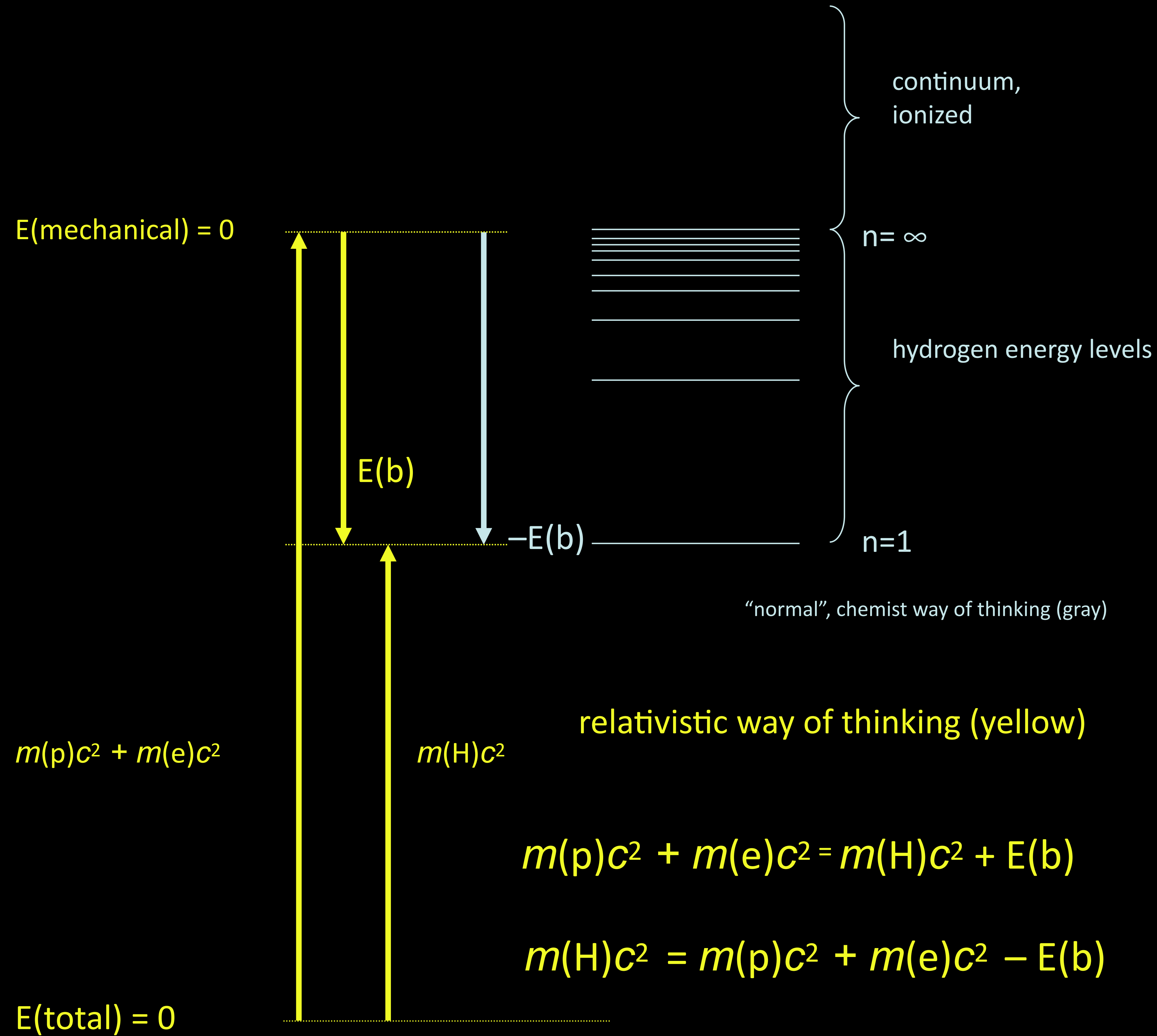
Remember from Chemistry:

What's it take to ionize* Hydrogen?

You must supply 13.6 eV

*make the electron free of the proton's influence

energy diagram for H



$$m(p)c^2 + m(e)c^2 = m(H)c^2 + E(b)$$

$$m(H)c^2 = m(p)c^2 + m(e)c^2 - E(b)$$

The mass of a hydrogen atom is LESS than the sum of $m_p + m_e$
No negative binding energy...just a "mass deficit" in the attraction of the P and e.
The energy is in the field.

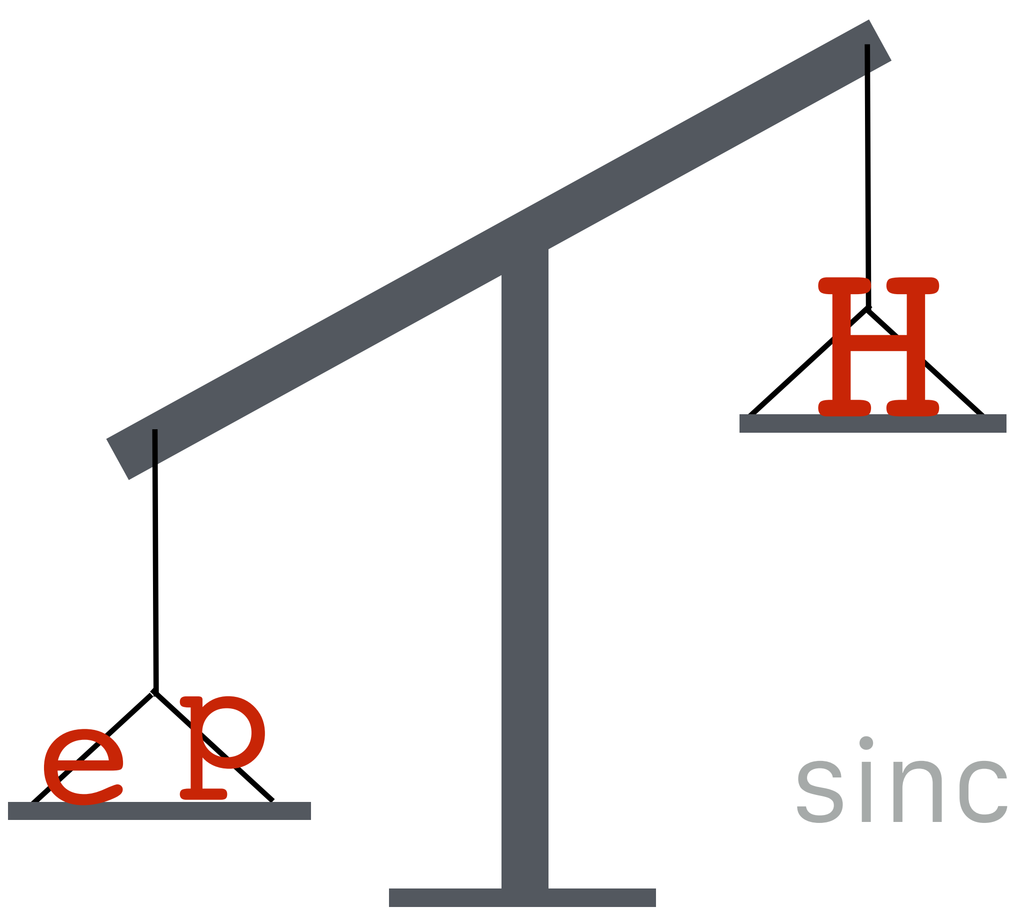
a hydrogen atom, take 2

weighs less than the components of a hydrogen atom

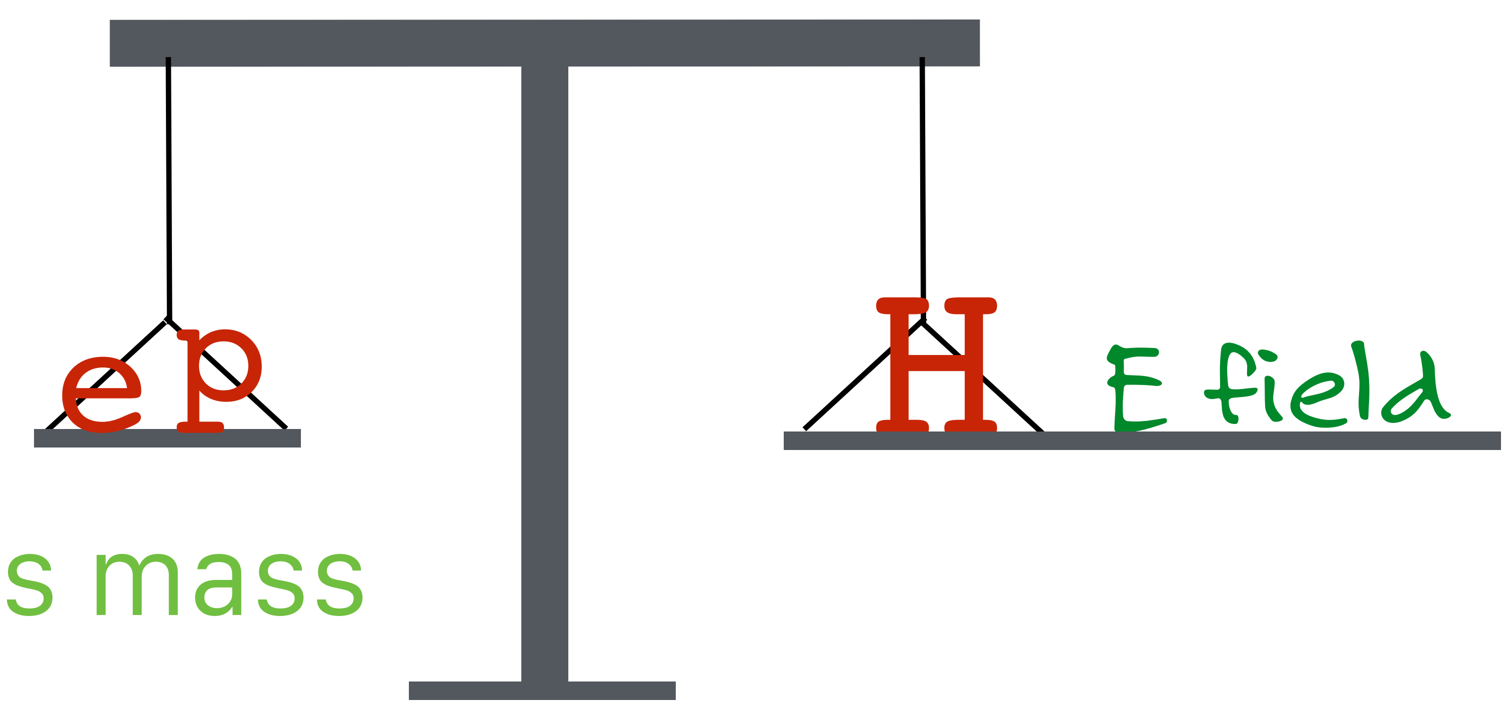
so it can't fall apart into its components

where is that "missing mass"?

in the energy of the Electric Field,



since Energy is mass

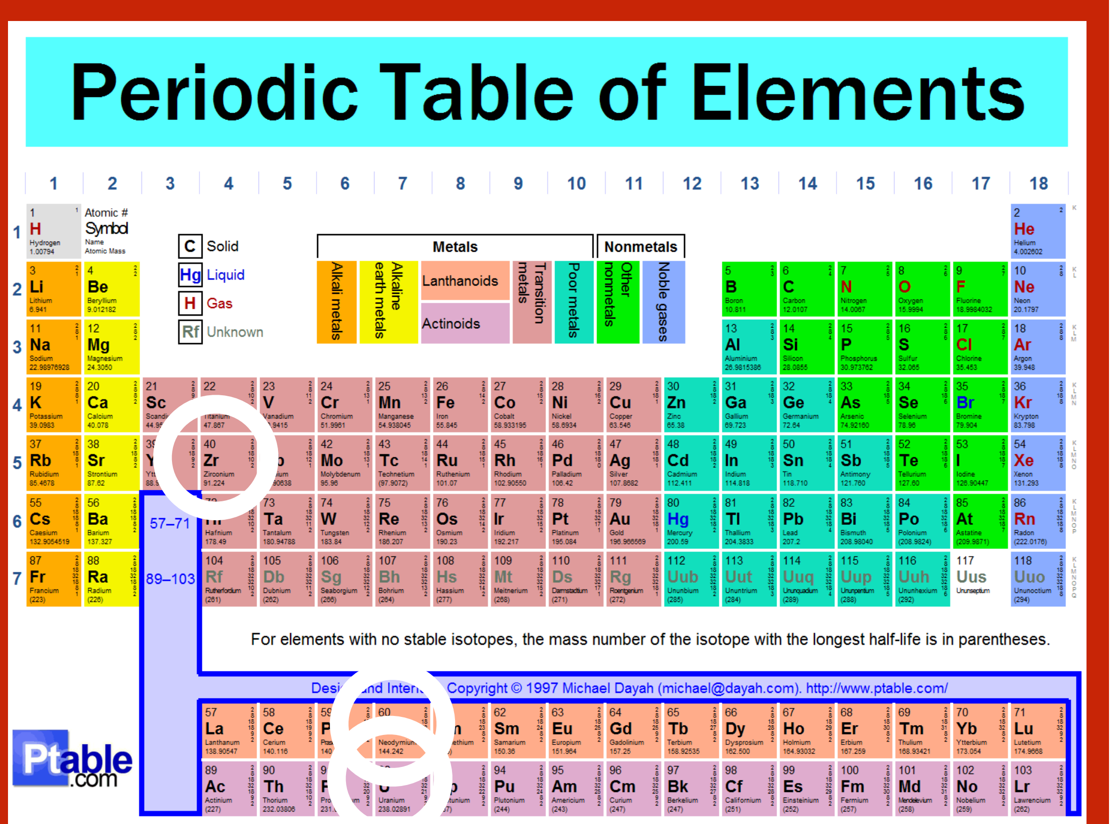


the ‘‘mass deficit’’ in nuclei

is observable and works for good and for ill.

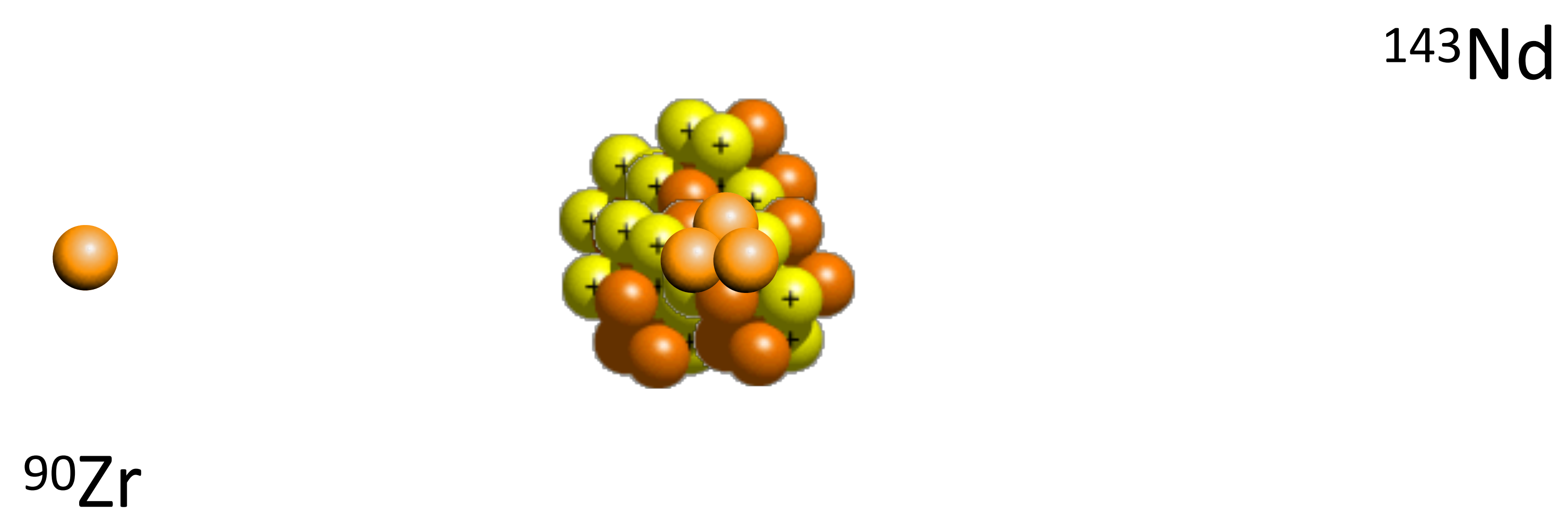
a bound system like an atom

but much stronger!



It happens many ways, here is one:

"Uranium 235" is a big nucleus of 92 protons and 143 neutrons



the M(^{235}U) < 143 x M(neutron) + 92 x M(proton)

so, it's "bound" like Hydrogen

But when a neutron tickles it...

the mass deficit in binding energy is released as K...which becomes heat in nuclear reactors

M(^{235}U) + M(neutron) > M(^{143}Nd) + M(^{90}Zr) + 3 x M(neutron)

by 200 MeV

1 gm ^{235}U releases 23,000 kW-h

about 25 households' energy needs

looky here...

two things to worry about

$$E_T^2 = \left(mc^2\right)^2 + (pc)^2$$

Energy and momentum are related for massless objects...

$$E = pc$$

What about the negative solution?

$$E_T = \pm\sqrt{(mc^2)^2 + (pc)^2}$$

so, how was this all received?

According to Einstein's sister,

…he anticipated a large reaction with much criticism

What he got at first was silence.

oh, a nice note from Max Planck asking for some clarification

then a seminar by Planck in Berlin which touched on Relativity…

- only then… a little professional attention, to "Prof. Einstein, University of Bern"

The first paper published on Relativity by not-Einstein:

also by Planck, who derived the relativistic momentum relation, $p = \gamma m v$

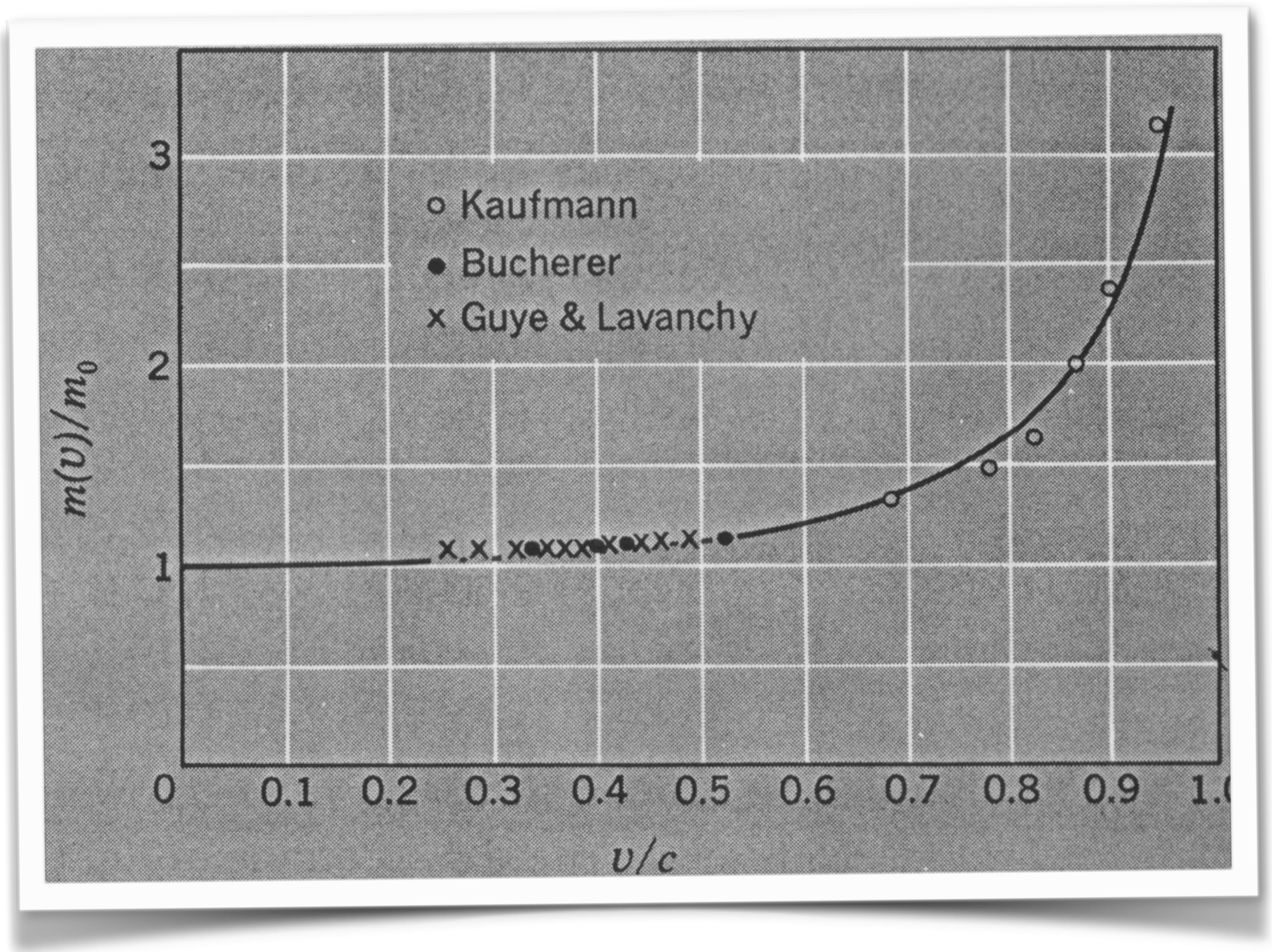
The 1908 Minkowski lecture, in which he worked out completely in modern form the mathematics of relativity and the spacetime view got people's attention

What about experiment?

the first experimental confirmation

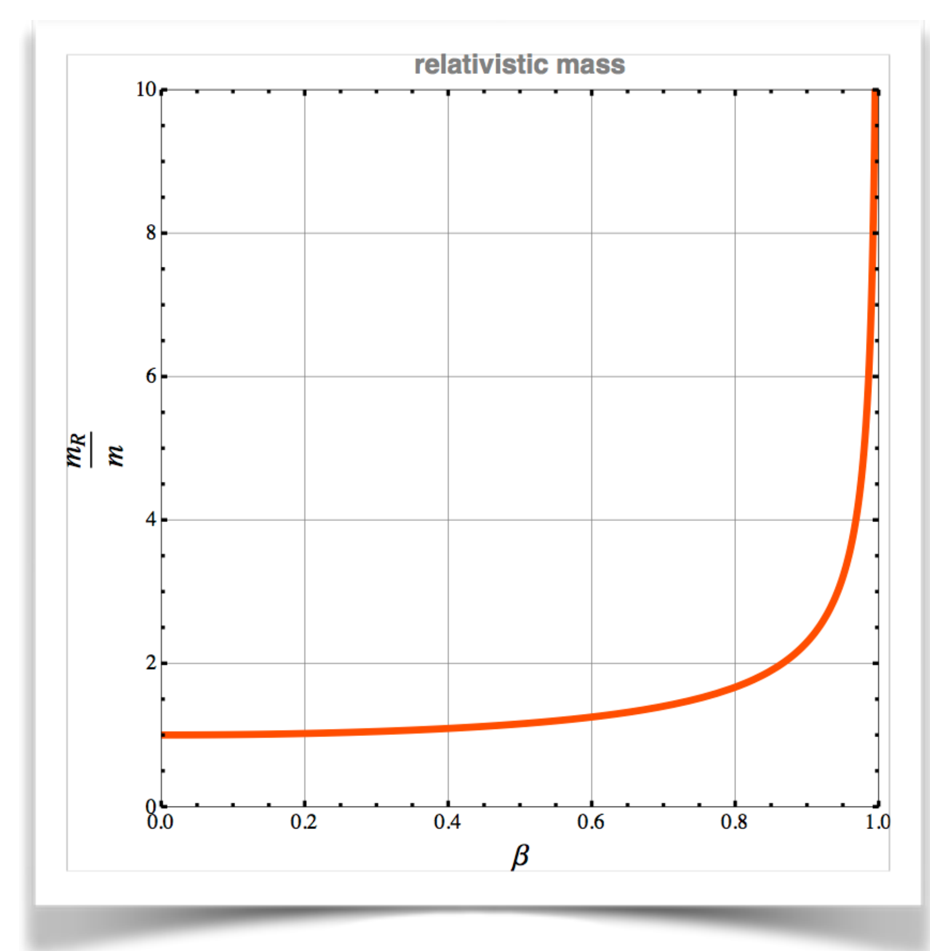
New experiments were done,

and by 1910, the results were:



These results are from 1910 for three experiments, and the curve is the special relativity prediction

From this point on relativity has become a part of everyday scientific and engineering life



Kaufmann lost again…Max Planck corrected his analysis

shift gears

special relativity → general relativity

Special Relativity

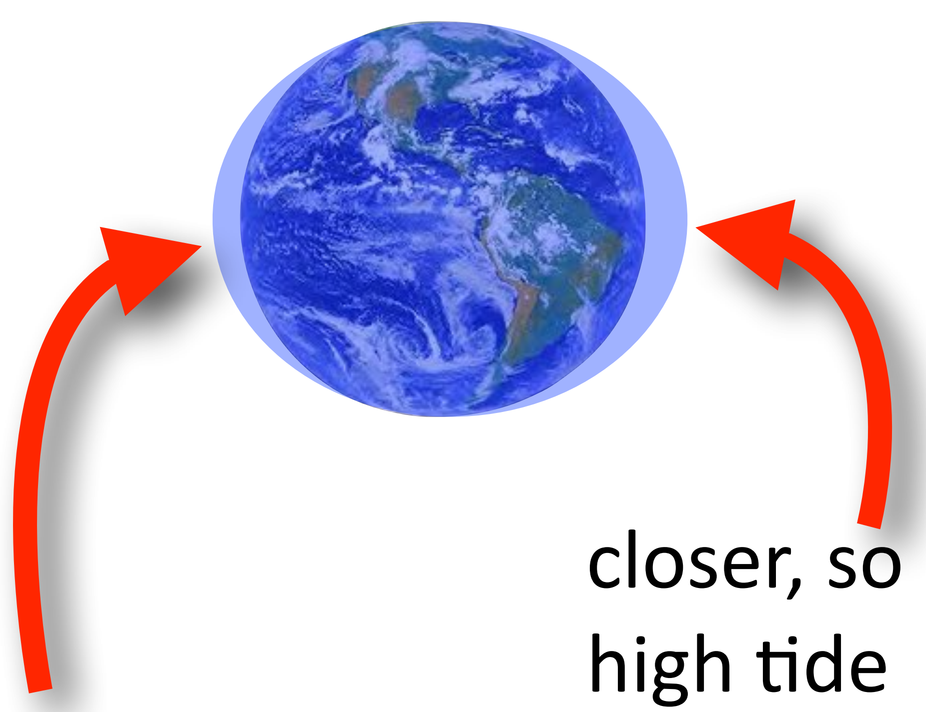
created a problem

What about most of the universe...?

Where gravitation is a fact of life?

In particular...the action-at-a-distance thing.

Think about the tides...caused by Moon



closer, so stronger,
high tide

further, so weaker,
low tide

Suppose Moon disappeared?

Would the tides flatten instantly?

That violates the rules of Special Relativity

what's worse

How do you deal with Newton's Universal Gravitational formula?

Masses appearing different from different frames?

$$F_{1,2} = G\frac{M_1 M_2}{R^2}$$

Start length-contracting the distance?

Worrying about Gravity led Einstein to

think hard about

SPACE and TIME

~~moving~~ coordinate systems

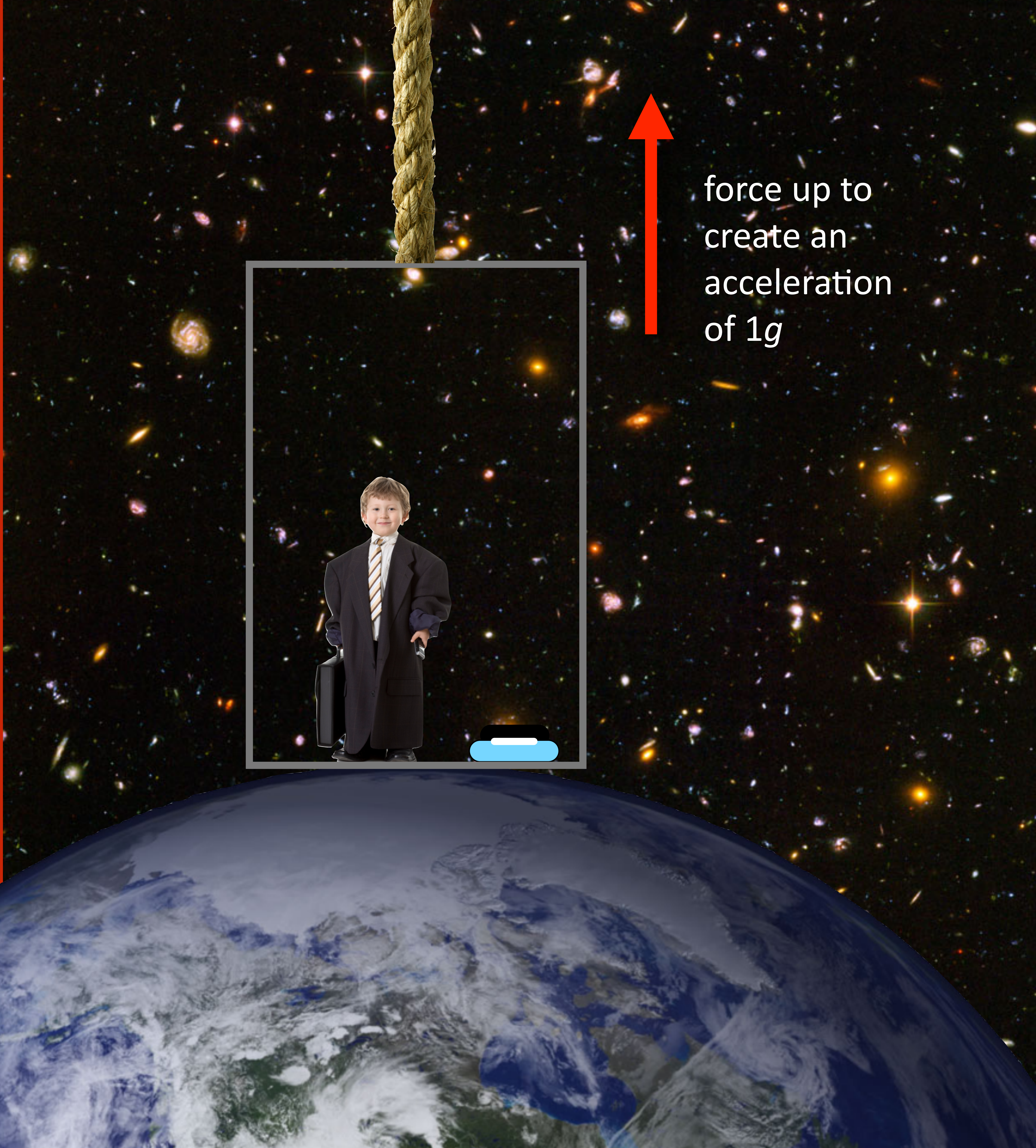
accelerating

stupid elevator trick, #1

gravitational attraction

stupid elevator trick, #2

gravitational attraction



Here comes a Relativity-like statement:

similar to Galileo’s ship-hold…

“you can not perform any mechanical experiment to tell you that a ship is moving at constant speed relative to the land.”

or Einstein’s…

“you can not perform any mechanical or electromagnetic experiment to tell you that a ship is moving at constant speed relative to the land.”

There is no mechanical or electromagnetic experiment he can perform

that would tell him that he was

1. being attracted by the Earth **due to gravity** or

2. being pulled **and accelerated** g with no gravitational field anywhere

There is no mechanical or electromagnetic experiment he can perform

that would tell him that he was

1. being attracted by the Earth **due to gravity** or

2. being pulled **and accelerated** g with no gravitational field anywhere

said another way

any effect in an accelerated rest frame

should occur in a rest frame at rest in a gravitational field