hi

Day 24, 05.10.2018

Quantum Mechanics 3

housekeeping

Gotta come to class

question about anything? I'll make a movie for you:

Quantum Mechanics:

Readings: Oerter, Cosmic Perspective, and Hobson

Hobson_QM1.pdf & Hobson_QM2.pdf are chapters 12 & 13 out of Hobson

Homework #11 is all from MasteringPhysics - due saturday, rather than friday

honors project began

https://qstbb.pa.msu.edu/storage/Homework_Projects/honors_project_2018/

contains:

- the first instructions: the plan & tutorial
- the second instructions – v2 uploaded, added a missing student
- the data, assigned by name in the second instructions

dates:

- complete first part, March 16
- analyze data by April 24 and hand in complete writeup at the final exam

where we are…by 1926

Planck/Einstein: light is both a wave and a particle

energies "quantized" $E = nhf \quad \lambda_\gamma = \dfrac{h}{p_\gamma}$

Rutherford/Bohr: the atom consists of a hard, positive nucleus surrounded by negative electrons

electronic transitions

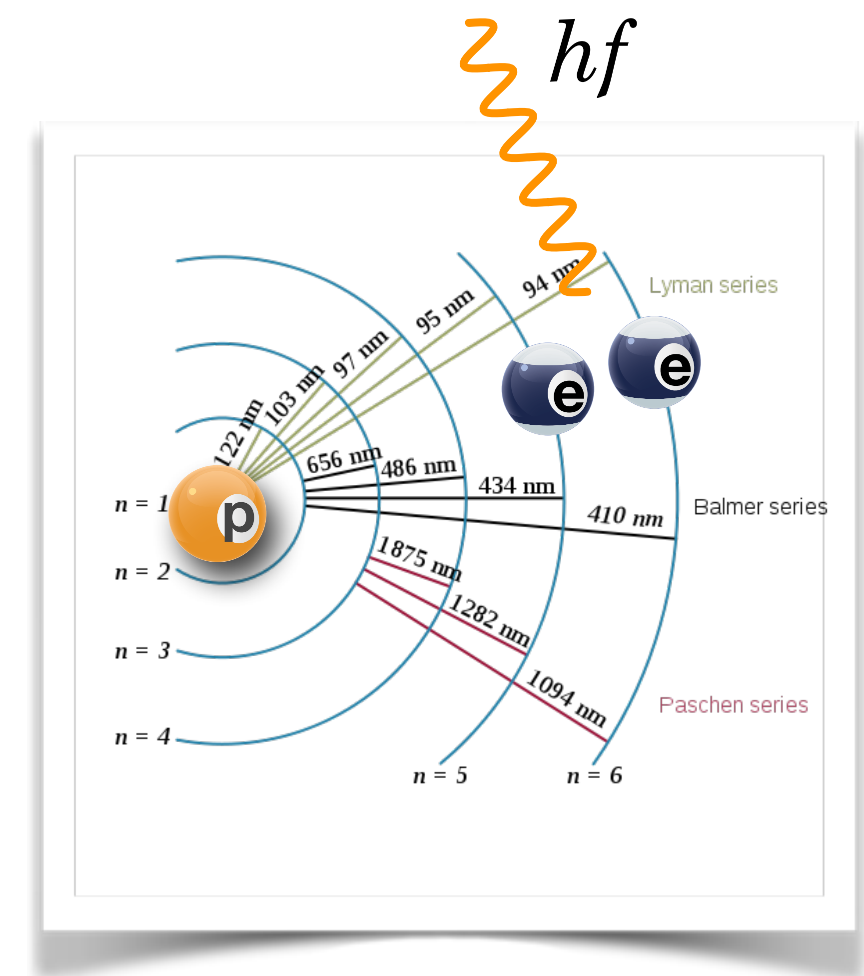
Rutherford: radioactivity is nuclei emitting alpha (2n2p), beta (e), and gamma "rays" (γ)

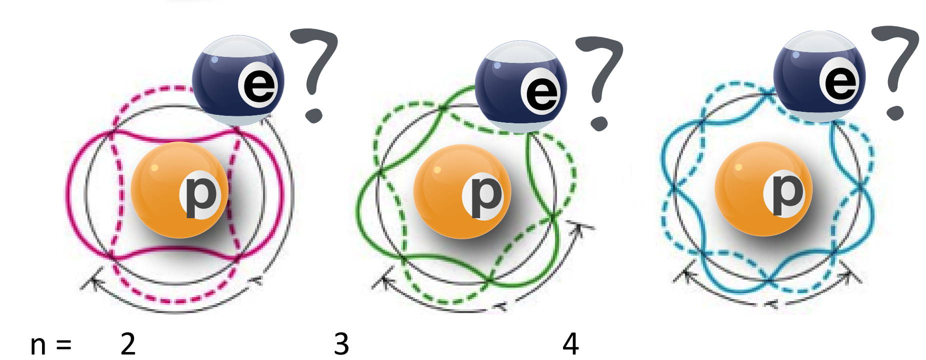
deBroglie: electrons are both waves and particles

momentum related to wavelength $\lambda_e = \dfrac{h}{p_e}$

Schroedinger: electrons' quantum behavior governed by the "wavefunction"

absolutely calculable, but a wave in a complex space ψ



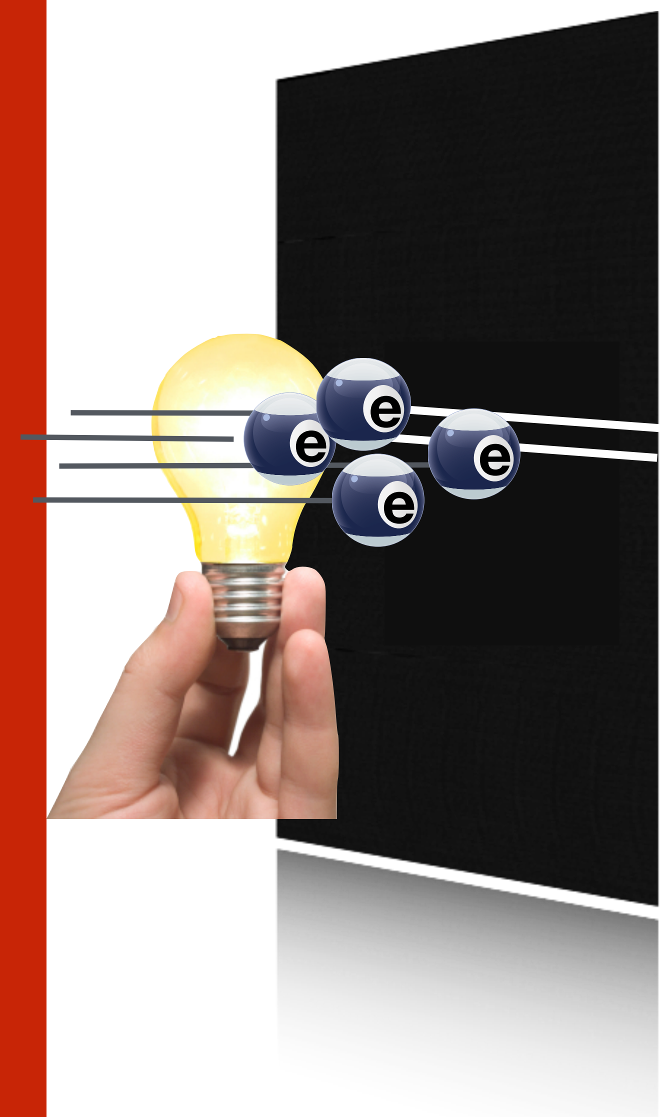


here's how it works

let

light or electrons

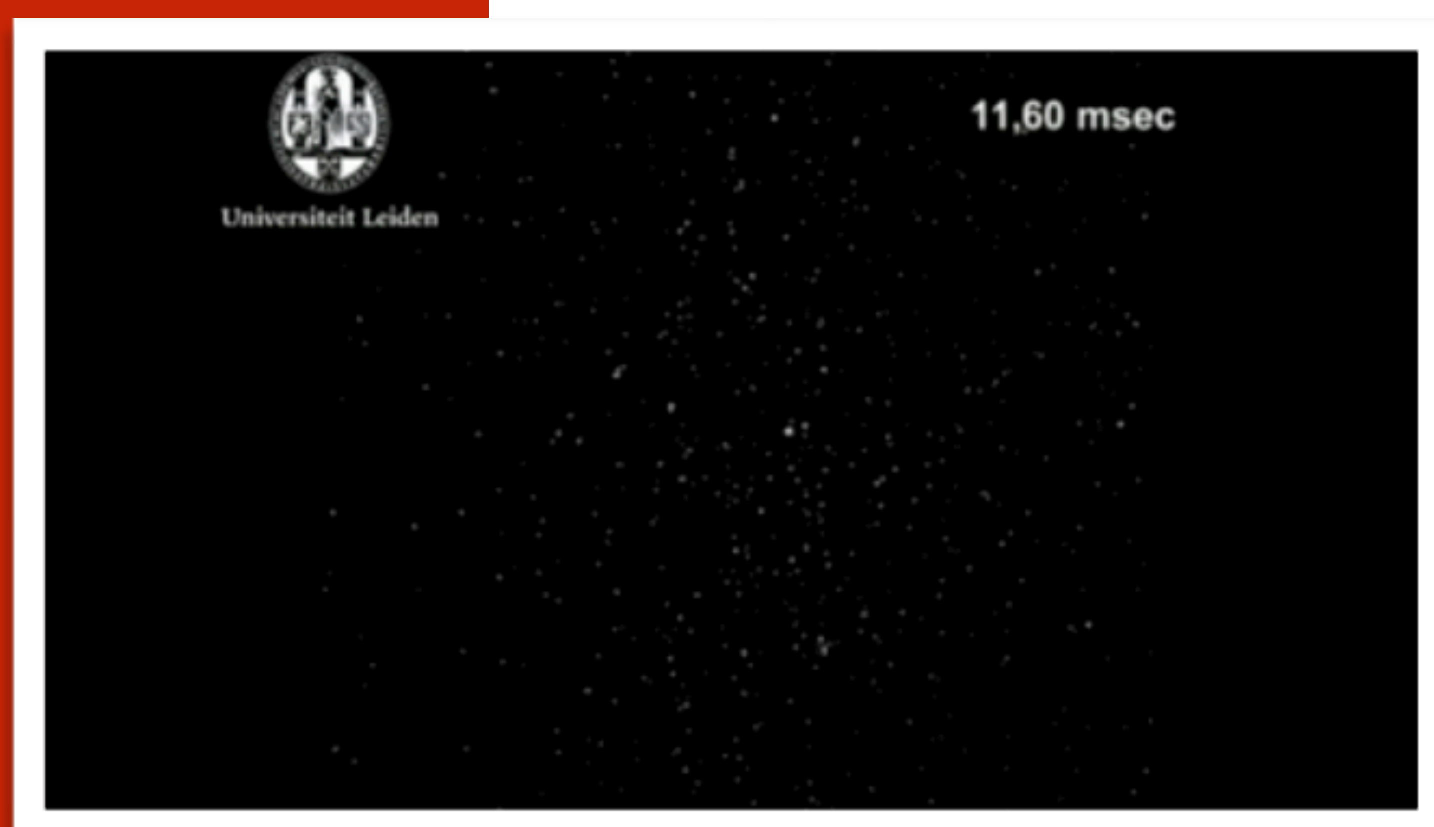
go through a double slit



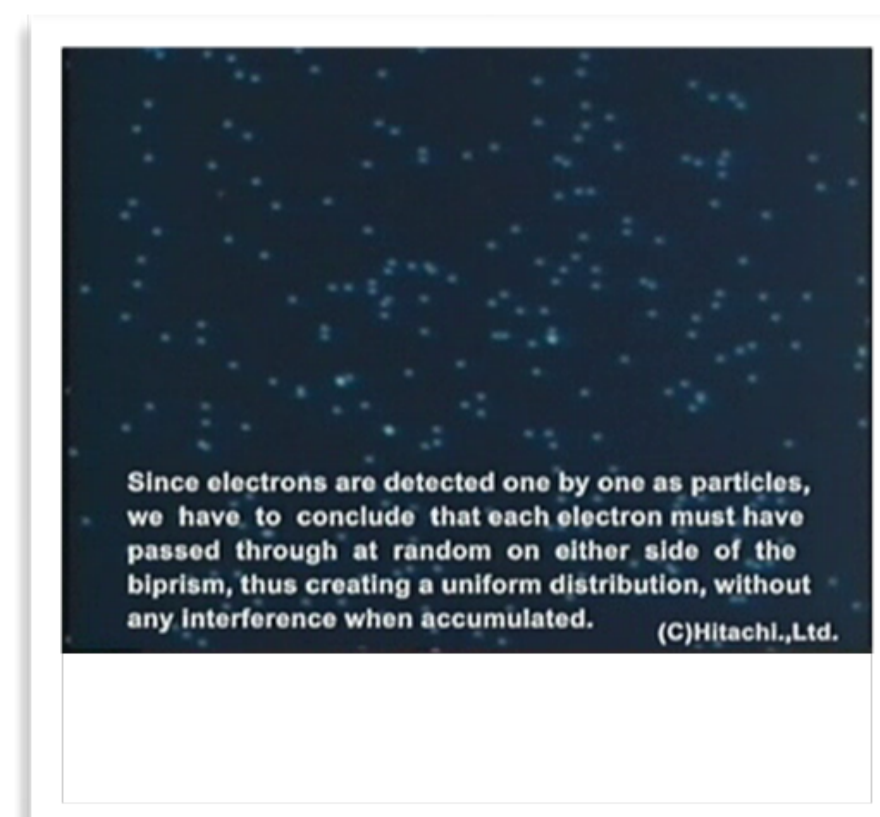
individual light particles

or individual electrons

photons:



electrons:



WHAT’S WAVING??

“wavefunctions”

...but they’re imaginary!

$$\psi(x, t)$$

Schroedinger had to work with real quantities

built from the **imaginary** quantum field function

With only a half-baked clue of what he was doing.

Remember what imaginary quantities are?

$$i = \sqrt{-1} \longrightarrow A = a + ib$$

has both real and imaginary parts

Nature... does "Real."

So, Schroedinger created a real number out of ψ

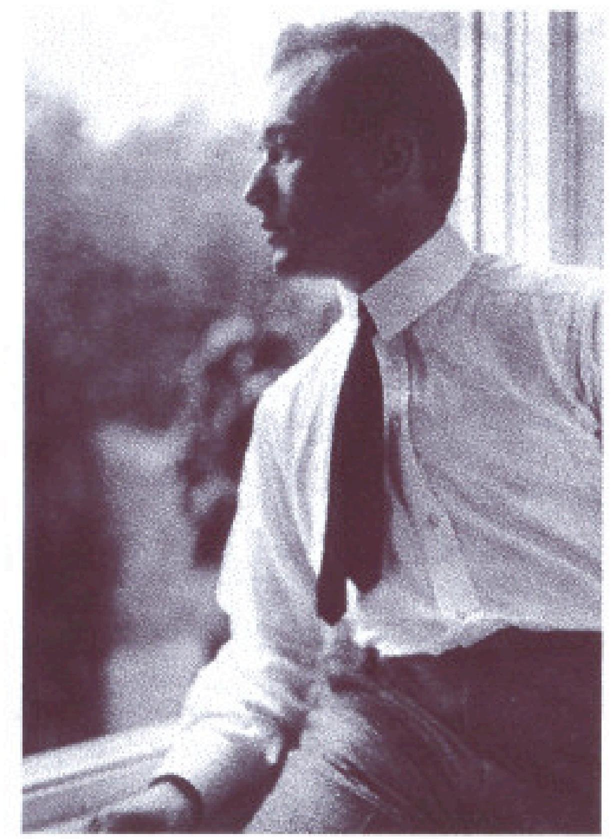
The "complex conjugate" of A is: $A^* = a - ib$

And a real combination of them is the "norm" $|A|^2$

$$AA^* = (a + ib)(a - ib)$$

Schroedinger thought that $|\psi|^2$ might refer to the distribution of electrons' electrical charge.

Sandy: Oh Danny, is this the end?
Danny: No Sandy. It's only the beginning.



"I am now convinced that theoretical physics is actually philosophy."

probably, it's probability

Max Born's inspired suggestion:

$|\psi|^2$ *is the **probability of finding the electron***

a measure of the likelihood that an electron will be at a given place at a given time…that's all we can know

then: no radiation problem…since the electron is not actually orbiting

We calculate the **shape of its probability density**

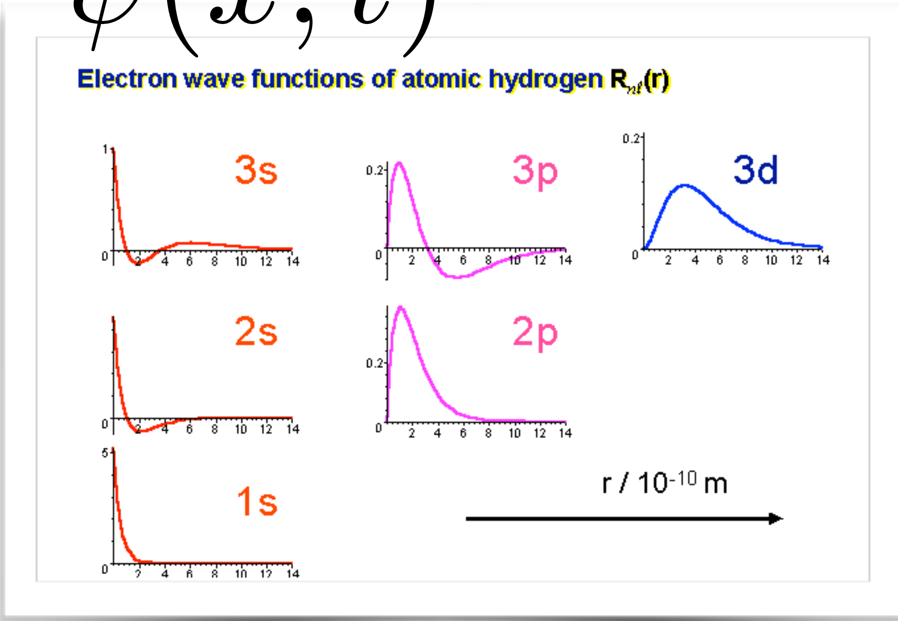
*a **probability***

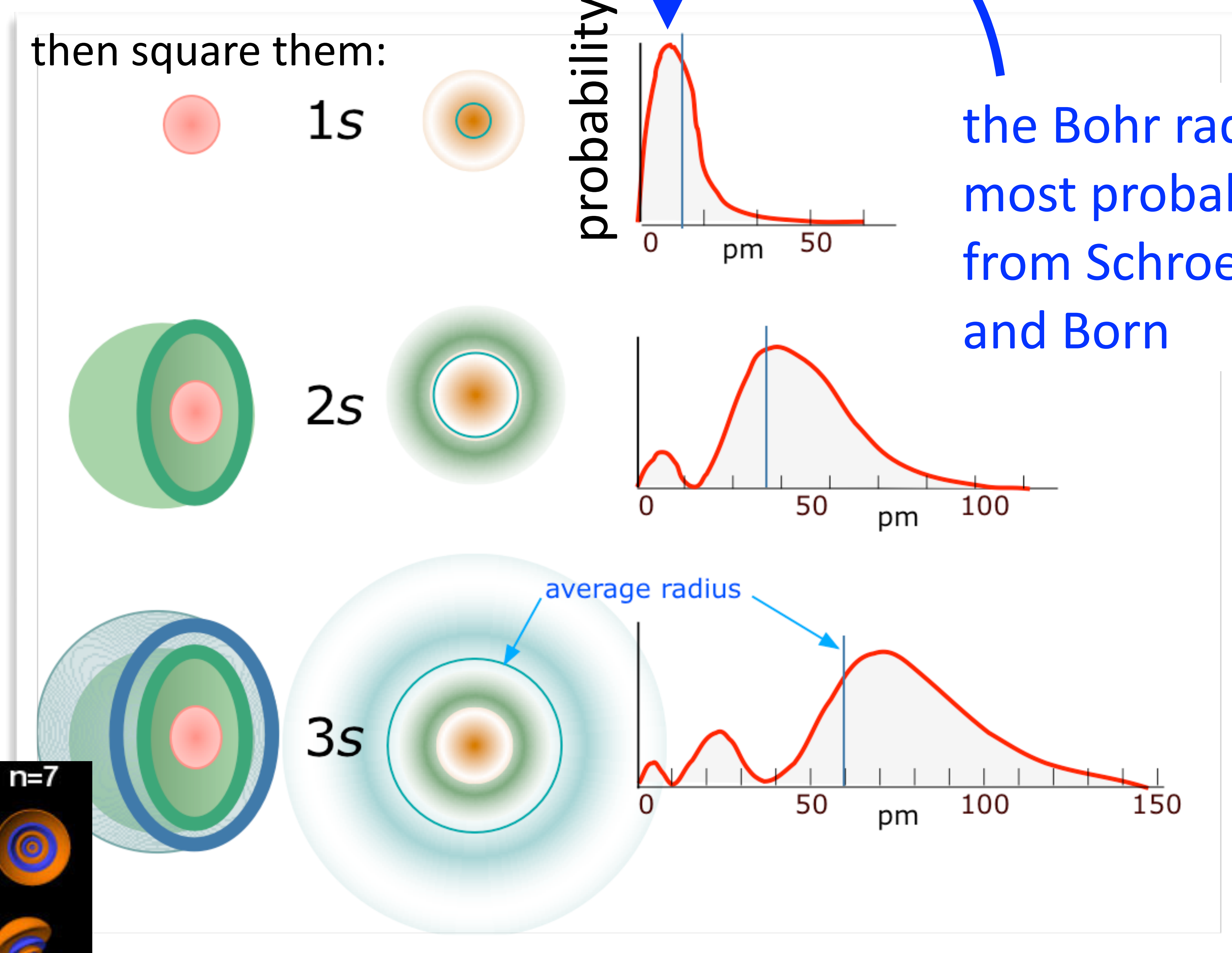
The concept of normal matter disappears, never to return

slice through the wavefunctions of Hydrogen

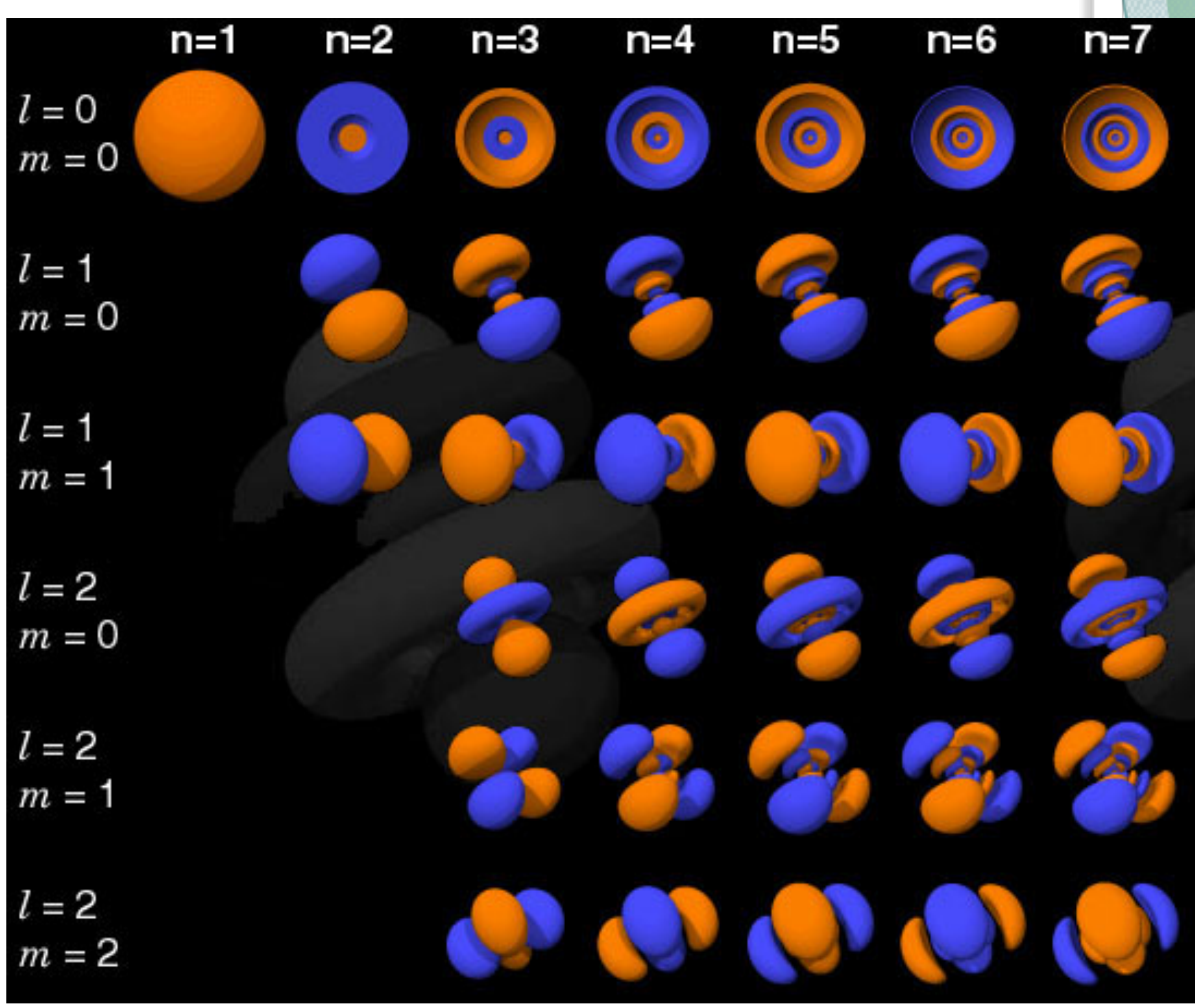
Solve Schroedinger equation
and get wavefunctions:

$$\psi(x,t)$$



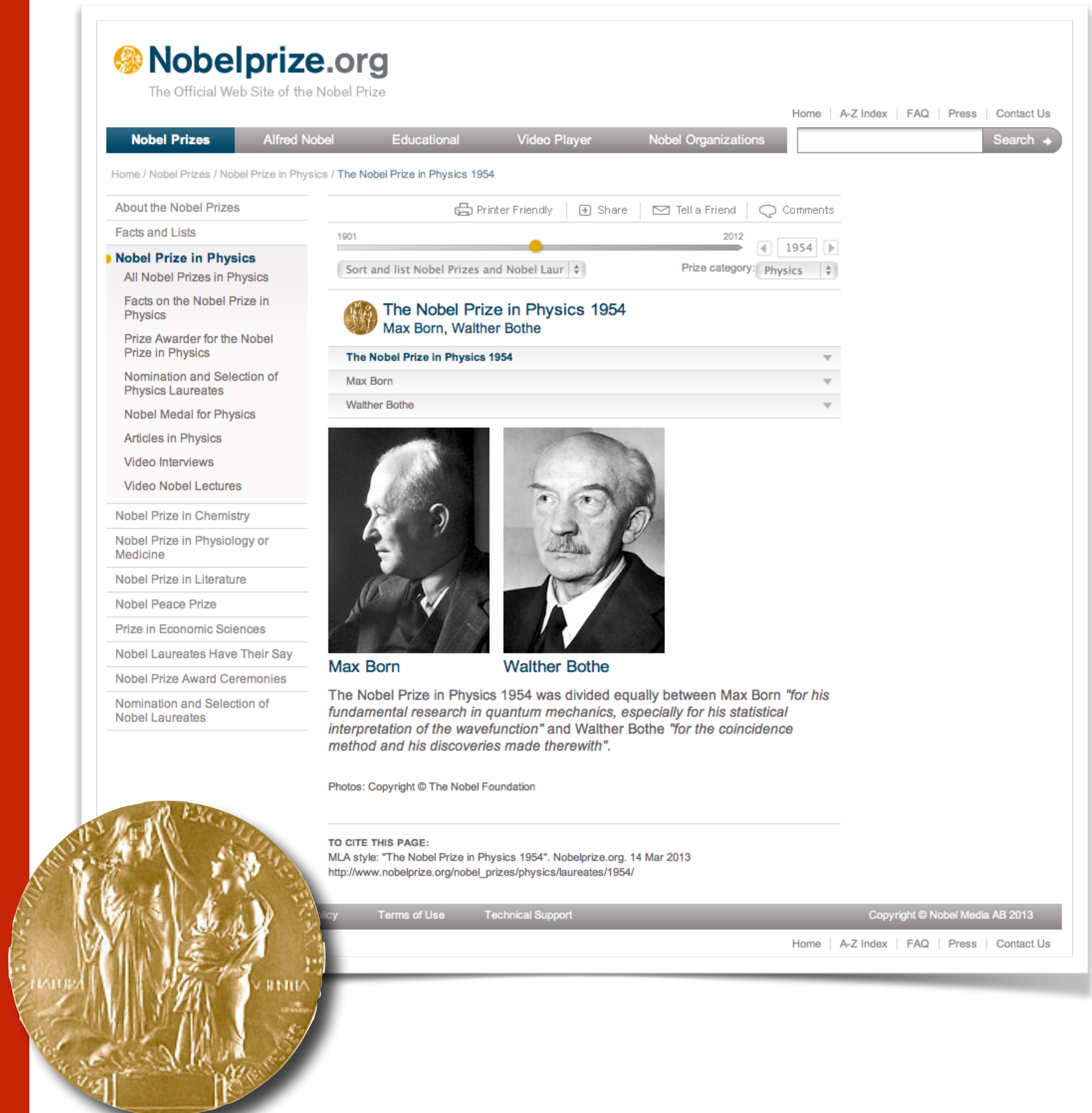


the Bohr radius - the most probable radius from Schroedinger and Born



finally

in 1954



I'm now uncertain.

This probabilistic interpretation stresses your intuition

intensely pursued by Heisenberg, who in the best Einsteinian tradition, asked a simple question:

what's involved in measuring something…?

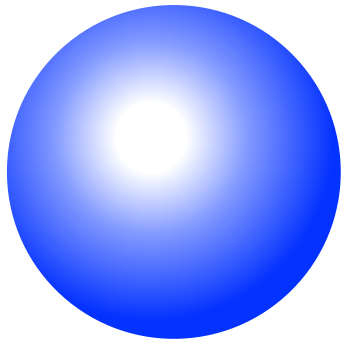
`critical is:` $\lambda = \frac{h}{p}$

it was hard enough

for photons

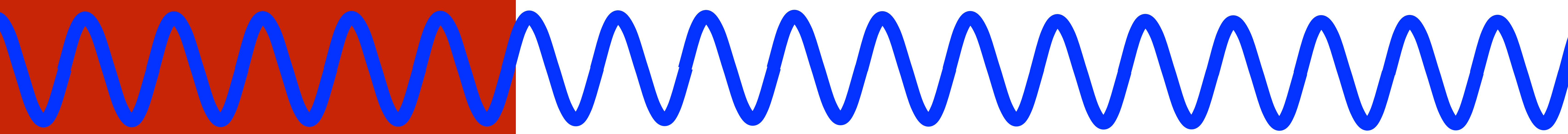
but for an electron?

A particle is HERE:



$$p = mv$$

A wave is EVERYWHERE:



The deBroglie hypothesis:

of given momentum

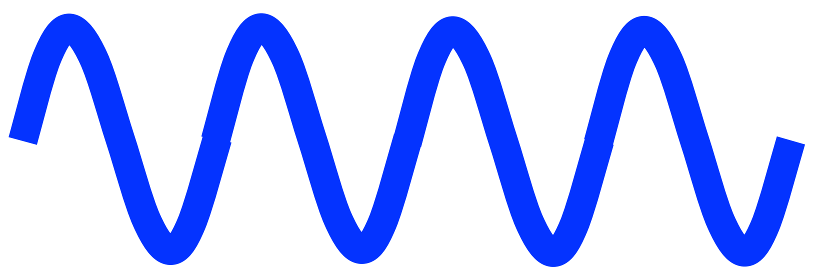
<u>also</u> has

a single wavelength

$$p = \frac{h}{\lambda}$$

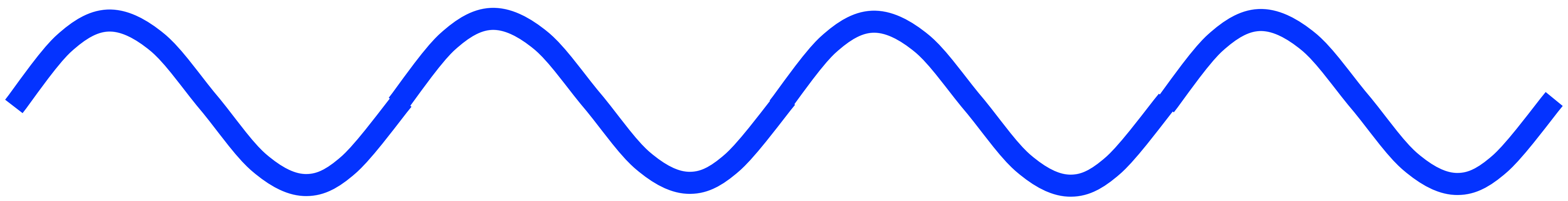
immediate implications

wavelength and momentum are inversely linked



$$p_1 = \frac{h}{\lambda_1}$$

immediate implications

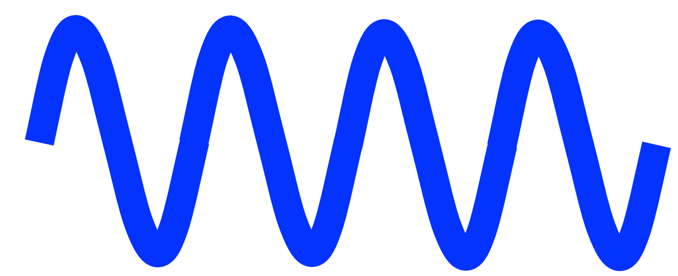


$$p_2 = \frac{h}{\lambda_2}$$

$$p_2 < p_1$$

long wavelength: low momentum

immediate implications



$$p_3 = \frac{h}{\lambda_3}$$

$$p_3 > p_1$$

short wavelength: high momentum

but a pure momentum:
one wavelength, one
definitive momentum

suppose we trap

an electron

Where’s the electron?

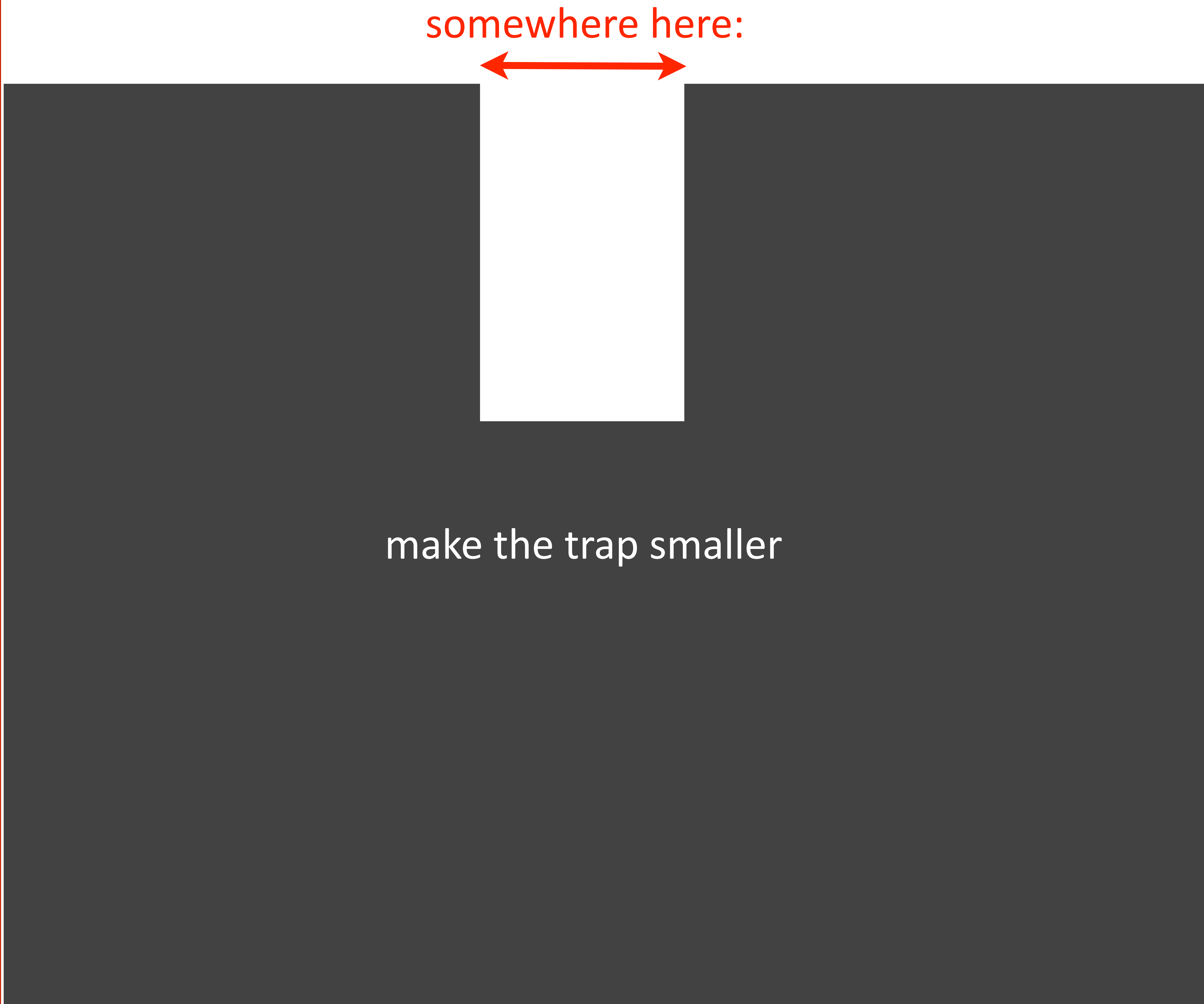
somewhere here:

how to locate it better?

suppose we trap

an electron

Where's the electron?



suppose we trap

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Where's the electron?

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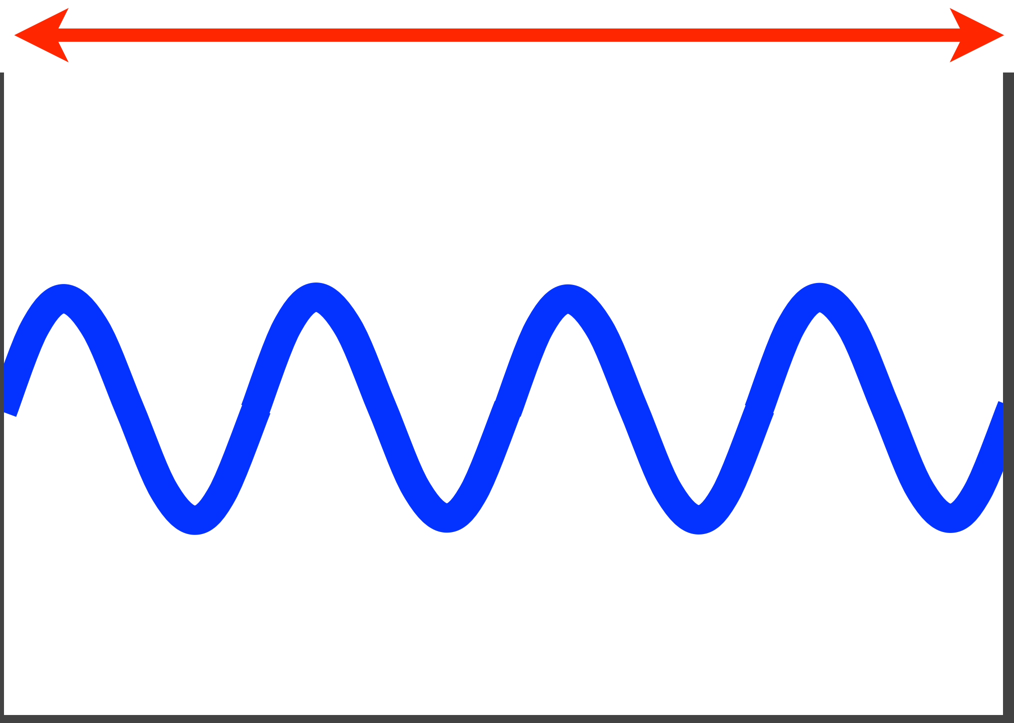

how to locate it better?

suppose we trap

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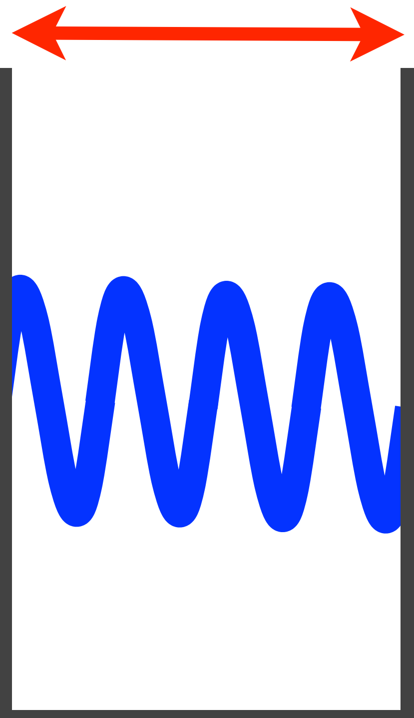
Where's the electron?

somewhere here:



make the trap smaller

$$p = \frac{h}{\lambda}$$

The wavelength is shorter...
So the momentum is higher!

an inevitable trade-off

in order to make the location more precise

you pay the price that its **speed is higher**

Heisenberg Uncertainty Principle

the Heisenberg Uncertainty Principle

was from 26 year old Werner Heisenberg

an enigma

inventor of many important concepts

did he save the west from a German nuclear bomb?

or the opposite?

Werner Heisenberg 1901-1976

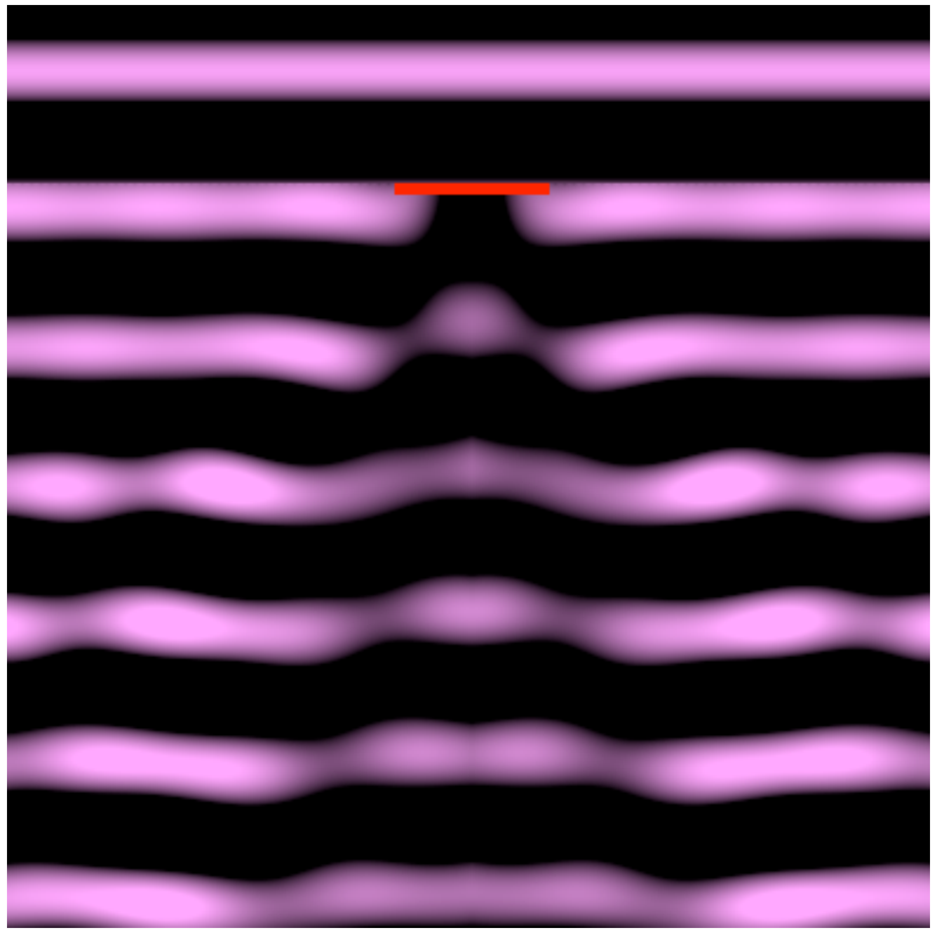
Heisenberg Uncertainty ... really!

how do you measure the trajectory of an object?

look at it in Time

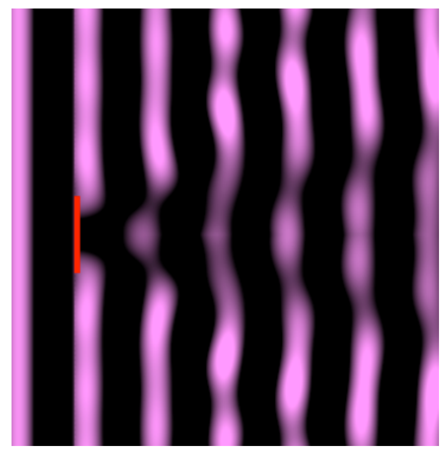
→ bounce light off it

Sweet spot for identifying an object:
need λ ~ size of the object



uncertainty - sometimes called “indeterminancy”

Try to “see” and electron. Electrons are small.
So...need tiny photon wavelengths.

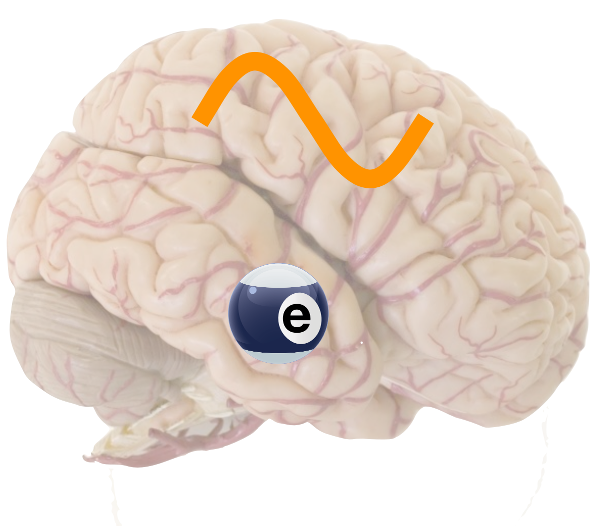


$\} \Delta x \sim \lambda$

So, make λ small to reduce Δx

But, $p = \frac{h}{\lambda}$ makes p **large**!

And, the photon-electron scattering transfers momentum: it kicks it a larger, unpredictable amount



Gedankenexperiment

Photon diffracts by the electron “barrier” and blurs the electron position by about the amount of the photon wavelength

so now knowledge of the electron’s momentum is blurred

$$\Delta p \sim \frac{h}{\Delta x}$$

$$\Delta p \Delta x \sim h$$

there is no way to beat this:

one cannot make a simultaneous arbitrarily precise measurement of position and momentum

the inverse relation between p and λ messes with you every time

$$p = \frac{h}{\lambda}$$

but here’s the hard part

this inability to determine position or momentum to arbitrary precision

is not about poor instruments

It. Is. About. Nature.

relation alert:

Heisenberg Uncertainty Relation

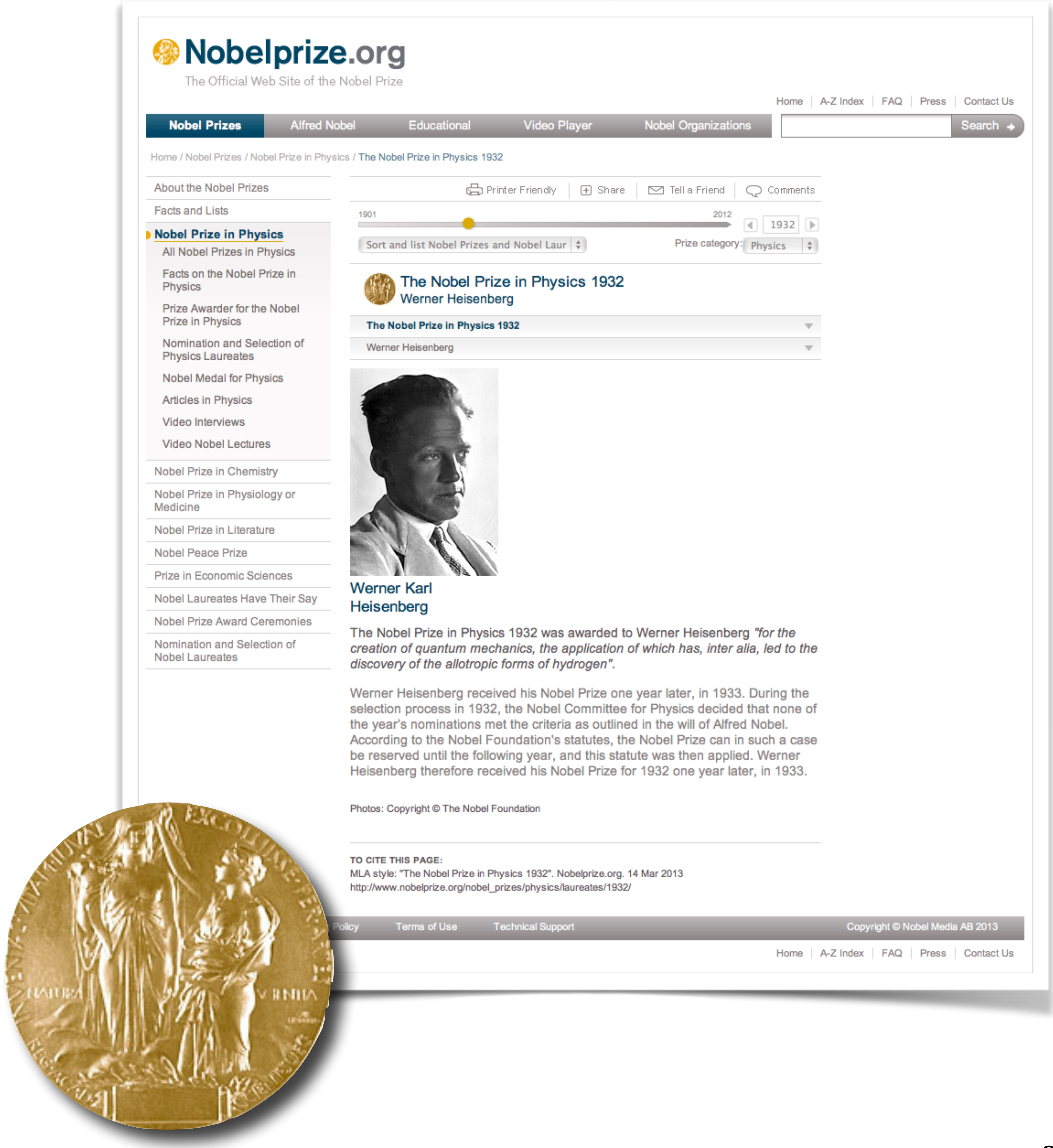
refers to: $\Delta x \Delta p \geq h$ & $\Delta t \Delta E \geq h$

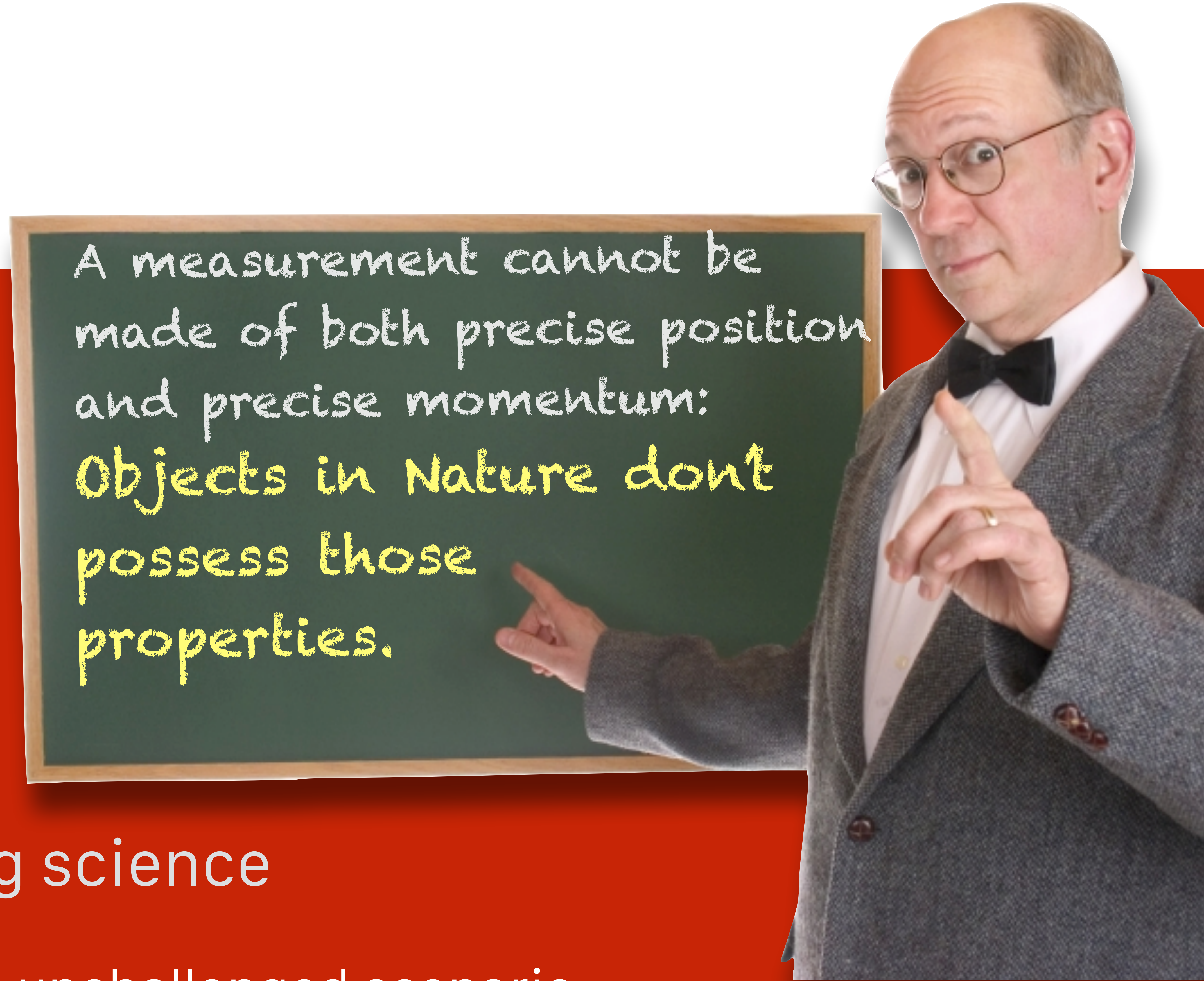
an inherent property of Nature

example: lots of things!

1932
Nobel

31 years old





a new way

Of thinking and doing science

we lose another classical, unchallenged scenario

there is no such thing as a precise trajectory

and a measurement is not isolated from the thing being measured

which is where new-age-y analyses of physics go off the rails

get real

I got pulled over for doing 105 mph*

could I argue that the officer had mistaken me for the other guy because, you know…Heisenberg?

* it was a different black Bimmer that had passed me a while back…

instead of midlife-crisis sports cars

how about:

a proton at 0.9c

what’s its position uncertainty?

$$\Delta p \Delta x \sim h$$

$$\Delta x = \frac{h}{p} \sim \frac{h}{m\gamma v} \sim 10^{-15} \text{ m}$$

about 1/3 the size of a nucleus

the whole story

for technical reasons, we use:

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

plus

the other form:

$$\Delta t \Delta E \geq \frac{h}{4\pi}$$

one more...

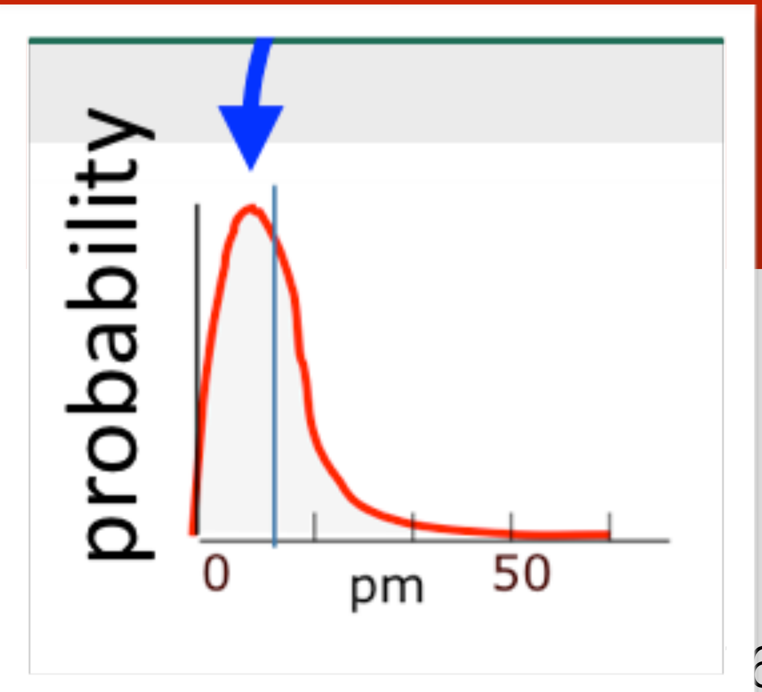
from the Bohr model, the speed of the electron is

~ 2 x 10^6 m/s – let's use non-relativistic momentum:

for Δp for an electron: $\Delta x \Delta p \geq \frac{h}{4\pi}$ $\quad \Delta x \sim \frac{h}{4\pi \Delta p} \sim 3 \times 10^{-11} m$

just about the Bohr radius!

So, the size of the atom is consistent with the electron being smeared all over the "fixed" Bohr radius.

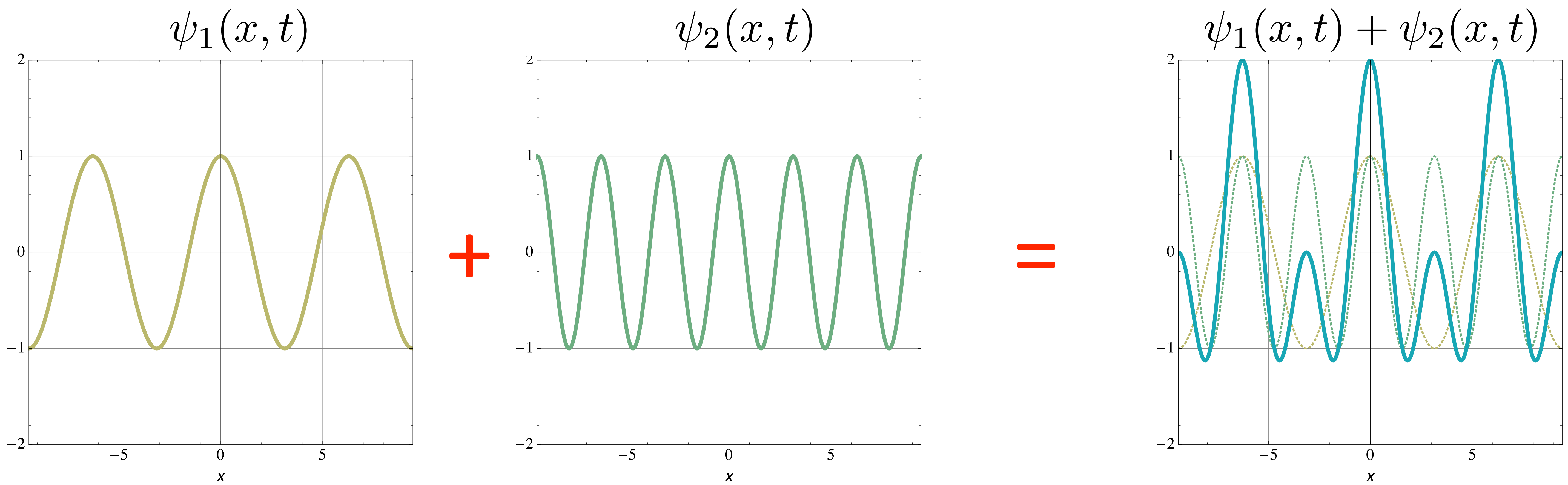


The “electron cloud” in a bound system is sort of...visualizable

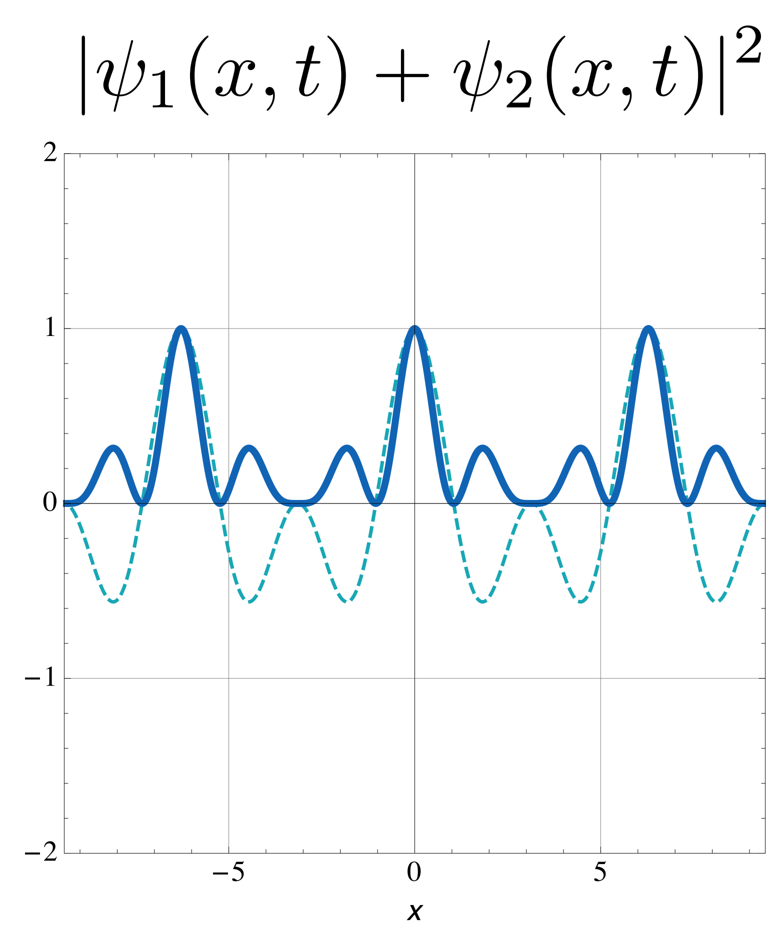
I’m dancing around a tough question

But, if particles are waves and if waves are “everywhere”

...what’s the “particle” in Particle Physics?



But, remember that what's real about the quantum fields is the square: $|\psi(x,t)|^2$



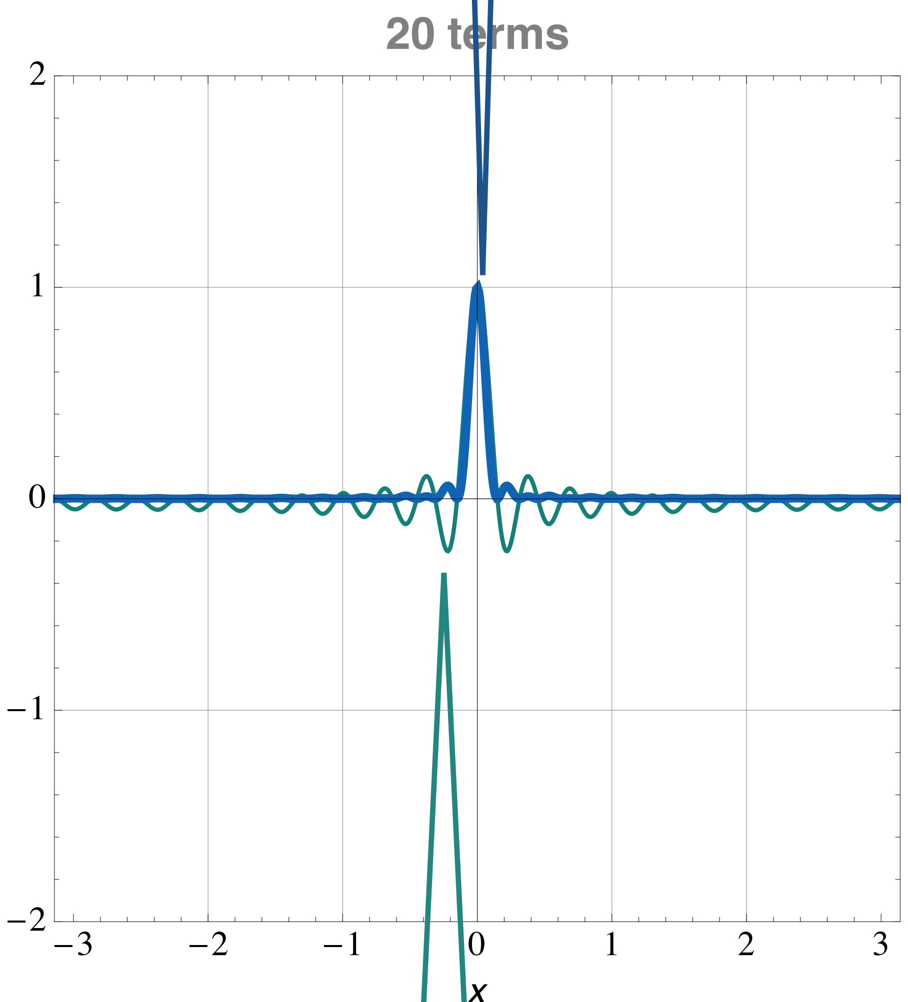
notice the peaking

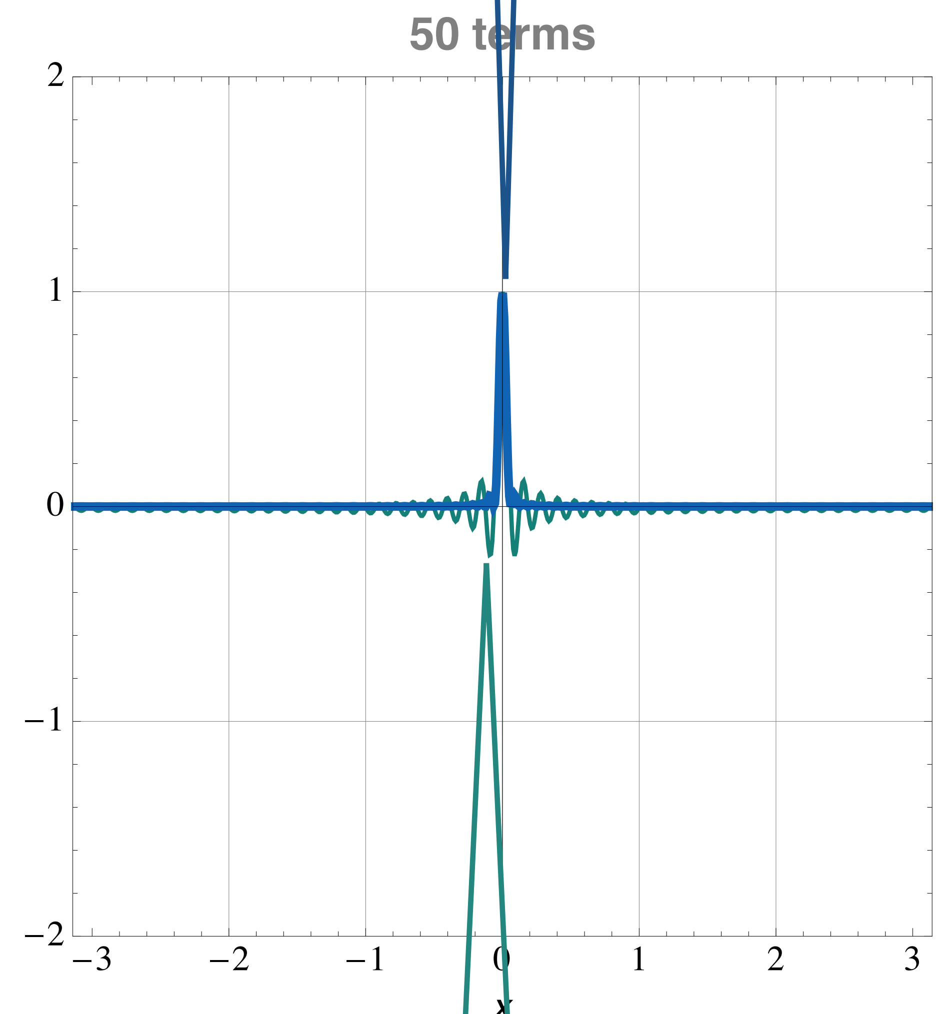
(I've changed the heights)

add quantum field functions - more terms

$$|\psi_1(x,t) + \psi_2(x,t) + ...\psi_{20}(x,t)|^2$$

$$|\psi_1(x,t) + \psi_2(x,t) + ...\psi_{50}(x,t)|^2$$





$$\psi_1(x,t) + \psi_2(x,t) + ...\psi_{20}(x,t)$$

$$\psi_1(x,t) + \psi_2(x,t) + ...\psi_{50}(x,t)$$

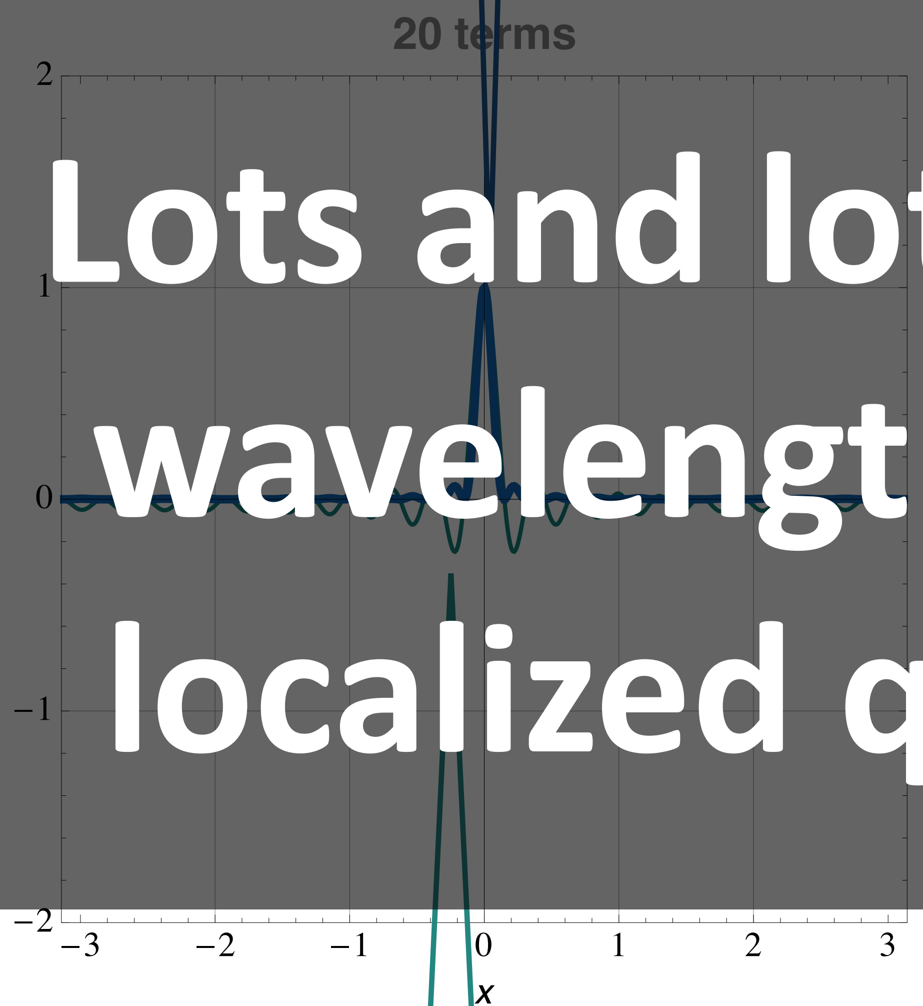
peaking is even more pronounced

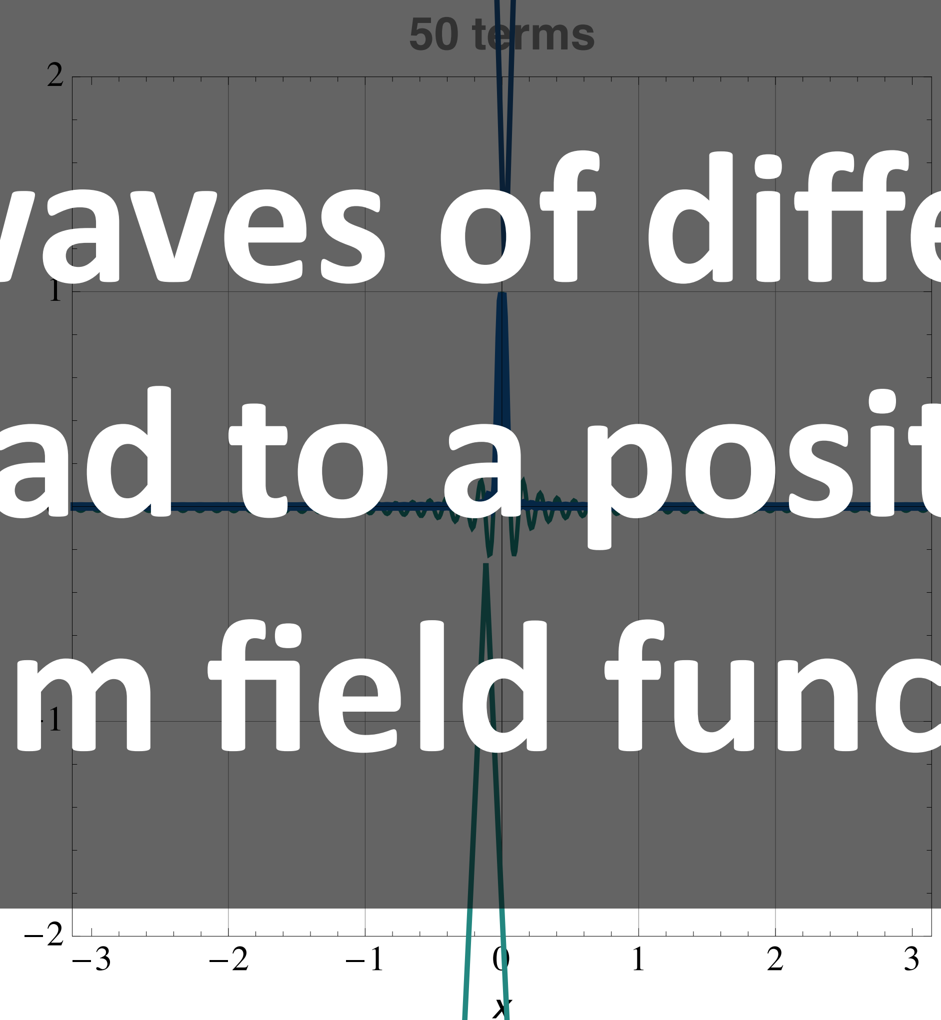
(I've changed the heights)

add quantum field functions - more terms

$$|\psi_1(x,t) + \psi_2(x,t) + ...\psi_{20}(x,t)|^2$$

$$|\psi_1(x,t) + \psi_2(x,t) + ...\psi_{50}(x,t)|^2$$





Lots and lots of waves of different wavelengths...lead to a position-localized quantum field function

$$\psi_1(x,t) + \psi_2(x,t) + ...\psi_{20}(x,t)$$

$$\psi_1(x,t) + \psi_2(x,t) + ...\psi_{50}(x,t)$$

peaking is even more pronounced

(I've changed the heights)

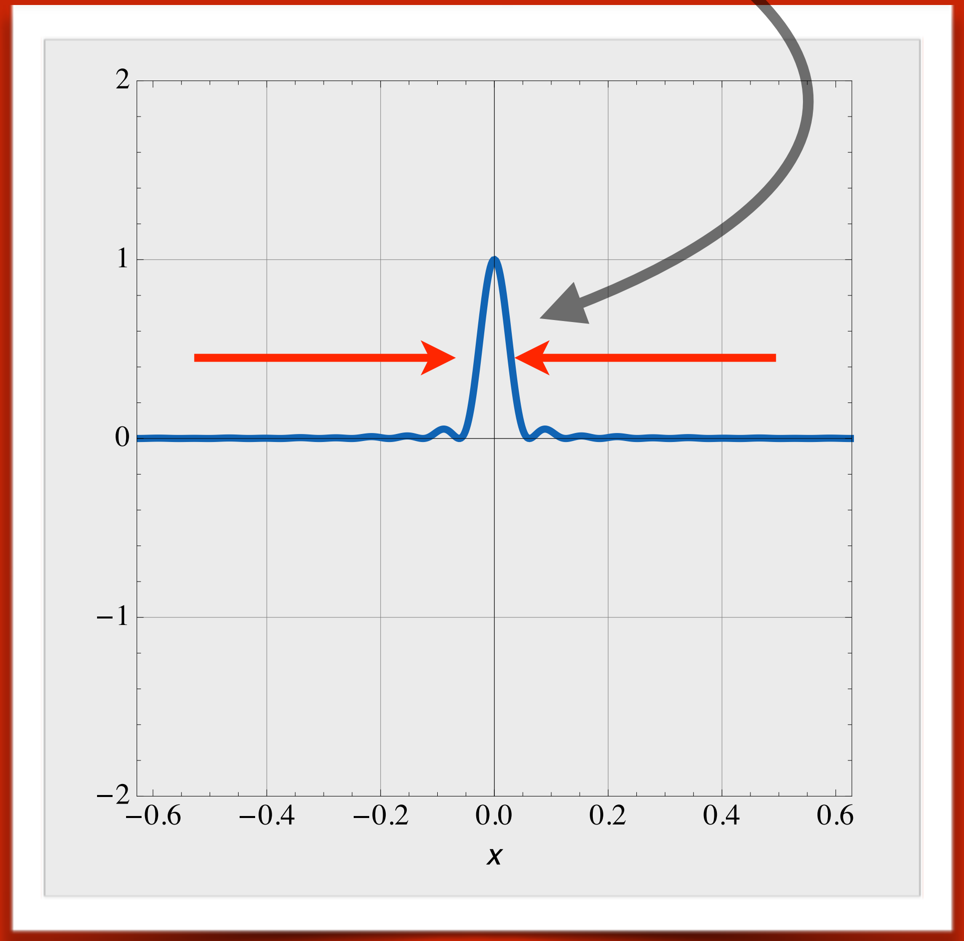
the wave combinations localize the state...with some spread in x

`waves of different wavelengths?`

$$p = \frac{h}{\lambda}$$

different momenta

Heisenberg Uncertainty Relation at work again



called “wavepackets”

all of the wave combinations means all of the momenta contribute: an spread in p.

the larger the momentum spread

the smaller the localization

“particles” are more particle-like at large momentum the larger the momentum spread

the smaller the localization

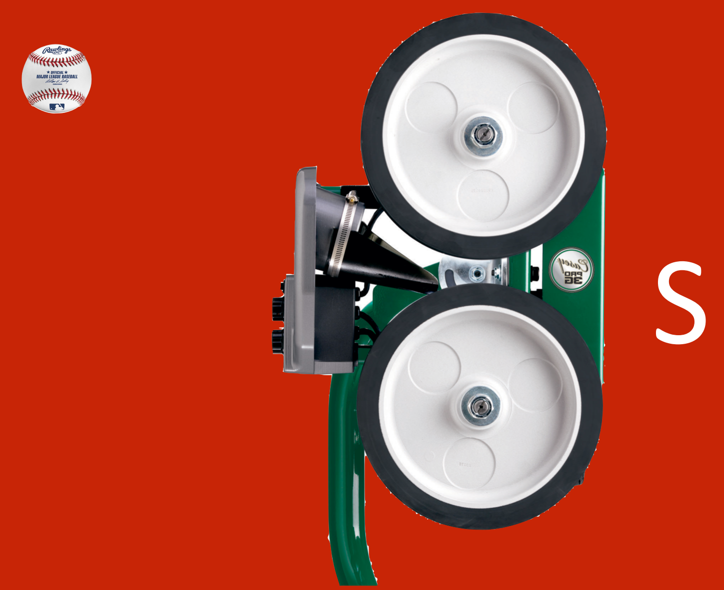
“particles” are more particle-like at large momentum

THAT’S WHY WE CALL IT “PARTICLE PHYSICS” and not “wave physics”

Nature’s little joke

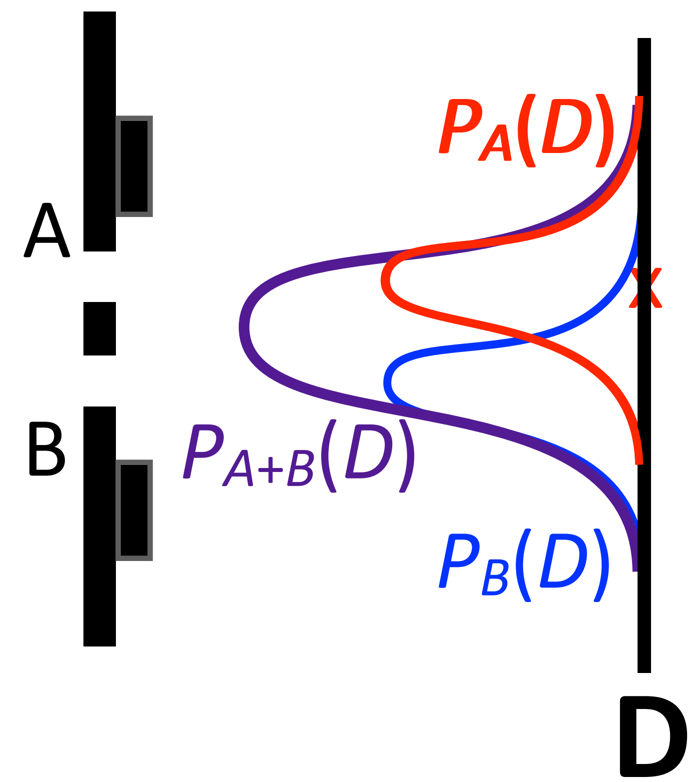
is encapsulated in a famous Feynman-description

a Gedankenexperiment…



two slit experiment

2 + 1 ways

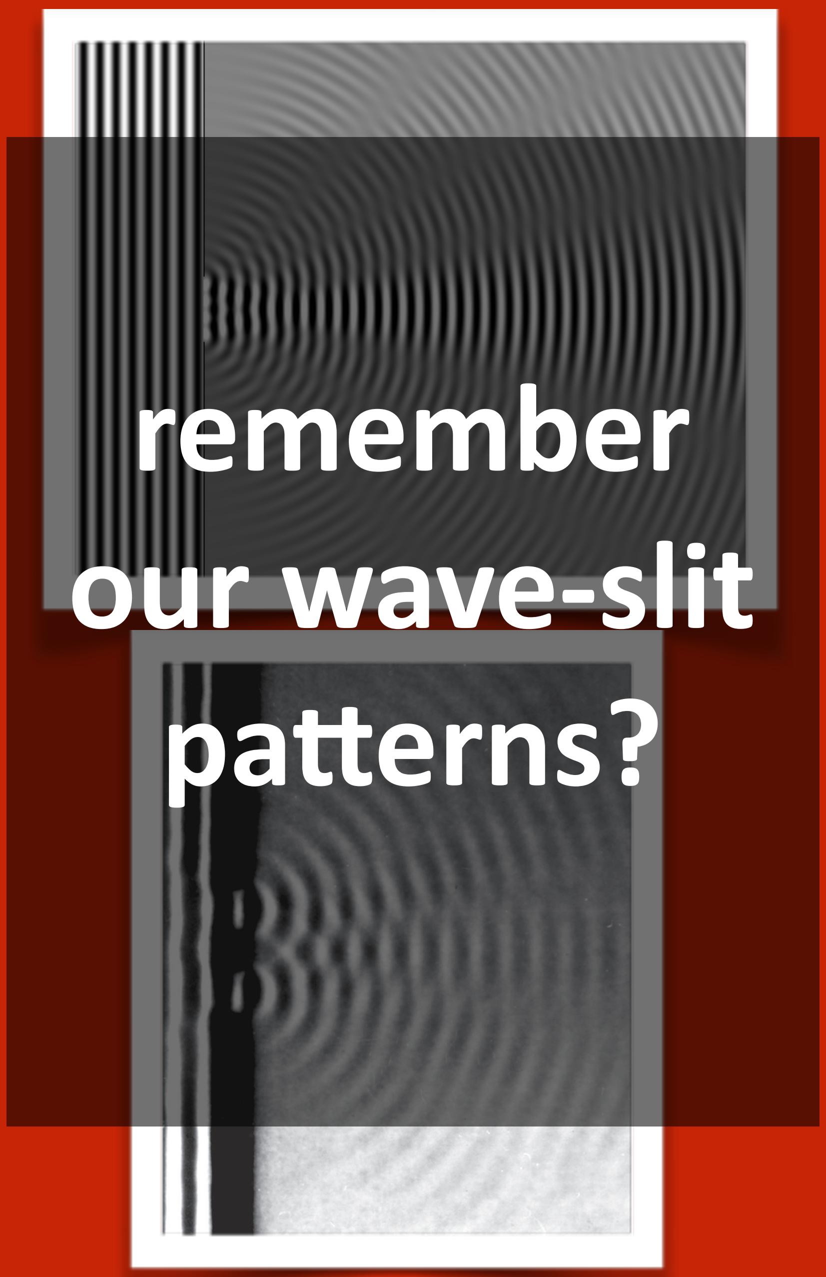


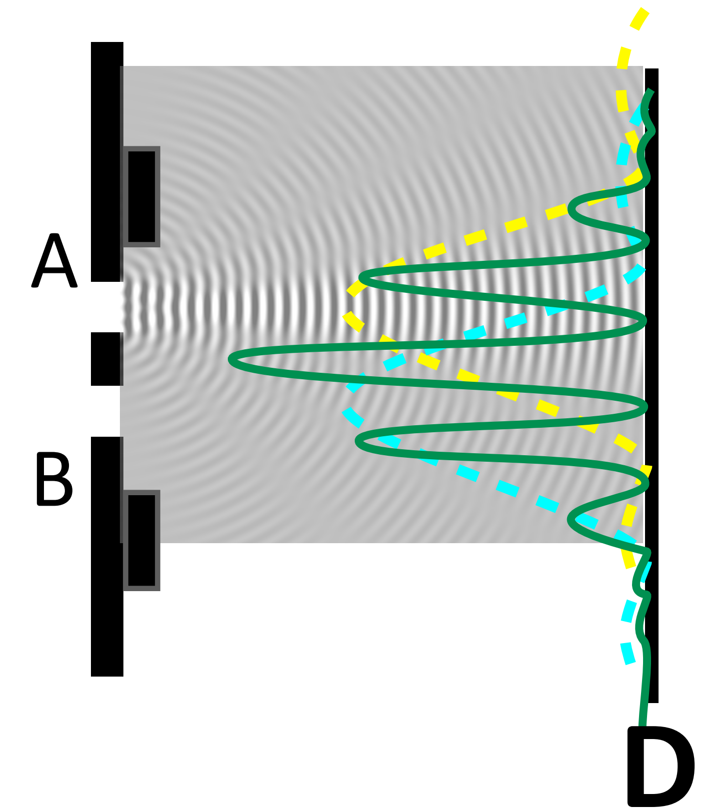
Two slit experiment with classical baseballs

$$P_A(D) + P_B(D) = P_{A+B}(D)$$

Like the "classical" situation of asking what is the probability of getting heads or tails in a coin flip...you'd add 0.5 and 0.5.

S

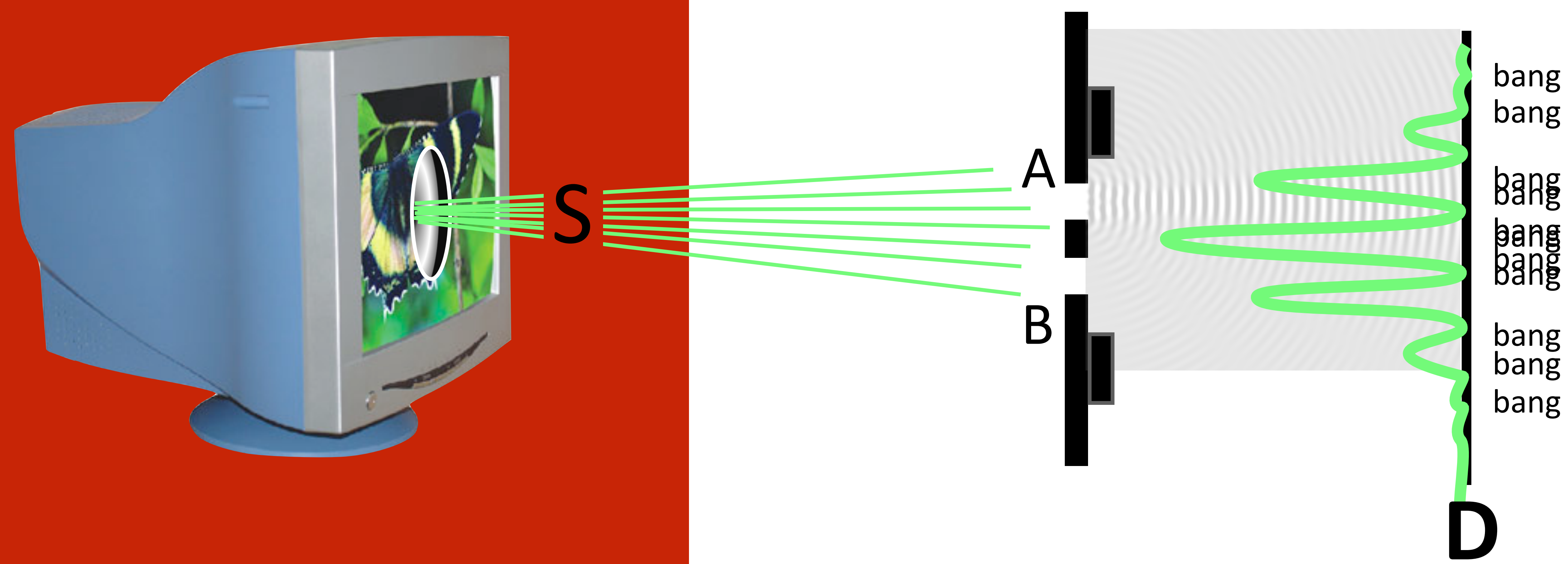




Two slit experiment with waves

$$P_A(D) + P_B(D) \neq P_{A+B}(D)$$

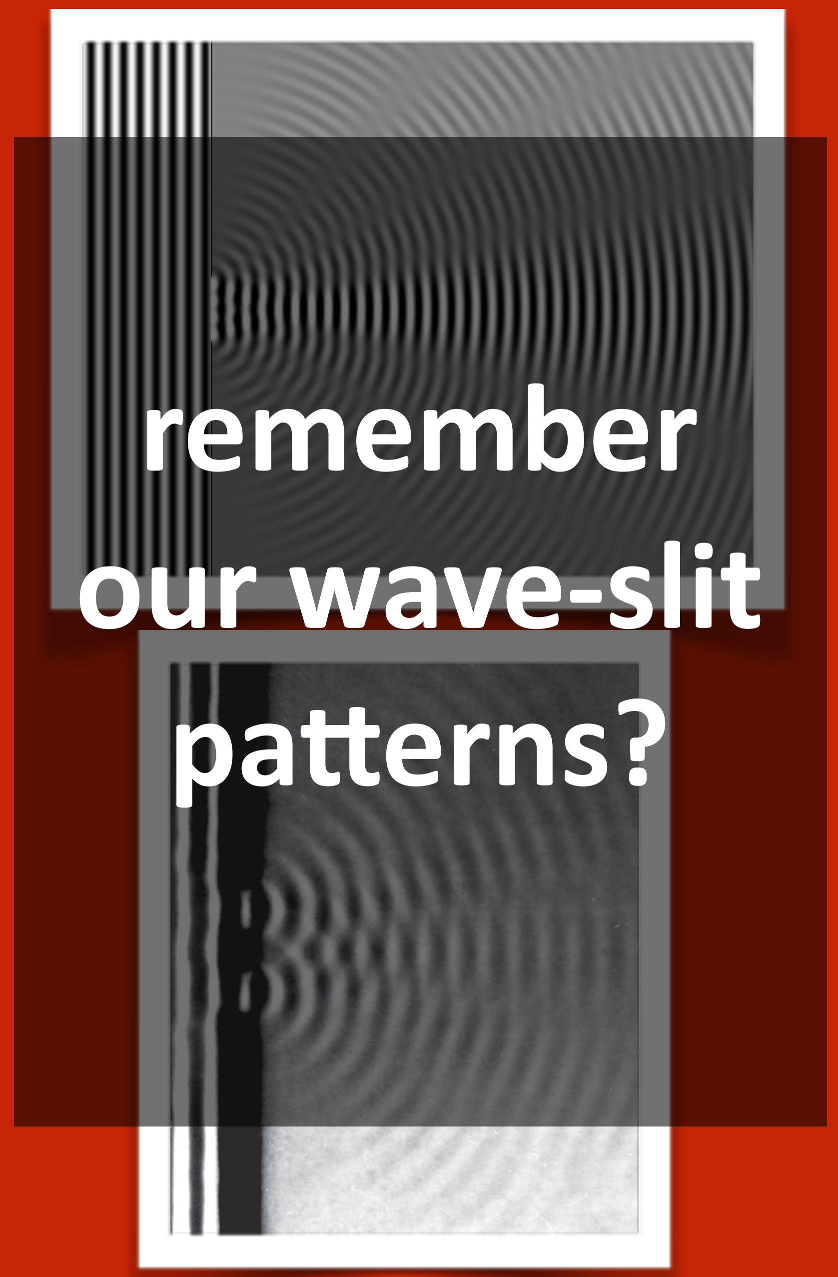
Interference causes the characteristic diffraction pattern



Two slit experiment with electrons?

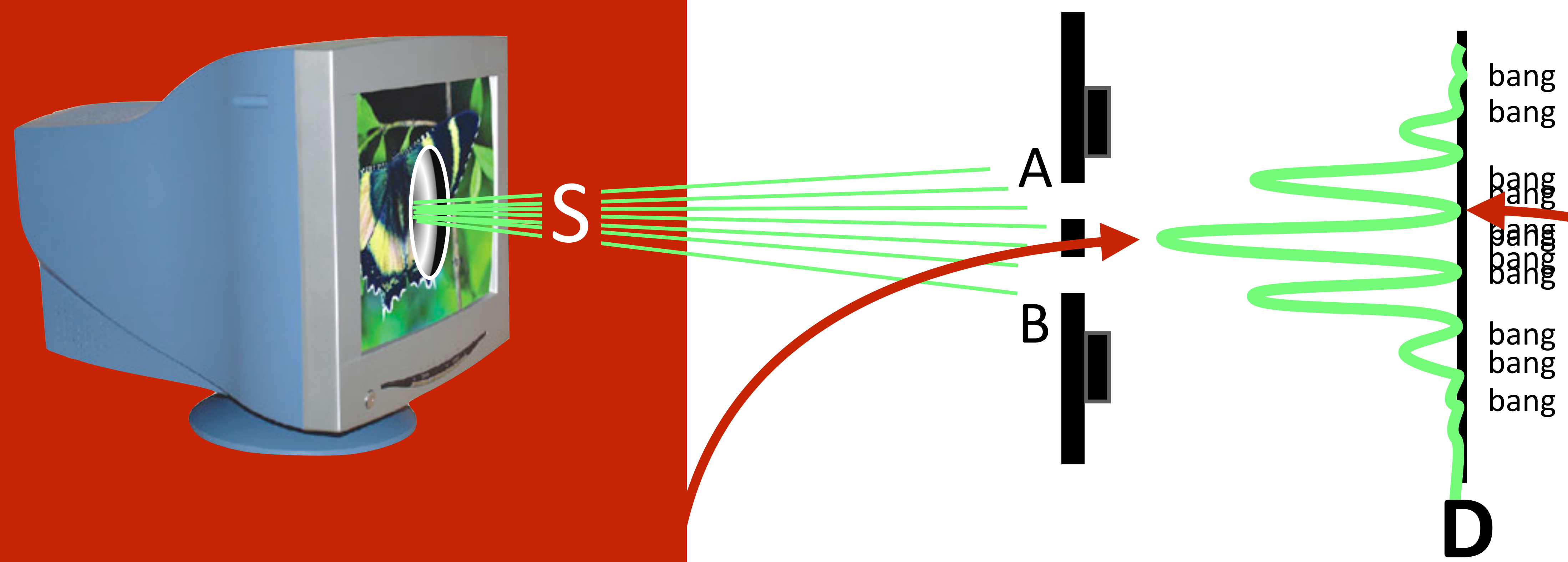
$$P_A(D) + P_B(D) \neq P_{A+B}(D)$$

Interference causes the characteristic diffraction pattern



Same result as for waves.

Maybe not a surprise given what's come before, eh?



probabilities don't add

it's the **quantum fields** that do the wavy-ness!

ψ

~~$P_A(D) + P_B(D) \neq P_{A+B}(D)$~~

$$P_D = |\psi_A + \psi_B|^2$$

$$P_D = \psi_A^2 + \psi_A^2 + \psi_A \psi_A^*$$

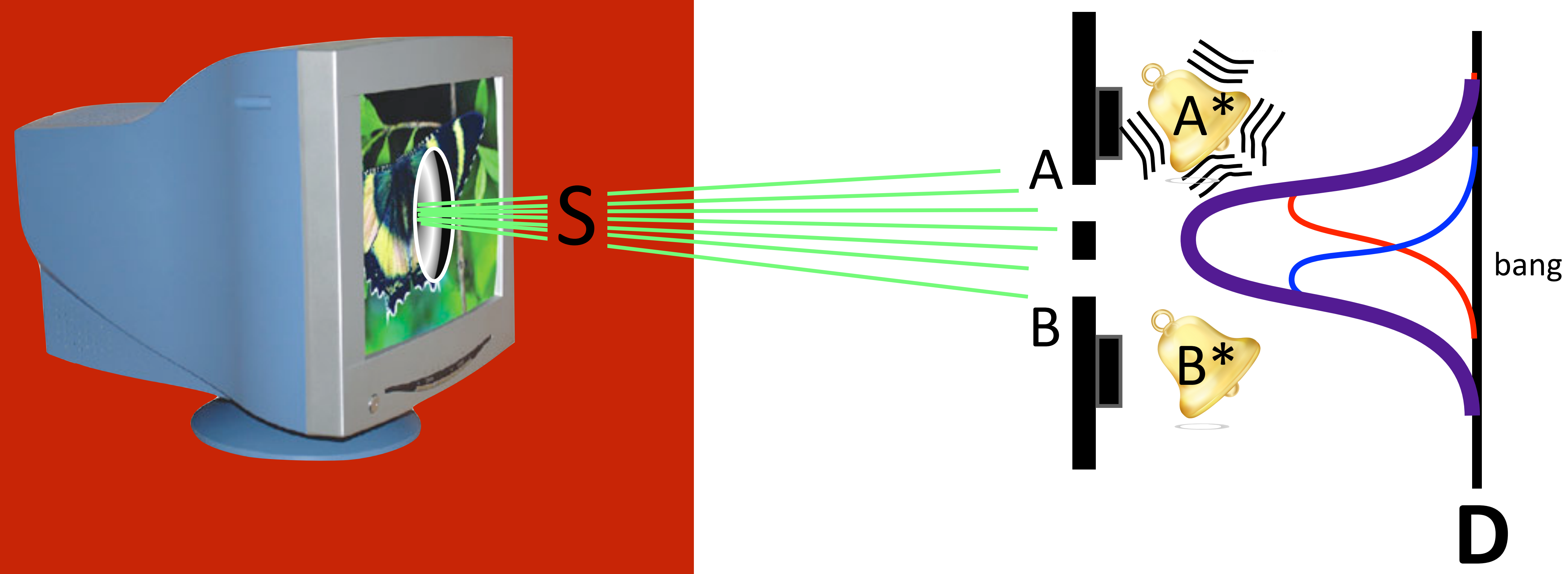
at some points this can be negative, sometimes positive

which gap did any electron come through?

okay…let's trick it

rig an alarm that sounds when an electron goes through a slit.

Hah!



Two slit experiment with electrons and an alarm?

So the sequence "S-A-A*-D occurred.

Every time A* rings - red curve. B* rings, blue curve.

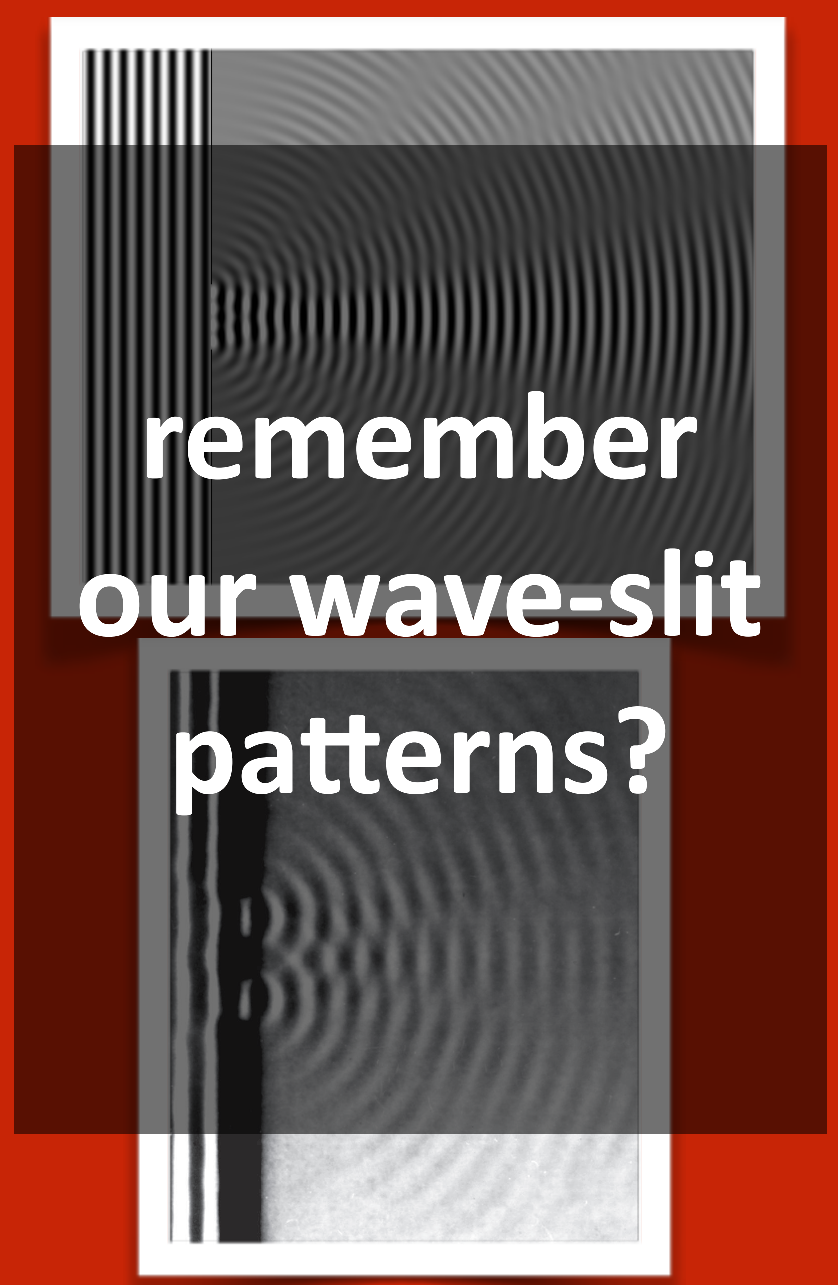
Same result as for baseballs.

Interference has gone away!!

Now: A* is a DISTINGUISHABLE event from B*

We specified the path...

and that changed the reality.



summarize

the classical situations

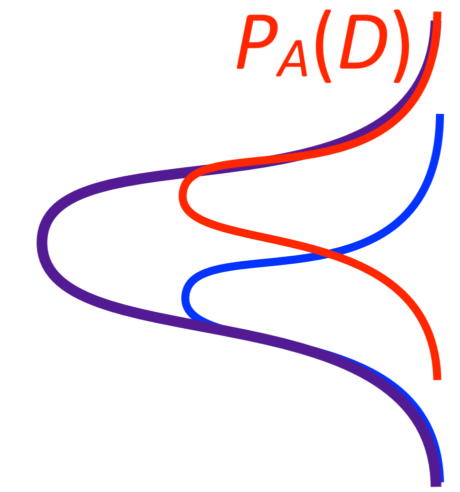
For **macroscopic objects**: outcomes add "normally":

The result of

whatgoesthroughA and whatgoesthroughB is

the sum of whatgoesthrough(A **or** B)

one or the other



For **waves**: outcomes interfere:

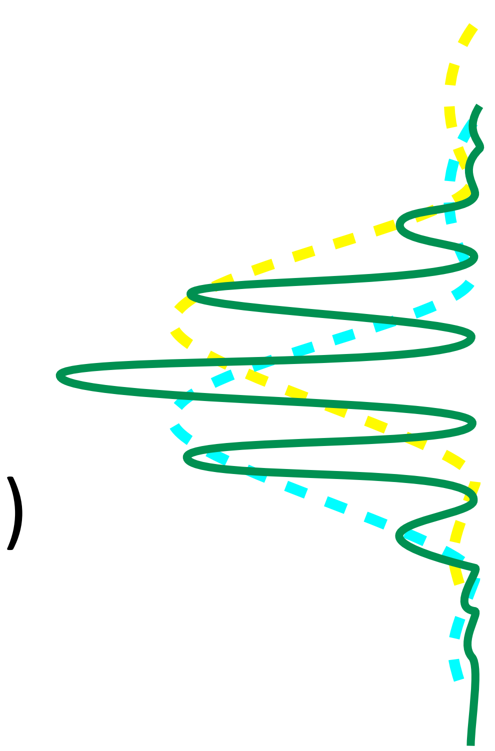
the result of

whatgoesthroughA and whatgoesthroughB is

the interference of whatgoesthrough(A **and** B)

both at the same time

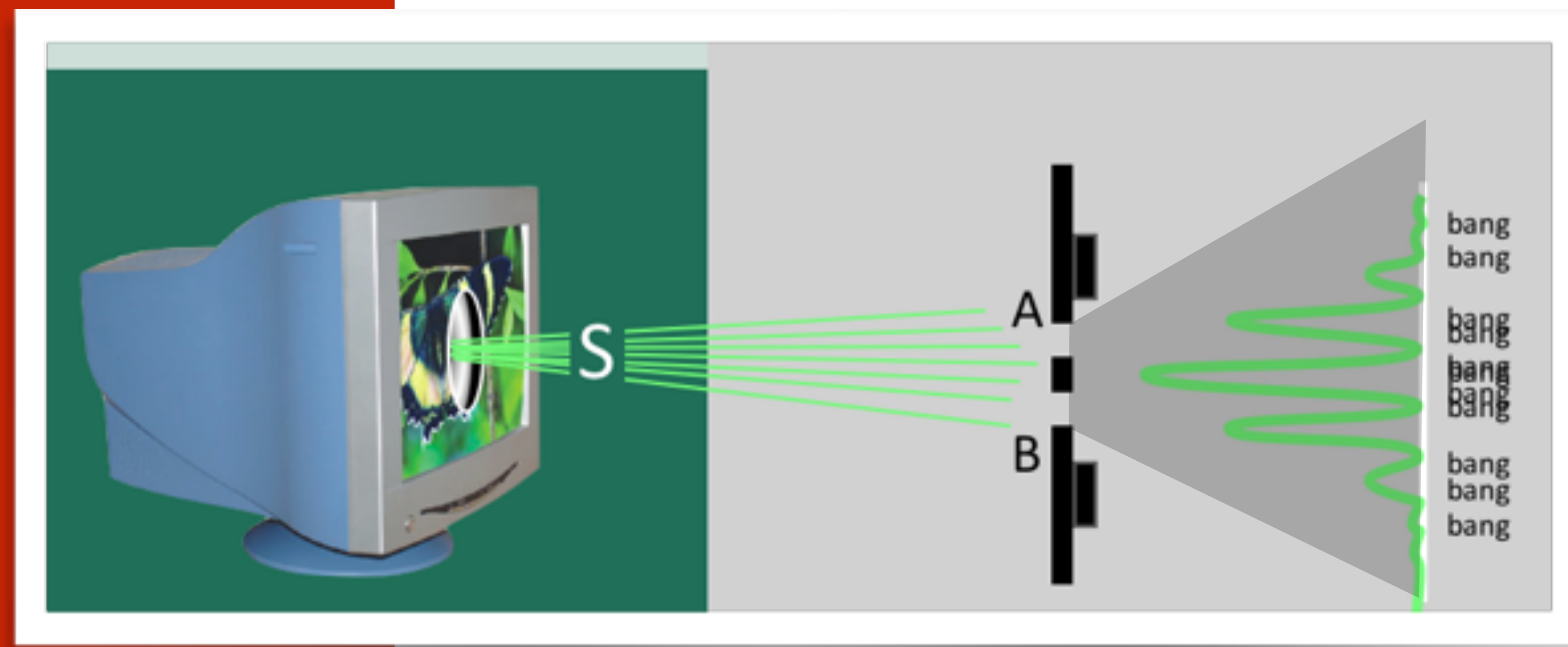
the waves interfere

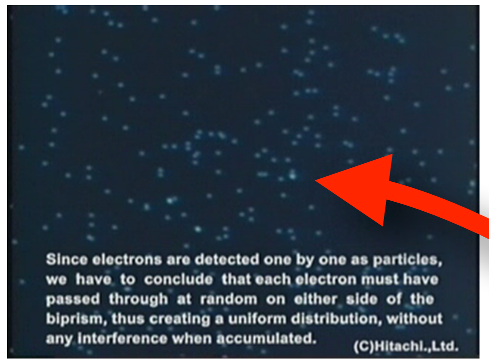


where is the electron?

it’s real only when you make a measurement

and your measurement can determine how it’s real





The electron is real at the screen.
it’s unambiguously...there.
the “bang” is a measurement

what about here?

We have to say that an electron:

- goes through both slits
- and is in a “superposition” state,
  *here of **both** the state* ψ_A *and the state* ψ_B

As soon as measurement is made...the superposition goes away and the potentiality becomes the actuality...according to the probabilistic prediction of the Schroedinger Equation.

what we can say is real

is now very tricky

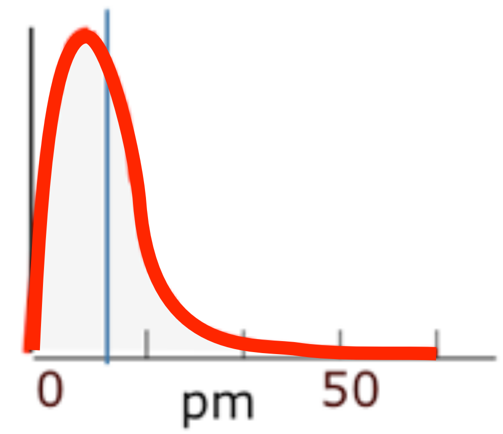
and not understood.

We know that quantum fields contain all of their potentialities

and a measurement "collapses" them into just one outcome

the concept of a "measurement" is totally not understood.

the wavefunctions are everywhere



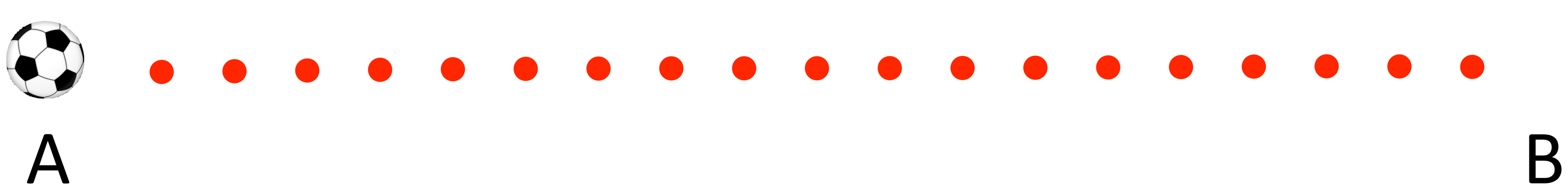
doesn't go to zero.

There's a probability that the electron in one of your water molecules might spend a brief time at the Louvre

spread out and overlapping

that's how molecules stay together

but...jeez. everywhere.



Something big...seems to have a definite trajectory

Something tiny...doesn't.

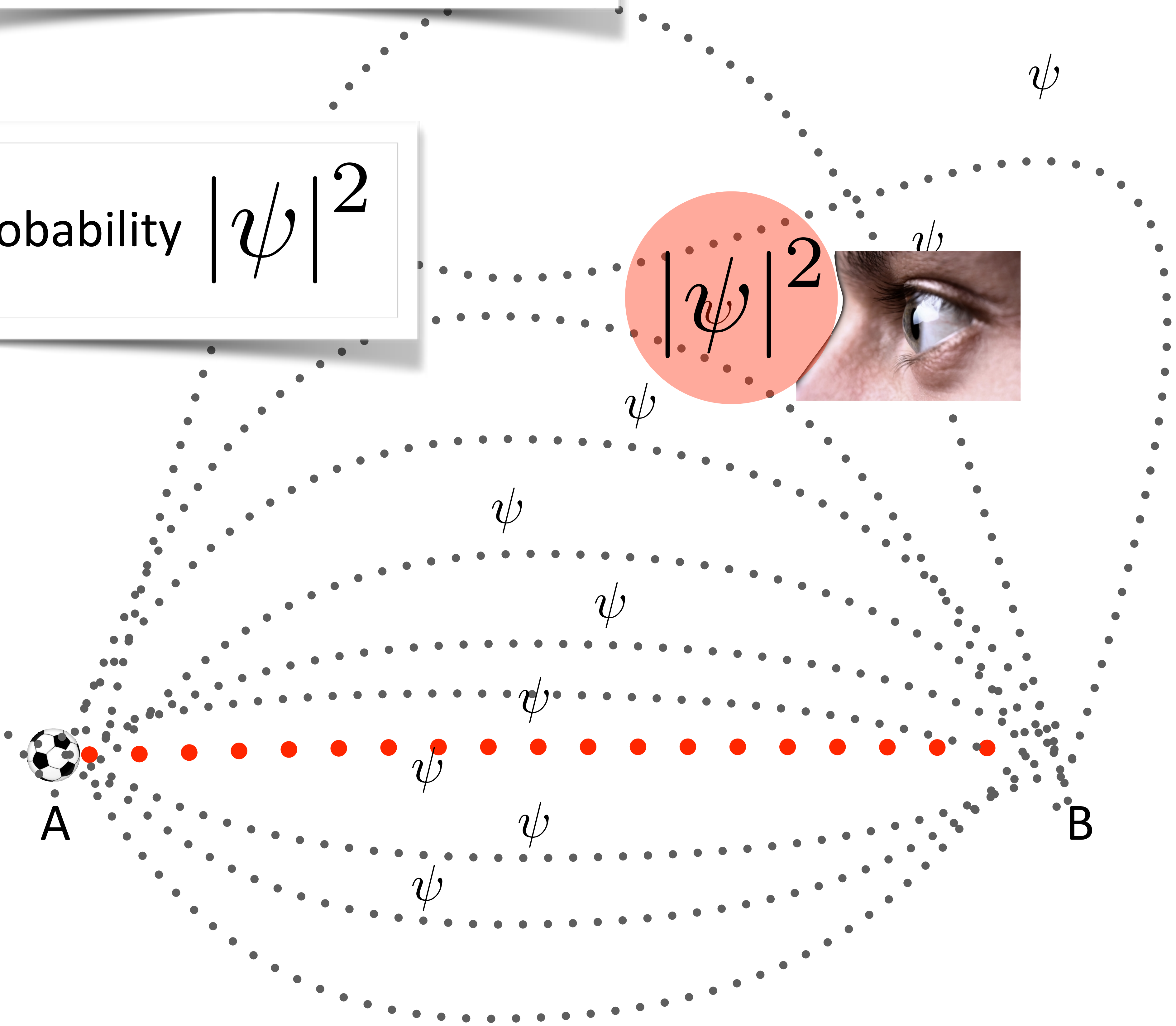
the wavefunctions are everywhere

They're waves, after all.

make a measurement....there

Only then is it real.

the electron is there with probability $|\psi|^2$



Feynman's picture was one of particles: which take all possible paths

We can calculate the wavefunction at any point, very precisely...it's completely deterministic

The trajectory of a big object?

Overwhelmingly probable quantum likelihood: the classical path

so where is a quantum

before it’s measured?

anywhere? everywhere?

yeah.

to take it to an absurd conclusion: the dreaded Schroedinger's Cat

proposed by Schroedinger as an absurdity in 1935

because he too had become disgusted with this own creation - he switched to biology!

Schroedinger must have been a dog person

Imagine:

a radioactive source,

Geiger counter, and

a glass bottle of a deadly poison

with a cat

in a box,

a weight drops on the glass, breaking it

after the first radioactive decay?

...dead cat.

Schroedinger must have been a dog person

Now imagine that the radioactive nucleus as a **half life of 10 sec**.

so, after 10 s,

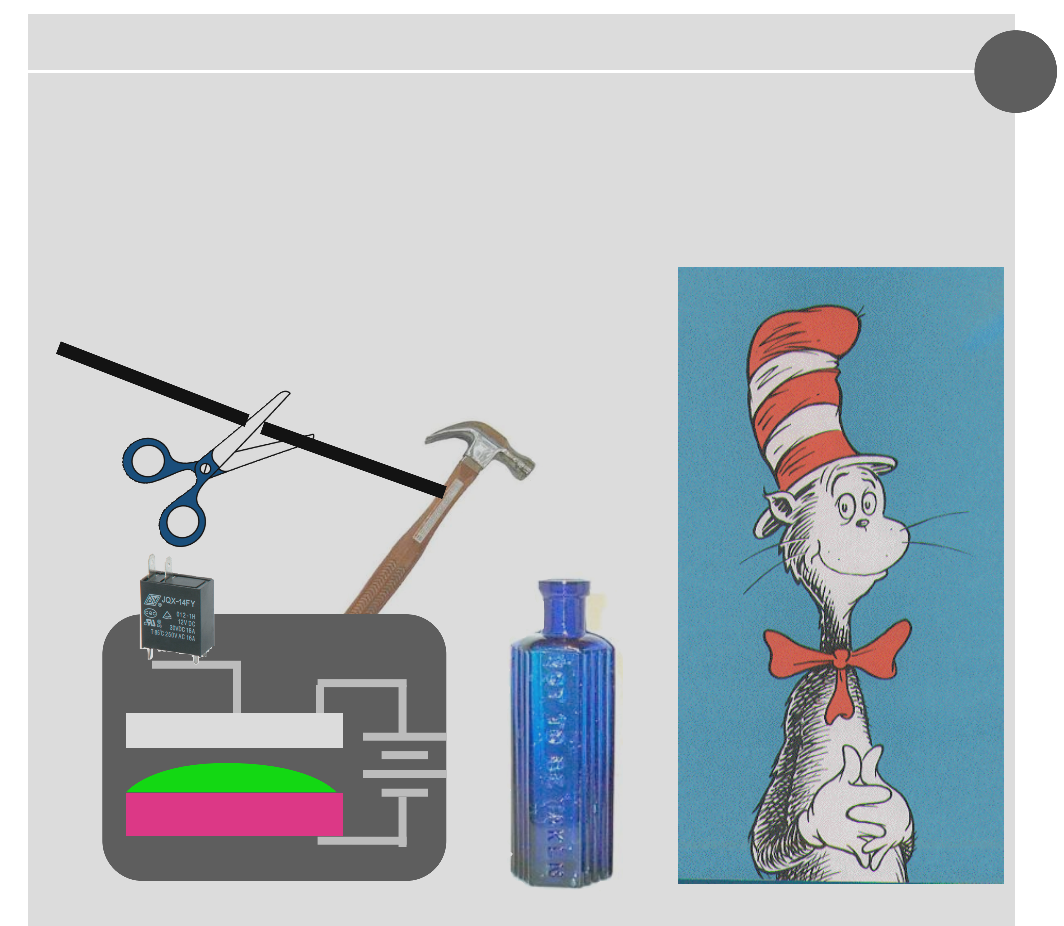
50-50 chance that it has decayed

Set it all up...wait for 10 seconds.

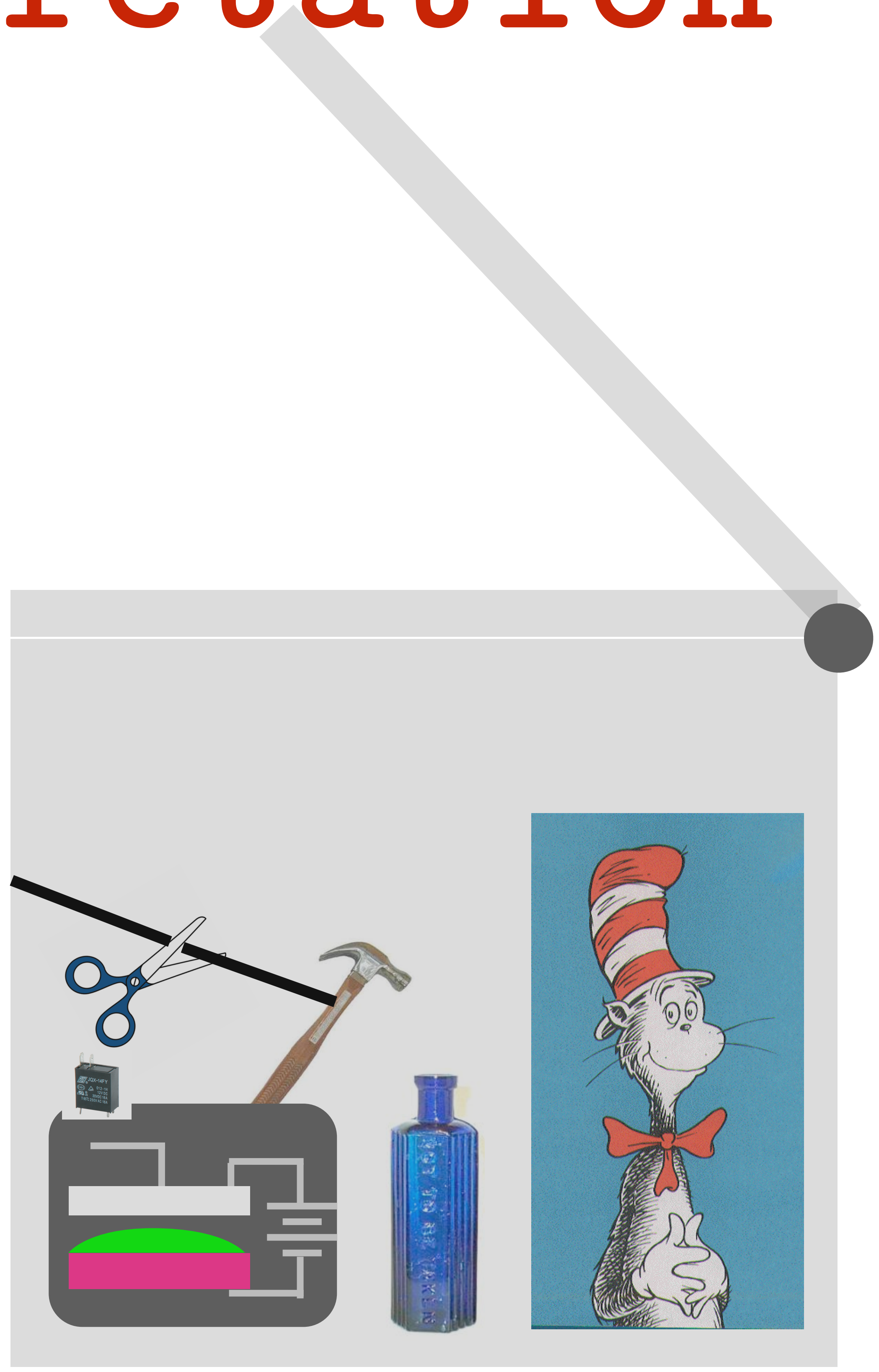
what is the state of the cat?

alive or dead?

or both?



‘‘Copenhagen Interpretation’’



It is meaningless

to speak of reality without a measurement

Entities have no definite reality

the cat is neither alive nor dead

or it is both

To know you must open the box

make a measurement

this is how we have to think about it:

before measurement: alive-dead state -
superposition state of both

after measurement: is **either** alive **or** dead

here's our house

just before painting last year

need to pick a color:

my wife says "red"

I say "blue"

COVER THE EARTH

SHERWIN-WILLIAMS.

quantum paint

I expect it to be:

purple

mixing red and blue

but the quantum mechanical paint

that I paid extra for?

can't "exist" in a superposition, mixed state.

Only one state.

sometimes it's red

but the quantum mechanical paint

that I paid extra for?

sometimes it's blue

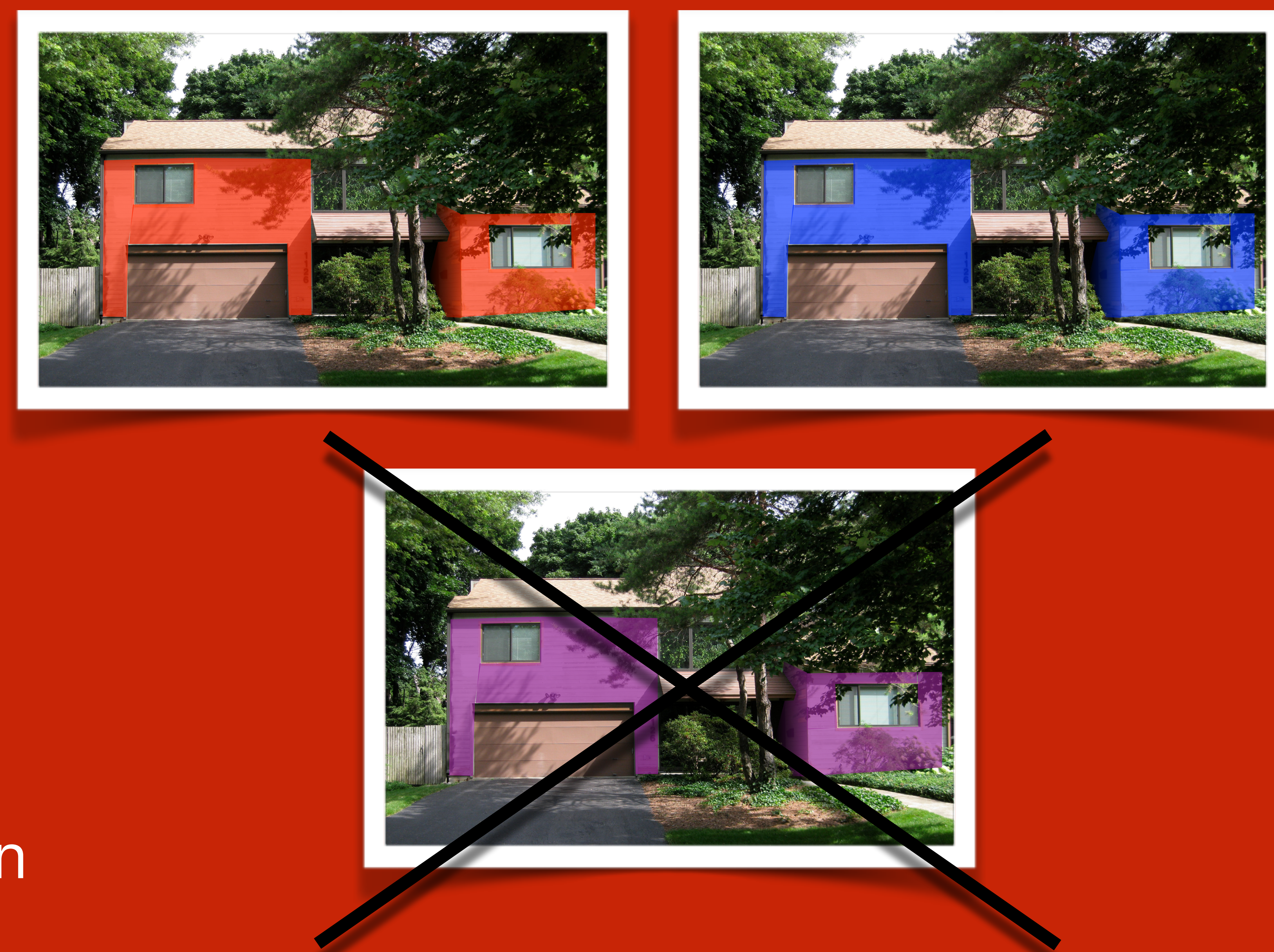
it’s never the mixture

that it potentially might be

one or the other

More red paint?

not redder...just red more often



the cat is either alive or dead, not both.

“I think I can safely say that nobody understands quantum mechanics.

Richard Feynman

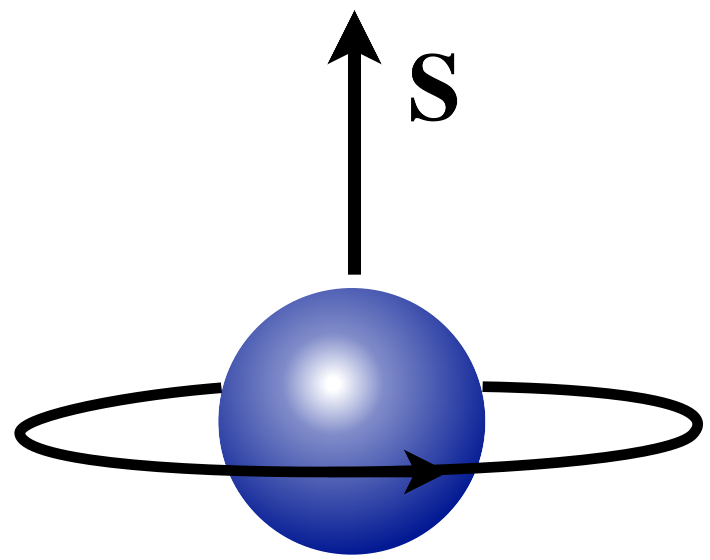
But we can calculate with Quantum Mechanics very, very well.

We’re all highly skilled Quantum *Mechanics*

OBTW: electrons are little magnets

They behave in a magnetic field as if they are little spinning current spheres

The electron **itself** is *like* a spinning charge...



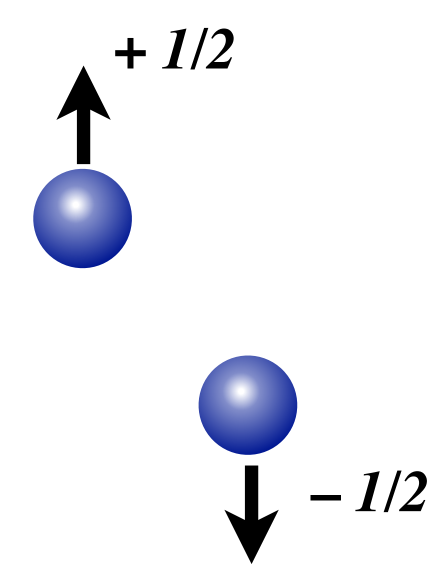
Electrons have an **intrinsic** angular momentum, "S": "spin"

$$S_z = m_s \frac{h}{2\pi}$$

But, the "spin" can only take on two values:

$$m_s = +\frac{1}{2} \quad \text{or} \quad m_s = -\frac{1}{2}$$

We say
"spin, plus 1/2" or "spin up"
and
"spin, minus 1/2" or "spin down"



The electron is NOT

a ball of spinning charge

its outer edges would have to move >> c

This is a quantum mechanical feature with no classical analog

Pauli Exclusion Principle:

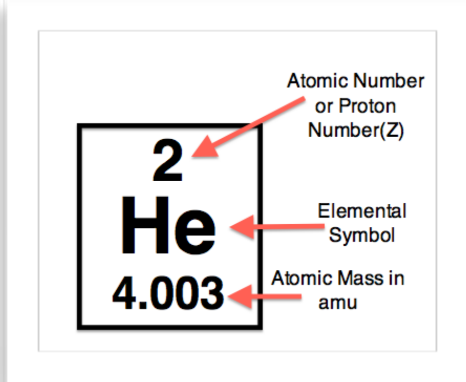
No two electrons can be in the same quantum state

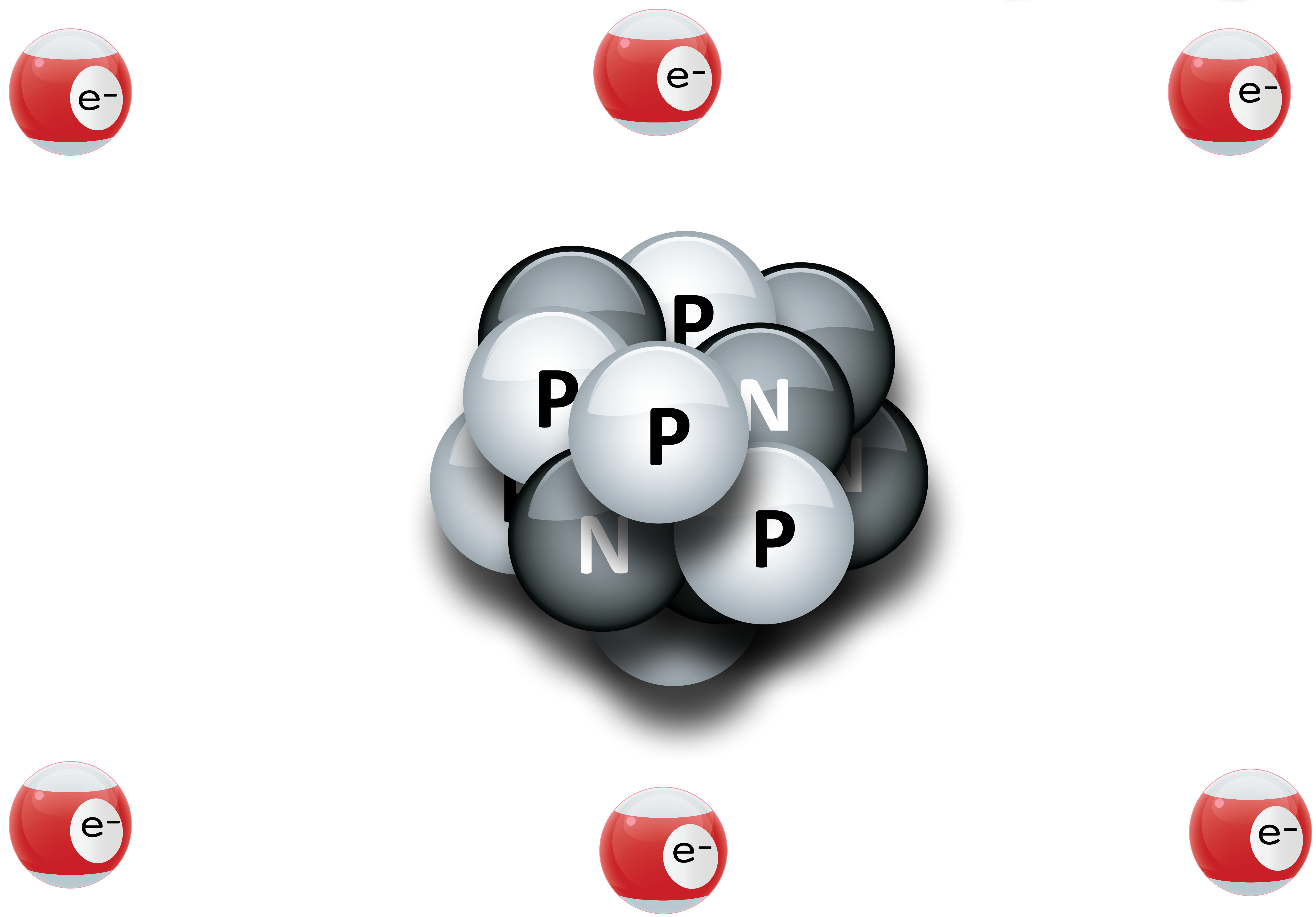
that is, have identical “quantum numbers”

...integers that characterize the atom

Carbon... 6 electrons, 6 protons, 6 neutrons:

$$^{12}_{6}\mathrm{C}$$



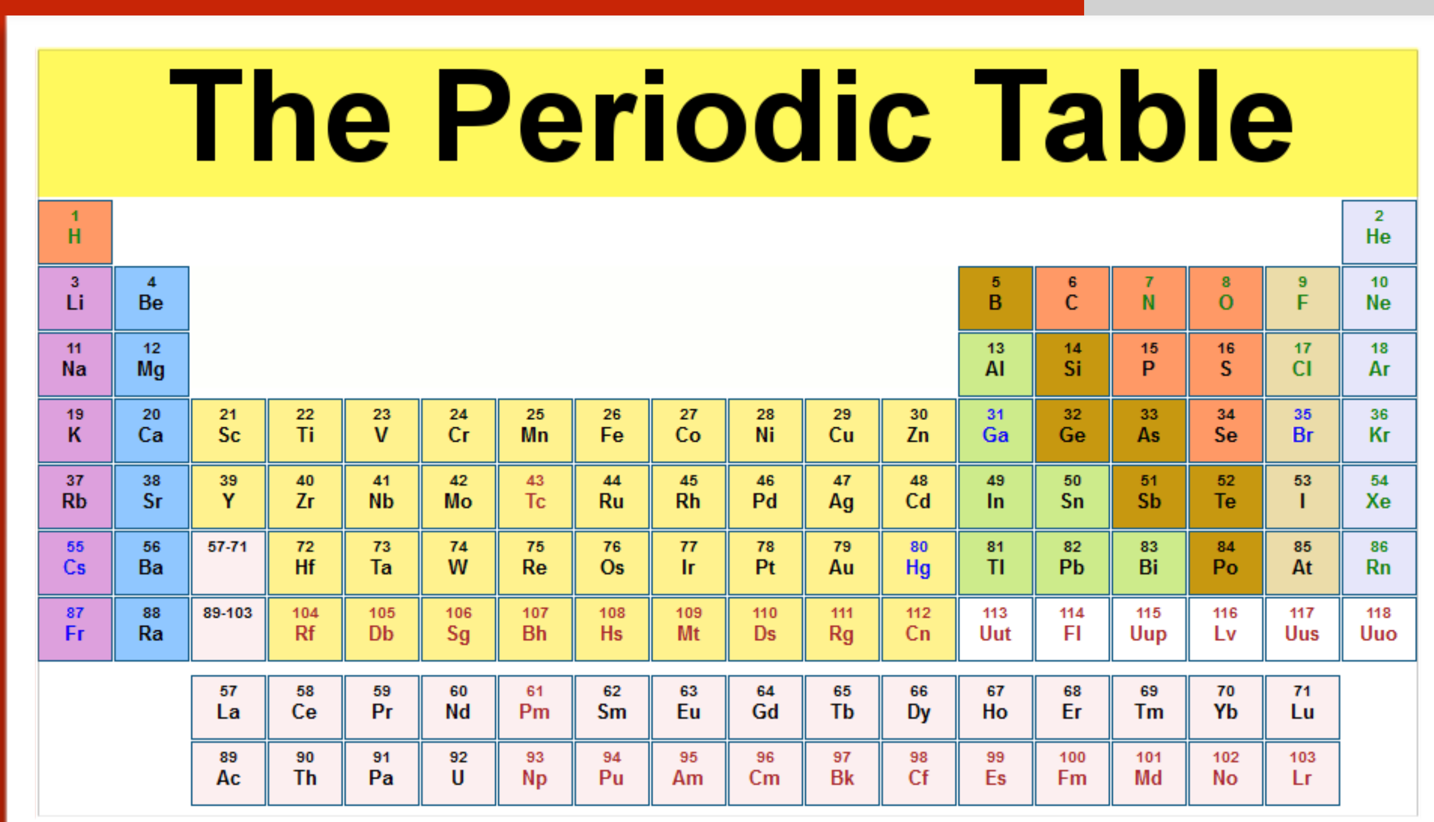


The Pauli Exclusion Principle

Explains it

& SPIN is the reason

"1s2 2p2 2p6 3s2 3p6..."

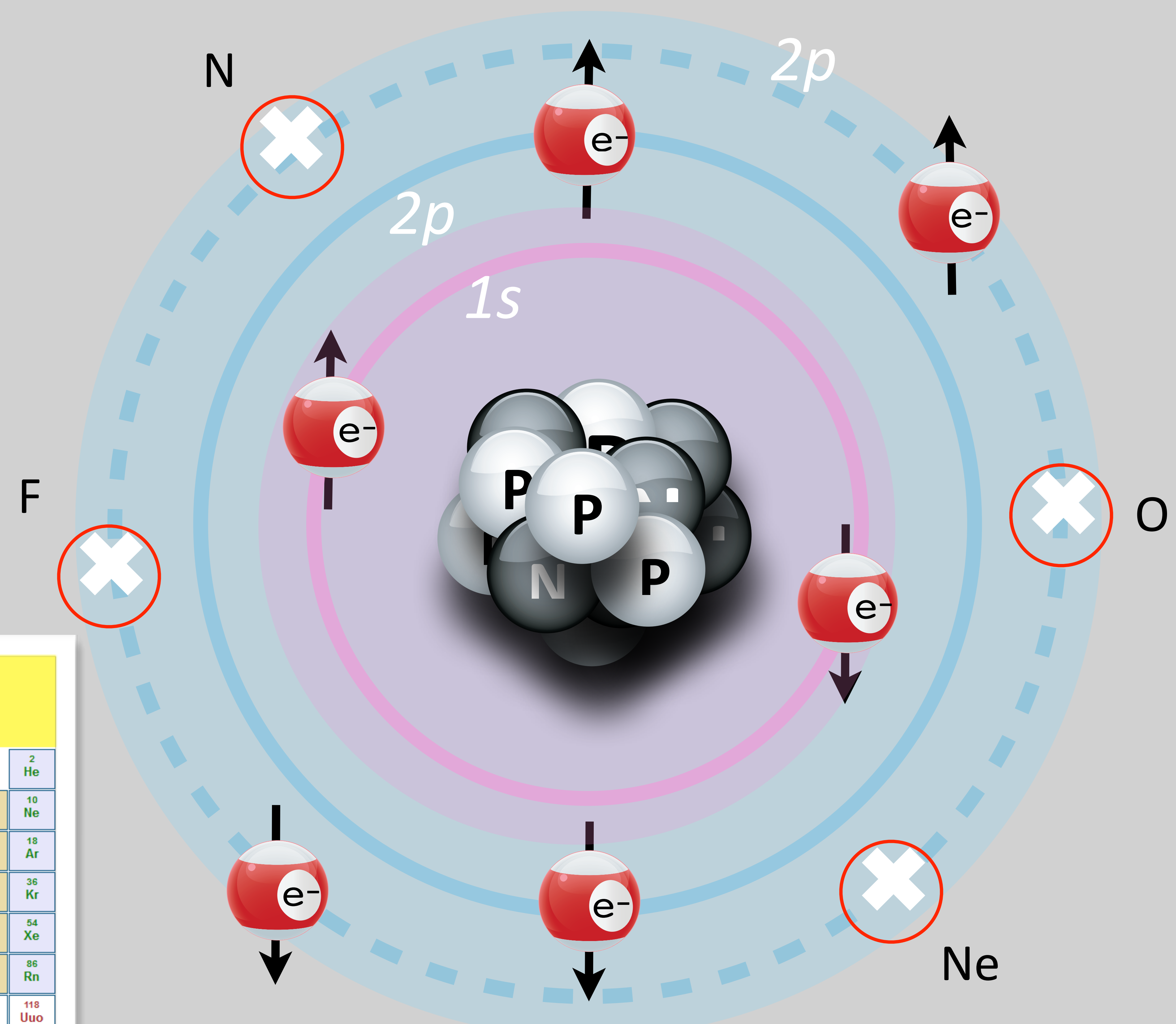


The Periodic Table

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57-71	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89-103	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo
		57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
		89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

How come Carbon *is* like:

The Pauli Exclusion Principle still works
...since **spin up ≠ spin down, so different quantum states**



The combination of Schroedinger, Pauli, Uhlenbeck and Goudsmit - explained the Periodic Table jargon alert: **fermion**

refers to:	any particle with half-integer spin
entomology:	from Fermi's theoretical work on the behavior of large numbers of Fermions
example:	electron, proton, neutron

jargon alert: **boson**

refers to:	any quantum object with integer spin
entomology:	from Satyendra Nath Bose, who worked on the effects of multiple boson aggregates
example:	photon, pion, Higgs Boson

spin is a defining quality of an electron

electron

symbol: e

charge: $-1e$

mass: $m_e = 9.0 \times 10^{-31}$ kg ~ 0.0005 p

spin: 1/2

category: fermion, lepton

particle: **proton**

symbol:	p
charge:	$+1e$
mass:	$m_p = 1.6726\times10^{-27}$ kg = 1 p
spin:	1/2
category:	fermion, hadron

again, an inherent angular momentum and a defining property of photons

particle: **photon**

symbol: γ

charge: 0

mass: $m_{\gamma} = 0$

spin: 1

category: boson, aka Intermediate Vector Boson